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OF

A. I. E. E. TRANSACTIONS

Vol. XXXVII, Parts I and II

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

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2. GENERAL THEORY

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme **Vol. xxxvii—1918, pp. 65-85**

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

A. C. Crehore **Vol. xxxvii—1918, pp. 261-298**

A Lecture.

No discussion

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Karapetoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

R. E. Hellmund **Vol. xxxvii—1918, pp. 589-626**

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.

By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.

Discussion incorporated with that of paper by B. G. Lamme on "A Physical Conception of the Operation of the Single-phase Induction Motor."

A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR

B. G. Lamme **Vol. xxxvii—1918, pp. 627-658**

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be produced by direct-current excitation. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

Discussion, (including that of paper by R. E. Hellmund), pages 659-702, by Messrs. J. Slepian, C. Fortescue, C. A. M. Weber, G. H. Garcelon, R. E. Hellmund, A. M. Dudley, B. G. Lamme, M. I. Pupin, E. F. W. Alexanderson, B. A. Behrend, L. W. Chubb, A. M. Gray, C. F. Scott and S. Haar.

A general discussion on theory.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue **Vol. xxxvii—1918, pp. 1027-1115**

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by

symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical coordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Discussion, pages 1116-1140, by Messrs. J. Slepian, C. P. Steinmetz, V. Karapetoff, A. M. Dudley, C. F. Scott, C. O. Mailloux and C. L. Fortescue.

A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

N. S. Diamant **Vol. xxxvii—1918, pp. 1141-1202**

It shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short-circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s. s. c. fundamental. See Fig. 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Discussion, pages 1203-1208, by Messrs. F. D. Newbury, R. E. Doherty, W. F. Dawson, C. M. Davis and N. S. Diamant.

A discussion involving criticisms of author's solution of short-circuit phenomena.

REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

R. E. Doherty and O. E. Shirley **Vol. xxxvii—1918, pp. 1209-1297**

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, making the calculation from design sheet data a matter of a few minutes.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by other authors, *transient reactance*.

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit.

Discussion, pages 1298-1340, by Messrs. C. J. Fechheimer, V. Karapetoff, F. D. Newbury, W. F. Dawson, C. L. Fortescue, H. R. Summerhayes, N. S. Diamant and R. E. Doherty.

A discussion involving a listing of various methods of determining reactance and their feasibility.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

H. B. Dwight **Vol. xxxvii—1918, pp. 1379-1400**

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. A set of empirical curves for straps is given from which approximate values of R'/R for any case may be read.

Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

Alexander Gray and J. G. Pertsch, Jr. **Vol. xxxvii—1918, pp. 1417-1424**

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic

Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

No discussion.

MAGNETIC PULL IN ELECTRIC MACHINES

E. Rosenberg **Vol. xxxvii—1918, pp. 1425-1469**

See abstract of preceding paper.

No discussion.

APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES

Waldo V. Lyon **Vol. xxxvii—1918 pp. 1477-1513**

It is shown that the flux distribution in the airgap of a synchronous machine consists of a series of component distributions that are simple harmonic wave trains, either stationary or moving at constant velocities. Methods are suggested for determining the effects of slots and the saturation of the magnetic circuit on the magnitudes of these component distributions. Expressions for the voltage generated and the power developed thereby are given. The theory is applied to the operation of a three-phase, synchronous machine under different conditions of load, both qualitatively and quantitatively. A table of comparative calculated field currents is appended.

Discussion, pages 1514-1517, by Messrs. N. S. Diamant and W. V. Lyon.

General discussion.

3. UNITS, MEASUREMENTS AND INSTRUMENTS

A NEW STANDARD OF CURRENT AND POTENTIAL

Chester T. Allcutt **Vol. xxxvii—1918, pp. 167-173**

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

Discussion incorporated with that of paper by C. A. Hoxie on "A Thermoelectric Standard Cell."

A THERMOELECTRIC STANDARD CELL

C. A. Hoxie **Vol. xxxvii—1918, pp. 175-182**

This paper considers a means of obtaining a secondary standard of e. m. f. by utilizing the e. m. f. of a thermocouple. The construction of the cell is discussed in detail, particularly the use of gas in the bulb.

A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown.

Discussion, (including that of paper by C. T. Allcutt), pages 183-188, by Messrs. C. H. Sharp, F. C. Stockwell, W. B. Kouwenhoven, P. G. Agnew, C. T. Allcutt, C. A. Hoxie and A. C. Campbell.

A discussion covering the making of a practical substitute for the standard cell and other efforts to obtain a standard e. m. f.

THE CHARACTER OF THE THERMAL STORAGE DEMAND METER

P. M. Lincoln **Vol. xxxvii—1918, pp. 189-210**

Following a detailed description of the principle and construction of the thermal storage demand meter the author shows wherein it always indicates what may be called "logarithmic average" rather than "arithmetic average" of power consumption, heretofore indicated by practically all demand meters. The inherent faults of the "arithmetic average" or "block interval" meter are described and examples given demonstrating that the thermal storage meter alone recognizes the true heating effect that fixes size of equipment and therefore cost that should be assessed against the customer.

Discussion, pages 211-219, by Messrs. C. I. Hall, F. V. Magalhaes, G. L. Hoxie, W. H. Pratt, M. G. Lloyd, A. S. Albright, H. D. James, H. L. Wallau and P. M. Lincoln.

This discussion consists of arguments from several viewpoints, for and against the thermal storage demand meter and its method of measuring current.

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson, C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

THE SECOMOR

A Kinematic Device Which Imitates the Performance of a Series-Wound Polyphase Commutator Motor.

V. Karapetoff **Vol. xxxvii—1918, pp. 329-354**

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

V. Karapetoff **Vol. xxxvii—1918, pp. 1018-1023**

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

H. B. Dwight **Vol. xxxvii—1918, pp. 1379-1400**

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin

strap. A set of empirical curves for straps is given from which approximate values of R'/R for any case may be read.

Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

4. INSULATION AND DIELECTRIC PHENOMENA

CORONA TESTS AT HIGH ALTITUDE

B. F. Jakobsen **Vol. xxxvii—1918, pp. 91-108**

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000. The method employed in the tests is described and the calculations are given. The results are compared to "Peek's law." Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather.

Discussion, pages 109-122, by Messrs. F. W. Peek, Jr., H. J. Ryan and B. F. Jakobsen.

A general discussion.

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F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

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A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

G. I. Gilchrest **Vol. xxxvii—1918, pp. 805-824**

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type.

A summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

Discussion, pages 825-832, by Messrs. C. Fortescue, C. F. Scott, V. Karapetoff, L. W. Chubb, S. Barfoed and G. I. Gilchrest.

A general discussion on insulator design with special emphasis on porosity, refraction of lines of force, etc.

LIGHTNING ARRESTER SPARK GAPS

THEIR RELATION TO THE PROBLEM OF PROTECTING AGAINST IMPULSE VOLTAGES

Chester T. Allcutt **Vol. xxxvii—1918, pp. 833-854**

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap".

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap

arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

THE OXIDE FILM LIGHTNING ARRESTER

Crosby Field **Vol. xxxvii—1918, pp. 881-890**

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt.

A general discussion.

5. ELECTRIC CONDUCTORS

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Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson, C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

G. J. Newton **Vol. xxxvii—1918, pp. 399-449**

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author treated the subject only in a general manner; the object of this article is to show, assuming average conditions, each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

Discussion, pages 450-461, by Messrs. A. A. Meyer, C. W. Rakestraw, H. L. Wallau, E. Friedlaender, E. B. Meyer, W. Sykes, F. M. Hibben and G. J. Newton.

A general discussion involving description of variations from practise as described by the author and arguments on the question of varnished cambric vs. paper insulated cables.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

W. H. Cole **Vol. xxxvii—1918, pp. 757-784**

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

Discussion, pages 785-790, by Messrs. J. B. Taylor, W. H. Cole, W. A. Del Mar, P. M. Lincoln, E. B. Meyer, R. W. Atkinson, J. R. Craighead and O. C. Traver.

A discussion on various factors involved in cable installations.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

E. B. Meyer **Vol. xxxvii—1918, pp. 791-800**

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

Discussion, pages 801-803, by Messrs. W. F. Dawson, E. B. Meyer, H. L. Wallau, W. A. Del Mar, J. B. Taylor, P. Torchio, J. A. Johnson and H. R. Summerhayes.

A general discussion.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

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Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

7. BATTERIES

A NEW STANDARD OF CURRENT AND POTENTIAL

Chester T. Allcutt **Vol. xxxvii—1918, pp. 167-172**

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

Discussion incorporated with that of paper by C. A. Hoxie on "A Thermoelectric Standard Cell."

A THERMOELECTRIC STANDARD CELL

C. A. Hoxie **Vol. xxxvii—1918, pp. 178-182**

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A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown.

Discussion, (including that of paper by C. T. Allcutt), pages 183-189, by Messrs. C. H. Sharp, F. C. Stockwell, W. B. Kouwenhoven, P. G. Agnew, C. T. Allcutt, C. A. Hoxie and A. C. Campbell.

A discussion covering the making of a practical substitute for the standard cell and other efforts to obtain a standard e. m. f.

9. ELECTRICAL MACHINERY AND APPARATUS

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme **Vol. xxxvii—1918, pp. 65-86**

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

RATING AND SELECTION OF OIL CIRCUIT BREAKERS

E. M. Hewlett, J. N. Mahoney and G. A. Burnham **Vol. xxxvii—1918, pp. 123-138**

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

Discussion, pages 139-165, by Messrs. E. M. Hewlett, J. N. Mahoney, H. R. Summerhayes, W. W. Willard, B. Jones, F. C. Hanker, P. Torchio, P. M. Lincoln, R. E. Doherty, F. L. Hunt, H. H. Dewey, H. D. James, N. L. Pollard, E. G. Merrick, P. H. Adams, I. M. Cushing, C. Lichtenberg, A. Collins, C. C. Garrard and G. A. Burnham.

A general discussion largely from the viewpoint of the importance of arriving at a standard method of rating circuit breakers as to capacity.

THE POLYPHASE SHUNT MOTOR

W. C. Korthals Altes **Vol. xxxvii—1918, pp. 295-327**

There exists a demand for a reliable, adjustable-speed, alternating-current motor, suitable for operation at a large number of speeds. The neutralized motor with shunt field control is analyzed and it is shown that it is not practical for commercial frequencies. The induction motor with commutator on the secondary side is discussed. The induction motor with commutator on the primary side offers the best solution for machine-tool motors. Its theory is discussed in detail, and a complete description is given of the mechanism required to shift the brushes and the new type of armature winding used.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory thae perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region whert commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Karapetoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

Robert F. Hamilton **Vol. xxxvii—1918, pp. 463-484**

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy

required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

Discussion, pages 485-489, by Messrs. A. M. Dudley, C. A. Menk, N. W. Storer and W. Sykes.

A discussion on relative importance of the requirements specified in rolling mill motor selection.

SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

J. D. Wright **Vol. xxxvii—1918, pp. 491-496**

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

Discussion, pages 497-501, by Messrs. E. Friedlaender, C. A. Menk, T. E. Tynes, H. S. Richardson, B. G. Beck, G. E. Stoltz and J. D. Wright.

A description of variations from authors practise in equipment and control.

NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

R. E. Hellmund **Vol. xxxvii—1918, pp. 539-626**

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.

By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.

Discussion incorporated with that of paper by B. G. Lamme on "A Physical Conception of the Operation of the Single-phase Induction Motor."

A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR

B. G. Lamme **Vol. xxxvii—1918, pp. 627-658**

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be

produced by direct-current excitation. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

Discussion, (including that of paper by R. E. Hellmund), pages 659-702, by Messrs. J. Slepian, C. Fortescue, C. A. M. Weber, G. H. Garcelon, R. E. Hellmund, A. M. Dudley, B. G. Lamme, M. I. Pupin, E. F. W. Alexanderson, B. A. Behrend, L. W. Chubb, A. M. Gray, C. F. Scott and S. Haar.

A general discussion on theory.

LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

Chester T. Allcutt **Vol. xxxvii—1918, pp. 833-854**

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap".

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows

the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

THE OXIDE FILM LIGHTNING ARRESTER

Crosby Field **Vol. xxxvii—1918, pp. 881-890**

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt.

A general discussion.

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

V. Karapetoff **Vol. xxxvii—1918, pp. 1018-1023**

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for

pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

J. J. Linebaugh and J. L. Burnham **Vol. xxxvii—1918, pp. 1341-1352**

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

Complete protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

Discussion, pages 1353-1365, by Messrs. C. L. Fortescue, F. C. Hanker, H. B. Dwight, F. Wunsch, F. D. Newbury, W. F. Dawson, J. F. Tritle, H. E. Trent and J. L. Burnham.

Description of various types of high-speed breakers and installations.

A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED

S. R. Bergman **Vol. xxxvii—1918, pp. 1405-1412**

The standing of this problem in the past is briefly reviewed. As far as is known to the author the type of machine described presents a new solution.

The theory of the machine is described and diagrams of connections given. Performance curves obtained from tests are shown.

A method is described whereby instantaneous regulation of the voltage is obtained, which method also secures approximately a constant voltage independent of the heating.

Finally there follows a discussion of the efficiency of the new machine as compared with a standard machine of the same speed and output.

Discussion, pages 1413-1415, by Messrs. J. G. De Remer and S. R. Bergman.

General discussion.

ELECTRIC WELDING—A NEW INDUSTRY

H. A. Hornor **Vol. xxxvii—1918, pp. 1519-1529**

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding

Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the ship-building industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson **Vol. xxxvii—1918, pp. 1531-1567**

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

11. POWER PLANTS AND CENTRAL STATIONS

EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

Lynn S. Goodman and William B. Jackson **Vol. xxxvii—1918, pp. 1-17**

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out. The way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

Discussion, pages 18-63, by Messrs. B. A. Behrend, Mr. Sykes, J. W.

Cowles, C. A. Adams, W. B. Jackson, F. A. Bryan, R. G. Hudson, M. Freund, B. J. Arnold, P. Torchio, P. Betts, W. S. Gorsuch, H. M. Hobart, D. C. Jackson, P. H. Bartlett, W. N. Smith, J. J. Harold, P. R. Moses, R. F. Schuchardt, F. A. Coffin, D. W. Roper, C. W. Pendell, C. A. Keller, F. H. Bernhard, J. W. Mabbs, A. Honegger.

A general discussion on the various phases of the central station problems.

AMERICA'S ENERGY SUPPLY

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 985-1009**

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically.

Discussion, pages 1010-1014, by Messrs. A. M. Schoen, B. A. Behrend, S. Barfoed, R. Bennett and C. P. Steinmetz.

A general discussion.

THE AUTOMATIC HYDROELECTRIC PLANT

J. M. Drabelle and L. B. Bonnett **Vol. xxxvii—1918, pp. 1367-1377**

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size without sacrificing complete control.

Discussion, page 1378, by Messrs. H. R. Summerhayes, J. J. Linebaugh, Mr. Bump, O. C. Traver, J. M. Drabelle and L. B. Bonnett.

General discussion.

ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

Frank G. Baum **Vol. xxxvii—1918, pp. 1471-1475**

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

No discussion.

ELECTRIC POWER GENERATION IN ONTARIO ON SYSTEMS OF HYDROELECTRIC POWER COMMISSION

Arthur H. Hull **Vol. xxxvii—1918, pp. 1607-1630**

Discussion, pages 1631-1636, by Messrs. H. U. Hart, W. G. Hewson, A. H. Hull, T. A. Worcester, E. M. Ashworth, H. R. Summerhayes, W. Mac Lachlan, P. A. Borden and F. A. Gaby.

Questions and answers on details of the system.

13. TRANSMISSION LINES

CORONA TESTS AT HIGH ALTITUDE

B. F. Jakobsen **Vol. xxxvii—1918, pp. 91-108**

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000. The method employed in the tests is described and the calculations are given. The results are compared to "Peek's law." Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather.

Discussion, pages 109-122, by Messrs. F. W. Peek, Jr., H. J. Ryan and B. F. Jakobsen.

A general discussion.

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson,

C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

W. H. Cole **Vol. xxxvii—1918, pp. 757-784**

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

Discussion, pages 785-790, by Messrs. J. B. Taylor, W. H. Cole, W. A. Del Mar, P. M. Lincoln, E. B. Meyer, R. W. Atkinson, J. R. Craighead and O. C. Traver.

A discussion on various factors involved in cable installations.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

E. B. Meyer **Vol. xxxvii—1918, pp. 791-800**

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

Discussion, pages 801-803, by Messrs. W. F. Dawson, E. B. Meyer, H. L. Wallau, W. A. Del Mar, J. B. Taylor, P. Torchio, J. A. Johnson and H. R. Summerhayes.

A general discussion.

APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

G. I. Gilchrest **Vol. xxxvii—1918, pp. 805-824**

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type.

A summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

Discussion, pages 825-832, by Messrs. C. Fortescue, C. F. Scott, V. Karapetoff, L. W. Chubb, S. Barfoed and G. I. Gilchrest.

A general discussion on insulator design with special emphasis on porosity, refraction of lines of force, etc.

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Questions and answers on details of the system.

110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER

S. Svenningson **Vol. xxxvii—1918, pp. 1653-166**

The paper deals with some remarkable construction recently completed by the Shawinigan Water & Power Company near Three Rivers, Quebec. The St. Lawrence river is crossed by transmission line wires on a span of 4800 feet, being the longest span in the world. Due to necessities of navigation clearance, the towers are 350 feet high.

The preliminary investigation leading to the adoption of this construction is outlined, and a general description is given of the design and construction of the towers, insulators and cables. The provisions for protection from ice and the method of sag calculations are also given.

Discussion, pages 1666-1670, by Messrs. C. W. Baker, S. Svenningson, E. V. Pannell, H. B. Dwight, H. C. Don Carlos, T. A. Worcester, F. J. Wyman and W. P. Dobson.

A general discussion on details of installation.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

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A general discussion.

RATING AND SELECTION OF OIL CIRCUIT BREAKERS

E. M. Hewlett, J. N. Mahoney and G. A. Burnham **Vol. xxxvii—1918, pp. 123-138**

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

Discussion, pages 139-165, by Messrs. E. M. Hewlett, J. N. Mahoney, H. R. Summerhayes, W. W. Willard, B. Jones, F. C. Hanker, P. Torchio, P. M. Lincoln, R. E. Doherty, F. L. Hunt, H. H. Dewey, H. D. James, N. L. Pollard, E. G. Merrick, P. H. Adams, I. M. Cushing, C. Lichtenberg, A. Collins, C. C. Garrard and G. A. Burnham.

A general discussion largely from the viewpoint of the importance of arriving at a standard method of rating circuit breakers as to capacity.

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Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

THE OXIDE FILM LIGHTNING ARRESTER

Crosby Field **Vol. xxxvii—1918, pp. 881-890**

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the

commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt.

A general discussion.

PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

J. J. Linebaugh and J. L. Burnham **Vol. xxxvii—1918, pp. 1341-1352**

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

Complete protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

Discussion, pages 1353-1365, by Messrs. C. L. Fortescue, F. C. Hanker, H. B. Dwight, F. Wunsch, F. D. Newbury, W. F. Dawson, J. P. Tritle, H. E. Trent and J. L. Burnham.

Description of various types of high-speed breakers and installations.

15. DISTRIBUTION SYSTEMS

DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

G. J. Newton **Vol. xxxvii—1918, pp. 399-449**

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author treated the subject only in a general manner; the object of this article is to show, assuming average conditions, each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

Discussion, pages 450-461, by Messrs. A. A. Meyer, C. W. Rakestraw, H. L. Wallau, E. Friedlaender, E. B. Meyer, W. Sykes, F. M. Hibben and G. J. Newton.

A general discussion involving description of variations from practise as described by the author and arguments on the question of varnished cambric vs. paper insulated cables.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

W. H. Cole **Vol. xxxvii—1918, pp. 787-784**

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

Discussion, pages 785-790, by Messrs. J. B. Taylor, W. H. Cole, W. A. Del Mar, P. M. Lincoln, E. B. Meyer, R. W. Atkinson, J. R. Craighead and O. C. Traver.

A discussion on various factors involved in cable installations.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

E. B. Meyer **Vol. xxxvii—1918, pp. 791-800**

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

Discussion, pages 801-803, by Messrs. W. F. Dawson, E. B. Meyer, H. L. Wallau, W. A. Del Mar, J. B. Taylor, P. Torchio, J. A. Johnson and H. R. Summerhayes.

A general discussion.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue **Vol. xxxvii—1918, pp. 1027-1118**

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by

symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical coordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Discussion, pages 1116-1140, by Messrs. J. Slepian, C. P. Steinmetz, V. Karapetoff, A. M. Dudley, C. F. Scott, C. O. Mailloux and C. L. Fortescue.

A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

16. CONTROL, REGULATION AND SWITCHING

SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

Robert F. Hamilton **Vol. xxxvii—1918, pp. 463-484**

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

Discussion, pages 485-489, by Messrs. A. M. Dudley, C. A. Menk, N. W. Storer and W. Sykes.

A discussion on relative importance of the requirements specified in rolling mill motor selection.

SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

J. D. Wright **Vol. xxxvii—1918, pp. 491-496**

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

Discussion, pages 497-501, by Messrs. E. Friedlaender, C. A. Menk, T. E. Tynes, H. S. Richardson, B. G. Beck, G. E. Stoltz and J. D. Wright.

A description of variations from authors practise in equipment and control.

THE AUTOMATIC HYDROELECTRIC PLANT

J. M. Drabelle and L. B. Bonnett **Vol. xxxvii—1918, pp. 1367-1377**

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size, without sacrificing complete control.

Discussion, page 1378, by Messrs. H. R. Summerhayes, J. J. Linebaugh, Mr. Bump, O. C. Traver, J. M. Drabelle and L. B. Bonnett.

General discussion.

17. TRACTION

EL RIFICATION OF THE MONTREAL TUNNEL ZONE

William G. Gordo **Vol. xxxvii—1918, pp. 1637-1648**

The author describes the electrification of the tunnel through Mt. Royal at Montreal which was built to give the Canadian Northern Railway entrance into the heart of the city. The tunnel is 3.1 miles long and the method of construction is described in detail.

The power is purchased and delivered to a substation near the west portal of the tunnel. The equipment of the substation is described. Details of the equipment and dimensions of both the locomotives and motor cars are given.

The catenary system, which is described in detail, has a number of unusual features due to special local conditions and the extremely low temperatures which sometimes prevail in Montreal.

Discussion, pages 1649-1652, by Messrs. W. G. Hewson, W. G. Gordon, W. A. Bucke, A. H. Hull, H. U. Hart, W. H. Mulligan, W. Mac Lachlan, W. M. Gifford and R. R. Stevenson.

A general discussion on details of the installation.

20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

Robert F. Hamilton **Vol. xxxvii—1918, pp. 463-484**

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

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ELECTRIC POWER FOR NITROGEN FIXATION

E. Kilburn Scott **Vol. xxxvii—1918, pp. 957-974**

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate-from-air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

Discussion, pages 975-983, by Messrs. C. P. Steinmetz, W. A. Del Mar, S. Barfoed, F. W. Sperr, Jr., and E. K. Scott.

A general discussion largely in support of the authors plan for cooperation between nitrogen fixation industry and by-product coke industry.

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

V. Karapetoff **Vol. xxxvii—1918, pp. 1015-1023**

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for

pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

ELECTRIC WELDING—A NEW INDUSTRY

H. A. Hornor **Vol. xxxvii—1918, pp. 1519-1529**

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the ship-building industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson **Vol. xxxvii—1918, pp. 1531-1567**

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL

J. B. Crane **Vol. xxxvii—1918, pp. 1671-1676**

This paper gives figures as to the power cost and current consumption of anthracite mines and the reasons for these being in excess of the requirements of bituminous mines.

Estimates are also given as to the additional coal that will be released by the electrification of the anthracite mines.

Illustrations are included showing representative installations of electric drive.

Discussion incorporated with that of paper by F. L. Stone on "Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating."

DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

F. L. Stone **Vol. xxxvii—1918, pp. 1677-1695**

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

Discussion (including that of paper by J. B. Crane), pages 1696-1708, by Messrs. G. Bright, C. W. Parkhurst, F. L. Stone, E. J. Cheney, W. I. Slichter, P. Kain, J. B. Crane, L. H. Rittenhouse, C. A. Adams and L. S. Randolph.

A general discussion with particular emphasis on figures for kw. hr. per ton mined of both anthracite and bituminous coal.

21. TELEPHONY AND TELEGRAPHY

INDUCTIVE EFFECTS OF ALTERNATING CURRENT RAILROADS ON COMMUNICATION CIRCUITS

H. S. Warren **Vol. xxxvii—1918, pp. 503-532**

A brief discussion of inductive interference in general is first given, including reference to the work of the Joint Committee on Inductive Interference in California. Inductive interference due to electrified railroads is then taken up and various possible means for reducing such interference considered. A description is given of four important installations of railroad electrification and the specific means adopted in each case for preventing interference, with the degree of success which has been met with.

Discussion, pages 533-538, by Mr. M. Latour.

Description of system proposed by Latour which allows telegraphic line to be operated in spite of strong disturbing alternating e. m. fs.

THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

Harold S. Osborne **Vol. xxxvii—1918, pp. 897-936**

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

Discussion, pages 937-944, by Messrs. V. Karapetoff, C. Fortescue, A. G. Chapman, H. Mouradian and H. S. Osborne.

A discussion on various methods of measuring "cross-talk."

22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

Lynn S. Goodman and William B. Jackson **Vol. xxxvii—1918, pp. 1-17**

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out. The way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

Discussion, pages 18-63, by Messrs. B. A. Behrend, Mr. Sykes, J. W. Cowles, C. A. Adams, W. B. Jackson, F. A. Bryan, R. G. Hudson, M. Freund, B. J. Arnold, P. Torchio, P. Betts, W. S. Gorsuch, H. M. Hobart, D. C. Jackson, P. H. Bartlett, W. N. Smith, J. J. Harold, P. R. Moses, R. F. Schuchardt, F. A. Coffin, D. W. Roper, C. W. Pendell, C. A. Keller, F. H. Bernhard, J. W. Mabbs, A. Honegger.

A general discussion on the various phases of the central station problems.

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme **Vol. xxxvii—1918, pp. 65-85**

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25

cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

A. C. Crehore **Vol. xxxvii—1918, pp. 261-293**

A Lecture.

No discussion

THE SECOMOR

A Kinematic Device Which Imitates the Performance of a Series-Wound Polyphase Commutator Motor

V. Karapetoff **Vol. xxxvii—1918, pp. 329-354**

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Kara-

petoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

ADDRESS BY PRESIDENT E. W. RICE, JR. AT A. I. E. E. MIDWINTER CONVENTION FEBRUARY, 1918

Vol. xxxvii—1918, pp. 391-397

A REVIEW OF ELECTRICAL ENGINEERING PROGRESS

E. W. Rice, Jr. **Vol. xxxvii—1918, pp. 703-714**

President's Address.

ENGINEERS AND THE WAR

Major General William M. Black **Vol. xxxvii—1918, pp. 945-956**

An Address.

No discussion.

ELECTRIC POWER FOR NITROGEN FIXATION

E. Kilburn Scott **Vol. xxxvii—1918, pp. 957-974**

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

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A general discussion largely in support of the authors plan for cooperation between nitrogen fixation industry and by-product coke industry.

AMERICA'S ENERGY SUPPLY

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 985-1009**

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically.

Discussion, pages 1010-1014, by Messrs. A. M. Schoen, B. A. Behrend, S. Barfoed, R. Bennett and C. P. Steinmetz.

A general discussion.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue **Vol. xxxvii—1918, pp. 1027-1115**

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A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

N. S. Diamant **Vol. xxxvii—1918, pp. 1141-1202**

It shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short-circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s. s. c. fundamental. See Fig. 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Discussion, pages 1203-1208, by Messrs. F. D. Newbury, R. E. Doherty, W. F. Dawson, C. M. Davis and N. S. Diamant.

A discussion involving criticisms of author's solution of short-circuit phenomena.

REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

R. E. Doherty and O. E. Shirley **Vol. xxxvii—1918, pp. 1209-1297**

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, making the calculation from design sheet data a matter of a few minutes.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by other authors, *transient reactance*.

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit.

Discussion, pages 1298-1340, by Messrs. C. J. Fechheimer, V. Karapetoff, F. D. Newbury, W. F. Dawson, C. L. Fortescue, H. R. Summerhayes, N. S. Diamant and R. E. Doherty.

A discussion involving a listing of various methods of determining reactance and their feasibility.

CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

Alexander Gray and J. G. Pertsch, Jr. **Vol. xxxvii—1918, pp. 1417-1434**

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

No discussion.

MAGNETIC PULL IN ELECTRIC MACHINES

E. Rosenberg **Vol. xxxvii—1918, pp. 1435-1469**

See abstract of preceding paper.

No discussion.

ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

Frank G. Baum **Vol. xxxvii—1918, pp. 1471-1475**

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson **Vol. xxxvii—1918, pp. 1531-1567**

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting

characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

F. L. Stone **Vol. xxxvii—1918, pp. 1677-1698**

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

Discussion (including that of paper by J. B. Crane), pages 1696-1708, by Messrs. G. Bright, C. W. Parkhurst, F. L. Stone, E. J. Cheney, W. I. Slichter, P. Kain, J. B. Crane, L. H. Rittenhouse, C. A. Adams and L. S. Randolph.

A general discussion with particular emphasis on figures for kw. hr. per ton mined of both anthracite and bituminous coal.

AMERICAN ENGINEERING RESEARCH

W. R. Whitney **Vol. xxxvii—1918, pp. 1709-1721**

Discussion incorporated with that of paper by R. A. Millikan on "Research in America after the War."

RESEARCH IN AMERICA AFTER THE WAR

R. A. Millikan **Vol. xxxvii—1918, pp. 1722-1734**

Discussion (including that of paper by W. R. Whitney), pages 1735-1746, by Messrs. C. E. Skinner, E. E. F. Creighton, P. G. Agnew, C. H. Sharp, E. P. Hyde, L. T. Robinson, R. A. Millikan, M. W. Franklin, C. A. Adams, J. B. Taylor and G. F. Gray.

A discussion including various suggestions on the solution of the research problem.

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EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

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ABSTRACT OF PAPER

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country, but the principles enunciated relate in their broad application to all kinds of public utility service.

The electric companies are facing a grave situation owing to the Government's needs and requirements, to the abnormal prices of labor, supplies and equipment, to the difficulty of retaining their trained employees, and to the present impossibility of satisfactorily financing extensions.

Owing to these conditions the electric companies may be forced to regularly operate their systems, during the continuance of such conditions, with reduced reserve in capacity of equipment and with partially trained operating forces, with accompanying reduction in efficiency of operation and reliability of service as compared with normal conditions.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out.

To work out the situation to the best interests of the public and the companies will require earnest cooperation between the Government and the companies. Although the situation is one requiring the determination of how the electric companies can best meet the power requirements for war manufactures, yet the way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

THE EFFECTS of war conditions in the public utility field are demanding at the present time the most careful consideration of both the operating and the regulatory bodies throughout the country. We have had a preliminary course of training, as it were, both in the experience of our foreign contemporaries

and in our own experience prior to and during the entry of the United States into the war. But there is a wide difference between the effect on the electric utility field of this country, with the nation in the status of a neutral with its industries independently meeting the demands of the domestic and foreign markets, compared with the conditions produced by the United States standing as a growing war power, collecting, directing and conserving its vast resources for the prosecution of the conflict.

War conditions are extraordinarily affecting every department of production and distribution of electrical power, and the results not only have to do with the present and immediate future, but the effects are likely to extend far beyond the termination of the war. How far these effects will reach into the future will depend not only upon the length of the war, but to a large extent upon their treatment during its progress, and how resolutely the situation is now faced by engineers, bankers, and government officials. It is therefore important for us to analyze these influences.

This paper deals more particularly with the effect of war conditions upon electric light and power service, but the principles relate in their broad application to every kind of public utility service.

The principal directions in which the effects of war conditions on electric service appear are:

1. In relation to operating:
   - (a) In increased salaries and wages paid for operating.
   - (b) In difficulty of retaining trained operatives, and conversely the need to operate with partially trained forces.
   - (c) In increased cost and difficulty of obtaining fuel and in reduction of its uniformity and quality.
   - (d) In increased cost of other supplies and materials for operation and maintenance.
   - (e) In the need for protecting the properties against enemy agents.
   - (f) In increased taxes.
   - (g) In possible decrease of consumption of electric power by ordinary customers.
   - (h) In possible changes of load factor.
2. In relation to extensions of plant:
   - (a) In the necessity in many cases for quickly caring for large accessions of permanent and temporary business.
   - (b) In increased cost over normal for plant required to care for additional business.

(c) In high cost for money and difficulty of obtaining it at any rate considered reasonable in normal times.

(d) In the difficulty of obtaining equipment in reasonable times of delivery.

Each one of the above factors has a direct bearing upon the cost and quality of electric service and any one of them arising under normal conditions would demand careful consideration and treatment, but when all of these factors arise coincidently and are affecting an industry which has been operating under circumstances requiring for stability and success the condition of steady and predictable markets for its purchases and sales, the results may be not only serious but even destructive if the appropriate measures of relief cannot be obtained.

The effects of the war conditions are being manifested not only in the matter of heavy increases in operating costs but also in the matter of extraordinary increases in cost for new plant required to care for added business. At first thought it might be concluded that the former apply only to the period of the war, but increases in wages for any cause usually result in all or a part of the increases remaining after the cause has been removed, also temporary changes in operating methods are likely to bring about modifications in the permanent methods of operation. The latter may increase the cost per unit of output incident to investment in plant, and therefore affect the cost of service not only for the period of the war but for the life of such plant, unless the increases in plant investment over normal can be amortized through increase in revenue during the period of the war and the period of readjustment which must succeed the war.

The effects of the war conditions have already increased the operating expenses of the electric companies of this country to the extent of over a hundred and sixteen million dollars per year, as hereafter shown. This points to the necessity of readjustment to the new conditions without delay, while at the same time requiring readjustment to abnormal labor conditions.

Any complete study of the effect of the increase in expense for operating wages and salaries on the operations of electric companies, caused by war conditions, is a difficult proceeding because every class of labor is represented, from the unskilled wage worker to the highest class of administrative official, and increases among these classes differ in their proportions, the highest paid men having received the smallest proportional increases, if any;

also because the wages paid for the same kind of labor as well as the increases involved vary throughout the country. Moreover, the growth of companies and accompanying increase in output per unit of labor, improvement in organization and in the science of economic labor application have tended to continually reduce the cost of labor required per unit of output, that would otherwise be required to produce equal or improved results. Although individual salaries and wages have been gradually increasing during past years, the labor cost per unit of service had been decreasing prior to the war. War conditions, however, have greatly affected this situation. The growing scarcity of labor in the ordinary occupations of peace, due to stoppage of immigration, demands for industrial workers for government work and drafts for the National Service, together with the necessity for enabling employees to meet the increased cost of living and the competition for retaining some experienced employees and of obtaining new employees, all combine to increase wage and salary scales. The employment of women has enabled the electric utilities to obtain an additional class of labor at a relatively low rate, but when women take the places of men, more women employees are required than the number of men replaced, and the war conditions tend to exhaust the supply of even this class of labor by offering wider fields of employment. The necessity for female labor to become more nearly self-supporting during war-times further tends to increase the female wage scale.

It is recognized that the labor cost to public utilities will further advance either by increases in the direct wage or by some system of war additions such as are being employed by the other nations at war. This system of war additions is generally recognized as a measure to compensate for the temporary increases in living costs during the war period and is adopted principally in order to facilitate the ultimate return to normal conditions.

It should be recognized that by normal labor conditions for the future, we do not mean the identical conditions existing prior to the war, but refer to the conditions which would normally have been attained had war conditions not intervened.

An analysis of the United States Census statistics shows that the increase in the average wages paid per employee (exclusive of general officers, managers and superintendents) during the ten years from 1902 to 1912 was 11 per cent. The actual increase in wages for like classes of labor may have differed from this figure,

since the evolution of electric generation and distribution apparatus has shifted the relation of unskilled and skilled employees, but it plainly shows how abnormal are the sudden great increases in wages which have occurred as a consequence of the war.

During the war period thus far, salaries of officers, managers and general superintendents have in general not greatly increased but increases in wages in the operating departments have ranged from 15 to 50 per cent and we are of the opinion that 25 per cent may be taken as the average increase thus far occasioned by the war.

The total salaries and wages paid to employees of electric companies throughout the United States make up about one-third of the total operating expense, including under the latter heading the cost of power purchased and the renewals and replacement expenses, but not interest on capital debt. Under normal growth from 1912, at the rate indicated by the growth during the previous ten years we find that the salary and wage disbursements of electric companies in the year 1917, had there been no unusual disturbance, should have amounted to $90,000,000 of which one-seventh would have been for general officers, managers and superintendents' salaries and six-sevenths for wages.

From this it is seen that the increase in wages of 25 per cent means an outlay on the part of the electric companies of $19,000,000 for the year.

In the matter of retaining trained operatives, the government's draft for war services and the war scale wages offered in industrial fields both contribute to deplete the trained operating forces of the electric companies. We believe the government will and should, as a matter of good business, not unduly hamper the carrying on of a service so essential to its needs, but many of the finest electrical employees have voluntarily enlisted and the draft has taken a share of trained employees. Also, although the permanent employment in the public utility field tends to make it in that aspect more attractive than more or less temporary employment of war industries, yet many of the best employees of the electric companies have not been able to withstand the temptation of the higher wages obtainable in the latter, associated with the idea of being closer to war activities.

This situation is already being felt not only in increased wage scales, but also in cost for training new operatives, and the continual change in employees will necessarily be reflected in a re-

duction in economy of operation and in some reduction in the quality of service not only in the supplying of power but also in the accounting departments, and in the new business departments as far as they are maintained. The effects of this condition will become more and more apparent as time progresses, and the utilities must be prepared to meet the conditions of depleted forces and untrained operatives. We are unable to estimate the effects of this condition in terms of dollars and cents, but it is evident that it is a condition that will require no small consideration on the part of those responsible for the operation and service of the public utilities.

The electric utilities meet the largest single item of increased expense in the fuel account. The gross cost of fuel of a given quality is made up of cost at the mine, cost of transportation, and cost of labor for handling; and, due to differences in mining costs, length of haul and purchasing facilities on the part of the utilities, the gross costs as well as the increases have differed materially among the utilities, and throughout the country. In addition to tremendous increases in costs per ton, the utilities are confronted with the necessity of accepting coal of inferior and non-uniform quality, which results not only in lower efficiency due to reduction in heating value but also in lower efficiency incident to operating boiler equipments with continually changing grades of fuel. These conditions are now being keenly felt in the more modern highly efficient steam generating plants.

The cost of fuel has an extremely important bearing upon the total cost of electric service. Estimates based upon the United States Census reports show that this item of expense for all the electric companies in the United States would have reached $50,000,000 for the year 1917 under normal conditions of the country, and would have amounted to about 60 to 65 per cent of the normal generating expense. Definite information as to the amount of increase in fuel cost for the whole country is not available, but from information obtained from various sections of the country, we arrive at the conclusion that on the average the cost per ton of coal to electric companies has increased a little more than 100 per cent on account of war conditions, and that 100 per cent is not far from correct. On this basis the increase of total cost due to the increased price per ton of fuel is $50,000,000. A conservative figure for the increase in tonnage due to lower quality and non-uniformity of grade is 10 per cent, which means an added increase of $10,000,000, making the total increase $60,000,000.

Reliability of service has already been adversely affected in some instances by the fuel situation, and the importance of maintaining continuous service especially in congested territory and in industrial centers producing war materials is obvious. The size of the reserve coal supply required to assure continuity of service is dependent upon the rate of use and the dependability, frequency and regularity of deliveries. Present conditions of coal production and transportation point to the need for more than normal reserve coal supply and its importance we believe fully justifies the abnormal expenditures which have been necessary in the purchase of coal for the purpose of maintaining the supply.

An estimate of the output from steam driven electric central stations which might have been expected for 1917 under normal conditions shows 13,000,000,000 kw-hr., and an average requirement of three pounds of coal per kw-hr. of output, shows the fuel requirements would amount to not over 20,000,000 net tons, which is approximately 3 per cent of the estimated output from the mines for 1917. It is thus seen that a relatively large reserve supply of coal in the hands of every electric company would tie up but a very small part of the coal supply of the country and this supply would be widely distributed over the country and to a certain extent would be in proportion to the populations and industrial importance of the several sections of the country.

The normal cost of materials and supplies other than fuel, used in operation and current maintenance of electric properties, makes up no small proportion of the total annual operating expense, which we estimate as a little over 15 per cent. The percentage increases in the cost of such materials and supplies due to war conditions have been enormous and extremely varied in amount and the determination of the average amount of these increases imposed on electric companies is very complicated, but the indications are that this amounts to as much as 75 per cent. Such an increase in this expense means an increase in expenditures in the neighborhood of $30,000,000 over normal expense for 1917.

The increase in the cost of materials and supplies may be expected to cause operating companies to curtail their repairs and current maintenance to the greatest possible extent but such reduction cannot be large if satisfactory quality of service is to be maintained. In the matter of supplies contemplated for improvements and replacements of plant, however, the difficulty in obtaining equipment in reasonable times of delivery and the

abnormal cost of any equipment that may be required naturally points to the advisability, and in many cases the necessity, of retaining present equipment in service until the exigencies of the situation make a change imperative, even though under normal conditions it would be advantageous to make the changes promptly, thus introducing an element of reduced economy in operation which under normal conditions would not be present.

The measures which must be taken to protect property from malicious interference by enemy agents comprise the development and maintenance of effective protective structures and lighting systems as well as special policing. The large capacity of individual units of generating equipment and the large capacity of transmission circuits of today increase the necessity for thorough protection owing to the large amount of damage that could be accomplished by an enemy agent if given the opportunity. The government action in restricting the activities of the enemy alien population is an important safeguard in this connection but the necessity for direct protective measures during the continuance of the war adds hundreds of thousands of dollars to the normal expense accounts of individual large electric corporations and runs into the millions in the total cost of service throughout the country. How many millions of dollars it amounts to we are not prepared at this time to say, but we expect the aggregate amounts to at least two or three million dollars.

In the matter of increased taxes we have purely a problem of caring for the increases over those of times prior to the war, and there is no way of predicting how large a factor this expense will become during the progress of the war or how far it will reach into the future. Estimates based upon the United States Census returns indicate that the 1917 taxes paid by electric companies might normally have reached $25,000,000. The proportion of gross revenue required for taxes has apparently been increasing year by year, having been slightly over three per cent in 1902, a little over three and one-half per cent in 1907, and nearly four and one-half per cent in 1912. Taxes on net income made up a very small proportion of the total tax in former years, so that changes in net income had little bearing on the amount of the taxes. This form of income taxation has had growing favor in legislative circles and is freely used by the government in its war tax program. An estimate of the amount of the expense which may be expected to be added to the cost of electric service

throughout the country by the operation of the net income tax law is obviously dependent upon the effect of war conditions on net incomes, and the effects of "tax free" bond clauses and taxes on Excess and Undistributed Profits clearly cannot be estimated. We may hazard a guess that the increase over normal expense will lie between $5,000,000 and $10,000,000 for the year 1917.

Summing up the foregoing amounts shows that the extra expenses now imposed on the electric companies on account of war conditions amounts to the immense aggregate per year, as follows:

Increased salaries and wages chargeable to operating.	$19,000,000
Increased cost of fuel	60,000,000
Increased cost of other materials and supplies	30,000,000
Increased taxes	7,500,000
	$116,500,000

This amounts to a quarter of the normal estimated gross revenue for 1917 of all the electric companies and it wipes out two-thirds of the sum that would have been available for interest, dividends and surplus. It does not include additional expenses caused by the difficulty of retaining trained operatives and the cost of protecting the properties against malicious interference, the magnitude of which we are unable to estimate. It puts the electric companies in a critical position, which is rendered more ominous by the impossibility of foretelling how much larger these extra expenses may become in future months.

It is to be expected that the American people will come to more fully appreciate the need for economies in every direction as the war progresses, and that their consumption of electric power for residential and ordinary commercial lighting will become materially reduced. This effect is now just beginning to be felt by the electric companies. It can result in only a relatively small reduction in the outputs of the electric companies but to that extent it will liberate generating capacity for use in war service and will conserve some fuel. It will, however, liberate relatively little distribution plant which can be used for war demands and will have substantially no effect upon total distribution expenses, so that a very large part of the reduction in gross revenue arising from such economies will appear as a reduction in the net income. As an economic consideration of the

war such economies are to be earnestly encouraged, but it is nevertheless true that so far as the electric companies are concerned they mean reduction in net income almost equal to the amounts by which the gross revenues are affected. It is not possible to estimate what the effect on the companies' revenues will be except to say that it apparently will run into the millions.

Before considering further the operating phases of the situation we will take up the effect of war conditions on extensions of plant since the two phases are so closely interrelated.

The effects of the war conditions are being manifested not only in the matter of heavy increases in the operating costs already pointed out, but also in the matter of extraordinary increases in cost for new plant required to care for added business. At first thought it might be concluded that the effects of the latter apply only to the period of the war, but the increased cost of new plant per unit of capacity manifestly affects the cost of service not only for the period of the war but for the life of such plant, unless some measures can be devised to amortize the excess first cost of new plant over the first cost of like plant in normal times.

Many electric companies are now confronted with the necessity of not only caring for their normal growth of business, but have had thrown upon them large demands for power arising from the increasing expansion in the manufacture of material of war, which includes almost every necessity of life, ranging from shipping to textiles and food stuffs, in addition to special war products like arms and ammunition. On this account the electric companies have found themselves obliged to prepare very rapidly to meet increased power demands of several hundred thousand horse power over their normal requirements.

When viewed from every standpoint, it will be seen that the economical central power generating station is the proper medium for the supply of the large power requirements arising on account of the war, since purchased power leaves the manufacturers of munitions and other war material free to devote their energies to the development and operation of their manufacturing plants without diverting any of their energies to the development of power plants or their operation; it provides the greatest possible amount of available power from a pound of fuel; it reduces the total capacity of power generating equipment necessary by taking advantage of the diversity between the power requirements of the various manufacturing establish-

ments; and it provides the greatest possible amount of available power per man occupied in the power generation field and thus diverts the least possible man power from the other needs of the nation. It also makes it possible for the power equipment manufacturers to concentrate upon the production of the larger units of equipment used by central electric stations, thereby making it possible to produce the largest capacity of equipment in the least time. And the centralization of electric power generation equipment, with comprehensive distribution systems, places it in position to most effectively care for the changes in power requirements of industrial establishments to be expected after the termination of the war.

These advantages of the central station power are so large that it is advisable for the Government to use every reasonable means to encourage the central station companies and discourage individual power plants during the war period.

It is to be expected that much of the business arising from the war will prove of a temporary character, so that the electric companies are faced with the double dilemma of finding themselves called upon to provide capacity to care for extraordinary accessions of new business, at costs for the equipment and installation far above normal prices, and this in the face of the fact that it is to be expected that in many cases some of the business will prove of temporary nature without equivalent other demands to take its place after the war.

The electric companies have so far provided for the needs of the situation splendidly, but this has been done largely at the expense of reserve capacity, as the financing of almost no large extensions, if any, has been accomplished since our country become a party in the war. There can be no doubt that the power demands on account of the war have not yet nearly reached their maximum and it will be necessary to devise some way by which it will be possible to care for these demands. This can be accomplished by the adoption and promulgation by the government of certain war measures; one being to make possible the discontinuance or curtailment when advisable particularly at the peak load periods of the electric companies, of power and lighting demands not necessary for prosecution of the war or for the safety and reasonable necessities of the population, and another being for the government to assume the responsibility for seeing that money for extensions required for war service can be obtained on reasonable terms.

The first named measure would also operate in the interests of fuel conservation.

We may show the effect of war prices on the electric light and power business in the aggregate. The United States Census of central stations shows that the total of the revenues received from operation and other sources by all central electric light and power systems (including both hydraulic stations and steam stations) in 1912 was in round figures $302,000,000 and the total operating expenses, including taxes, and renewals and replacement expense, but not including interest on debt, was $184,500,000, leaving a total income of $117,500,000. The reported cost of construction and equipment was $2,176,000,000. Extension of these totals to the present year 1917 shows that under normal growth the total revenues in 1917 would have reached $475,000,000 and the operating expenses, including taxes and renewals and replacement expense would have reached $290,000,000 making the total income before deducting interest on debt, $185,000,000. Estimating the reported cost for construction and equipment would have grown to $3,500,000,000, an increase of 60 per cent in five years, the above income would represent 5.3 per cent of this cost of construction and equipment. If no other factors entered into the problem besides increases in cost of operation, and assuming these increases effective over the whole year, the fuel expense as before pointed out, would increase $60,000,000 for 1917, other supplies $30,000,000, labor expense $19,000,000 and taxes $7,500,000, representing an aggregate increase of operating expenses for these items of $116,500,000. This is an increase of 40 per cent in operating expenses, and it reduces the divisible income to $68,500,000, which amount is equivalent to less than 2 per cent on the cost of construction and equipment. This percentage is still lower in the case of the steam-electric systems of the country taken alone, and additional expenses for training new employees and the lowered efficiency of such employees, the cost of special policing, etc., reduce the amount still farther.

There are certain operating economies and changes which might be adopted by the companies if forced to it by war conditions. The service rendered the customers might be curtailed for example by discontinuance of lamp deliveries and free minor repairs, by less prompt attention to troubles and complaints, thereby reducing the number of "trouble" employees, by bi-monthly or tri-monthly billing and the like; the canvassing and

promotion departments might be almost entirely dispensed with, with consequent reduction in meter testing and setting expense; and advertising expense might be eliminated. In the storeroom, increased cost for material points to more than merely salvaging scrap material. It forces the use of old and shopworn supplies and non-standard equipment, and previously used wire and cables. While the use of such equipment may render the service less reliable, it may become an unfortunate necessity.

The current expense accounts may also be reduced by postponement of plant repairs which would normally be made at once; but it is well recognized that the longer repairs are put off the more they cost, and while current expenses may for a time appear lower, the cost in the long run is apt to be increased.

The net results of such economies might amount to as much as 10 to 15 per cent of the normal operating revenue of the electric companies, and every consideration should be given to them, but they are offset by increases in expense which have not been included in the amounts named.

There is a phase of the situation which has not been referred to in the preceding but which in some cases is already being felt, and is likely to be generally felt by electric companies. This is the improvement in load factors arising from putting the less essential industries on an off peak basis and the fact that war industries give an exceptionally long hour demand, owing to the intensive character of war manufacture. This produces two quite distinct results, one being in the direction of reducing the maximum loads which may be carried, particularly on underground cables and to some extent upon generating equipment, and the other being in the direction of decreasing the expense, other things being equal, per kilowatt-hour of total output. How much effect the first result may have it is impossible to predict but it may have marked effect in cases where the load factors become very high, and it is already showing.

A part of the increased cost on account of war conditions has been provided for by many companies by the introduction of coal clauses in contracts for large light and power service. These clauses have taken various forms from the simplest in which the increased (or diminished) charge is figured at a given amount for each dollar of change from a stated cost of coal per ton, to the more elaborate form in which the increased charge is figured upon the relation between the present day cost and

the cost in a prior period for the coal required for performing the same amount of work.

But even if coal clauses could be satisfactorily applied to all electric rates, which they cannot, they would be only a partial remedy for the present conditions since the increase in cost for coal is only about one-half of the total increase in cost due to the war conditions. Each electric rate requires modification to take into account the other factors as well as coal entering into the cost, so as to increase the total revenue approximately 25 per cent, in order to bring the same income to the electric companies of the country to cover interest and surplus as would have been expected for 1917 under normal conditions. It should be noted, however, that a correctly adjusted coal clause more fully meets the situation for contracts with high-tension customers than with the general average of small customers in serving which there are large expenses for labor and materials required in the processes of transformation and distribution.

What has been pointed out in the foregoing regarding the effect of war conditions on central station electric service is also applicable to the cost of power produced by private power plants. The immensity of this field is seen by reference to the United States Census of Manufacturers for 1914, in which the total primary power reported as used in this field aggregated 22,500,000 horse power (exclusive of isolated electric plants for office buildings, hotels, etc.) of which 15,700,000 horse power was comprised of steam driven equipment and only 3,900,000 horse power was in the form of purchased electric power. The central station steam and water-driven electric generating capacity in 1912 was only 7,500,000 horse power, with a probable 9,000,000 horse power in 1914.

The high costs and difficulty of obtaining new equipment and operating supplies, produced by war conditions, and the increased demands on the industries, draw the manufacturer to the central station for either his whole or additional supply of power. Here is a fertile field for conservation of fuel, through the supply of this power by central electric companies but it is evident that under present conditions the electric companies cannot afford to add such load except at fully compensatory rates based on war costs, and on long term contracts, and that they are practically barred from undertaking such service owing to the financial situation, except where the cost of additions of equipment has already been financed.

In the extension of the central station into this field and especially in the acceptance of temporary or auxiliary loads, there is the menace to the central utility of being burdened with unused and abnormally expensive station capacity at the end of the war period. This menace lies not only in the possible decrease of manufacturing activity but also in the possibility that the user may return to private plant operation with the re-establishment of normal markets. It would seem unfair that the future inhabitants of an industrial territory who had been called upon to furnish large amounts of war material for defense of the whole country should alone bear this burden of unused capital. A more equitable distribution of the burden might, for example, lie in special rates for such service, long-term contracts, or in capital subscriptions or deposits made by the power users to carry the investment until such time as the plant not used after the war period is put into active operation again.

Considering the output of power by the industrial plants using steam power, which do not now purchase electric current, estimated on the basis of the capacity of equpment as reported for 1914, operating at the equivalent of full load for a sixth of the time, the total horse power output would amount to 23,000,000,000 h.p -hr. It is safe to say that at least three-quarters of this is such that the exhaust steam cannot be effectively used for heating purposes and there would be a possible saving of at least 1½ pounds of coal per horse power hour through service of this three-quarters from central steam-driven electric stations, making a total saving in fuel of 13,000,000 tons of coal per year under the industrial plant output for the year 1914. The saving would be much greater when considering only the more modern and economical central stations. The same considerations as above apply to the field of isolated building and hotel electric plants where conservation of coal amounting to millions of tons could unquestionably be effected.

We have heretofore given principal consideration to the effect of war conditions as increasing the demands for service. We have not as yet in this country much experienced the opposite effects of economies forced by war conditions. In England, the danger from air raids and the necessity for conserving coal have materially decreased and in some cases almost wiped out the street lighting service furnished by many companies, and government regulations together with economies practised by the customers have very materially reduced the domestic and commercial lighting loads. It is obvious that such losses of business

coupled with increased cost for supplies and labor increase the expense per unit of output far beyond the point where it may be neutralized by any economic practises of the supplying company. The result has been quite a universal increase in rates, in some cases flat percentage increases of the same amount for light and power, in other cases differing percentage increases for light and power, and in still others increases depending upon changes in cost of fuel. These flat percentage increases have varied from less than 10 per cent to as high as 50 per cent over the rates in effect prior to the war, London rates having been increased 50 per cent according to the London *Electrical Review*.

As to the effect of war conditions in this country on ordinary domestic and commercial light and small power consumption for other than manufacturing purposes, the rate of adding new residence and commercial customers is on the whole falling off due to personal economies of the public and generally fewer extensions of lines by the companies, and to decrease in building activities, and the like. It does not appear that the lighting customers in this country have as yet inaugurated any general lighting economies sufficient to be seriously felt by the companies, and the companies have on the other hand shown a disinclination to increase the rates to small consumers. The drain on our resources will, however, be felt more and more as the war progresses, and it is to be expected that the domestic and commercial customers will decrease their consumption of current.

Government action toward conservation of our resources during the war may result in the adoption of plans such as the "Day Light Saving Plan" now in considerable use in Europe. The effect of such action on the cost of electric service is substantially the same as results from voluntary economy practises, in that the use of current is reduced without full equivalent reductions in expense for generating and distributing plant required or in the expenses connected with operation. While some forms of rates in common use are designed to return, first, as much as practicable of those costs of service not dependent upon the quantity of current used, commercially applicable rates cannot be so nicely balanced as to make the appropriate returns under all conditions. It is obvious that any action directed toward general lighting economy, while conserving our national resources and possibly releasing some plant for uses more essential to carrying on the war, results in higher relative cost per kilowatt-hour of current furnished and this points toward higher rates to the public.

It is evident that increased expense for service arises in every department of the business; in operating labor and supplies and taxes, in protection of the property, and in cost for extensions of plant. And the latter is not only affected by abnormal first cost for equipment and its installation but also by the present difficulty of obtaining money for such purposes at other than exorbitant rates as compared with normal. In the matter of labor expense the cost of service is increased not only on account of increase in wages but by the need for training new employees to the service and the necessity of operating with partially trained employees. These last conditions are likely to be increasingly felt with each new draft of men for the armed forces or other direct government service. The increases in cost of electric service on account of the war conditions are so great that rates for service which were equitable at the beginning of the war, are in some cases now not covering the operating expense, and where companies are being loaded with war business the new business in many cases may become a serious menace to the company, which can only be overcome by taking into account the war conditions in determining the rates to be charged.

Notwithstanding that wages of operating men are higher than ever before, yet the necessity for using many partially trained men in the operation and inspection of electrical systems makes for a lowering of reliability of service and this coupled with the need in some cases to utilize all equipment to its limit with reduced reserve capacity has a further effect in this direction as well as in the direction of reduced operating efficiency. Also it is to be expected that with present conditions added transmission lines will not be installed if the present circuits can possibly carry the loads, even though under ordinary conditions additional circuits would be promptly provided, and like considerations apply to distribution systems with a consequent tendency toward reduced efficiency and a wider range of voltage variations than is now considered good practise. It seems proper that regulatory bodies should take into account these considerations in their requirements for electric service during the period of the war.

It should be recognized that the electric companies while serving the best interests of the public broadly may in some cases be obliged to operate with impaired forces and to utilize their equipment during the war to such an extent that they may be unable to fully maintain present standards of service.

DISCUSSION ON "EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE" (GOODMAN AND JACKSON); BOSTON, MASS., JANUARY 8; NEW YORK, N. Y., JANUARY 11 AND CHICAGO, ILL., JANUARY 14, 1918.

DISCUSSION AT BOSTON

B. A. Behrend: The question of conservation of limited natural resources should be viewed in this crisis differently from the way in which we have looked upon it before the war. It is more essential that work be done and results accomplished than that resources be husbanded. It is equally essential that people should realize that the waste of coal for heating purposes in public buildings, hotels and railroad trains during a spell of warm weather takes the coal which should be used during a time of extreme cold. Now is the time to urge the reduction of temperatures in places where human beings aggregate to 65 deg., and orders to this effect should be issued by the Fuel Administration. A man or a woman should be marked as unpatriotic who does not care to comply with so obvious and sane a regulation.

I suggest the drafting of all men below the age of forty-five. If they cannot as yet be used for any government activity, they should be held in readiness, and once drafted and properly "labelled" they should be under the same obligation and responsibility as any soldier. Besides its being unpatriotic to strike or refuse to undertake work assigned or work to be done in the regular course of business, it should be made an offense. Thus, strikes of painters or carpenters or tool makers or moulders should be made impossible by an enrollment of every man under forty-five years of age or even fifty years of age.

The nation and the hammer blows which this nation must deal cannot have the strength of the whole American nation unless every man does his duty. However important economy is at this time or at all times, the increase of production is of even greater importance to the vital issue.

Mr. Sykes: I approach this subject from a somewhat different standpoint than Mr. Jackson as my interest has been centered with the consumers of power. During the last few years I have had a good deal of work to do in trying to persuade large industrial plants to use central station power and the progress that has been made in this direction in the last five years has been very marked. I have been particularly interested in power for steel mills where there are very large customers who think nothing of a 5000 or 10,000-kw. load. All their units are large power consumers. Now the situation at the present time from the standpoint of the consumer is a very disturbing one and some of the plants in which I am interested have been very much hindered in their operations due to the irregularity of central station service. Now those of us that are connected with the electrical business understand some of the problems of the

central station. However, the customers of central stations very often do not. During the past few years we have been gradually educating the large consumers of power to buy central station energy. We have been told that the one thing that did not go up in price was central station power, whereas coal and everything else went up. Now, we must recognize under present conditions that there must be an increase in price for power. I think large consumers in time will readily pay whatever is a reasonable amount when the central station makes out its case.

From the standpoint of the power consumers, the central stations must run. They do not care how the central station does it, but they must have current to run their mills. It is particularly important at the present time to remember that we are facing interruptions of power all the time, this at a period when we can least afford to have such interruptions. New industrial plants are springing up all over the country. Most of them have not the time to build power stations, nor could they get the equipment. The central station must carry the load necessary to manufacture munitions.

Speaking of maintenance I think we will find before long that electrical manufacturers are going to be in a very serious condition when it comes to supplying repair parts to maintain central station service. Unless some action is taken by the Government or other agency to keep the electrical manufacturers running enabling them to take care of their customers, especially when machinery breaks down, it will not be a question of a few days or weeks, but we will be lucky to resume operation at all. Anybody with electrical manufacturing companies can see this in the offing. In Germany, of course, there are harder conditions than we face for they cannot repair plants. There is no necessity for us to reach that state, but people like electrical manufacturers seem to have all kinds of burdens thrust upon them. Their plants are being turned into munition plants and the electrical situation is pushed into the background. Unless we can hold all organizations together and make parts as they may be required, I think there will be times when very serious shutdowns will occur.

I do not see how load factor is going to trouble us very much, although the load is certainly going to change round. In relation to off peaks. I am from Pittsburg and we do not have much in the way of off peaks there. We are working nights now. We have had our meatless days and our wheatless days, but we have also had powerless days. However, that is a thing which is being corrected with the aid of the United States Government.

I cannot see how central stations can increase their efficiencies at all with the whole industry being rearranged. I can only see decrease in efficiency. All manufacturers are facing a decrease in efficiency. We are losing some of our best

men because of the war and we cannot replace these men by poor men and at the same time get efficiency. We will face a comprehensive decrease in efficiency all along the line.

J. W. Cowles: I can hardly undertake to speak from the standpoint of generation or even distribution of electric power to-night, but my thoughts naturally turn to that phase of the business which we term "Service." We know that to the central station industries to-day the manufacture and distribution of our products, important as these phases of business are, are yet by no means the only ones and that during the past few years we have been coming to the point rapidly of realizing that it is service which we are selling in the broadest sense, and that there is a great deal to do with our service rendered than merely the product of generation or manufacture. The problems confronting us interest me particularly in the economies that can be introduced and must be introduced in connection with our service to our customers. I am passing by the generation and distribution problems simply because they do not happen to be quite as near to me as service. When it comes to service rendered to 100,000 or more customers the problems are gigantic and to me the problem to-day is how far can we and must we curtail below the standards formerly considered essential and all-important. We are all seeing very marked curtailments in service rendered. Grocery stores not only cease to come around to take our orders, but they have cut their deliveries in two or more. The department stores have done likewise in all directions. How far must we go in that same direction toward reducing our service efforts, all of which cost money and add a very considerable increment to the operating expenses of companies. Those of us who depend on train or electric railway transportation certainly know to their sorrow what the curtailments and decreased service privileges are in every direction. We may not accept all of these curtailments gracefully, as we all doubtless do our share of complaining and growling on our way to and from the office, but at the same time we all appreciate that it is the inevitable and that all of our criticisms against the particular railroad or the particular trolley line that we may be patronizing are unwarranted. We are coming to recognize the fact that we must accept a little more graciously these hardships, and they are nothing less than hardships. So in our own line of business it seems to me that we must accept the inevitable and curtail our service efforts trying to cultivate in the minds of our friends that they must expect less from the electric light companies in the same way that they are expecting less from other industries or concerns with which they have dealings. No one appreciates more than the central station man these problems of to-day and the study that is being constantly given to them certainly bespeak the patience and consideration of all our friends and acquaintances during these most perplexing times.

C. A. Adams: There is one aspect of this subject which is immensely interesting to me, namely, its broad aspect. We are dealing with a problem which is not in the ordinary sense technical; it is largely an economical problem. We find more and more that we can no longer be mere engineers in the old sense of that word but that we must be citizens in the broadest sense and take our part in the solution of the big social, economic and political problems, many of which are the direct result of the work of the engineer, and can be best understood by the engineer. This paramount duty is being emphasized by war conditions and should be faced with courage and fair-mindedness. This is the time for men with vision. Who more than the engineer should have this vision?

There are two or three little points in the paper which have caught my attention. The author says, "A conservative figure for the increase in tonnage due to lower quality and non-uniformity of grade is 10 per cent, etc." I think that this is a very considerable under estimate of the percentage increase. Coal as it comes from the mines now is not picked over as it used to be; the percentage of ash has gone up in some cases from 6 per cent to 10 per cent and even to 20 per cent. This means not only an increased tonnage in proportion to the increase in ash, but also lower boiler efficiency and larger boiler capacity. I have heard it stated very recently by central station men that this increase in ash, slate, stone, etc., in the coal and the resulting lower boiler efficiency were sufficient to increase the total consumption by 50 per cent rather than 10 per cent and that the number and capacity of boilers in operation sometimes was greater by about 40 per cent as comppared with the ordinary conditions of a few years ago when the coal was of superior quality. I merely pass these figures along as they do not come from first hand experience.

There is another figure quoted concerning which there is some question in my mind and that is the average of four pounds of coal per kilowatt hour. With our enormous central stations operating at so much higher economy, I should be inclined to put this average at nearer three and one-half pounds.

W. B. Jackson: As to the situation in Europe, and how these problems have been met; I cannot speak of them in much detail particularly as to Germany as I find it very difficult indeed to obtain authentic information. In the case of England, the Government has not taken any official stand in the matter, but as I have stated, prices for power have been raised very greatly indeed. Several of the largest companies in London having had their prices raised 50 per cent flat. In Birmingham the prices have been raised 30 per cent flat, with a possible later additional raise in price.

In regard to the isolated plants; it would be rather presumptious to say that all isolated plants are necessarily going to disappear. This question was discussed in the paper, where you will remember we referred to the fact that probably three-

quarters of the boilers used in the isolated plants may be considered as not necessary for heating purposes. It is certain that there are a very great number of isolated plants today that have no economic or other proper reason to exist.

DISCUSSION AT NEW YORK

F. A. Bryan (by letter): I think Mr. Goodman covers the subject very thoroughly. Plants operating wholly by water power are not, of course, affected to as great an extent as those operating by fuel alone, while plants operating with water power, with steam reserve, as is our plant, are affected very considerably, although not as much as if we depended entirely on fuel. Our situation is one where we depend both on steam and water power, the steam plant having been until quite recently almost entirely a reserve and being built as an emergency or reserve plant, we did not go into the refinements for cheap operation required in a continuously operated steam plant. The interest on the investment for such a plant over-balanced the cost of occasionally operating the plant to its full capacity. Now we are confronted with this condition. Due to the high cost of fuel, customers who depended partially on our service have increased their demand materially and manufacturers who have not heretofore used our service at all have immediately demanded service. It has placed a load on us far in excess of our anticipations, increasing very materially the amount of energy required of our steam plant and we are, therefore, compelled to use larger quantities of fuel than ever before and perhaps are not in a position to use it as economically as we could had we anticipated and prepared for such demands. I think that you will find this situation true of all combined hydraulic and steam plants. The same argument can be applied to the steam-heating end of the central station business. Where two years ago our New Business Department put forth their greatest effort to have our mains loaded up, we now find that those residing along our mains are not only requesting steam heat, but demanding it and at a time when the price of steam is practically equal to the cost of fuel required to develop it.

The fact that conditions exist as outlined in Mr. Goodman's paper are undebatable. The question with the central station operator today is "What is the remedy?" Business is being presented to him unsolicited in greater quantities than ever before and his plant is being abnormally loaded, while at the same time, his ability to secure additional capacity is almost prohibited both on account of cost and delivery of equipment.

Without this additional equipment, *Service*, such as he has so long boasted of is bound to be impaired and with impaired service, even during war conditions, what is going to be the effect upon his ultimate business? Our position in this matter where large customers are now coming on our lines, is to advise them that in case of any interruptions or lack of capacity on the part of our plant, they must necessarily be the first ones to

have their service discontinued but even this, of course, will not remedy the possible interruptions of service which our old and regular customers are bound to have.

Assuming that the central station can purchase additional capacity to take care of increased demand, the cost as compared with costs a year ago, taking into consideration the actual cost of the additional equipment, plus the increased cost of money, is approximately twice the cost under normal conditions. With the State permitting a 7 per cent and 8 per cent return on value under normal conditions, what is going to be the condition of the utility when times return to normal and the value of equipment inventoried is appraised at about one-half of cost. In other words, unless the States change their attitude, the central station will actually lose money on such betterments. In the face of such a possibility, is the central station justified, even if they can get delivery on equipment, in purchasing under existing conditions in order to take care of the increased demands? Furthermore, is a central station, as long as they have auxiliary equipment, under existing rates and costs, justified in loading up this equipment with this abnormal business to the detriment of their service to their regular customers?

I do not see any remedy for the predicament of the average central station today other than the raising of rates to offset the tremendous increase in cost. Nor do I see any legitimate reason why commissions should refuse to increase the rates to offset these abnormal increases and I believe that the public will not protest against a legitimate increase in rates when the central station can show that their costs have so tremendously increased. The increase of rates, as I see it, is the only way in which the public utility can expect to survive the present abnormal conditions.

R. G. Hudson (by letter): It is a well established fact that the purchasing power of a dollar is less in time of war than in time of peace. The sale of any commodity during war time at a peace price infers that the commodity costs no more to produce, that the commodity was made before the war, that the commodity is of inferior character, or that the producer is content with a smaller profit.

The cost of producing electric energy is unquestionably greater now than in normal times, none of the electric energy now offered for sale was made before the war and it is obviously impossible to offer an inferior brand of energy. The continuance of the normal rates for electric energy during war time therefore implies that the producer must receive a smaller profit.

In the paper just presented it is estimated that the average return in 1917 on the capital invested in public service plants supplying electric light and power must be less than 2 per cent while in normal times the return would have been over five per cent. The directors of corporations operating plants under average or worse conditions are then confronted with the task of deciding whether the stockholders are not entitled to a greater

return upon their investments, how long their plants may be operated under present conditions with any balance of profit and, if the revenue is to be increased, how to accomplish that end.

It is reasonable to presume that our public service commissions must sanction a higher return to investors than can be given to them in some cases under present conditions. The attitude of the federal government is reflected by the recent recommendation of the President that the railroads under government control should receive as compensation the average of their net operating incomes during the past three years. Directors would then seem to be supported by representatives of the public in any effort to make a fair return to the stockholders during the war.

To operate a public service plant with the revenue barely exceeding the operating cost under the present conditions of steadily advancing costs of labor and materials would furthermore be precarious and unwise since public service corporations do not enjoy the privilege accorded the private corporations of accumulating large emergency reserves from current revenue. There is then no alternative in critical cases but to increase the revenue.

In view of the prevailing tendency of the public to economize in all expenditures an increase in rates does not necessarily mean a proportional increase in revenue since it also increases the tendency to use less power. In conjunction with an increase in rates an earnest effort must be made to increase the number of consumers taking off-peak service.

The ideal solution of the problem would be obtained by increasing the number of consumers to the extent that, with each consumer taking less load than usual, the total load on the system would be unchanged and the increased revenue would be proportional to the increased rate. Such a solution takes into account the psychological effect of an increased rate on the consumer in times of enforced economy, the necessity for the conservation of coal, the desirability of maintaining the status quo of the plant, and the problems associated with the reconstruction period after the war. The promotion department was never more important and the field of promotion, while narrowed by an increased rate, must constantly broaden as the condition of the isolated plant becomes more critical.

The author of this paper also raises the distinct and separate question regarding the manner of meeting the threatened shortage of coal and labor. To adjust this situation the public must be instructed openly regarding the existing conditions and the possible consequences. Federal authorities must be convinced of the need for coal and exemption boards of the scarcity of skilled labor. In the absence of governmental regulation skilled laborers who leave their positions to enlist or to take up more remunerative work may only be restrained from such action by building up popular sentiment against it. The watch engineer and the switchboard operator must be made to feel

that they serve their country as effectively in their accustomed positions as among the military forces. The opportunity should be given such essential workers to enlist in an industrial Legion of the United States and to wear appropriate insignia indicating their loyalty to the nation.

Mortimer Freund (by letter): The central electric generating station industry is so vital a feature of our economic life that its continued welfare is of prime importance during the critical times which now confront us. Our determination to "win the war" can only be realized by placing and maintaining in healthy condition the constituent parts of our social and economic structure. If the central station industry is up against it, to use the vernacular, or approaching this state, it behooves the engineering profession to lose no time in investigating the causes and the remedies and to make sure that the latter are applied without delay.

With this in mind, I do not believe it proper to allow to pass unquestioned the authors' statement that "the economical central power generating station is the proper medium for the supply of large power requirements arising on account of the war", and further that "it is advisable for the Government to use every reasonable means to encourage the central station companies and discourage individual power plants during the war period." Both of these statements are by far too sweeping in character and, as a matter of fact, are only justifiable for such cases, where a careful and disinterested consideration of all the circumstances, will warrant such a conclusion.

It seems to me that for the second statement might better be substituted the following: It is advisable for the Government to encourage all consumers of fuel to use every effort to fulfill their heat, light and power requirements by such means as will utilize fuel most economically and to do away with all existing wastes which are preventable.

There are industrial plants for example, where a great part if not all of the electricity used is virtually a by-product, due to the utilization of exhaust steam from the electric generating units. I have in mind a large industrial plant which up to 1915 operated two separate boiler plants, the output of one of which was utilized almost entirely in pipe coils for drying. The exhaust steam from the electric generating plant was wasted in the non-heating season and only partially used during the heating season. Substitution of purchased electricity had been suggested. Since 1915 the exhaust steam has been utilized in place of live steam, the use of one of the boiler plants has been discontinued and the actual cost of electricity in view of the use of the exhaust steam is far less than the best price which outside service can offer. The action on the part of the management of this mill in undertaking the changes necessary to permit of this, has not only paid them well on the investment made, but has benefited the country to the extent of reducing their fuel consumption. Surely the authors do not recommend that

the Government discourage such an individual generating plant, although such a conclusion might be drawn from their statement.

This calls for a pertinent inquiry as to whether or not certain of our central stations are in an unhealthy condition because they are suffering from "indigestion" caused by "biting off more than they can chew." In recent years a number of the central stations have been very energetic in conducting campaigns for the shutting down of private power plants. This procedure in itself is quite commendable, if carried out on a sound basis. But the sale of electricity at ridiculously low and unprofitable rates merely for the purpose of swelling the list of "Private Power Plants Supplanted" to appear in advertisements in the daily press does not appear to be based on sound ecnomic principles. Cases have been recently reported where even industries necessary for the conduct of the war have been hampered due to their having abandoned their private plants for central station service, which is now unable to meet the increased demand.

An engineer, who recommends to his client the installation of a private power plant, where conditions do not so warrant, merely for the sake of increasing his professional fee, is very properly denounced by reputable engineers in a no uncertain manner. A man who installs a private power plant, merely as a show place, where sound engineering does not warrant such a plant, is wasting the natural resources of the country and is to this extent actually an enemy of society. By the same token is the central station management which encourages the shutting down of private plants just for the sake of increasing its own prestige, irrespective of the fact of whether or not real economy is secured (and I include under this term the consideration of continuity of service), no less an enemy of society, because it is a business policy to do so.

I hold no brief for the private power plant. There are unquestionably many private power plants now in operation which should be supplanted by central station service. It has been my own practise to recommend central station service in all cases, excepting those where the installation of a new private power plant or the continuance of an existing plant will show a substantial saving in the cost of operation or other advantages of substantial worth. The private power plant is not an obsolete idea, as some have been trying to tell us. There are cases where it results in the most highly efficient production of heat, light and power possible. Each case should be decided upon its own merits and not on the basis of sentiment. Sweeping generalities are in my opinion unwarranted and misleading.

Central station service and the private power plant should not be in conflict, especially at the present time. Each has its proper sphere and conditions may have arisen or will arise in particular instances to cause one service to supplant the other. It has been and will be to the mutual advantage of many central stations and private power plants to co-operate, by exchange of service at different times of the day or during different seasons

of the year. Friendly competition and co-operation should exist between central station management and the management of private power plants, so that maximum efficiency in operation may be maintained and the greatest benefit accrue to the general public. This is a policy in keeping with the spirit of the times and necessary for the conservation of our resources.

B. J. Arnold: This whole question, as you all know, is a very serious one. I was talking today with the president of one of our railroads, and he told me that they were within three hours of having to shut down the railroad at one time, on account of a shortage of coal. They fortunately got relief, but they have to shut down sections of the track now in order to keep the service going. I have no doubt that similar conditions exist with other companies in the vicinity of New York and elsewhere, although I do not happen to know the details of those cases. Consequently, it behooves every one of us to do everything we can to conserve what fuel we have or can get, and every manufacturer of electric current I have no doubt is doing his best to make every pound of coal go as for as he can.

One possible way in which the manufacturer of current can help the situation—and incidentally I may say I am not the first man to think of it, but I might emphasize it—would be, with the consent of the public service commissions—because I surmise that consent will have to be obtained—to cut off certain kinds of service during certain periods of the day, as one of the authors has pointed out, and run manufacturing plants, engaged on munitions work, and any other work that can be utilized, during the daylight period, so that the capacity of the plant can be utilized at night for lighting; in other words, distribute the capacity of the plant over the twenty-four hour period by dividing up the classes of work into the kinds of work that can be utilized in the respective periods of the day, and take full advantage of the total possible capacity of the plant.

Some of us who have to do with electric railroads buy power from a different source, and we can buy power from our relay systems, our emergency sources, during certain periods of the day for a much lower price than we can during the peak load, and in that way we can utilize at reasonable cost what surplus power we buy during these light load periods or daylight periods. That principle, applied in a general way to the power producing properties and the power using industries will help out in the general situation.

That is not anything new, but is something we may have to do. The greatest thing we have to consider today is what will we do with our railroads on account of the transportation problems, and I want to emphasize this point. Some of us have been talking, for some time, upon the necessity, or advisability, at least, of Government control of the operation of the steam railroads, and I may say that we thought that applied also to electric railways, but not to such an extent, for the reason that

the electric railway companies have shown more of a disposition to cooperate and coordinate their interests than the steam railway companies have; but the great lesson we have to learn from the presidential action which has recently taken place in the Government taking over the operation of the railways is that it creates conditions whereby the steam railway companies are going to learn by a great example the advantages of coordinating their interests and operating their terminals jointly; that is, taking the freight and passenger service from one set of tracks, which are heavily laden, and placing a part of it on others that are less heavily laden, and getting the advantage of utilizing the entire railroad investment of the country to the best advantage, and avoiding additional lines or extensions, where they have heretofore been thought to be necessary, to enable the railroads to compete with each other. That proposition will apply to trunk steam lines everywhere.

We have been laboring in Chicago for years to get the steam railroads which enter that city fully to utilize the main trunk terminals tracks as they now exist rather than to attempt to individually extend or widen them, and we are making some progress. As you know there is another such situation right here in New York. In other cities which we have gone into in the past we have also found this disposition on the part of the railroads, to build side tracks into new industries to get their portion of the business. There are certain cases where the steam railroads have cooperated and agreed on joint tracks in some of the smaller cities, but as a general proposition the steam railway companies endeavor to get their own side tracks into the industry, so as to compete for the business.

I think that is wrong, and I believe that one of the great advantages of governmental control of these roads at the present time is going to be that it will put into operation a system which will bring about the coordinated operation of the railroad terminals, and that we will never go back to the old plan, because when the economies that can be affected by such a system of operation are once recognized, the railroads will take advantage of and continue that method of operation, even when they get the roads back into their own hands.

I do not want to blame the steam railway men, as we know that some of them have attempted to co-operate in the past, to a certain extent, but their conditions, and prejudices and certain laws have worked against them—the Sherman Law had been the principal difficulty in the way—and they have been unable to bring about the shipments of freight as effectively as they could have done, if they had not been handicapped by this law and by their own prejudices, in some instances. It has been necessary to have this governmental step in order to get the problem in the condition it is in today.

I think the same thing that applies to steam railway terminals and steam railway trunk lines can be made to apply more effectively, perhaps, to the production of power than it has been app-

lied in the past, although in the last ten years the tendency has been toward the elimination of the small power producing plant and the placing of the load on the central station, which could produce the power for much less per kilowatt-hour distributed.

There in no question in my mind that where you can utilize the steam not only for heating, but also have some use for that heat, aside from merely heating in the winter time, there is an advantage in having an isolated plant. For instance, in a hotel, where you need steam for cooking all the year round under such conditions as that, the isolated plant, in my judgment, will be superior to the central station for power, that is, it can produce its own heat and electrical energy cheaper than it can buy it from the station. That is the only instance, in my experience, where I found that it would work out in that way, otherwise, it is generally cheaper to buy electrical energy from the central station.

Philip Torchio: The problem before us tonight is of national importance. Col. Arnold has just made reference to the parallel of the railroads. I believe that the national investments in public utilities, represent, at least ten per cent, and probably fifteen per cent of the railroads, so that they are of an importance deserving the attention of all people. Incidentally, the production of primary power equals the importance of all railroads, in so far as the consumption of coal of the country is concerned.

It is a strange position in which the public utilities are placed in abnormal times such as we are passing through, probably in normal times it is almost as bad. While the costs of production and operation increase, public utilities are not allowed to adjust their prices accordingly, as all the other commercial industries are allowed to do. So that the public utilities are facing the situation of increased costs, without authority of readjusting the revenue for their service. It is obvious that a situation of this kind cannot exist indefinitely. What the solution should be is hard to outline in a meeting of this kind—I do not know that it is the proper function of engineers to advance schemes to solve financial problems. It is, however, the duty of the engineer, to clearly visualize the economic factors of an engineering problem. It is perhaps one of the saving graces in this war that we seem to crystalize our ideas, to get a clearer insight into broad problems, to get a quicker perception of the true fundamental ethics of social and economic problems, than was ever done before in normal times.

In these days, we can visualize broad economic questions without, perhaps, being accused of doing it for selfish motives—we can visualize the best interests of the country, not only for the period of stress through which we are passing, but also for the future development of the country. To outline this goal is principally the function of engineers. The engineering profession can command a full knowledge of the various details and factors necessary to evaluate and solve these problems.

The power problem is brought home to all of us, especially here in the East, by the lack of fuel. There is an appeal made

now that every means possible should be used to economize fuel. All of the advocates of the development of potential water powers are at work advising that the country should promptly put into use the water powers in the country.

Now, it is essential that we really shoud see what the country ought to do. I want to give you a mental picture of the situation in the country as affecting power and the use of coal. The United States in the year 1915 consumed 540,000,000 tons of coal. Of this amount practically one-half, or 270,000,000 tons were used to produce primary power, and the other half was used to produce heat. In addition, there was produced through hydroelectric developments about five per cent of all the primary power used. The 270,000,000 tons of coal generated power at an efficiency of five per cent. The other 270,000,000 tons used for heating gave an efficiency of fifty per cent. Our efforts should be concentrated on bringing about improvements in these efficiencies. In the production of primary power we can eliminate the consumption of coal by using potential water powers. The potential water powers of the country are practically two-thirds in the Mountain and Pacific States, and the other one-third are in the remaining states. If all of the potential water powers of the country east of the western and mountain states were utilized in full, they would replace, it is estimated, about fifty per cent of all the primary power used in the States east of the Western States. It will, therefore, always be necessary, with the ever increasing use of power, that the nation will need, even after all of the water powers are utilized, to generate a considerable bulk of our power from coal, as far as the mind can now penetrate the future of sources of power. Therefore, it would seem that the steam central stations of the nation, east of the western States, are to be a prominent and permanent feature of the national development.

Let us neglect, for the time being, the exploitation of these water powers, and let us confine our study to what we can accomplish in the saving of a part of that 270,000,000 tons of coal that we used in 1915, and probably 350,000,000 that were used last year, for the production of primary power. I said that there was an efficiency of five per cent. If we had today central stations developed on a scale equal to what would be necessary for the development of potential water powers, we would be able today to generate practically four times the energy which was generated with the coal which was used in 1915. Excluding the power that was generated by central stations in 1915, which represented probably 15 per cent of all the primary power used, we could save in the generation of the 85 per cent remaining, three-quarters of the coal used. In addition if we installed systems developed on a unified and large scale to reach out throughout the country to different States, we could by the use of such modern equipment as we would add to our present stations, to a very large extent carry the off peak loads, that are now carried on less efficient old type apparatus.

I estimate that this incidental saving would represent probably one-third of all the coal used by central stations, which will be a clear saving on top of the other large and main saving that I have mentioned before. This is looking into the future, when the coal must be conserved at any cost. This would represent a saving, as I said, of three-quarters of the amount of coal that is being used with old fashioned methods of steam generation.

Now, the nation requires certain products, which cannot be produced except by the use of cheap power, or by importation from other countries. I am referring to materials like aluminum, caustic soda, the nitrates, ferro silicon, electric steel, etc.

There is an immense need in the country for the products of these different industries, which require very large bulk of cheap power in the development of the products. Now, if the country has the water powers, and the country must have these other necessities, it seems to me logical that the water powers should be dedicated to such national uses rather than using the water powers to distribute in small quantities throughout the territories in the field where the steam central station, as I said before, must operate in the future, and will have always to operate. The reason is this,—if you take away from the steam central station the desirable load, that might be put on an economical water power development, the operating cost of that steam central station will increase. That is an economic fact that central station men very clearly recognize, but often engineers at large do not appreciate this fact, that if we transfer the economic load from one station to another, and we keep operating the other station for reserve, we may increase the operating costs with the lesser load so much that we wipe out the economy obtained by the transfer of portion of the load to the other source of supply.

I am not going to take more time in analyzing this problem. I hope that this subject will be more fully discussed; it is of national importance. Nine years ago, in reviewing the future of central stations, I made a forecast, to which one of the leaders of the industry paid the ambiguous compliment of "poetic remarks." The conditions created by this war make it imperative that we visualize the future for our guidance in the present. I am not old enough to indulge in quoting myself, but as the forecast I then made embodies a life study of the power problems, and is timely in this connection, I may be permitted to do so:

"In closing this forecast I would say that in the progress of the gradual uplifting of the industrial efficiency of the country, it is the province of the central station companies to furnish to the nation all of the power required by it in all of the commercial industrial and agricultural needs. With thousands of central stations dotting the map of the United States from east to west and from north to south with a closely woven network of power wires connecting the heaviest centers to the most humble hamlets, is it not reasonable to expect that this wonderful organization should ultimately command and control within this terri-

tory the generation and distribution of power for everything that lights, or heats, or operates machinery, or propels vehicles or trains?

"To carry out this work successfully, the industry will require professional, operating and business men of broad knowledge, sound views and superior ability, thoroughly qualified to be leaders in their respective fields of activity."

It is hoped that this subject, with the correlated subject of utilization of potential water powers, may be thoroughly threshed out from a broad national point of view in all of its technical and economic details, so that the findings of engineers may be of sure guidance to legislators in their future actions regarding one of the most vital requirements of the country.

Philander Betts: It was about three years ago that some of the public utilities began to feel the effects of the war conditions—I refer more particularly to the quality of service that was being furnished rather than the cost. It was not so long after that, however, that some of them began to feel the effect of changes in cost, and something more than a year ago some of the electric light companies in New Jersey filed with the New Jersey Commission proposed increases in rates. These proposed increases in rates were based on increased cost for coal, largely due, not so much to prices paid for the coal itself, but due to the increased cost in getting the coal to the plant. Since that time, conditions have grown worse and have extended to other utilities.

Something more than a year ago a meeting was held in this building of a committee called to consider the advisability of cooperating, or bringing about a system of cooperation, between the owners and operators of all the large power plants in New York City and vicinity. That meeting was not brought about due to any fear at the time of increased cost, but because of the thought that it was necessary to protect against the possibility of interruptions in service. The result of the work of the committee was to determine methods by which each of the companies could obtain service from the other companies, in case of emergency, including full information with regard to available men, materials for repairs, and places where emergency connections could be made between the various systems, and this information and like information was all very carefully made up and placed in the hands of each company.

At about the same time a meeting was held in the Engineers Club of the representatives of all of the large water companies, and the larger municipalities of northern New Jersey to discuss the same subject. In addition to discussing what would have to be done in case one company or one municipality had to call on the others for help, plans were evolved and put into effect last spring, nearly a year ago, providing for police protection. I may say that police protection does not appear, on the face of it, to cost very much, but in the case of at least one water company, which has not spent very much in its ordinary opera-

tion, I happen to know, that because of police necessities its operation expenses were very materially increased. That one water company has already been granted an increase in its rates by the Commission. The wisdom of having such co-operation between the water companies was shown in the last two weeks when it became necessary for some of the larger water companies in northern New Jersey to help out the city of Jersey City.

Bringing the matter down a little closer, a number of the companies have found it necessary to file with the New Jersey Commission—I bring this home to New Jersey, a sister State, because these facts have come before us, these are facts which have already affected the people at large—provisions for increases in certain rates, due partly to increased cost for labor, but very largely to the uncertain conditions with regard to coal supply and the growing increases in coal cost. A much more serious condition confronts us, though, and has confronted us in the last few days. A large company in New Jersey—and its representative here can tell you the details—has found it necessary within the past week to cut off probably forty per cent of its power load, due to the fact that it simply could not get the coal at any price under any condition. In cutting off that load, however—we all know that loads are not segregated in a way that makes it possible to cut off certain classes of service—all we can do is to cut out certain localities, without reference to what there may be in that locality, and doing this has caused a cessation in the manufacture of many things needed in the progress of this war.

The situation is more serious even than we thought, in a way. Hundreds of men and women, and young men and young women, have appealed to the Municipal Employment Bureau of Newark for employment, due to the closing down of works where they have been employed. All of those employed in these works that have been shut down, temporarily—we hope—have been turned away by the Municipal Employment Bureau with the statement that it was not wise to merely give them temporary employment in some other place which would probably be affected the same way a few days later. It is rather a pessimistic picture, but it is better, I think, that we face the situation, that we begin to realize what we will have to face.

In one of the leading papers published in this City yesterday, there is an article referring to the necessity for classifying the uses of fuel, in the first place, among other things, was placed the schools, that it was primarily necessary to furnish coal for the schools. The newspaper thought that that was not a proper classification, but it may have been a proper classification for New York City, due to the fact that a great many of the poorer people in the City had no heat at all in their homes, and the only way the children could be comfortable was by keeping them in school. If that is true, then the schools ought to be temporarily arranged so as to accommodate the parents as well as the

children during at least a portion of the day. If that is a fair statement of the conditions, then it may be wise to include the schools of New York City in that first class.

But it seems to me the vital thing is to keep things going, and employment is one of the vital things. Employment involves the continuation of the operation of machinery, and therefore it seems to me that the machinery in the shops and factories doing productive work of a certain character must be given the preference. If that is done, then the products that are needed will be forthcoming, and the people themselves will have that employment which is necessary to continue them in their well being.

There is something more which will have to be done, however, at least for some little time, and that is there will have to be a classification with regard to the needs of uses for power. Much has been said about electric signs and things of that kind, but electric signs constitute a small proportion of the load that is worrying the electric power company. There are a great many things for which power is used which will have to be temporarily either laid aside entirely or at least curtailed. This possibly will entail the greatest governmental regulation perhaps, that has ever been attempted, but it will be just as necessary as the imposition of the draft law or as the census which we found was necessary. I think we will have to recognize this, and all who have anything to do with the operation of utility plants, particularly with the business end of them, will have to take systematic steps to acquaint the existing customers and prospective customers of the company with the conditions, with a view to cutting down those uses of energy which interfere with the proper operation of those things which inure to our well being.

W. S. Gorsuch: The subject of cost of producing power is a very broad and important one which must be studied from both a financial and economical viewpoint. Especially is this true under the present conditions which make it imperative to carefully analyze and examine power costs, and to look into the possibility of increasing efficiencies.

The increase in cost of labor, material and fuel, also a readjustment of operating conditions, have all had a marked influence upon the cost of producing power. I will briefly discuss in a general way, the magnitude of these effects with particular reference to some of the large first-class power plants.

One of the elements entering into the cost of power which has been materially affected by war conditions is the cost of labor. The authors state that any complete study of the effect of the increase in wages of the electric companies, caused by war conditions, would be a difficult task. The labor situation in the power stations, however, can be more readily analyzed than that of the entire system of any utility.

The increase in wages, reduction in working hours, difficulty in retaining efficient men, training new men, all have been reflected in the cost of producing power. In many instances

the utilities have been obliged to employ very inefficient labor at much higher rates than were paid efficient men before war conditions existed. The employment of inefficient labor and the enormous labor turn-over, have resulted in a reduction in economy of operation and an increase in cost of power. This situation has been particularly felt in the boiler-room where the largest portion of the cost of producing power is involved in the handling and burning of coal.

The cost of a large portion of the labor in the power station has increased as much as 55 per cent. Taking into account the direct increase in wages and the increase as a result of a reduction in working hours. I believe that a fair average increase for all labor in the power station is about 30 per cent. That is, considering all the labor charges in power stations of appreciable size, the average increase in cost of operating and maintenance labor over the period just preceding the war, is about 30 per cent.

The problem of reducing working hours is one that the utilities will be confronted with for some time to come. The adoption of a six-day week schedule for every employe in the power stations, necessitates an increase in the number of men employed, and involves a corresponding increase in labor costs for the same output. It would probably be unfair to charge all such increases in cost to war conditions. However, the present conditions are responsible for hastening the adoption of a reduction in working hours.

While the present increases in wages are liable to remain after the return to normal conditions, it is more than probable that we will be in a position to employ a more efficient class of men than at the present time, and that there will be a marked reduction in labor turn-over.

Another item involved in the cost of generating power and which forms a smaller percentage of the total cost of operation and maintenance than labor cost, is Miscellaneous Materials and Supplies. The increase in cost of supplies and material other than coal, for operation and maintenance of the power station, has been very erratic, and it would be a difficult matter to give a reasonable average figure representing the percentage increase. There has been an increase in the cost of many of the repair parts as high as 90 per cent. The effect of such enormous increases in this class of material is to curtail in repairs, where such can be done without interfering with the economy of operation.

By far, the largest and most important item involved in the cost of producing power is that of fuel. The cost of coal like the cost of labor varies so materially throughout the country, that it is difficult to obtain sufficient reliable data to determine an average figure which will apply to all generating plants of appreciable size. Increases as high as 100 per cent and over are on record. Neglecting the quality of coal, I believe that an increase of from 65 to 70 per cent represents a fair average figure for many stations.

The cost of coal has an extremely important bearing upon the cost of power, representing about 70 per cent of the total cost of production of a first-class steam power plant, prior to the enormous increase in cost of coal due to war conditions. For illustration, consider a plant which prior to the war was generating power for 0.40 cents per kilowatt net output, with a coal cost of 0.28 cents per kilowatt hour net output. With an increase of 65 per cent in cost of coal per ton, the cost of coal per kilowatt hour net output will be 0.462 cents, or more than the total cost of operation and maintenance before the increase in cost of coal. On a basis of an increase in coal cost of 65 per cent per ton, and an average increase in total power station labor of 25 per cent and an average increase in miscellaneous material of 50 per cent the cost of coal will constitute 74.3 per cent of the total cost of production, instead of 70 per cent as stated above. The above illustration is on the basis of plant efficiency and B. T. U. per pound of coal remaining the same.

Many utilities have been obliged to accept coal of inferior and non-uniform quality at a considerable increase in cost per ton. The use of such coal results in lower efficiency, and particularly so, when we are operating under conditions of constantly training new and unskilled men who do not realize what it means to burn coal efficiently. The poorer the coal with respect to ash the higher will be the loss of combustible through the grates, which loss forms an appreciable percentage of the yearly waste. This has been of much concern to some utilities who are paying very high prices for coal which contains a high percentage of ash. In the foregoing illustration we considered coal of the same B. T. U. per pound, with an increase in price of 65 per cent per ton. To show the effect on the cost of power with coal costing 65 per cent more per ton and containing less B. T. U. per pound, let us consider two coals, A and B, and apply the "Figure of Merit" (TRANS. A. I. E. E. 1913, page 1648.) which represents the B. T. U. per dollar after correcting for ash, and B. T. U. lost in refuse.

Figure of Merit:

$$= \frac{\text{B.T.U. per ton (moist)} - \left(\frac{2240\% \times \text{ash}}{100} \times \text{B.T.U. per pound of refuse}\right)}{\text{Price per ton, (2240 lb.) (\$)}}$$

Coal	B. T. U. per pound moist	Price per ton, 2240 lb. $	Percentage of ash in coal as received.	B. T. U. per pound refuse	Figure of merit
A	14,500	3.50	5	3,500	9,170,000
B	13,000	5.77	10	4,000	4,890,000

From the Figure of Merit of each grade of coal given in the above table, it will be seen that 87.5 per cent more B. T. U. is obtained from coal A. That is, if coal B is used which costs

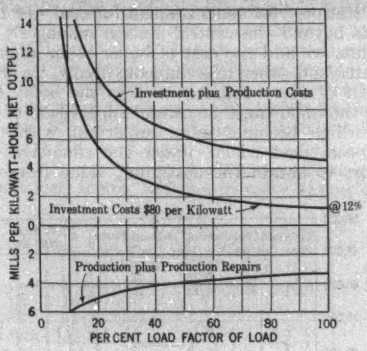

FIG. 1

65 per cent more per ton than coal A, there will be a corresponding increase in cost of coal per kilowatt hour of 87.5 per cent. Apply this increase to the cost of coal and allowing for an in-

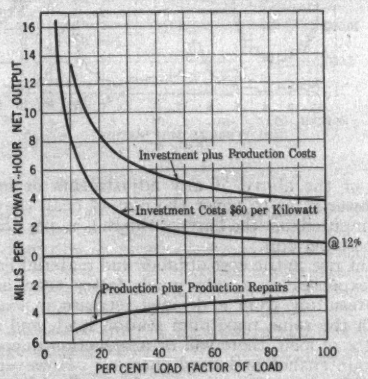

FIG. 2

crease of 25 per cent in the cost of labor and an average increase in miscellaneous material of 50 per cent the cost of production including operation and maintenance is increased from 0.40

cents to 0.6845 cents per kilowatt net output, an increase of 71 per cent. The cost of coal per kilowatt net output representing 76.6 per cent of the total cost of production. That is, out of every dollar spent for operation and maintenance cost in the power station, *76.6* cents is spent for coal, the market price of which is beyond the central station manager's control. If to this figure, we add the cost of boiler room labor, neglecting cost of water and other miscellaneous items, we will find that *84* cents out of every dollar is used in the boiler room. This brings out the importance of careful supervision of boiler room operation which becomes more important with the increase in cost of coal, and which, under the conditions cited, calls for an effort to obtain the maximum value from each pound of coal.

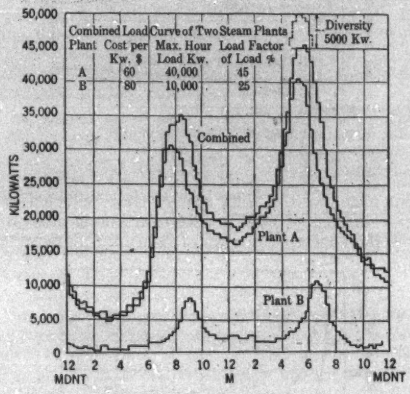

FIG. 3

In view of the above, if any adjustments or increases are made in power rates to cover the cost of coal, they should be made or computed on the basis of the B. t. u. per dollar and not increased cost per ton.

The recent rise in the cost of labor and material has increased operating expenses at a rate much faster than any possible economics resulting from a decided increase in load factor of load. With the same maximum station load, and an increase in load factor of load, there will be a reduction in cost of operation and maintenance and fixed charges. However, the economy effected by operating at a much higher load factor, may not be sufficient to compensate for the very large increase in the cost of coal and labor. This feature is brought out very clearly by referring to the curves in Fig. 1 which are fairly representa-

tive of the cost of power for a plant costing $80 per kilowatt before war conditions existed. Consider the plant operating at 40 per cent load factor under normal conditions as to cost of labor and coal. The total cost of power will be 0.700 cents, 0.425 cents of which represents cost of operating and maintenance, and 0.275 cents fixed charges. If now there should be an increase of 50 per cent in the cost of coal per ton, having the same B. t. u. per pound, and an increase in labor of 25 per cent, and at the same time the load conditions readjusted so as to operate at, say, 55 per cent load factor, the total cost of production will be 0.746 cents, 0.546 cents for operation and maintenance, and 0.200 cents for fixed charges. That is, operating at 55 per cent load factor, with an increase in cost of coal and

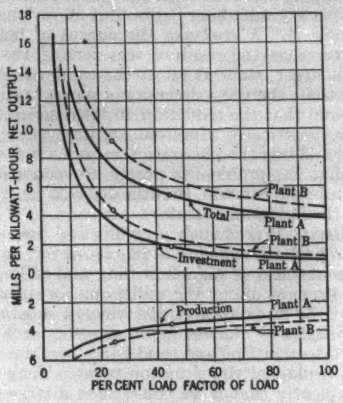

FIG. 4

labor of 50 per cent and 25 per cent respectively, the total cost of power per kilowatt hour net output is 6.5 per cent higher than when operating at 40 per cent load factor with no increase in cost of coal or labor. The difference is still greater in a plant costing $60 per kilowatt, as can be seen by referring to Fig. 2. On the other hand, with a higher load factor, the volume of business is increased, and the increase in the cost of coal may be, to a large extent, offset by the sale of power, depending upon the rate charged.

Much has been said of late about closing small isolated plants and transferring the loads to central stations. Aside from the spirit of patriotism, there is one vital question which concerns the smaller company, and, that is, can the larger power plant generate power cheaper than the smaller plant, taking every-

thing into consideration? The smaller plant will have to carry fixed charges on all its investments whether it generates its own power or purchases power from another source. The question then is, can the central station sell power to the smaller plant at a price equal to or less than the cost of operation and maintenance of the smaller plant?

To illustrate, let us consider two plants, one having a maximum hour load of 40,000 kw. and operating at 45 per cent load factor of load, and a smaller plant having a maximum hour load of 10,000 kw. and a load factor of 25 per cent, the maximum hours occurring at different times. To see what would happen when these two load curves are combined, the smaller has been added to that of the larger plant, as shown in Fig. 3. The corresponding cost of operation and maintenance, also fixed charges per kilowatt hour net output at various load factors are shown in Fig. 4. A study of the curves of both Figures 3 and 4 will show that the result of transferring the load of the smaller to the larger plant as far as the total generating cost is concerned, is that, there is a diversity of power station demands of 5000 kw., and that the reduction in total generating cost per kilowatt hour, practically wipes out the investment cost of the smaller plant. While the illustration represents a special case, it is evident that the problem is purely an economic one, which requires careful analysis and, that the decision must be made on the merits of the individual case.

H. M. Hobart: War is such a cruelly sad business that one is reluctant to find any benefits in it. However, as regards this matter of the business of manufacturing and selling electricity, this war has brought about the millenium, or at any rate, has brought about the foundations upon which a substantial millenium can be based. Far seeing engineers, such as Messrs. Ferranti and Merz in England, and Mr. Torchio in America, had long ago explained the situation to us. They had demonstrated to us clearly that if we could have a large enough load of a suitable character the cost of electricity could be halved and halved again, but unless we bring the cost of electricity down to so low a figure as a fraction of a cent, they foretold and figured out clearly, that we cannot hope to get the satisfactory large load. On the other hand, without the large load it is impossible to bring down the cost. What has this tragedy of war done in that respect? It has forced on us the large load of very desirable characteristics.

The authors have, it seems to me, emphasized only one side of the picture. There is a great deal of truth in the side which they portray, and portray so vividly, but there is this other side. The authors do not give even a suggestion that there is this other side, and in the discussion speaker after speaker has echoed their cry of the threatened ruin of the electrical manufacturers. The cost of electricity is based chiefly upon three things: cost of fuel, cost of wages and amount and char-

acteristics of the load. If the cost of fuel is great, it is a handicap; if the cost of wages is great, with relation to the intelligence and activity of those to whom these wages are paid, it is a handicap; but chief of all, for years we have emphasized as the heaviest handicap the smallness of the load. The smallness of the load has been agreed to be the heaviest handicap.

What have the authors done? They have left out the denominator in their paper, and that denominator is the output in kilowatt-hours. We must take into consideration the fuel per kilowatt of output, the wages per kilowatt-hour of output, and the capital and overhead charges per kilowatt-hour of output. The output is going up tremendously. In peace time it would lead us to extravagance. We would buy much more machinery for the generating stations. We now simply cannot. We use to better effect that which we have, and trust to pulling through, and thus reach conditions which enormously decrease the overhead charges per unit of output. The large load also decreases to a slight extent the fuel charge per unit of output, and very largely decreases the wages charge per unit of output. As the years go by, the charge for wages will become less. In every progressive electrical undertaking the managers pride themselves on the increasing utilization of machinery and the decreasing number of men employed per kilowatt-hour of output. The boiler house is an utterly different place today from what it was ten years ago. I consider that all these aspects of the subject should be introduced into the discussion.

There are serious handicaps on which the authors enlarge. But let us turn to England. The authors refer to the situation in London. They say that in an issue of the *London Electrical Review*, the date, of which they do not give, it was stated that there had been a 50 per cent increase in rates. London is an exception—it might well be—there are seventy little stations, equipped years ago, and it is a hard burden on the antiquated machinery in these times. Go to the north of England, the Tyne or the Clyde, or to the Midlands. In undertakings in those districts I should be very much surprised to find that there had been any considerable increase in rates. Any increase in the rates for electricity in these districts will have been very slight if there has been any at all. That is because they are equipped with modern plants. Take the case of the Tyne, dozens of stations all feed into one network, and all sorts of sources of power contribute to a common network. The output is enormously increased by the munitions works that are being put up, and the great activity in shipbuilding and in other lines. For the same reasons an enormous increase in the output is taking place here.

Again, it is only a matter of conjecture, but I think if we were to analyze the output of any large concern manufacturing dynamos or motors, it is very likely we should find three-quarters of the output is for private customers and one-quarter of

the output for the electricity supply corporations. It is that ratio which counts, and while I have not here with me any figures to prove the case that the manufacturers of electricity are exactly in clover, I do maintain that both sides of the situation ought to be equally carefully examined before we arrive at a definite conclusion.

D. C. Jackson: There is one point in the situation which has not been brought out in the discussion, and which is scarcely touched on in the paper. Many years back it was the habit of people to say that that nation would be the greatest which had the greatest supply of coal, lime and iron ore. Now, that is too materialistic to be convincing, in view of the fact that modern civilization rests upon the principle of Christianity, but there is a good deal of truth in it, after all, when it is analyzed. Civilization of the Twentieth century depends upon those characteristic structures which abbreviate space and annihilate time, that is, on transportation and intercommunication, and these do depend upon the railroads and the means of communication, which in turn distinctly depend upon the supply of coal and iron ore, from which steel may be made. I will not argue that point further, because of lack of time, but I will turn directly to the particular and extraordinary circumstances in which we now rest.

We have taken upon ourselves a laboring oar in the most remarkable and the biggest enterprise the world has seen, and it is one that we have entered upon with the idea firmly fixed in our minds (and doubtless it is right) that it is for the welfare of the world and for the betterment of civilization. Under the circumstances, it is wise to analyze exactly where we are at. War in the old days, at one time, consisted of a man and a horse, with an iron housing, and a couple of armorers to take care of it, but it has become an entirely different matter. For instance, in our Civil War, a very critical situation was met by a single instrument, built in record time, as I remember, three months, and that was Ericsson's Monitor, and one or two other such instances may have provided the solution of the situation in the Civil War. But if we look at the matter today, we will see that a single submarine, or a dozen submarines, will not solve the situation for one group of belligerants, nor will a single flying machine, or perhaps ten thousand flying machines, solve it for the other. The problem has become, apparently, a problem of production on a very rapid and at the same time, economic scale;—rapid, that it may meet the requirements of the situation, and economic, so that it may meet the possibilities of expenditure of funds, I mean in regard to the raising of funds, rather than the mere outlay of them.

That brings us directly to the point of the central station as an important factor in production. There is no question about the necessity of our further expanding our industries, and very greatly expanding them, whether the war goes on

for a long time or not, because if we do not expand them we are assured of a long war. The rapidity of expansion of our industries, with the economical production of mechanical power necessary for their welfare is inevitably bound up with the development of the central station to a much larger degree than we are now utilizing in the war, and it is a factor of great importance that our national authorities should take this fully into account, which they have not yet apparently done. It is a part of the duty of electrical engineers in the present juncture to push that particular feature along with the utmost of their enthusiasm and their force.

I will speak of one other point, and that is the shortage of labor. I have been, at a great expenditure of time and mental effort, serving on the District Board which has in Boston the classification of industrial workers with respect to the draft. I have become convinced that there is a very great difficulty in getting satisfactory industrial workers of skill and character who will stay at home and do their work, because such men are full of vigor and full of enthusiasm and are rather likely to desire to go into the great enterprise directly. At the same time a very able and intelligent labor leader said not long ago—"There is no shortage of labor. There are twenty thousand idle hands in the State of Massachusetts." Massachusetts has a population of three million people, and has eight thousand square miles of area. It is not a very great factor in the United States—not so much in those lines, but in some lines it is. I asked him whether he was sure that his statement was correct. He said—"I am sure, but the difficulty is not the number of men, it is the distribution." I told him that I had heard a lot about distribution, that the men were always fifty miles away when you wanted them. He said—"That is not what I mean, but distribution in trades. The large proportion of bricklayers in the State are idle at the present time, but there is a frightful dearth of sheet metal workers." Can not we do something to translate bricklayers, who are skilled men with their hands, and some with their brains, into sheet metal workers. There is a point worthy the consideration of engineers.

P. H. Bartlett (by letter): This paper will undoubtedly be regarded as a notable summing up of the present very unsettled and uncomfortable situation of many of the electric light and power properties. The subject has not only been analyzed quite thoroughly from the viewpoint of the problems of cost and operation confronting the industry, but the arguments contain suggestions and possible remedies looking to the solution of some of these problems. Intelligent discussion or contructive criticism is difficult, however, on questions or problems which have arisen so suddenly and more or less unexpectedly and from which the industry is in many instances still struggling to extricate itself. Changes in policies which have become traditional, reorganization of ideas and the alterations in methods

and practise necessary to quickly meet the conditions arising in the present emergency have left some of us at least more or less at sea as to the present status. The labor question, however, while very much accentuated at this time, is an ever present one, and a few observations on this subject may be timely.

With the larger operating companies, more perhaps than with the smaller companies, the question of labor turnover has been critical. A recent analysis of figures, complied by several departments of a large operating company, for the year 1917, show, in one instance a turnover of 570 employes out of a total of 547 on the payroll at the beginning of the year or 106 per cent with a total of 498 employes at the end of the year or a net reduction of 49. Naturally all of the positions were not affected, some of them being filled once and others several times, the analysis showing that the positions affected by the labor turnover were 270 out of the total of 547, or a turnover percentage of positions affected of 211 per cent. In analyzing this in more detail, there was one group of 90 highly trained technical men in which the labor turnover during the year was 73 or 81 per cent, leaving a net total at the end of the year of 86 employes; the positions affected by the turnover in this case being 39 or a turnover percentage of positions affected of 187 per cent. In another similar group of 96 trained and semi-trained technical men, the turnover during the year was 96 or just 100 per cent, the number of positions affected by the turnover in this case being higher, namely 56 or 172 per cent. Of the 547 employes in these departments on January 1st, 1917, over 100, or approximately 20 per cent, have entered the military or naval or other service of the government, and of the 498 employes on the payroll at the end of the year, 60 are girls who have replaced men, some of them on what might be called semi-technical work.

While the conditions which have created the present unprecedented demand for labor of all classes have also resulted in a decreased activity in certain other directions, which has tended to reduce to some extent the work to be performed by some of the groups covered by the above figures, and while economics in work in other directions have enabled a certain reduction in some divisions of the force, or rather the necessity of filling certain vacancies has been eliminated, nevertheless, there has been the general and usual increase of from 15 to 20 per cent in essential work which must be accomplished. Considering the more or less great effort to maintain these forces, together with the fact that a large percentage of the existing employes are more or less inexperienced, the results are not such as to encourage optimism.

The causes for such an unusually high percentage of turnover are apparently due to several reasons, some of which have been mentioned. For instance:

First: High remuneration or wages paid by other industries.

Second: Military reasons.

Third: Many of the positions are of such a nature that they are difficult to fill even under normal conditions,

Fourth: The generally unsettled condition and unrest among all classes of labor.

Referring to the first reason, while an certain upward revision of salary must be anticipated, it is obvious that the power company whose rates are regulated by law, and were originally predicated on cost of labor, material and operation in general based on normal trade conditions, is unable to pay salaries equivalent to those of other industries, manufacturers and distributors whose rates can fluctuate with and do reflect every rise in the market, nor can they compete in this respect with the very necessary and essential, but what might be termed mushroom industries, born and fostered under the present national emergency and whose life, in many cases, will be equal only to the life of the emergency and who are in many instances working directly or indirectly on government contracts with a guaranteed overhead and a percentage on the cost. Nor would it appear to be wise to attempt to meet apparently inflated rates of wages, which in the event of a downward trend later on would result in expensive and top-heavy organizations, or the undesirable expedient of reductions to again meet normal conditions. The alternative is that the utility must accept in the rank and file, raw material of little or no experience and begin on a larger scale the more or less laborious process of training men for their particular work. This again adds to the percentage turnover, due to the weeding out of undesirable or unfit material, and also in many instances to the fact that as soon as the employee becomes partially trained along some particular line which makes him valuable in some other industry, there is apt to be another vacancy in the force.

Referring to the second cause mentioned, namely, loss of men due to their entering the military, naval or government service, either by volunteering or draft. This has been in some units quite serious, irrespective of the optimistic view taken by our government officials. It has been stated that about 80 per cent of the industrial and agricultural workers of the country are not subject to call, being either above or below the age limit or on account of dependents and that only 3 per cent have thus far been taken from the industrial or agricultural fields, and while granting this, the fact, remains, as before stated, that over 20 per cent of the trained men, with an average of over four years' service in one department of one power company, have already entered the government service—the majority of them volunteers—and many more will be taken in the next draft unless the industrial claims asked for are recognized. Among the reasons for this high percentage of loss, it may be noted that the industry is young and its great growth has necessitated the employment of young men of high character and ability who can be trained for the particular work in which

they are engaged, and these are the character of men who are usually the first to volunteer or who, on account of age, fall within the draft limits. The industry must expect, however, to lose a certain percentage of their forces to the government service and this is not unduly alarming, except perhaps in isolated cases where temporary inconvenience will result, providing that the balance of the forces can be maintained.

The third cause mentioned is merely accentuated under present day conditions. There is probably no type of labor which has to be more carefully selected than employes of public utilities. The work is exacting and responsible, and the duties of quite a large percentage of the total require them to be in more or less close touch with the customer and this applies equally from the office boy who may answer the telephone to the highest class salesman or the maintenance, testing or operating staffs, and when through scarcity of available labor the selection becomes more limited, the turnover is naturally greater as the unfit are weeded out.

The fourth reason, viz., the generally unsettled condition of labor, is particularly accentuated at this time due, no doubt, directly to the war itself and its effects. The wage rates at the present time are probably higher than they have ever been. The laboring class, the organized mechanical trades and in practically every walk of life, except perhaps among the so-called middle class, more money is being earned than ever before and every employee feels the impulse and many are changing their positions or becoming floaters temporarily while the prosperity lasts.

Such a constant hiring, trying out, training or eliminating of new employes results in high cost to the companies which it is difficult to express in dollars and cents, due to error and inefficiency of the new men, to waste of materials, to time and supervision of the older employes necessary in training the new men and to other reasons which cannot be isolated.

Under normal conditions the cost for taking on and training a new employe has been expressed in various companies as varying from $30.00 to $75.00 per man in addition to his salary, and the general inefficiency arising from the introduction of new labor, even when very carefully supervised, is well recognized, but this is, of course, one of the problems that must be met.

The technical employees of the power companies are a highly specialized class and on account of their training and experience are in great demand, and while a reasonable upward revision of salaries from time to time may be necessary, other means must be taken and arguments advanced, as to how these organizations, so vitally necessary at this time, should not only be held together but extended to meet the additional demands that may be made upon them.

The maintenance of these organizations may present more difficulties in those localities where the war industries are center-

ing than in others, due to the enormously increased demand for men, and in such cases the results pyramid in their effect, as the greater the demand for labor the greater becomes the demand for, and the use of, power which can in most cases only be supplied by the power company. That the amount of available labor varies considerably in different cities is evidenced by the fact that an advertisement for young men for the electrical industry in one city produced several hundred replies, from which some very good material was secured, whereas an equally attractively worded and spaced advertisement having an equally wide distribution in another city results in only 25 to 30 replies, the results from which were negligible.

In groping for a remedy for these conditions we have found as yet no formula to follow. Experiments which may seem to offer remedies for particular cases must be tried out, and while the situation may appear to be discouraging, especially in view of the responsibilities resting on these organizations, it is well to remember that we are not facing, at least at this time, any such labor shortage as other countries must have met and managed to overcome.

The use of girls and women, in the event of a very acute shortage of man power is, of course, one of the first thoughts to suggest itself and one of our prominent engineers has recently been quoted as saying that he could, providing that the women would undertake it, perform the actual operation of his plants with female labor, with the exception of the rougher work requiring more or less physical endurance. Women have, of course, invaded the field in many power companies in the past year, but probably only in positions which have heretofore been recognized as open to this class of labor, namely, for clerical or other kindred duties. From information obtained from a number of large companies who have not heretofore employed women and who are now employing them in considerable numbers, and bearing out the statement of the paper this evening, it would appear that it requires four girls to replace three men. In one instance, however, it has been found that six girls on semi-technical work, after they became sufficiently experienced, were able to replace eight men. This was probably due to the small number of girls, permitting a very careful selection, and also due to the fact that the work proved irksome to men who were ambitious and who permitted their attention to be distracted by other more interesting work going on around them. This condition would probably not obtain in many cases, but is one instance where female labor is doing more work and doing it more accurately and at less cost than it was done by men.

The use of boys from the senior classes of Manual Training and High Schools, after school hours, on certain limited classes of work has been tried out with some success and further experiments in this direction are in preparation. The importation

of colored linemen from the South has also met with some success as has also the hiring of inexperienced younger men and placing them under intensive training for some weeks, but these measures, as with others before mentioned, provide only for specific divisions of the work and do not take care of the trained forces so essential.

One operating engineer with whom the matter was discussed stated:

"The plan which I have tried to carry out in order to safeguard against serious troubles with our labor forces has been, briefly, to keep in closest personal touch with my direct assistants and supervising heads. These men are kept satisfied, their troubles either personal or business, are my troubles and I see that a solution is reached in every case that arises which is mutually satisfactory. In other words, I have tried to stay so close to my assistants and their troubles that there is no possibility of any unrest or dissatisfaction however small in the beginning that is not known by me and corrected before it reaches the stage where it would be past correction. On following this plan you will recognize that it only means giving greater attention to a fundamental principle that should govern all successful departmental work.

I recognize the fact that there is a great labor unrest today which must be reckoned with. The unrest is most apparent in the lower classes of labor and among all classes not capable of thinking clearly for themselves. This spirit, however, will spread in any organization of labor, reaching in many cases men who under ordinary conditions would be immune from such influences. As I view it, a department head must start downward in the scale of labor using every means at his command, giving the maximum of his help, advice and spirit to counteract, as far as possible, the spread of unrest and dissatisfaction which reaches upwards from his lowest scale of labor.

Our efforts thus far have been gratifying and I feel confident that our organization is a unit down to the labor class. This class very frankly we cannot hope to satisfactorily cope with and we are using our efforts merely in replacing the men as rapidly as they leave.

My own feeling in the matter is that I know the job that is ahead during the period of this war, and every man I am able to keep in personal touch with knows what I know—that we've got to see it through as gracefully as our abilities and the means at our command permit."

It is obvious that for the effective carrying on of the work and for the maintenance of the great rank and file of the organizations, reliance must largely be placed in the department heads or in the hearts of the men in immediate supervision. The man who can keep in closest personal touch with his men, will in most cases keep the men close to him, in spite of immediate financial considerations.

The necessity of keeping as nearly intact as possible the supervising forces, who can in turn hold a more or less large percentage of the rank and file and thus provide a nucleus for the training of the new men is of paramount importance and this should extend down to the smallest subdivisions of the organization.

The retaining of these supervising forces of power companies, even down to the minor subdivisions, is not such a difficult problem, as these staffs are usually composed of men who have grown up in the service or have been connected with it for many years, and if they are competent and enjoy the confidence of the management, will remain loyal to the industry, and will not be tempted by the offers of other concerns which may exist today and are gone tomorrow.

The importance of vocational training and the retention of and building up of the personnel of the organizations we are discussing should not be overlooked, and in fact would seem to be of greater importance now than ever, and we have before us the example of our own army and the shipbuilding industry which are possibly the greatest examples of vocational training ever undertaken. It may be argued that vocational training is for peace and not for war times, and it has been argued that to attempt to continue vocational training at this time merely results in the utilizing of the services of urgently required men to train others, who when trained are snatched away by other industries. Vocational training must, however, be continued in one form or another and if this training is confined strictly to the experience required by the men to perform the immediate duties for which they are employed, the results cannot fail to be beneficial.

This discussion has shown that the labor turnover in some power companies at least is very great and an endeavor has been made to point out some of the reasons for it and some possible remedies. How far these remedial measures will hold, in the event of a long continuation of the war, with greatly extended drafts for military and other government purposes, and with the further anticipated drafts on labor for war industries, which must grow in proportion to the military establishment, is problematic. Such a condition would manifestly require drastic readjustments, extraordinary effort and revolutionary changes in the direction of energy and these problems must be met as they arise.

W. N. Smith: I want to make one suggestion, prompted by one or two remarks in the early part of the discussion,—that is, the matter of pooling private interests.

We are up against the most fearful emergency civilization has ever known, and drastic readjustments from former practise must undoubtedly be worked out in our country as well as in other countries before the conflict is finished.

In these industrial districts which are served very well in ordinary times by the public service corporations, there are many private corporations owning manufacturing plants of one kind and another, large and small, some very large, which have good sized power plants at their disposal. While this subject is so broad that it can only be touched on lightly in the brief time at my disposal, it does seem to me as though there

were a possibility of commandeering for the benefit of the community certain of the private plants, even if only for service during certain hours of the day, for the helping out of the general power situation.

This will have to be done of course through some state or federal agency which would be clothed with broad powers necessary to exercise jurisdiction over a situation involving such large interests.

If a general arrangement like this could be worked out it will also have the effect of enabling an increase of economy of operation in these private manufacturing plants, many of which use very large quanties of coal. Some of them use it very wastefully because of the fact that their commercial profits are not often largely dependent upon the cost of power, that is to say, if the cost of their power is not a very large percentage of the operating expenses, they are apt to ignore those finer economies in the generation and use of steam, which are a matter of every day thought and discussion among central station men and manufacturers of electrical apparatus.

I have been up against this sort of thing myself in a practical way for the last two years and I know how careless the average private plant operator is apt to be about the consumption of his fuel.

In New Jersey or in the Brooklyn district, or for that matter in any large manufacturing city there are some plants which have a good big outfit of boilers, engines and electric generators that could be turned to practical use to help out the community in time of stress, when the public service corporations cannot meet all of the demands upon them, and it seems to me that it is up to us as engineers, and up to the community in general, to consider such a possibility as that of a pooling of generating machinery in private plants, where the same could be used to help out a shortage in the power usually furnished by public service corporations.

Such a proposition as this would be a suitable field for the exercise of jurisdiction by local Public Service Commissions, but in any event there would have to be some Board of Engineers or experts who after taking a census of the available power equipment in a district would go through everything very carefully as regards the availability of private plant equipment and its application as auxiliary to the central station system. In New Jersey for instance, there have been no electric lights on some of the streets recently. There would no doubt be found a considerable number of power plants which might with a little additional apparatus perform that lighting service and turn loose a considerable amount of additional power besides. A great many of the manufacturers power plants already have veteran trained employees in the boiler and engine rooms day and night, so that the element of labor is already provided.

There are many engineering conbinations possible for dealing with such a situation as now confronts us, which is likely to be a continuing, rather than a temporary emergency. I believe that it will be found possible to utilize existing electrical apparatus by emergency methods readily available, without waiting a year for new steam turbines.

John J. Harold: I have been interested in problems in connection with this war in three continents,beginning with Africa, then Europe, and finally in this country. The labor phases of the situation, in particular, I have been able to understand as I was Secretary of the Witwatersrand Trades and Labor Council for two years and three months, and my views on that aspect of the subject may be of interest to you.

In order to understand the prices of labor and materials rapidly going into a state of unstable equilibrium, let us go back to what happened in Sweden. Sweden exchanged American goods for German gold until the second embargo fell and they found themselves in possession of German gold which they could not eat. European gold has been coming to this country for the purchase of munitions and for the sake of the "easy" money the producers of food and coal started to make shells and fuses, just in such a way as your bricklayers in Massachusetts could be taught to do sheet metal work. Indirectly they were employed by European belligerents who were consuming American supplies, which they had ceased to produce. It will not be very long before they find out that they cannot eat gold and they will begin to wonder why the purchasing power of the dollar is falling so low.

More than this, prices are going up because of the instability of labor—the man secures employment, works a few days, and then will secure a leave of absence to look for another job. He does not break with his last employer until he lands the new position. The best way to combat this proposition is for employers not to employ any one at least without the knowledge, if not with the consent, of his previous boss. After a year of war England passed the Munitions Act, whereby it was illegal for any of the commandeered factories to employ a mechanic who had left his employment in another munitions works without his employer's consent. Such a man had to remain idle for six weeks, before he could take up another job. To the Irish and Colonials, this did not apply. All the Irishman had to do was to go home to Ireland and reengage there for employment in England. I do not imagine that this principle could be applied to the United States, because although the British Government knew the amount of leeway that an English workman could stand, under similar circumstances the Yankee mechanic would take his yacht down to Florida for the prescribed period.

But what could be done is to grant a bonus of ten per cent on all the wages earned for thirteen consecutive weeks, payable at the end of the fourteenth week. This would tend to stabilize the labor turnover.

Now, we come again to the big question of the load factor. I would suggest that the power companies grant a rebate of, say, ten per cent on the price of all current consumed at all hours of the day other than hours of the peak load. In this way, the manufacturer, who must have power at any price, is able to offset the ten per cent increase in wages that he is obliged to pay for night shift work. Although you will not be able to make the peaks disappear, they will, at least, be rendered less noticeable.

Philip Torchio: I would like, before the discussion is closed, to give an answer to Mr. Hobart's question as to the possibility of the cost of power not having really increased. I made some rough figures of the production costs in the last few months of the lighting companies in New York. The production costs, the operating costs alone, increased 95 per cent, and if I include operating costs, plus fixed charges, the total cost increased 50 per cent.

P. R. Moses (communicated after adjournment): It is undoubtedly true that costs have increased and are continuing to increase, but these have been largely offset by an increase in revenue, and the writer believes if the public utility will change its policy in connection with private power plants a sufficient increase of revenue, which will not involve additional capital expenditure, can be obtained to offset the increased cost.

A plan as to cooperation between central stations and private power plants has been suggested by me during the past several years, and in a paper presented before the American Society of Mechanical Engineers on April 5th, 1917, it was pointed out how much less capital would be required to take care of peak loads if the private plants were used to supply this demand than if the peak loads were taken care of by public utility plants.

The utilization of localized private plants supplying electricity and exhaust steam for heating is a ready-to-hand means of meeting the increased demand on the central station while at the same time it affords a method of reducing the fuel required for a stated number of kilowatt hours during the winter season, as the electricity obtained from the private plant during this period is as is well known practically a by-product of the heating.

The low cost of the private power plant installation allows taking care of the peak load at barely one-sixth the cost of taking care of this load by increases in the central station including feeders, substations, etc.

There is already available large percentages of spare capacity in existing power plants which by proper cooperation with the central station could be fully utilized, for the purpose of taking care of peak load demands.

I am of the opinion, however, that under present fuel and labor conditions, during such periods of the year when exhaust steam may not be fully utilized and during light load periods, during six or seven months of the year in this latitude, private electrical plants should be shut down, and the current derived

from a central source, the public utility company being required to supply this current at a price which will enable the consumer to profitably purchase, and at the same time cover the expense of supply plus a reasonable profit, but without allowance for depreciation on plant or other fixed charges on investment, because supplying the offpeak and summer load as suggested would not require additional investment by the public utility company, except to a very minor degree for meters and connections, and this might be properly charged to the individual consumer, thus avoiding any investment by the public utility company.

It is evident from the published figures of the large public utility plants that such off-peak load could be profitably supplied at from one and one-half to two cents per kilowatt hour in cities like New York, and at a much lower cost where overhead transmission and high voltages are available.

If our public utilities will take up this question in the submitted outline of cooperation with the view to utilizing the private plants to their utmost economic extent, which means to use them during the period when fuel and labor can be doubly used, they would greatly benefit themselves, enormously improve their load factor, increase their revenue, and what is of far greater importance, effect an immense reduction in the amount of fuel burned.

Discussion at Chicago

R. F. Schuchardt: We will all agree, I am sure, that the situation as set forth so ably by Mr. Jackson is of considerable importance to all of us, and to engineers and financiers especially. To a central station man the paper reads very much like a description of the condition in his own company, which merely illustrates that the situation is pretty universal throughout the country. Lest, however, all of you here rush to the stock market to unload your utility securities tomorrow, we should take into consideration that there is also a bright side. Mr. Jackson has pointed out one of the most important of the brighter sides, which is the widespread recognition of the real value to our country of this economic industrial development, the central station, and a general knowledge of the benefits of centralization must react advantageously to the industry.

Also, we are now putting more of our investment to use, or in other words, we are using our reserve to earn money. This might be expressed as living on our fat. We must live on the fat, as we can't get additional financial food, for reasons which Mr. Jackson has pointed out. Also, the public mind is getting somewhat used to everything going up in price or else coming down in quality, and so it may be easier to satisfy the kicker, where kickers are numerous, which of course is not the case in our industry.

For the engineer, however, there seems to me to be a very bright side. It is true there is a good deal more work required

of the engineer at this time, but he who at this time doesn't want to do more work than he did before is a pretty sorry sort of a man. In fact, it is a sort of compensation for those of us, who are not privileged to take a more active part in the war, to be able to carry some of the usual burdens of those who do go.

Also the engineer must now use more resourcefulness. Instead of following precedents, he must set new ones. This should arouse renewed enthusiasm and enable him to increase the quality of his work. The engineer, instead of thinking more particularly of kilowatts, must now think more of the relations between those kilowatts and the dollar.

F. A. Coffin: An important condition facing all of us is the growing difficulty of financing extensions to meet new customers. We are rapidly approaching a condition where only such new capital expenditures are justified as are directly necessary to the prosecution of the war. It is probable that in addition to the deterring influence of the high cost of obtaining new money we will have strict regulatory action by federal authorities. All of this vitally affects expenditures for line and service extensions which have hitherto been an inseparable part of the process of attaching new business. This necessitates a radical revision of rules with respect to line extensions and requires that during these times new customers will be required to finance at least the difference between normal cost and present abnormal costs in supplying them service.

The second aspect of importance is economy in the conduct of the sales department. I do not share the speaker's pessimism that the war leads to a curtailment of the use of electric service. Early this year we dispensed with all but the nucleus of our sales organization and it is of interest to note that the increased business which we hoped to achieve by the end of the year through the usual sales methods has all come to us unsolicited. The high cost of fuel has no doubt made many industrial enterprises more ready to inquire for electric power supply and a part of the business gained has been secured through the momentum of past sales efforts. But, even discounting the effect of the fuel shortage, there is still a normal increase in the demand for service.

The third aspect to which I desire to direct attention is the question of character of service. The user of electric energy in America receives more in auxiliary service rendered by the Engineering departments, trouble departments, lamp renewal department, etc., for the rates he pays, than the customer in any other country. We have naturally taken great pride in the completeness of the service we furnish, but there is no gainsaying that it imposes a very substantial expense which is probably not justified in these times. The Food Administrators are now recommending extra charge for delivery of parcels from other mercantile institutions and there is no reason why similar modification should not be made in our rules with respect to delivery

of lamp renewals, appliances, etc., or that the amount of gratuitous assistance given by engineering departments should not be gauged by the necessities of the situation rather than a mere endeavor to accommodate the customer.

Finally, and by no means the least important duty of the sales department, lies in co-operation with the fuel conservation program of the Government. Lightless nights perform an admirable function of impressing upon all customers the necessity of conservation and no doubt the sum total of little economies effected will help the nation to save coal. But we would be deceiving ourselves as well as the public if we lost sight of the fact that the actual amount of fuel in the power house saved by this method is indeed very small, being a mere fraction of one per cent of the total coal consumed by the central station and an infinitesimal part of the coal utilized by the entire community. There is much to be done besides enforcing and thoroughly co-operating with the rule with respect to lightless nights. Perhaps the most important economy which can be achieved would be the organization by the fuel administrator of a voluntary inspection service for industrial plants, charged with making definite, practical recommendations on the proper insulation of steam pipes, satisfactory maintenance of boiler settings to avoid air leakage, scale elimination in boilers, care in banking and firing boilers, avoidance of general illumination of unoccupied factory space, possible savings effected in shutting down machinery when not required, etc., and the general elimination wherever possible of the non-condensing plant. In such a work the commercial engineering department could do effective work. It is not necessary for me to point out that such assistance by the central station is a labor of patriotism. We are naturally not interested in laboring to make the isolated plant more efficient. Our interest in normal times is to shut it down and sell it service. But the necessities for fuel conservation are so great and pressing and the possibilities in this field are so substantial that no effort should be spared to cooperate to the greatest extent.

D. W. Roper: Some years ago in preparing a paper on line transformers for the N. E. L. A., and attempting to justify the use of lightning arresters for the protection of transformers, I had some difficulty, and in order to find a sufficient amount on the correct side of the balance sheet which I drew up for the purpose, I had to go into the question of continuous service and place a value on interruptions to service, that is, on the amount which the company would probably care to spend to prevent an interruption. In doing this I consulted a number of department heads around the company and found a very wide range of opinions, varying from something more than the loss of revenue to 100 times the loss of revenue. That, of course, is a very wide figure, but placing, as I remember it, a value of about fifteen or twenty times the loss of revenue as the value

to the company of the prevention of interruption, then the cost of lightning protection for preventing that interruption was warranted.

Since that time I find that the company's estimate of the value of an interruption has undergone some very wide changes with the times. For example, in the most prosperous times we have had, say in the last ten years, the company had been willing to extend their lines to reach the new customer when the estimated revenue from the customer over a period of say two years would cover the investment required for line extension. And at the same time where there was an equivalent amount of business already on the line or a group of old customers the company would be willing to incur a reasonable expenditure for a reserve supply, if that customer or group of customers would warrant it. And in those most prosperous times a large number of such connections were made so as to insure continuity of service.

But nowadays if customers want service from the Edison Company, they either must be on the lines or their revenue must be sufficient in one or two months, instead of two years, to pay for the cost of the extension. That shows immediately how hard they are struggling for new business, if that is the word to use. By the same sign you can gather how much they are doing in the way of providing for reserve connections in the case of the failure of the original supply. That is to say, they are doing nothing along that line at all.

In the most prosperous times, if there was any trouble with your lights or motors the company would, on request, send their trouble man to your premises. If the interruption to your lights was caused by some minor defect in the wiring or in the appliances or the sockets or plugs or switches, the repair man would, if he could do it in a half hour or so, make the repairs for you, sometimes temporarily, but he would make the repairs so as to enable you to get service, and then he would advise the customer what to do in the way of making permanent repairs. Now the company makes a minimum charge of thirty-five cents for sending a repair man to the premises, if the trouble is found to be on your premises. In addition, a further charge is made for any time over the first fifteen minutes. So the initial cost is thirty-five cents for going, and the regular rate for any time used beyond the fifteen minute limit. That applies also to heating appliance repairs in apartment buildings.

There was once a sort of perpetual guarantee on the cords for electric flat irons. The practise now is to charge the customer for the cost of repairs. That is about seventy-five cents, I believe, for the replacing of an electric iron cord. And the same applies to various other devices, toasters, broilers, etc. Those were all repaired if it could be done by the repair man, but not including, of course, the replacing of the heating apparatus.

Lamp bulbs have for a number of years been delivered free.

Of course the ordinance requires that they be furnished free, but it is not required that they be delivered free, and it is probable that within the next few weeks the company will make a charge for the delivery of them similar to that charged for answering a trouble call which I have just described. But there will of course be the option that the customer can get his own lamps free upon applying to one of the numerous delivery stations which the company will establish in various parts of the city.

On the question of station extensions some of you may have noticed the suggestion made by Mr. Budd of the Elevated Railroads to a Commission in Washington some weeks ago. His suggestion was, in effect, that the working hours of various lines of industry be adjusted so they do not all begin and end their working hours at the same time, requiring a very sharp transportation peak at that time, and that if the hours of starting and quitting could be spread over three or four hours in the

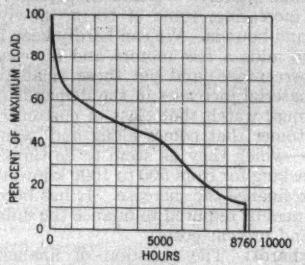

FIG. 5—ANNUAL LOAD CURVE OF COMMONWEALTH EDISON COMPANY.

morning and three or four in the afternoon instead of only one or one and one-half hours, then the peak in labor as well as the peak in power requirements could be very materially reduced.

Some years ago in making studies of the load of the Edison Company I prepared some annual load curves, which were of the form illustrated by Fig. 5. Any point in this curve shows the total number of hours during the year that a particular load has been carried. At that time the upper portion of the curve was investigated in considerable detail and it was found that the upper 25 per cent of the station capacity produced about one-half of 1 per cent of the kilowatt-hour output during the year. From this we can conclude that if Mr. Budd's suggestion could be put into effect so as to spread out the railway peak there would probably be no great difficulty in reducing the peak of the railway load by 25 per cent and if this were done, then the generating station supplying this railway load would probably not need any extensions during the period of the war.

***W. S. Gorsuch** (by letter): For text of this discussion see page 34.

R. F. Schuchardt: With relation to the fuel it must be remembered that the generating cost is not the entire cost and that when fuel increases 70 per cent the total cost per kilowatt-hour delivered is not increased by this amount. The fixed charges on the investment are often quite equal to the generating costs; in other words, if the generating stations were to be shut down and all employes were to be dismissed, about one-half of the cost would keep right on.

***F. A. Bryan** (by letter): For text of this discussion see page 22.

***R. G. Hudson** (by letter): For text of this discussion see page 23.

***M. Freund** (by letter): For text of this discussion see page 25.

C. W. Pen Dell: We men who have sales work find that our chief duty at present is to ward off those industries that are wanting additional power. We find at present that all of our customers are increasing, some of them slowly and some of them by leaps and bounds. We can hardly keep from supplying a customer who adds a few motors and increases his demand only a small percentage, and yet those small percentages are making very material increases in the demands on some of our lines, and we must watch that class of customers very closely. The larger customer that comes along and wants service for a steel furnace—which they all seem to be thinking of at the present time—asking for from 500 to 1000 kilowatts in capacity can be asked to finance the increase. It has become quite general that customers be required to finance the difference between pre-war costs and present costs.

R. F. Schuchardt: The question of financing extensions, is a very interesting one. Most companies require some form of advance deposit either equal to the first two months' bills, as mentioned by Mr. Roper, or to an amount on some other basis as has been described. But the line is only a part of the extension. The point is reached sooner or later when the substation back of the line is loaded to the full extent, or the generating station has no more reserve. Then this curve of Mr. Roper's serves as a new inspiration to the power salesman to go out and try to fill in, not at the peak, but in that big valley of low load. The resultant improvement in load factor, as has been indicated is one of the very beneficial effects which the situation is bringing about for the central station. You are all, of course, acquainted with the fact that in England interconnection of adjoining systems is being promoted by the Government.

C. A. Keller: Recently I have had occasion to make a study of increased values of generating stations and substations on account of present high prices and some of the figures probably

*Discussions presented at both New York, January 11, and Chicago, January 14.

would be of interest to you, particularly since they bear out some of the statements of Mr. Jackson.

	Percent increase in value today as compared with normal pre-war times.	
	Individual	Weighted Average
Generating Stations:		
Buildings	35	45
Boiler equipment	50	
Turbines and auxiliaries	55	
Electrical	35	
Substations:		
Buildings	35	30
Electrical	20	
Storage Batteries	50	
Line Extensions:		
Conduits and tunnels	30	50
Poles, cross arms and pins	60	
Transmission line conductors	50	
Overhead distribution conductors	80	
Underground distribution conductors	50	
Services	80	
Transformers:	30	30
Total Weighted Average		43

These figures are of course very approximate; the pre-war costs being based upon varying market prices during the fourteen years preceding the war.

F. H. Bernhard: But Mr. Jackson has not pointed out a great many simple remedies. He does mention a few in which governmental action might be of advantage, including the possibility of increasing the rates. But in regard to the matter of increasing the rates, while that matter has been taken up in England to the extent of increases ranging from thirty to fifty per cent, a great many American central station companies have, and are doing their utmost to prevent horizontal increases in rates. In the central station business propaganda it has been a very strong argument to show that central station service has been steadily decreasing in cost in comparison with steady increases in costs for living and other expenses, and a desperate effort is being made to continue that downward trend. It is impossible to say whether practically all companies may not have to show an upward price curve before they continue for any length of time. However, Mr. Roper has pointed out quite a few features

that may be not only considered but actually carried out, such as the employment of women and disabled soldiers.

This curve of Mr. Roper's is especially interesting, and it seems to me that the entire electrical industry should be actively boosting anything that will help to spread out the peak load, because it must be remembered that it is not the central station industry alone that is going to be benefited by spreading out the peak load, there are the electric railways, the traction lines. We have had in the past week a very forceful example of the importance of the urban transportation lines maintaining means of traffic—some sort of traffic—in the community.

Congestion usually existing during the rush hour periods has been very bad. During last week it was much worse due to heavy snow diverting all traffic to the street car tracks. Isn't it possible, not through merely gentle suggestions, but possibly from a concerted effort through bodies like the Association of Commerce and even through the City Council, to bring about a realization of the importance of staggering hours of employment? There are a great many business houses, factories and department stores closing at 5:30, between that and 6:00. So why can't we spread that time out at least two hours, thus relieving the peak load on all means of transportation.

R. F. Schuchardt: I am astonished that more of the gentlemen that have discussed the paper have not mentioned the daylight saving plan. One of the incidental effects of the war is the fact that the economies on which the central station business is built are also being appreciated in their application to other business.

John W. Mabbs: The great item in the production of power is fuel. The question up to us is how to solve the fuel problem. The cause for the present excessive price of fuel is given by the coal man as shortage of cars, inability of the railroads to transport the coal and the high price of labor.

The Government has taken over the railroads and by strict, judicious, and energetic business methods the thousands of cars, both loaded and empty, that usually stand for indefinite periods of time on the sidings could be kept moving and would provide ample facilities for transporting and handling all the coal required. This would furnish the cars necessary to run the mines full time or even overtime, instead of half or two-thirds time as at present.

If the excuse of the coal man, of shortage of cars and transportation is eliminated, he comes back with the excuse that he cannot get the necessary labor. As the Government has the right and the power to take over the railroads and draft men to go into the trenches, it also has the right and authority to take over the coal mines, *the country's natural resources*, and draft the men to work in them.

This would put a stop to a most outrageous profiteering in one of the necessities of life, and would not only go far to solve the power question but would be an untold benefit to every man, woman and child in this country.

The natural resources of the country should be conserved and operated for the benefit of all the people, and not for the enrichment of a few.

A. Honegger: The existing war conditions abroad and subsequent war emergencies at home have created enormous political and economical changes in our industries, the effect of which as regards public utilities has been eminently portrayed in the paper presented by Mr. William B. Jackson. The facts as they stand have been brought before us in concrete form and complete in most all details, and the results show clearly the urgent necessity for taking measures to meet the unusual conditions.

In the discussion of the paper most speakers have endeavored to seek solutions in proposing means of obtaining economies by improving efficiencies and by charging for incidental services not directly employed in producing and delivering electrical energy. All of these will no doubt have some bearing on the question and will bring some results to the satisfaction of the consumers as well as of the operator.

However, we must be aware of the fact that the operating percentage, the part of the gross revenues which is expended for fuel, labor and supplies, has increased so tremendously that the net has on the average been reduced to a point where the payment of the interest on actual moneys invested has reached an alarmingly close limit. This seems to me the important point brought to our attention by the paper and in part by the discussion.

This condition can only be remedied by an adjustment of rates, by an increase of the units of the gross revenue.

There are existing a number of important institutions, created for the purpose of adjusting rate questions and executive bodies establishing rates for the safeguarding of both the consumer and the operator, the operator referred to being the financiers and managers of public utilities. All of these have made great endeavors in the past to solve the individual problems to the best interest of the parties they represent, but today we are confronted with a far different situation which must be treated from a different standpoint and which must receive a just solution.

Our industries must keep on providing the fixed interest rate on moneys invested, if we shall avoid calamities of national importance and this can only be done by increasing light and power rates. The justice of increasing rates can clearly be illustrated by facts and it is up to the managers to come to the foreground with these facts and approach the institutions having jurisdiction over rates for just increases. Difficulties will no doubt be encountered, but the presentation of facts will surmount these obstacles. The present war has broadened the minds of all clear thinking Americans in a remarkable degree and this will not only help our cause, but should spur those of us most greatly affected to immediate and concerted action. A result must be obtained whereby public utilities will be placed

on the basis of all other industries, at least to the extent of producing revenues adequate to paying interest rates on actual moneys invested.

Considerable action has been exhibited in different sections of the country and to my knowledge results have been obtained in certain communities in Iowa and Nebraska, where City Councils and Boards of Trustees have granted increases in rates on the basis of increased costs. A fair way of fixing rates in this newly created period is to have rates established to slide on a scale with increased fuel costs. In the same manner there have been contracts made for selling power in bulk to large consumers and power companies. This will bring public utilities more on the basis of other industries which have the privilege of increasing their selling prices in accordance with rise of raw material prices with the exception that adjustments made and sanctioned by institutions above mentioned will safeguard the acknowledged interests of the consumers prefixedly in case of public utilities.

I suggest that action be taken on a large scale; the facts demand immediate alterations if we want to avoid further losses and amend the present precarious status.

A further point of importance brought to our attention by the paper and discussions is the effect on provisions for improvements and extensions. The funds of the nation are required for the successful carrying out of the war and in this no doubt all public utilities and their fiscal agencies have heartily co-operated with the National Government and will continue to do even more so in the future. This of course, has curtailed the possibilities of larger extensions and new undertakings. It would be unpatriotic to stimulate such undertakings while the funds are needed in the great National Undertaking. Nevertheless there are certain immediate improvements and extensions which have been sanctioned as necessities, and while the ordinary channels for the supply of funds for such purposes have been fairly well throttled, it may be well to look for new sources of supply for provisioning and financing such authorized improvements. These will be found in localities and occupations mostly benefited economically by war activities and it is of interest to certain public utilities to note that recently the consumer in farming communities and also communities with war industries has become an investor in securities. Is it then not logical to seek funds for immediate and necessary improvements by selling securities to your consumers and tie over until conditions become normal; this will no doubt be beneficial and provide a relief to the extent of the possible activities achieved in this direction. It will also add a friend in every consumer who owns your local securities and establish friendly relationship much needed and sought for.

W. B. Jackson: You will feel that I have painted a gloomy picture, but I wish to say that I am an optimist with the rest of you, because as expressed in the earlier of my remarks, I

have no doubt whatsoever, that this situation will be worked out with fine results from the standpoint of all.

Referring to Mr. Coffin's remarks, I am much interested in what he had to say. In the course of his valuable remarks he says that he does not agree with the statements in the paper relating to the economies in use of current to be expected, but I am inclined to believe that he did not catch the intent of the paper in this, namely, that our residence and ordinary commercial customers will normally and very properly economize in the use of current. I cannot imagine a residence customer today not doing his best to economize in the use of current, as compared with what he would have used two or three years ago; nor of a man using commercial service not making endeavor to economize such as he has never made before.

I have recently seen a concrete case where such economies have been clearly shown in both lighting and small power load. The curve of total current used per annum, taken for the end of each succeeding month, showed a fair increase in total current consumed on account of increase in number of customers, but the use per customer showed a decrease. This was beautifully shown by the fact that the average amount paid per kilowatt hour showed an increase, which meant no small decrease in the amount of current taken per customer.

From Mr. Gorsuch we might gather the impression that the increase in coal cost is the only important factor to be considered but from our analysis of the situation we find that the increase in the cost of coal is only approximately a half of the total increase. It is therefore seen that to obtain any true visualization of the situation as a whole it is necessary to take into consideration all of the items involved, as we have done.

With reference to the reduction of peak loads as compared with the average loads; that is one of the things that we must naturally expect will be done. It would seem that this can only be accomplished through government action, so far as present customers on the lines are concerned, but you can readily see how the valleys will be filled if the non-essential industries are required to keep off of the peaks but permitted to operate during the valleys.

In this paper we have taken the situation as nearly as we can as it stands today, and have pointed out the lines along which the changes are likely to come,—the beneficial changes and the detrimental changes. We have not endeavored to predict far into the future. My hope is that we have accomplished what is needed by such a paper, namely, that we have given you a visualization of the situation, so that you can appreciate how serious it is and how important it is for us to support and help those who are responsible for the correct solution of the problems involved. If we do get behind them, we can be just as confident that these problems will be solved satisfactorily, as we are that we will come out of the war victorious.

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THE TECHNICAL STORY OF THE FREQUENCIES

BY B. G. LAMME

ABSTRACT OF PAPER

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This covers, to a certain extent, the merits and demerits of each frequency as then understood, and the reasons each one met certain pressing commercial conditions at the time it was brought out. This is followed by a discussion of various alternating-current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

The persistent developments of the designing engineers gradually overcame the limitations in various types of 60-cycle apparatus so that eventually the 60-cycle system in its application approached more and more closely to the 25-cycle and, in the end, has taken the lead.

The outcome of the battle of the frequencies was determined far more by the conditions in the operating field than by the exploitation of any particular system by designing engineers. As a consequence, the energies of the engineers were directed exclusively toward overcoming the defects and limitations of the systems and not expended in fighting each other.

THE STORY of how and why the various commercial frequencies came into use and then dropped out again, in most cases, is not primarily the story of the frequencies themselves, but of the various uses to which the alternating current has been applied.* In other words, fundamental changes in the application of alternating current have led to radical changes in the frequencies. Some of the applications which have had a determining factor on the frequency of the supply system are as follows; incandescent lighting, transformers, transmission systems, arc lighting, induction motors, synchronous converters, constructional conditions in rotating machinery, and operating conditions. A brief consideration of these items individually,

*It should be distinctly understood that this paper covers only the story of American development.

from the present viewpoint, indicates that while some of them had, at one time, very considerable influence in determining frequency conditions, yet, in a number of cases, the original reasons have disappeared through improvements and refinements, as will be described later.

At various times the following standard frequencies have been in use in this country, namely, 133⅓, 125, 83⅓, 66 2/3, 60, 50, 40, 30 and 25 cycles per second. These did not appear chronologically in the order given above, and a few odd frequencies in a few special applications are omitted.

In the following, the various frequencies will be considered more or less in the order of their development and basic reasons will be given for their choice, and the writer will endeavor to show why certain of them have persisted, while others have dropped out. It will also be shown why the commercial situation has first tended strongly toward certain frequencies and afterwards swung toward others.

133 and 125 Cycles

In the earliest alternating work, the whole service consisted of incandescent lighting, and the electric equipment was made up of small high-speed belted single-phase generators and house-to-house distributing transformers. As the transformers were of small capacity and as their design was in a very crude state, it was believed that a relatively high frequency would best meet the transformer conditions. A choice of such an odd frequency as 133⅓ cycles per second, is due to the fact that in those early days (1886 to 1893) frequencies were usually designated in terms of alternations per minute. One of the earliest commercial generating units constructed by the Westinghouse company had a speed of 2000 rev. per min. and had eight poles. This presented a fairly convenient constructional arrangement for the surface-wound type of rotating armature, which was the only one recognized at that time. The speed of 2000 rev. per min., with eight poles, gave 16,000 alternations per minute, or 133⅓ cycles per second according to our present method of designation. Thus the earliest frequency in commercial use in this country was fixed, to a certain extent, by constructional reasons, although the house-to-house transformer problem apparently indicated the need for a relatively high frequency. The Thomson-Houston company adopted a standard frequency of 15,000 alternations per minute, (125 cycles) instead

of the Westinghouse 16,000, but the writer does not know why this difference was made. However, the two frequencies were so close together that practically they could be classified as one.

At this time, it should be borne in mind, there were no real transmission problems, no alternating-current arc lighting, no induction motors and the need for uniform rotation of the generators was not recognized. The induction motor, in its earliest stages, came in 1888 and considerable work was done on it in 1889 and 1890, but it required polyphase supply circuits and comparatively low frequency and, therefore, it had no connection whatever with the then standard single-phase, 133⅓ and 125-cycle systems. The synchronous converter was also unheard of (one might say almost undreamed of) at that time.

60 Cycles

In 1889 or 1890 it was beginning to be recognized in this country that some lower frequency than 125 and 133⅓ cycles would be desirable. Also about this time direct-coupled and engine-type alternators were being considered in Europe and it was felt that such construction would eventually come into use in America. It was appreciated that in such case, 133⅓ cycles would present very considerable difficulties compared with some much lower frequency, due to the large number of poles which would be required. For instance, an alternator direct driven by an 80-rev. per min. engine would require 200 poles to give the required frequency and such construction was looked upon as being practically prohibitive. About this time Mr. L. B. Stillwell, then with the Westinghouse company, made a very careful study of this matter of a new frequency, in connection with the possibilities of engine-type generators, and after analyzing a number of cases, it appeared that 7200 alternations per minute (60 cycles per second), was about as high as would be desirable for the various engine speeds then in sight. Transformer constructions and arc lighting were also considered in this analysis. While it was deemed that a somewhat higher frequency might be better for transformers, yet a lower frequency than 60 cycles was considered as possibly better for engine-type generators. A compromise between all the various conditions eventually led to 60 cycles as the best frequency. However, while this frequency originated about 1890, it did not come into use suddenly, for it was impossible to introduce such a radical change in a brief time. Moreover, the direct-coupled

or engine-type generator was slow in coming into general use and, therefore, there was not the necessity for the introduction of this low frequency in many of the equipments sold from 1890 to 1892. However, by 1893, 60 cycles became pretty firmly established and was sharing the business with the 133⅓-cycle systems. It should be borne in mind that, at this time, the adoption of this frequency was not considered as a direct means for bringing forward the polyphase induction motor, for the earlier 60-cycle systems, like the 125- and 133⅓-cycle, were all single-phase. Also, it was then thought that the polyphase motor would possibly require a still lower frequency and, moreover, the polyphase system was looked upon as in a class by itself, suitable only for induction motor work. At that time the introduction of polyphase generators for general service was not contemplated. This followed about two or three years later.

In 1890 the Westinghouse company, which had been developing the Tesla polyphase motor, laid aside the work, largely on account of there being no suitable general supply systems for this type of motor. The problem was again revived in 1892, in an experimental way, with a view to bringing out an induction motor which might be applied on standard frequencies such as could be used in commercial supply circuits for lighting and other purposes. It should be understood that at this time such circuits were not in existence but were being contemplated. In 1893, after the polyphase motor had been further developed up to the point where it showed great commercial possibilities, the best means for getting it on the market were carefully considered. It was decided that the best way to promote the induction motor business was to create a demand for it on commercial alternating-current systems. This meant that, in the first place, such systems must be created. Therefore, it was decided to undertake to fill the country with polyphase generating systems, which were primarily to be used for the usual lighting service. It was thought that, with such systems available, the time would soon come when there would be a call for induction motors. In this way experience would be obtained in the construction and operation of polyphase generators and the operating public would not be unduly handicapped in the use of such generators, compared with the older single-phase types.

An early example of this new practise was in the 2000-kw. polyphase generating units used for lighting the Chicago World's Fair in 1893. Here the single-phase type still persisted, as each

generator unit was made up of two similar frames placed side by side, but with their single-phase armatures displaced one-half pole pitch from each other so that the combined machine delivered two single-phase currents displaced 90 degrees from each other. It was considered that each circuit could be regulated independently for lighting service, and polyphase motors could be operated from the two circuits. These generators (at that time the largest in this country) were designed in 1892 and were of 60 cycles. These, therefore, indicate the tendency at that time toward lower frequency and polyphase generation, although commercial polyphase motors were not yet on the market.

25 CYCLES

At the same time that 60 cycles was selected as a new standard it was recognized that at some future time there would be a place for some much lower frequency, but it was not until two years later that this began to narrow down to any particular frequency. In 1892 the first Niagara electrification, after several years consideration by eminent authorities, had centered on polyphase alternating current as the most desirable system. The engineers of the promoting company had also worked out what they considered the most suitable construction of machine. This involved 5000-h. p. units at 250 revolutions per minute. Prof. George Forbes, one of the engineers of the company had furnished the electrical designs for a machine with an external rotating field and an internal stationary armature. His design used eight poles, thus giving 2000 alternations per minute, or $16\frac{2}{3}$ cycles per second. Quite independently of this, the Westinghouse company, in 1892, had been working on the development of synchronous converters, using belted 550-volt d-c. generators with two-phase collector rings added. The tests on these machines had shown the practicability of such conversion and had even proved at this early date, that the converter copper losses were much lower than in the corresponding d-c. generators. Thus it is an interesting fact that the first evidence of this important principle was obtained from a shop test rather than by calculation. The writer, from an analysis of the tests, which were made under his immediate direction, concluded that the armature copper losses must be considerably lower than in the same machine used as a d-c. generator. He also brought the matter to the attention of Mr. R. D. Mershon, then with the Westinghouse company, and the problem was then worked out mathematically by him

and the writer, in two quite different ways, but with similar results, showing that the converter did have actually very much reduced copper losses.

As a result of this work of the Westinghouse company on the synchronous converter, it was decided that, to make such machines practicable, some suitable relatively low frequency was required. This appeared to be about 30 cycles. About this time the construction of the Niagara generators was taken up with the Westinghouse company to see whether it would construct these machines according to the designs submitted by the promoting company's engineers. These designs were gone over as carefully as the knowledge of such apparatus, at that time, permitted, and many apparent defects and difficulties were pointed out. The Westinghouse company then proposed, as a substitute, a 16-pole, 250-rev. per min. machine (the speed being definitely fixed at 250 rev. per min.). This gave $33\frac{1}{3}$ cycles or as near to the Westinghouse proposed 30-cycle system, as it was possible to get. Then many arguments were brought forward, pro and con, for the two machines and frequencies. Prof. Forbes' preference for $16\frac{2}{3}$ cycles was based partly on the possibilities it presented for the construction and operation of commutator type motors, just as with direct-current circuits. The Westinghouse contention was that this frequency was too low for any kind of service except possibly commutator type machines. Tests were made with incandescent lights and it was found that at $33\frac{1}{3}$ cycles there was little or no winking of light, while at $16\frac{2}{3}$ cycles, the winking was extremely bad. Tables were also made up, showing the limited number of speed combinations at $16\frac{2}{3}$ cycles for induction motors, in case such should come into use. This showed how superior the $33\frac{1}{3}$ cycles would be as regards such apparatus. It was also brought out that synchronous converters, when such became commercial, would be much better adapted for the higher frequency, as the choice of speeds would be much greater. From the present viewpoint the arguments appear to have been much in favor of the Westinghouse side of the case.

As a consequence of all this discussion the suggestion was advanced by some one, that a 12-pole, 250-revolution machine, (that is, 3000 alternations, or 25 cycles), might meet sufficiently the good qualities of both of the proposed frequencies and would thus be a good compromise. In consequence a 12-pole, 25-cycle machine was worked up by the Westinghouse company and

eventually this frequency was adopted for the Niagara generators. Afterwards, while these generators were being constructed it was brought out pretty strongly that the great advantage of this frequency would be in connection with synchronous converter operation, but that it was also extremely well adapted for slow-speed engine-type generators, which were then coming into use. In consequence of the prominence given this frequency it was soon adopted as a standard low frequency, especially in those plants where synchronous converters were expected to form a prominent part of the system.

However, while 60 and 25 cycles came into use, as described above, it must be recognized that they had competitors. For instance, $66\frac{2}{3}$ cycles (8000 alternations or one-half of 16,000) was used to a considerable extent by one of the manufacturing companies. Also 50 cycles came into use in certain plants and, to a certain extent, is still retained, but has become the standard high frequency of Europe. Instead of 25 cycles, the Westinghouse company advocated 30 cycles for some of its plants, largely because with the 20 per cent higher speeds permissible with such frequencies, the capacities of induction motors could be correspondingly increased and also incandescent lighting was more satisfactory. However, it was soon recognized that the $66\frac{2}{3}$ and 30-cycle variations from the two leading frequencies of 60 and 25 cycles were hardly worth while, and they were gradually dropped, except in plants already installed. A brief attempt was made at a somewhat later period to place 40 cycles upon the market as a substitute for both 25 and 60 cycles. This was done under the impression that 40 cycles would give a universal system for arc and incandescent lighting, transmission, induction motors, synchronous converters and about everything else. This frequency possessed many merits and it was thought, at one time, that it might win out, but apparently the two other frequencies were too well established, and the 40-cycle system eventually lost ground.

The problem of the frequencies finally narrowed down to the two standards, and these two were accepted because it was thought that they covered such entirely different fields of service that neither of them could ever expect to cover the whole. In other words, two standards were required to cover the whole range of service. It was recognized that 25 cycles would not take care of alternating-current arc lighting and that it was questionable for incandescent lighting in general. In other ways,

such as suitability for engine-type construction, application to induction motors and synchronous converters and transmission of power to long distances, it met the needs of an ideal system, as then understood. Also, in parallel operation of engine-type alternators, which was one of the serious problems of those days, the 25-cycle machines were unquestionably superior to the 60-cycle ones, due to the lesser displacement of the e. m. f. waves with respect to each other with a given angular variation in the engine speeds. However, although the 25-cycle system presented so many advantages, it could not take care of the lighting business, and, therefore, could not entirely dominate the situation.

As regards 60 cycles, it was felt that this could handle the direct lighting situation in a very satisfactory manner and was possibly better suited for transformers than 25 cycles, although there were differences of opinion in this matter, especially when it came to the larger capacities. It was reasonably well adapted for induction motors in general, but not for very low speeds. In matters of transmission and in the operation of synchronous converters it was thought to be vitally defective.

From the above consideration it would appear that the 25-cycle systems presented the stronger showing as a whole and, therefore, there was a decided tendency toward this frequency, except in those cases where lighting directly from the alternating-current system was considered of prime importance. In those systems, such as many of the Edison companies, where low-voltage three-wire direct current was used from synchronous converters, the tendency was almost solidly toward the 25-cycle system. In those days the central station, which had been committed to the 60-cycle system so deeply that it could not change, was looked upon with commiseration. Sixty-cycle plants were looked upon, to a certain extent, as a necessary evil. In fact, so strong was the tendency toward 25 cycles that in many cases 25-cycle plants were installed for industrial purposes, where 60 cycles would have been better. The 25-cycle synchronous converter development advanced by leaps and bounds and the machines were so good in their operation that it was believed that 60-cycle converters could never be really competitive with them.

On the other hand, in those large plants, which were so "unfortunate" as to have 60 cycles installed, many apparent makeshifts were adopted to meet the various service requirements.

In arc lighting, incandescent lighting, transformers and motors there was no need for makeshifts. However, in conversion to direct current, one of the greatest difficulties appeared. There were many who advocated motor-generators for this purpose, largely because the 60-cycle converter was thought to be impracticable, in spite of the fact that the manufacturing companies were putting them on the market. The 60-cycle converter at that time bore a bad name. It is now recognized that many of the faults of the early 60-cycle synchronous converter operation were not in the converters themselves, but were, to a considerable extent, in the associated apparatus. Low-speed engine-type, 60-cycle generators were not always adapted for operation of synchronous converters. In fact, in numerous cases such generators would not operate in an entirely satisfactory manner in parallel with each other, and yet when it was attempted to operate synchronous converters from these same generators the unsatisfactory results were not blamed upon the generating system but upon defects of the converters themselves. Unfortunately, defects in the generating and transmission systems usually appeared in the converters as sparking and flashing, and such troubles naturally would be credited to defects in the construction of the converters themselves. In fact, in those days, 60-cycle converters were expected to do things which now are considered as absurd. For instance, in one case in the writer's knowledge a 60-cycle synchronous converter was criticized as being a very badly designed piece of apparatus, due to serious flashing at times. Investigation developed that this converter was expected to operate on either one of two independent 60-cycle systems with no rigid frequency relation to each other. The converter in service was thrown from one system to the other indiscriminately, and sometimes it flashed in the transfer and sometimes it did not. The machine was considered to be "no good" because it would not always stand such switching.

At one time the writer stood almost alone in his belief that the 60-cycle synchronous converter presented commercial possibilities sufficient to make it a strong future contender with the 25-cycle machine, provided proper supply conditions were furnished and certain difficulties in the proportions of the converter itself were overcome. One basis for his contention was that in some of the 60-cycle plants, where the generator rotation was quite uniform, the converters were evidently much superior in their operation to other plants, using low-speed engine-type

generators with considerable periodic variations. In such plants the hunting tendency of the converters was very greatly reduced, with consequent improvement in sparking and general operation. It was early recognized that hunting was a very harmful condition, both in 60- and 25-cycle synchronous converters, but whereas it was a relatively rare condition in 25-cycle plants it was much more common with 60 cycles. However, the operating public was not particularly concerned whether the trouble was in the generating plant or in the converters themselves, as long as such trouble existed and was not overcome. Very early in the synchronous converter development it was found that hunting would produce sparking or flashing at the commutators of the converters. However, even in those plants where there was no hunting apparent, there was difficulty at times due to flashing, especially with sudden change of load, which resulted in temporary increase in the d-c. voltage. This was a difficulty which was inherent in the converter itself and could not be blamed entirely upon the generating or transmitting conditions, for 25-cycle machines were practically free from this trouble under similar conditions of operation. Investigation developed the fact that this flashing trouble was due largely to unduly high value of the maximum volts between commutator bars. This difficulty was recognized long before it was overcome, simply because certain physical limitations in construction had to be removed. There were two ways in which the maximum volts per bar could be reduced, namely, by increasing the number of commutator bars per pole and by decreasing the ratio of the maximum volts to the average volts per bar, that is, by increasing the ratio of the pole width to the pole pitch, but both of these involved structural limitations in the allowable peripheral speeds of the commutator and the armature core. Here is where a little elementary mathematics comes in. The peripheral speed of the commutator is directly proportional to the distance between adjacent neutral points on the commutator, and the frequency. Therefore, with a given frequency the distance between the adjacent neutral points is directly proportional to the peripheral speed. Thus, with a commutator speed of 4500 ft. per min. which was then considered an upper limit, the distance between adjacent neutral points on a 60-cycle converter is only 7½ in. (19 cm.) This distance is thus fixed mathematically and is independent of the number of poles or revolutions per minute, or anything else, except the peripheral

speed and the frequency. With this distance of $7\frac{1}{2}$ in., (19 cm.), about the only choice in commutator bars per pole was 36, giving an average of $16\frac{2}{3}$ volts per bar on a 600-volt machine, and nearly 20 volts per bar with momentary increase of voltage to 700, which is not uncommon in railway service.

However, it is not this average voltage which fixes the flashing conditions, but it is the maximum voltage between bars, and this is dependent upon the average voltage and upon the ratio of the pole width to the pole pitch. Here is where one of the serious difficulties came in. The pole pitch is directly dependent upon the peripheral speed of the armature core and the frequency. Therefore, in a 60-cycle machine, if the peripheral speed is fixed, the pole pitch is practically fixed. For example, with an armature peripheral speed of 7200 ft. per min., (which was considered high at that time,) the pole pitch becomes 12 in. (30.48 cm.), regardless of any other considerations. and here was where a most serious difficulty was encountered. If a sufficiently wide neutral zone for commutation was allowed the interpolar space became so wide that there was not enough left for a good pole width. For instance, if the interpolar space was made 6 in. (15.24 cm.) wide, in order to give a sufficiently wide commutating zone to prevent sparking or flashing, due to fringing of the main field, then this left only 6 in. for the pole face. With this relatively narrow pole face the ratio of the maximum volts to the average volts was so high that with the 36 commutator bars per pole the machine was sensitive to arcing between commutator bars, thus resulting in flashing. By widening the pole face this particular difficulty would be lessened or overcome, but with the fixed pole pitch of 12 in. (30.48 cm.) the neutral zone would be so narrowed as to make the machine sensitive to sparking and flashing at the brushes. Thus, no matter which way we turned we encountered trouble. Obviously there were two directions of improvement, namely, by increasing the number of commutator bars, thus reducing the average voltage, and by increasing the pole pitch. thus allowing relatively wider poles with a given interpolar space. These two conditions look simple and easy, but it took several years of experience to attain them. When we have reached apparent physical limitations in a given construction, especially when such limitations are based upon long experience, we have to feel our way quite slowly toward higher limitations. For instance, in the case of the 60-cycle converters we could not boldly jump our

peripheral speeds 20 to 25 per cent higher and simply assume that everything was all right. We first had to build apparatus and try it out for a year or so. Troubles, due to peripheral speed, do not always become apparent at once, and thus time tests are necessary. Therefore, while the peripheral speeds of the 60-cycle synchronous converters were actually increased 20 to 25 per cent practically in one jump, yet it took two or three years of experimentation and endurance tests before the manufacturers felt sure enough to adopt the higher speeds on a broad commercial scale. Thus, while the change from the older more sensitive type of 60-cycle converter to the later type occurred commercially within a comparatively short period, yet the actual development covered a much longer period.

Let us see now what an increase of 25 per cent in the peripheral speeds actually meant. As regards the commutator, the number of bars could be increased 25 per cent, that is, from 36 up to 45 per pole, which was comparable with ordinary d-c. generator practise. In the second place, an increase of 25 per cent in the peripheral speed of the armature core meant a 15-in. (38.1-cm.) pole pitch, where 12 in. (30.8 cm.) was used before. Assuming, as before, a 6-in. (15.24-cm.) interpolar space, then the pole face itself became 9 in. (22.8 cm.) in width instead of 6 in. (15.24 cm.) or an improvement of 50 per cent. In fact, this latter improvement was so great that some manufacturers did not consider it necessary to increase the number of commutator bars, although in the Westinghouse machines both steps were made.

The above improvements so modified the 60-cycle converter that it began to approach the 25-cycle machine in its general characteristics. It was still quite expensive compared with the 25-cycle, due to the large number of poles, and its efficiency was considerably lower than its 25-cycle competitor, on account of high iron and windage losses. However, due to the need for such a machine it was gradually making headway, in spite of handicaps in cost and efficiency.

Almost coincident with the initiation of the above improvements in the 60-cycle converter, came another factor which has had much to do with the success of this type of machine. This was the advent of the turbo-generator for general service. As stated before, one of the handicaps of the 60-cycle converter was in the non-uniform rotation of the engine-type generators which were common in the period from 1897 to about 1903 or 1904. But, about this latter date, the turbo-generator was making

considerable inroads on the engine-type field and within a relatively short period it so superseded the former type of unit, that it was recognized as the coming standard for large alternating power service. With the turbo-generator came uniform rotation and this at once removed one of the operating difficulties of the 60-cycle converters. However, in the early days of the turbo-generator, 25 cycles still was in the lead and many of the earlier generators were made for this frequency, especially in the larger units. But it was not long before it was recognized that 60 cycles presented considerable advantage in turbo-generator design due to the higher permissible speeds. In the earlier days of turbo-generator work, this was not recognized to any extent, as the speeds of all units were so low that the effect of any speed limitations was not yet encountered. For instance, a 1500-kw., 60-cycle turbo-generator would be made with six poles for 1200 revolutions, while a corresponding 25-cycle unit would be made with two poles for 1500 revolutions. This slightly higher speed at 25 cycles about counterbalanced the difficulties of the two-pole construction compared with the six-pole. However, before long, more experience enabled the 1200 r.p.m., 60-cycle machine to be replaced by 1800 revolutions, and a little later by two pole machines at 3600 revolutions. This, of course, turned the scales very much in the other direction. In larger units, however, the advantage still appeared to be in favor of 25 cycles, but in the course of development, 1500 revolutions was adopted quite generally for 25-cycle work, and this was the limiting speed, as such machines had only two poles, or the smallest number possible with ordinary constructions. On the other hand, for 60 cycles, 1800 revolutions was adopted quite generally for units up to almost the extreme capacities that had been considered, consequently the constructional conditions in the large machines swung in favor of 60 cycles. Therefore, with the coming of the steam turbine and the development of high-speed turbo-generator units, the tendency has been strongly toward 60 cycles. This, with the greater perfection of the 60-cycle converter, had much to do with directing the practise away from the 25 cycles.

However, there were other conditions which tended strongly toward 60 cycles. In the early development of the induction motor, the 25-cycle machines were considerably better than the 60-cycle and possibly little or no more expensive. However, as refinements in design and practise came in, certain important advantages of the 60-cycle began to crop out. For instance,

with 25 cycles there is but little choice in speed, for small and moderate size motors. At this frequency a four-pole motor has a synchronous speed of only 750. The only higher speed permissible is 1500 revolutions with two poles, and it so happens that in induction motors the two-pole construction is not materially cheaper than the four pole, consequently the principal advantage in going to 1500 revolutions was only in getting a higher speed where such was necessary for other reasons than first cost. However, in 60 cycles the case is quite different, where a four-pole machine can have a speed of 1800 revolutions, synchronous, a six-pole 1200, an eight-pole 900 and a ten-pole 720 revolutions. In other words, there are four suitable speed combinations where a 25-cycle motor has practically only one. Moreover, with the advance in design it developed that these higher speed 60-cycle motors could be made with nearly as good performance as with the 25-cycle motors of same capacity, and at somewhat less cost. However, leaving out the question of cost, the wider choice of speeds alone would be enough to give the 60-cycle motor a pronounced preference for general service.

However, there is one exception to the above. Where very low-speed motors are required, such as 100 rev. per min., the 60-cycle induction motor is at a considerable disadvantage compared with 25 cycles, or this has been the case in the past. It is partly for this reason that the steel mill industry, through its electrical engineers, adopted 25 cycles as standard some ten or fifteen years ago. At that time, it was considered that in mill work, in general, there would be need for very low-speed motors in very many cases. However, due to first cost, as well as other things, there has been a tendency toward much higher speeds in steel mill work, through the use of gears and otherwise, so that part of this argument has been lost. However, there still remain certain classes of work where direct-connected very low-speed induction motors are desirable and where 25 cycles would appear to have a distinct advantage.

In view of the above considerations, steel mill work has heretofore gone very largely toward 25 cycles, particularly where the mills installed their own power plants. However, in recent years there has been a pronounced tendency toward purchase of power, by steel mills, from central stations, and the previously described tendency of central stations toward 60 cycles has forced the situation somewhat in the steel mills, particularly in those cases where the central power supply company can furnish

power at more reasonable rates than the steel mill can produce in its own plant. This, therefore, has meant a tendency toward 60 cycles in steel mill work, even with the handicap of inferior low-speed induction motors. But, on the other hand, remedies have been brought forward even for this condition. The great difficulty in the construction of low-speed, 60-cycle induction motors is in the very large size and cost if constructed for normal power factors, or the very low power factor and poor performance if constructed of dimensions and costs comparable with 25 cycles. In the latter case the extra cost is not entirely eliminated because a low power factor of the primary input implies additional generating capacity, or some means for correcting power factor on the primary system. However, in some cases it is entirely practicable to correct the power factor in the motors themselves by the use of so called "phase advancers" of either the Leblanc or the Kapp type. Such phase advancers are machines connected in the secondary circuits of induction motors and so arranged as to furnish the necessary magnetizing current to the rotor or secondary instead of to the primary. In this way the primary current to the motor will represent largely energy and the power factors can be made equal to, or even much better than in, the corresponding 25-cycle motor; or, in some cases, the conditions may be carried even further so that the motor is purposely designed with a relatively poor power factor, in order to further reduce the size and cost, and the phase advancers are made correspondingly larger. In those cases where the cost of the phase advancer is relatively small compared with the main motor, there may be a considerable saving in the cost of the main motor and then adding part of the saving to the cost of the phase advancer.

One difficulty in the use of phase advancers is found in the variable speeds required in some kinds of mill work. In those cases where flywheels driven by the main motors are desirable to take up violent fluctuations in load, it is necessary to have considerable variations in the speed of the induction motor, in order to bring the stored energy of the flywheel into play. Unfortunately this variable speed in the induction motor is one of the most difficult conditions to take care of with a phase advancer, so that here is a condition where the 60-cycle motor is at a decided disadvantage.

Thus it may be seen from the above that even in the steel mill field, where the induction motor has the most extreme applications, there is quite a strong tendency toward 60 cycles, due to the purchase of power from central supply systems.

There remains one more important element which has had something to do with the tendency toward 60 cycles, namely, the transmission problem. In the earlier days of transmission of alternating current, 25 cycles was considered very superior to 60 cycles due to the better inherent voltage regulation conditions. At one time, it was thought that 60 cycles had a very limited field for transmission work. However, a number of power companies in the far west had installed 60-cycle plants, principally for local service and with the growth of these plants came the necessity for increased distance of transmission through development of water powers. At first it was thought they were badly handicapped by the frequency, but gradually the apparent disadvantages of their systems were overcome and the distances of transmission were extended until it became apparent that they could accomplish practically the same results as with 25 cycles. Part of this result has been obtained by the use of regulating synchronous condensers. It is a curious fact that the possibility of synchronous motors used as condensers for correction of disturbances on transmission systems, has been known for about 25 years, but it is only within quite recent years that they have come into general use as a solution of the transmission problem, and largely in connection with 60-cycle plants. In 1893 the writer applied for a patent on the use of synchronous motors as condensers for controlling the voltage at any point on a transmission system by means of leading or lagging currents in the condenser itself. A broad patent was obtained, but there was no particular use made of it until it had practically expired.

Another improvement came along which still further helped to advance 60 cycles to its present position, namely, the use of commutating poles in synchronous converters. The principal value of commutating poles in the 60-cycle converters, has not been so much in an improvement in commutation over the older types of machines, as in allowing a very considerable reduction in the number of poles with corresponding increase in speed, resulting in reduction in dimensions. As a direct result of this increase in speed the efficiencies of the converters have been increased. If, for instance, the speed of a given 60-cycle converter can be doubled by cutting its number of poles to one-half, while keeping the same pole pitch and the same limiting peripheral speed, then obviously the amount of iron in the armature core is practically halved and, at the same magnetic densities the iron loss is also practically halved. Also with the same

peripheral speed and half diameter of armature the windage losses can be decreased materially. Thus the two principal losses in the older converters have been very much reduced. There have also been reductions in the total watts for field excitation, and in other parts, so that, as a whole, the efficiency for a given capacity 60-cycle converter has been brought up quite close to that of the corresponding 25-cycle machine, even when the latter is equipped with commutating poles. This gain of the higher frequency compared with the lower is due to the fact that the lower-frequency machine was much more handicapped in its possibilities of speed increase, and furthermore, the iron losses and windage represented a much smaller proportion of the total losses in the low-frequency machine. This improvement in the efficiency of the 60-cycle converter together with the lower losses in the 60-cycle transformer as compared with the 25-cycle, has brought the 60-cycle equipment almost up to the 25-cycle, so that the difference at present is not of controlling importance. This development has given further impetus toward the acceptance of 60 cycles as a general system.

Formerly a serious competitor with the 60-cycle converter was the 60-cycle motor-generator. This was installed in many cases because it was considered more reliable and more flexible in operation than the synchronous converter. Both of these claims were true to a certain extent. However, with improvements in the synchronous coverter the difference in reliability practically disappeared, but there remained the difference in flexibility. In the motor-generator set, the d-c. voltage could be varied over quite a wide range, while in the older 60-cycle rotaries the d-c. voltage held a rigid relation to the alternating supply voltage. However, with the development and perfection of the synchronous booster type of converter, flexibility in voltage was obtained with relatively small increase in cost and minor loss in economy. This has been the last big step in putting the 60-cycle converter at the front as a conversion apparatus, so that today it stands as the cheapest and most economical method of converting alternating current to direct current. Moreover, while the 25-cycle synchronous converter has apparently reached about its upper limit in speed, there are still possibilities left for the 60-cycle converter.

In line with the above it is of interest to note that for units of 1000 kw. and less, the 60-cycle converter has nearly driven the 25-cycle out of business from the manufacturing standpoint.

For the very large size converters, 25 cycles still has the call, but largely in connection with many of the railway and three-wire systems, which have been installed for many years; that is, the growth of this business is in connection with existing generating systems. However, the 60-cycle converter, in large capacity units, is gaining ground rapidly and it is of interest to note that the largest converters yet built, namely, 5800 kw., are of the 60-cycle type.

One most interesting point may be brought out in connection with the above described "battle of the frequencies", namely, it was fought out in the operating field, and between conditions of service, and not between the manufacturing companies. This is a very good example of how such matters should be handled. Here the engineers of the manufacturing companies were expending their efforts to get all possible out of both frequencies, and consequently development proceeded apace. When 60-cycle frequency seemed to be overshadowed by its 25-cycle competitor, the engineers took a lesson from the latter and proceeded to overcome the shortcomings of the former. It was no innate preference of the designing engineers that has brought the higher frequency to the fore; it was the recognition that it had greater merits as a general system, if its weak points could be sufficiently strengthened; and, therefore, the engineers turned their best efforts toward accomplishing this result.

It must not be assumed, for a moment even, that because 60 cycles appears to be the future frequency in this country, that 25 cycles was a mistake. Decidedly it was not. In reality it formed a most important step toward the present high development of the electric industry. Many things we are now accomplishing with 60 cycles would possibly never have been brought to present perfection, if the success of the corresponding 25-cycle apparatus had not pointed the way. The success of the 25-cycle converter, and the high standard of operation attained, gave ground for belief that practically equal results were obtainable with 60 cycles. Therefore, the 25-cycle frequency served a vast purpose in electrical development; it was a high class pacemaker, and it isn't entirely out-distanced yet.

There has been considerable speculation as to what two standard frequencies would have met the needs of the service in the best manner, and would have resulted in the greatest development in the end. It has been claimed by some, that 50 and 25 cycles would have been better than 60 and 25. In the earlier

days possibly the former would have been better, but as a result both standards might have persisted longer. In any case, the general advantages would have been small. In one class of machines, namely, frequency changers, consisting of two alternators coupled together, the 25-50 combination would certainly have been advantageous.

Again it has been questioned whether 30 and 60 cycles would not have been a better choice. This was the original Westinghouse choice of frequencies, but not on account of frequency changers. As stated before, it was felt that 30 cycles could do about all that 25 cycles could, and would give an advantage of 20 per cent higher speed in motors and converters, with correspondingly higher capacities. Also for direct coupled alternators, the two-to-one ratio of frequencies would fit in nicely with engine speeds, in many cases. Possibly, from the present viewpoint, the choice of thirty cycles, would have longer retained the double standard.

Something further may be said regarding the 40-cycle system, brought out by the General Electric Company. This contained many very good features, for the time it was brought out. It was then believed that if the 60-cycle frequency was retained, the double standard was necessary. The 40-cycle system was an attempt to eliminate this double standard. It apparently furnished a better solution than 60 cycles then promised for the synchronous converter problem, and was a fair compromise in about everything else. But it came too late, for the 25-cycle system was too firmly entrenched, and for further development, the designing engineers preferred to expend their energies in seeing what could be accomplished with 60 cycles, as this seemed to present greater possibilities than either 25 or 40, if it could be sufficiently perfected. Thus the 40-cycle system probably missed success due to being just a little too late.

As to 50 cycles, it was stated that this is still in use to a limited extent. Most of the 50-cycle plants in this country are in California. Such plants were started during the nebulous period of the frequencies, and have persisted, to a certain extent, partly because certain 60-cycle apparatus could be easily modified to meet the 50-cycle requirements. Also, as 50 cycles is the standard in many foreign countries to which this country exports equipment, the use of 50 cycles in some home plants has not been unduly burdensome from the manufacturers' standpoint.

In addition to the preceding, there have been certain classes

of electric service which have depended upon frequency, but which have not been a determining factor in fixing any particular frequency. Among these may be considered commutating types of a-c. apparatus. The first a-c. commutating motors of any importance, which appeared, were, of course, the 25-cycle, single-phase railway motors. These as a rule have operated from their own generating plants, or from other plants through frequency-converting machinery. One exception in the railway work may be noted in the use of 15 cycles on the Visalia plant in California. There is a pretty well defined opinion among certain engineers experienced in such apparatus that some low frequency, such as 15 cycles, would present very considerable advantages in the use of single-phase railway motors in very heavy service, such as on some of the western mountain roads. Here the problem is to get the largest possible motor capacity on a given locomotive, and the main advantage of the lower frequency would be in allowing a very materially higher capacity within a given space. This does not imply reduced weight or cost compared with the 25 cycles, but simply means greater motor capacity. With the modern, more highly developed, single-phase types of railway motors, it would appear that there may be very considerable possibilities in 15 cycles.

Outside of the railway field, there has been more recently a development of various types of a-c. commutating apparatus, principally in connection with heavy steel mill electrification work. Such apparatus has been largely in the form of three-phase commutating machines and these have been used principally in connection with speed control of large induction motors. As these regulating machines are usually connected in the secondary circuits of induction motors, the frequency supplied is represented by the slip frequency. Consequently where the slip frequency never rises to a large percentage of that of the primary system, such commutating motors are applicable without undue difficulties. Such motors, presumably are better adapted for 25-cycle mill equipments than for 60-cycle, but due to the tendency, already described, for steel mills to go to 60 cycles on purchased power, it has been necessary to build these three-phase commutating motors for the regulation of 60-cycle main motors, in many cases.

There is still another class of service, which has come in recently, where the choice of frequency is of much importance, but where there is no great necessity for adhering to any standard,

namely, in heavy ship propulsion by electric motors. As each ship equipment is a complete system in itself, and as it cannot tie up with other systems, there is not any controlling need for maintaining any definite frequency or voltage. Except in similar vessels, there is little chance for duplication in parts, as the various equipments vary so much in size and capacity. In consequence it has been found advisable, at least up to the present time, to design each propulsion equipment for that frequency which best suits the generator and motor speeds, taking into account the various operating conditions and limitations, such as the different running speeds, steaming radius, etc. In consequence, different maufacturers bidding on such equipments may specify different frequencies, depending upon the constructional features of their particular types of apparatus. At the present time with the relatively small amount of experience obtained with the electrical propulsion of ships, it looks as if it would be a considerable handicap to attempt to adopt some standard frequency for all service. Later, with wide experience, it may be possible to adopt some compromise frequency, which will not unduly handicap any of the service.

Conclusion

It has been the writer's intention to show that, as a rule, the choice of frequency has been a matter of most serious consideration, based upon service conditions at the time. Moreover, in view of the wide range of conditions encountered, it is surprising how few frequencies have been seriously considered in this country. Occasion has arisen, times without number, where an obvious solution of a given problem would lie in modification of the frequency to allow the use of apparatus and equipment already designed, but the engineers of the manufacturing organization have steadily held out against such policy, regardless of the apparent need of the moment. The swing of the pendulum from 60 cycles to 25 cycles and back, has covered a period of many years and, therefore, cannot be considered as a fad of the moment, but is the result of well defined tendencies, backed by the best engineering experience available. As a rule no manufacturer has made any particular frequency his "pet," but all have worked to develop each system to its utmost.

DISCUSSION ON "THE TECHNICAL STORY OF THE FREQUENCIES" (LAMME), WASHINGTON, D. C., JANUARY 18, 1918.

H. B. Brooks: Mr. Lamme's instructive paper appeals strongly to those of us who have followed the development of the a-c. art from the beginning.

In addition to the early frequencies of 133 and 125 cycles, I recall that the Slattery a-c. system, made by the Fort Wayne Jenney Electric Light Co., was operated at 140 cycles. That was the day of the "system;" each inventor who designed an alternator felt in duty bound to design a complete system, including generators, transformers, station instruments, switches, cutouts, sockets, and in some cases the lamps also.

Mr. Lamme has not referred to the attempt which was made at an early stage of the art to limit the frequency to a single value, namely, zero. I refer to a rather short but bitter attempt by the advocates of d-c. distribution to have the alternating current outlawed on the ground of its alleged great risk to life.

It may be of interest to glance at the frequency situation abroad. In Italy, which is poor in coal but rich in water power five frequencies are in use, of which 42 cycles is in the lead, with 50 cycles a close second. Around Rome, 46 cycles is used, and there are also some 16 and 25-cycle installations throughout the country. The Italian Electrotechnical Association is planning a campaign to standardize frequencies.

London is probably the worst example of the independent growth of small detached generating systems with no thought of possible future interconnection. Merz and McLellan studied the situation and made a report in 1914 to the London County Council, from which it appeared that there were in the central area of London 41 generating stations representing 31 systems and 8 frequencies. The report advocates the adoption of 50 cycles as a standard frequency for London.

In contrast to the medley of plants and frequencies in London, the cities of Hamburg and Berlin each have electric supply from a single company. The frequency of the Berlin system is 50 cycles. The business in Paris is practically all in the hands of a single company, operating at 25 and 42 cycles.

A single standard frequency would have great advantages. It would permit the interconnection of transmission systems wherever this might be expedient. This question of interconnection is an important one in England at the present time, as the demands for power for war industries have overloaded some systems which are not readily able to tie in with others for mutual assistance.

A standard frequency would benefit the manufacturer by reducing the number of designs, patterns, and stock of generators, transformers, motors and meters. The user would benefit from the consequent reduction of cost, and would be able to move equipment from one locality to another without finding it inapplicable to the new supply conditions.

From the standpoint of export trade, it would be desirable to have an international standard frequency. While the attainment of this ideal appears to be far off, it is a matter which may well be kept in mind by the national engineering societies and the International Electrotechnical Commission, in order that so far as possible the trend of standardization in the different countries may be along converging lines.

I would like to ask Mr. Lamme whether the higher peripheral speeds which have greatly contributed to the success of the 60-cycle converter are made possible by the use of materials of greater tensile strength, or by improved methods of utilizing the materials heretofore available.

B. G. Lamme: The improvements have been almost entirely in the design of the machines. We are using practically the same materials that were used in the older machines, but we are proportioning the parts in such a way that they are better sustained, that is, we know more about the stresses and how to take care of them.

G. S. McCumber: I have one question that I wanted to ask. Mr. Lamme referred to the chief disadvantage of the 60-cycle system, I assume that he intended increased voltage drop and the lower power factor, but I wanted to know also whether there was any disadvantage due to such other factors as additional difficulties in insulation, corona losses or anything of that kind.

Also one other question. Reference was made to the fact that in heavy ship propulsion a variable frequency system was employed, and I wonder if he would state for us what the standard frequencies or the normal range frequencies are on which that system was based.

Robert Dalgleish: Mr. Lamme in speaking of 60-cycle converters brings to my mind the question as an operating engineer—the problem of the 25-cycle converter, which most all of us are now using. The question arises, is the 60-cycle converter in the high voltages, such as 600, in railway operation as successful as it is in the lighting voltages; and is the possibility of the future of the railway turbo-generators—and naturally the converter—to be 60-cycle instead of 25? I can appreciate that the companies who are at present equipped with 25-cycle turbo-generators are naturally going to stick to that. With the question of future installations, where the installation is entirely new, will it be an installation of 60-cycle or 25-cycle turbo-generators?

Mr. Hunt: I would like to ask Mr. Lamme what the present limit of size of the 60-cycle rotary converter is, and what are the chief limitations in design that makes that size the limit.

B. G. Lamme: Before I answer any of the questions brought up, I want to bring out more clearly what it meant to the manufacturing companies to adhere to a standard frequency. You may not appreciate, unless you have been in the manufacturing business, how easy it is to change the frequency, in many cases.

For, instance suppose a manufacturer has built a 60-cycle, 3300-volt machine at 150 r. p. m. and somebody comes along and wants a 100-r. p. m., 2200-volt machine of somewhat less capacity. The 60-cycle machine already constructed may fit the new conditions in everything except frequency. If a frequency of 40-cycles would be acceptable, then the old machine could be used without any change, but it would introduce a special frequency. Thousands of cases have come up where we could take some existing machine and meet the customer's condition, if it were not for the frequency. I have seen many cases where an existing machine was all right to meet new conditions except that it would have two poles too many or too few, and it looked so easy to make a minor change in the frequency instead of in the machine. In such cases, however, if the customer could not change his speed conditions, the manufacturer would change his design rather than modify the frequency.

There has been an enormous amount of money spent in making such changes, but the engineers of the manufacturing companies feel that they have really spent less money in making them than they would have spent in the end if they had allowed changes in frequency; for every change, no matter how small, means confusion in the end. The engineers of the manufacturing companies have been stubborn on that matter and for a good reason. That is why we have had so little variation in the frequencies in this country.

It has been asked whether the 60-cycle converter in the higher voltages, such as 600 for railway service, is as successful as it is in the lighting voltages. In reply to this I will say that, as the tendency for the last few years has been largely toward 60 cycles in new plants, which may be called upon to handle railway service as well as lighting, it has been necessary to develop the 60-cycle converter for 600-volt railway service up to the point where it compares quite favorably with similar 25-cycle machines. This is indicated by the fact that below 1000-kw. capacities there is but relatively small business in 25-cycle converters; whereas, there is quite a good business in 60 cycles. Above 1000 kw. there is a very strong tendency also toward the 60-cycle converter. In fact, while the 25-cycle rotary converter business is quite large in the units above 1000 kw., yet such machines are sold very largely to 25-cycle plants already installed, or to extensions of such plants.

As to the limit of capacity for which 60-cycle converters can be built, one answer is that the real limit is in the pocketbook of the customer. Such machines can be built of almost any capacity, if there is a demand for them. We could make 10,000-kw., 60-cycle converters today, if anybody wanted them, just as we could make 10,000-kw., 25-cycle machines. The largest 60-cycle converter is of about 5500 kw. compared with 4000 kw. for 25 cycles. Therefore, the 60 cycle is already ahead of the

25 cycle, as far as maximum capacity is concerned; but from the quantitative standpoint the large 25-cycle converters are still ahead because there were so many 25-cycle generating plants installed in the past entirely for railway work through rotary converters.

As to Mr. McCumber's questions regarding the relative disadvantages of voltage drop, insulation difficulties, corona losses, etc., in 60-cycle transmission, I will say that I am not an authority on transmission difficulties, but I understand that all the above troubles were quite serious in the early days, and that much doubt was expressed regarding the possibilities of maintaining satisfactory voltage conditions at the end of a 60-cycle line. Apparently they feared poor regulation more than any other feature. Possibly some of the difficulties in the early times worried the engineers more in the prospect than in the realization, for, apparently, when they came to operate such 60-cycle lines, they did not have as much trouble as was expected and the regulation trouble was overcome in a number of cases by the addition of synchronous condensers.

I remember, not so many years ago, that all of us pitied the Californians, for their adoption of 60 cycles as a general standard, in transmission work. It was thought that they had made a serious mistake, but now they are right in line, although they probably didn't foresee it. They had committed themselves to 60 cycles and they couldn't very well change.

As to the basic frequency used for ship propulsion, as such are variable frequency systems, it may be said that there is no definite frequency except for the highest speed which the apparatus will stand. A number of the battleship equipments have used 35 cycles for the full speed condition with variation from this down to half value or even less. On some of the other vessels as high as 50 and 60 cycles are used. The choice of frequency has been largely dependent upon the motor-speed combinations required.

Presented at a meeting of the San Francisco Section of the American Institute of Electrical Engineers, San Francisco, Cal., January 25, 1918.

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CORONA TESTS AT HIGH ALTITUDE

BY B. F. JAKOBSEN

ABSTRACT OF PAPER

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000.

The method employed in the tests is described and the calculations are given. The corona losses were found by subtracting the core and copper losses of the transformers and the line copper losses due to the line charging currents from the total loss measured. Core-loss tests were made on the transformers in order to check these against the shop tests. The highest voltage experimented with was 72,300 when a corona loss of 153 kw. was found, or a little over two kw. per mile of line.

The results are compared to "Peek's law" and a wide divergence found, amounting to over 800 per cent at about 66,000 volts and a loss of 58 kw.

Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line, and the equation of the loss-voltage curve obtained is the same as was established by Faccioli, and shows this relation to be logarithmic and not quadratic for the voltage range investigated.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather and this shows no appreciable difference from tests made during fair weather.

INTRODUCTION BY FRANK G. BAUM

There has been installed in Peru under the writer's direction a hydroelectric plant having about 120 mi. (193.1 km.) of transmission line, running from 12,000 to 14,000 ft. (3,657.6 to 4,267.2 m.) elevation, with most of the line at about 13,000 ft. (3,962.4 m.).

I considered this an excellent opportunity to make some actual corona tests at high altitude, as these would be of value to the profession. The results are shown in the report of a test on 68.4 mi. (110 km.) of three-phase line by Mr. B. F. Jakobsen, electrical engineer.

The results are encouraging, in that the actual losses are less than expected, and as this fact influences to some extent

the size of wire to be chosen, it is seen that the matter is of practical importance to the electrical profession.

Probably there is no line which has more severe lightning conditions than this one. And no doubt the high altitude and consequent light air pressure allows atmospheric discharges that would not occur at lower altitudes for the same cloud potential conditions, for the same reasons that we have corona at high altitude for less voltage than at low altitude.

While not directly related to corona it may be of interest to know that the transmission has been freer from lightning troubles than anticipated. One line about 90 mi. (144.8 km.) long is operated at 44,000 volts, Y-grounded, (this line was formerly delta operated as mentioned in the test report) and another line about 20 mi. (32.1 km.) long is operated 25,000 volts, delta-ungrounded. Aluminum lightning arresters and horn gaps are used. The Y line is more free from lightning troubles than the delta line, but the amount of trouble on either line has been almost negligible.

AT THE request of Mr. Frank G. Baum of San Francisco some corona tests were made in April 1914 in Peru, South America on a newly constructed transmission line. The tests were made on that part of the line which runs from the power house at Oroya (12,250 ft. or 3,733.8 m. elevation) to the smelter substation at La Fundicion (about 14,000 ft. elevation), see profile Fig. 1.

Several tests were made at frequencies near normal (60 cycles) and as these tests gave practically identical results, only two are given herewith, one for fair weather and one for rainy weather.

A frequency of 61.1 cycles was chosen as this was the highest at which the water-wheels could be operated under control of their governors.

Power Plant Data. At the power plant there are three 5000-h.p. generating units, each consisting of two impulse-type waterwheels direct connected to one 60-cycle, 2300-volt, 3-phase star-connected, ungrounded-neutral generator. For each generating unit there is provided three 1000-kv-a. 2300—55,000-volt transformers, connected in delta on both sides. All instrument transformers were in the 2300-volt leads between the generators and the transformers.

Transmission Line Data. The line consists of two three-phase circuits of No. 1 B. & S. stranded copper wire and a grounded steel cable ¼ in. (6.3 mm.) diameter, carried on one steel-pole line and arranged as shown in Fig. 2. Each circuit was given two complete transpositions between Oroya and La Fundicion and the distance from Oroya to La Fundicion along the line is 68.4 mi. (110 km.) and the normal operating voltage is 44,000.

The line constants[1] are as follows:

Resistance of one wire, 68.4 mi. at 20 deg. cent....	46.2
Reactance of one wire, 68.4 mi. at 61.1 cycles.....	54.6
Impedance of one wire, 68.4 mi.....	71.6
Susceptance of one wire, 68.4 mi. and 61.1 cycles..	387×10^{-6}

One unit only was used in these tests, but during all the tests about 2500 kw. at a power factor of about 0.6[2] was carried on

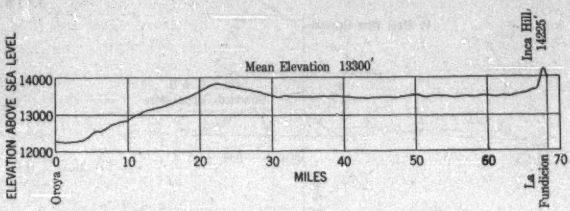

FIG. 1—PROFILE OF LINE, OROYA TO LA FUNDICION

one of the other units and transmitted to the smelter substation over one of the two circuits.

Transformer Data. The name plates of the transformers had been lost and the shop-test data could therefore not be applied to each transformer. The resistance of the three high-tension windings in series was measured at 21 deg. cent. with a Wheatstone bridge, and the average taken. The resistance of the low-tension windings was taken as the average of the nine transformers from the shop report, as the Wheatstone bridge was not adaptable for measuring such small resistances. The impedance drop was assumed equal to the average for the nine transformers taken from the shop-test report, as it could not readily be measured with the available equipment. The average im-

1. TRANS., A. I. E. E. 1911 Vol. XXX, Part III, p. 2264.

2. The 1500-h.p. synchronous motor provided at the smelter substation to improve the power factor was not ready for operation when the tests were made.

pedance drop at 60 cycles is 4.105 per cent; assuming 2.105 per cent for the high-tension winding and 2.0 per cent for the low-tension winding, the transformer constants at 21 deg. cent. and 61.1 cycles, with 2300—55,000-volt connection and expressed on the high-tension side, are: High-tension winding:

$$R = 11.45; \quad X = 62.6, \text{ and } Z = 63.6;$$

Low-tension winding:

$$r = 12.5; \quad x = 56.65, \text{ and } z = 57.95$$

Meters and Instrument Transformers. The instruments used were standard switchboard instruments furnished with the Oroya switchboard and the current transformers for the ammeter and the indicating and integrating wattmeters were 1200-5 amperes

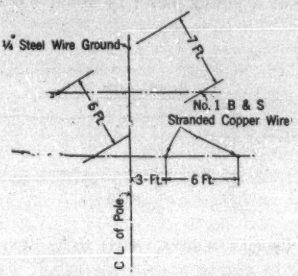

FIG. 2—ARRANGEMENT OF WIRES ON TRANSMISSION LINE

ratio. In order to augment the readings on these instruments, 300-5-ampere current transformers were connected in series with the 1200-5-ampere current transformers on the 2300-volt side and in parallel on the instrument side, thus increasing the readings in proportion of 1 to 5. No power factor indicator was available.

Method of Tests. Before starting the corona tests, a test was made to determine the transformer exciting current and the core loss for the range of voltages and frequencies to be employed. In this test the generator current (equals $\sqrt{3}$ times transformer exciting current), generator voltage, kilowatts and frequency were measured.

For the corona tests these same quantities were measured and the corona loss obtained by subtracting the transformer core

losses, transformer copper losses and the line copper loss due to the charging current from the total generator output.

Calculations. Fig. 3 shows the diagram of connections and Fig. 4 the corresponding vector diagram, wherein:

E_g = generator voltage
E_t = transformer core voltage
E_L = line voltage at generator end
I_{ch} = charging current per wire
I_{co} = corona current (equivalent sine value)
I_L = line current per wire
I_t = transformer exciting current
I_g = generator current

all expressed on the high-tension side and all voltages between wires.

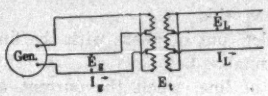

FIG. 3

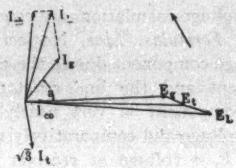

FIG. 4

The current I_{co} due to corona is quite small in all these tests; at 72.3 kv. the corona loss is 153 kw. and the equivalent sine current[3] in phase with the voltage is

$$I_{co} = 153 \div 1.73 \times 72.3 = 1.22 \text{ amperes}$$

The corona current has therefore been entirely neglected in the computations.

Referring to the vector diagram Fig. 4, the line charging current is very nearly 90 deg. out of phase with the voltage[4] and for voltages above 50 kv. the generator current due

3. See oscillographs by Bennett, A.I.E.E., TRANS., Vol. 32, p. 1787 and cyclogram by Prof. Ryan, p. 1825 and remarks under 8 on p. 1827, Vol. 32.

4. At 72.3 kv. the charging current is 16.3 amperes and the line loss for 3 wires $\frac{16.3^2 \times 46.2}{1000} = 12.2$ kw. and the resulting power factor 0.0975 and angle = 84° 20′. For smaller voltages this power factor is still smaller.

to the transformer exciting currents is very nearly 90 deg. out of phase with the generator voltage, see Fig. 7. The generator current during the corona test is also nearly 90 deg. out of phase with the generator voltage (the current leads), except for the higher voltages when the generator current is small, (see Fig. 8) as the line current and $\sqrt{3}$ times the transformer exciting current more nearly balance each other. Since also the transformer reactances are several times larger than the corresponding resistances, and the drop due to the transformer resistances, with the currents involved, is comparatively very small and at right angles to the generator voltage, it follows that the vector relations shown in Fig. 5, may be substituted for those shown in Fig. 4. All this simplifies the calculations very materially without introducing any appreciable errors, as far as current and voltage calculations are concerned.

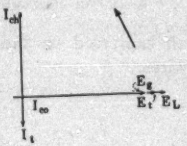

FIG 5

Formulas. Line. Neglecting the voltage component due to the charging current and the line resistance, as this component is very nearly 90 deg. out of phase with the line voltage and comparatively very small. Let

E_r = voltage at receiver end of line, when the current at receiver end is zero, then[5]

$$I_{ch} = \frac{E_r}{\sqrt{3}}\, b \left(1 - \frac{b\,x}{6}\right) \text{ and } E_r = \frac{E_L}{1 - \dfrac{b\,x}{2}} = 1.0106\, E_L \qquad (1)$$

b = line susceptance for one wire and voltage to neutral,
x = line reactance for one wire;

Combining:

$$\left.\begin{aligned} I_{ch} &= \frac{E_L \times b}{\sqrt{3}\left(1 - \dfrac{b\,x}{2}\right)}\left(1 - \frac{b\,x}{6}\right) \\ I_{ch} &= 0.226\, E_L \text{ amperes.} \end{aligned}\right\} \qquad (2)$$

Line Loss Due to Charging Current. Assuming the charging current per unit length of line to be constant, or in other words,

5. Hagood, A. I. E. E., TRANS., Vol. 32, p. 868.

that the current on the line is, at any point, directly proportional to its distance from the receiver end of the line, then;

$i_{ch} = I_{ch} \frac{s}{S}$, where i_{ch} = charging current s miles from the receiver end of the line and

S = total length of line.

I_{ch} = line charging current at generator end.

$$\text{Loss on three wires} = 3 \int_0^S I_{ch}^2 \frac{s^2}{S^2} \frac{r}{S} \, d\,s = I_{ch}^2 \times r \qquad (3)$$

$$= \frac{2.36 \times E_L^2}{1000} \qquad \text{kw.}$$

Voltage rise in low-tension transformer windings:

$$E_t - E_g = I_g \times x \div 1.73 = 0.0327\, I_g \qquad (4)$$

Voltage rise in high-tension transformer windings:

$$E_L - E_t = I_{ch}\, X \div 1.73 = 0.00816\, E_t \qquad (5)$$

In (4) and (5) all voltages are in kilovolts and for leading currents.

Power loss in low-tension transformer windings:

$$\left(\frac{I_g}{\sqrt{3}}\right)^2 \times \frac{3\,r}{1000} = 0.0125\, I_g^2 \qquad \text{kw.} \quad (6)$$

Power loss in high-tension transformer windings:

$$\frac{I_{ch}}{\sqrt{3}} \times \frac{3\,r}{1000} = 0.585\, E_L^2 \div 1000 \qquad \text{kw.} \quad (7)$$

Combining (3) and (7), the loss on the line and in the high-tension windings together is

$$2.945\, E_L^2 \div 1000 \qquad \text{kw.} \quad (8)$$

Core Loss Test. The result of this test is plotted in Fig. 7; the relation between the integrating wattmeter readings and the indicating wattmeter readings is plotted in Fig. 6. The indicating wattmeter readings have been corrected as per curve *B* Fig. 6 for use in Fig. 7. The frequency was 61.1 cycles.

Remarks on Wattmeter Accuracy. As already stated two current transformers with different ratios were in series on the 2300-volt side and in parallel on the instrument side. Full rated current on the 1200-5 ampere current transformers is about 50 amperes (on the high-tension side) and 12.5 amperes for the 300-5 ampere current transformers.

At 55 kv. and 61.1 cycles, the core loss for three transformers is (Fig. 7) 24.5 kw.; the exciting current times $\sqrt{3}$ is 2.65 amperes.

Three times the average core loss of the nine transformers at 60 cycles is according to the shop test report 21.94 and the average for the exciting current times $\sqrt{3}$ is 2.75 amperes. The core loss for the nine transformers varies between a minimum

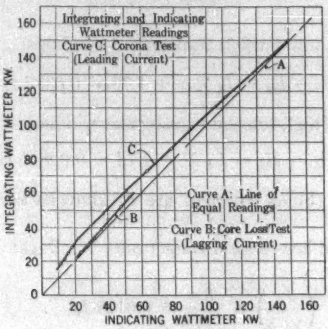

FIG. 6

of 6.9 kw. and a maximum of 7.81 kw., while $\sqrt{3}$ times the exciting current varies from 2.16 to 3.41 amperes.

The influence of the frequency upon the core loss may be estimated roughly, as follows:

$$\left.\begin{aligned} \text{voltage} &= e = c\,f\,B \\ \text{core loss} &= p = k\,f\,B^{3.34} \end{aligned}\right\} \qquad (9)$$

where,

c and k are constants,
f = frequency,
B = magnetic flux per unit area of core, and

the exponent 3.34 is calculated for values between 50 kv. and 60 kv. from Fig. 7, by assuming that the core-loss curve in Fig. 7 follows the formula $p = k\,f\,B^n$.

From (9) it is found, that the core loss at 61.1 cycles is about 96.5 per cent of the core loss at 60 cycles; the core loss as measured was 24.5 kw. at 61.1 cycles and would accordingly have been 25.4 kw. at 60 cycles. The shop tests[6] give for the three transformers with highest core loss only 22.62 kw. or about 11 per cent less than found in the above test.

This discrepancy may be due solely to the instruments and their transformers and in that case would indicate a correction angle of about 40 min. and that the resultant secondary current of the current transformers lead the primary current, *i.e.*, the generator current.[7] In this case, we could expect wattmeter readings about 11 per cent low for the same current and voltage, but a leading current.

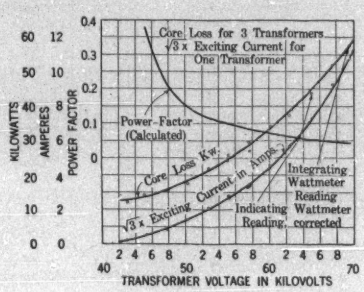

FIG. 7

However, the values for core losses given in the shop reports are for a sine form of the e.m.f. wave. When operating on open circuit, the generator e.m.f. wave is practically a sine curve, but when the transformers are connected to the generator and with the star-delta-delta connection used here, it is likely that a fifth harmonic in excess of the amount demanded by the hysteresis cycle[8] appears in the transformer and generator currents, causing a fifth harmonic to appear in the e.m.f. waves with a corresponding flattening of the voltage crest[9] and a corres-

6. The shop reports give (60 cycle, 55,000 volt):—7.07; 7.20; 6.90; 7.39; 7.40; 7.81; 7.30; 7.41 and 7.325 kw. respectively for the nine transformers in the Oroya power plant.

7. Robinson, A. I. E. E., TRANS., Vol. 28, page 1005.

8. Janet, Lecons d'electrotechnique générale, 2 Ed., Vol. 2 p. 191.

9. Curtis, A.I.E.E., TRANS., Vol. 33, p. 1273 and Figs. 10, 11 and 12.

ponding increase of core loss for the same effective voltage. If, as a limiting value, the e.m.f. wave form was rectangular, the core loss would be about $1.11^{3.34}$ or 51 per cent in excess of the core loss for a sine wave with the same effective voltage and same frequency.[10]

TABLE I.—CORONA TEST. 61.1 CYCLES.

"*Corona Test.* April 23rd 1914. Test started at 1.45 p.m. and finished at 2.45 p. m.; temperature at Oroya in shade outside of Power House was 60° fahr. and barometer was 48.3 and 48.2 cm. respectively at 1.45 and 2.45 p. m. Barometer at La Fundicion was 42.15 cm. No temperature was taken at La Fundicion, but may safely be assumed to be the same as at Oroya. Weather at Oroya was very fair with bright sun; at La Fundicion sky was somewhat overcast; no rain at any place along the line, as far as known."

(Values given for high-tension side of transformers.)

Generator voltage in kv.	Generator current in amperes	Indicating wattmeter kw.	Integrating wattmeter kw.
40.5	9.2	11	18.7
43.0	9.61	14	21.6
45.5	10.0	16	
45.3	9.97	16	24.9
47.8	10.24	18	
47.3	10.2	18	28.2
50.1	10.49	22	
50.0	10.48	22	33.5
52.9	10.58	29	
52.8	10.58	30	40.5
55.1	10.5	37	
55.1	10.5	37	47.9
57.9	10.2	51	
60.0	9.71	64	61.0
62.1	9.1	78	73.2
64.5	7.95	108	86.5
66.9	6.7	140	
69.3	5.1	183	
71.6	3.6	237	
69.3	5.15	177	
66.9	6.98	135	
64.5	8.12	105	
62.1	9.24	77	
59.7	9.81	64	
57.2	10.29	50	
55.1	10.24	41	
52.6	10.58	34	
50.3	10.5	28	
64.6	7.7	111	126
67.2	6.31	149	149

The integrating wattmeter readings were calculated from the formula: Kw. = 11,520 R/T where R is the revolutions of aluminum disk for T seconds. The readings were taken for one revolution of disk and the time, varied from 123 to 15.4 seconds and was taken by a stop watch.

10. Kapp, Transformatoren für Wechselstrom and Drehstrom, 2 Ed. p. 23.

TABLE II*.—CORONA TEST. 61.1 CYCLES

	Generator voltmeter-volts	Generator ammeter-amperes	Indicating wattmeter readings-kw.	Integrating wattmeter readings-seconds
1	84.8	1100	55	123
2	90.0	1150	70	106
3	95.3	1195	80	92.2
	94.9	1192	80	
4	100.0	1225	90	81.5
	98.9	1220	90	
5	105.0	1254	110	68.5
	104.7	1252	110	
6	110.9	1265	145	56.6
	110.8	1265	150	
7	115.5	1258	185	48.0
	115.4	1258	185	
8	121.2	1220	255	37.6
9	125.7	1163	320	31.4
10	130.0	1038	390	26.6
11	135.0	950	540	
12	140.0	800	700	
13	145.0	610	915	
14	150.0	430	1185	
15	145.0	615	885	
16	140.0	834	675	
17	135.0	970	525	
18	130.0	1105	385	
19	125.0	1175	320	
20	119.8	1230	250	
21	115.5	1255	205	
22	110.3	1265	170	
23	105.3	1257	140	
24	135.5	920	555	19.8
25	140.8	755	745	15.4

*Measurements as actually taken and from which Table I was derived.

All readings are actual instrument readings, so that the ammeter and wattmeter readings are to be multiplied by 1/5 and the voltmeter reading by 20 in order to obtain actual values.

From Table I, the values plotted in Figs. 8 and 9 were calculated by the formulas given previously.

As a check on the correctness of the measurements, the calculated charging current is shown in Fig. 8 as the straight line and the points are the measured generator currents plus $\sqrt{3}$ times the transformer exciting currents taken from Fig. 7 for the corresponding voltages. The agreement between the measured values and the calculated currents is quite good.

Curve A in Fig. 9 is the total loss measured; the integrating-meter readings have been used without correction, but the indicating-wattmeter readings have been corrected according to curve in Fig. 6, where the integrating and indicating-wattmeter readings are plotted against each other.

This was done because the integrating meters could be

read with more accuracy and also because it was assumed that the integrating meters would measure the loss more accurately. When the indicating meter readings were accepted as accurate, it was found that the transformer copper and core loss

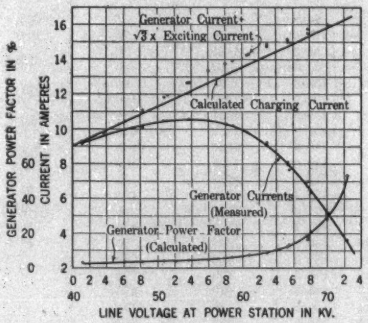

FIG. 8

plus the line charging current copper loss (curve B in Fig. 9) exceeded the total loss (curve *A* in Fig. 9), for the voltages around 40,000 volts. And this, of course, is impossible.

Curve *B* in Fig. 9 is the sum total of all the losses, except the corona loss, calculated from the test and the given formulas. Curve *C* is the corona loss, the difference between curve *A* and *B*.

Comparison of Test with Published Formulas. A.—Peek's Law of Corona, A. I. E. E., Vol. 30, p. 1894, equation (8). $m_0 = 0.87$; $g_0 = 21.1$ kv. eff.; $r = 0.42$ cm.; $s = 183$ cm.;

Line kv.	Peek's law form. (9) kw.	Curve *C* Fig. 9 kw.	Ratio
50.5	0	6.0	
54	22	9.6	2.29
58	100	17.5	5.72
62	236	31.5	7.49
66	428	57.5	8.31
70	678	105.5	6.42
72.3	848	153.0	5.54

$b = 45.93$ (mean value); $t = 16.4$ deg. cent.; $\delta = 3.92 \times 45.93 \div (273 + 16.4) = 0.622$; distance = 110 km. E = voltage between wires in kv.; then for this line,

$$P = 1.78\,(E - 50.5)^2 \quad \text{kw.} \qquad (10)$$

The original test data were submitted to Professor Ryan in June 1914 and Fig. 9A was made at that time by Professor Ryan. The close agreement between the two curves suggested further analysis of the losses found on the Oroya line and these are given below.

Referring to Fig. 9A Professor Ryan wrote:

"In Fig. 6, page 342 of the A. I. E. E., 1911, TRANSACTIONS Faccioli gives the corona losses found in fair weather on the 63-mi. transmission from Shoshone to Leadville of the Central Colorado Power Co. at an average elevation of 8500 ft. and a maximum of 12,000 ft. This line has a length of 102 km. while the Oroya-smelter line has a length of 110

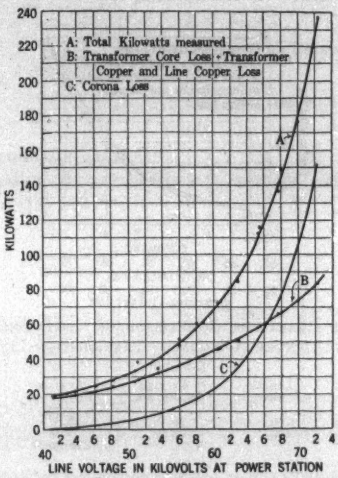

FIG. 9

km. The Shoshone line developed a corona loss of 51.5 kw. at 95.8 kv.; the Oroya line 55 kw. at 65.6 kv., *i. e.* one-half kilowatt per kilometer in each case. For the purpose of comparison, we may assume that corona was definitely started at this amount of loss, viz.: one-half kilowatt per kilometer of line. This is really about the only value in regard to corona formation that approaches some degree of definiteness when related to voltage, line capacity, barometer and temperature, aside from the fact that the amount of corona loss is largely dependent upon $(E - E')$ rather than E, wherein E' is the line voltage that produces a corona loss of one-half kilowatt per kilometer and E is any higher line voltage. It follows therefore, that the corona was started on the Shoshone line at 95.8 − 65.8 = 30 kv. higher than the Oroya line. Other things being equal, the amount of corona loss is due to the amount of voltage that exceeds

corona starting voltage. Let us call such voltage that exceeds corona starting voltage, *Corona Voltage*. Then in Fig. 9-*A*:

I. Oroya-smelter transmission corona voltage and corona loss; length 110 km. average elevation 13,300 ft., maximum 13,800 ft.

II. Shoshone-Leadville transmission corona voltage and corona loss; length 102 km.; average elevation 8500 ft.; maximum 12,000 ft.

I have mounted also the corresponding total line voltages. In comparing the two curves, the differences in length of lines and in the elevations must be kept in mind. We know how to allow for the difference in length but we are not sure that we know exactly how to allow for the differences in altitude. Peek came to the conclusion that corona loss is inversely proportional to the density of the atmosphere. Such conclusion is reasonable. The corresponding composite factor for reducing

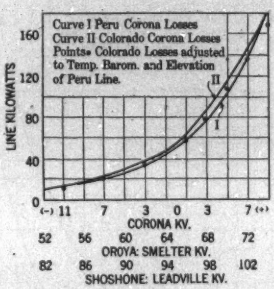

FIG. 9A—CURVE CHART FOR COMPARISON OF OROYA-SMELTER (PERU) AND SHOSHONE-DENVER (COLORADO) TRANSMISSION LINE CORONA LOSSES

Location of Line	Length km.	Average barom. cm.	Average temp. deg. cent.	Authority
Peru.....	110	45.9	16.4	Jakobsen report Apr. 27, 1914
Colorado.....	102	52.0	5.0 (Assumed)	Faccioli, TRANS. A. I. E. E., 1911

the Shoshone-Leadville corona loss to a corona loss corresponding to the Oroya-smelter altitude, barometer and temperature, thus becomes 0.93. In Fig. 9-A, the points enclosed with small circles were located on corresponding ordinates of the Shoshone-Leadville corona loss curve by means of the proportional dividers set at 0.93. You will note that they are in close agreement with the corresponding Oroya-smelter-transmission corona-loss values. Some difference in the forms of the curves must exist because of the differences in the mountain profiles traversed by the corresponding lines. Then, too, the instruments must, inevitably, have been out by small amounts as instruments of the type that had to be employed usually are. Neither cause, however, is a basis for expecting much difference between the two curves."

Analysis of Oroya-Smelter Corona-Loss Curve. In Fig. 10 the square root of the corona loss, equal to $\sqrt{p}$, is plotted against the

line voltage. As this line is not a straight line, the loss curve is not quadratic. Assuming, however, that the loss curve from 69 kv. upwards is quadratic, we get[11] using the $\Sigma\ \Delta$ method:

$$p = 0.765\ (E - 58.23)^2 \qquad \text{kw.} \qquad (11)$$

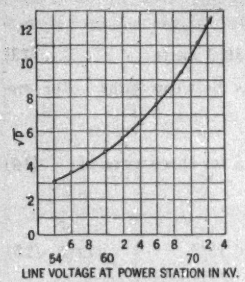

FIG. 10

Plotting the line voltage from 56 kv. upwards against the logarithm of the corona loss from Fig. 9, gives a straight line, and by the $\Sigma\ \Delta$ method is found:

$$\log p = 0.0662\ E - 2.6 \text{ or}$$

$$p = \epsilon^{0.1525\ (E-39.2)} \text{ kw.} \qquad (12)$$

where ϵ = the base of the natural logarithms.

This equation holds good for the whole range and it is the same equation found by Faccioli.[12]

The following shows the close agreement between the test and formula (12) above:

Line voltage kilovolts	Equation (11) kw.	Curve *C*. Fig. 9 kw.
44	2.1	1.5
46	2.6	2.5
48	3.0	4.0
50	5.1	5.5
52	8.7	7.0
54	10.2	9.6
56	12.9	13.0
58	17.4	17.3
60	23.4	23.2
62	32.4	31.5
64	43.6	42.3
66	58.9	57.5
68	79.4	77.5
70	109.5	105.5
72	144.5	146.5
72.3	151.5	153.0

11.

Kilovolts	Fig. 9 kw.	$\sqrt{\text{kw.}}$
69	90	9.48
70	105.5	10.26
71	124	11.13
72.3	153	12.37

12. $p = 9.5\ \epsilon^{(0.089\ E-9.75)}$. A. I. E. E., TRANS., Vol. XXX p. 349, particularly curve *B* Fig. 10.

B.—Prof. Ryan's formula, A. I. E. E., TRANS., Vol. 30, p. 64.

Equation (3) on p. 69, Vol. 30 gives the voltage at which the loss due to part corona is about equal to the loss due to convection, and this last named loss is given by the equation at the bottom of p. 67, Vol. 30, viz.:

Loss per 1000 ft. of single-phase line

$$= p = 2 \times f^2 \times E^2 \times 10^{-6} \text{ watts} \qquad (13)$$

where, f = frequency and E-kilovolts between wires. For one wire therefore,

$$P = 4 \times f^2 \left(\frac{E}{2}\right)^2 \times 10^{-6} \text{ watts; and for three wires of} \qquad (14)$$

a three-phase circuit,

$$P = 4 \times f^2 \times E^2 \times 10^{-6} \qquad \text{watts} \qquad (15)$$

For 61.1 cycles and 68.4 miles of any three-phase line,

$$P = 5.48\ E^2 \qquad \text{watts} \qquad (16)$$

The critical corona voltage is, equation (3) p. 69, Vol. 30

$$E_{crit.} = 0.455 \times k \frac{17.9\ b}{459 + t} \frac{d + 2\ a}{C} \qquad (17)$$

where, C = 1.16 times capacity of 1000 ft. (304.8 m.) of single phase line in microfarads, and a is obtained from Fig. 2 of Prof. Ryan's article. The constant k is a factor selected by judgment guided by the results of practical tests and experience. See Tables I and II on pages 70 and 71 and remarks on p. 68, Vol. 30; select $k = 0.75$;

In formula (17) above:

$d = 0.33$ in.; $a = 0.069$ in.; $b = 18.1$ in.; $t = 61.6$ deg. fahr.: $D = 72$ in. and

$$C = 1.16 \frac{0.00368}{\log\left(\frac{D}{r}\right)} = 0.00161 \text{ microfarad}$$

From (17) above: $E_{crit.} = 61.6$ kv.

From (16) above: $P = 20.8$ kw.

The kilovolts in Fig. 9 corresponding to $2 \times 20.8 = 41.6$ kw. is 63.9 kv., as against the 61.6 found by Prof. Ryan's formula.

Bearing in mind the errors likely to occur particularly on the lower part of the curve, this is certainly as good an approximation as could be expected.

In order to show the influence of the constant k of Prof. Ryan's formula, the following table has been computed:

k	E form. (15) kv.	2 × convection loss form. (14) kw.	Corresponding line voltage from Fig. 9 kv.
0.72	59.4	38.6	63.4
0.73	60.0	39.4	63.5
0.74	60.9	40.5	63.7
0.75	61.6	41.6	63.9
0.76	62.5	42.6	64.1
0.77	63.5	43.1	64.2
0.78	64.4	45.4	64.4
0.79	65.1	46.3	64.6

It is seen from above, that the constant k does not largely alter the result and any value between $k = 0.72$ and say $k = 0.82$ will give very good results.

The following test was started at 7.30 p.m. during heavy rain at La Fundicion; light rain, off and on, at Oroya. Probably only local showers along the line. Temperature outside Oroya power house 14.5 deg. cent. at 8.17 p.m.; barometer at Oroya 48.3 cm. and at La Fundicion 42.85, both at 7.30 p.m.

Generator voltage kv.	Line kv.*	Generator current Fig. 8 amperes	Generator† current amperes	Corrected‡ ind. wattmeter readings kw.	Curve A Fig. 9 kw.	Frequency
58.6	59.4	10.1	10	60	65	60.8
61.9	62.7	8.95	9.1	83.5	86	
64.6	65.3	8.00	8.04	110.7	110.5	
66.9	67.7	7.30	6.96	138	140	
62.1	62.9	9.10	9.35	84.5	81	61.0
65.0	65.8	7.8	8.04	118	115.1	
67.2	68.0	6.5	6.78	148	143.5	
69.5	70.2	5.0	5.4	193	184	
71.5	72.2	3.6	4.22	237	234	
73.2	73.9	2.8	3.58	285		

*By interpolation from other test.
†A smaller scale ammeter used than in the first test.
‡Using Fig. 6.

At 71.8 kv. at Oroya a slight brush discharge was visible at the end of the lightning arrester horns connected to the line; the same was the case at the smelter substation. No visible corona was on the line either at Oroya or at the smelter. The night was quite dark with a new moon behind heavy clouds.

ACKNOWLEDGMENTS

In conclusion I wish to express my appreciation to Mr. F. G. Baum, Mr. J. P. Jollyman and Professor Harris J. Ryan for suggestions in the preparation of this article.

DISCUSSION ON "CORONA TESTS AT HIGH ALTITUDE" (JAKOBSEN), SAN FRANCISCO, CAL., JANUARY 25, 1918.

F. W. Peek, Jr. (by letter): I wish to call attention to a paper, "Comparison of Calculated and Measured Corona Loss Curves," published in 1915, where the laws of corona from my 1911 paper are discussed and applied to measurements made on practical and experimental lines in different parts of the country. Mr. Faccioli's Shoshone measurements are included in this comparison. I will quote directly from this paper:

"The law of corona takes the general form

$$p = c\,(e - e_0)^2$$

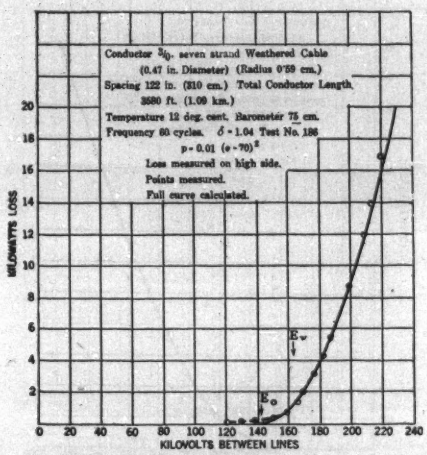

FIG. 1—OUTDOOR SINGLE-PHASE EXPERIMENTAL LINE. (TESTS MADE BY F. W. PEEK, JR., SCHENECTADY, 1910)

which means that under conditions otherwise constant, the loss varies as the square of the applied voltage above a certain critical voltage, e_0 or e_d. The critical voltage, e_0, is called the *disruptive critical voltage*. Visual corona does not start at the *disruptive critical voltage*, but at some higher voltage e_v, the visual critical voltage. Both of these voltages have been calculated and marked on the curves. Theoretically, if the conductors were perfectly smooth, no loss should occur until the visual critical voltage, e_v, is reached, when the loss should suddenly take a definite value. For e_v, and higher voltages, the loss should follow the quadratic law. In practise, due to dirt, points,

etc., brushes occur on the conductors at voltages lower than e_v. Between e_v and e_0, due to these brushes, there is a loss. This loss practically follows the quadratic law for large weathered conductors, where e_v and e_0 approach each other in value. For small conductors and especially new conductors with fairly clean surfaces, *the loss between e_v and e_0 falls below that of the quadratic law*, as shown in Figs. 2 and 3. If the conductors were highly polished there would be very little loss until the voltage e_v is reached. At this voltage the loss would suddenly take a definite value very nearly equal to that calculated by the quad-

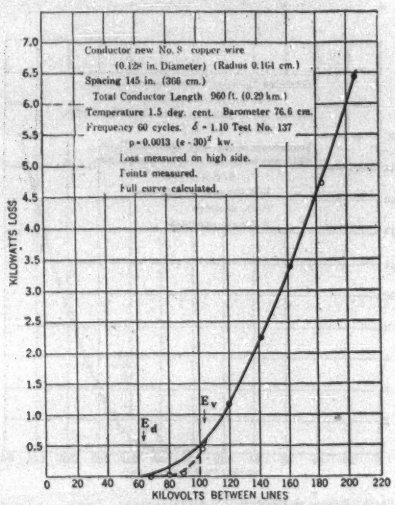

FIG. 2—OUTDOOR SINGLE-PHASE EXPERIMENTAL LINE. (TESTS MADE BY F. W. PEEK, JR., SCHENECTADY, 1910)

ratic law with e_v as the applied voltage and e_0 as the critical voltage in the equation.* In all cases the loss follows the quadratic law above e_v. The part of the loss curve between e_0 and e_v will thus vary from day to day depending upon the chance condition of the conductor surfaces. Over this unstable section of the curve the loss follows the probability law."

$$p = q\,\epsilon^{-h(e-e_0)^2}$$

Excess loss due to irregular irregularities, as spots, points, etc.

*"e_0 or ed must always be used as the critical voltages in quadratic law."

"It is of *practical importance only to know the limits* of the *loss at this part of the curve*; e_0 *should generally be the limit of the voltage on practical lines, as otherwise storm losses become excessive.*"

"Thus, from the considerations above, the quadratic law should be *closely followed in all cases for voltages higher than* e_v. For *the part of the curve between* e_v *and* e_0 the loss is unstable and dependent upon chance surface conditions, dirt spots, etc. as follows:

1. For large weathered cables, such as are used in practical lines, the quadratic law is closely followed.

2. For new conductors, and especially small ones, the loss at voltages lower than the visual critical voltage, e_v, falls below the quadratic law."[1]

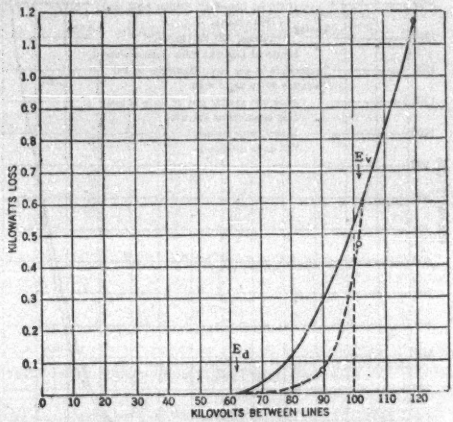

FIG. 3—LOWER PART OF FIG. 2 TO AN ENLARGED SCALE

There are, thus, two critical voltages: the *disruptive critical voltage*, e_0, used in the loss equation, which corresponds to a constant gradient of 30 kv. per cm. max., and the *visual critical* voltages, e_v, the voltage at which visual corona starts. e_v is always higher than e_0. For perfectly polished wires, the loss would occur as shown in Fig. 5, suddenly reaching a definite value at e_v and following the quadratic. Between e_v and e_0 the loss is unstable and is caused by spots, points, dirt and other

1. "Law of Corona and Dielectric Strength of Air." F. W. Peek, Jr., A. I. E. E. TRANSACTIONS, 1911, page 1892, part III. Also "Law of Corona II". F. W. Peek, Jr., A. I. E. E. 1912, page 1051, part I. "Comparison of Calculated and Measured Corona Loss Curves," F. W. Peek, Jr., A. I. E. E. TRANSACTIONS 1915, page 269, Part I.

irregularities, and not "sub corona", "part corona", etc. Between e_v and e_0 the loss may be greater or less than the quadratic, as shown in Figs. 1 to 4. For practical sizes it generally approximates the quadratic over the whole range. This was fully explained in my 1911 paper.[2] In practical lines it is generally not advisable to operate above the e_0 voltage. It is thus of great importance to be able to determine this voltage accurately.

In calculating the losses, etc. on the Oroya line I find

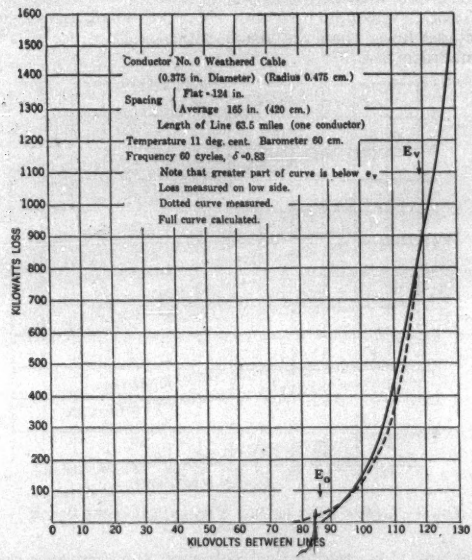

FIG. 4—SHOSHONE—LEADVILLE TRANSMISSION LINE, THREE-PHASE (TESTS MADE BY FACCIOLI.)

$$p = c\,(e - e_0)^2 = c\,(e - 29)^2$$

$$e_0 = \frac{50}{\sqrt{3}} \text{ kv. to neutral.} \quad e_v = \frac{75}{\sqrt{3}} \text{ kv. to neutral.}$$

Or between lines

$E_0 = 50$ kv. *Disruptive critical voltage*
$E_v = 75$ kv. *Visual critical voltage*

2. Also "Dielectric Phenomena in High-Voltage Engineering," page 143, chapter V.

Mr. Jakobsen's measurements were all made below the visual critical voltage or on the unstable part of the curve where the probability and not the quadratic loss law is followed. Mr. Jakobsen does not mention this fact in making comparisons, although he uses these measurements mainly to show that the quadratic is incorrect. His Fig. 9, curve C, indicates about 50 kv. for E_o, as calculated above. It is quite probable that with a new line the loss for voltages lower than the visual corona voltage would come lower than that indicated by the quadratic law. This is shown by my Fig. 2, where the $E_o - E_v$ range of voltage is greater than that used by Mr. Jakobsen. With large weathered conductors the loss between E_v and E_o should be as in my Figs. 1 and 4. It is difficult to estimate the accuracy of Mr. Jakobsen's measurements. With the apparatus that he had available, a slight phase displacement caused by the current

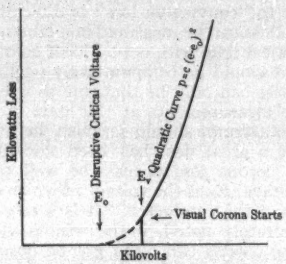

FIG. 5—CORONA LOSS FOR PERFECT CONDUCTORS.

NOTES—The loss for a Perfect Conductor is Shown by a Heavy Line. It is Zero up to E_v When it Suddenly Reaches a Definite Value and Follows the Quadratic Law. For the Actual Loss in Portion Below E_v See Figs. 1, 2 and 4.

transformer, potential transformers, etc., at the low power factors would cause very large error. It is quite possible, however, that the loss for the voltage range measured was actually lower than that predicated by the quadratic law.

In my Fig. 4 the dotted curve represents the losses measured on the Shoshone-Leadville line by Mr. Faccioli. The full curve is plotted from a direct calculation of the starting voltage and loss by my corona formulas. Contrary to the conclusions reached by Mr. Jakobsen from his Fig. 9A the check is good. As is usual in practical lines with large weathered conductors, the quadratic law approximates the loss even for voltages lower than e_v. Fig. 9A does not show that the Oroya and Shoshone losses check. No comparison is actually made by Mr. Jakobsen. The two curves have simply been slid together without any regard as to what the relative starting voltages should be. It only means that the two curves happen to have approximately the same slope for the range plotted. To show the fallacy of

this method, I have plotted Fig. 6. The drawn curve is the calculated Shoshone curve. The points are the measured Shoshone and Oroya losses and the calculated Oroya losses, all "good checks", measured by the standards of Mr. Jakobsen's Fig. 9A. It thus does not seem rational to attempt to compare loss curves for different conductors by tying the curves together at some arbitrary loss per mile. It will be found that two curves so tied together at one temperature and pressure will differ widely at some other temperature and pressure.

Mr. Jakobsen's results given in the table on page 105 in connection with equation (**11**), should naturally be expected. A number of other equations would fit equally well over the limited range. This should be especially expected from an exponential equation. When an equation is written, however, it should be rational. The probability law is the rational equation for an approximation of the excess loss due to irregular irregularities.[3]

The reference to "convection loss" is difficult to understand. Assuming such a loss can be measured in a laboratory on polished wires, on a practical line, with actual local corona loss at points and dirt spots, it would be comparatively negligible. Below e_v the loss in practise can only be that due to irregularities. The accuracy of the measurements at this part of the curve is so doubtful that no attempt should seriously be made at checking a "law". Since a great deal has been said about the visible and non-visible corona loss it may be well to explain. The visual corona voltage, E_v, is the voltage at which corona suddenly appears all around the conductor. It is a *very definite* voltage and can be accurately noted, if the wire is viewed in a very dark room. The corona can often not be observed out in the open, however, even on a "dark" night, at much higher voltages. Any appreciable loss below this voltage is due to brushes at roughened and dirty points along the conductor. These brushes are also visible in a dark room but are not readily observed on a line out of doors. All corona loss is thus really due to visible corona or brushes. Below the visual corona voltage the loss is due to the chance brushes at roughened or dirty spots.

---

3. The loss below the voltage e_v can probably be best represented by the sum of two equations, a quadratic giving the loss due to the regular irregularities, and the probability law giving the loss due to the irregular irregularities.

Thus $$p\,K\,(e - e^1_0)^2 + q^{-h(e^1_0 - e)^2}$$

Irregular irregularities may be spots, points, dirt, etc.

Regular irregularities may be the strands in a cable, a weathered surface, a ridge, etc.

For voltages above e_v, all irregularities become practically regular and

$$p = c(e - e_0)^2$$

For small conductors e_0^1 may be larger than e_0. K is smaller than c since it may represent only part of the surface.

The sum of the curves above may be approximated by an exponential.

In the last table in Mr. Jakobsen's paper are given loss measurements made during a storm. These measurements are practically equal to those made in fair weather. This can be explained in only one of two ways. Either the meter arrangement is such that the indication is the same regardless of the actual loss, or the storm was local and confined to a very small portion of the line. There is no doubt whatever that corona loss is greatly increased during rain. This is substantiated by actual measurements and by theory.

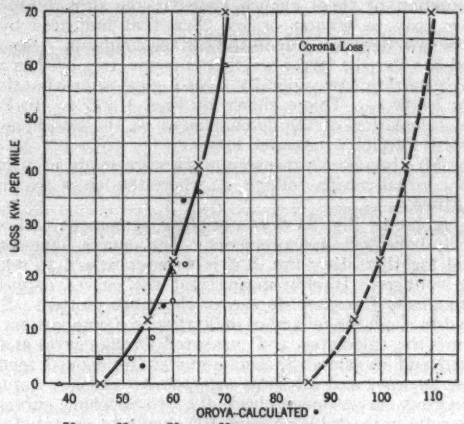

FIG. 6

NOTE—The Four Curves Have Been Moved Together Along the X Axis as in Mr. Jakobsen's Report Fig. 9A to Give the Appearance of a Check—The Dotted Curve is Drawn in to Indicate how This was Done—Actually, no Comparison is Made by This Method.

It would be extremely interesting if Mr. Jakobsen could make measurements on different sizes of conductors at voltages above the visual corona voltage at these high altitudes. This would probably have to be done with a testing transformer on short lengths of line.

I have been very much interested in the data obtained, however, which seem to check the calculated e_0. The irregularity loss on this part of the curve may also, for a good surface condition, fall below the quadratic as Mr. Jakobsen has found. I do not agree with Mr. Jakobsen in his interpretation of the data, however.

To Summarize:—

There are two critical voltages which must be considered, the disruptive critical voltage, e_0, used in the quadratic law, and the visual critical voltage e_v. e_v is always higher than e_0. In practise it is generally not desirable to operate above the e_0 voltage.

The loss at voltages above e_v always follows the quadratic law.

For voltages lower than e_v there would be no appreciable loss for perfect wires (Fig. 5). In practise there is a loss on this part of the curve due to brushes at irregularities. The rational approximation of these chance losses is the probability law. This loss may be greater or less than that indicated by the quadratic law, depending upon the surface condition. See Figs. 1, 2 and 3. For the larger weathered conductors used in practise, the quadratic law generally gives a good approximation at voltages below e_v. This is shown by Figs. 1 and 4. In Fig. 4 a direct calculation of the corona losses on the Shoshone line agrees with Faccioli's measured losses.

All of Mr. Jakobsen's measurements were made at voltages below the visual corona voltage, e_v, where the losses are due to irregularities.

Mr. Jakobsen's Fig. 9A does not show agreement with Faccioli's Shoshone loss measurements. The curves have simply been slid together along the X axis without regard to relative starting voltages. It only means that the curves happen to have practically the same slope over the range plotted. Fig. 6 is plotted to show the fallacy of making such a comparison. In this figure the calculated and measured Oroya curves and the calculated and measured Shoshone curves are moved together in the same way, and all show an *apparent* check. Actually, the two Oroya curves do not check; the two Shoshone curves do.

The results in the table on page 105 should be expected. An approximation of this curve should be an exponential. A number of other equations could equally well be made to fit.

Referring to the calculations on page 106, if there is a convection loss, it must be relatively so small compared to the actual corona loss at irregularities, that it is negligible.

The losses during rain given in the table on page 107 practically agree with the fair weather loss. This is contrary to all other measurements and to theory.

In estimating the cost of the loss of power per year due to corona it must be remembered that the loss varies from day to day due to changes in temperature and pressure. A given line with very high losses during the hot summer days may have no loss for the greater part of the year. It is instructive to plot a time-loss curve for a year. The rain losses may be very high during periods of precipitation, but when put on a yearly basis may be a very small part of the total loss. At the part of the curve near the critical voltage, e_0, the loss will be considerably affected by surface conditions not under control

of the operator, due to dust, fog, sleet, etc. Such conditions can only be estimated. We have found the quadratic law to give the best average estimate of these losses.

Harris J. Ryan: Mr. Peek characterizes my comparison of the forms of the Faccioli and Jakobsen high-altitude corona line-loss curves by saying that, "the two curves have simply been slid together without any regard as to what the relative starting voltages should be." Precisely the opposite is the case. The curves were brought together for their form or "law" comparison by allowing for the difference in their corona starting voltages. See text of the present Jakobsen paper, page 103: "For the purpose of comparison we may assume that the corona was definitely started at this amount of loss viz.: one-half kilowatt per kilometer of line." "It follows, therefore, that the corona was started on the Shoshone-Leadville line at 95.8 − 65.8 = 30 kv. higher than the Oroya line." The two curves were therefore given a common origin at the definite initial corona loss of 0.5 kw. per kilometer of line and at the corresponding line voltages of 52 and 82 kv. On Peek's authority the values of the Shoshone line losses were corrected so as to correspond to the values that would have occurred at the barometric pressure and temperature of the Oroya line by using the multiplier 0.93. I know of no more rational proceeding in the circumstances.

In his contribution to this discussion Mr. Peek has stated and restated that the non-visible coronas are controlled by "irregularities" and "irregular irregularities". Ten years ago Foote in his Michigan 110 = kv. transmission studies and practise proposed that a small definite loss due to corona, say 0.5 kw. per mile of transmission line be adopted to determine the corona starting voltage. It seemed to me then* and the results of many studies since cause me to continue to think that Foote's proposal is a good one and should be adopted, viz.: That for practical purposes the corona starting voltage should be taken as the value which causes a loss due to corona of 0.5 kw. per kilometer (0.8 kw. per mile) of transmission line.

Because of the work of Peek and of others we have an abundant knowledge of the visible corona losses. Engineering practise is much concerned with the non-visible losses and knowledge thereof is scanty. Mershon's results obtained in 1905 at Niagara Falls and a few results obtained from tests made on actual transmission lines are all that we have. Much more should be done. The factors involved are many and the work is difficult. The present paper is most welcome. It is to be hoped that more of its kind will follow. The present out-look is that these non-visible corona losses must be handled empirically.

B. F. Jakobsen: I shall consider Mr. Peek's remarks under three headings, as follows:

1. Range of test voltage at Oroya.

*Conductivity of the Open Atmosphere, TRANS. A. I. E. E., Jan. 1911.

2. Accuracy of Oroya tests.
3. Comparison of calculated and measured corona losses.

1. *Range of Test Voltage at Oroya.* The maximum voltage that could be obtained at Oroya by raising the generator voltage, was 72,300 volts. At this voltage no corona was visible on a dark night, either at Oroya at 12,200 ft. (3.72 km.) or at the Smelter at about 14,000 ft. (4.27 km.). The corona loss was about 150 kw. for one circuit or 300 kw. for the two circuits. Mr. Baum states that power is worth ½ cent per kw-hr. and the loss due to corona would therefore amount to $13,000 per year. The copper in both circuits from Oroya to Smelter is 550,000 lb. (249,475 kg.) and it cost about $140,000 in place; therefore we could afford to double the amount of copper rather than accept the corona loss of 150 kw. per circuit. *The maximum voltage used in the test was therefore considerably above any voltage that could be considered as operating voltage.*

It is the corona loss at voltages below the visual corona voltage that is of importance in transmission line work, since it is important to know how high one may safely go with the line voltage under certain conditions, rather than knowing what the corona loss would be at a voltage several times higher than any possible operating voltage.

2. *Accuracy of Oroya Test.* Faccioli made some tests in order to check the accuracy of his corona tests (see page 347, Vol. 30, Part I) and as these have a bearing on the accuracy of the present test, I shall quote from Faccioli's paper, regarding the Denver-Boulder single-phase test:

> Since the power factor of the readings was very low it was considered advisable to raise this power factor by using an artificial load (a water box) in multiple with the low-tension winding of the step-up transformer which energized the line. The measurements were taken as follows:
>
> The transformer only was connected to the system and readings taken, then the line was connected to the transformer and readings taken at the same voltages as before. The difference between the kilowatts in the two cases gives the total losses of the line. Additional measurements were made, first, with the transformer and auxiliary load; second, with the transformer, auxiliary load and line. The utmost care was taken to keep the auxiliary load constant during these two sets of measurements. The introduction of the auxiliary load proved unnecessary as the results obtained with and without the extra load were the same.

At the maximum voltage employed at Oroya, *i. e.* 72,300 volts, the load on the wattmeters was 1,185 kw. (see Table II of my paper); this instrument load is five times the actual load (Table I and Fig. 9 of my paper) on account of the current transformer connections employed (see "Meters and Instrument Transformers" page 952). The generators are 3750 kw., so that the load on the wattmeters was about 30 per cent of the generator capacity; the power factor on the wattmeters was 0.54 (see Fig. 8 of my paper). The two wattmeters (indicating and

integrating) agree at this voltage (see Fig. 6 of my paper), and the measured current agrees closely with the calculated current (see Fig. 8 of my paper.) *There is thus no reason to expect an error of more than a very few per cent at the most, while if Mr. Peek's formula was correct, the error committed would be about 550 per cent.*

3. *Comparison of Calculated and Measured Corona Losses.* In Mr. Peek's paper "Comparison of Calculated and Measured Corona Coss Curve", A.I.E.E. Vol. 34 (1915) Part I, page 269, only a single test made on an actual transmission line is considered, and this is Faccioli's Shoshone-Leadville test, reported in the A. I. E. E. TRANSACTIONS Vol. 30 (1911) Part I, page 337. The profile of this line is shown on page 79 of Vol. 30 Part I. Referring to this particular test, Mr. Faccioli said, page 344:

> Although the Shoshone-Leadville tests were taken on a section of line 63.5 miles (102 km.) long, still the curves of Fig. 6 cannot be used to determine the critical voltage or the law which connects losses to kilovolts because the voltage at the far end of the line is higher than the voltage at the power house and because of the different altitudes through which the transmission line runs. For this reason some experiments were conducted on the Denver-Boulder section of the system.

Fig. 4 of Mr. Peek's paper (Vol. 34, page 269) is reproduced as Fig. 4 in this discussion. This figure shows the close agreement between the loss measured by Faccioli and the Peek formula, but it is quite misleading, because Mr. Peek has used the temperature and the barometer for Shoshone, as given by Mr. Faccioli on page 344, Vol. 30 (1911) Part I, with practically no corrections, although Shoshone lies at only 6000 ft. (1.8 km.) while the maximum point on the line lies at 12,000 ft. and the average for the whole line being about 8500 ft. (2.6 km.). Faccioli gives: temp. = 55 deg. fahr. (12.8 deg. cent.); barom. = 24 in. (61 cm.) while Mr. Peek has used 11 deg. cent. and 60 cm. barometer and Professor Ryan used 5 deg. cent. and 52 cm. barometer in Fig. 9A.

I do not know where Professor Ryan obtained the barometric pressure which he used, but using the following standard formula:

$$H_2 - H_1 = (18{,}400 + 70 \times t_m) \times \log\left(\frac{P_1}{P_2}\right)$$

where $H_2 - H_1$ = difference of altitude in meters,
P_2 and P_1 = corresponding barometric heights in millimeter of mercury;
t_m = average temperature of air between the two elevations, in centigrade;

with $H_2 - H_1 = 2500$ ft. (763 meters), $P_1 = 610$ mm. (24 in.) and $t_m = (12.8 + 5) : 2 = 9$ deg. cent., I find $P_2 = 555$ mm., instead of the 60 cm. used by Mr. Peek and 52 cm. used by Professor Ryan.

Using the temperature and barometric pressure as given by

Professor Ryan, $\delta = 0.724$ instead of 0.83 as given by Mr. Peek in Fig. 4. With $m_0 = 0.87$ (Peek's law of corona, Vol. 30, Part III, page 1890), $E_0 = 74.2$ kv. between wires, and the loss for three wires at 60 cycles and 63.4 miles (102.5 km.) as per equation (6), page 1894, Vol. 30, Part III, is:

$$P = 0.981\,(E - 74.2)^2 \text{ kw.}$$

This gives the following table for the comparison between the measured Shoshone-Leadville line loss (Faccioli Fig. 6, page 342, Vol. 30, Part I) and Peek's formula:

Voltage between lines in kv.	Loss by Peek's formula. kw.	Measured loss. kw.	Ratio	Square root of measured loss
85	114	25	4.56	5.0
90	246	30	8.2	5.46
100	655	115	5.7	10.72
110	1260	360	3.5	19.0
120	2060	1120	1.84	33.5

The square root of the loss is plotted against the voltage between wires in Fig. 7 and this shows a curve exactly like the one found for the Oroya line and plotted in Fig. 10 of my paper, and it shows that the loss measured does not follow the quadratic law, for if it did, this line would be a straight line.

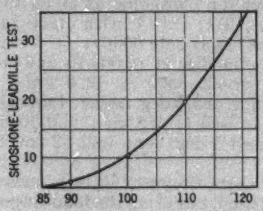

FIG. 7

Since the corona loss is very much affected by barometric pressure and temperature and since these were measured only at one point (the lowest point along the line), this test is decidedly unfit for comparisons.

The increase of voltage towards the open end of the line, as mentioned by Mr. Faccioli, in addition to the higher altitude at the open end of the line, would increase the losses. In comparing a standard formula with these measurements, allowance must be made for this, or the comparison is quite worthless.

Faccioli measured the corona losses by first measuring the transformer core loss + low-tension copper loss and then connecting the transformers to the line and measuring the loss again

at the same voltage. The difference between these two losses he took as the corona loss at that voltage, see page 340 near top, Vol. 30, Part I. In this way the high-tension transformer copper loss and the line copper loss, as given by my formula (**8**), was included with the corona losses, and this method would give a corona loss larger than the actual loss.

In order to get an idea of the magnitude of the various losses involved in the corona loss tests at Oroya, Fig. 8 was plotted. Curve *A* represents the core loss of the three transformers; Curve *B* is the high-tension transformer copper loss + the line copper loss (equation (**8**) of my paper), the line copper loss amounting to about 4/5 of the curve *B* loss, equations (**3**) and (**7**). The low-tension copper loss varies as the generator current,

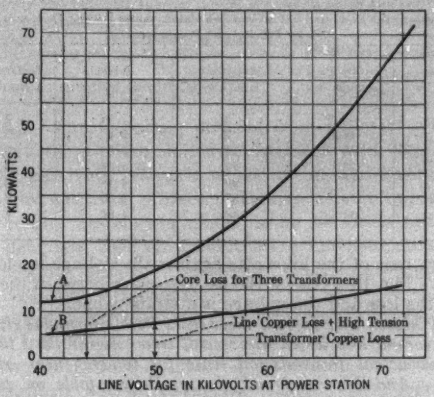

FIG. 8

Fig. 8 and its maximum value is only 1.4 kw. At 40 kv. it is 1.1 kw. and at 70 kv. it is 0.3 kw. The copper loss in the low-tension transformer windings added to the two curves *A* and *B* in Fig. 8, would give the total loss curve, Curve *B* in Fig. 9 of my paper.

The Denver-Boulder single-phase line test made by Faccioli and shown in Fig. 9, page 347, Vol. 30, Part I is considered by Mr. Faccioli as being the more accurate test and it would therefore form a better basis for comparisons than the Shoshone-Leadville tests. The constants given by Faccioli are: distance 27.6 miles (44 km.); line No. 1 B & S. diameter = 0.33 in.; spacing = 124 in; barometer 24.4 in. (62 cm.) and temperature 62 deg. fahr. (16.7 deg. cent.). These two last are taken as the average of 24.1 in. and 24.7 in.; and 49 deg. and 74 deg. fahr. as given by Faccioli on page 345, Vol. 30 and no corrections were

made for the small differences in barometric pressure and in temperature due to the difference in elevation between the substation at Denver and the average altitude of the line. The profile is shown on page 79, Vol. 30. The average altitude is given by Faccioli as 5300 ft. (on page 344, Vol. 30), but this does not agree with the profile, which is probably correct and which shows about 5900 ft. (1.8 km.).

Taking $m_0 = 0.87$, Peek's formulas give $E_0 = 85.8$ kv. between wires and the loss for the two wires (single-phase):

$$P = 0.199\,(E - 85.8)^2 \text{ kw.}$$

Faccioli derived the following formula for the loss for voltages above 100 kv. (see his Fig. 11 on page 351, Vol. 30):

$$P = 0.036\,(E - 80)^2 \text{ kw.}$$

Comparison between measured values and calculated values for the Denver-Boulder test by Faccioli and Peek's formula.

Voltage between lines in kv.	Loss by Peek's formula. kw.	Measured loss. kw.	Ratio	Square root of measured loss.
90	3.5	3	1.17	1.73
95	16.8	7	2.4	2.65
100	40	12	3.33	3.46
105	73	19	3.84	4.36
110	117	28	4.18	5.3
115	170	37	4.59	6.09
120	234	48	4.86	6.94

Square root of the kilowatts against the voltage is plotted in Faccioli's Fig. 10. Above 100 kv. the curve between the square root of the loss and the voltage is a straight line.

It will be seen that in both these cases Peek's formulas give values for the corona loss which are several times higher than the losses found on actual transmission lines for voltages near operating voltages. And this was found also at Oroya, table on page 102 *of this volume.* That these test voltages were below the visible corona voltage is quite true, but that does not detract from the fact that we are mostly interested in knowing the losses which are likely to occur at or near the actual operating voltages.

It is very important, as Mr. Peek says, to be able to determine E_0 (the disruptive critical voltage), but I cannot see why Fig. 9 of my paper shows this to lie at about 50 kv., since the loss starts at about 40 kv.

On long high-tension transmission lines carrying large amounts of power it will most likely be economical to allow a certain percentage loss as corona loss and in order to do this, the corona loss below the visual corona voltage should be investigated on actual transmission lines.

Regarding the losses during rain, which Mr. Peek contends should be considerably larger than the fair weather losses, I had no means of knowing the weather conditions except at the end of the line, as I stated in the paper.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.

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RATING AND SELECTION OF OIL CIRCUIT BREAKERS

BY E. M. HEWLETT, J. N. MAHONEY AND G. A. BURNHAM

ABSTRACT OF PAPER

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

THERE IS an increasing demand from engineers and operators for a more uniform statement from the various manufacturers with reference to the rating and recommended selection of electrical protective equipment. It appears that simple concise statements might easily be made that would convey definitely the desired information, but there are so many variables which enter into the selection that a simple statement to cover all cases is impossible.

The object of this paper is, (1) to discuss the interpretations of the A. I. E. E. rules covering the rating of oil circuit breakers, (2) to discuss the factors involved in the proper selection of oil circuit breakers, and (3) to suggest average system short-circuit characteristics which can be used for selecting oil circuit breakers for certain systems.

It is hoped that the interpretation given and the data proposed for oil circuit breaker selection will meet with the approval of those persons interested in this problem or give rise to suggestions leading to improvement.

The subject has received careful consideration by the Standards Committees of the American Institute of Electrical Engineers and several papers on this subject[1] have been presented

1. Rating of Oil Circuit Breakers, by E. M. Hewlett, TRANSACTIONS, A. I. E. E., 1916; Rupturing capacities of Oil Circuit Breakers, by S. Q. Hayes, TRANSACTIONS, A. I. E. E., 1916; Rating of Oil Circuit Breakers, by G. A. Burnham, TRANSACTIONS, A. I. E. E., 1913.

to the Institute, all of which have resulted in bringing about a clearer understanding of the various expressions and methods used in connection with the rating of this class of equipment.

Circuit breakers are classified according to their rated pressure, rated current, rated frequency and interrupting capacity.

Systems may be classified according to their normal operating pressure, normal current, normal frequency and current transients.

The *rated pressure* (voltage) of a circuit breaker is the greatest normal pressure in r.m.s. volts between any two wires of any circuit to which the breaker should be connected.

The Standardization Rules of the American Institute of Electrical Engineers require that oil circuit breakers for voltages above 600 volts withstand a dielectric test[2] of 2.25 times rated pressure plus 2000 volts for 60 seconds. Although not stated in the rules we infer that it contemplates a test with the apparatus under dry conditions.

The *normal operating pressure* of a system is the greatest pressure in r.m.s. volts ordinarily maintained between any two conductors.

The rated current of a circuit breaker as defined by the A. I. E. E.[3] is "the normal r.m.s. current which it is designed to carry." This rating is covered by the following rule:[4]

"Temperature Tests—Rated current at rated frequency shall be applied continuously until the temperature becomes constant. The maximum temperatures of the various parts shall not exceed the following when the ambient temperature of reference is 40 deg. cent.:

Contacts in air.....60 deg. cent.
Oil and contacts therein.....70 deg. cent.
Coils (See sections 376-379 incl.)[5]
Other parts (see section 392)[6]

Contacts in air may be subjected to an ultimate temperature at 70 deg. cent. for periods of short duration.

2. Rule 755. June 1917, supplement to A.I.E.E., Standardization Rules.
3. Rule 752, June 1917, Supplement to A.I.E.E. Standardization Rules.
4. Rule 754, June 1917, Supplement to A.I.E.E. Standardization Rules.
5. The rules referred to herein, limit the maximum permissible temperature rise of coils to temperatures determined by the insulating materials used.
6. The rule referred to above reads in part as follows: "All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material may be operated at such temperatures as shall not be injurious in any other respect."

"The Institute recognizes the inherent decrease in capacity of switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit breakers is, therefore, based on sufficient maintenance to keep the temperature within the specified limits."

The *normal current* in a circuit of an electrical system is the rated current in r.m.s. amperes for which that circuit is designed.

The actual current may vary through wide limits from day to day and at different seasons of the year. The upper limit for continuous operation or the rated current is, however, fixed by the capacity of the conductors as determined by the maximum allowable temperature at which the conductors and their insulation may be operated.

The *interrupting* (rupturing) *capacity* of a circuit breaker as specified by the Standardization Rules[8] of the American Institute of Electrical Engineers is:

"——the highest r.m.s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times."

It is recognized that factors anticipated in the above rule as the "prescribed conditions" may affect the interrupting capacity of the breaker. Such factors are discussed under the section: "Present Interrupting Capacity Rating."

The "stated intervals" and "specified number of times" at a given current and pressure determine the *duty* imposed upon the breaker. The breaker interrupting capacities in r.m.s. amperes published by various manufacturers are based on an assumed duty, *i.e.*, that the breaker will interrupt its rated r.m.s. current two times at a two minute interval and then be in condition to be closed and carry its rated current until it is practicable to inspect it and make necessary adjustments.

The duty, including a statement of the "prescribed conditions" therefore, places a limit on the interrupting capacity of a breaker and any change in duty or prescribed conditions will necessarily affect the rated interrupting capacity.

Previous Interrupting Capacity Ratings

It has been the practise in the past to state the interrupting capacity of circuit breakers in terms of the total alternator capacity in kv-a. at a specified reactance. In rating a circuit

8. Rule 753, June 1917, Supplement to A. I. E. E. Standardization Rules.

breaker in these terms, consideration was given to the short-circuit characteristics of the machines together with the characteristics of the circuit breakers and relays.

The "arc kv-a." ratings as previously listed were derived by multiplying the interrupting capacities in amperes by an assumed value of pressure. This assumed value was considered as the probable pressure that would be re-established on the bus immediately after the short circuit was cleared or that occurring during the clearing. The "arc kv-a." rating, based on the assumption that the re-established bus pressure will be normal, can be obtained for three-phase circuits by multiplying the interrupting capacity of the breaker in amperes by the normal pressure in volts of the circuit to which it is connected, and by the factor 1.73 divided by 1000.

It is to be noted that the interrupting capacity rating specified by the A. I. E. E. in r.m.s. current anticipates normal pressure to be re-established. Systems having characteristics such that the re-established pressure during short circuit will be higher than normal, will require a larger breaker.

As power stations have increased in capacity, and transmission and distribution net works extended, considerable reactance is introduced between the alternators and the point of short circuit which limits the current appreciably, and the total alternator capacity is no longer a measure of the severity of the short circuit. Service considerations are becoming more severe and larger and more expensive circuit breakers are required. A method of rating circuit breakers that will allow of more accurate selection for a wide range of conditions is desirable.

Present Interrupting Capacity Rating

The rating of a circuit breaker in r.m.s. current interrupted at normal operating pressure simplifies the selection of a proper breaker for a given service condition. Such a rating makes comparative tests possible if a sufficient amount of power is available.

In the A. I. E. E. rule establishing this rating it is qualified by the words "prescribed conditions". It is generally recognized, indicated by test and by the operation of circuit breakers in service, that the power factor or the stored electrostatic and magnetic energy of the system are among the conditions affecting the interrupting capacity at a given r.m.s. current. During the current-opening periods an arc is established, and

the current and voltage relations during this period are much more complicated than the simple phase-angle relation covered by the statement of power factor. Furthermore, the arc may be re-established under transient voltage conditions, still further complicating the phenomena. The theoretical and empirical data available on the effect of these conditions on the work done by the breaker are not at present adequate for the authors to prescribe any particular power factor for test, or to suggest any method for general use, to take into account the power factor and energy storage characteristics for all systems. These factors are of special importance when a breaker of relatively small interrupting capacity is connected to a large system in such a way that the breaker may afford the only outlet for the stored energy of the system. Such influences, while extremely difficult to take into account, will not differ widely in average systems. They are taken into account in a general way in the factor of safety employed in the rating of a breaker and their effects need not be considered in ordinary individual problems.

Selection of breakers for unusual conditions, large systems, or those involving large investment, should be checked with the manufacturers.

For average systems, a determination of the r.m.s. current that will flow at the instant the contacts part, irrespective of power factor or circuit conditions, will enable one to select the proper circuit breaker.

Determination of Short-Circuit Current

In order to determine the r.m.s. current that the circuit breaker will be required to open, an analysis of short-circuit phenomena on an alternating-current net work is necessary, and it may be desirable to call attention to some of the conditions under which circuit breakers may be called upon to function.

Short-circuiting a system at any point permits an abnormal current to flow immediately in that system. The amount and persistency of this current rush depend upon the characteristics of the synchronous apparatus connected to the system at the time, and upon the impedance in circuit between the synchronous apparatus and the point of short circuit. The value of r.m.s. current which the circuit breaker will be called upon to interrupt, will depend upon the length of time that elapses between the start of short circuit and the parting of the contacts as will be seen from the following analysis of short circuits.

The greatest transient disturbance of the system which can occur at the point of application of the circuit breaker when the system is short-circuited, governs the selection of a suitable breaker. A diagram showing the current flowing in one phase of the external circuit when a system is short-circuited under ordinary operating conditions, is shown in Fig. 1. In this diagram O is the origin of the co-ordinates and is taken at the instant at which the short circuit occurs. $O\,X$ is the axis of abscissas and the abscissas represent time. $O\,Y$ is the axis of ordinates and the ordinates represent current. $C\,D$ is a curve passing through the maxima of the wave of the total current and $E\,F$ is a curve passing through the minima. $A\,B$ is a curve

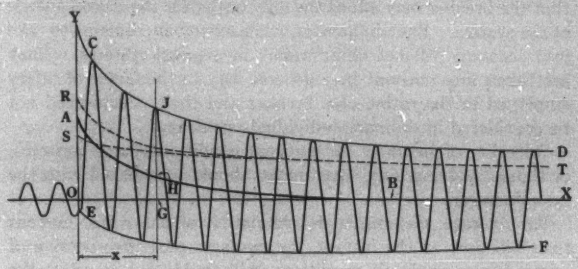

FIG. 1

Generator characteristics—illustration of behavior of generator current when short circuited from full load at 0.8 power factor

which cuts the vertical everywhere midway between $C\,D$ and $E\,F$.

The wave of total current whose crests lie along curve $C\ D$ and $E\,F$ and whose ordinates are measured from the axis $O\,X$ may be regarded as having two components, namely,

1. A direct component
2. An alternating component.

The direct component is represented at any time by the ordinate to the curve $A\,B$ or at the time x by the ordinate $G\,H$.

The alternating component is a wave whose crest value at any time is the difference between the ordinates to the curves $C\,D$ and $A\,B$. This difference, at the time x, has the value $H\,J$. The r.m.s. values of this alternating component are shown on curve $S\,T$. At any instant this component is considered to have the same r.m.s. value as an alternating wave of constant ampli-

tude whose crest value is represented by one half the distance between curves *C D* and *E F* at that instant.

The r.m.s. value of the total current wave under short circuit at any instant is the square root of the sum of the squares of the value of the *direct component* and the *effective* value of the *alternating component* at that instant. It is represented by the curve *R. T.* This r.m.s. value of the total current at the time of parting of the circuit-breaker contacts is used in making circuit-breaker applications.

The r.m.s. current at any point of a system under short-circuit conditions is affected by the following factors.

1. The total kv-a. reactance and transient characteristics of the synchronous machines connected to the system.
2. Number, reactance, resistance, capacitance and arrangement of all circuits over which power can be supplied to the point of short circuit.
3. The kv-a., arrangement, resistance, reactance and capacitance of all reactors and transformers through which power can be supplied to the point of short circuit.
4. Contact resistance at the short circuit.
5. The nature of the short circuit, whether single-phase or multiphase.
6. The kv-a. and power factor of the load being carried at the time of the short circuit.
7. The point of the pressure wave at which the short circuit was established.
8. The use of automatic voltage regulators.

The short-circuit transient for systems may be determined by test, by calculation or, less closely, by assumption. Obviously, the determination by test for all circuits of a large system is expensive and involves considerable time and interruption to service. This will be practicable in but few cases. The determination by calculation is also a matter of considerable labor but is feasible if only the important factors listed above are considered. Practical approximate selection, sufficiently accurate for many cases can be made by using only reactance and an accepted group of time-current decrement curves.

Suggested decrement curves are shown on Figs. 2 and 3 and their ordinates on Table I.

These curves are based on the following assumptions: Transient characteristics for alternators of normal design determined from oscillograph tests: That the effect of capacitance and resistance is neglected: That the contact resistance at short

circuit is zero: That the alternator is carrying full load 80 per cent power factor: That the short circuit was established at the point of the pressure wave corresponding to maximum possible

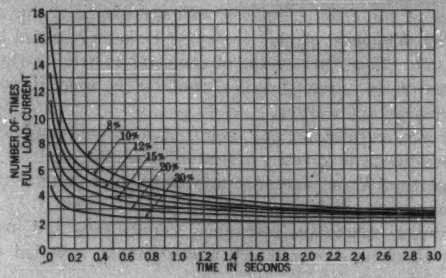

FIG. 2

System short-circuit characteristics—8, 10, 12, 15, 20 and 30 per cent total reactance based on total kv-a. rating of synchronous machines

Time-current curves—r. m. s. current in terms of total full-load current of machines—initial full load at 0.8 power factor assumed

instantaneous current: That no automatic voltage regulators are used.

These curves differ from those that have been usually considered in the past for two reasons: first, r.m.s. values are used

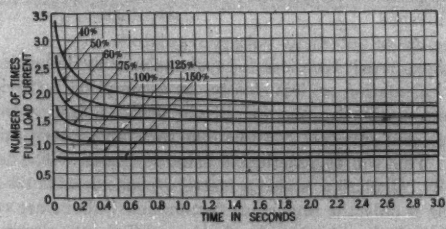

FIG. 3

System short-circuit characteristics—40, 50, 60, 75, 100, 125 and 150 per cent total reactance based on total kv-a. rating of synchronous machines

Time-current curves—r. m. s. current in terms of total full-load current of machines—initial full load at 0.8 power factor assumed

instead of peak values; and second, the effect of the increased flux existing under the load condition assumed has been taken into account.

The effect of using r.m.s. values instead of peak values is to

TABLE I—SHORT-CIRCUIT CURRENT FACTORS.

Reactance*	8%	10%	12%	15%	20%	30%	40%	50%	60%	75%	100%	125%	150%
Elasped time in seconds from start of short circuit	Current Factors expressed as number of times full-load current.†												
0.05	13.91	11.16	9.59	7.68	6.04	4.03	3.01	2.40	2.00	1.58	1.17	0.92	0.77
0.08	11.78	9.54	8.25	6.66	5.27	3.59	2.74	2.21	1.86	1.50	1.13	0.90	0.76
0.10	10.94	8.89	7.68	6.23	4.97	3.41	2.63	2.13	1.81	1.46	1.11	0.89	0.76
0.15	9.16	7.54	6.57	5.40	4.38	3.08	2.42	2.00	1.71	1.41	1.09	0.89	0.76
0.20	8.24	6.80	5.97	4.95	4.06	2.92	2.30	1.92	1.66	1.38	1.08	0.88	0.76
0.25	7.55	6.28	5.54	4.63	3.82	2.79	2.23	1.87	1.63	1.36	1.07	0.88	0.76
0.30	7.03	5.88	5.19	4.39	3.67	2.70	2.18	1.84	1.60	1.34	1.06	0.88	0.76
0.40	6.27	5.30	4.74	4.03	3.40	2.57	2.10	1.79	1.57	1.32	1.06	0.87	0.76
0.50	5.74	4.91	4.40	3.80	3.23	2.48	2.04	1.75	1.54	1.31	1.05	0.87	0.76
0.70	4.99	4.34	3.93	3.45	2.98	2.34	1.96	1.70	1.51	1.29	1.04	0.87	0.76
1.00	4.25	3.77	3.47	3.11	2.73	2.21	1.88	1.65	1.48	1.27	1.04	0.87	0.76
1.50	3.63	3.31	3.08	2.82	2.53	2.10	1.81	1.61	1.45	1.25	1.03	0.87	0.76
2.00	3.20	2.98	2.82	2.63	2.39	2.03	1.77	1.58	1.43	1.24	1.02	0.87	0.76

*Reactance expressed in per cent based on total kv-a. rating of synchronous machines. This includes both internal reactance of machines and reactance of external circuit reduced to the above basis.

†Rated full-load current based on maximum continuous kv-a. rating of synchronous machines. When the equivalent reactance of line, reactor, transformer, or combination of these expressed in per cent based on the total synchronous machine rating exceeds 150 per cent, the current to be interrupted may be determined directly from the reactance. This is due to the fact that under those conditions the generator reactance and the time of opening of the breaker may be neglected.

appreciably reduce the ratio between short-circuit and rated amperes. For example, assuming 10 per cent reactance and a short-circuit current containing the maximum possible direct component, the ratio of the peak value of the first alternation of the short-circuit current to the peak value of rated current is roughly twenty. Under the same conditions the ratio of the r.m.s. value of the first alternation of the short-circuit current to the r.m.s. value of the sinusoidal rated current is roughly seventeen.

The effect of using the flux at rated voltage and the assumed load instead of at rated voltage and no-load is to increase the short-circuit current by a somewhat less percentage than the alternator reactance percentage. This effect in alternators of low reactance relatively is unimportant but assumes increasing importance as the alternator reactance increases.

The characteristic shapes of the time-current decrement curves have been arrived at by analysis of alternator tests including oscillograph studies of short circuits occurring when the alternators were excited to full voltage and were carrying various loads at various power factors.

In the curves for total reactances up to and including 20 per cent, the reactance is assumed to be wholly within the alternator and for higher values of reactance the alternators were taken at 20 per cent and due allowance made by calculation for the effect of the external reactance. In the latter case, if alternators of other reactance had been assumed the results would have been somewhat different but the error is not large enough to be of practical importance.

The final values of the current *i.e.*, the sustained short-circuit current, have been assumed in accordance with experience and tests and are based on the behavior of machines of normal design.

The study of representative oscillograms of short-circuit tests showed that in most cases the direct component disappeared within 0.5 second and that the transient portion of the alternating component disappeared within 3.0 seconds. These time values have therefore been used in constructing the characteristic curves.

Several alternators with the same reactance and synchronous impedance will not necessarily have the same rate of r.m.s. current decay. This has been considered in constructing the characteristic curves and they may safely be taken as representing the greatest r.m.s. currents that will be given by modern alternators of normal design.

These curves are applicable for selecting circuit breakers for systems as follows

1. Single machines without external reactance.
2. Single machines in combination with external reactance.
3. Multiple machines with no external reactance.
4. Multiple machines in combination with external reactance.

Examples

In order to illustrate the use of these curves in making oil circuit-breaker selections, the following examples are given:

Example 1. (Arrangement of apparatus shown in Fig. 4.) Alternator rating 5000 kv-a., 2300 volts three-phase, 1250 amperes. Breaker contacts part in 0.25 second after start of short circuit. From table, under 12 per cent reactance, we find that at 0.25 second the current will be 5.54 times normal, therefore

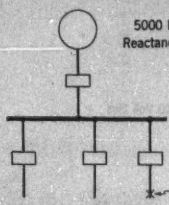

Fig. 4

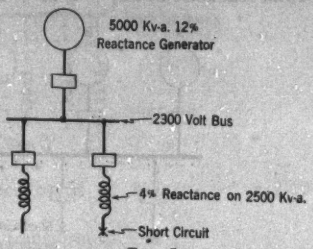

Fig. 5

the short-circuit current equals 5.54×1250 amperes $= 6950$ amperes.

Example 2. (Arrangement of apparatus shown in Fig. 5.) Alternator rating 5000 kv-a., 2300 volts, three-phase, 1250 amperes. Feeder rating 2500 kv-a. Feeder reactance 4 per cent based on 2500 kv-a. Breaker contacts part in 0.25 second after start of short circuit.

Alternator reactance based on 5000 kv-a.	=	12 per cent
Feeder " " " " "	=	8 per cent
Total " " " " "	=	20 per cent

From the table under 20 per cent reactance, we find that at 0.25 second the current will be 3.82 times normal, therefore the short-circuit current equals 3.82×1250 amperes $= 4780$ amperes.

Example 3. (Arrangement of apparatus shown in Fig. 6.)

Alternator *A* rated 2000 kv-a., 2300 volt, three-phase reactance = 8 per cent

Alternator *B* rated 5000 kv-a., 2300 volt, three-phase reactance = 12 per cent

Alternator *C* rated 8000 kv-a., 2300 volt, three-phase reactance = 16 per cent

Total alternator kv-a., 15,000

Normal current based on 15,000 kv-a., 2300 volts = 3760 amperes.

Breaker contacts part in 0.4 second after start of short circuit.

Alternator *A* reactance based on 15,000 kv-a. = 60 per cent
" *B* " " " " " = 36 per cent
" *C* " " " " " = 30 per cent

Total reactance at bus = 1/60 + 1/36 + 1/30 = 12.9 per cent.

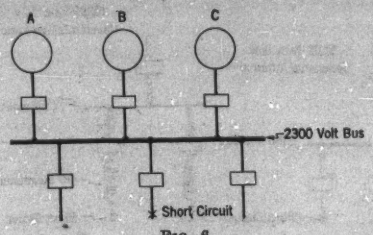

FIG. 6

From the table, interpolating between 12 per cent and 15 per cent reactance, we find that at 0.4 second the current will be 4.53 times normal; therefore, the short-circuit current equals 4.53 × 3760 amperes = 17,030 amperes.

Example 4. (Arrangement of apparatus shown in Fig. 7.) Same as example 3, excepting that power is distributed over 2500 kv-a. feeders in which are installed current limiting reactors having a reactance of 3 per cent based on 2500 kv-a.

Breaker contacts part in 0.4 second after start of short circuit.

Total alternator reactance based on 15,000 kv-a. = 12.9 per cent
Feeder " " " " " = 18 per cent

Total " " " " " = 30.9 per cent

From the table using 30 per cent reactance, we find that at 0.4 second the current will be 2.57 times normal: therefore, the

short-circuit current equals 2.57 × 3760 amperes = 9650 amperes.

Example 5. (Arrangement of apparatus shown in Fig. 8.) Breaker contacts part in 0.1 second after start of short circuit.

Alternators same as for example 3.

Transformer banks each 7500 kv-a.; reactance = 6¼ per cent based on 7500 kv-a.

Lines: 20 miles of 1/0 copper; reactance = 5.5 per cent based on 7500 kv-a. at 60 cycles and 44,000 volts with 4-ft. conductor spacing.

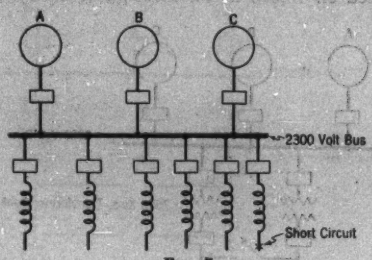

FIG. 7

Total alternator reactance based on 15,000 kv-a. = 12.9 per cent.

Parallel reactance of step up transformers based on 15,000 kv-a. = 6.25 per cent

" " " lines based on 15,000 kv-a. = 5.5 per cent

" " " step-down transformers based on 15,000 kv-a. = 6.25 per cent

Total reactance..... = 30.9 per cent

From the table, using 30 per cent reactance, we find that at 0.1 second the current will be 3.41 times normal. The normal current based on 15,000 kv-a., 11,000 volts, three-phase = 788 amperes; therefore, the short-circuit current equals 3.41 × 788 amperes = 2690 amperes.

Example 6. (Arrangement of apparatus shown in Fig. 9.) Breaker contacts part in 0.4 second after start of short circuit. Conditions same as for example 5 except that a 475-kv-a., 2300-volt feeder has been added to the low-voltage distribution.

Transformer reactance, 3 per cent based on 475 kv-a.
Total reactance up to 11,000-volt bus
based on 15,000 kv-a. = 30.9 per cent
Feeder transformer reactance
based on 15,000 kv-a. = 94.5 per cent

Total reactance based on 15,000 kv-a. = 125.4 per cent

From the table, using 125 per cent reactance, we find that at 0.4 second the current will be 0.87 times normal. The normal current based on 15,000 kv-a., three-phase 2300 volts = 3760

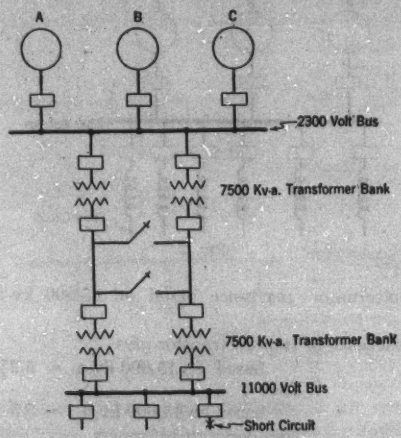

FIG. 8

amperes, therefore the short-circuit current equals 0.87 × 3760 amperes = 3270 amperes.

With reactance of 125 per cent or higher values, the alternator portion of the total reactance becomes of small importance. In example 6 for instance, we have a total reactance of 125.4 per cent. The alternators in this example have a reactance of 12.9 per cent. If the reactance of the external circuit only is considered we have a total reactance of 125.4 per cent − 12.9 per cent = 112.5 per cent.

The short-circuit current on this basis would be

$$\frac{100}{112.5} \times \text{normal, or } 0.890 \times 3760 = 3340 \text{ amperes.}$$

Comparing this value with the 3270 amperes obtained by considering the alternator reactance we have an error of approximately 2 per cent.

If the total reactance is greater than about 125 per cent or 150 per cent, the error will be even less than 2 per cent. For values of external reactance in excess of the table values the alternator reactance may be omitted.

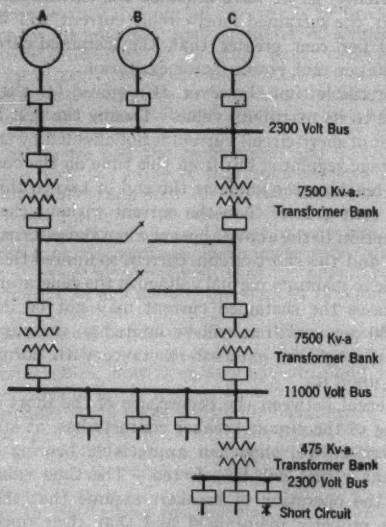

FIG. 9

PRECAUTIONS

The data given for the selection of oil circuit breakers is applicable only to average systems. Therefore, a short discussion of other factors requiring separate or more detailed attention seems worthy.

Automatic voltage regulators may introduce system transients differing from those which occur on systems not so equipped.

When the alternators are equipped with automatic voltage regulators such regulators will increase the excitation after a short circuit in the endeavor to hold normal voltage on the bus

bars. The maximum voltage which can be obtained from the exciters will ordinarily be not more than 50 per cent greater than that required at full load, 80 per cent power factor on the alternators. Under short circuit, the alternator terminal voltage is reduced, hence the resultant flux density in the alternator iron is also reduced. A given increase in excitation, therefore, produces a proportionate increase in current flowing in the short circuit. Hence, as we have assumed the excitation to increase 50 per cent, the sustained short-circuit current will be approximately 50 per cent greater than the sustained current due to full-load 80 per cent power factor excitation.

An appreciable time, however, is required for the excitation to increase to its maximum value. During the first half second the amount of short-circuit current is not affected by the presence of the voltage regulator, but from this time on the current curve is higher, reaching the value at the end of two to three seconds of 50 per cent greater than the current without the regulator.

An exception to the above appears when the external reactance is so high and the short-circuit current so limited that the regulator is able to maintain normal voltage at the generator terminals. In such cases the sustained current may not be increased as much as 50 per cent, but will be limited to the current which will pass through the external reactance with normal voltage impressed upon it.

The interval between the occurrence of the short circuit and the parting of the circuit breaker contacts has, as study of the selection curves will show, an appreciable bearing on the oil circuit breaker that will be selected. The time values usually given for the operation of breakers assume that the breakers have been properly maintained and that their operation will not be impaired by factors resulting from neglect of maintenance or adjustment.

It is believed that the manufacturers of oil circuit breakers should publish information similar to that outlined in this paper to serve not only as a guide in the application of electrical protective devices but also to assist in bringing about a more uniform selection and a better understanding of the various expressions and methods used in connection with the rating of this class of equipment.

DISCUSSION ON "RATING AND SELECTION OF OIL CIRCUIT BREAKERS (HEWLETT, MAHONEY AND BURNHAM), NEW YORK, FEBRUARY 15, 1918.

E. M. Hewlett: What we have been struggling with is to make a "Bench Mark" from which we can all start and work out a common language. In the past the folks who were working on generators, on reactances, on transformers and on the lines outside, had each their own problems and their own equations, which caused much confusion, and this confusion was further added to because they did not all use words in the same way.

Then, as to the quantity and character of amperes used—when we were called upon to interrupt larger capacities as the generator grew larger, we asked the question, what is the current we are to interrupt? We were told by the calculators that it was about three times full-load current,— but we knew we were experiencing currents much larger, that is to say, we felt this even before we had the oscillograph and before we could tell anything from the swing of the ammeter, etc. Finally we obtained an oscillograph and discovered that we had swings that ran up many times,—ten times or so. But then in figuring these amperes, sometimes figured from the zero line, sometimes symmetrically and sometimes unsymmetrically, we found it very difficult to get an agreement, thus the papers presented in Boston a year or so ago started the work of bringing the terms together.

In our work on this paper with the different companies, we have looked over the various generator curves and determined their characteristics, how they average, etc., and have found that it was possible to do a thing that we did not think would be possible, that is, to make a general characteristic curve that was close enough to be within a negligible error. We realized that there may be machines above and below that curve, but consider those as special conditions, and they should be recognized and considered as such.

Now, as to the testing equipments, and the different tests to determine the rupturing capacity, etc., we have been getting larger tests and tests on different plants.

Of course, it is very difficult to get the use of the line and the plant to test 100,000 kw., etc., so that we have only been able to establish a series of tests on smaller capacities, and have had to depend on the tests in operation, incidental short circuits, etc., and the readings taken at that time and observations made that would indicate the limits of switches, so as to give a rating.

We hope later to be able to make tests and to check up and settle definitely all of these different ratings. The work, however, will be carried on as rapidly as possible, and the line extended as capacities are available and as different tests are made available.

The information that we get in reference to the effectiveness of the circuit breakers, etc., is very conflicting. We find that one man will say, "Why, yes, it works fine. It opened the

circuit." Then you ask him, "Well, what happened to the circuit-breaker." He will reply, "well, we got the circuit breaker fixed up before we could get the trouble fixed up, but that was all right." Another man will say, "We had a great deal of trouble." Probably he had blown out a half pint of oil and that had bothered him, so that the different viewpoints of the user are not standardized, either.

The kilowatt rating that we formerly used was a great convenience up to a certain point, and we will have now a kilowatt rating to be used on transformers; that is, a transformer limit, a rating that will show the size of transformers that can be used with a given breaker, to limit the capacity and make the breaker of easier application.

If we can get any constructive criticism and help as to what can be done in the way of getting tests, and in standardizing so that we can use the information we have, that is, the fundamental characteristics and r. m. s. current values, we will feel as though quite a step will have been made, and other papers will be brought out from time to time that will finally put the industry in this respect more in the shape of a science than of an art.

J. N. Mahoney: In reference to the data suggested, in this paper, as the basis of comparison of "Interrupting capacity" rating between the different manufacturers. Some may question the data, as not being derived from service practise. We state the device obeys Institute Rules regarding "duty" rating by interrupting specified power two times at a two-minute interval. In practise there are all degrees of requirement, ten operations some want, and others are satisfied with a single successful interruption. The natural trend of practise is that requirements are not constant—they change from operation to operation, whereas the condition specified by the Institute rule is necessarily constant, such as a given maximum of amperes. We assume that the usual short-circuit current may be only a fraction of the current interrupting guarantee specified which contemplated maximum conditions. In practise we have found that interrupting the actual maximum two times at a two-minute interval permits of more operations under average conditions. The current is less, maybe one-half, and you get approximately ten operations with little depreciation of the breaker details, as the "short" is out on the line and not at the breaker. I mention this so that it will not be inferred that two operations are the only duties the breaker is good for. It is good for all the operations you want as long as the oil and contacts are in reasonable condition.

Henry R. Summerhayes: In the paper there is mentioned the present interrupting capacity rating, and in connection with that rating the words "prescribed conditions" are used. I want to make some comments on "prescribed conditions." They were presumably put in the Institute Rules as to interrupting

capacity in order to allow conditions to be described, when we knew enough about the theory of the circuit breaker operations to describe conditions under which tests could be made. The theory is not in a very satisfactory state. Of course, it is generally recognized that the work imposed on the circuit breaker is the kv-a. used up or dissipated in the circuit breaker during the time it is opening, and another element that may appear to have a bearing on that and at first sight a very simple relation is the power factor of the current flowing in the short circuit.

If the current wave and the voltage wave are in phase, 100 per cent power factor, and if according to observations, as circuit breakers ordinarily do, the circuit is opened at the zero of the current wave, the voltage wave is at that time zero, and there is little tendency to establish the arc. If, on the other hand, you have 90 deg. lag., so that the current ceases to flow at the moment when the voltage is at a maximum you apparently have the worst condition, the tendency to re-establish the arc, due to voltage being high.

In the case of most of the short circuits the reactance in the circuit predominates, and the power factor is low, ordinarily varying from zero, or nearly zero to about 20 per cent. With that variation perhaps the voltage wave is fairly flat, and at nearly the maximum voltage, so that ordinary variations of the power factor, theoretically, should not make much difference in the working of the circuit breaker, but if the power factor is very high, then the work is apparently less. That is based on the assumption that the current and voltage during the period of circuit breaker opening follow a sine wave. From this point of view it would appear that the work imposed on a circuit breaker would not vary much for power factors from zero to about 20 per cent, because if the current and voltage during the period of circuit breaker opening follow sine waves, and the current ceases to flow at zero of the current wave, the voltage tending to re-establish the arc will be nearly at the maximum point of its wave, for a variation of power factor from zero to about 20 per cent.

However, the arc in the breaker may set up oscillations in the system superimposing high frequencies on the simple sine waves with reestablished voltages in the circuit breaker higher than the circuit voltage, and with varying phase relation between the voltage and current, so the simple relations no longer exist, and it is difficult to determine by test or in any other way the actual power factor existing at that time.

It is generally assumed that the presence of high reactance and low power factor in the circuit increases the electromagnetic storage of energy which may have to be dissipated in the circuit breaker by these oscillations, and therefore may present a severer condition for the breaker to handle than when the current in the circuit is limited chiefly by the resistance which

absorbs the energy of oscillation. According to that view, the observed fact that the circuit breaker has an easier time when the power factor is high may depend on the absorption of the energy oscillations by resistance; that is, the resistance in the circuit predominates rather than being dependent directly on the power factor. The power factor during short circuit may also be affected by the resistance in the arc.

Due to these considerations it would appear, as stated in the paper, that the data available are not sufficient to state how much the interrupting capacity of the breaker is affected by the relative resistance, reactance and capacitance of the circuit. Apparently we have not enough data to prescribe at what power factor the tests should be made.

As the authors point out, the average circuit has parallel feeders and the breaker will seldom be required to dissipate all of the stored energy of the system; in other words, when you have parallel feeders with more or less non-inductive load, it is thought that these act as a resistance, and take up part of the energy. If you set up test conditions, with all of the generators running, and a single feeder, and have the circuit breaker on that feeder, it is a more severe condition than if the breaker opens the same amperes, but with three feeders running in multiple.

It is believed that average systems are generally similar as to the conditions of stored energy, electrostatic and electromagnetic, so that the test conditions are generally more severe than the service conditions, and the curves given in the paper will probably be safe for nearly all conditions.

W. W. Willard: The statement is made that the accurate calculation of the currents that will result from short circuit is usually not practicable, because of the dearth of data for the purpose, and also because of the alternator power; and certain suggested diagrammatic curves, shown in Figs. 2 and 3, are offered as a means of determining the excess currents that will result in varying cases from short circuits.

It is worth while to note, however, that under "Precautions," the data given for the selection of oil circuit breakers are applicable only to average systems. We are also reminded that the selection of circuit breakers for unusual conditions, large systems, etc., should be checked with the manufacturers. One of the sets of conditions under which this would seem to be desirable is suggested. This suggestion reminds us that the curve indicated for this purpose should take into account the nature of the short circuit, that is, whether it is single-phase or multiphase. There are several ways in which a short circuit can occur on a system, especially on a polyphase system, and one of these is a case in which all of the legs of the circuit are involved.

One of them is a common case, in which only two legs are involved, and another is the case where a single leg of a circuit is involved in a ground on a system which is Y or star-connected,

with ground neutral; or in the case of 3-phase, 4-wire circuits, in which there is a short circuit from one of the phases to the neutral.

The curves proposed in this paper have been drawn in such a way as to include a factor of safety sufficient to provide for the case of a short circuit in which all of the legs of the circuit are involved, and also for the majority of cases where single-phase short circuits occur on polyphase systems, notwithstanding the fact that a single-phase short circuit occurring at the terminals of a polyphase generator may and usually will result in higher values for the sustaining current, that is the current which will exist for a lapse of say, two or three seconds, than would be the case if all the legs of the circuit had been involved in the short.

It is believed also that the curves are drawn high enough to provide for the common case of a ground on one leg of a Y-connected system, with grounded neutral, or of a short circuit from one leg to neutral, unless the neutral is brought out from all of the generators, or all of the generators grounded with current limiting resistance or reactance. If only one of the generators is grounded, and there are several generators, so that the reactance of the grounded generator, figured on the accurate capacity of the generators, is within the limits of the table, such a generator might operate as a ground to a limited reactance. If, however, no such condition exists, and it is desirable to apply a circuit breaker between a generator having a grounded neutral and a generator bus, that is to say, on the generator side of the first transformer bank, the current resulting from a short circuit of that kind will probably be high enough to make it necessary to exercise extra precaution, and to consider that one of the exceptional cases, that will require us to learn the magnitude of such currents from other sources than the curves given in this paper, as the curves given are hardly suitable for that purpose.

Bassett Jones: I speak from the point of view of the man who has to advise another man how to invest his money in circuit breakers. The paper presents a very definite, easily understood method of determining the duty that may be imposed on an oil circuit breaker. Except in a very general way, it does not determine a method of selecting any particular variety of breaker. This latter problem has always been shrouded in deep mystery, particularly in view of the fact that so many varieties of breakers have been devised to meet such a multitude of special conditions, that the manufacturers' catalogues and published ratings are not always of use. By this I mean that even when the duty required has been determined by the method outlined in the paper, it does not necessarily follow that the proper or rather the most economical type of breaker can be selected directly from a catalogue or hand book.

A case develops in which with, say, a rated full-load capacity of 12,500 kv-a., 10 per cent reactance in each generator con-

nected to the bus, and with what may seem a reasonable per cent of added reactance in the bus, a short-circuit value of current in a feeder figured in this manner would lead the uninitiated to select a breaker costing, say, $500, when actually a considerably cheaper breaker, with one or another of the multitude of standard reinforcings will meet the conditions. Thus, we find a manufacturer carrying in stock, say, twenty-five different varieties of a single type of circuit breaker, because the consumer has imposed at least this number of operating conditions which the manufacturer must meet if he is to make a sale and keep the business. Each time the manufacturer has sought a new way out of his dilemma by devising a new variety of reinforcing, or a slight change in design which will enable him to get away with the sale. And, withal, he probably takes a chance at that—a chance that will most hurt the consumer if the dice fall the wrong way.

This sort of thing results only in harm to all parties concerned. It works an undoubted hardship on the manufacturer, for in time the cost to him of maintaining so many standards, any one of which may have relatively few sales a year, becomes prohibitive. This means more capital invested, and when we get right down to brass tacks it is the consumer who invests his own capital in the manufacturers' stock room. The term "stock room" here covers a multitude of sins.

In a paper recently presented before the Schenectady Section, I drew attention to the economy both to consumer and manufacturer that could be achieved by "Standardization of Demand" in the special case of the low-tension distributing systems in industrial plants. The same remarks apply in a broader sense to distributing systems of all kinds. The distributing system is to be considered as including everything engaged in carrying energy from the generator to the point of conversion or utilization—switchboards as well as transmission equipment.

The problem of "Standardization of Demand" is obviously up to the consumer and his advisers rather than to the manufacturer.

Fred C. Hanker: The rating of circuit breakers has been specified as the highest r. m. s. current that can be interrupted under certain conditions. It is this basis of specifying rupturing capacity which makes it necessary to determine the value of the short-circuit current before the breaker can be properly selected for a specific application.

The authors have given in Figs. 2 and 3 a series of curves for various values of total system reactance. They have, for convenience, tabulated these data.

With this information it will then be possible to properly apply a given breaker, after the method of tripping and the value of feeder reactance in the system is determined.

The elapsed time can be ascertained with accuracy once the method of tripping is adopted and the speed of the breaker

is specified. Furthermore, the reactance of the system can be calculated from the characteristics of the apparatus under consideration.

This, then, leaves the short-circuit transients of the system to be determined in one of the ways outlined by the authors, so that the value of the r. m. s. current at the time of opening the breaker can be secured.

Obviously, from the number of factors affecting this characteristic, it is not a simple calculation, and it is always debatable whether all of the variables have been given proper weight or consideration. Even granting the ability to predetermine this characteristic, the best criterion of its accuracy is a check against actual results obtained from oscillograph tests.

The curves as given in Figs. 2 and 3, showing the relative values of r. m. s. current in terms of the total full-load current of machines, and under short-circuit conditions, check, reasonably well, the r. m. s. current values determined from actual oscillograms taken on a-c. generators when short-circuited at full voltage. After the first fraction of a second following short circuit, the actual curves fall slightly below the proposed curves, due to the fact that the oscillograms were taken for short-circuit conditions at no load, and full voltage, whereas the proposed curves are based on full-load conditions, at 80 per cent power factor.

From this check it is the opinion that the curves are satisfactory and represent with safety the short-circuit currents on three-phase short circuits that may be expected from normally designed generators.

The curves that we used in making this check covered a wide variety of apparatus, as they included both high-speed turbo-generators as well as normal speed and low speed water-wheel machines. They do not take into consideration extreme power factor operation, as this has been specified at 80 per cent in the paper. Obviously, for lower power factor operation, where greater excitation is required, the sustained short-circuit values may be somewhat higher. These cases are rather rare, and would be taken care of without difficulty if they are once specified.

Another point is in the sustained value as given in the curves. They cover a certain range in values as given at three seconds. If the machine alone had been considered it might have been preferable to construct the curves on the basis of a given sustained value, but when you take into consideration the external reactance which occurs in all cases, probably above 15 per cent or 20 per cent, and in a large number of cases above 10 per cent and 12 per cent, that sustained value would be different for different conditions.

Philip Torchio: The subject of the selection of circuit breakers of large rating is of relatively great importance in but a few instances where the plants have very large capacity, and, as the

authors have specified, on such occasion a special study must be made jointly and in cooperation between the manufacturer and the user.

I shall not refer to that type of circuit breaker, except by making a general statement that as far as I know we have not today available a circuit breaker of sufficient capacity to open the short circuits on our larger systems with a short circuit in proximity to the generating stations. In such instances limiting reactances have to be used to confine the "short" within the rupturing capacity of the circuit breakers available.

In the development of the circuit breaker, we have apparently attempted to follow the example of the man who plays his fiddle by main force. We are forcing our circuit breakers to do all the work, and probably we are expecting too much in that direction; still, it is wise to keep on studying the problem, in the hope that some day we will find a circuit breaker that will be of unlimited rupturing capacity.

When it comes, however, to the general use of circuit-breakers —which represents 95 per cent of the circuit-breaker applications—for those cases the paper before us is applicable, and is a step in advance, and should be followed up by further standardization, so that, as some of the previous speakers have already pointed out, the user could more readily get at the values of the ratings of the circuit breakers offered by different manufacturers.

I also want in this connection to give a note of warning with reference to following out the suggestion in the paper that if we select a certain circuit breaker for certain existing conditions, we must remember that the conditions of the supplying electric systems change very materially with the growth of the system. The man who makes a selection must also have a view into the future, and not be guided only by what the conditions are today, or what he expects next year, but he should possibly foresee what the conditions may be in several years.

In my personal experience, I find that many times we are confronted with a proposition that today could be handled with a certain circuit breaker, which is relatively cheap, say costing $300. If we could foresee the conditions which will probably prevail three or four years from now, we would see that that circuit breaker should either be supplemented by a limiting reactance or a larger circuit breaker used. In a great majority of cases you will find that the larger circuit breaker is more economical than the combination of the small circuit breaker and the limiting reactance.

In customers' installations, it may be, however, impracticable to put in the larger circuit breaker—as, having only one circuit breaker on the premises, it would be preferable to avoid the operation of a complicated mechanism. The mechanism that would be perfectly feasible in the substation of an operating company might not be so in an industrial plant or on customers'

premises. In that case the limiting reactance, in combination with the small circuit breaker, in many instances, while it may be more expensive in first cost, I believe will be found to be preferable. If you take such point of view, broadly you will standardize the circuit breaker, and you will not have so many circuit breakers to select from.

Referring specifically to the paper, where it is stated—"Although not stated in rules, we infer that it contemplates a test with the apparatus under dry conditions." I imagine that the outlines mean that it was filled with oil, in accordance with the Standardization Rules of the Institute. The wording is not entirely clear to me as to what the dry condition means.

Referring to paragraph, "The breaker interrupting capacities in r. m. s. amperes published by various manufacturers are based on an assumed duty, *i. e.*, that the breaker will interrupt its rated r. m. s. current two times at a two-minute interval and then be in condition to be closed and carry its rated current until it is practicable to inspect it and make necessary adjustments." I suppose it is assumed that also it is in position to open the circuit a third time. That was our understanding—that is, when closed, it would be still in a position to open.

J. M. Mahoney: Open mechanically, but not automatically, under the same conditions.

Philip Torchio: Quoting again, "Systems having characteristics such that the reestablished pressure during short circuit will be higher than normal, will require a larger breaker," refers to circuit breakers at the ends of long distance transmission lines.

In reference to the dying away of the transient current in the direct component of the current—at least mention might be made—that in a system where they operate without Tirrill regulators, but with rheostatic control in the field, the rheostat resistance in series with the field would tend to accelerate the dying out, because it absorbs more rapidly the energy that is dissipated while in places where the Tirrill regulator is used, with less resistance in the field, probably there would be slower dying out.

In the section referring to the guarantees and heating in a general way we must depend on the manufacturers to arrange the guarantees, but I think there ought to be more co-operation, to get a little more uniformity, so that the user can have clear before him information for the selection of the circuit breakers needed.

Paul M. Lincoln: I want to call attention to one paragraph which is particularly interesting to me, it is that referring to the classification of systems, in which the authors say: "Systems may be classified according to their normal operating pressure, normal current, normal frequency and current transients." It is that last item, the "transients" that is the unknown quantity

in this classification. It was absolutely unknown to me at the time I entered the operating business some twenty five years ago with the Niagara Falls Power Company. In perfectly good faith, when we came to figure our short-circuit currents at that time, we figured the currents we were going to get on short circuit were the same we got on the short-circuit test made in the test room, and which amounted to two or two and one-half times the normal full-load current. When we got short circuits in practise we found evidence of the fact that the currents were not limited to two or two and one-half times full-load current but were many times that. Such things as the tearing apart of the windings, the breaking of windings, the throwing of cables around the power station, and similar things, gave evidence that the currents were not limited to the two or two and one-half times full-load current that we expected, and it was then that this question of transients began to have a real meaning for us.

Our knowledge of what transients are has, of course, increased in the time between that experience, back in 1896 and 1897, and the present time, but I think even yet there is much we have to learn about the matter of transients.

The authors speak of the use of reactances, and there is a more or less indefinite term used in connection with the rating of reactance. Reactances are usually rated in percentages of the normal voltage, but it is not often stated how much current these reactances are supposed to carry for that given stated percentage of reactance voltage, and the stating of that current is an absolute essential in the rating of the reactance. The percentage stated in terms of the normal carrying capacity of the reactance, or in the normal carrying capacity of the apparatus back of the reactance or in some definite manner is an absolute essential in arriving at the rating of any reactance.

On this question of the rating of circuit breakers, let me say that the manufacturers are at all times giving to the user of these circuit breakers the best they have. Whether or not the best they have is good enough to take care of the job of the user, there is only one way of determining and that is by trial.

As Mr. Hewlett pointed out, it is impossible for the manufacturer to go to the user of the apparatus, particularly if the user is on the order of 100,000 kw., or more, and ask him for the loan of his plant for the purpose of testing circuit breakers.

That is out of the question, because the user of that apparatus is not justified in taking the risk which would be involved in the testing of the circuit breakers under the actual operating conditions. As a consequence, the manufacturer must get his information for designing circuit breakers by the observation of short-circuit conditions that occur, and he must gather that information as it is presented to him from time to time. Consequently the question of just what breaker is necessary, just how big it should be, how much oil to use, and all those things

that are necessary to take care of given conditions, particularly when those conditions approach the limit of existing plants, must necessarily depend upon the closest possible cooperation between the manufacturer of the apparatus on the one hand and the user of the apparatus on the other. It is only by that kind of close cooperation between the two that the question can be ultimately worked out.

R. E. Doherty: I should like to make some comments that may be useful in applying the curves shown in Figs. 2 and 3. There is a large difference between the sustained short-circuit current of generators which were built 10 or 15 years ago, and those which have been built in more recent years. These curves show sustained values which correspond approximately to the maximum that would be found in machines which have been built during recent years by one of the manufacturing companies, whereas for older machines the curves show values which are more nearly representative of the average. That is, the older machines may have higher sustained short-circuit currents than indicated by the curves, and should therefore be treated as special cases.

It may be of interest that the difference between the old and the more recent generators is largely the result of the introduction of the automatic voltage regulator. Before the advent of the regulator, generators were required to have good inherent regulation, that is, low armature strength, relative to the field strength. That means high sustained current. With regulators, this restriction is removed, and for reasons of economy the armature is made stronger, which means lower sustained current.

Fred L. Hunt: It is not understood, that the authors have suggested any more definite means of rating circuit breakers than has already been done by the rules and practises to which they refer. Operating engineers will find all the information and data given in this paper to be most valuable and highly practicable, but it is believed that these engineers desire very much to have a more definite method of rating circuit breakers, presented. Nothing, so far as is known, which is now published by the manufacturers would enable the operating engineer to choose from the published data a circuit breaker suitable for one of the definite conditions outlined in the examples in this paper, assuming that he had the information regarding his own system shown in the examples.

Under present conditions the operating engineer must leave entirely to the manufacturer the question of deciding what circuit breaker is suitable for his particular conditions, without having any means of determining how well he has met the requirements specified, or of comparing the factors of safety which may be allowed by different manufacturers in their recommendations for the same switching requirements.

All will probably agree that the manufacturers are well qualified to recommend the circuit breakers that should be used for

any given set of conditions, but if circuit breakers are to be put on anything like the same basis, so far as comparative ratings are concerned, as is now the practise with other electrical apparatus, in order that operating companies may have an independent means of comparing the recommendations made by various manufacturers, and of testing under a fixed set of operating conditions the apparatus produced by various manufacturers, a system of ratings should be established which would be definite with regard to all the conditions which are pointed out in this paper as affecting the rupturing capacity of a switch.

Eight items are mentioned, all of which affect the current flowing to a short circuit at any point in a system. This does not mean, however, that there are necessarily that many different interpretations of a given rating for the rupturing capacity of a circuit breaker.

It is suggested that the manufacturers set down a definite rupturing capacity for a given switch, describing in as much detail as they choose the exact conditions upon which the rating is based, and that they then furnish curves or other data which will show the variation of this rupturing capacity due to—

First: Variation of the re-established bus voltage.

Second: Variation of power factor of the current flowing into the short circuit.

Third: Variation of the ratio between generator reactance and line or system reactance.

Fourth: Variation in the total capacity of synchronous equipment connected to the system.

Fifth: Variation due to nature of the short circuit, whether single-phase, three-phase, or between one phase and ground.

This will then leave to the engineer who applies the circuit breaker the duty of determining the transient characteristics of his synchronous machines, the time setting of his relays, the total impedance of his circuits leading to the point of short circuit, the resistance of the short circuit, the power factor of the load being carried at the time of the short circuit, and the bus voltage to be re-established after the opening of the circuit.

The data suggested above to be supplied by the operating engineer could all be obtained from other sources than from the oil switch designer.

Some such system of rating as that suggested above is required in order that operating engineers may be able to select switches for their circuits in the cases at least of all systems which do not come under the heading of "average systems", and it seems reasonable to ask the manufacturers to supply such information. It also seems desirable that the authors of this paper should define what is meant by an "average system".

The writer has recently had occasion to study switching capacities for a system in which a switch in one substation is apparently required to open a short-circuit current of approximately 40,000 kv-a., whereas a switch in another similar sub-

station not far away is required in the case of a short circuit to open 145,000 kv-a., and at another substation short-circuit conditions may require the opening of 270,000 kv-a. It would be interesting to know whether such a system would be considered an "average system".

Herbert H. Dewey: It is gratifying to the transmission engineer, to know that we have reached the point where we are talking about the same kind of amperes. If now we accept the fact that the circuit breaker designers have chosen the ampere-rating in rupturing capacity that their switches are good for, it is up to the power and transmission engineer, the man who makes the practical application of these switches, to determine how much current we have to rupture in a given case.

Some points in connection with determining this current have always been somewhat uncertain. We obtain from the manufacturer the reactance of the generator, transformer, etc., usually in per cent, and we know that a ten per cent reactance generator will give ten times normal current on short circuit. It has never been definitely understood by us all however what this ten times normal current is. One man suggests that it is ten times normal symmetrical, and it may be double that. The paper discusses the different values of current obtained on short circuit and defines the current to be used in determining the duty imposed on an oil circuit breaker as the r. m. s. current at the time the circuit breaker opens. The application engineer must determine the r. m. s. current at the point of short circuit by some suitable method.

On simple systems, which the paper purports to cover, that is an easy proposition. It is merely a question of adding up the reactances in series or multiple, as they occur on the system. When we have obtained the final result we have a reactance of, say, twenty per cent on the rating of the total generating capacity, and by referring to our curve we find that we get so many r. m. s. amperes at a given time.

The next point to determine is how soon the circuit breaker is to open. That will be a pretty definite thing because of its location. We use certain relays that must be selective. We use on the generators, as a rule, non-automatic switches. If we have to choose a generator switch, we consider the time it will take for the operator to open the switch, which means constant value of current. If we have a differential relay, which is expected to open the generator circuit in case of breakdown of the generator itself, that means we have a very quick-operating relay, so that the duty on the circuit breaker would be quite different in the two cases.

Mr. Torchio and Mr. Lincoln pointed out that we are going to have very great increases in the capacity of our systems, and it is increasingly important that we have some definite method of determining the amount of short-circuit current that will flow to a fault. Existing stations have grown away beyond

their original capacity as planned, and existing systems have tied in with other systems, so that we are getting enormous blocks of power concentrated within short distances.

The increase in the capacity of circuit breakers necessarily has gone up by leaps and bounds, and we hardly know where it is going to stop. As a matter of fact, it would seem as though there were some possibility of reaching a limit, that is not that of the rupturing capacity of the circuit breaker. It is, of course, possible to build circuit breakers of enormous size, and it is conceivable that we could reach practically unlimited capacity in the circuit breaker, one that may be large enough and strong enough for any situation. It is probable there will not be a demand for this, on account of the damage done at the point where short circuits of these enormous capacities are concentrated. We have ruptured, perhaps, 500,000 kv-a. up to the present time. We have found the bus bars pulled from their supports, the cables damaged, pulled off the wall, and great damage done in other ways outside the circuit breaker itself, so it is probable that these features will be limiting ones, rather than the extreme rupturing capacity of the circuit breaker.

There is another point in answer to Mr. Hunt's question, as to what the scope of these curves is, or, in other words, what is an average system. The authcrs have put down in this paper examples of what they consider average systems, and you will notice they are all confined to one generating station. When we come to deal with a number of generating stations we get into complications in calculating the current involving the use of Kirchoff's Laws which make the problems very difficult. We have in our department in the General Electric Company in Schenectady a calculation device which will take into account the distribution of current between stations and that makes it very simple to solve a problem of this kind. It is practically the only way it can be done, because with several generating stations the apparent reactance of each line is modified by what is coming in from others, and it is difficult to obtain the direction of flow of current in each individual circuit. By the use of apparatus of this kind, the tables and curves given in the paper would become universal, and any one could calculate to a fair degree of accuracy the short-circuit current on practically any complicated network.

H. D. James: I wish to call attention to this statement, "These factors are of special importance when a breaker of relatively small interrupting capacity is connected to a large system in such a way that the breaker may afford the only outlet for the stored energy of the system." It is very proper that this discussion should have been confined largely to breakers applied to large systems that had large capacities behind them. However, a very large class of applications are for the protection of electric motors, and come in a class of smaller breakers. Ninety to 95 per cent of such applications are limited by the distributing transformer, so that the power is limited.

There is danger of trouble, however, in power stations, and other large installations, where the power behind the breaker is large. This is particularly true where the overload protection is included as part of the control equipment. One way of taking care of that is to introduce a time element in the overload capacity of the control equipment or breaker protecting it, so that the feeder breaker will take the load. Unfortunately, the feeder breaker also has a time-element introduced in many cases, so that care must be taken to see that the time-element protecting the motor from overload is longer than the time-element of the breaker back of it, if the power is greater than the circuit breaker capacity of the control equipment.

N. L. Pollard: The paper presented contributes a great deal toward clarifying the determination of system rating, but does not assist in the selection of proper oil circuit breakers to meet these conditions, even after determining them.

The sections of the Standardization Rules covering the rating of oil circuit breakers are very indefinite, and do not in any way assist in a selection, unless supplemented by further information furnished by the manufacturer, as recommended in the closing paragraph of the paper under discussion. Such information has been entirely omitted from the paper, although it is a well-known fact that extensive tests covering rupturing capacities of various types of breakers have been made.

This subject is of such vital importance to the profession that further investigation seems advisable, and I would therefore suggest that a committee be appointed, composed of operating as well as manufacturing engineers, who should attack the subject from the standpoint of determining the relative merits of the various design factors. These factors should then be given definite values with regard to the rupturing capacity of the breaker and the results tabulated and embodied in the Standardization Rules.

The maximum rupturing capacity of the oil circuit breaker in amperes at rated voltage, should be stamped on the nameplate, in addition to the normal rating.

E. G. Merrick: The typical curves given in the paper are based on three-phase short circuits; the results are sufficiently accurate also for single-phase conditions if instantaneous values are considered but cannot be applied in all cases to sustained current calculations.

If a Y-connected, three-phase alternator with perfect "damper" is short-circuited successively across three terminals, two terminals and one terminal to neutral—the excitation remaining constant—the sustained currents will be approximately in the ratio of 100, 175 and 300. These values are modified more or less depending on the extent to which the single-phase armature reaction is annulled by the damping action of the field, so that for a standard salient-pole machine without amortisseur winding we find that the currents are in the ratio of approxi-

mately 100, 130 and 200 and for a salient-pole machine with amortisseur winding, or for a turbo-alternator with solid forged rotor, they are in the ratio of 100, 150 and 250. That is, in the former case, the single-phase short-circuit current will be 1.3 to 2.0 times the three-phase value and in the latter case 1.5 to 2.5 times—the excitation remaining constant.

The single-phase values for short circuits between terminal and neutral can in general be disregarded, in view of the fact that where several machines are operating in parallel, it is customary to solidly ground the neutral of one generator only, or if all machines are grounded, to parallel the individual neutrals through grounding resistances.

If the short circuits occur on the line side of a transformer in series with the alternator, the single-phase values of sustained current, with constant generator excitation, are again different from the three-phase values, the magnitude of the difference depending on the ratio of sustained generator reactance to transformer reactance and also whether the transformer is connected delta-delta or delta-star. These values may again be considerably modified if the transformer is feeding a transmission line whose characteristics are such as to give high values of charging current for the single-phase conditions.

Although the single-phase values of current may be, in certain cases, greater than the three-phase values, it does not necessarily mean that the switch duty has been increased. If the single-phase short-circuit occurs between terminals of generator or transformer, the higher value of current is more or less compensated for by the fact that there are now twice as many active switch contacts in series (charging current neglected).

In view of the usual margin of safety in oil circuit breakers and the fact that they are generally set to operate before sustained conditions are reached, their selection can ordinarily be made on the basis of three-phase values of short-circuit current. The sustained condition is important, however, in certain cases and should be made the subject of a future contribution to this subject.

P. H. Adams: Does this paper bring us any nearer to a uniform method of rating oil circuit breakers than we were a year ago? The operating engineer can determine his short-circuit current values, either by test or calculation, but on what basis can he select the oil circuit breaker for rupturing this current? There are on the market today, oil circuit breakers with fanciful ratings, having tin cans for oil tanks. How can we discover the truth in regard to our oil circuit breakers when such conditions exist?

There are ten or more well-known factors entering into the design of an oil circuit breaker, which govern its rupturing capacity. Modesty has apparently prevented the authors from touching this side of the question, therefore I agree with Mr. Pollard, that a committee should be appointed to investigate

this matter further and if possible, assign definite values to these design factors from which the rupturing capacity may be determined.

The rupturing capacity in amperes at rated voltage should appear on the name-plate of each oil circuit breaker, and the circuit breaker should rupture this current two times at a one minute interval and then be in condition to be closed and carry its rated load until it is practicable to inspect it and make necessary adjustments, without regard to generator or bus capacity.

If this condition cannot be met, then the Committee should divide oil circuit breakers into several classes, based on generator capacity and clearly define the design factors governing each class.

This is the kind of information the operating engineer desires and is what the title of this paper would lead one to expect to find outlined in it. I thoroughly agree with the closing paragraph of the paper that manufacturers should publish more information on the subject but this will be of little value unless the Institute adopts a standard of comparison. It is disappointing to hear Mr. Hewlett say this is the kind of information we cannot get.

Ira M. Cushing: The authors start in by speaking of normal pressure, normal frequency, and all conditions are normal until they begin to talk about the duty of the circuit breaker, and then all conditions become extremely abnormal.

It seems to me the circuit breaker has two, if not possibly three duties to perform. First, it is a device for disconnecting a circuit, or possibly disconnecting the apparatus. That is a very important duty, and in connection with that duty is the second one, namely, the ability to carry normal current. Then there is the third duty, that of opening a circuit under abnormal conditions.

I believe that this paper, to an outsider or possible purchaser of oil circuit breakers, is almost an alarmist paper—he would mentally throw up his hands and say—"I don't dare have one on my system." Therefore, I think this word "duty" should possibly be modified, or there should be a paragraph or two added to the paper, describing what the oil switch or oil circuit breaker should do, besides the opening under abnormal conditions.

Possibly the words "prescribed conditions" are not well chosen. We should use the word "abnormal conditions." You speak of prescribed things as something you expect might happen quite frequently, but when you come to speak of abnormal conditions, then you are looking for something that is different, and that is, of course, what is spoken of in the paper.

At the end of the paper the authors practically nullify the entire thought by admitting that everything that has gone before covers systems that do not have voltage regulators. It

is my impression that many systems have voltage regulators, and especially systems that have a large lighting load, and for that reason it would appear to a person, not familiar with the apparatus, that the figures given would not apply at all to the average system.

J. N. Mahoney: Mr. Torchio mentioned the probable condition of the apparatus after it performed the duty specified. One of the well know disturbances presented by an oil breaker in operating under near maximum conditions, is to throw some oil. It is self-evident that if the amount of oil thrown is great, it limits the ability to take care of the next successive operation of the breaker. As a matter of fact, that is one of the points that is a controlling feature.

The breaker may open twice, undertaking the maximum duty for which it is guaranteed, and the oil level may be so reduced that if operated a third time you could not expect to get the same standard of performance, as in the first instance. You might operate it at a smaller power duty, and so on down with each successive operation, and in this way you could operate it successfully a large number of times; the first time at the maximum. With a sufficiently small power duty, the breaker might be made to operate indefinitely even in its depreciated condition.

The breaker blowing off a tank, breaking a tank supporting frame, or causing an unusual mechanical destruction of some other kind, is not contemplated as being within the accepted practice, or definition of successful "two-time operation."

The second point in the depreciation, you might say, of operation, is the arcing "tips", or the contacts forming the arcing members. While the conducting contacts may not be injured, the arcing contacts may be consumed considerably. Therefore it is undesirable to operate at the maximum duty any more than a given number of times, until the circuit breakers are inspected and repaired. The device may be closed again, and service maintained for an indefinite period, that is, until the normal inspection period. That contemplates, assuming the breaker is an automatic one, that if it is subjected to two successive maximum operations as stated in the guarantee, the breaker would be made non-automatic after the second operation. It will then be just a conductor, or disconnecting switch, and the automatic protection will be afforded by other means in the circuit. Usually, as Mr. Hewlett remarked, it may be that destruction beyond the breaker, caused by the short circuit gives you all necessary time, even after one operation, to readjust the breaker, before it is necessary or desireable to close it.

The question has been raised by several of the speakers, as to what constitutes an average system. It simply means an average radial system, as practically all general service systems are. Such a system does not have its load in one block, with

a single controlling breaker. The difference is not because of any condition of power factor in the system, but the fact, in the single breaker system, there is practically but one outlet for the stored energy, that is the breaker itself. I think that will clarify the situation.

Philip Torchio: The point I make I think is rather important: After the second closing, if you require the breaker to do some future interupting work,—do not close it, but leave it there for the normal inspection. Its condition is that it is not immediately capable of opening against its rated powers.

J. N. Mahoney: That is the understanding.

Chester Lichtenberg (communicated after adjournment): The rating and application of oil circuit breakers requires an intimate knowledge of the phenomena taking place not only in the external electrical circuit, but also in the breaker. It is essential to understand not only how a circuit behaves under transient electrical conditions, but also how the oil circuit breaker performs its energy dissipating and circuit interrupting functions under these conditions.

Consider an oil circuit breaker. It consists essentially of a pair of separable contacts immersed in an oil bath. Suppose this simple breaker to be connected in an a-c. circuit carrying a steady current at a steady pressure and steady power factor. Now assume the breaker contacts to part, and to continue in motion with approximately constant velocity until the circuit shall have been interrupted.

Under these conditions it is found that when the breaker contacts part, an arc is formed. This arc gasifies that portion of the oil in the immediate neighborhood of the contacts, and a spherically shaped globule of gas is formed around them. The pressure in the globule depends on the amount of energy which the breaker is called upon to dissipate, and under extreme conditions it may become as high as several thousand kilograms per square millimeter.

One effect of the pressure in the gas globule is to raise the level of the oil in the bath. It is for this reason that all well designed oil circuit breakers have a so-called "air buffer" between the top of the oil and the oil vessel cover or lid.

As the contacts move further apart, the arc lengthens, and due to internal pressure, the gas globule grows larger. The globule retains its spherical shape, though until the contacts move out of it, or it encounters the side walls of the containing vessel.

When the moving contact passes out of the gas globule, the arc follows, and a path is provided for the relief of the pressure stored in the globule. This causes the globule to burst explosively, and by this action the arc is blown out and oil tended to be ejected from the vessel.

In the interruption of most electrical circuits this is the only action that occurs during a successful operation of the breaker. In the interruption of some circuits, however, there is an added

step. The bursting of the first globule, initially surrounding the contacts, is followed by an elongation of the arc through fresh layers of oil, and the formation therein of new gas globules having, however, much less internal pressure than the first. These new globules burst in turn without interrupting the circuit, till finally one of them succeeds in blowing out the arc and interrupting the current flow.

The formation of gas globules, their subsequent enlargement, distortion, and bursting are shown in Fig. 1. This photograph was taken with the successive image camera described in a previously contributed discussion. It is typical of hundreds of photographs which have been taken and studied. The arc images represent exposures made at intervals of about 0.005 second. The first image is at the right end of the lower row. The second, third, fourth, and fifth are adjacent to it on the left in the order named. The sixth is at the right end of the lower but one row. The seventh is at the left of it. The twelfth is at the right end of the third row, and so on. They show quite clearly the different stages in the interruption of a current carrying electrical circuit by an oil circuit breaker.

The bursting of the gas globules when an oil circuit breaker is clearing a circuit, is followed by external evidences of distress. Some of these are as follows: (1) Dull thud; (2) Ejection of oil; (3) Ejection of flame; (4) Straining or bursting of oil vessel. The circuit interruption is also indicated to have taken place explosively by oscillograms contributed in previous discussions. These show that the current is diminished with increased rapidity as the interruption progresses successfully, until the end is approached with great suddenness or abruptness.

This explosive interruption of the circuit has a very important bearing on the application of oil circuit breakers. As I pointed out during a previous discussion, and as is stated in the paper, not only must the characteristics of the circuit to which a breaker is to be connected be known, but also the characteristics of the breaker itself must be known. The explosive action of the breaker introduces an element of wide variablity in the operation of the breaker, and makes it difficult to predetermine its performance with great accuracy.

It is a well known fact that the characteristics of generators, motors, transformers, transmission lines, and other parts of electrical circuits may be determined by test and calculation quite accurately, *i. e.* within 2 or 3 per cent. It is also a fact, though not nearly so well known or recognized, that the performance of an oil circuit breaker cannot be predetermined with an accuracy closer than 15 per cent. This fact requires especial emphasis at this time since many incorrect breaker applications are made because a knowledge of the limits of accuracy of the calculation of systems and breakers is not general.

The limit of accuracy of the predetermination of breaker ratings is an important item at this time also because the paper

FIG. 1 [LICHTENBERG]

under discussion proposes rules for the rating and application of these devices. These rules have been derived from an analysis of a large amount of data accumulated during over twenty years of experience. In this, the proposed circuit breaker application rules follow the precedent established in the development of empirical rules for the calculation of other electrical machines. The great difference is, though that whereas most other electrical machines and devices are calculated for ordinary or steady circuit conditions, the oil circuit breaker interrupting capacity must be computed for extraordinary or transient circuit conditions. Therefore, while apparatus for steady conditions can be built to within 2 or 3 per cent of the design calculations, oil circuit breakers frequently have interrupting capacities more than 50 per cent outside the design calculations.

One other factor requiring attention is the application of breakers to growing systems. A breaker has a finite capacity, while compared thereto the ultimate capacity of the system may be infinite. The limitation of the breaker, while known, is not always taken into account as the system grows. Finally a point is reached in the expansion where the breakers are quite inadequate and a failure results. The possible future growth of a system should always be considered at the time the breaker equipment is planned, and while it is not recommended that the initial installation of breakers be suited for the ultimate development, it is suggested that the arrangement of units be such that if the system grows beyond the limits of the circuit interrupting equipment initially installed, this may be moved to other parts and replaced by larger breakers with a minimum of expense.

A. Collins (communicated after adjournment): It would be of interest to know whether the authors are of the opinion that the factors suggested should be increased if a three-phase generating plant is short-circuited across one pair of phases only, as this may have some bearing on the breaking capacity of switches in which a separate tank is employed for each phase.

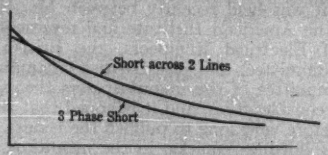

FIG. 2

The curves given are presumably based on three-phase short circuits, and it would be expected that although the short-circuit current across one pair of phases is actually about 13 per cent less than the three-phase short-circuit current, owing to the decreased armature reaction, the resultant field will decay

less rapidly, thus producing characteristic curves with a more gradual slope, see Fig. 2.

Under the heading of "Precautions" there are one or two other factors which may tend to increase the stress on the switch which has been selected by the use of the data provided.

It is conceivable that a high resistance fault may be sufficient to operate the protective gear, and energize the trip coil, and that this fault may develop into a dead short circuit at the instant the switch contacts separate. With maximum assymetry, or as the authors describe it, maximum possible direct component in the short-circuit characteristic, the current at the instant of break will be much greater than that given by the figures in Table 1; for example, with 10 per cent reactance the peak value of the short-circuit current would be 20 times the peak value of the full-load current, and the ratio of the r. m. s. values would be approximately of the same order.

In addition there is the doubling effect in transformers and reactance coils, resonance and other phenomena.

It is important that switches, particularly those near to the generating plant should be selected for the most severe conditions that may reasonably be expected, but I should like to hear the authors' opinion as to how far they consider provision should be made, if any, for such contingencies as those just mentioned.

Charles C. Garrard (communicated after adjournment): There can be no doubt that, at the present time, the circuit breaker forms a limiting feature in the construction of very large power stations. The application of power limiting reactances has rendered the super-power house possible. Such reactances are, however, costly and, in themselves, undesirable pieces of apparatus. The lower limit of size above which reactances are necessary is fixed to a very great extent by the safe rupturing capacity of the oil circuit breakers; the higher this is, then the greater may be the capacity of the power house without the addition of the reactance coils.

This subject is at the present moment occupying the attention of the standardizing authorities in this country. A conference dealing with it was held recently between Mr. H. M. Hobart representing the American Institute and representatives of the Institution of Electrical Engineers, the British Engineering Standards Committee and the British manufacturers. In addition a committee of the manufacturers themselves has the matter under consideration, and it is hoped that this committee will be able before very long to publish the results of its deliberations. The following remarks, in the meantime, are made in the writers' personal capacity.

In the first place as regards the pressure test I am not quite certain what is meant by "dry conditions;" I presume it is intended that the insulators should be free of moisture but that the oil tanks should be filled to their normal extent with oil. If so I am in agreement, unless of course the circuit breaker is for

out-door service in which case the test should be taken under rain-conditions.

I think the limit of temperature rise of contacts in air of 20 deg. cent. (ambient 40 deg. cent.) is too low. For large apparatus especially, it is difficult to work with such a low limit, moreover I can see no reason at all for such a small figure. I think 30 deg. cent. is quite safe. Again the temperature limit of 70 deg. cent. for oil is unduly conservative for large current circuit breakers. This limit might be retained for sizes up to and including 2000 amperes; above this however a temperature rise of the oil of 40 deg. cent. is not unreasonable. The temperature should be measured at the top of the oil.

The most important point of the whole discussion is of course the rupturing capacity. It follows from the authors' arguments that a rating in terms of rupturing current is recommended. In this the authors agree with the German standardizing authorities. The original V. D. E. suggestions for the rating of oil-circuit breakers proposed a kilovolt-ampere rating (see E. T. Z. 16-2-11, 1911, page 171), but this was afterwards changed to an ampere one. There are, however, very considerable advantages in the adoption of a kilovolt-ampere rating. Really, of course, there is no essential difference between this and the authors' system. The ampere rating is simply converted into the kv-a. rating by multiplying by the rated pressure and the factor 1.73 and dividing by 1000. For rapid calculations, however, and for the comparison of circuit breakers on different voltages, the kv-a. rating, in the writer's opinion, is the better one. There can be little doubt that, probably without introducing any error greater than that inherent in the underlying assumptions made by the authors, that the kv-a. rating (rupturing capacity) of a given breaker is independent of the voltage. Thus the same circuit-breaker (if the insulators be suitable) may be used either on a 6000 or a 3000-volt circuit. With the ampere rating it would be necessary to give this one circuit in such circumstances two different ratings for its rupturing capacity which would be likely to cause confusion. Moreover, to the present writer, it seems more logical to rate in terms of kv-a. It is after all the power flowing which is ruptured. It is the absorption of power within the circuit-breaking device over the time taken for the rupture, *i. e.* the total energy absorbed in the oil, which does the damage. The theoretically correct rating would therefore be in kilowatts rupturing capacity; this, however, introduces questions of power factor etc., as to which no exact statement can be made. It seems, therefore, that the kilovolt-ampere rating is the best under the circumstances. Certainly in this country the present tendency is towards this conclusion.

As regards the prescribed duty I am in agreement with the suggestion that the breaker should interrupt its rated rupturing current (or, as I prefer, its rated kv-a.) twice in succession, and

then be in condition to be closed and carry its rated current until it is practicable to inspect it and make any necessary adjustments. I do not think however a two-minute interval should be called for between the two operations; it should be allowable that these should follow each other rapidly as this is in accordance with practical operating conditions.

The definition of breaking capacity, therefore, which I would suggest is as follows: "The breaking capacity of an oil circuit breaker should be given in kilovolt amperes based on the rated pressure and the current at the instant of break. The rating of an oil circuit breaker shall be the maximum kilovolt amperes the circuit breaker will break twice in succession, and after this be capable of carrying normal current."

The curves and examples given by the authors as aids in the selection of an oil breaker for any particular duty are most valuable and will be of great service to the user and manufacturer. I am inclined to think however that the method as a whole is too complicated for practical use. It must be remembered that at the present time no generally accepted system whatever is in use. The step from this state of affairs to the very complete statement suggested by the authors is a very large one. A simpler proposal is, I consider, feasible, and while such would constitute a really very great advance over present day practise, it would be more likely to receive general support on account of its greater ease of application.

As an example of what I mean I would point out that the authors' method entails a statement of the time taken by the circuit breaker to open its contacts. This generally under practical conditions, is unknown. Moreover it is a very variable quantity, depending on the electrical and mechanical conditions of the controlling and operating mechanisms, and, in the case of electrically operated breakers, on the voltage of the operating battery. Practical considerations would therefore require an average assumption to be made in nearly all cases.

Seeing therefore that it will nearly always be necessary to make an assumption it seems to me unwise to complicate the issue in an attempt to secure a greater scientific exactitude than the original assumption will allow. I think that the following simple assumptions for example will probably be entirely sufficient.

$$\left.\begin{array}{l}\text{Kilovolt amperes to be broken by}\\ \text{generator circuit breaker}\end{array}\right\} = 0.6\ \frac{(\text{Rated kv-a.} \times 100)}{\text{percentage generator reactance.}}$$

It will be seen that the factor 0.6 has been assumed to allow for the rapid fall of the short-circuit current from its high initial value before the circuit breaker has time to operate. Moreover the doubling effect has been neglected.

For oil circuit breakers at the ends of feeders it will be prob-

ably quite sufficient to assume constant busbar voltage in the generating station and calculate the short circuit taking into account the impedance of the feeder. When dealing with feeders I do not think it right to neglect the resistance; this is especially so with underground cables.

I would also like to suggest that the examples given in the paper should be amplified by considering what happens when the oil circuit breaker interrupts a ring main, and also the effect of running synchronous motors.

Applying the simple formula given above to an alternator having 8 per cent total reactance we get kv-a. to be broken = 7.5 times rated kv-a. This value on the authors curves (Fig. 2.) is obtained at 0.25 seconds from the time the short circuit begins. It is obvious that if the time taken by the circuit breaker to operate be shorter than 0.25 seconds, the factor 0.6 is too small. The question really resolves itself to determining what factor represents most closely the general results of experience.

I would like to suggest in conclusion that this simple statement of the usual case should be considered by the American Standardizing Authorities with the object of using it for the usual run of work and reserving the more detailed data of the authors for use on special occasions.

G. A. Burnham: Mr. Collins' question as to whether the authors had taken into consideration a single-phase short circuit on three-phase generating plants, was, I believe, fully covered by Mr. Willard's remarks. The authors feel, however, that for average application the curves give a sufficient factor of safety to take care of this condition.

With reference to Mr. Collins's statement wherein a high-resistance fault may be so long developing into a so-called "dead short circuit" that the maximum current might occur at the instant when the switch contacts separated, is one of the factors which must be eliminated in the selection of breakers for average application, or else the breakers would have to be selected on the basis of instantaneous tripping, and this might mean for average conditions, a too conservative application.

Mr. Garrard has contributed an interesting discussion. It is possible that the words "dry conditions" in the absence of any remarks with reference to the dielectric tests for switching equipment for outdoor service might lead one to believe that the dry dielectric test was to be made without oil in the tank. This, however, is not the information which it was desired to convey. It is found necessary to differentiate in the dielectric tests for apparatus used for indoor service and for outdoor service. Apparatus for indoor service is to be tested under dry conditions, that is, the insulators and structure would be dry, but their tanks would be filled with oil to the proper level.

This difficulty will undoubtedly be cleared up in the near

future, as there is before the Standards Committee at the present time recommendations for the dilectric test of apparatus to be used indoors and also apparatus for use out of doors.

I do not entirely agree with Mr. Garrard that the kv-a. rating or rupturing capacity of a given breaker is independent of the voltage. There seems to be quite a difference of opinion in America in reference to this point. It is undoubtedly true within reasonable limits, but circuit breakers designed for a given kv-a. interrupting capacity for a given voltage probably would not interrupt the energy in a circuit of equal kv-a. capacity on a very low voltage. It would be necessary, therefore, to give different kv-a. ratings for the same circuit breaker when used on voltages differing widely from the maximum rated voltage.

If all generators had the same reactance and time-current decrement curves, and all transformers had the same reactance, irrespective of their kv-a. capacity, and that it would be assumed that the kv-a. interrupting capacity of a circuit breaker was dependent only on the product of the current and voltage at the instant of the parting of the circuit breaker contacts, the kv-a. basis of interrupting capacity would undoubtedly offer advantages. These assumptions have evidently been the basis of Mr. Garrard's recommendations for determining the kv-a. to be interrupted by generator circuit breakers. It would seem, however, that circuit breakers when called upon to trip within a second to a second and a half after the occurrence of a short circuit would not be applied as conservatively in many cases as would be required, and, on the other hand, in many instances would be very much larger than required.

I fully agree with Mr. Garrard that resistance should not be neglected when dealing with feeders alone; as it has a very modifying effect upon the short-circuit current, particularly as he mentions on underground systems. However, when considering feeder circuits in conjunction with reactors, transformers and the reactance of generators, the effect of resistance as to the value of the short-circuit current is usually relatively small.

Mr. Jones' and Mr. Torchio's remarks concerning standardization are of great interest to the manufacturer. Certainly, if we can arrive at some form of standardization, if only applying to structure, mounting, etc., it will relieve the manufacturers of a great deal of special engineering and hasten the production of standard apparatus.

Standardization is a most important factor. If engineers, in laying out their plans, gave this matter more serious consideration, I can assure you it would be of great economic value, particularly in the moderate sized installation.

In reference to using factors to modify values representing the interrupting capacity of oil circuit breakers where conditions cause widely different re-established voltages, I do not believe it is possible with the data at hand.

I might answer Mr. Hunt and Mr. Pollard on the question of knowing more about the various factors mentioned and the relative effect which they have on the circuit breaker interrupting capacity. At the present time there is no suitable apparatus that would allow one to determine with any degree of accuracy the relative effects of the various factors mentioned. It is a question of a long field experience, or, in other words, careful observation of the particular piece of apparatus under widely varying conditions. The circuit breaker manufacturer would welcome any suggestions that would be of assistance in giving more information pertaining to the action of oil switching equipment. Mr. Pollard and Mr. Hunt also brought up the question that even after the paper had been presented and even after these curves were published by the manufacturers, it would then be practically impossible to make application of them. I think with the general publications as they are today that it is true, to a certain extent, but in the newer publications which all manufacturers are preparing, proper information will be available. There will undoubtedly be listed the minimum tripping time of each type and form of circuit breaker, also the minimum tripping time of various forms of relays that are associated with circuit breakers in order that the total lapse of time may be determined. It will then only be necessary for the customer to determine his relay settings to arrive at the maximum time from the instant of short circuit to the parting of the contacts, and, knowing his reactance, pick out the current value from the transient current curves.

I believe that this will take care of a great deal of the difficulty that we are encountering at present.

Mr. Dewey's and Mr. Willard's remarks cover the question of average systems. It cannot be described in a few words, but I believe these curves, with the exception of the few conditions pointed out, will apply in probably 75 per cent of the applications.

I want to emphasize Mr. Lincoln's statement in reference to cooperation between the user and the manufacturer. The manufacturer tries to get all possible information pertaining to the system to assist the customer in making proper application. A closer cooperation will result in a better understanding of the situation, not only from the standpoint of the customer with reference to the manufacturer, but of the manufacturer in reference to the customer's viewpoint.

The real object of this paper was to state as clearly as we could terms which could be used so that when each of us got through with a problem and assumed the same conditions, we would arrive at the same conclusion. As Mr. Hewlett pointed out, we desire to establish a bench mark to be used by all, and this will allow a more consistent conclusion in the analysis of the various circuit breaker problems.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.

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A NEW STANDARD OF CURRENT AND POTENTIAL

BY CHESTER T. ALLCUTT

ABSTRACT OF PAPER

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

THE increasing use of potentiometers in commercial service, especially in connection with the measurements of temperature by means of thermocouples, makes it very desirable to secure a substitute for the standard cell usually required by these instruments. The serious shortage of standard cells caused by the war has emphasized this need. Furthermore, it is being recognized that it is poor economy to use a costly precision standard in a commercial potentiometer which reads to three significant figures at the most.

In order to eliminate the standard cell from the type of thermocouple potentiometer used in measuring the temperature of electrical machines, it has become quite common practise to use a calibrated current-measuring instrument for setting the current. A long-scale galvanometer, in connection with a suitable shunt, is used as a deflection instrument for setting the current in the potentiometer circuit. After setting the current, the same galvanometer is used as a null instrument for balancing the thermocouple e.m.f. against the potentiometer. The chief objection to this practise is the fact that it necessitates the use of a very high-grade galvanometer. A long-scale galvanometer, suitable for use as a deflection instrument, is necessarily much more costly and more delicate than a short-scale instrument of the same sensitivity. For this reason it is desirable to provide a means for setting the current which will not

only eliminate the standard cell but will also remove the necessity for a deflection instrument.

Description of Proposed Standard

The new standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements is essentially a Wheatstone bridge which will balance for but one value

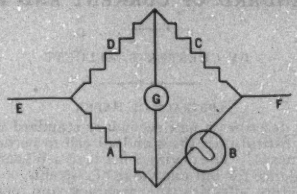

Fig. 1

of current. Referring to Fig. 1, the device consists of a Wheatstone bridge comprising three constant resistances, A, C and D and a fourth element B, whose resistance is a function of the current flowing through it. In practise, the resistance element B consists of an evacuated glass bulb which encloses a fine filament of a material having a relatively high temperature coefficient of

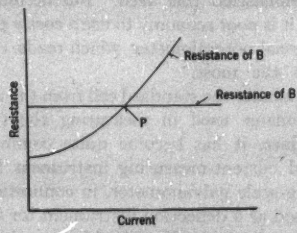

Fig. 2

resistivity. A galvanometer G is connected across the bridge and the leads E and F are connected to an external source of direct current (not shown). For the purpose of illustrating the action of the bridge, let us assume that the resistances C and D are equal. It is then obvious that the bridge will balance when the resistance of B is equal to the resistance of A. Fig. 2 shows graphically the relations involved. The bridge will balance

when the current through the branch $A\ B$ of the bridge is equal to the abscissa of the point P. We thus provide a simple null method for setting the current in a circuit at a predetermined value.

The Variable Resistance Element—Construction and Characteristics

The practicability of the device depends on securing a variable resistance having the desired current-resistance characteristics, together with a high degree of permanence. For use with potentiometers, it is necessary to have the bridge balance at a rather low value of current, 20 milliamperes being the usual value for portable potentiometers. The necessary properties of the variable resistance element may be most readily obtained by using the construction already referred to; *i.e.*, a fine filament in a vacuum bulb. It was found possible to obtain resistances in this manner whose value would change very rapidly with small changes in current, even with currents of a few milliamperes. A number of bulbs have been made which would more than double in resistance when heated by the passage of 0.005 ampere. Fig. 3 shows a typical current-resistance curve of such a bulb. It will be seen that for currents of more than two milliamperes the resistance increases very rapidly with the current.

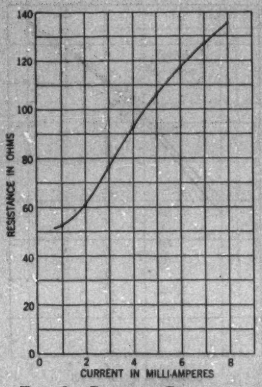

Fig. 3—Current-Resistance Curve—Bulb G

Most of the bulbs experimented with by the writer had platinum filaments approximately 0.0005 cm. in diameter and from 1 to 3 cm. in length. Silver coated platinum wire (Wollaston wire) was used, in order to facilitate the handling incident to mounting the filament in place. After mounting the filament, the silver coating was removed by means of nitric acid, leaving the platinum core exposed.

The bulbs were evacuated to from 10^{-4} to 10^{-5} mm. of mercury. Past experience with high vacua has shown that even with pres-

sures as low as 10^{-5} mm., a very high degree of permanence may be obtained. Furthermore, at pressures as low as 10^{-4} mm., the change in heat conduction from the filament with changes in pressure is extremely small. In order to secure the permanence of vacuum referred to above, it is necessary to observe the usual precautions, such as heat treatment of the glass, etc., during the process of evacuation.

Another operation necessary for securing the requisite degree of permanence is "seasoning" the filament by passing, for a considerable time, enough current to heat the filament to a bright red heat. Twenty-four hours of seasoning was found to be

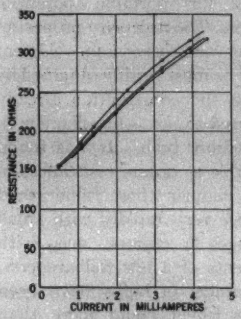

FIG. 4—CURRENT-RESISTANCE CURVES

Curve A—New bulb.
Curve B—24 hours later.
Curve C—After 24 hours' seasoning.

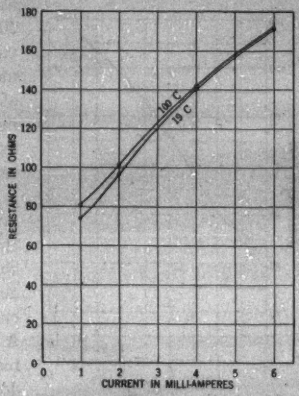

FIG. 5—CURRENT-RESISTANCE CURVES—BULB *H*—SHOWING EFFECT OF AMBIENT TEMPERATURE

ample. In every case the filament should be heated to a temperature considerably above the temperature at which it is to operate. Fig. 4 shows the effect of seasoning the filament on the characteristics of a bulb. Curve *A* shows the current-resistance characteristics of a new bulb. Curve *B* is plotted from data taken 24 hours later, while Curve *C* was taken after 24 hours seasoning. Further tests at a later date followed Curve *C* with great exactitude.

Another factor which might be expected to detract from the permanence of the bulbs is the possibility of a gradual change in the resistance of the filaments due to evaporation. As, in

every case, it is proposed to operate the filaments below red heat, no trouble from this source need be feared.

In order to observe the effect of ambient temperature on the current-resistance characteristics of a bulb, the data given in Table I were taken. Fig. 5 presents part of these data graphically. It will be seen that for the larger values of current the effect of ambient temperature is very small. For example, at six milliamperes the temperature coefficient is about 0.007 per cent per deg. cent. change in ambient temperature. For some classes of service this temperature coefficient is negligible. In

TABLE I—TESTS ON BULB H SHOWING EFFECT OF AMBIENT TEMPERATURES.

Temp. in oven	Current through bulb	Resistance of bulb
19°C.	0.001 amperes	74.08 ohms
19 "	0.002 "	96.5 "
19 "	0.004 "	140.6 "
19 "	0.006 "	171.1 "
44°C.	0.001 amperes	75.33 ohms
45 "	0.002 "	97.65 "
46 "	0.004 "	141.05 "
46 "	0.006 "	171.3 "
65°C	0.001 amperes	77.26 ohms
65 "	0.002 "	99.0 "
64 "	0.004 "	141.7 "
64 "	0.006 "	171.9 "
102°C.	0.001 amperes	81.05 ohms
101 "	0.002 "	101.5 "
100 "	0.004 "	142.0 "
100 "	0.006 "	172.1 "

any case, however, it may be exactly compensated for by giving one of the fixed resistances of the bridge with which the bulb is used a proper temperature coefficient, thus making the current for which the bridge will balance entirely independent of room temperature.

The degree of permanence possible is shown by the data given in Table II. These data are the results of a series of observations on a bulb, covering a period of several weeks. It will be noted that for the smaller values of current there are differences in the values of the resistance that may be accounted for by differences in room temperature, while for larger values of current the effect of room temperature is apparently negligible.

Characteristics of New Standard

Figs. 6 and 7 show the sharpness of balance that can be obtained in a bridge using a bulb of the type described. These

TABLE II—RECORD OF TESTS ON BULB G.

Observation No.	Room temp.	Resistance at different values of current.				
		0.001	0.002	0.004	0.006	0.008 amperes
1	26.5	52.6	60.7	92.6	116.9	134.9
2	25.	52.5	60.6	92.5	116.8	134.9
3	23.5	52.2	60.4	92.5	116.9	134.9
4	24.	52.4	60.5	92.5	116.9	134.8
5	25.	52.5	60.7	92.7	116.9	134.9
6	24.	52.4	60.5	92.5	116.8	134.9
7	24.5	52.2	60.4	92.5	116.8	134.9
8	27.	52.4	60.5	92.6	116.9	134.9
9	27.	52.5	60.6	92.6	116.9	134.9

curves give the unbalanced voltage of the bridge (*i. e.*, the voltage tending to send current through the galvanometer) as a function of the current flowing through the bridge. The sharp-

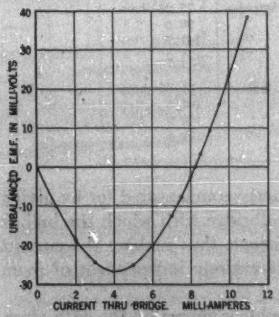

Fig. 6

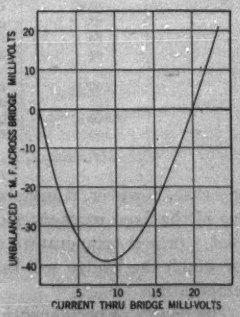

Fig. 7—Characteristics of Bridge Using Bulb *H*

ness of balance may be increased by using two bulbs connected in opposite arms of the bridge, but it has been found that a sufficient degree of sensibility may be secured with but one bulb.

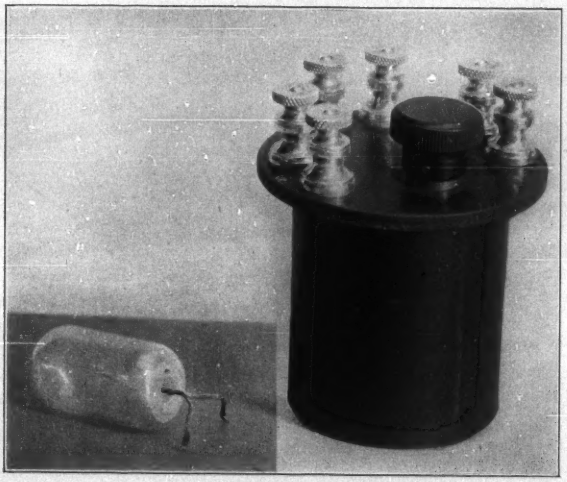

FIG. 8 [ALLCUTT]

Fig. 8 is from a photograph of the bridge whose characteristics are given in Fig. 7. This illustration also shows a bulb similar to the one used in constructing the bridge. The bridge was designed to balance with a current of 20 milliamperes. In addition, leads were brought out having just one volt potential difference between them when the bridge is balanced. A rheostat is mounted in the same box with the bridge. Fig. 9 is a diagram of connections of the device. In using this standard, a dry cell is connected to the binding posts marked *B* and a galvanometer at *G*. Then the rheostat is adjusted until the galvanometer shows no deflection. When the bridge has been balanced in this manner there is one volt between the terminals marked *E*. The instrument is thus a very simple standard of both current and e.m.f.

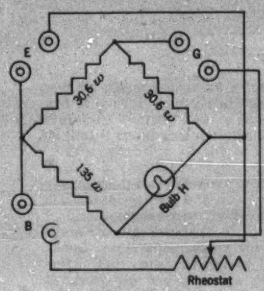

FIG. 9

For use with potentiometers, the current-setting bridge is permanently connected in series with the potentiometer circuit. Means are provided for throwing the galvanometer either across the bridge or in the circuit leading to the unknown e.m.f.

CONCLUSION

The work done in connection with the development of this new standard has demonstrated that it may be relied upon to maintain an accuracy of ± 0.1 per cent. The sensibility to current changes is ample, as it is very easy to set the current to within 0.1 per cent with the type of galvanometer usually supplied with portable potentiometers. The device may, therefore, be regarded as a suitable substitute for the standard cell for practically all classes of service outside of precision laboratory work.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.

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A THERMOELECTRIC STANDARD CELL

BY C. A. HOXIE

ABSTRACT OF PAPER

This paper considers a means of obtaining a secondary standard of e. m. f. by utilizing the e. m. f. of a thermocouple. The standard thermo cell is fundamentally a standard of current, in that it requires a definite value of current to function properly.

The operation of the cell consists in balancing the potential across a resistance against the thermoelectric e. m. f. of the thermocouple. This requires a definite value of current through a filament which is a source of heat for the thermocouple.

The temperature coefficient has different values, depending upon the temperature of the heating filament. Means are provided for compensating for the temperature coefficient of the cell. The construction of the cell is discussed in detail, particularly the use of gas in the bulb.

A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown. The standard cell has been successfully applied to potentiometers designed for thermocouple work. Further experimental work on this cell is now under way.

MOST engineers are familiar with the uses and limitations of standard cells of the Clark or Weston types. Those who have used either of these cells know that they will not function at freezing or boiling point temperatures, and are easily damaged if an appreciable current is drawn, as by accidental short circuiting.

Though the "Thermal" cell about to be described is not, strictly speaking, a primary standard or source of e. m. f., as is the Clark or Weston cell, it is at least free from the above drawbacks. It may be more properly classed as a secondary standard, its value being determined by comparison with a primary standard. In fact it is just as legitimate to call it a standard of current as of e. m. f., and perhaps a little more so, for the reason that it is simply a combination comprising a resistance, a thermo junction and a heater wire arranged in such a manner that it requires a current of a certain definite value in order for it to function properly. A standard value of e. m. f., however, may

be obtained by taking the drop across a suitable resistance placed in the circuit.

To be more explicit, the standard thermal cell in its present form consists of a glass bulb containing a thermo junction T, (see Fig. 1) a heating filament H which is not in contact with the junction, the balancing resistance R and the drop resistance R_1. These are mounted in a case with binding post connections on the top. (See Fig. 2.)

Operation

If a battery and galvanometer are connected as shown in Fig. 1, and the current adjusted to a critical value the galvanometer will indicate a balance. The reason for this may be readily understood by reference to Fig. 3. The curve a represents the

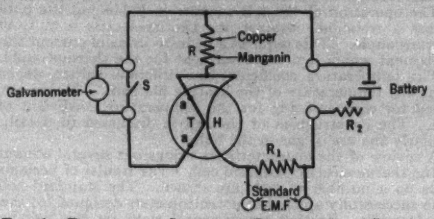

Fig. 1—Diagram of Standard Thermo-Cell Circuit

T = Thermo junction
H = Heating filament
a = Cold end
R = Balancing resistance
R_1 = Drop resistance
R_2 = Rheostat
s = Galvanometer short-circuiting switch

drop of potential across the balancing resistance R as a function of the current through the heater H. The curve b shows variation of the thermal e. m. f. of the junction with the current through the heating filament. These curves indicate that the thermal e. m. f. is approximately proportional to the second power of the current through the heater, while the drop across the resistance R varies directly. The point of intersection C of these curves is that point at which the thermal e. m. f. equals the drop of potential across the balancing resistance. Therefore by adjusting the current through the heating filament to this critical value, we have a means of obtaining a definite potential drop across the resistance R_1. In order to obtain a high value of thermal e. m. f. it is desirable to heat the filament to a high

FIG. 2 [HOXIE]

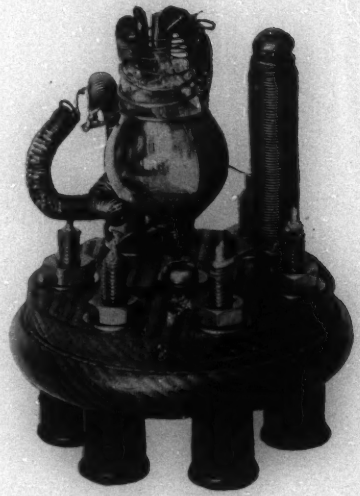

FIG. 4 [HOXIE]

temperature. The maximum current that can be used without effecting the constancy of the cell, is that which will bring the filament to a dull red heat when viewed in a dark room. Under actual operating conditions the filament current is generally maintained somewhat below this value to prevent damaging the cell due to an accidental increase.

In many of the tests with these cells a lead storage battery having a maximum variation in e. m. f. from 2.4 to 0.8 volts has been employed. By reducing the filament current to about 10 per cent below the reddening point, the cell can be constructed so that 2.4 volts applied to its terminals will not damage the cell. The adjustment to the critical balancing current is then secured by means of a rheostat in the battery circuit.

Temperature Coefficient. Since the drop and balancing re-

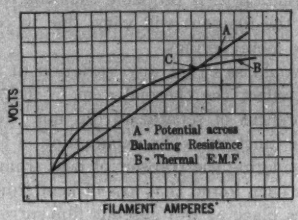

FIG. 3—CURVES SHOWING RELATION BETWEEN POTENTIAL DROP ACROSS BALANCING RESISTANCE AND THE E. M. F. GENERATED AT THE THERMO JUNCTION

sistances are constructed of wire which has a negligible temperature coefficient, any change in the potential across the drop resistance must be due to a variation of the thermal e. m. f.

It is found that when the cell has been adjusted so that the current necessary for a balance is sufficient to bring the filament to a dull red heat, the cell has no measurable temperature coefficient. If adjusted to any smaller value of current a negative coefficient results. This is due to the fact that up to a certain point, a rise in the surrounding temperature increases the thermal e. m. f. of the couple for a given difference in temperature between the cold and hot ends. Apparently this critical point is reached when the filament attains a dull red heat.

The fundamental cause of the temperature coefficient of the cell is, as previously stated, the positive temperature coefficient of the thermal e. m. f. If a portion of the balancing resistance

is constructed with the proper amount of some metal having a positive temperature coefficient, the increase in thermal e. m. f. is balanced by an increased drop across the balancing resistance, hence it becomes unnecessary to vary the filament current. We have therefore compensated for the temperature coefficient of the cell.

Construction

Considerable experimental work was necessary to determine the proper construction of the various parts of the thermal cell. The results of this work will now be discussed.

Heating Filament and Couple Wires. The heating filament and junction wires are kept under tension in order to maintain a constant position with respect to each other. The tension is produced by small spirals located at the base of the bulb. Even with this construction, it is probable that the relation between the heater and the filament will vary, due to contraction and expansion of the metal springs.

At first we would say that any variation in this separation would seriously effect the thermal e. m. f.. Experiments show that small differences of separation did not effect the e. m. f. generated at the junction. The explanation of this phenomena is found by considering the transfer of heat from small wires in gases.* It has been found that loss of heat from wires by free convection takes place as if a gaseous film surrounded the wire and the heat was transferred through this film by conduction. It has also been determined that the thickness of the film is independent of the temperature but is dependent upon the diameter of the wire. Evidently if we assume other factors constant, we will obtain a steady transfer of heat from the heating filament to the junction, if we arrange the junction so that it always remains within the gaseous envelope surrounding the heater.

The characteristics of both junction and heater wire must be carefully considered. The heater wire should be of small diameter and of a material having a high resistance and capable of withstanding considerable strain when heated to a cherry red. Tungsten wire 0.07 mm. in diameter was found to meet these requirements and gave the proper temperature with a current of about 40 milliamperes. Experiments indicate that cells constructed with a heater wire of larger diameter do not remain

*"Convection and Conduction of Heat in Gases" by Irving Langmuir, *Physical Review*, 6-12, vol 34, p. 408.

constant. A heating filament of about 1 cm. in length and having a resistance of approximately 12 ohms has given good results.

The couple wires should have as small diameter as possible in order to decrease the conduction of heat, reduce the lag to a minimum and to permit reaching a maximum temperature. The couple and heater wires are mounted on a single glass stem and after an adjustment of their relative positions has been made, this working unit is placed in the glass bulb.

Use of Gas in Bulb. If we produce a high degree of exhaustion in the bulb a low thermal e. m. f. must result. Conversely, if this bulb is filled with gas under pressure, the junction will develop a high thermal e. m. f. the temperature of the filament being the same in both cases. In the first case the transfer of heat by convection is practically eliminated, leaving only the loss of heat by radiation from the small heater wire. This is negligibly small up to a temperature of several hundred degrees. In the second case, the heat is transferred so rapidly by convection that a high current value is required for the heater. Besides affecting the thermal e. m. f., the amount of gas is an important factor in the operation of the cell. Experiments have shown that if the amount of gas exceeds a certain value the cell becomes very sensitive to even slight changes in position. The thermal e. m. f. increases as the junction approaches the point directly above the heating filament.

Nitrogen gas under a pressure of about 2 cm. has given good results. It permits the operation of the cell in any position and a transfer of heat sufficient to produce the required e. m. f. In the construction of the cell, the bulb is first exhausted to about 0.4 microhms. It is then heated to approximately 350 deg. cent. to eliminate moisture, at which time the gas is admitted and the bulb sealed off. A second filament, heated to high temperature for a few minutes is used to eliminate the impurities in the gas.

Aging of Filament. After the bulb has been assembled, the current is passed through the filament, raising its temperature to a point considerably beyond the final operating value. This ages the filament and increases the permanency of the cell. (Experiments were made to determine if it were possible to age the heating filament wire before placing in the bulb. This did not seem to produce the desired aging effect, even when heating the filament to a high temperature in gas. Possibly the wire

was aged, but the handling necessary to assemble it in the bulb, may have resulted in mechanical strains sufficient to nullify the aging.) Tests are then made to determine the proper values of R and R_1, also the amount of positive temperature coefficient metal to be inserted in the balancing resistance.

Connections Between Bulb and Binding Posts. Any lag in the temperature of the balancing coil with respect to that of the heater and the couple wires is prevented by spiralled connections, as shown in Fig. 4. By spiralling the connections from the bulb to the binding posts on the top of the cell the heat conducting path is lengthened, so that changes in the surrounding temperature have a uniform effect on all parts of the cell.

Characteristics

The e. m. f. obtainable from this cell is not limited as in other types. The drop resistance may be adjusted so that the potential across it can have any desired value, usually one volt. Any combination of e. m. f. values however, may be obtained by taking taps from the drop resistance.

Temperature Coefficient. As previously shown the temperature coefficient may be reduced to zero.

Effect of Temperature. The surrounding temperature may vary from values below 0 deg. cent. to over 100 deg. cent. without the slightest injury to the cell. This is due to the fact that no liquid is used in its construction.

Effect of Short Circuit on Cell. When these cells are properly constructed they cannot be damaged by short-circuiting any of the external connections.

Accidental Increase of Filament Current. An excessive current through the filament may change the standard value of e. m. f. but will not necessarily destroy the usefulness of the cell. This effect will be considered under "Results of Tests."

Position of Cell. The position of the cell does not affect the value of the standard e. m. f.

Strength. The cells are readily portable and will stand severe shocks without injury. They can be easily shipped if ordinary precautions are taken in packing.

Accuracy. Due to the fact that a null method is employed in balancing the cell, the observation error will be a minimum one. The principle sources of error are the temperature coefficient of the cell and thermal e. m. f's. other than the e. m. f. generated at the junction. The error due to thermo e. m. f's. in

the galvanometer circuit may be reduced to a minimum by short-circuiting the galvanometer circuit at the standard cell. The galvanometer will then indicate the true zero which must be used in obtaining the balance of the cell. Furthermore, any error in the thermo-junction circuit introduces but half the per cent error in the potential measured across the drop resistance. As previously stated this is due to the fact that the thermal e. m. f. of the junction circuit is proportional to the square of the current through the filament, whereas the potential across the drop resistance is directly proportional to this current. A standard thermo cell that is properly constructed and operated should remain constant to at least 5 parts in 10,000. It is probable that in view of experiments now under way that the constancy of these cells will be greatly improved.

Permanency Tests. In Fig. 5 is shown the results of tests on standard thermo cells 1 to 8 inclusive. These tests cover a period of 14 to 23 months and are representative of a considerable number of cells. The following table gives the results of these tests:

Cell No.	Original e. m. f.	Maximum variation. Parts in 10,000	Avg. variation from orig. e. m. f. Parts in 10,000	No. of tests	Period covered by tests. Months
1	0.9890	2.0	0.0	27	23
2	1.0199	5.0	1.4	21	14
3	1.1294	3.0	1.0	16	15
4	1.0645	4.0	1.1	20	16
5	1.2004	4.0	1.4	14	14
*6	1.0376	3.0	0.2	5	4
†6	1.0371	4.0	0.4	14	11
7	1.1833	6.0	2.1	14	12
*8	0.9828	5.0	2.0	17	15
†8	0.9791	4.0	1.6	9	7

*Before filament temperature was accidently increased to reddening point.
†After filament has been brought up to reddening point.

Results of Tests. The curves for cells 6 and 8 indicate the effect of increasing the filament temperature above the reddening point. As previously stated, this produces a permanent increase in the filament resistance. It follows that when the filament resistance is increased, the heater current must be decreased to obtain the new balancing point *C*. See Fig. 3. This decrease

in current of course results in a smaller potential across the drop resistance. It is interesting to note, that after the heating filament resistance has changed, the cell continues to function properly except that a lower value of e. m. f. is obtained, assuming of course, that the drop resistance is not changed. In the event of an accident of this nature the standard e. m. f. may be restored to its original value by adjusting the drop resistance.

Conclusion

In conclusion, it is believed that the characteristics of the standard thermo cell justify its use in many measurements in which standard cells are now required. This cell has been successfully used with potentiometers designed for thermocouple

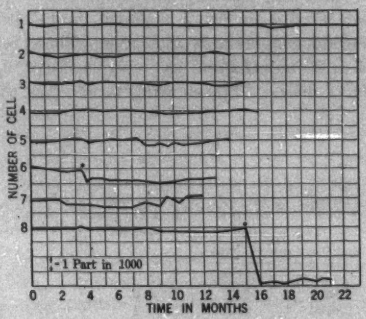

Fig. 5—Curves Showing Constancy of Typical Standard Thermo Cells

*Filament current accidentally overheated at this point

work. During the past year several of these potentiometers have been in constant use, and were operated by unskilled labor. During this period no trouble has been experienced that could be charged to the standard cell. Several cells have been burned out through the carelessness of the operator. In every case of a damaged cell it has been found that either the galvanometer or the slide wire or both were destroyed.

Very little experimental work has been done during the past year due to conditions imposed by the war. There is still considerable work to be done on the cell some of which is now under way.

DISCUSSION ON "A NEW STANDARD FOR CURRENT AND POTENTIAL" (ALLCUTT) AND "THE THERMOELECTRIC STANDARD CELL" (HOXIE). NEW YORK, FEBRUARY, 15, 1918.

Clayton H. Sharp: These papers indicate the efforts which have been made to satisfy a need in the field of potentiometry. One reason why the need has become acute is indicated by Mr. Allcutt in the beginning of his paper where he says that the standard cells are hard to get, and he also asks—"Why should we use a standard cell which is capable of very high precision in work which does not require such very high precision, and is not capable of such high precision?"

I think one answer to that is that it is not necessary that the

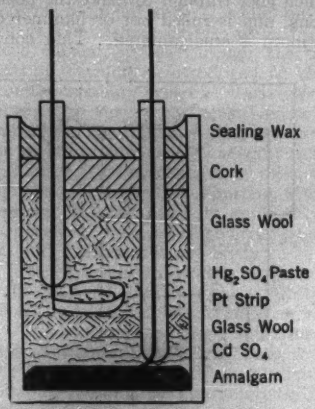

FIG 1.—SIMPLE FORM OF WESTON CELL

cell which is used should be of such high precision in order to have a standard which is suitable for that kind of work.

Now, to make a Weston standard cell, as a primary standard of electromotive force, requires a very great degree of care and many precautions in the preparation of the chemicals, the assembling of the cells, and in their aging and treatment thereafter. That is for the purpose of making a precision standard. But if you take the materials that go into a Weston standard cell, that is, purified cadmium sulphate, purified mercurous sulphate, and distilled mercury, more or less as they are, and assemble them in a cell, you get something which is not a primary standard, but which gives an electromotive force very close to the electromotive force of the primary standard, which is reasonably constant, which has the low temperature coefficient of the Weston standard cell, and which will serve very well indeed for

this grade of potentiometry, and is not subject to the objection of being very expensive or very delicate.

Such cells we have been using for some time in certain classes of our work. They are constructed as shown in the Fig. 1. The amalgam of mercury and cadmium is at the botton of the tube, which may be a vial or something of that kind. There is no glass blowing about it, except to seal the platinum wires in the ends of the glass tubes. On top of the amalgam is a layer of crystals of cadmium sulphate, and on that some glass wool and on top of that, the paste of mercurous sulphate rubbed up with metallic mercury and cadmium sulphate crystals wetted with a saturated solution of cadmium sulphate. The end of the platinum wire which goes through to make the positive terminal is left rather long, and is rolled out or hammered out thin, and then the platinum is amalgamated. That constitutes what is

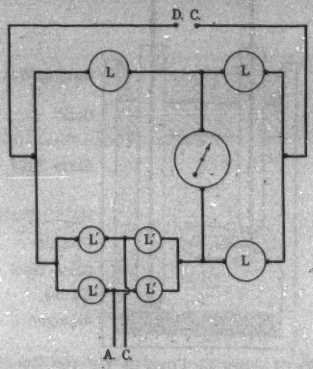

FIG. 2

ordinarily the mercury electrode of the cell, and is simply coiled up in the paste. More glass wool is put in and the entire contents are wetted with saturated solution of cadmium sulphate and sealed in with sealing wax on a cork. That cell is not a primary standard, but it is pretty good for low accuracy potentiometer work, and does not cost much. Anybody can make it. That is another way out of this difficulty, not so neat and ingenious as those which have been described, but still worth knowing about.

I was very much interested in Mr. Allcutt's application of what might be called the bolometer principle to produce a standard e.m.f. A good many years ago they used to use a similar arrangement under the name of the Howell bridge, a bridge which consisted of three arms of zero temperature coefficient wire, and the fourth arm made of an untreated carbon filament lamp. The untreated carbon filament having a large negative

temperature coefficient, the bridge would be in balance at one value of the current or voltage upon it, and an indicator of that kind was in considerable use in the early days of the industry. More recently, with the advent of tungsten lamps, that idea has been applied in photometry. The current in the photometer lamp is maintained at proper value by taking advantage of the positive coefficient of the tungsten filament, making the lamp one arm of a bridge. The balance can be indicated by a galvanometer or a telephone receiver.

This is something we have tried also—to make an a-c.-d-c. arrangement on this principle. We have made a bridge of seasoned tungsten lamps as shown in Fig. 2. Each of these arms is composed of a single lamp L, while the fourth arm is itself an auxiliary Wheatstone bridge made up of the four lamps L^1. Direct current passes through the main bridge, while the alternating current is introduced at the two equipotential points on the auxiliary bridge. Variations in the a-c. voltage are reflected with very great sensitiveness in the d-c. measuring instrument. The difficulty with the arrangement is to get a selection of lamps sufficiently constant and so balanced that the bridge is insensitive to changes in room temperature, and insensitive to changes in the direct current which is passing through it. However, it may have its application in certain cases. The sensitiveness of these bridge arrangements is really very remarkable, and I think it is not likely to be appreciated until one does a little experimental work with them.

F. C. Stockwell: Mr. Allcutt proposes a substitute for the standard cell, referring presumably to the Weston (cadmium) Secondary Cell. The occasion for using a substitute arises from a serious shortage of standard cells, and their relatively high cost. The device described is an ingenious solution of the problem. It has, however, the inherent characteristic of substitutes in that it is inferior to the original in certain important particulars, viz., simplicity, convenience, accuracy and permanence. "Avoid substitutes" is a good slogan because it is good sense. If standard cells are difficult to obtain from the manufacturer, they are not difficult to set up and they are not expensive.

I had occasion in the latter part of 1914 to set up some cadmium cells at Stevens Institute of Technology for the use of senior students to take the place of the commercial form of secondary standard. Four cells were constructed of the ordinary "H" form. Chemicals of the highest grade (C. P.) were used. The specifications followed were those of the Bureau of Standards, but no attempt was made to carry out the refinements there described for the preparation of the different ingredients. It should be borne in mind that when the greatest care is exercised these cells are reproducible to within 2 or 3 parts in 100,000. As a practical matter chemicals can be used in the form purchased, and with ordinary care the cells are readily set up in a short time. This is not a difficult operation; the simplest apparatus only is required. One of the cells was made from a

solution, amalgam, and paste that had not been freshly prepared. Notwithstanding this wholly unscientific procedure, this cell after a few weeks seasoning has been indistinguishable from the others in its characteristics. The cost of the cells for chemicals and glassware was about two dollars each. The materials are readily obtainable now at about 50 per cent increase in price.

These home-made cells have been used for three years for a period of 6 to 8 weeks each year, and they have occasionally been subjected to severe abuse at the hands of students, such as reversal of polarity, and use as a current supply for Wheatstone bridge measurements. The cadmium cell has, however, remarkable recuperative qualities, after a short period recovering its original e.m.f. It may fairly be assumed, I believe, that these cells have operated under conditions which are more severe than would be ordinarily encountered in technical work.

These cells have been calibrated at the beginning of each college year to determine their fitness for further use. The

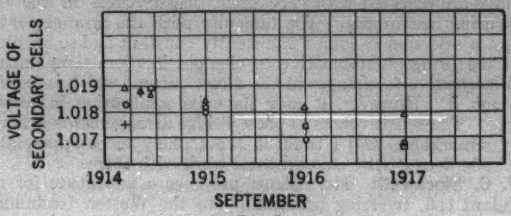

FIG. 3

results are shown in the accompanying plot, Fig. 3. It will be noted that their e.m.f's are now about 1/10 per cent lower than when the cells were prepared.

The experience with the cells which I have described raises the question whether there is a real need for a substitute for the cell as a secondary standard of e.m.f. An authoritative opinion could hardly be based on tests upon so small a number of cells. I believe that it would be very desirable to have recorded the experience of others who have prepared secondary standards in order that this question may be satisfactorily answered.

W. B. Kouwenhoven: I think that these two new devices will find a considerable field of usefulness in the university and technical school, and also in certain branches of the profession where it is not desirable to use a small portable standard cell of the Weston type.

I should like to ask Mr. Allcutt what is the effect of overloading his cells, that is, suppose the cell is heated to incandescence by some accidental short circuit? In Mr. Allcutt's aging curves, in connection with Fig. 4, he makes the statement: "Curve *A* shows the current resistance characteristics of a new bulb. Curve *B* is plotted from data taken twenty-four hours later,

while Curve *C* was taken after twenty-four hours' seasoning. Further tests at a later date followed Curve *C* with great exactitude." Has he any data showing the effect of overloading on the characteristics of the cells?

P. G. Agnew: There is one rather important advantage in both of these types of cells which I think neither author has mentioned, and that is, a very low resistance cell can be readily constructed. In fact, the values given by both authors are lower than that of the unsaturated, and decidedly lower than the saturated Weston cell. That is a point which is of importance in the question of the deflection potentiometer, in which it is desirable to use a pivoted rather than a suspended galvanometer.

C. T. Allcutt: Dr. Sharp has brought out the fact, which possibly is not as well known as it should be, that standard cells can be made of a degree of precision suitable to the service in which they are to be used. Mr. Stockwell made a similar remark.

In connection with the question of convenience brought out by Mr. Stockwell, if we consider this current balancing bridge as a unit with the portable potentiometer, we see that the necessary operations in balancing the potentiometer are the same as when balancing against the standard cell, that is, the galvanometer is thrown across the bridge, which is permanently connected in series with the potentiometer bar, and a balance is obtained which sets the current in the potentiometer bar at the desired voltage, and then the galvanometer is thrown off into the circuit in series with the unknown e.m.f., and the measurement is obtained.

In considering the relative cost or difficulty involved in obtaining a bridge of this type and a standard cell, we must consider, in addition to the ordinary type of standard cell, the chemical type; it is necessary in the potentiometer to provide the resistance through which the drop in voltage is obtained, that is compared to the e.m.f. of the standard cell, and that must be charged up to the standard cell in its connection with the portable potentiometer.

For example, if we consider a potentiometer unit, we will find that they comprise a solid wire and a current balancing bridge. That is, the resistance would be manufactured approximately equal. The bulbs can be made with a characteristic within ten per cent. Having given these two resistances, which are comparatively simple, and may be of wire, if necessary, we have then this one resistance which is necessary to adjust in order to fit the whole potentiometer for service. In case a standard cell is used, we have a potentiometer wire and another resistance, and provision for connecting the standard cell across that resistance. The current is varied until the drop through this resistance is equal to the e.m.f of the standard cell.

In the manufacture of the potentiometer, this resistance must be carefully adjusted to give the proper current to balance

against the standard cell. So we consider, then, the standard cell with this resistance in comparison with the bridge in the unit.

That cuts down somewhat the relative difficulties involved in the two, that is, the bulb itself becomes the equivalent of the bridge in this case.

Mr. Kouwenhoven has asked two questions; first, concerning the effect of overload. That is, perhaps, best answered with reference to Table II which is a record of a test on one of the bulbs. It is carried up, as you will notice, to currents of 8 milliamperes, at which current the filament was bright red hot, almost as bright as the ordinary carbon filament lamp, and the reproduction of these curves, through a considerable period of time, showed that for short intervals, at least, during the time taken in running the test, the filament could be heated to a bright red heat, without causing any change in its characteristics.

The record of the test given in Table II is not of the same bulb of which I showed the seasoning curves Mr. Kouwenhoven refers to in Fig. 4. I show some further tests in Fig. 6, with great exactitude. I have not the data of this particular bulb. After these other curves were taken it was burned out shortly afterward, so that the test did not cover the full period of time covered with certain of the other bulbs, but if you read these data given in Table II, they are to the same effect, showing the degree of precision with which the bulb will follow the curve after seasoning.

C. A. Hoxie: Mr. Kouwenhoven asked about the cold end. Both the cold ends are within the bulb. The copper wire is very short. It extends from the center of the bulb, and about one-quarter inch away— a half millimeter or a little over in each side is the cold end—and it is all contained within the bulb, so it is pretty definite at all times.

A. C. Campbell: (communicated after adjournment): Some years ago I described (Proc. Inst. Elec'l Engineers p. p. 10-11, Vol. 30, 1901) an arrangement for measuring current which is identical in principle with the main part of Mr. Hoxie's device. Instead, however, of obtaining a balance for only one definite current, in my apparatus a potential contact connected to the galvanometer could slide along the resistance R and thus a considerable range of current could be measured. The practical results which Mr. Hoxie has obtained with his cell are most interesting.

Mr. Allcutt does not seem to know that thermal standards such as he has been working with have been known for many years past. I described the construction and use of such standards in a paper read before the Institution of Electrical Engineers (London) (Proc. I. E. E. p. 12, Vol. 30, 1901), and in the discussion Professor Callendar mentioned that some years earlier he had used similar standards and found that a good degree of accuracy was obtainable. It is interesting to know that the thermal methods are coming to the front once more.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.

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THE CHARACTER OF THE THERMAL STORAGE DEMAND METER

BY P. M. LINCOLN

ABSTRACT OF PAPER

Following a detailed description of the principle and construction of the thermal storage demand meter the author shows wherein it always indicates what may be called "logarithmic average" rather than "arithmetic average" of power consumption, heretofore indicated by practically all demand meters. The inherent faults of the "arithmetic average" or "block interval" meter are described and examples given demonstrating that the thermal storage meter alone recognizes the true heating effect that fixes size of equipment and therefore cost that should be assessed against the customer.

THE ADVENT of the thermal storage wattmeter naturally raises a question concerning the character of the quantity that is measured by that device. The object of the following pages is to discuss this question and particularly to analyze the "logarithmic average"—the quantity measured by any thermal storage meter—and compare this quantity with the arithmetical average which is the quantity measured by practically all previously existing types of demand wattmeters.

Let us first consider the fundamental reasons for measuring maximum demand. Briefly stated, the incorporation of maximum demand in a rate for electric service is an attempt to assess upon the user of that service his proper share of the annual cost of the equipment necessary for giving the service. Let us assume a concrete case as an example. Assume, for instance, that the consumption of a given customer is 1000 kw-hr. per year. If this load is taken at a perfectly steady rate throughout the entire year it means a steady consumption of 114 watts continuously. The amount of equipment to supply the load as thus taken is fixed by this continuous load of 114 watts. But now let us assume that our customer insists on taking his entire year's supply in a single day. Instead of equipment to supply 114 watts, we must now provide equipment to supply 41.7

kilowatts; that is, the equipment must be 365 times as large as before. Let us go further and assume that our customer insists on having his entire year's supply in thirty minutes; this would mean an equipment able to deliver 2000 kw. for one-half hour. Obviously, the cost of the equipment for this condition would be enormously greater than that for the supply of 114 watts continuously, and it is only just that the customer that takes his entire year's supply in a day or an hour should pay more for his service than the one who distributes his demands more

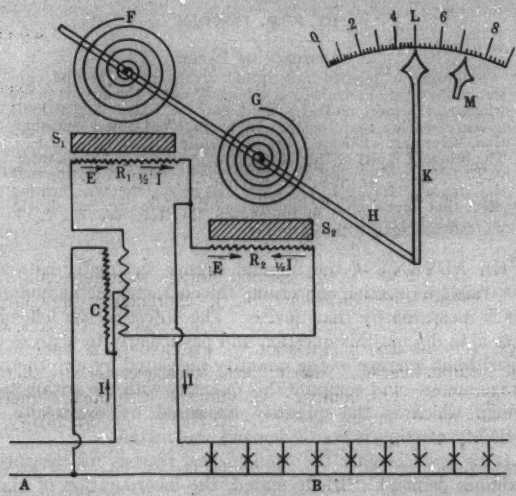

FIG. 1

evenly. Our illustration is, of course, exaggerated, but the exaggeration is one of degree and not one of kind. It is the object of the demand rate to recognize automatically this variation in the equipment necessary with variation in load factor and to assess this equipment cost against the customer. Also, it is the "maximum demand" and not the kilowatt hours of consumption that determines the amount of equipment that must be installed to carry a given customer's load. It is to determine this "maximum demand" that demand meters are used.

It may be well at this point to give a brief description of the thermal storage wattmeter as now being built.*

Referring to Fig. 1, A is a circuit feeding a load B. C is a small transformer incorporated within the meter with its primary across the circuit A. In series with the secondary of this transformer are two equal resistances R_1 and R_2. A current is, of course, set up on these resistances that is proportional to the voltage of the circuit A. The load current is also caused to circulate through these same resistances in the manner shown in Fig. 1, being taken into the middle of the secondary of the small transformer and being taken out at the connection between resistances R_1 and R_2. These two currents—one the secondary current, due to the presence of the voltage and the other due to the passage of the load current—are additive in one of these resistances and subtractive in the other, and the difference in the heating effect of the two resultant currents is proportional to the watts of the load B.

If we represent the current that passes through the resistance R_1 and R_2 due to the presence of the voltage by E, and the load current therein by I, the resultant current in one of these resistances is $E + I$, and in the other $E - I$. The losses are of course, proportional to the squares of these currents and the *differences* in these losses is proportional to the product $E\,I$. This holds true, independent of power factor and wave form, as shown in the paper above referred to.

F and G represent two spiral springs made from bimetallic strip, attached rigidly to their casings at the outer ends and to a common shaft H at their inner ends. These bimetallic springs tend to coil up on an increase in temperature (due to the difference in temperature coefficient of the two metals of which they are composed), but, since the two springs are wound in opposite directions, no movement of the shaft H will take place unless there is a *difference* in temperature between F and G. The shaft H, therefore, will not turn with changes in atmospheric temperature or with any other condition that causes both springs to maintain the same temperature, but will respond only to the *difference* in temperature caused by the difference in the losses in resistances R_1 and R_2. S_1 and S_2 represent diagram-

*The complete theory of the thermal storage wattmeter is given in the author's paper read before the American Institute of Electrical Engineers, Oct. 8, 1915, entitled "Rates and Rate Making". (TRANSACTIONS A. I. E. E. Vol. 34, pages 2175 to 2214.)

matically the thermal storage of the cases in which the bimetallic springs F and G are enclosed. Due to this thermal storage, the wattmeter does not respond instantly to a change in load but always indicates the logarithmic average load over the time period immediately preceding the instant of observation, the length of this time period being determined in part by the amount of thermal storage in the cases, shown diagrammatically at S_1 and S_2. K is a pointer attached to shaft H and traveling over the scale L. M is a loose pointer which shows the highest excursion of pointer K since last reset.

Fig. 2 is made from a photograph of a graphic meter of the thermal storage type with the cover removed, showing the working parts. The two cylindrical cases each containing a coiled bimetallic spring may be observed at the top of the instrument. The thermal storage capacity of these cases is so designed that it requires thirty minutes for them to acquire 90 per cent of their final temperature on a steady application of load. The working parts of the indicating meter is a duplicate of that shown in Fig. 2, except for the omission of clock, paper rolls, etc.

A thermal storage meter thus constructed always indicates what may logically be called the "logarithmic average" of the power consumption during the particular time period immediately preceding the instant of observation. Quoting the language of the paper above referred to on this point, the indications of a thermal storage wattmeter "will not be due to the watts passing at that instant, as is the case with the indications of an indicating wattmeter of the usual type, but will be the resultant of all the wattage flow that has passed, each instant of past flow having a value influenced in respect to its time proximity by a logarithmic law. This resultant is not an average in the commonly accepted sense of that word. When we use the word average in its commonly accepted sense, we assume that each instant of time over which the average is taken has equal weight. In the resultant that is obtained by a heat storage meter, each instant of time has not an equal weight, but the influence of each instant decreases with its remoteness in point of time, and the degree by which the watts during any instant influences the total indication is proportional to e^{-Kt}, where e is the base of Napierian logarithms, K is an adjustable constant, and t is the time measured backward from the instant of observation. For want of another name, let us call the resultant thus obtained by means of thermal storage the 'logarithmic average.'" The

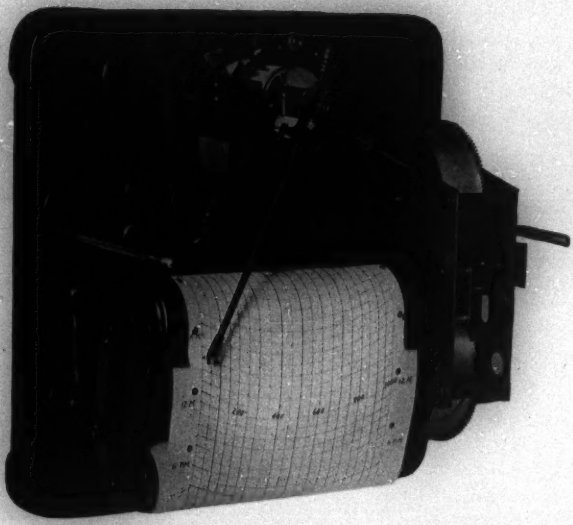

FIG. 2 [LINCOLN]

foregoing quotation indicates the nature of the quantity that is measured by the thermal storage demand meter. In a subsequent paragraph of this paper, a further quotation will be made from my previous paper showing how the "logarithmic average" of a given load may be calculated.

Heretofore, practically all demand meters have indicated in terms of the arithmetical average. This has followed from the fact that the basis of practically all previous demand meters has consisted of a standard watthour meter coupled with some timing device by means of which the integrated value of the load under measurement is obtained over a series of short time intervals. A further mechanism is provided so as to record the highest one of these successive blocks of energy. This type of meter is known as the "block interval" meter.

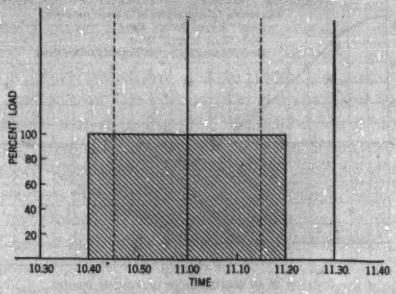

FIG. 3

One of its inherent faults is that it may split an isolated peak of load and therefore become indefinite in its indications. Reference to Fig. 3 will show the reason for this. Suppose we have an isolated block of load that comes on at say 10:40 a.m. and lasts until 11:20 a.m.—such a load for instance as would be involved in the pumping out of a small drydock. Suppose, further that we are using a "block interval" meter with a thirty minute time period to measure the maximum demand of this load. If the time intervals of this meter happened to begin and end on the even half hours—that is, if it integrated the load first from 10:30 to 11:00 and then from 11:00 to 11:30—it is evident that the maximum quantity indicated during any one period would be much less than if the meter periods began and ended on the even quarter hours. It is also evident by inspec-

tion that this indefiniteness of indication begins when the duration of the block of load is less than 60 minutes and that when its duration is less than 30 minutes, this "coefficient of indefiniteness"—if we may coin that term—becomes 50 per cent; that is, a load peak of less than thirty minutes duration may be entirely integrated within a single meter period or it may be divided equally between two adjacent periods depending upon the instant of time when these meter periods begin and end. There have been various suggestions of methods to overcome this fault but so far none of these suggestions has borne fruit.

A second and more serious fault of the "block interval" meter is that for isolated blocks of load it does not measure the

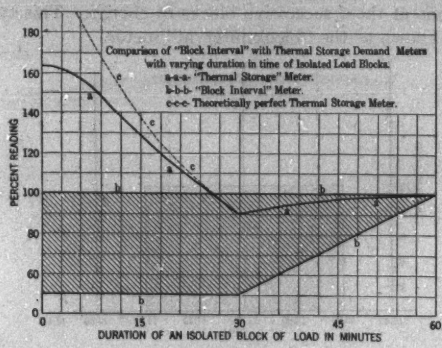

FIG. 4

true value of its heating effect on the equipment that serves the load and therefore does not measure the true value of the duty on that equipment. This matter is further treated in subsequent paragraphs. There has been no suggestion of any method by which the "block interval" meter may overcome this fault and there seems to be no possibility of such a suggestion.

So long as the loads are steady over long periods of time, the arithmetical (block interval) and the logarithmic (thermal storage) averages are exactly the same. It is only when the duration in time of a block of load begins to come down to the time period of the meter that there is an appreciable difference between the two types. To assume a concrete case again, suppose that service is being sold on the basis of the maximum

demand over a 30-minute period. So long as the duration of an isolated block of load exceeds one hour (twice the meter period) the "block interval" and thermal storage meters will give the same results for all practical purposes. Theoretically, for periods of load duration greater than twice the meter period (sixty minutes in our concrete case) the difference between the two types is less than one per cent. For load duration less than 60 minutes, the comparison between the two types of meter is shown in Fig. 4.

The cross hatched area in Fig. 4 indicates what may be called the "area of indefiniteness" of a "block interval" meter of 30-minutes time period; for isolated blocks of load of less than 60 minutes duration, the indications of the "block interval" meter may fall anywhere within this area. On the other hand, the thermal storage meter is perfectly definite in its indication. Each time a given load of given time duration is applied to this type of meter, it gives the same indication.

However, this indication differs from that of the "block interval" meter and the comparison between the two types is given in curve *A A A* in Fig. 4. For a 60-minute block of load, there is a difference between the two types of only one per cent. As the block continues to decrease in time of duration, the thermal storage meter continues to decrease in indication compared to the "block interval" meter (assuming that the "block interval" meter is reading its maximum) until with a 30-minute block of load it reaches 90 per cent. As the time of load duration continues to decrease below 30 minutes, the indications of the thermal storage meter increase until with very short applications of load it indicates about 163 per cent of the maximum of the "block interval" meter and 326 per cent of its minimum.

There will be some one who will at first be constrained to comment adversely on the fact that the thermal storage meter reads higher than the "block interval" meter for all load durations less than about 26 minutes, and that when the load durations are very short, this discrepancy is so large. The action of the thermal storage meter in this respect is, however, entirely defensible. Referring, for instance, to the same concrete example we used above, suppose our customer with a yearly consumption of 1000 kw-hr. insists on taking his entire year's supply in one minute. He would obviously use energy during this one minute at the rate of 60,000 kw. Electrical equipment can, of course, be grossly overloaded for so short a time as one minute, but even

with the greatest overload imaginable, the amount of equipment to carry 60,000 kw. for one minute is more than that to carry 2000 kw. for 30 minutes. The "block interval" meter would recognize no difference whatever between these two conditions, since the arithmetical average of the two loads when taken over a 30-minute period is exactly the same. The thermal storage meter, however, discriminates automatically against the short-time high-peak load and gives a result which in any event follows the same kind of a law as does the heating of the equipment that

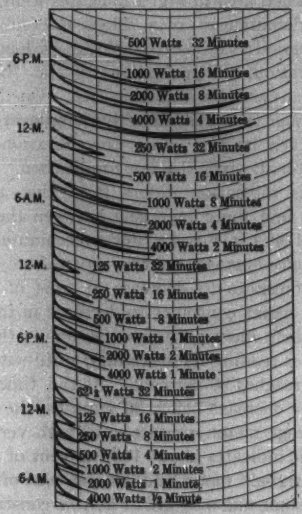

FIG. 5

furnishes the power. In short, both the "block interval" and the thermal storage meters recognize the effect of load factor so long as the duration of the block of load is equal to or greater than the meter period. However, when the load duration is less than the meter period, the "block interval" meter recognizes no difference while the thermal storage meter continues to recognize the heating effect of a given block of load entirely independent of its duration in time. It is the *heating effect* that fixes the size of the equipment, and, therefore, the cost

that should be assessed against the customer. It follows, therefore, that the indications of the thermal storage meter are more logical for rate making purposes than are those of the arithmetical average or "block interval" meter.

In this connection, the results of tests shown in Figs. 5, 6, 7, 8 and 9 will be of interest. These show the results of a number of test runs made with a graphic meter of the thermal storage type. These tests were made on the instrument shown in Fig. 2. The time period was 30 minutes; that is, the application of a steady load would bring the meter indication to 90 per cent of final at the end of 30 minutes, 99 per cent at the end of 60 minutes, 99.9 per cent at the end of 90 minutes, etc. The paper speed was one-quarter inch per hour and the full scale was 1000 watts. Fig. 5 shows the result of four series of tests. In the first series shown at the right hand bottom of Fig. 5, 2000 watt-minutes were applied to the meter, each test of the series starting at zero. In the first test of the first series the whole 2000 watt-minutes were put in in one-half minute; that is, power was applied at the rate of 4000 watts (four times normal load) for one-half minute. After the meter had had time to return to zero, the second test of the series was applied, viz., 2000 watts for one minute. The third block was 1000 watts for two minutes; then follow 500 watts for four minutes, 250 watts for eight minutes, 125 watts for 16 minutes and 62.5 watts for 32 minutes. If these same tests had been applied to a "block interval" meter they would all have given the same results (barring the "indefiniteness" mentioned in a preceding paragraph) if measured on a 30-minute meter except, of course, the last test of the series, which would have been lower than the others in the ratio of 30 to 32. I think there are very few who will contend that the customer who insists on taking his entire half hour's requirements in a half minute should not pay a larger demand rate than the one who spreads it out evenly throughout the half hour.

The second series of tests, shown in Fig. 5, is the same as the first except that the amount of power is doubled, 4000 watt-minutes being applied instead of 2000. No tests were made that involved applying load at a rate greater than four times normal. Similarly, in the third and fourth series of tests, 8000 and 16,000 watt-minutes were applied respectively, with the results shown. Comparing the four series of tests with each other, it will be seen that doubling the watts for a given time period doubles the meter indications, as, of course, might have been anticipated.

Comparing now the individual tests of a given series and comparing these results with those deduced from purely theoretical considerations, we find that the quantity indicated by the actual

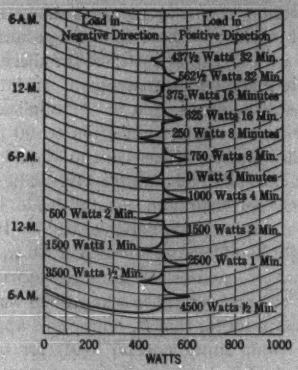

FIG. 6

meter when subjected to isolated short time blocks of load is not as great as the true logarithmic average. Referring to Fig. 4, for instance, the curve *AAA* and the area *BBB* compare the

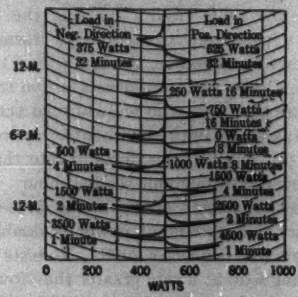

FIG. 7

actual thermal meter with the equivalent "block interval" meter. If the thermal meter had followed a true logarithmic law, the comparison would have been shown by the dotted curve *CCC*

instead of the solid curve *AAA*. The reason for this and a discussion thereof will be set forth in a later paragraph of this paper.

Figs. 6, 7 and 8 are identical with Fig. 5 insofar as the value of the load fluctuation is concerned, but the fluctuation is made with the meter starting from and returning to the *half* load point instead of the zero load point. In Fig. 6, for instance, there is shown a series of tests that are taken with the application of the following schedule. The meter was operated at 500 watts for a long enough period for the pen to take up the 500 watt position; then, 4500 watts was applied in a positive direction for one-half minute and the load was returned there-

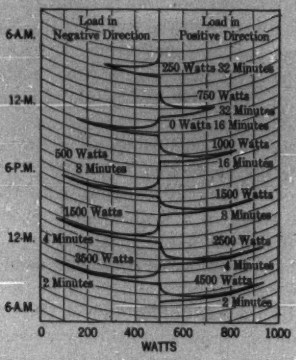

FIG. 8

upon to this 500-watt point. After the return of the pen to the steady 500-watt position, 3500 watts was applied to the meter in a negative direction for one-half minute, the load then being returned to the 500 watt point. This caused the pen to travel in a negative direction by the same amount as the first application caused it to travel in the positive direction. Also, it will be noted that the travel of the pen in both these tests is the same as in the first test of the first series in Fig. 5. The remainder of the tests in Fig. 6 are according to the following schedule, it being understood that after the application of the scheduled load, the load was in each case returned to the 500-watt point:

2500	watts	in	the	positive	direction	for	1	minute.
1500	"	"	"	negative	"	"	1	"
1500	"	"	"	positive	"	"	2	minutes
500	"	"	"	negative	"	"	2	"
1000	"	"	"	positive	"	"	4	"
0	"	"	"			"	4	"
750	"	"	"	positive	"	"	8	"
250	"	"	"	"	"	"	8	"
625	"	"	"	"	"	"	16	"
375	"	"	"	"	"	"	16	"
562.5	"	"	"	"	"	"	32	"
437.5	"	"	"	"	"	"	32	"

In other words, this schedule is a repetition of that shown in Fig. 5, except that it goes both ways from the 500-watt point instead of only one way from the zero point. A comparison of the various readings indicates that a given departure in load from the 500-watt point gives exactly the same result as the same degree of departure from the zero point—a result that might have been expected.

Figs. 7 and 8 show the same thing as Fig. 6 except that the loads are respectively twice and four times those in Fig. 6. These results may be compared directly with those in Fig. 5. Fig. 9 shows the result of a load schedule exactly similar to Fig. 5 except that the point of departure is made the full-load or 1000-watt point instead of the zero point as in Fig. 5, or the 500-watt point as in Figs. 6, 7 and 8. Figs. 5, 6, 7, 8 and 9 indicate that a given departure for a given time from the previous steady condition always gives the same result independent of where that previous steady condition has maintained the pen.

In the foregoing tests are given the results of applying isolated blocks of load to the thermal storage meter. The question now naturally arises, suppose the blocks of load are not isolated, but follow each other before the meter has had time to return to zero. The series of tests shown in Figs. 10, 11, 12, 13 and 14 were undertaken to answer this question. Fig. 10, for instance, shows three series of tests. In the first series, (shown at the right hand side of Fig. 10) 1000 watts were put on the meter and kept on for one minute; the load was then thrown completely off for one minute, that is, the time cycle was two minutes long. Both the load and the time of application were then accurately kept on a repetition of this load schedule for about one and a half hours. It will be noted that the meter pen came to a steady value of 500 watts just as if a steady load of 500 watts were

applied instead of a full load and zero load during alternat minutes. The second series of tests shown in Fig. 10 is exactly similar to the first, except in point of time of power on and power off, being a two-minute interval instead of one minute; that is, the time cycle was four minutes long. In the third series, the time cycle of load on and load off is ten minutes long, five minutes on and five off. Referring now to Fig. 11, which is a continuation of Fig. 10, this shows three similar series of tests, the difference being that the time cycles are 20

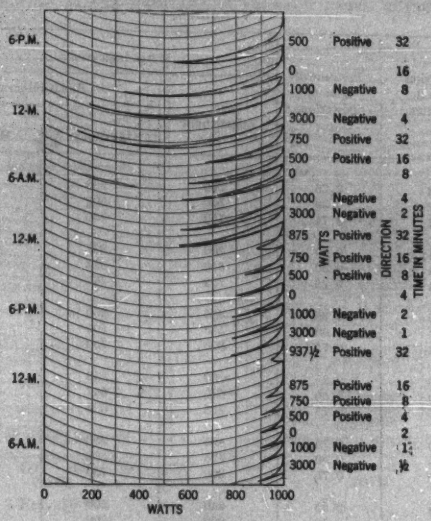

FIG. 9

minutes, 30 minutes and 60 minutes, respectively. In each of these power schedules, the average use of power is of course, at the rate of 500 watts. With the two-minute cycle the successive blocks of power blend into each other so that the result is the same as if 500 watts were applied continuously. When time of the cycle is increased to four minutes, the pen responds slightly to successive blocks of load application; the total travel of the pen being perhaps two per cent of the total scale. When the time of the cycle becomes ten minutes, this travel increases to

about ten per cent of total scale. With a 20-minute cycle, this travel becomes about 25 per cent; with a 30 minute cycle about 45 per cent and with a sixty minute cycle, 80 per cent. If we compare the thermal storage meter with the "block interval" meter for these various load conditions, we will note the rather curious fact that for certain time intervals, the "block interval" meter has an "area of indefiniteness," while for others it has not. Figs. 12, 13 and 14 show a number of series of tests on power

TABLE I.

Duration of cycle minutes	Per cent of time of power on	Meter indications.		
		Logarithmic	Block interval	
			Maximum	Minimum
2	0.25	250	250	250
2	0.50	500	500	500
2	0.75	750	750	750
4	0.25	260	267	250
4	0.50	510	533	500
4	0.75	755	767	750
10	0.25	285	250	250
10	0.50	550	500	500
10	0.75	800	750	750
20	0.25	360	333	250
20	0.50	640	667	500
20	0.75	855	833	750
30	0.25	425	250	250
30	0.50	725	500	500
30	0.75	890	750	750
60	0.25	640	500	250
60	0.50	900	1000	500
60	0.75	980	1000	750

cycles of the same length as in Figs. 10 and 11—viz., the cycles are of two minutes, four minutes, 10 minutes, 20 minutes, 30 minutes and 60 minutes. However, in Figs. 12, 13 and 14, the power is kept on during one-fourth and three-fourths of the time instead of one-half of the time as in Figs. 10 and 11. The resulting pen traces are highly interesting and instructive. Tabulating all of these results we arrive at the comparison between the two types given in Table I.

The two minute, ten minute and thirty minute cycles all give an arithmetical average of 250, 500 or 750 watts as the case may be, independent of the point in the cycle where the meter period begins. The other time intervals vary over the limits assigned in Table I, depending on what point in the power cycle the meter period begins.

It is evident from an inspection of Figs. 10, 11, 12, 13 and 14 as well as the foregoing table that the values indicated by the thermal storage meter increase in a definite, logical and consistent manner as the time period between peaks is increased

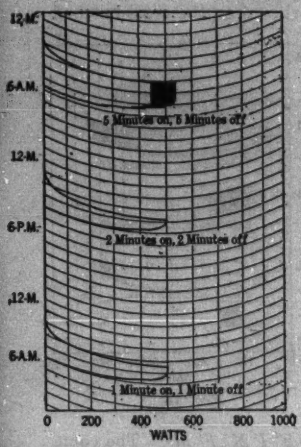

FIG. 10

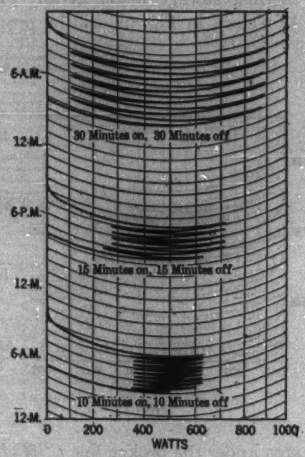

FIG. 11

while the "block interval" meter gives results that are indefinite, illogical and inconsistent under the same conditions. In other words, the thermal storage meter recognizes the maximum heating effect of a given load application of any character, while the "block interval" meter does not.

It may be of interest at this point to make a brief analysis of the action of the thermal demand meter and show the reasons for its departure from indicating a true logarithmic average for isolated short time loads as was referred to in a previous paragraph. Fig. 15 is a reproduction of Fig. 6 taken from the au-

thor's A. I. E. E. paper of October 8, 1915, and referred to in a previous paragraph. This shows how the "logarithmic average" of a given load may be calculated from purely theoretical considerations. Quoting from that paper, page 2195: "Suppose we have a load constantly varying with time as indicated by the broken line *C H D E I K F M*. If we apply a thermal storage meter to this load of such characteristics that it requires an hour for it to attain 90 per cent of its final indication, the cooling (or heating) curve of that meter will follow the law indicated by curve *A*. The quantity that will be indicated by such a thermal storage meter at any given instant (for instance, at 12 o'clock in Fig. 6), will be proportional to the cross-hatched

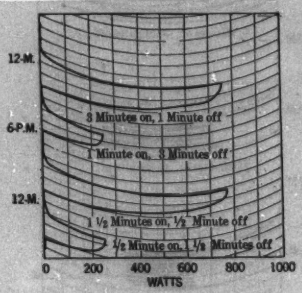

FIG. 12

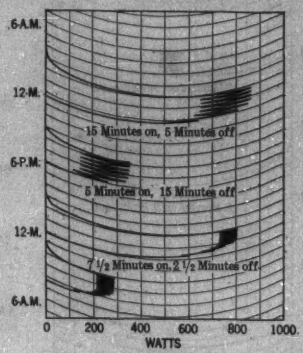

FIG. 13

area under the broken line *C′ H′ D′ E′ I′ K′ F′ M′*. The value of the ordinates of this cross-hatched area at any instant are proportional to the value of the power ordinate at that instant reduced by the ratio of the ordinate of curve *A* at that instant to the maximum ordinate *O G* of curve *A*. If we can just imagine this curve *A* as continually sliding along the power curve, the quantity which it measures will always be proportional to an area that is secured at each instant, just as Fig. 6 shows it at the instant of 12 o'clock.

"If our meter is a ten-minute meter instead of one-hour meter—that is, if it takes only ten minutes to cool down or heat up to within 10 per cent of its final value—the quantity that will be measured will be proportional to the cross-hatched

area under the broken line $K'' F'' M''$. In this case, the ordinates of this area are reduced in accordance with the logarithmic curve B instead of A; the curve B comes down to 10 per cent of its initial value in ten minutes instead of one hour as is the case of A."

The above shows the method of calculating the indications of a theoretically perfect thermal storage meter. For short time applications of load, however, the actual meter is not theoretically perfect. The principal cause for this departure is the fact that diffusion of heat throughtout the mass of a meter

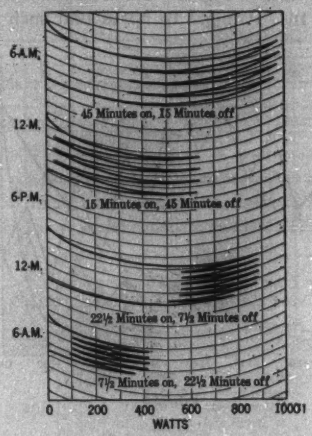

FIG. 14

element is not instantaneous. Consider, for instance, the application of a one-minute load to such a meter. Test shows that it takes nearly five minutes after the load is thrown off before the pointer reaches its maximum position. The application of this isolated block of load heats one meter element and cools the other from the previous steady condition. This wave of heat change, of course, originates in the resistance, but to effect the meter must pass to the spiral bimetallic springs. That is, the heat must first pass from the resistances to the casings enclosing the springs, then the air inside the casings is heated and this, in turn, heats the bimetallic springs. The heat must get

from the resistances to the bimetallic springs before the meter will respond. This process takes time. During this time some heat that has been put into the resistance escapes from the casing partly by radiation and convection and partly by conduction back through the lead wires. This action leads to no departure from theoretical when the load is steady but only when the load is an isolated block. The amount of this departure can readily be found from the results of the tests shown in Figs. 5, 6, 7, 8 and 9 and the comparison of the actual meter with the theoretically perfect meter is shown by comparing the solid and dotted lines in Fig. 4. This comparison is also shown directly in Fig. 16. It might be noted that such departure as

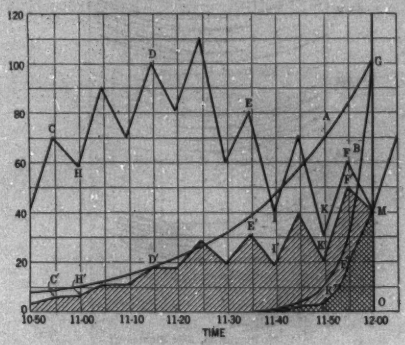

FIG. 15

there is from the theoretically perfect meter causes the actual meter to read lower than the theoretical. This is the "safe" position. If any device used in determining a customer's bill, favors the company supplying the service, it can be successfully attacked by the customer. If the contrary is true, it cannot. Hence, the departure noted is on the "safe" direction.

An objection has been raised to the thermal storage meter in that it reads higher on an increasing load than it does on a decreasing load, the kilowatt hours and the time of application being the same in both cases. The tests shown in Fig. 17 were undertaken to find the value of this discrepancy. In this figure, the first test (beginning at the right hand side) was made by applying 10 per cent load during the first minute, 20 per cent

during the second minute, 30 per cent during the third, etc., until ten periods of one minute each had been applied, the last one minute period being 100 per cent load. In the second test, exactly

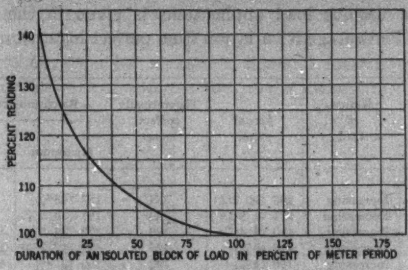

FIG. 16—THEORETICALLY PERFECT THERMAL METER COMPARED TO ACTUAL PERFORMANCE

the same load schedule was applied, except that the time of each application was made for two minutes instead of one. In the third test, this time period was increased to five minutes. In

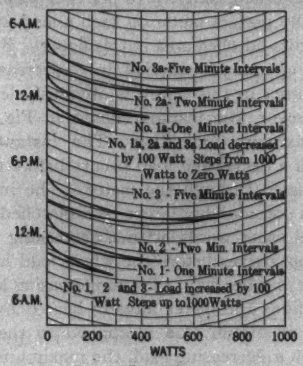

FIG. 17

the next series of tests, the load was made to decrease with time instead of increase; during the first time interval, 100 per cent load was applied, followed during the next interval by 90 per cent load, then 80 per cent, etc., until during the tenth interval 10

per cent load was applied. The time intervals were one minute, two minutes and five minutes as in the first series. The comparison of the actual thermal meter, the theoretically perfect logarithmic average meter and the arithmetical average meter for these various load applications is given in the following table. (Percentage is variation from perfect logarithmic meter)

Time interval. Minutes	Nature of load	Actual meter	Theoretically perfect meter	Block interval meter	
				Maximum	Minimum
1	Increasing	295 (10.2%)	328	187 (43%)	93 (71.5%)
1	Decreasing	275 (1.8%)	280	187 (33.3%)	93 (66.7%)
2	Increasing	480 (9.4%)	529	373 (29.5%)	187 (64.7%)
2	Decreasing	425 (0%)	425	373 (12.3%)	187 (56%)
5	Increasing	760 (4%)	792	750 (5.2%)	450 (43.2%)
5	Decreasing	630 (0.2%)	631	750 (9.6%)	450 (28.8%)

The reason for the difference in indication between increasing and decreasing loads is readily seen by reference to Figs. 18, 19 and 20. In Fig. 18 the method of analysis shown in Fig. 15 is applied to a load increasing by 10 per cent steps. The reading of a theoretically perfect meter on such a load would be proportional to the cross-hatched area in this figure. Fig. 19 shows the same method of analysis applied to a decreasing load. The cross-hatched area in this figure is obviously less than that in Fig. 18. However, the instant chosen in Fig. 19 is not the instant of maximum indication. The instant of maximum indication with a decreasing load occurs before the entire block of load has passed through the meter. The instant shown in Fig. 20 gives a considerably larger cross-hatched area than that in Fig. 19. In other words, with a decreasing load, the maximum indication arrives before the whole load has been put in. Or, to put it in another way, with an increasing load, each increment of load finds the meter already heated by the preceding load and the maximum load is applied to the hottest meter element. With a decreasing load, the maximum load is applied to the coldest meter condition and the maximum temperature arrived at is not as great as with an increasing load. This action is entirely defensible since exactly the same action takes place in the equipment that serves the load. An increasing load heats up transformers, cables, generators, etc., more than does a

decreasing load, although the kilowatt-hours and the time of application are exactly the same in each case.

The question may properly be raised as to the proper time period to use in the measurement of maximum demand. At present, the practise of various public service companies varies in this respect over a very large range. One minute is the minimum time duration for maximum demand measurement that the author is aware of and one hour is the maximum. Between

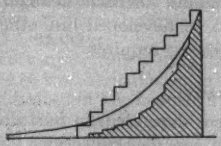

FIG. 18

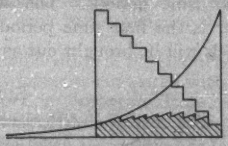

FIG. 19

these limits a large number of time periods have been proposed and used. So long as it is recognized that equipment cost is the element that dictates the maximum demand portion of a customer's bill, the use of time periods of less than about thirty minutes cannot be justified, since no part of a normal equipment for supplying electric service to a customer has heat storage characteristics that will cause it to arrive at 90per cent of its final temperature in less than thirty minutes and many of the items of such

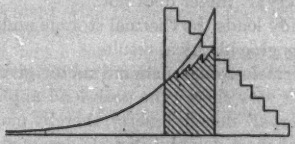

FIG. 20

an equipment have a much longer time period. In many cases, short-time periods for maximum demand have been adopted for the purpose of penalizing the customer with a high short-time peak. The thermal demand meter does this automatically and, therefore, there is not the same reason for using short periods when measuring demands with a thermal meter that there is when using the "block interval" meter. For steady loads, it does not matter whether the demand is on a one minute or a

one hour basis, the result is the same. The average generator, transformer or cable has heat storage characteristics that usually require a time considerably in excess of 30 minutes for them to arrive at 90 per cent of their final temperature when a steady load is applied. A 30-minute meter, therefore, is about the minimum time period for maximum demand that can be justified on the score of assessing equipment costs against the customer and the tendency of the future will undoubtedly be toward longer time periods. The 30-minute thermal demand wattmeter is the first time period to be developed but other time periods will be brought out as occasion requires.

Summary

1. The cost of electric service is dependent in part upon the cost of the equipment necessary to provide that service.

2. The cost of the necessary equipment to a given customer depends upon his maximum demand and not on his kilowatt-hours of consumption.

3. The thermal storage demand meter gives a perfectly definite indication independent of the character of the load applied, while the "block interval" demand meter becomes highly indefinite on short-time peak-load applications.

4. For short-time peak-load applications the thermal storage demand meter follows a law of the same nature as the heating effect of the load application upon the service equipment, while the "block interval" meter does not.

5. For steady loads the thermal storage and "block interval" demand meters give identical results.

6. The thermal storage demand meter gives a much higher indication for a very short-time peak-load application than does the "block interval" demand meter, thereby making unnecessary the adoption of short demand periods designed to penalize such high peak loads.

DISCUSSION ON "THE CHARACTER OF THE THERMAL STORAGE DEMAND METER." (LINCOLN), NEW YORK, FEBRUARY 15, 1918.

C. I. Hall: A very large part of the justification for the use of a thermal type of demand meter is based upon the idea that the primary rate should be proportioned to the thermal capacity of the equipment required to serve a customer. Since the thermal capacity of the various units of the installation depends very largely, or entirely, upon the current flowing rather than the wattage of the circuit, would it not be very much more logical to employ a thermal ammeter for such measurements than a thermal wattmeter?

It is very interesting to note the modification of the characteristic curve of the thermal storage wattmeter as compared with the curve indicated in Mr. Lincoln's paper on "Rates and Rate Making" of October, 1915. At that time in the discussion there was presented a curve on the characteristics of three different types of demand meters, the first being the Wright demand, the second the logarithmic average as taken from Mr. Lincoln's paper, and the third the characteristic of a thermal ammeter.

The approach of the Lincoln thermal demand meter described at that time to the theoretical logarithmic curve was very close on account of the very rapid dissipation of heat throughout the measuring apparatus.

In connection with the modifications which have been made by the use of a bi-metallic strip where the heat, which is proportional to the wattage, is applied to one end, the deviation from the theoretical logarithmic curve is considerably greater, and by taking the curves as presented in the present paper, it is interesting to note that the curve on the thermal storage wattmeter is now entirely coincident with the curve of the thermal ammeter that was given at that time.

All of the data which have been presented to indicate the differences of indication between the thermal type of demand meter and other types of demand meters are based upon single customer loads. It is, of course, academically interesting to know what these variations are, but no account in this discussion has been taken whatever of the diversity factor. Mr. Ferguson in a paper presented before the Association of Edison Illuminating Companies many years ago stated that the diversity factor was one of the prerogatives of the central station, one of its inalienable rights. In other words, a high diversity factor assists the central station in all of its work and in its rate making.

Therefore, in the discussion of the application of metering devices to actual problems, it is essential to consider the diversity factor as paramount. When so considered, it will be found invariably that any academic differences of individual customer loads are gradually widened out, and that the general effect is the bringing together of the characteristics of various types of meters.

In this connection I believe it would be very interesting to have at some later time a discussion of the measurement of actual load conditions when metered by the various types of demand meters operating upon various characteristic loads.

F. V. Magalhaes: It would be interesting to know what the performance of the instrument would be under varying external temperature conditions. Mr. Lincoln dismisses this point with the statement that the operating shaft will not turn with changes in external temperature. There are no actual figures given showing the temperature coefficient of the instrument.

He compares the results obtained by his instrument with one type of commonly used demand meter only and ignores entirely other types of demand meters and in particular one which has been available and in successful operation for several years. I refer to what is known as the Ingalls device. The indications obtained by the Ingalls device permit the demand to be calculated readily and accurately to any minute, and do not confine the demand to predetermined clock intervals.

Mr. Lincoln's comparison between the performance of his instrument and what he calls the Block-interval-type meter is entirely unfair to the Block-interval-type meter, as he shows absolutely no results obtained from the instrument itself on an actual test, but merely states its theoretical or calculated performance on an assumed load. He certainly is well aware that no load in practise follows cyclically the set intervals which can be imposed on a test load. It is a matter of record, as demonstrated by tests extending over long periods, that the block-interval-type meter indicates the actual maximum demand of the circuit to within the instrument's error of registration. It is possible to impose on this type of instrument a short test run with a load definitely controlled within certain time limits, so that the instrument, although it will show with accuracy the maximum demand within its rated interval, will not show the highest figure that may have occurred in the short test period. This is by no means the fair or proper way to test such an instrument, and misleading conclusions should not be drawn from narrow test results so obtained.

Mr. Lincoln is very strongly in favor of standardizing the maximum demand interval for the entire country. This is a laudable desire, but it is very much of a question as to whether his desires in the matter are not colored more by the fact that his instrument is inherently a half-hour instrument rather than by a desire to settle the maximum demand troubles for the entire industry. Mr. Lincoln had some years' experience at Niagara Falls and is undoubtedly familiar with the one-minute and other special maximum demand periods which have in the past and still continue to be effective in that district.

There are other localities where contracts embodying a demand period of one hour have been in effect for years. It is possible to find still other localities where the demand period

may be anything between one minute and one hour and the period justified by local conditions which could not be altered merely by a desire to standardize to some other period.

G. L. Hoxie: On the first page of Mr. Lincoln's paper he gave the reason for the maximum demand indicator, as follows: "The incorporation of maximum demand in a rate for electric service is an attempt to assess upon the user of that service his proper share of the annual cost of equipment necessary for giving the service."

Now, it seems to be assumed in this paper that the annual cost of the equipment necessary to give the service which a customer uses is a function of the heating of that equipment due to such service. The limiting thing, however, may well be the maximum peak capacity of the generating stations of the company furnishing the current. It is only rarely that maximum possibility of service is determined by the capacity of equipment located outside of the generating or sub-stations. The maximum peak capacity of a generating station is not usually limited by heating. I believe in the case of a modern turbine that the number of kilowatts you can take out depends to a greater extent upon the steam end than upon the electric end, and a similar situation exists with many water power units, not with all, of course.

However, the principal comment, I wish to make is along the line of that of a previous speaker, and is in connection with diversity. Mr. Lincoln has used a very striking illustration of a customer taking one thousand kilowatt hours per year, and, if the service all be taken in one day, he takes current at 365 times the rate, as if his use were spread out over the year, and the inference is that his use of the company's apparatus costs the company 365 times as much as if the service were so spread out.

To carry that same illustration further, let us assume that there are 364 other customers, each taking current on a different day. In that case, so far as the production of the electricity is concerned, there would be no difference between the load of the 365 customers, each taking current on a different day, and one customer taking the same current every day.

It seems to be assumed in the paper that the heating effect shown by the customer's maximum demand meter, taken by itself, is the quantity which should be considered, but as a matter of fact, it seems to me it is the effect upon the generating stations of the company, due to this particular customer's draft of energy, combined as it is with the demands of all the other customers of the company; in other words, the customer's maximum demand, in itself, is not a question with which the generating company is necessarily concerned. The maximum demand of a particular customer *which occurs synchronously with the maximum demand upon the generating stations*, is the one thing which, it seems to me, must be considered, and therefore I would suggest modifying clause No. 2 in Mr. Lincoln's summary so that it will read:

2. The cost of the necessary equipment to a given customer depends upon his demand *coincident with the station maximum demand* and not on his kilowatt hours of consumption.

That of itself eliminates the use of the one-minute peak for the ordinary customer. The one-minute peak of the ordinary customer will be smoothed out long before it reaches the generating station. In the case of a customer like a railway company, which may have a demand which is a large percentage of the total demand of the company, possibly a one minute peak may be an important quantity.

W. H. Pratt: There is a question I would like to ask, suggested by the sentence, "On the other hand, the thermal storage meter is perfectly definite in its indication. Each time a given load of given time duration is applied to this type of meter, it gives the same indication." What would be the comparative reading of two instruments, of different capacities, when put on the same load, for instance, an instrument of 1000 kw., and another one of 2000 kw., given a load of, roughly, say 900 kw., or 800 kw. It is, of course, necessary at times to use a device such as is available, but that might not be the device chosen were all capacities available.

It seems to me that there is no serious objection to using the logarithmic law of performance of demand meters for considerable classes of work. It seems to me also that it is a mistake to make a virtue of this logarithmic law, to place a special emphasis upon it, unless the law is pretty closely followed, and I judge from the figures given in this paper that it is not, in fact, that the departure is wide.

M. G. Lloyd: The objection which the author has spoken of to this meter in regard to the differences in registration with increasing loads and decreasing loads would not, on the face of it, be any objection if the load under consideration were the only load on the generating station. The fact that this meter very closely reproduces, in the effects within it, what is taking place in the generating apparatus, as regards heating, would make that matter take care of itself. However, in the case of a central station utility, one does not usually have a condition of that kind. The load of a particular customer is simply imposed upon a comparatively large load in the generator, in which case the effect is not reproduced in the generator, and consequently there is some objection to that characteristic in the meter.

Another factor which is to be considered and to which the author has referred, is the time taken for a generator or other apparatus to reach its final temperature, unless his assumptions are that one is starting from a condition of no load, and of course that does not apply to any particular customer's load being thrown on the line; and it is for that reason I think, that the central station does find it important to consider comparatively brief intervals of demand rather than demand covering periods of a half hour or longer.

I should like to ask the author whether this instrument was made up in a form to be indicating rather than recording, in the sense of giving a graphic record.

A. S. Albright: We have had the opportunity of making some tests on the indicating form of the type "RH" meter. After various laboratory tests, the "RH" meter was installed on a typical factory load, together with two other demand meters, both of the graphic integrating type, and readings of the three meters were observed weekly for several months. The curves obtained, given in Fig. 1, show the comparative performance of the two types of meters on this installation. The thermal meter shows a higher demand in all cases. The break in the curves is caused by skipped readings.

We have had no experience with the graphic form of Mr. Lincoln's meter.

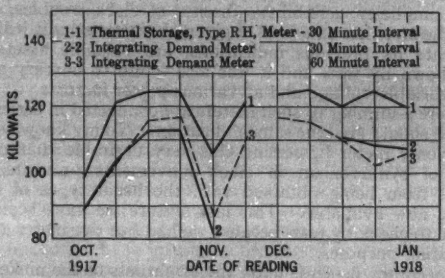

FIG. 1—COMPARISON OF "THERMAL STORAGE" WITH INTEGRATING DEMAND METERS ON A TYPICAL FACTORY LOAD

H. D. James: There is another use for this thermal storage meter which I wish to bring out, because I believe it has an important field, and that is in connection with individual pieces of apparatus. I have particularly in mind the motor. Many motors are operated on an intermittent load. The operator is, in many cases, desirous of knowing what is the average heating effect of this load on the motor. If he has an ammeter in front of him, and it varies back and forth, he must make a mental integration of the heat. If, however, we place a time-demand-overload meter on the motor, he does not know what is going to happen until the motor shuts down. If, however, he has a meter which indicates the heat going into the motor—a 30-minute time interval is long enough for most applications—he will have in front of him an ammeter which is giving him a true integration of the heat in his motor, and continually indicating what is going on, and when the needle reaches a predetermined point

his motor will shut down, and stay shut down a predetermined time, before he can start it up again.

Such a device is very desirable, and I think it is one of the devices the trade has been working toward for a number of years. We have the thermo-couple in the large generator, but so far it does not seem as though the thermal couple for the small apparatus can be produced commercially. We have a time-element-overload relay, which is commercial, but it has only a short time interval.

If we have a thermal integrating meter which is useful for not only determining the charge the customer should pay for the power he consumes, but also as a protection for his motor, and which will allow the customer to operate the motor close to its maximum, and still keep within the proper heat limit.

H. L. Wallau (by letter): The thermal type of demand meter is sure to have a promising future, once its operating characteristics are thoroughly understood.

Its simplicity, reliability, absence of clock troubles, coupled with the fact that on a load of given characteristics it will always duplicate its previous indications, are valuable assets. This was shown by test, and it was also found to be very accurate with variable voltage and at various power factors.

Where a number of such meters are connected in series with a load all should give the same indication. To my knowledge, this is the only meter possessing this very desirable characteristic. Lack of synchronism in clock movements prevents similar results from being obtained with the usual types of demand meters now available. This last feature we have been as yet unable to check by tests because we had but one meter available for testing purposes.

Its accuracy depends upon the brightness of the nickel plated heater casings remaining unimpaired. Just what effect the exposure of this type of meter to fumes and gases in industrial establishments will have, remains to be seen.

The author has discussed its principles, acknowledges that on heavy loads of relatively short duration its registration is above the arithmetical mean, but points out that it does truly follow the heating characteristics of the energy supply devices and therefore that this is a point in its favor.

It is self-evident that if every consumer of a class pays his just share of the cost of delivering current to him, there will be no discrimination in favor of any, and that the ultimate effect must be to allow of rate reductions to the class, as a whole, more rapidly than if some losses must be absorbed due to inequalities in charges favoring short-hour users. Commercial expediency may modify the application of this principle somewhat, but its truth remains unchanged.

As regards the period during which the maximum demand should be measured, considerable difference of opinion is yet manifest. One large company uses an hour period and reads

the demand weekly. The average of the four readings is billed as the monthly demand. The object of this is to prevent the undue penalization of a consumer for some abnormal demand which might accidentally be made on a certain day, and which is not typical of the general operating characteristics of his load.

In my judgment the meter with a time period, such that 90 per cent of the load will be indicated in the interval for which the demand is taken under the contract, is preferable to that in which only 63.2 per cent of the load would be indicated in that period.

P. M. Lincoln: Mr. Hall refers to the thermal capacity of the equipment, and states that on the basis of thermal capacity that the meter used should be a current meter, an ammeter rather than a wattmeter. That is true within limits. It is not true that apparatus in general has a temperature in proportion to the amperes carried. If all of the heat were due to the passage of the current, that statement would be true, but in electrical apparatus in general, only a part of the heat is due to the passage of the current.

In large turbo-alternators, the armature copper loss amounts to only 10 or 20 per cent of the total, and consequently the heating of that device is not proportioned, by any means, to its ampere capacity. The same is true of other pieces of electrical apparatus which are used, even the cable. The heating of cables depends almost wholly, to be sure, on the current carried, but the next paper indicates that a considerable proportion of the losses which take place in cables is due to the element depending on the voltage, so that an element which depends on current alone is not sufficient to insure the heating of electrical apparatus in general. It must be something which takes in elements other than mere amperes.

I maintain, therefore, that the wattmeter will probably in general come nearer measuring the true capacity of the apparatus than will an ammeter, although the ammeter on exactly the same principles is available, and will undoubtedly have a considerable part in the future of demands as they are measured.

Another thing called attention to is this matter of the time of diffusion, the time it takes for heat to diffuse through the meter element. Mr. Hall's view of that feature and mine seem to be different. I consider the time of diffusion as a distinct handicap. I think that Mr. Hall considers it an advantage, and there is a distinct difference of opinion between us on that particular point. I wish I could produce a device in which the time of diffusion was zero.

It was found that a true logarithmic law would avoid the error shown in Fig. 16 of my paper. It is the time taken for the diffusion of the heat throughout the meter limit which gives rise to departure from the true logarithmic law, and therefore gives rise, in the case of the thermal meter, for analyzing the true logarithmic average as analyzed in Fig. 16.

I wish it were possible, therefore, to avoid the time of diffusion throughout the meter element. I consider that time of diffusion as a distinct disadvantage. Possibly as things go along, we will find means of overcoming it.

Mr. Magalhaes asks concerning the temperature coefficient of the meter. I have made a good many measurements of the temperature coefficient, and the temperature error is so small that it is pretty hard to make an accurate determination of it. In general, I can say that it is about one-tenth or one twentieth of a per cent for each degree of change in centigrade temperature. That is not a very large change, but as I say, it is a difficult thing to measure because it is so small.

Mr. Magalhaes speaks of a device which he calls the Ingalls device. I am not prepared to comment on that, because I am not familiar with the device he mentions.

Mr. Hoxie and several other gentlemen have objected to my conclusions, in that I have taken the cost of the customer's equipment as being proportional to the maximum demand of that customer. Mr. Hoxie and the others who referred to that point spoke specifically of the generator capacity, and if we consider generator capacity only, they are more or less correct in their conclusions. However, it is true that the majority of the cost of electrical equipment for serving the customers is not in the power house. It is out on the line. I think if the cost of equipment for serving customers is analyzed they will find that the cost of equipment at the customers' end amounts to a good many times the cost of equipment in the generating station, and the cost of the equipment at the customer's premises is something that depends on the customer's demand. The diversity factor takes care of that if it goes back into the system, and it is true, as pointed out, that the maximum demand of the customer does not affect the generating capacity to anything like the same extent that it affects the apparatus which is placed at the premises of the customer.

You must remember in this connection that the customers are very many, and that the separate pieces of apparatus to serve them run into thousands of pieces, and the aggregate cost of these pieces in general is considerably higher than the cost of the generating equipment to serve these customers. That consideration, therefore, leads me to quite a different conclusion than if one is considering the generating equipment only.

Mr. Hoxie spoke of my illustration of the customer who insisted on taking his entire year's supply in a single day, and said that if there were three hundred and sixty-five customers, each of whom took his load on one day of the year, that my illustration would not apply, but my illustration does apply, because each of these customers would have to have equipment enough to take care of the whole output of the generating station during that day, which would be practically as much equipment as there is in the generating station, and it would mean practically

multiplying the equipment of the customer by 365, which would certainly be a large increase in the total cost of power for that station. I cannot agree with Mr. Hoxie on that point.

Mr. Pratt asked concerning the difference in indication, for instance, of a 1000-watt wattmeter as compared to that of a 2000-watt wattmeter. There is no difference. It does not matter what the capacity is, a 2000-watt wattmeter will indicate the load as much as a 1000-watt wattmeter. There is a difference, however, in the indication of two different times. If we had a 30-watt wattmeter, and compared that with the indications of our demand wattmeter, the indications of these two would not necessarily be the same, and would depend on the character of the load. If the loads are the same, they would give the same results. If the load had high peaks, the meter with the short time element would give higher indications than the one with the longer time element. That depends on the application of a logarithmic law.

Mr. Albright speaks of a number of tests made in Detroit of this device, and states that the indication of the 30-minute thermal demand, as compared with meters of the same kind, and as compared with meters of a different type, taken through a number of weeks, shows that the thermal meter gives a higher demand in all cases, and that is about what one would expect, and would indicate that the particular load he has it on is not a steady load, but one with a number of peaks in it. As I recall it, he applied it to a machine shop load in which the peaks are rather marked, and it was only to be expected that the thermal demand meter would give higher results in the case of such a load than the wattmeter.

Concerning the application of a device of this kind as a motor protection, as spoken of by Mr. James and Mr. Atkinson, that is an entirely practical application, and I look for that application to be developed to a considerable extent in the future.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.

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MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

BY F. M. FARMER

ABSTRACT OF PAPER

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves.

The discussion includes:

(a) The theory of excessive internal dielectric loss as accounting for cable failures at local "hot spots."

(b) The advantages of plotting data with logarithmic scales.

(c) A comparison of results obtained by computation from single-phase measurements with those obtained by direct measurement with three-phase potential.

The conclusions drawn are:

(a) The power loss in the dielectric in a three-conductor cable under actual three-phase conditions can be readily measured in the laboratory with specimens ten feet long.

(b) No special apparatus is necessary for such measurements other than a reflecting high sensitivity wattmeter.

(c) Apparently the power loss in the dielectric cannot in all cases be accurately calculated from data obtained in single-phase tests although it is highly probable that for all practical purposes the discrepancy would not be serious. Further investigation is necessary, however, before final conclusions on this point can be stated.

(d) While the method of determining power losses in the dielectric directly by three-phase measurements involves more complication in preparation and slightly more time in the actual measurements, it has the important advantage that the results are conclusive and not subject to the uncertainty which pertains to results calculated from single-phase measurements.

THE CARRYING capacity of a cable is determined by the temperature at which the dielectric strength becomes dangerously low or at which deterioration takes place at an abnormal rate. If the temperature which the materials in the cable will safely withstand is known, the carrying capacity of the cable is fixed when the hottest part of the cable reaches this limiting temperature.

Until two or three years ago the only factors which had been given serious consideration in determining the temperature of the cable were the $I^2 R$ losses in the conductor and the thermal conductivity of the surrounding media. In other words, it was assumed that the copper loss was the only source of heat that need be considered and that the temperature which would be reached would depend simply upon the amount of heat thus generated and the rate at which this heat was carried away. However, careful investigations of cable failures, for which there was no obvious explanation, seemed to indicate that there were local hot spots which were not due to any outside cause. Attention was then drawn by investigators of insulation problems to the possibility of power losses in the insulation being responsible for abnormal temperature rises. Investigations made by Hochstadter[1], Clark and Shanklin[2], Bang and Louis[3] and others indicated that, under operating conditions which could be considered as normal, these losses might become sufficiently high to raise the temperature of the insulation to the destruction point. The subject of power loss in cables has therefore been attracting much attention and it is particularly pertinent at the present time because of the necessity for operating cables at the maximum possible capacity in order to meet the demand for more power. Furthermore, the amount of the power loss in dielectrics, although small, may in the aggregate be sufficiently large to justify consideration from the economic standpoint when new installations are contemplated at the present prices of materials and labor.

Object of Paper. The object of this paper is to describe the methods used in making tests at the Electrical Testing Laboratories for the purpose of determining the power loss in the dielectric of samples of paper-insulated, lead-covered cables 10 feet long, the loss being measured at various voltages and various temperatures. While nothing absolutely new or particularly novel is presented, it is felt that some detailed information in regard to these tests may be of service to engineers who will be called upon to make tests of this character.

1. M. Hochstadter—"Twisted Cable," *Electrotechnische Zeitschrift*, November 25th, 1915, page 617.

2. W. S. Clark and G. B. Shanklin—*Insulation Characteristics of High-Voltage Cables*, TRANS., A. I. E. E., 1917, page 447.

3. A. F. Bang and H. C. Louis—"The Influences of Dielectric Losses on The Rating of High-Tension Underground Cables," TRANS., A. I. E. E., 1917, page 431.

The power loss in the dielectric of three-conductor cables is probably most frequently determined by making single-phase measurements and then computing the corresponding three-phase loss on the basis of certain assumptions. That is, the power loss in the dielectric between the conductors (conductor loss) and the loss between the conductors and the sheath (belt loss), is measured with single-phase potential at the appropriate Y or delta voltage. The loss with three-phase potential is then calculated on the assumption that these two losses are the same with three-phase potential as with single-phase potential and that their sum is the total loss. Hochstadter[4] pointed out the possibility of this assumption being open to question because of the complex nature of the electrical field in the insulation between the conductors when subjected to three-phase potential.

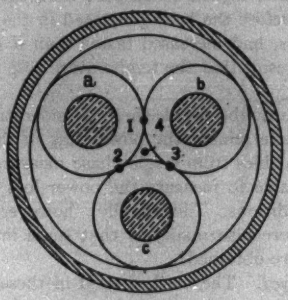

FIG. 1

Referring to Fig. 1 which represents diagrammatically a cross-section of a three-conductor cable, the dots 1, 2 and 3 indicate the points of contact between the various conductor insulations on lines connecting the centers of the three conductors. The dot 4 is the center of the cable and may in three-phase operation, be assumed to be at ground potential. If delta potential, E, is applied to the three conductors, a, b and c, the potential between different points will be as follows:

$$\text{Volts } a-4,\ b-4,\ c-4 = \frac{E}{\sqrt{3}}$$

$$\text{Volts } a-1,\ b-1,\ a-2, \text{ etc.} = \frac{E}{2}$$

4. M. Hochstadter—"Twisted Cable," *Electrotechnische Zeitschrift*, November 25th, 1915, page 617.

In other words, points 1, 2 and 3 are not at the same potential and therefore there is a tangential electrical stress in the insulation between the conductors which does not exist when making single-phase measurements. Consequently a power loss might be anticipated in the dielectric in this part of the cable insulation which would not be determined by such a measurement.

That the loss per unit volume of insulation between conductors may be actually higher than elsewhere, is suggested by evidences found by Clark and Shanklin and others in their investigations of cable failures,—that is, the insulation between the conductors was charred although the insulation between the conductors and sheath was apparently uninjured. As it is well known that the power loss in most insulations increases rapidly with temperature, it seemed logical to suspect that the loss per unit volume, and therefore the heat generated in the insulation between conductors, had increased faster than it could be dissipated, with the result that the temperature rose to the destruction point.

Because of the doubt which these considerations cast upon the method of determining the power loss in the dielectric of three-conductor cables from single-phase measurements, it was considered advisable to measure the power loss directly under three-phase conditions. Incidentally, however, single-phase measurements were also made, so that some interesting comparative data were obtained.

Method Employed. The method used in these tests was the simple three-wattmeter method for measuring three-phase power. That is, the cable was Y-connected, with the sheath as the neutral, and a wattmeter was connected to each phase in turn. The algebraic sum of the three quantities thus obtained is, of course, the total power dissipated. The high potential which was applied to the cable was obtained with three 15,000-volt, 200-watt, potential transformers connected in Y. The wattmeter was a reflecting dynamometer instrument, the current circuit being connected between the grounded neutral point and the high-tension winding of the transformer in the phase being measured, while the potential circuit was connected to the low-tension winding of a step-down potential transformer connected across the same phase.

The actual measurements were not, however, accomplished as easily as this description might indicate despite the simplicity of the general method. In the first place, the power factor is

as low as four or five per cent at moderate temperatures. Consequently, the effect of inductance in the potential circuit of the wattmeter and the phase angle in the step-down potential transformer is very marked and would produce serious errors if not properly taken into account. In the second place, the voltages used are so high and the power being measured so very small (as low as one watt) that losses through paths in parallel with the cable, that is leakage losses to ground from the test circuits, are likely to be relatively large and great care must be taken to eliminate them.

The effect of the inductance in the potential circuit of the wattmeter and the phase angle of the potential transformer was eliminated by the usual method of shunting a variable portion of the series resistance with a condenser as indicated in the diagram in Fig. 2. Thus the resultant equivalent inductance of

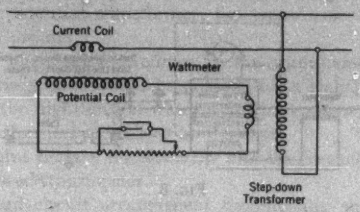

FIG. 2

the circuit is compensated for by the introduction of a proper amount of capacitance. Exact compensation can be determined by the following methods:

(a) Measure the inductance of the wattmeter potential circuit and the phase angle of the transformer. Then compute the capacitance and shunt resistance necessary from the relation $L = C R^2$, where L is the equivalent inductance of the circuit in henrys, C the capacitance of the condenser in farads and R the shunt resistance in ohms. This method requires a very careful measurement of the phase angle of the transformers, a measurement which involves either special apparatus or standardized transformers.

(b) Connect a high-voltage air condenser as the load in place of the cable and adjust the resistance which is in shunt with the condenser until the wattmeter shows no deflection. The accu-

racy of this method depends upon the power factor of the condenser. Normally an air condenser may be assumed to be a perfect condenser and therefore have zero power factor, but at high potentials the corona discharge from the circuits and from the condenser plates may introduce a power component unless great care is taken in the construction and proper means are provided to eliminate this corona discharge. However, when done in the careful manner employed by Shanklin[5] it is probable that this method is the best one for adjusting the compensation.

In the tests discussed in this paper, the compensation was determined by method (a) and checked by method (b), slightly modified. It was not found possible, with the only apparatus available to eliminate leakage entirely from the test circuits and consequently the deflection of the wattmeter could not be reduced to zero. The compensation was therefore so adjusted that connecting various amounts of capacitance to the circuit

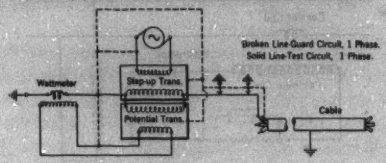

FIG. 3

did not change the deflection. Incidentally, this capacitance was obtained by suspending from the test circuit a number of metal cans about six inches in diameter and 15 inches deep, which were conveniently at hand. Since the adjustment could be made at potentials well below the part where corona discharges would take place from these cans, it was not necessary to take special precautions on this account.

As previously stated, the cable was tested by applying Y voltage to the conductors, the lead sheath being connected to the neutral and to ground. Obviously, all points of contact with each high-tension circuit provided leakage paths to ground in parallel with the insulation of that particular conductor of the cable, so that the power measured at the terminals of the high-tension winding of the transformer would include the

5. G. B. Shanklin, "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," *General Electric Review*, Vol. 19, page 842 (1916).

power expended in these leakage paths. A guard circuit was therefore provided for each of the three phases as indicated by the broken lines in Fig. 3 which shows diagrammatically the arrangement for one phase. This guard circuit consisted of a metallic connection between each of the following:

(a) A piece of tin or tin-foil inserted between ground and each insulator supporting the high-tension circuit, moderate insulation being provided between the metal and ground.

(b) A few turns of fine bare copper wire around the insulation of the conductor about two or three inches from the end (the lead and wrapper insulation of the cable having been cut back several inches in the usual manner to permit separation of the ends of the conductors) which served to pick up any leakage current over the surface of the insulation between the conductor and the lead.

(c) The iron cases of the step-up power transformer and the step-down potential transformer, the transformers being insulated from ground.

(d) The low-tension windings of the transformers. This is necessary because it would be possible for a leakage current to pass from the high-tension winding to the low-tension winding of the transformers as well as to the cases.

(e) A point *between the wattmeter and the grounded end* of the step-up power transformer.

This guard circuit arrangement theoretically eliminates any current from the wattmeter current circuit that does not pass through the insulation of the cable except, of course, that directly through the air due to corona. In these tests, however, the potentials were always below the corona value.

As previously stated, it was not found possible to eliminate leakage currents entirely but it is probable that they were entirely due to insufficient insulation between all parts of the guard circuit, including the circuits to which it is connected, and ground. In fact, there was a partial ground in one phase of the generator which necessitated connecting the low-tension side of the power transformer to ground instead of to the guard circuit. Consequently leakage from the high-tension winding to the low-tension winding of the power transformers could not be properly shunted by the guard circuit.

A special smooth-core generator was used in these tests and oscillograms made under test conditions showed that the wave shape was substantially a sine curve under all conditions.

Suitable switching arrangements were provided so that the current circuit of the wattmeter could be quickly shifted from one phase to another and a similar provision was made for changing the potential circuit from one phase to another. Fig. 4 shows diagrammatically the circuits used in the test and the switching arrangements employed.

Power Measurements. The power in each phase was measured as already indicated, the total power to the test circuits and the cable being taken as the algebraic sum of these three measurements. The cable was then disconnected from the high-tension leads, leaving all of the rest of the test circuits exactly the same,

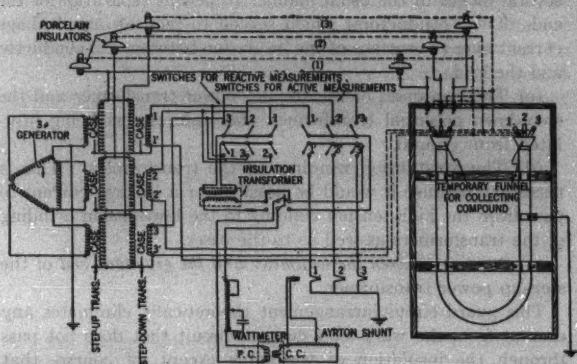

FIG. 4—DIAGRAM OF CONNECTIONS FOR DIELECTRIC POWER LOSS MEASUREMENTS—THREE-CONDUCTOR CABLES WITH THREE-PHASE POTENTIAL

and the measurements repeated. The latter gave the "leakage" loss. The difference between these two quantities was taken as the power loss in the dielectric of the cable.

Power Factor. The power factor was obtained by measuring the reactive volt-amperes. This measurement was made by simply shifting the voltage circuit of the wattmeter through 90 degrees by connecting to the proper delta voltage instead of the Y voltage.

Direct-Current Resistance. The resistance between the various combinations of conductors and lead sheath was measured in the usual manner with a sensitive reflecting galvanometer at 700 to 800 volts.

Capacitance and Alternating-Current Resistance. These quantities are obtained by computation from the above data,—the alternating-current resistance from the power and the capacitance from the reactive volt-amperes.

Heating of Cable. The heating of a section of cable 10 feet long to a temperature as high as 125 deg. cent. uniformly throughout its length is not the simple problem that one might expect. The first scheme considered appeared to be a very simple solution until it was tried. A sheet iron pipe about 10 inches in diameter with the cable suitably supported at its center was constructed and low-voltage current circulated through the pipe. It was expected that this scheme would give uniform heating, provide convenient control of the temperature and make the determination of the temperature of the cable a relatively easy matter because of the symmetrical arrangement of heat source with respect to the cable. The first difficulty experienced was the heating at the joints in the pipe due to the greater electrical resistance at these points. Also, there was an excessive cooling at the ends. The pipe was then wrapped with asbestos and a layer of iron wire wound thereon and through this wire current was circulated—additional turns being provided at the ends to compensate for the cooling effect. This scheme was satisfactory insofar as obtaining uniform temperature conditions was concerned, but it was found that at the high temperatures too much compound was being lost from the cable at the ends. It was therefore evident that some provision had to be made to prevent this loss of compound.

The iron pipe was abandoned and a simple oven was constructed consisting of a wood box lined with asbestos of such a size that the cable when bent in the form of a U having about 2.5 feet radius could be mounted in the box in a vertical position. The expansion of the compound was taken care of by soldering a sheet tin funnel to the lead at each end of the cable and into this funnel the compound could expand on heating and flow back again into the cable on cooling. Heat was provided by large incandescent lamps so located by experimental trials as to give the most uniform temperature, two fans being installed to assist in attaining this condition. Switches outside of the box provided control of the amount of heat being supplied and therefore of the temperature.

The temperature of the cable was measured by seven thermocouples soldered to the lead sheath. No difficulty was found in

getting these thermocouples to agree within 3 deg. cent. at a temperature of 100 deg. cent. One of the couples was connected to a curve-drawing instrument for the purpose of conveniently showing when a cable had reached constant temperature with

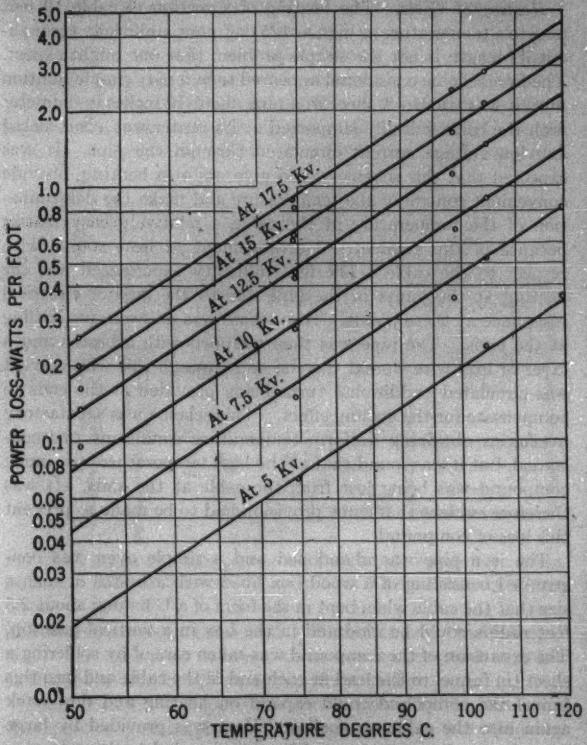

FIG. 5—POWER LOSS VS. TEMPERATURE AT VARIOUS TEMPERATURES—CABLE *A*—(LOGARITHMIC SCALE)

any given condition of heating. The final test of thermal equilibrium was the wattmeter indication, the power loss in the dielectric, being very sensitive to temperature. Final loss measurements were not taken until the wattmeter indication had been constant for at least thirty minutes.

Typical Data. While this paper is primarily a discussion of the method of measurement of dielectric losses in cables, it is

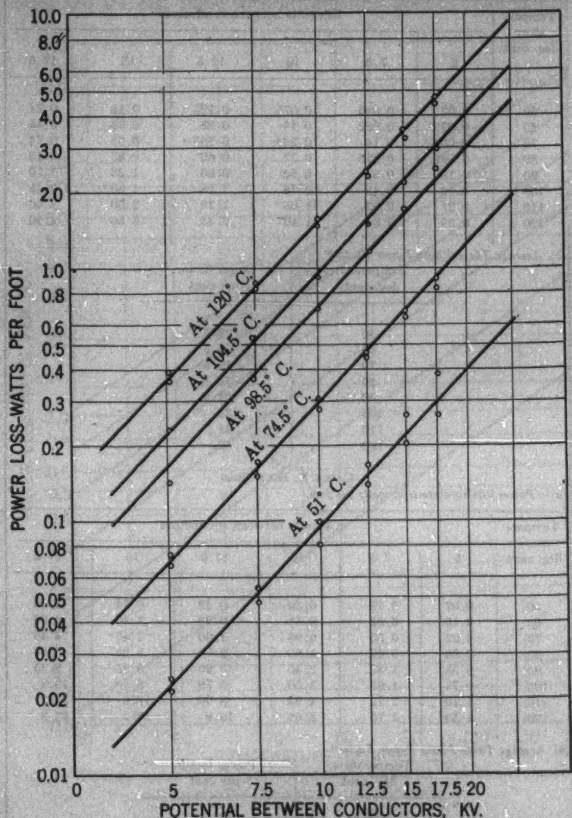

FIG. 6—POWER LOSS VS. VOLTAGE AT VARIOUS TEMPERATURES—CABLE *A*—(LOGARITHMIC SCALE)

thought that some typical data which have been obtained will be of interest. Table I gives summaries of the power loss in the dielectric and power-factor data for two cables, one having a

TABLE I.

CABLE A, LOW LOSS

(a) *Power loss in dielectric, watts per foot.*

Temperature deg. cent.	Kilovolts between conductors					
	5	7.5	10	12.5	15	17.5
50	0.02	0.045	0.076	0.125	0.18	0.24
60	0.035	0.082	0.14	0.23	0.33	0.44
70	0.058	0.14	0.235	0.385	0.55	0.74
80	0.092	0.215	0.37	0.60	0.83	1.15
90	0.135	0.32	0.55	0.90	1.25	1.70
100	0.19	0.45	0.78	1.28	1.80	2.45
110	0.27	0.63	1.08	1.75	2.50	3.40
120	0.35	0.84	1.40	2.35	3.30	4.40

(b) *Average Three-Phase Power Factor**

Temperature, deg. cent.	Power factor, per cent
50	3.5
60	6.5
70	10.5
80	15.
90	21.5
100	31.
110	41.
120	52.

CABLE B, HIGH LOSS

(a) *Power loss in dielectric, watts per foot.*

Temperature deg. cent.	Kilovolts between conductors					
	5	7.5	10	12.5	15	17.5
50	0.07	0.18	0.30	0.49	0.74	1.08
60	0.16	0.32	0.58	0.93	1.40	2.00
70	0.22	0.55	0.98	1.60	2.45	4.45
80	0.36	0.89	1.60	2.60	3.85	5.55
90	0.58	1.35	2.40	3.90	5.85	8.35
100	0.78	1.95	3.50	5.70	8.50	12.3
110	1.10	2.75	4.95	8.00	12.0	17.2
120	1.50	3.75	6.65	10.8	16.2	23.2

(b) *Average Three-Phase Power Factor**

Temperature, deg. cent.	Power factor, per cent
50	10.
60	21.
70	32.5
80	44.
90	55.5
100	66.
110	72.
120	75.

*Computed from ratio of total three-phase watts to total three-phase reactive volt-amperes, the figures given being the average for all voltages.

much higher loss than the other. The data as actually obtained on samples 10 feet long are shown graphically in Figs. 5 to 11 inclusive—the first four being curves plotted with logarithmic scales and the last three with arithmetical scales. The figures

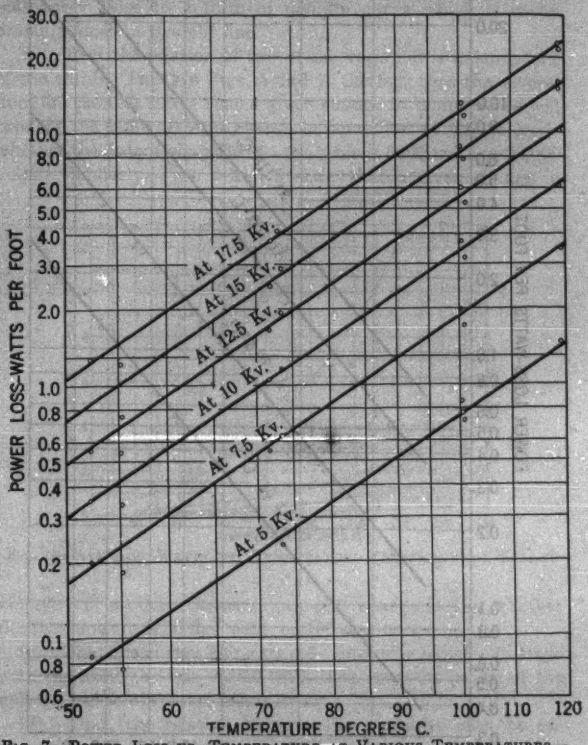

FIG. 7—POWER LOSS VS. TEMPERATURE AT VARIOUS TEMPERATURES—CABLE *B*—(LOGARITHMIC SCALE)

given in the tables were taken from these curves at fixed interval temperatures in order to make comparisons more convenient. It is obviously not possible in tests of this character to adjust the temperatures to exactly predetermined values.

Use of Logarithmic Scales. The general relations of the variables in experimental data are quickly comprehended when the

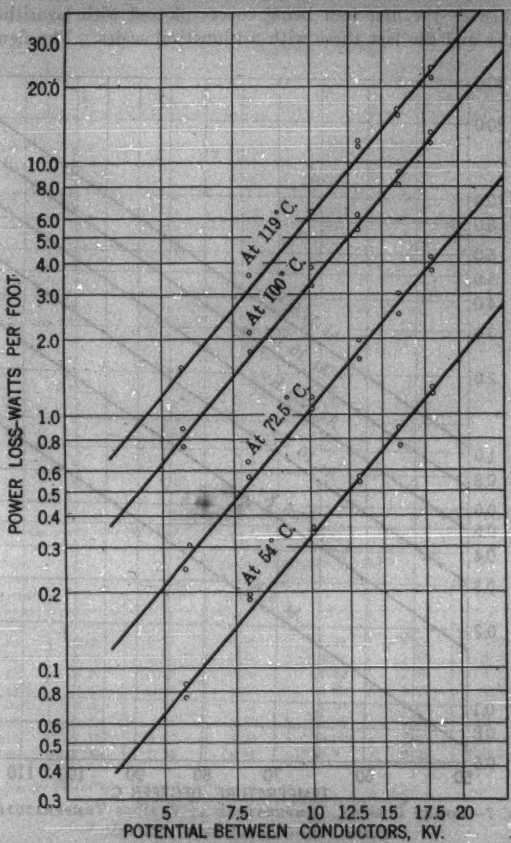

FIG. 8—POWER LOSS VS. VOLTAGE AT VARIOUS TEMPERATURES—CABLE *B*—(LOGARITHMIC SCALE)

data are presented in the shape of curves plotted in the usual way with rectangular co-ordinates and arithmetic scales. But when something more than simply the general relation of the

variables is desired, curves plotted with rectangular co-ordinates and logarithmic scales frequently have many advantages. For example, the curves plotted in Figs. 5, 6, 7 and 8 show:

(a) That no serious error was made because the points fall reasonably close to a form of definite curve which is easily drawn, namely a straight line.

(b) That the relation of two of the variables is independent of the third. Thus, in Figs. 5 and 7, the fact that the several lines are parallel shows that a given change in temperature will produce the same per cent change in power loss of the dielectric whether the potential applied to the cable is 5000 volts or 20,000 volts. Similarly, Figs. 6 and 8 show that a given change in

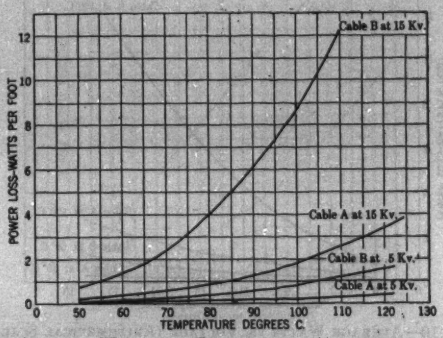

FIG. 9—AVERAGE WATTS VS. TEMPERATURE (ARITHMETICAL SCALE)

voltage will produce the same per cent change in loss whether the temperature is 50 deg. cent. or 100 deg. cent.

(c) That the curves being straight lines which make an angle with the axes, the loss varies with some constant power of the voltage and temperature respectively.

(d) That, the relation between the variables may be expressed by the equations:

$$W \text{ (at a given potential)} = k\,T^{n_t} \text{ and}$$

$$W \text{ (at a given temperature)} = k\,E^{n_e} \text{ where}$$

W = power loss of dielectric in watts, T = temperature in degrees centigrade, E = potential in volts, n_t and n_e = exponents and k = constant.

The value of the exponents is readily computed from the relations

$$n_t = \frac{\log \frac{W_1}{W_2}}{\log \frac{T_1}{T_2}} \quad \text{and} \quad n_e = \frac{\log \frac{W_1}{W_2}}{\log \frac{E_1}{E_2}}$$

where W_1 and W_2 are the losses corresponding to temperatures T_1 and T_2, or to voltages E_1 and E_2.

The exponents for cables A and B referred to in the preceding tables and also for two other cables are given in the following

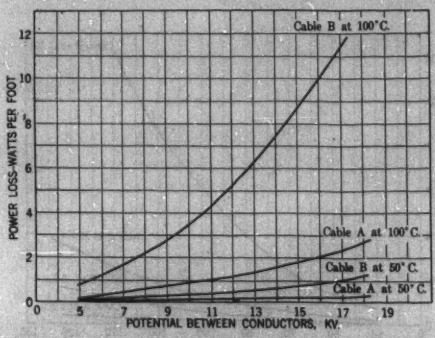

FIG. 10—AVERAGE WATTS VS. VOLTAGE (ARITHMETICAL SCALE)

tabulation. The power loss in the dielectric per foot at 80 deg. cent. and 15,000 volts is given for purposes of comparison.

Cable	n_t	n_e	W
A	3.36	2.07	0.85
B	3.54	2.34	3.85
C	3.77	2.01	1.76
D	4.26	2.16	2.62

n_t = exponent for temperature change, n_e = exponent for voltage change, W = power loss in the dielectric per foot in watts.

Three-Phase vs. Single-Phase Tests. Single-phase measurements were made on a number of specimens in addition to the

three-phase measurements. These single-phase measurements included the following, all measured at Y voltage:

(a) R_1, R_2, R_3 = power loss in the dielectric between each conductor and the other two conductors connected to lead sheath, that is referring to Fig. 12, $R_1 = W_1 + W_4 + W_5$, $R_2 = W_2 + W_4 + W_6$ and $R_3 = W_3 + W_5 + W_6$.

(b) R_4 = power loss in the dielectric between all three conductors connected together and the lead sheath, that is (Fig. 12) $R_4 = W_1 + W_2 + W_3$.

If it is assumed that the conditions with three-phase potential

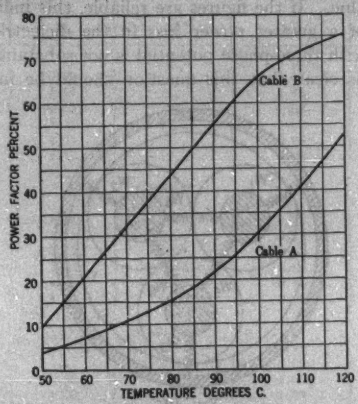

FIG. 11—AVERAGE POWER FACTOR VS. TEMPERATURE

are the same as in these single-phase tests, a value for three-phase loss can be computed from the relation

$$W \text{ (total watts)} = \frac{3\,(R_1 + R_2 + R_3) - R_4}{2}$$

The three-phase loss corresponding to single-phase conditions has been calculated on this basis for a number of cables and the ratio of the measured three-phase loss to computed loss is given in Table II.

These figures do not appear to settle definitely the question of the relation of measured and calculated losses. But if it is assumed that the variations or apparent inconsistencies have no

significance and that all of the figures for a given cable can be averaged, the results in the order of the cable designations are 1.015, 0.968, 1.010 and 0.967 respectively. It is questionable, however, if the variations are merely accidental because the data for the calculations were taken from straight line curves plotted on logarithmic paper. Furthermore, in some cases there seems to be a more or less definite change in the ratio with voltage especially in the case of cable *D*.

The most significant thing shown by these figures is that the measured value is very frequently considerably *lower* than the calculated value. If the figures are reliable, this indicates that the theory of a greater power loss in the dielectric between conductors with three-phase potential is not substantiated. In any case it is clear that this question should be investigated

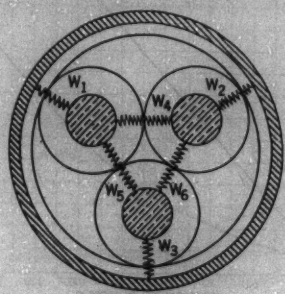

FIG. 12

further although it is probable that, for, all practical purposes, the power loss may be determined from two single-phase measurements without serious error.

Application of Data. The value of tests of this character may be largely relative because the results do not necessarily apply to operating conditions. In these tests the cable was at a uniform temperature throughout, while under operating conditions the temperature of the dielectric varies from a maximum at the conductor to a minimum at the lead sheath, so that the power loss per unit volume of insulation is higher near the conductor than it is next to the lead. However, Bang and Louis[6] found this gradient to be very small (lead temperature 86 deg., copper temperature 89 deg.) under conditions closely duplicating those in

6. loc. cit.

service. Of course if the temperature gradient is as small as these figures indicate, then it may be assumed without serious error that the entire cable is at the temperature of the lead and if this temperature can be measured, the total losses in the insulation can be computed if a laboratory test of a sample of the cable

TABLE II—RATIO OF MEASURED TO CALCULATED THREE-PHASE POWER LOSS IN DIELECTRIC

Potential between conductors, kilovolts	Temperature, deg. cent.			
	50	75	100	120
		CABLE A		
5.0	0.79	....	1.03	....
7.5	1.07	....	1.02	....
10.0	0.91	....	1.03	....
12.5	1.16	....	1.02	....
15.0	1.11	....	0.96	....
17.5	1.11	....	0.98	....
		CABLE B		
5.0	1.26	0.86	0.91	0.79
7.5	1.11	0.80	0.81	0.93
10.0	1.12	0.87	0.88	0.92
12.5	1.13	0.88	0.89	0.99
15.0	1.18	1.02	0.90	0.98
17.5	1.11	0.93	0.97	1.03
		CABLE C		
5.0	....	1.05	1.06	0.96
7.5	0.96	0.94	1.03	1.02
10.0	1.03	1.00	1.09	0.95
12.5	0.91	0.93	1.04	0.95
15.0	1.07	1.10	1.11	1.00
17.5	....	....	....	....
		CABLE D		
5.0	0.66	0.78	0.89	....
7.5	0.88	0.85	0.82	....
10.0	0.98	0.89	1.02	....
12.5	1.04	0.97	0.97	....
15.0	1.08	1.02	1.03	....
17.5	1.19	1.15	1.18	....

has been made. However, this question of what are the actual temperature conditions in practise is very important and in order to get data under conditions approximating working conditions, tests are now being made with a length of cable 20 feet long installed in a concrete conduit in the laboratory. The dielectric

power losses will be measured from time to time while the cable is carrying its rated current and rated voltage until constant thermal conditions have been reached. Incidentally, various temperatures will be measured including the copper by increase in resistance, and the lead by thermocouples. It is hoped that these data together with those already obtained for the same sample under other conditions will allow operation of the installed cable, which this sample represents, at maximum carrying capacity without danger of failures due to hot spots.

Conclusions

The following conclusions may be deduced from the preceding discussion:

(a) The power loss in the dielectric of a three-conductor cable under actual three-phase conditions can be readily measured in the laboratory with specimens ten feet long.

(b) No special apparatus is necessary for such measurements other than a reflecting high-sensitivity wattmeter.

(c) Apparently the power loss in the dielectric cannot in all cases be accurately calculated from data obtained in single-phase tests although it is highly probable that for all practical purposes, the discrepancy would not be serious. Further investigation is necessary, however, before final conclusions on this point can be stated.

(d) While the method of determining power losses in the dielectric directly by three-phase measurements involves more complication in preparation and slightly more time in the actual measurements, it has the important advantage that the results are conclusive and not subject to the uncertainty which pertains to results calculated from single-phase measurements.

These measurements have been successfully made up to potentials corresponding to 30,000 volts between conductors. At higher voltages it is probable that corona losses from the test circuits would soon begin to give trouble so that a guard circuit to take care of losses through the air in addition to those directly to ground would have to be provided.

The precision obtained in these measurements was probably not very high, perhaps not over five per cent but this was quite sufficient for the purpose in mind at the time, which was primarily to determine the approximate power loss in the dielectric of the samples at various temperatures. The other data obtained were incidental. It is believed, however, after considerable

experience with this method, that by taking particular care in the construction and arrangement of the apparatus, all of the important dielectric data of a cable can be obtained with a relatively high order of precision.

Much of the material in this paper was obtained in connection with a series of tests made for Mr. W. H. Cole, Supt. of Street Engineering, Edison Electric Illuminating Company of Boston, and the author wishes, in closing, to express his particular appreciation of Mr. Cole's courtesy in permitting the use of a portion of the information and data obtained in that investigation.

DISCUSSION ON "MEASUREMENT OF POWER LOSSES IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES" (FARMER), NEW YORK, FEBRUARY 15, 1918.

Henry W. Fisher: At the top of the second page Mr. Farmer says: "Until two or three years ago the only factors which had been given serious consideration in determining the temperature of cables were the $I^2 R$ losses in the conductor and the thermal conductivity of the surrounding media." I think I will have to differ with Mr. Farmer in that respect, because as far back as the year 1907 I commenced investigations along this line, and at that time there had been papers written abroad on the same subject, and as a result of these investigations, in our manufacture we did take into account the dielectric losses, and undoubtedly other manufacturers have done so, too. For years back we have taken into account the heat due to dielectric loss as well as $I^2 R$ loss when figuring the carrying capacity of high-voltage cables.

As a result of these experiments with saturated paper, cables were constructed in which petroleum products were used in large proportions. Such cables were made as early as 1910, and some of these cables had very low dielectric loss.

About that time, it was found that some of the yellow varnished cambrics had very high dielectric loss. Some had a power factor as high as 20 per cent at ordinary temperatures. That meant an investigation leading to an improvement in the quality of varnished cambric. The above explanation will show that years ago the dielectric properties of cables were greatly improved by the use of a mineral-base compound, although this subject has come prominently before the Institute only within the last year or so.

R. W. Atkinson: The method used by Mr. Farmer is very similar to one of the methods we have used in our laboratory, and it is very fortunate for those who may be interested in making these measurements to have the data at hand. There is one thing that a person should be cautioned against, and that is, that not all of the precautions necessary in that sort of work can be described in a short paper. There is one small feature of that kind that undoubtedly was taken care of in Mr. Farmer's measurements, but it might easily lead to serious error if it were forgotten. The Ayrton shunt used in connection with the current coil of the wattmeter makes an exceedingly useful device. It gives a wide range for the instrument, and properly used it is entirely satisfactory. If, however, the coil of the wattmeter has too much inductance, or the Ayrton shunt is inductive (if these are of small magnitude, compensation may be made), serious power factor errors will result.

One difference, not at all fundamental, in the measurements, as we have them made, as compared with Mr. Farmer's measurements, is that we have found it most convenient to use a high non-inductive resistance right across the line in the same

place he used his potential transformers. Each method has its own advantages and disadvantages. One disadvantage of the high resistance method is the amount of power that is taken and the fact that a large transformer is needed. If the transformer is available, however—I mean if the transformer furnishing the power for the test is large enough—the method becomes very satisfactory. Phase errors of the resistance, which may not be absolutely non-inductive, can be taken care of in the same way that the transformer errors can be taken care of, and will be so small—at least we have found them so—that they can be easily taken into account.

In regard to logarithmic paper—we have used paper which is logarithmic in one direction and arithmetical in another direction, and plot the temperature in the arithmetical direction. If you wanted to plot the temperature of zero degrees on the logarithmic scale, you would be pretty far off at the left hand, and besides that if even on these particular cables a little lower temperature had been reached, the departure from a straight line would have been very material, but in the case of other cable samples, it will be found that the straight line, using the logarithmic or semi-logarithmic paper, straightens out the curve in every case and is very useful for the reasons suggested.

In the three-phase measurements and three-phase losses, as calculated from single-phase measurements in Table II, I noticed in the first column that the values alternate, low, high; low, high; low. The author tells us these were taken from straight line curves. I wonder why they came out just as they did. We have ourselves found power factors in the case of the three-phase measurements lower than the single-phase. The particular conditions where the three-phase power factors as measured are absolutely different from those calculated from single-phase measurements, are where the power factor or capacity changes with the voltage. If there is a material change in either capacity or power factor with voltage, then no simple theoretical way of calculating three-phase measurements exists. The fundamental assumption in deriving the formula for the calculation falls down when the power factor or the capacity changes with voltage. To illustrate how far off we may get in the case of the single-phase calculation, the calculation from single-phase measurements may give a negative loss three-phase, when the loss is actually very large. We have found an empirical method which gives good results.

C. W. Davis: I would like to emphasize particularly the point Mr. Fisher made that dielectric loss tests, particularly on cables, are not new. There has been a tremendous amount of work done along this line for years, and its importance has been well understood by some cable manufacturers, at least both here and abroad, although I happen to know of many cases where attempts have been made to give information on

dielectric losses to prospective cable buyers, that it has been looked on as more or less superfluous and decidedly academic.

I think it is rather more nearly in accord with the facts to say that it is only in recent years that people have begun to realize that a few, if not all, cable engineers have been working along the right lines, and have been talking about things that have real significance. Around about 1900 some work along this line was done abroad. A little later, Monash published his very important contribution to this subject, and Fisher, in 1907 in the A. I. E. E. TRANSACTIONS gave the results of some of his work on cables and wires. These tests were called power factor tests, and not energy loss tests, but the terms are practically interchangeable.

Later on a tremendous amount of work was done abroad, particularly in Germany, as well as in this country, and there have been many excellent papers on dielectric loss measurements on cables and condensers in the foreign and domestic technical press and in the bulletins of the U. S. Bureau of Standards. Perhaps the best bibliography on the subject up to 1912 was included as an appendix to a paper presented to the British Institute of Electrical Engineers by Mr. E. H. Payner, in 1912.

Mr. C. E. Skinner, some years ago made use of a high-voltage electrometer for dielectric loss measurements, and while I do not recall that he published any results of dielectric loss measurements in cables, his instruments were entirely suitable for the work; and a similar instrument and a slightly modified method were used by Mr. Rayner for his work.

It is not desirable to limit the methods of test, the main thing is to get results. We have used bridge methods, the dynamometer methods, and the electrometer methods, and have been measuring dielectric loss at high voltage as well as low voltage since 1907.

For five years we have used an electrometer like the one described by Mr. Rayner and Mr. Skinner. We have had four different instruments working at different voltages and working satisfactorily. They are exceedingly simple to work when you know how to work them, but exceedingly difficult when you do not know.

The fact that entirely satisfactory results can be obtained on apparatus of such diverse nature is sufficient evidence in itself that we should not be limited in the methods of measurement. In the electrometer, in particular, you have an exceedingly sensitive piece of apparatus, and while instruments of the sensitivity we have used are doubtless not necessary for regular cable measurements, it is of interest to know that instruments of such sensitivity as will readily measure the losses at from 2 kv. to 20 kv. in condensers and cable specimens ranging from about 0.0005 to 0.001 microfarads have been used daily by us for years with entirely satisfactory results.

Cable specimens 24 in. (60.9 cm.) in length and of such small capacity obviously present considerable difficulties in the way of accurate measurements, greater difficulties than are met with in measuring cable lengths of ten or more feet.

The air condenser, as a method of checking, is a very useful one. We have used air condensers for checking purposes for about four years. Our first air condensers were a series or battery of concentric vertical tubes with guarded ends. These were very satisfactory, except that they took up much room. About three years ago we built plate air condensers, much like those recently described in the paper of Shanklin and Clark, and these have been constantly used for checking since that time. Some of our methods are counter-checked from outside sources. The Westinghouse Company on several occasions measured high-voltage condensers for us by still different methods in order that we might have another check upon our results.

D. DuBois: Mr. Farmer makes reference to a paper by Hochstadter in which the electrical stresses in the center of a cable are studied. Hochstadter goes further than Mr. Farmer and states that the stresses between points 1, 2 and 3 in Fig. 1 are 87 per cent of the voltage between conductors. This calculation as Mr. Farmer states is based on the assumption that point 4, in the center of the cable, can be considered fixed at ground potential. I do not believe that this assumption is correct nor do I believe that such high electrical stresses are at any time found between the points in question. In a three-phase cable there is always a neutral plane passing through the cable and this plane rotates with the phase rotation. As it rotates it bends and straightens and at certain times passes through the center point but at other times it moves away towards those conductors of lower potential.

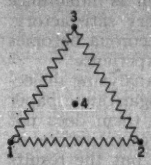

FIG 1.

This shifting of the neutral plane greatly modifies the internal stresses and it is therefore not surprising to see by the Table II that their effect in increasing the losses is not of great importance.

The fact that cables have been found with their central cores badly charred and the other insulation unburned has been explained as due to these central stresses but such charring can be explained very simply.

Any cable, no matter how good the insulation, if hot enough will be burned out by dielectric losses. Mr. Farmer shows that the losses vary with about the 4th power of the temperature and therefore a few degrees difference in temperature make a great difference in the losses. Any cable carrying current is hotter in the center than at any other place. It follows that the center is the first place to reach the condition where

heat production exceeds heat loss. When that condition is reached there is a great increase of heat production and a great increase of temperature in the center of the cable. It therefore follows that any cable failing through dielectric loss will first burn at the center.

In cables where dielectric loss is high it is difficult to make tests without the dielectric becoming heated during the tests and in three-phase tests a cable would become slightly hotter in the center. This would cause a greater loss in the center, which might erroneously be considered as due to excessive voltage strains in the center. It may be that in the Table II the definite ratio change in the case of cable *D* could be accounted for by internal heating at the higher voltages during the three-phase tests.

G. B. Shanklin: We are glad to see that Mr. Farmer has developed a method of correcting for phase-angle errors by calculation and individual measurement of the different inductances. We have never actually tried this method in our laboratory, looking at it in the light of a correction from the short end of a very long lever. An air condenser that automatically takes care of the combined errors such as inductance angles, corona and line leakage has proved very satisfactory. Mr. Farmer's limit of 30 kv. above which line corona becomes a serious factor, is fully verified by our own experience.

In regard to the method of working up and analyzing dielectric energy loss data—Plotting the values on logarithmic paper and drawing an average straight line through them does not give results of great analytical value. For quantitative or empirical purposes this method does very well but the results have no physical significance. We did not begin to make real progress in studying dielectric energy loss until we reduced watts and current to the more fundamental values, capacity and resistivity per unit length or per unit volume. These fundamentals clearly bring out characteristics that would be otherwise entirely overlooked.

We have wanted for some time to get an accurate comparison between three-phase loss calculated from single-phase readings and measured three-phase loss, and regret that Mr. Farmer did not cover this more in detail. It is possible to get a much higher degree of accuracy with single-phase measurements than with three-phase and we have always felt that this more than compensates for any slight discrepancy between calculated and actual three-phase values.

There should not be any great difference between the calculated and actual three-phase loss in good grade cable. This difference is due to the excess introduced by higher tangential stresses in the central triangle of the cross sections when three-phase voltage is applied. Surface leakage caused by tangential stresses is negligible in good grade insulation. It is only in poor grade material that it is appreciable. The poorer the

cable, then, the greater the difference should be between measured and calculated three-phase losses.

I believe that every one feels that the time has come when all cable specifications should contain a clause limiting the dielectric energy loss to some specified value. The Transmission Committee of the Institute has been working towards this end for some time and in the desire to be of help to them I have given the question considerable thought.

Our experience is that any impregnated paper cable having a power factor higher than 35 or 40 per cent at 100 deg. cent. will not withstand reasonably severe operating conditions. This is a simple rule to follow and one not at all difficult of attainment. The power factors and losses are high enough at 100 deg. cent. to largely eliminate those errors that prove such serious factors at lower temperatures. Unless some such simple clause as this is adopted I do not believe energy loss specifications would prove much of a success, for energy loss measurements require more time, care, patience and money than is generally available as a commercial proposition.

John L. Harper (Read by Mr. J A. Johnson): So far as I am aware, the method of test proposed and carried out by Mr. Farmer is the only such test made by parties not interested in some particular manufacturer or some particular customer, and Mr. Farmer's laboratory is now the only independent laboratory equipped for this form of work which is impartially open to both manufacturers and purchasers for determining comparative dielectric characteristics of cable.

I have recently divided up an order for cable between five of the large manufacturers of this material, and propose to have samples from each of these manufacturers tested by an independent laboratory for the purpose of determining a comparison between the different makes of cable. In case this independent test were made in Mr. Farmer's laboratory, the condition may possibly arise where a manufacturer would consider that the form of test used by Mr. Farmer did not give true results. With this probability in view, the writer hopes that representatives of the manufacturers may break through the protective crust with which they have surrounded the accomplishments of their respective plants in regard to dielectric conditions and enter into a free discussion upon the merits of the form of test proposed by Mr. Farmer.

The problem of the purchaser and user of cables is to make a cable carry the greatest amount of power possible with the least loss and the greatest safety from interruptions. The two major forms of loss in a cable may be put under the heads of "Copper Losses" and "Dielectric Losses", the copper losses being but to a slight extent under the control of the manufacturer while the dielectric losses are to a greater extent under his control. If it were true that a manufacturer could produce a cable which would operate with practically no dielectric losses,

this cable might, and probably would, be an entirely useless thing for the transmission of power, as, in choosing an insulation for the elimination of dielectric losses, some form of insulation might be used that would produce an impracticable and useless cable from other standpoints; therefore, it is the desire of purchasers of cables to know how far the manufacturers can go in the elimination of dielectric losses and at the same time produce a cable satisfactory as to cost, safety and continuity of operation.

Having arrived at this point, it then devolves upon the user to so install and operate his cables that the best results may be obtained.

If, by uniform test, it is established that a decrease in the operating temperature of a cable produces a smaller dielectric loss, the user should either keep his loads below the amount producing high or dangerous temperatures or should remove the heat formed in the cables by artificial means, thus allowing a much greater transmission of power over them than would otherwise be possible.

That the user can greatly increase the capacity of cables by the removal of heat and the consequent decrease of the dielectric losses, is illustrated by an incident in the writer's experience. During the hot season of 1916, a certain conduit was operated with 1,977,000 cir. mils per leg but would carry no more than 22,000 kw. without burnouts from overheating, while this same conduit, with the copper increased to 2,627,000 cir. mils, carried 42,000 kw. without any burnouts from overheating, the temperature being kept down by either submerging or spraying the cables with water. Thus, with the conduit system increased 33 per cent in copper, the load was increased 86 per cent, and without any burnouts at all from overheating the cables.

It is my sincere hope that the presentation and discussion of the method of test proposed by Mr. Farmer may result in the Institute adopting and incorporating in its Standardization Rules some uniform method of test for dielectric losses which could reasonably be used by all manufacturing companies, and which, although it might lack entire perfection for the absolute determination of dielectric losses, would still be sufficiently correct for their determination within commercial requirements.

This would leave the manufacturing companies free to adopt such additional forms of test as they might wish for their own investigation and research, and would, at the same time, give protection to manufacturers accomplishing the best results when in competition with a poorer product tested by a different method which might indicate characteristics better than those which would have been shown by the standard method.

W. H. Cole: I am principally interested in the correct interpretation of the results of the tests made under Mr. Farmer's

supervision, and what they mean to an operating company. About seventeen samples were included in the work done for the Boston Company during the past year. As a rule not less than two specimens were taken from each manufacturer's product. Practically all the specimens were picked out of our own stock and, therefore, are representative commercial samples.

As to the carrying capacity of a three-phase cable, there seems to be differences of opinion with reference to the existence of so-called tangential pressures and equalizing currents in the dielectric, but if there is anything in the theory that there are equalizing currents resulting from tangential pressures, the carrying capacity of a cable may be limited by the temperature at which the distribution of voltage stresses in the dielectric, become so altered from that obtaining at a lower temperature, as to materially over-stress certain parts of the dielectric. It is possible that this critical temperature in the usual type of three-phase cable construction, may be substantially lower than the governing temperature for actual and permanent deterioration of the insulating material.

While the correct interpretation of the results heretofore obtained in dielectric loss tests is of great importance, more elaborate investigation is necessary to determine at what temperature a given type of cable construction may be operated before any localized acceleration of dielectric heating takes place, the effects of which are now recognized by many cable engineers. I refer to the cumulative heating of the center of a three-phase cable, generally supposed to be accelerated by the flow of tangential or equalizing currents, apart from the heating due to radial leakage current through the dielectric.

In general it may be said that any reduction of losses in cable dielectrics is of an order of importance equivalent to any progress in the direction of increasing the conductivity of the cable conductors. If the latter improvement were a possibility, it is obvious that no effort would be spared, to secure it.

L. L. Elden and J. A. Walton (by letter): The effect of dielectric losses in limiting the carrying capacity of three-conductor cables has received little consideration in cable specifications until recent experience has forced a tardy recognition of their importance.

The data submitted by Mr. Farmer support that of other investigators and should be studied carefully by users of similar cables. In many cases such study will be found to offer a satisfactory solution of many cable failures which have been experienced in the past, and at the same time, indicate methods which may avoid similar occurrences in new installations.

An analysis of the results of all tests of this character reveals one important fact, viz: that the insulating materials (paper and impregnating compounds) used by the various manufacturers of standard three-conductor transmission cables vary widely in dielectric losses at all temperatures and more particu-

larly at the higher temperatures within permissible operating limits.

These results are undoubtedly due to the varying methods of manufacture and to the perfection of manufacturing processes, as much as to the materials themselves. That wide variations exist in the product of a single manufacturer is clearly shown by tests on samples of similar cables manufactured at different periods, sometimes only a few months and sometimes several years apart.

These conditions may be unavoidable, but where the differences are so wide and the losses so excessive in some cables, as to lead to their destruction within a relatively short period of operation under normal conditions, the inference must be drawn that there was something wrong in their manufacture.

It is therefore important in dealing with the effect of dielectric losses, that the constants developed from a single sample of cable be not applied to the product of another manufacturer.

It will be found preferable and more reliable to determine the characteristics of each lot purchased and from such tests determine the loading conditions under which each cable may be operated.

The dynamometer-type wattmeter developed by one of the large manufacturing concerns provides a fairly accurate method of measuring dielectric losses.

If this procedure is followed and routine tests are made for dielectric losses on each cable in service, about twice each year under nearly the same conditions, it will be found possible to keep a careful check on the condition of each cable, and to anticipate failures in many cases, when an increase in dielectric losses is found. This has actually been done in the practise of the company with which the writers are connected, it having been found possible to locate and replace sections of cable operating at higher temperatures than adjacent sections of the same cable before failures have occurred. Examination of the sections removed, revealed serious damage to the insulation from high temperature, due entirely to high dielectric losses caused by deterioration of insulating materials at the existing temperatures.

These conditions are sometimes found to be plainly evident to the touch by comparing the temperature of two sections of a cable each side of a joint, the cable all being made by the same manufacturer.

These losses have been found under test, to vary 1000 per cent between two samples of cable of the same make when excited at operating potential (13,800 volts), but carrying no load. The effect of such high losses on the carrying capacity of a cable may be left to the imagination.

It is further suggested that each cable in service should be confined to the product of a single manufacturer if possible and that repairs or replacements should preferably be made

with similar cable or at least with cable possessing characteristics not inferior to the original cable.

From time to time representatives of operating companies, representatives of manufacturers and other investigators have presented data on the carrying capacity of three-conductor cables under various conditions and it is noted that Mr. Farmer is conducting some tests of this character for the purpose of determining the temperature constants of the cables under test.

Here again it is important not to apply the results of such tests on the product of one maker to that of another, as the

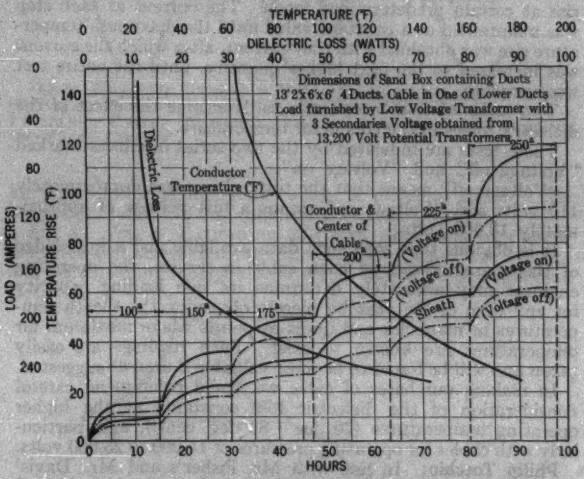

FIG. 2—CURVES SHOWING TEMPERATURE CHANGES AND DIELECTRIC LOSS IN SHORT DUCT SYSTEM (13 FT.) UNDER VARIOUS LOADS
3-Conductor, 4/0 Paper Cable. Length, 33 ft. By resistance coil method.

difference in dielectric losses may be responsible for wide differences in the temperature gradient between the lead sheath and conductors. Variations of fifty per cent on the same thickness of insulation have been noted in this respect.

It is further noted in this connection that although the A. I. E. E. Standardization Rules specify certain limiting temperatures for conductors insulated with paper, the manufacturers of this product are not in accord as to the operating temperatures for which they will guarantee their product, in some cases these limits being materially lower than that permitted by the rule in question.

Under such conditions it seems still more desirable to limit the cable used in each line to the product of a single manufacturer in order that the capacity rating of each line may be determined and based upon the same material.

Recent experience resulting from a study of cable failures has led us to study the temperature rise in 4/0 three-conductor $^7/_{32}$ in. × $^7/_{32}$ in. paper-insulated cables, as used on the system with which we are connected.

The result of this study as far as it has progressed with one manufacturer's cable is shown in Fig. 2.

These tests were conducted to determine the temperature rise at certain predetermined loads. The current at each step was maintained at a constant value until the maximum temperature rise was obtained with that current, after which the current was increased to the next higher quantity, until the entire test was complete.

Two series of tests were made to determine the effect of the dielectric losses in the conductor temperature.

These tests are indicated by the two series of curves marked "Voltage off" and "Voltage on".

"Voltage off" means that the test voltage was approximately 110 volts and "Voltage on" means a test voltage of approximately 13,600 volts.

It is obvious that with such data available and a knowledge of the characteristics of each cable in service, the operating engineer may accurately determine its safe loading capacity for any time of the year by merely observing the sheath temperatures in manholes and ducts at points where maximum air temperatures are known to exist. Such readings are easily taken and will be found to be most valuable if used as suggested.

In making purchases of cable we would recommend careful consideration of the dielectric loss constants at the higher operating temperatures (70 deg.—80 deg. cent.), and particularly with cables for operating pressures of 15,000 to 25,000 volts.

Philip Torchio: In line with Mr. Fisher's and Mr. Davis' information on measurement of dielectric losses in cables, I wish to state that I first measured not power factor, but watt loss per foot of cable, on paper and rubber-insulated cables, in 1901, and the results of my investigation were reported in a paper read before the Association of Edison Illuminating Companies at its meeting in 1902. Abstracts from that paper quoted before this Institute last June, pointed out specifically that the dielectric losses in cables were a function of the voltage, the frequency, and of a not yet determined function of the temperature. I believe that those tests were the first investigation ever made of these factors affecting the design and operation of cables.

With the aid of Figs. 3, 4, 5 and 6 I will now make a brief reference to the paper of Mr. Farmer, calling attention to the fact that in measuring losses in dielectric materials we find

conflicting results which at times are inexplicable until we analyze further the conditions of the test. I will point out the importance of the shape of the voltage curve by showing results obtained under different conditions of test.

Fig. 3: This is a test, 60-cycle, which shows a ratio of the

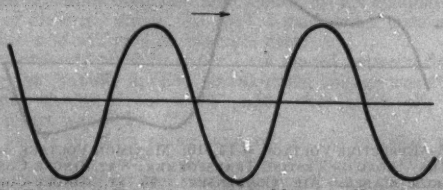

FIG. 3—EFFECTIVE VOLTAGE = 14910; MAXIMUM VOLTAGE = 20,800; 60 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

average voltage to maximum voltage of 14,900 to 20,800. The losses per foot at 100 deg. cent. are 0.8 watts.

Fig. 4: This is a curve on 25-cycles, which was very much distorted and shows that the ratio between the average voltage and the maximum voltage was 14,900 to 25,000. The losses were

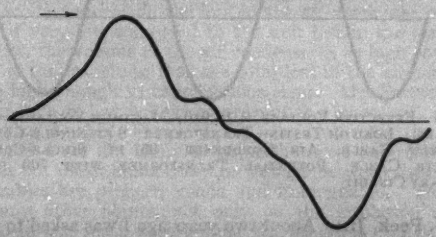

FIG. 4—EFFECTIVE VOLTAGE = 14,910; MAXIMUM VOLTAGE = 25,400; 25 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

1.20 watts, at 100 deg. cent. which are higher at 25-cycles than they were at 60-cycles.

Fig. 5: The next curve is also 25-cycles and by modifying the curve we reduce the losses to 0.68 watts at 100 deg. cent.

Fig. 6: The last curve, which is a 60-cycle curve, showed the average to be 14,921 and with this form at 60-cycles, we reduce the losses to 0.77 watts at 100 deg. cent.

We found that the distortions in the above voltage curves were caused by the transformers and regulators which stepped up and controlled the voltage supplied to the cable. This dis-

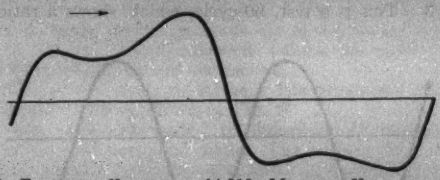

FIG. 5—EFFECTIVE VOLTAGE = 14,910; MAXIMUM VOLTAGE = 21,600; 25 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. AIR CONDENSER: 95 FT. SINGLE-CONDUCTOR STANOLITE CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

tortion was dependent upon the current supplied which in turn is dependent upon the voltage and amount of cable connected to the equipment.

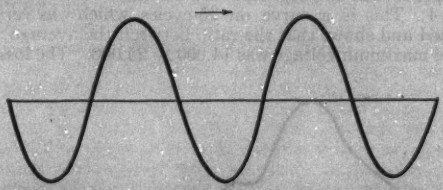

FIG. 6—EFFECTIVE VOLTAGE = 14,910; MAXIMUM VOLTAGE = 21,600; 60 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. AIR CONDENSER: 95 FT. SINGLE-CONDUCTOR STANOLITE CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

F. W. Peek, Jr.: About two years ago I was asked to investigate the cause of many cable failures on a large system. It was thought that these failures were due to high-frequency disturbances, because the failures were generally accompanied by lightning arrester discharges. An examination of the records showed that practically all of the failures were confined to a single make and type of cable. Everything but the load chart pointed to overheating as the probable cause of the trouble. An examination of some of these cables at practically no load showed them to be quite warm—obviously due to high dielectric loss.

Dielectric loss measurements on sections of these cables showed that the dielectric loss was of the same order as the full-load

copper loss. The high-frequency disturbances were caused by the failures and were not the cause of the failures.

An examination of these cables showed them to be very loosely wrapped, and showed that the compound had migrated, leaving parts dry and parts still well impregnated. Such a condition will tend to cause high loss and failure by an unequal distribution of dielectric stress. In 1914, I made some tests to illustrate this, and will give an example here:

Twenty sheets of plain paper were placed between flat electrodes—the "rapidly applied" break-down voltage was 10 kv.

Twenty sheets of the same paper were oiled—the break-down voltage was 16 kv.

Ten sheets of plain paper, and ten sheets of oiled, making a total of twenty, were tested. The break-down voltage was 8 kv., or less than for the twenty sheets of plain paper.

Of course the nature of the compound is also of great importance. Loss measurements made at the time a cable is purchased will not anticipate deterioration of compound, or migration of compound. These are best anticipated by the chemist, and an examination of the construction of the cable.

Mr. Farmer's three-phase measurements are very interesting and valuable. In making loss measurements at low power factor, great care is necessary to correct carefully for any angular displacement. As Mr. Farmer points out, if the air condenser method of making phase-angle correction is used at high voltages there is danger of error due to loss in the condenser. This is so, because while the loss in the air condenser per unit volume is very low compared to the unit loss in the insulation under test, the volume of the air condenser is so high compared to that of the insulation that the total loss in the air condenser may be high enough to cause serious error. Mr. Farmer made the correction at low voltage to prevent this error.

Referring to Mr. Farmer's Fig. 1: Naturally the points 1, 2, 3 and 4 are not likely to be at the same potential. This fact does not mean break-down or leakage between these points if the cables are properly made and impregnated. There are also many other points, not only on the insulation surface, but also in the space surrounding the insulated cables, not at the same potential. The unit stress is probably generally high at 1. The fact that the surface 1–2 is burned does not mean that there has been leakage over this surface; it may mean break down between 1–4 and 2–4. The values given for these stresses are not necessarily correct.

Referring to the method of heating the cables for test, a very good way is to bell the ends, seal them, and immerse in hot oil.

I have found that the loss in good insulation may be expressed by*:

$$p = afe^2 = bfg^2 \times 10^{-6} \text{ watts per cu. cm.}$$

*"Electrical Characteristics of Solid Insulations", F.W. Peek, Jr. *G. E. Review* 1915, November.

The constants a and b vary with the temperature, thus

$$b = kt^n$$

$$\text{or} \qquad p = k_1 fe^2 t^n = kfg^2 t^n$$

Where the insulation contains occluded moisture, the loss equation takes the form

$$p = bg^2 (f+c)$$

This often makes it appear that the exponent of voltage is greater than the square. It will be noted that most of Mr. Farmer's values of this exponent are very close to 2.

The method of reducing from single phase to three phase seems complicated and artificial. It would have been very interesting if Mr. Farmer had given curves on the same cables for single phase and three phase.

In selecting cables it is important that in addition to the voltage tests, examination be made of the tightness of wrap, quality of paper, and of the compound. Certain compounds have been found by experience to be undesirable because of probable chemical change, unreliability, and bad temperature characteristics. Unfortunately, poor cables cannot always be reliably anticipated by loss temperature measurements. The loss measurements will, however, be of use to show that the cable is in good condition at the time of purchase. The same result can be more simply accomplished by applying voltage and measuring temperature. Much can, however, be done by examination of paper and compound.

Excessive, long time high-voltage tests should be avoided.

I believe high-voltage direct current offers a good method of making cable tests. I quote from my paper:* "The Effect of High Continuous Voltages on Air, Oil, and Solid Insulations."

"There is considerable difficulty in practise in making a-c. voltage tests on long lengths of cables, due to the size of the apparatus, which is necessarily large on account of charging current. The necessary kilovolt-amperes often amount to several hundred. The wave shape is often distorted by the leading current, the apparatus is difficult to move about, etc. D-c. tests would eliminate these difficulties, as very small apparatus would be required, providing such tests would detect faulty sections, etc. It cannot be said that a given d-c. voltage is equivalent to a given a-c. voltage. The above tests indicate, however, that faulty sections of cable could be equally as well located by d-c. tests as by a-c. tests. In cases of cracks, etc., the air- or compound-filled space would be broken down at the same maximum voltage on direct current or alternating current. In case of a fault due to moisture, the breakdown would apparently take place at about the same voltage, alternating current or direct current. In the case of a cable in good condition there

*A. I. E. E. Trans. Vol. XXXV, Part II, p. 800.

would be much less likelihood of injury by direct current than by alternating current of the same maximum voltage. A d-c. voltage equal to the maximum of the a-c. test voltage would, therefore, seem suitable for such tests."

Resistance measurements should be considered.

M. G. Lloyd: In regard to the difference between the single-phase and three-phase measurements, I see but three possible causes for such a difference. One has been mentioned by Mr. Atkinson, that due to the fact that parts of the dielectric were stressed differently in the single-phase test from what they were in the three-phase, and with varying power factor in the electric field strength, and with power factor perhaps varying differently in different specimens, we have no definite relation between the two.

Another reason for the difference might be lack of homogeneity in the dielectric material, so that measuring one part of the cable would not give a value proportionate to that which would be obtained if it were all subjected to the same stress.

There is still a third possibility. I think of it in connection with the analogy with magnetic hysteresis, that is, the loss occasioned in magnetic material when it is subjected to a varying magnetic field. Let me recall to your mind that the loss in magnetic material when placed in a rotating magnetic field is different from the loss when it is placed in an alternating magnetic field of the same maximum value. The difference there is generally in this respect,—that in low densities the loss is greater in the rotating field and very low in the alternating magnetic field, but at high densities the loss is lower in the rotating field and greater in the alternating magnetic field. Magnetic hysteresis cannot take place unless the permeability varies with the flux density of the material. Similarly, dielectric loss cannot easily take place without the dielectric constant varying with the strength of the electric field. It seems to me quite possible there may be some analagous effect, that the loss in the dielectric material in the rotating alternating field may be different from that in the simple reversing field, and I suggest that as a thing which may be investigated by those who are experimenting in this subject.

James R. Craighead: Mr. Farmer states that he considers the precision of the measurements to be about 5 per cent. The variations of the ratios shown in Table II are decidedly in excess of a 5 per cent limit, while the average test value is within 1 per cent of the calculated. Averaging the values at each voltage shows a curious characteristic by which the ratio goes up with the voltage by a more or less irregular curve, from about 0.92 to about 1.07. Can Mr. Farmer suggest an explanation of this?

Comfort A. Adams: In the first place, we do not have a homogenous dielectric. We have different materials—paper, cambric, and impregnated compound. We have some parts

more thoroughly impregnated than others. We have some parts that will dry with air space or gas space, and with consequently increased loss. There are different temperatures in different parts of the cross-section; for instance, a much higher temperature in the center than on the outside. This changes the effective a-c. resistance, changes the losses per unit volume, changes the distribution of energy throughout, and accounts, in some part at least, for the great difficulty in comparing different cables.. It is not anything like as simple a matter as you might imagine.

Also, I think there is evidence which will lead us to believe that very minute quantities of moisture make more difference in the cable losses than some other things that we look after carefully.

Finally, this is not only a very important subject, one of the most important in transmission problems, but it is one in the investigation of which the ground has scarcely been scratched, and in which we have an immense amount to learn. We are dealing with problems and analyzing them on the assumption of certain approximate homogeneities, which do not exist, and we have got to get down, way down, to the bottom of things in a degree we have never done before.

F. M. Farmer: In regard to the statement in the paper which a number of speakers have taken exception to, where I intimate that nothing has been done on this subject up to two or three years ago, perhaps my wording was not very fortunate. Obviously, what was meant is that nothing has been done which has aroused serious interest among operating engineers until two or three years ago, although the subject has been very active since that time. We all know that there has been a tremendous amount of work done on the general subject of dielectric losses during the last fifteen years, but we also know that very little information has been made available to the profession at large, particularly in connection with underground cables. If the manufacturers have done a great deal of work, as stated by the representatives here, they have until very recently kept the accumulated information to themselves.

As to Mr. Fisher's point in regard to the carrying capacity of cables being limited by the dielectric strength, it is obvious that as the temperature goes up a point will be reached where destructive action takes place and the cable will break down due to the dielectric strength becoming nil. In other words, if the insulating materials used in the cables did not suffer physical deterioration at high temperatures, the carrying capacity would be limited by the permissible loss in a given installation rather than by the "safe working temperature" as determined by the physical characteristics of the material.

Mr. Atkinson stated that there are a great many points in the way of precautions in tests of this character that have not been mentioned in the paper. Of course it goes without saying

that all of the precautions that are usually taken in measurements of this class must be carefully observed. I simply tried to point out some of what might be called the extra precautions which have to be taken care of in this particular class of electrical measurements.

Mr. Davis' point in regard to the desirability of different investigators using different methods is very good. It is not necessary or desirable that everybody use the same method. What is essential is that the results obtained by the different investigators mean the same thing to everybody.

Mr. DuBois' conclusion that there is nothing in the Hochstadter theory may be quite correct. I simply stated this theory as a reason for making comparison between single-phase and three-phase tests. If there is anything in the theory, that is, if there is an extra loss due to tangential stresses, it ought to show up in the ratio between the two quantities. I am convinced that actual tests do not show any such loss of any considerable magnitude and naturally an experiment is usually the ultimate proof. In other words, the results we are getting in these peculiar cable failures are due to the cause which has been discussed here this afternoon, namely that the internal insulation naturally reaches a higher temperature than the rest of the insulation and therefore the loss is increased faster at that point, so that ultimate destruction takes place in the internal insulation long before it does in the outer insulation. In other words, destruction is probably not due to a higher dielectric loss per unit volume produced by tangential stresses.

In reference to Mr. Shanklin's question in regard to the formula which has been given for the three-phase loss calculated from single-phase tests, I think he has forgotten to consider the proper voltage for each single-phase loss. In a single-phase test, the voltage distribution under the application of three-phase potential cannot, of course, be duplicated because in the latter case the belt insulation is subjected to Y voltage while the insulation between conductors is subjected to delta voltage. Therefore, in calculations from single-phase measurements, this should be taken into account by assuming that the loss is proportional to the square of the voltage. Possibly it would have been clearer if the formulas in paragraph A under "Three-phase vs. Single-phase Tests" had been expressed as follows: (for single-phase tests made at Y voltage)

$$R_1 = W_1 + \frac{W_4}{3} + \frac{W_5}{3}, \text{ etc.}$$

Mr. Torchio's data showing the effect of wave form are most interesting. However, it is obvious that measurements of this character should not be undertaken without first investigating this point pretty carefully and being sure that the wave form is substantially a sine curve. It is one of the first precautions to be observed.

In reference to Mr. Craighead's inquiry I have no suggestion to offer. I believe that some of the apparent discrepancies which are most pronounced at the lower temperatures in the first cables tested are due to the temperature not being held for a sufficiently long time and not being measured with sufficient precision. This table was given merely to show that there is no very great difference between the three-phase losses calculated from single-phase tests and those obtained by direct three-phase measurement and is not given with any idea that the ratios obtained are precise. Although for all practical purposes these results show that there is no internal loss, still I feel that this phase of these measurements should be investigated further, knowing as we do now that the measurements could have been made more precisely.

Lecture before the 6th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 16, 1918.

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SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

BY ALBERT C. CREHORE

LAST year you were privileged to listen to an account of recent experimental observations in connection with atomic phenomena, I may say some of the fundamental facts of the physical world, coupled with an account of the way some of these facts are now generally interpreted in terms of "Radiation and Atomic Structure." The interpretation was principally in terms of the current Rutherford-Bohr theory of the atom.

It is the interpretation of physical observations that has engaged the speaker's attention for several years, and it has seemed to your committee that the time is ripe to present to you certain conclusions that have resulted. It goes without saying that no theory has ever yet been proposed that enables us to account for all the facts of observation by steps of logical deduction from certain assumed premises. Nevertheless, there is every reason why this high ideal should ever be kept before us. Unless we aim higher than we expect to attain, we will not accomplish so much as we expect.

I would not have you understand from this that no real progress toward the desired goal has been made. On the contrary, I believe it to be very great. How can we estimate the progress that has been made? Throughout the history of science we have seen one theory giving place to another sometimes in rapid succession. This has had a very depressing effect upon a certain class of people, who do not comprehend the real trend of it. I would impress upon you that this is one of the chief earmarks of progress. We cannot attain the realization of the ultimate goal at once, and, therefore, the only way to approach it is by a series of successive approximations. Each nearer approach involves of necessity some changes in preceding ideas.

It even seems probable that the fundamental hypothesis of atomic structure as composed of electrons, positive and negative, is entirely correct, and that the reason why we are not yet able

to account for more of the observed phenomena is because we have not discovered the proper processes to follow in reasoning from the simple to the complex. I suspect that this is the truth of the matter, and, when you comprehend more fully the complexities of most of the processes involved, you will experience no difficulty in agreeing with me.

The discovery of the electron and the isolation of it, so that its properties might be measured, marked a new era in our conception of matter. The fact that each electron always carries an electrical charge of the same amount is most significant. It has led by natural steps to the present electrical theory of matter. It was also most natural to begin to apply to these electrons, as representing electrical charges, the electromagnetic theory that is known to apply to other electrical charges, a theory that had been developed by a generation of men long before the electron was known, as the chief exponent of which the name of Maxwell stands preeminent. This was done, of course, but it was soon found that the existing form of electromagnetic theory was inadequate to cope with single isolated charges moving with considerable velocities through space.

Electromagnetic theory itself, therefore, became the object of much more careful study. The result has been that we have witnessed, even in our day, a gradual evolution of electromagnetic theory. Maxwell would hardly recognize the form in which electromagnetic equations are expressed today. The new thing that has been introduced into the theory is the conception that the effects of a charge moving through the ether are propagated out in all directions from it with a velocity which is characteristic of the ether, the same velocity as that of light. This introduces important differences into the equations, sometimes referred to as "retarded potentials."

Some progress was made by the use of this theory in interpreting atomic phenomena, but there were certain observations, particularly connected with radiation from gases, that simply refused to be brought into line in any way that could be conceived. On the strength of this Max Planck proposed what is now known as his "quantum theory." It stands today as one of the boldest proposals ever suggested in physics. He originally arrived at the theory through a consideration of the radiation of energy from gases, but it has since been shown to have a very general application in all atomic phenomena. The boldness of his proposal may be seen from the following. Finding that

electromagnetic theory was inadequate, as he conceived of it, he proposed to lay it aside and not use it. He then gave reasons for his proposal that, whenever energy is radiated anywhere from matter, the energy is transferred in units, or in quanta, so that the total energy transferred is some integral multiple of a fixed minimum of energy, the quantum.

Let it suffice to say that the application of Planck's quantum theory to radiation straightened out the difficulty immediately. Perhaps it would be better to say transferred the difficulty from one place to another, for difficulty there is and will be until the fundamental cause of the necessity for units or quanta of energy is comprehended. However, progress by the use of this new idea was very marked in every direction as regards atomic phenomena, The value of the unit or quantum of energy has now been experimentally determined. It is excessively small, but, when a quantity can be measured at all, its existence is not questioned. Its existence is now regarded as experimentally established.

The present trend of physicists is to generalize this idea, and to say that energy is essentially of an atomic nature, being composed of units in the same way as matter is so composed. This generalization implies that no flow of energy ever takes place that does not go in exact multiples of this unit. It seems to the speaker that there is danger in thus generalizing in advance of definite proof, or, at least, in letting our thoughts be guided as though this general proposition were true. A similar generalization was made without proof some years ago in physics. I refer to the principle of the conservation of matter, which has been published in most textbooks on physics of the older school. We have latterly been obliged to retract this. We do not know now that it is true. We cannot assert, however, that the contrary is true, namely, that matter is being created and destroyed. We simply do not know. We do know that the individual atoms do not remain intact as they were formerly thought to do, and that some of them are actually going to pieces, giving out parts of themselves and losing weight. Whether or not the electrons that they give out always enter into some other piece of matter, and thus make for the ultimate conservation of matter, it is not possible to say. We cannot say that the sum total of electrons in the universe remains constant.

You will easily comprehend that the effect of Planck's theory has been to divert attention from electromagnetic theory, since he was forced to lay it aside. This very act, and the fact that substantial progress was made by so doing, naturally brought

electromagnetic theory into disrepute, so far as its application to electrons in atoms is concerned. It is fortunate that he had the courage to do it, because the result has been that we are in possession of valuable information about the atoms, which would otherwise now be unknown.

It must be admitted, however, that it will never be satisfactory to the human intellect to leave the situation in the state we find it today. The solid foundations upon which we must seek to found an atomic hypothesis should in some way be connected with the properties of the electron and its connection with the ether of space. These are fundamental parts of electromagnetic theory, which expresses in mathematical language just this connection between an electrical charge and the ether, that is, between matter and the ether. It even seems possible, nay indeed probable, that some good reason for the existence of a quantum of energy may yet be discovered by the help of electromagnetic theory, as being connected with the very constitution of the atoms themselves. This hope alone is sufficient incentive to urge us to continue the search by the use of this theory, at least to such a point that it can be proved the search is vain, a most difficult undertaking.

You perceive from the above remarks, when taken in connection with the title of this address, "Some applications of electromagnetic theory to matter," the difficult nature of the role that the speaker has assumed. Having come to a decision to apply electromagnetic theory, so far as possible in all strictness to the electrons in the atoms, for the purpose of learning where it gives a good account of itself, and where it requires some modification, I have gradually been led to the conclusion that comparatively little has heretofore been done in this direction, and that the results thus far attained are of so great value that a great deal more should be done.

It is the under dog that has the sympathy of the crowd, especially if he shows sufficient life to put up a plucky fight. During the present phase of the wave of progress in physics, electromagnetic theory, as applied to the atoms is like the under dog. It is on the defensive, and there are strong reasons why those who are striving to save it should have the sympathy of the crowd.

II. Rutherford-Bohr Atomic Theory

With this introduction you may be better prepared to listen to the following account of some applications of electromagnetic

theory to matter. As above stated the Institute had the opportunity last year to hear an address upon topics of absorbing interest to physicists, which included some account of the Rutherford-Bohr theory of the atom. This theory is, no doubt familiar to you, but some of the chief features of it will form the starting point of my remarks tonight. According to their theory an atom is a structure composed in part of a charge of positive electricity, the center of which is the geometrical center of the atom. This charge is supposed to have small dimensions compared with the size of the atom, or even of the negative electron. Around this nucleus one or more negatively charged electrons are supposed to be describing approximately circular orbits, the radii of which are very large as compared with the radius of the electron itself.

There may be several electrons describing the same orbit at the same time. They would then distribute themselves at equal intervals around this orbit on account of the mutual repulsion they have for each other, thus forming a ring of electrons. The atoms may be composed of several such rings all circulating around the same nucleus. Atoms, in the ordinary states of matter, have no resultant charge, and it is supposed that in this case the sum of the negative charges of the electrons is equal to the positive charge of the nucleus, thus making the total zero, when the atom is said to be neutral. Under special circumstances atoms may be caused to lose or to gain one or more electrons than they have when neutral, and then the atom manifests a resultant charge, positive if electrons are lost, negative if gained. It has been observed that such atomic charges are always some exact multiple of the charge on one electron, but it may be either positive or negative.

There are certain remarkable features in this theory introduced by Dr. N. Bohr, relating to the changes which take place in such an atom when it is disturbed by any cause. He finds that the radius of the orbit of the electrons changes from one fixed value to another, there being a series of possible radii; but, when the change begins, it does not stop until it arrives at one of the other possible radii, or, in other words, the radii must change by sudden jumps. This is a direct result of an application of the quantum theory above referred to to the atoms; for, by it, the change in energy must take place in quanta. Bohr has found as a result of his premises that there is a change in the kinetic energy of the electron in passing from one of these fixed

orbits to another, and the size of the orbits has been calculated from *electromagnetic theory* by taking this change in the energy always as an exact multiple of the quantum of energy. The atom will then radiate energy when the electrons fall in towards the nucleus from one fixed orbit to another, and they absorb energy when they go outwards from the nucleus.

There are several assumptions that have been introduced in order to arrive at these results. First there is no radiation of energy at all when the orbit remains in one of the series of fixed positions, although the electrons are in rapid motion. Second the ordinary equations of electromagnetic theory apply to the electrons when they are in one of these fixed orbits not radiating energy, but that they do not apply when they are changing over from one radius to another and radiating energy. There have thus come to be recognized several states of the atoms, where the radii are fixed, known as the steady states, and where energy is neither radiated nor absorbed. All other states than these exist only during the radiation or absorption of energy.

III. Two Great Natural Divisions of Matter

There is thus made a great natural division of all existing physical phenomena. The atoms are either in one of these steady states or they are not. To illustrate, we may consider that most of the matter in the solid crust of the earth is in the steady state, or, more strictly, it would be if it were at the absolute zero of temperature not radiating any heat. It is quite conceivable to think of the temperature of the earth as being lowered to the absolute zero without disturbing in any way the real constitution of the rocks of which the earth is composed, or of the temperature of a crystal being at the absolute zero. As an example of atoms not in the steady state, we may think of substances during the process of undergoing any chemical reactions, where a change of partners takes place with an accompanying transfer of energy among the atoms and molecules. This includes all vital processes in organic substances, where chemical reactions are continually taking place under certain special laws of guidance or causation. This latter class involving energy transfer includes most of the phenomena of the greatest interest to man, but the former class, the steady state, is of the utmost importance even if somewhat more restricted in variety. We owe to it the home in which we live, the solid structure of the earth. It also includes the large class of metals and building

materials generally, in which the engineer is particularly interested.

It may be repeated, that, in this first class, the steady states, the Bohr theory, and other theories, assume that the ordinary electromagnetic theory is applicable to the electrons in an atom. The view is taken that it is not applicable to the second class involving energy changes in the Bohr theory, but it might be a better position to take to say that no one has as yet seen just how to apply the theory than that it is not applicable.

If we, therefore, confine our attention to those states of matter where there is no energy transfer, the first class, where the electromagnetic equations are said to apply even by those who indorse in every detail the Bohr theory of the atom, we ought to be able to make a beginning of progress in interpreting some of the physical properties of this class of matter by the correct application of electromagnetic theory. It is time enough to begin a consideration of the other necessarily more complicated class, where there is a transfer of energy, and where the applicability of the electromagnetic equations is at present in doubt, after we have achieved some sort of results in the simpler case. It seems as if this simpler case should naturally precede any consideration of the other, for one reason, because any success in the one is almost sure to shed some light upon the methods of procedure to be adopted in the other.

It is precisely to these steady states of matter, where the applicability of electromagnetic theory is admitted, that the speaker has been endeavoring during the past several years to apply the present form of electromagnetic theory in a more rigid manner than it has been applied heretofore, and it is to a consideration of some of the results of such application that your attention is directed in these brief remarks.

IV. Electromagnetic Theory

To begin with a few remarks upon the current form of electromagnetic theory seem to be required. Engineers may be disposed to associate the words "electromagnetic theory" with that form of the theory which comes into the subjects of Electricity and Magnetism, as it is taught in the technical schools. There is a very great difference, however, between the general form of the theory and that part of it which is commonly needed for a complete understanding of all of the phenomena usually met with in electricity and magnetism. You will see at once the

reason for the difference when you consider that you are always dealing with great numbers of electrons instead of with single isolated ones. An electric current is a continuous stream of electrons often moving in closed curves, which evidently simplifies matters very much, and in every practical case, you will find that a simplified form of the theory is all that you require. For this reason it seems fair to assume that most of you are not very well acquainted with the general form of the theory where single electrons are concerned. The example referred to by the use of the electric current may make the difference clear. Imagine a closed circuit of any shape carrying a steady current. There is a magnetic field set up in the whole region surrounding the circuit, and at each point of this field the magnetic force is a directed quantity, but the point I wish to make is that it is constant at each point of the field. Now imagine all but one of the procession of electrons circulating around the circuit to be removed in some way, so that a single electron instead of a multitude passes around the same circuit. It is evident to anyone that the magnetic force at the point in question cannot be constant as before, but must be pulsating in character with a fundamental period equal to the time of revolution in the circuit. The equations of the general electromagnetic theory must be such that they will give a complete description not only of the variable magnetic force at the point in question with time but of the electric force as well, for, when a magnetic force varies, as it must, it is always accompanied by an electric component, which in such a case as cited will also vary. The very great simplification in the force introduced by having the stream of electrons is, I think, made clear by this example.

The general electromagnetic equations in current use are of too complicated a nature to introduce into a talk of this kind, but I think that some things may be said of them which will serve our purpose. They are necessarily expressed in the language of vector analysis as always dealing with variations of directed quantities in space, and no attempt to put them into other mathematical language that we possess will ever be successful. In fact, until the vector language was developed it was hopeless to cope with such problems. Let us now try to get a definite picture of some of the fundamental facts of electromagnetic theory as applied to just two electrical charges, two electrons if you please. Imagine them isolated from the rest of the physical universe if you can. If each of them is at rest, the

ordinary electrostatic force, with which you are familiar, is all that we have to consider, a repulsion in the direct line joining the charges inversely as the square of the distance between them and directly as the product of the charges. But, let them be set into motion. We may now picture a disturbance, sometimes called an electromagnetic wave, as travelling out in all directions with the velocity of light from the charges. If the distance between the charges is considerable, it may be quite a time before the wave from the one reaches the other charge, 8 minutes if one is in the earth and the other on the sun. In 8 minutes the first charge will have moved a considerable distance if its velocity is great. If its velocity were 1/100th of that of light, it would have travelled 1/100th of the distance from the earth to the sun in this time. So, when the wave that left it in its first position actually reaches the second electron, the first electron is not very near to the place that it was. In what direction would the force on the second electron due to the first then be? Evidently not in the original direction it had when they were both at rest. Electromagnetic theory purports to tell us just what the force is at any time on the one charge due to the other, that is the instantaneous force, for it is evidently continually varying.

You will, I think, see from this that the force on the second electron is entirely due to this electromagnetic wave which emanates from the first electron. This wave must travel in a medium, and so electromagnetic theory is founded upon the real existence of an ether fixed in space, and the force upon the second electron is really due to some connection that it has with the ether. One of the fundamental equations in electromagnetic theory expresses in mathematical language this fundamental connection that there is between an electrical charge and the ether. Much more might be added on this topic but, if you are to hear the things I have set out to say, we must pass over this phase of it.

V. First Application of Electromagnetic Theory

It will be understood that electromagnetic theory is perfectly general in that it expresses the force at a single instant of time only and does not prescribe at all how the two charges in question shall move relative to each other. We have been led to believe for many reasons that electrons revolve in approximately circular orbits in the atoms. The Bohr theory above referred to assumes this as the natural stable condition. Granting that

this is true, it must be evident to you that the first step in the application of electromagnetic theory to atoms should be the solution of the problem of finding the force that one electron when revolving at a uniform rate in a circular orbit, exerts upon a second electron revolving in a different circular orbit. These orbits must also be fixed in space in perfectly general positions, because the electrons in different atoms are, of course, situated in every possible manner.

It has been the speakers contribution to this subject to solve this problem in its most general form, no terms of any kind having been omitted. It may not, at first, be apparent to you that any solution is required, or rather, perhaps, of what a solution consists, since the general theory gives the force of one of these electrons on the other. But you will see with some further consideration that the equation we want must contain several quantities, the radii of the two orbits in question, the values of the constant velocities of the electrons in the orbits, the location of the position of the center of the second orbit in space with respect to the first, that is the x, y and z coordinates in ordinary geometry, and finally the relative positions of the planes of the two orbits, whether parallel or not. All of these quantities must find a place in the final solution, and of course none of them finds a place in the original general equation for the simple reason that it is general, not specifying any orbits or any positions for them. It requires three full pages to print this equation, but events have shown that it contains most important consequences when applied to electrons in atoms. It is some of these that I propose to discuss. The question probably occurs to you how could it have been claimed before that electromagnetic theory had been applied to electrons until the solution of this general equation had been obtained. This is a part of my contention in support of the view that these claims have not been very well founded, and that there remains much of value to be done.

VI. Rings of Electrons

In giving an account of them I shall not follow the order in which the results were obtained, but prefer to review them, as it were, with hindsight instead of foresight. Naturally one of the first things to do with this equation is to apply it to the particular case where the two orbits in question exactly coincide, the two electrons then being in different places in the same orbit, for, in this way we may learn something of the properties of

rings of electrons, of which the atoms are supposed to be composed.

A very great simplification is naturally made in the general equation when the two orbits coincide because the coordinates of the center of the second orbit referred to the first are all zero, and the speeds of the two electrons are the same. So the equation reduces to a very manageable form. The *total* force of the second electron on the first may then be conveniently resolved into two components, the one along the radius of the orbit, and the other along the tangent line perpendicular to the one. In general, neither of these components of the force is zero, and, as we make the velocity of motion less and less, the force reduces to the ordinary electrostatic force.

a—Two Electron Atom. If there are only two electrons in the ring, it is evident that they must take positions at the extremities of the same diameter of the orbit on opposite sides of the nucleus of the atom, which may now be imagined at the center. For a stable position, both the radial component of the force and the tangential component must vanish, otherwise there would result both a change in the radius of the orbit and an acceleration along the orbit, neither of which conditions represent a steady state. In such an atom, consisting of a nucleus and just two electrons, I have calculated the total radial force on one electron, including that due to the nucleus, that due to the other electron and that due to the centrifugal force of the electron itself. This is shown in the curve *I* on the slide. The equation for the radial force is

$$F_\rho = e^2\left(-\frac{7}{4\,a^2} + \frac{\beta^4}{2\,a^3}\right) + \frac{m_o v^2}{a}.$$

There are two radii at which the radial force vanishes, the one about 2×10^{-10} cm. and the other at 1.85×10^{-8} cm. In between these values the force is negative, signifying an attraction of the electron towards the nucleus, and, for all values greater or smaller than this, the force is positive, signifying repulsion. From this it is evident that the larger root corresponds to an unstable position, and the smaller root to the only stable position. This result entails some very radical consequences as regards atomic theory, some of which must be referred to here.

In the first place it makes the order of magnitude of the radius of the orbit of the electrons 10^{-10} instead of the much larger order 10^{-8} cm. given by the Bohr theory.

In the second place, by omitting the forces due to the *motion* of the rings of electrons, the $e^2 \beta^4/2 a^2$ term in the equation, and by taking into account the electrostatic forces only, and balancing these against the centrifugal forces as Bohr does, the ordinary conditions for planetary motion as in a solar system obtain, except that there is repulsion instead of attraction between the planets. He has made use of the following theorem which applies quite generally to all such systems "In every system consisting of electrons and positive nuclei, in which the nuclei are at rest and the electrons move in circular orbits with a velocity small compared with the velocity of light, the kinetic energy will be numerically equal to half the potential energy."

It is the use of this theorem that has to be abandoned because of the neglect of the forces due to the motion of the ring. By the help of this theorem Dr. Bohr was led to adopt the equal angular moment of momentum hypothesis for each and every electron in all atoms, which he stated as follows, "in any molecular system consisting of positive nuclei and electrons in which the nuclei are at rest relative to each other and the electrons move in circular orbits, the angular momentum of every electron round the center of its orbit will in the permanent state of the atoms be equal to $h/2\pi$, where h is Planck's constant." This theorem is derived directly from the preceding theorem and fails when it fails.

The reason for obtaining such different results by a strict application of electromagnetic theory is apparent. The difficulty at the time Dr. Bohr first published this theory in July 1913 was that there was no available expression for the forces due to the motion of the ring, but now, the general equation referred to above has supplied this. It appears that there is a term in the force $e^2 \beta^4/2 a^2$ due entirely to the motion of the ring, that coming from the magnetic component in the general equation, which varies inversely as the third power of the radius, and directly as the fourth power of the velocity in terms of that of light. This term has not been pointed out before. It is a repulsive force and very small at distances anything like 10^{-8} cm. but, because it varies as the inverse cube of the distance, it is evident that it rapidly increases the smaller the radius, and must overtake in size any term that varies inversely as the square of the distance, namely the electrostatic forces that Bohr used. It is where the balance between these two kinds of terms is effected that we obtain the stable position of the orbit.

For complete equilibrium, as was stated, the tangential force of one electron upon the other must also vanish. This subject has been discussed in that now classical work on "Electromagnetic Radiation," by G. A. Schott. He has derived an expression for the sum of the forces upon one electron in a ring due to all the others in the ring, but it is not given in a form that is easy to use, involving as it does the use of Bessel's functions, and summations. However, our two results are based upon the same premises, although expressed in different form, and they should agree. They do agree in so far as a comparison has been possible, which gives some assurance that both are correct. Now these tangential forces can never be zero as Schott points out, unless we also take into account the force that the one electron exerts upon itself. The other electrons in the ring exert forces on the one in such a direction as always to accelerate the motion of the one electron, thus doing work upon it. The one electron exerts an exactly equal and opposite force, and thus makes the total tangential force zero. In other words it requires some energy to keep the ring in motion. Strangely enough, however, it requires much less the greater the number of electrons in the ring.

b—Regulation of Speeds of Rings in Electromagnetic Theory.

In electromagnetic theory, it is not possible to conceive of a single electron moving alone through the ether, sending out from itself a wave of energy in all directions and still keeping up an undiminishing velocity, unless the electron is itself changing in some way. There must be a source of this supply of energy, to supply the energy radiated, and lost to the electron. Schott has accounted in a beautiful manner for the regulation of the speed of electrons in rings. The regulator acts through the medium of these tangential forces, and Schott has shown that the peculiar nature of these forces is such that the velocity of the electrons in a ring will remain substantially a constant, or subject to very small variation for comparatively great changes in the radius of the orbit. This refers to the actual, not to the angular velocity.

It is not easy in a talk of this kind to carry you along so that the reasons for certain conclusions have much force, partly because there are many related matters that have a direct bearing upon any given one. Suffice it to say here that there are strong reasons to believe that the regulation of the speeds of rings of electrons is accomplished in the manner indicated by

electromagnetic theory as pointed out by Schott. The ground is taken here that this loss of energy by radiation is excessively small, too small to be detected by any means as yet at our disposal, and that no observations connected with the quantum theory are in the least affected by its existence.

c—Velocity of the Ring.

d—Line spectra and Rydberg's Constant.

Let us next consider some of the consequences of the law of radial force which you have seen in the curve. It may now be stated that, in order to get this curve numerically, the velocity of the electron in its orbit had to be known for the different radii, and, in accordance with the above statement, the velocity in the orbit has been taken as approximately constant for all radii between the two roots of the equation. The velocity of an electron in any ring, according to these ideas, may always be taken equal to $p \frac{\pi^2 e^4}{h^2}$, where p is the number of electrons in the ring, e the charge of the electron, and h Planck's constant. The justification for the use of this value is that it leads to precisely the same equations for line-spectra of hydrogen and helium as Bohr gives, which agrees very closely with observed spectra. It may be remembered that the derivation of Rydberg's constant (the constant connected with light spectra) in terms of the charge on the electron and Planck's constant has been one of the most cogent reasons for adhering to the Bohr theory. The expression for Rydberg's constant is

$$K = \frac{2 \pi^2 m_o e^4}{h^3}$$

The force of the argument is that, when the values of the constants on the right determined by physicists in a number of different ways are substituted in this expression, we obtain a number 3.294×10^{15}. The Rydberg constant, obtained by entirely independent observations on light spectra is found to be 3.290×10^{15}. Unless this expression were based upon a truth the chances are that no such agreement as this would be found. I will not be able to devote the time here to a demonstration of the following statements, but will merely state that the modifications above discussed lead to precisely the same expression for Rydberg's constant as the above, including also the equations for the line spectra of hydrogen and helium. The new ideas are thus not a step backwards, but,

when it is stated that I get in a similar manner to the above an expression in terms of the properties of the electrons for the Newtonian constant of gravitation that agrees just as closely with the value of this constant as determined by the astronomers, it will be seen that this, when added to the Rydberg constant greatly strengthens our position, thus being a step forward.

e—Energy of Separation of Ring from Nucleus. We may easily calculate from the force-curve the energy that is required to separate the ring of electrons from the nucleus from one radius to another separating both electrons together. This energy curve is shown as curve II. A minimum point of the energy curve occurs where the force is zero. From this point outwards we have to supply the energy to pull the electrons away from the nucleus and from each other, but beyond the larger root (not seen on the chart) where the force changes sign to a repulsion, the system will do work upon the electrons. Suppose, for example, that we supply from some outside source enough energy to separate the electrons from their normal orbit at 2×10^{-10} to 1.85×10^{-8} cm. The electrons will arrive there with little or no velocity, all of this energy having been used up, but, after that, the repulsive force of the system begins to act, and it will accelerate the outward speed of the electrons until the fixed amount of energy which corresponds to the separation from 1.85×10^{-8} to an infinite distance has all been converted into kinetic energy.

VII. Electrons Ejected From Atoms With High Velocity

It is calculated, in the particular case of the ring of two and a nucleus of two, that the velocity thus acquired is nearly 1/20 of the velocity of light. With a ring of four and with a larger nucleus, the velocity will be greater than this.

In radioactive substances it has been experimentally observed for some years that electrons are projected out from them with velocities that approach very close indeed to that of light, 98 or 99 per cent of it. In the atoms in their normal condition it has never been supposed that the electrons have velocities anything like as great as that of light, and this theory has now given a valid reason why there may be a great increase in the velocity when electrons are ejected from atoms.

VIII. X-Ray Spectra

In the Bohr theory as has been stated above, there are supposed to be several steady states of the orbits which differ

according to the values of an integer, τ. In his theory the radii of these orbits increase with τ, as the squares of the integer, 1, 4, 9, 16, etc., and there is supposed to be a definite change in the kinetic energy of the electron from orbit to orbit. All of these results came about due to the neglect of the forces due to the motion of the electrons. When these are taken into the account, we have no change in kinetic energy from orbit to orbit, but the energy change is of the nature of potential energy, the work done in separating the electrons against the radial forces shown by the curve. It may be stated that we now get a very different series of radii from those of the Bohr theory. The radius of the orbit for all values of τ from one to infinity never departs very far from the minimum point of the energy curve. We can find these radii corresponding to any value of τ by drawing a horizontal line across at a height just above the minimum point of the curve equal to the known value of the total energy to separate the electron to an infinite distance from the nucleus. Such a line is drawn in the second slide, which shows this same minimum point of the energy curve to an enlarged scale. Lines are drawn for $\tau = 2, 3$ and ∞, $\tau = 1$ corresponds to the minimum point itself. Each line cuts the curve in two points, and each point of intersection corresponds to the proper radius of the orbit for that value of τ. Instead of a single value of the radius as in the Bohr theory there are two possible values.

I fully realize that it is not possible to make the reasoning on this matter appeal to you in the brief time at my disposal, but these results carry such important consequences that I could hardly avoid referring to them. They throw much light upon an experimental observation that has received no good explanation in terms of atomic theory. I refer to the charts of the x-ray spectra of the elements as observed by Moseley. These were shown here last year I believe. Moseley shows a group of four lines close together which he has named the α, β, ϕ and γ lines in the L series, and in the K series, the α and β lines. There has never been any clear understanding why there should be such a grouping of lines in the x-ray spectra. This theory gives a reason for it, and enables us to approximate to the positions of the lines. Bearing in mind that the orbital velocity is constant for varying radii, and that the velocity in a circular orbit is equal to $2\pi a n$, it is evident that the orbital frequency is inversely proportional to the radius in this theory. So also is the x-ray frequency inversely as the wave length, λ. If,

then, the x-ray frequency is connected with the orbital frequency in a proportional manner, which is very probable, then the x-ray wave lengths should be proportional to the radii of the orbits. A comparison between the observed wave-lengths for the element europium, which happens to be the heaviest element for which Moseley has shown all four lines in the L-series, and the radii as estimated for values of $\tau = 1, 2, 3$ and ∞ is shown in the next slide. The general agreement is too close to be purely accidental.

In support of this view of the origin of this group of lines is the fact that they all run along in a parallel fashion without any cross-overs or intersections. You can see by the way the spacings are derived from the energy curve that any intersections of the lines in Moseley's chart would be fatal to this view of the matter. There is another interesting matter that this explanation involves. The radii for $\tau = 1$ are slightly larger than for $\tau = \infty$, so that the radii decrease slightly instead of increase for an increase in τ, but the amount of the total change is quite small. That is, we use the values on the left of the minimum point of the energy curve instead of those on the right. In the Bohr theory the radii increase with τ, and as the squares of the integers, as 1, 4, 9, 16 etc.

IX. Ionizing Voltages

Without attempting here to give any derivation of it, I will merely state that the theory gives a formula of a very simple kind for the voltages that are required to ionize a gas and to start radiation. This is limited as yet to gases having atoms with single rings of electrons like hydrogen and helium. A considerable amount of experimental work has been done recently on hydrogen on account of the interest in it from the standpoint of the Bohr theory. The voltages required the tear electrons off from the nucleus and to drive them entirely away from the atom, thus leaving the atom charged and making what is called on ion of it, can be measured experimentally. The theoretical formula is

$$V = 3.3844 \times \frac{p^2}{\tau},$$

where p is the number of electrons in the ring and τ an integer. This subject alone is large enough to occupy the whole evening, if it were gone into in detail, so we must content ourselves with

saying that the theory gives all of the values approximately for hydrogen that have been observed, including a very recent value between 15 and 16 volts which the Bohr theory gives no account of. It also gives values for helium much nearer to the observed values than the Bohr theory. It seems, however, that helium has not been so exhaustively investigated experimentally as hydrogen in this respect.

X. Theory of Crystal Structure

Let us now pass to some results of a very different kind that have been derived from the general equation. I refer to the theory of crystal structure. By means of the x-rays you are aware that it has been possible to find the exact location of the centers of the atoms of which a crystal is composed. This method was discovered by Prof. Laue, and first published in a paper by Laue, Friederick, and Knipping. I shall confine these remarks to a few of the simplest forms of crystals belonging to the cubic or isometric system. In all of these the general arrangement of the atoms is in equilateral triangles in a plane. The whole crystal is built up by many separate planes of atoms all similarly arranged in each plane. I have brought here a number of models to help give you an idea of the space lattice arrangement in four of the important classes of crystals belonging to the cubic system. Without models it is not very easy to make much progress in understanding crystal structure. In each one of them you will observe that the fundamental arrangement is the equilateral triangle. The several planes of such triangles are related in several possible ways, which makes the characteristic differences in the resultant space lattice.

a—Directions of Axes of Atoms in a Crystal. According to our ideas of atoms each has an individual axis of revolution, being determined by the common plane of the orbits of the electrons revolving around the nucleus. If we fix the attention upon just two atoms whose axes point in different directions, the equations of electromagnetic force show that there is a tendency of the one atom to turn the plane of that of the other until they become parallel to each other before complete equilibrium is established. Now the angle between the directions of the axes of the two orbits is one of the quantities that is contained in the general equation, and unless we know the directions of the axes, we cannot calculate the force by means of that equation.

Fortunately, it has been possible to find the directions of all

the axes of the atoms because of the symmetry of the crystals, coupled with the knowledge that each atom in the whole structure tends to turn the direction of the axis of rotation of every other atom. Any one atom feels the turning effect of all the others, and it is only the resultant of them all that determines the position of any given axis. Now, without knowing how much the tendency to turn the axis is, it is evident that if three atoms, which are just alike, are placed at equal distances from the given atom having their axes properly directed, they may annul one another and have no turning effect upon the given atom. Without going into the history of all the troubles that presented themselves before a solution of this question was found, I have a model which shows you an arrangement of the axes of the atoms in one of the equilateral triangular planes, and can prove to your satisfaction that this particular arrangement makes the total turning moment hold each axis in the crystal in just the position indicated by this model. In this model there are only four directions of the axes. One quarter of them all is parallel to one given direction, a second quarter to another, and so for the whole. These four directions are the directions of the medial lines of a regular tetrahedron, or, which is the same thing, the four diagonals of a cube. I have placed colored strings in the models, one string passing through each atom to indicate the direction of its axis in exact accord with the plane model. You will find upon studying the models that each of the parallel planes has atoms with their axes arranged exactly like the plane model. Taking all these planes together you see that every atom has the direction of its axis fixed. But, again, you will observe that each model can be turned in any one of four different positions so that a different set of planes each time makes up the whole crystal. And no matter in which of these four ways you turn the model the relative directions of the axes is precisely the same as in the plane model. In doing this we have really counted each atom four times, and every time the direction of its axis is right. Had some different arrangement of the axes been adopted for the plane model it would not have turned out that you could turn these models in any one of four directions and then found that the individual planes were the same as in the original model. In other words the possible number of arrangements of the axes in any plane is very limited, and it is my belief that this is the only possible arrangement that will satisfy the condition that the turning moments shall be zero.

b—Order of Magnitude of the Radii of the Orbits. Having found the directions of the axes, but not before, it is possible to apply the general equation to determine the total force on an atom due to all the rest in the crystal. When the expression for the force thus obtained is equated to zero for the stable equilibrium condition, the only two unknown quantities that remain are properties of the atoms themselves, viz., the sum of the squares of the radii of the orbits of the electrons in the atom in question on the one hand, and the sum of the products of the radius by the velocity for each.

In other words, starting from the known dimensions of the space-lattice forms in certain crystals, and knowing the kind of atoms there are at each point of the lattice, we may, by an application of electromagnetic theory, arrive at some knowledge of the sizes of the orbits of the electrons themselves within the atoms.

Without giving you the process in detail, the results may be briefly stated. The sum of the squares of the radii found in this way show a gradual increase with heavier and heavier atoms according to a definite curve, but the order of magnitude of the radii lies between 10^{-10} and 10^{-9} cm. This agrees well with the value found in an entirely different way for hydrogen.

c—Some Kinds of Atoms Show Two Possible Values of Their Radii. This theory has been applied to twenty different crystals, minerals, containing in all as many separate elements in different combinations. The same element, say chlorine or sulphur, occurs in several of these crystals. It may be regarded as a confirmation of the theory that the same values are obtained for the same kind of atom no matter in which crystal it occurs. This brings out a point of very great interest, because the above statement is not universally true. For example, the element sulphur occurs in four of the crystals, zincblende, $Zn\,S$, iron pyrites, $Fe\,S_2$, manganblende, $Mn\,S$, and galena, $Pb\,S$. We get the same value for the radii for the sulphur atom from three of these crystals, but a different value in galena, $Pb\,S$. The numerical relation between the sums of the squares of the radii in the two cases is $2^{4/3}$, the sulphur in galena being of smaller radius than the others. In a similar manner several other crystals give different values for the same element, and they always differ by precisely this same factor $2^{4/3}$. The result is very suggestive as indicating that the same atom may have at least two different states, differing in the size of the radii of the

orbits, but without affecting the weight. It is possible to see by looking back at the equation for the radial force for the single ring of two electrons, where there were two roots at each of which the radial force is zero, that the more complex equation for atoms with several concentric rings might give more than two roots, and of these there might be more than one stable position for the radii. This is a thing that we might expect from our knowledge of atoms in chemistry. There are different kinds of sulphur atoms, having different valencies, and so with some other atoms. It is most important to observe though that the weights of these atoms are the same. Electromagnetic theory throws much light upon this matter, as we shall presently show.

d—Rigidity and Bulk—Modulus. We will conclude the remarks on crystals with the statement that the forces which have been obtained give a very good reason for the rigidity of crystals, and the resistance they offer both to change of shape and change of volume. The estimated value of the bulk modulus is of the same order of magnitude as the experimental values.

XI. Atoms at a Great Distance

There is another matter to which the general equation has been applied, which I have left until the last, although it would have been most natural to place it first, as being perhaps the simplest application of the equation. The question is what effect will two atoms have upon each other according to the equation when the distance between them is very large, say one centimeter or more. It may be shown that all of the electrostatic forces between two neutral atoms at a great distance exactly cancel out until we come to terms involving the inverse sixth power of the distance. Each atom has in effect a zero charge at these great distances, and the only part of the forces that we have to calculate arises from the *motion* of the electrons in their orbits. In the final analysis, therefore, to get the whole mechanical force between the atoms, we have to find the average mechanical force that one single electron in the second atom exerts upon another in the first atom. The total force is evidently merely the sum of a number of such expressions, the number depending upon how many different pairs of electrons there happen to be in the two atoms considered. If it happens, as it does, that the force due to any one pair of electrons gives us a quantity that is *not* cancelled out by that due to another pair, we obviously get a

real resultant force between the two atoms. We know that such a force exists in fact, the gravitational force, and we should be interested to compare the force given by the equations with the actual known force.

a—Problem of Two Electrons. Average Force. In order to solve this problem for two electrons only it was necessary to obtain the average force between the two electrons taken over a very long time, a large number of revolutions. The only way as yet found to do this is to develop the equation into an infinite series of powers of the distance between the centers of the two atoms, and then average the separate terms. The result is that the series contains all powers of the inverse distance between centers, beginning with the first power, and so on. Moreover, the force is not in the direction of the line joining the centers of the two atoms. This is easily understood when it is remembered that the force between the two electrons in the original equation is not in this direction.

b—Force Resolved Along the Line Joining the Centers of the Orbits. We are interested, however, in the component of the total force that is in this direction, and hence the total force has been resolved in the direction of the centers. The part at right angles to this does not interest us because it will all be cancelled out by the action of the electrons in other atoms turned in every possible direction.

Now it results in the process of resolving the force along the center line, that the first term of the series, the inverse first power term, cancels out leaving it to begin with the inverse square, and higher powers. Moreover, because the distance between the atoms is assumed to be great, all of the inverse higher powers are evidently negligible, so that the total force varies inversely as the square of the distance between the atoms. The resulting force-equation is of sufficient interest to give in full.

$$F = \frac{1}{2} e_1 e_2 \beta_2^2 \{1 - (- X \sin \alpha + Z \cos \alpha)^2\} r^{-2}. \qquad (1)$$

Here e is the charge on the electron; β_2 the ratio of the velocity of the second electron, the force of which we are getting, to that of light; X and Z are direction cosines determined by the position of the center of the second orbit; α the angle between the directions of the two axes of rotation, and r the distance between the centers of the orbits.

c—Simplification of Result in Case of Two Masses of Matter. Let us apply this equation to two electrons, one in each of two

atoms of any gas, liquid or solid, crystals excepted. At any given moment the axes of the two orbits may be turned in any possible directions relative to each other, so that the theory of probabilities must be applied, because it is the force on the average that we require. When this is done, it is shown that the quantity within the brace becomes 2/3, and hence the average force on the first electron due to the second becomes the very simple expression

$$F = \frac{1}{3} e_1 e_2 \beta_2^2 r^{-2}. \tag{2}$$

This expression is in the form of the law of gravitation in two respects. First, it always gives an attraction, and second, it varies inversely as the square of the distance. Before inquiring into the magnitude of the force, let us apply the equation to the only kind of matter not included above, namely crystals.

d—Attraction Independent of the Orientation of the Axes of a Crystal. We have seen that there are but four different directions for all the axes of the atoms in a crystals so far as the theory of crystals has been developed. Would we obtain the same result as before using these four directions only? If not, the theoretical attraction between two crystals would show some differences at the same distance when they are turned about in different directions. The experimental evidence is that the gravitational attraction shows no difference in any direction, and, consequently, if the force is to be identified with that of gravitation, it is necessary that the terms involving the directions of the axes in this parenthesis should give the same result as above even in the case of a crystal. It has been possible to show that they do, and this circumstance is due to the fact that the axes in the crystal are all parallel to the four medial lines of a regular tetrahedron.

e—A New Geometrical Theorem Obtained from Physical Considerations. There is an interesting incident in this connection. The equation showed that the following geometrical proposition must be true if the force is to be independent of the orientation of the crystal. "If through any point in space four lines be drawn making equal angles each with any other (that is parallel with the four medial lines of a regular tetrahedron) and, if from a second point in space, at a distance r from the first point, the four perpendiculars be drawn, one to each of the said four lines, then the sum of the squares of these perpendiculars is constant for all points at the same distance

from the first point." The quantity within the brace of equation (1) is equal to the square of one of these said perpendicular lines.

The truth or falsity of this proposition was unknown at the time of arriving at the conclusion that it must be true if this is the proper form for the gravitational force. It has since been proved to be true,* and this has added a new geometrical theorem to the list, as it was unknown to the mathematicians to whom it has been submitted. This incident of a mathematical theorem deduced in this physical way helps to show that we may be on the right track in understanding the cause of the force of gravitation.

We see, therefore, that the force tallies in every way with the gravitational force except in the matter of magnitude, which will next be considered.

f—Magnitude of the Force. As you have seen above in considering hydrogen, we have a knowledge of the probable speed of the electrons, and there is nothing else in doubt that we need for calculating the force, since the charge on the electron is known. Using these known speeds, the force calculated from this equation comes out about 10^{21} times greater than the actual value of the gravitational force. This is an enormous error and cannot be accounted for by assuming any mistake in the velocity of the electrons used. The result is very significant at the same time because the calculation indicates a force 10^{21} times greater, *not less*, than the actual existing attraction. It is a more fortunate result than it would have been had the calculated force come out the smaller of the two because then gravitation might easily be attributed to something else, but, as it stands, we are forced to question the truth of the hypotheses that have entered into the case. What are they? They are really but two. First, that the atoms are structures consisting in part of electrons revolving in circular orbits, or approximately circular orbits as you please. Second, that the modern Lorentz form of electromagnetic equations applies to the case.

We are not yet ready on the strength of any electromagnetic theory to abandon the idea that electrons describe approximately circular orbits within the atoms. It is easier to believe that some modification in the present form of electromagnetic theory is required. Let us look at the expression for the average force upon one electron in a circular orbit due to another a little more attentively. We have already seen that the expression gives a good

*The proof is due to Prof. F. W. Owens of Cornell University.

account of itself in many ways. It represents always an attraction, and not a repulsion, and obeys the inverse square law. It also shows that, when such forces are summed up for two crystals, the result is independent of the orientation of the crystals. In fact, it has the right look in so many ways that I have been led by natural steps to suppose that some very important factor has been omitted somehow from the Lorentz theory.

g—Equation of Force Unsymmetrical. You see by the expression (**2**) that the speed β_1 of the electron, upon which we are getting the force is not present, indicating that the force does not depend upon the velocity of the first electron. If we should write down the force of the first electron upon the second we would merely put β_1^2 in place of β_2^2. Therefore, unless both these velocities were the same, the force of the second on the first would not be the same as the first on the second. This has led me to suspect that the square of the velocity of the first electron should also appear in the force-formula. The order of magnitude of this is 10^{-4}, so that this alone could not possibly account for the discrepancy of 10^{-31} above mentioned. It reduces the discrepancy, however, to 10^{-27}. Now the mass of the electron itself at slow velocities is $.90 \times 10^{-27}$ grams. If the mass of the electron is also included, equation (**1**) takes the very symmetrical form

$$F = \frac{m_o}{2K} e_1 e_2 \beta_1^2 \beta_2^2 [1 - (- X \sin \alpha + Z \cos \alpha)^2] r^{-2}, \quad (3)$$

and the average force on one single electron due to another is

$$F = \frac{m_o}{3K} e_1 e_2 \beta_1^2 \beta_2^2 r^{-2}, \quad (4)$$

where m_o is the mass of the electron at slow velocities, and K, the specific inductive capacity of the ether, equal to unity, has been introduced for the sake of keeping the dimensions of the equation right.

h—When a Factor is Introduced to Make the Equation Symmetrical, the Force-Equation Agrees With the Law of Gravitation. By means of this equation we may easily show that the magnitude of the attraction agrees to a surprising degree of accuracy with the attraction obtained from the law of gravitation. It may be applied to any two masses of matter equally well, but I will illustrate it by calculating the attraction between two average atoms of hydrogen, because the atomic theory gives the velocity of the electrons in the atom. When the formula is to be applied

to two masses of matter containing a large number of electrons, it is only necessary to replace the β_1^2 by the $\Sigma_1 \beta^2$, and the β_2^2 by $\Sigma_2 \beta^2$, the summations being taken of the velocities squared of all the electrons in each of the two bodies in question. The formula for the total attraction between the two masses then reads

$$F = \frac{m_o}{3K} e_1 e_2 \Sigma_1 \beta^2 \Sigma_2 \beta^2 r^{-2}. \tag{5}$$

We have seen above that the value of β^2 for any single ring of electrons is $p \frac{\pi^2 e^4}{c^2 h^2}$, and the sum of β^2 for all the electrons in the ring is p times this. If we are writing the attraction between two atoms of hydrogen, the two summations in (5) are the same, and we get by substitution of the above value

$$F = \frac{m_o}{3K} p_H^4 \frac{\pi^4 e^{10}}{c^4 h^4} r^{-2}, \tag{6}$$

where the p_H denotes the number of electrons in the hydrogen atom.

If this force is to be identified with the gravitational force, it should be numerically equal to the attraction as given by Newton's law, which is usually written

$$F = k\, m\, m'/r^2. \tag{7}$$

Putting m_H in place of m and of m', as denoting the mass of the hydrogen atom, this becomes

$$F = k\, m_H^2\, r^{-2}. \tag{8}$$

Equating (6) and (8), as being different expressions for the same force, the r^{-2} cancels out, and, dividing through by m_H^2, we find a value for k

$$k = \frac{p^4}{3} \frac{m_o}{m_H^2} \frac{\pi^4 e^{10}}{c^4 h^4}. \tag{9}$$

The K is omitted because it is unity and does not affect the numerical value, and also because it is not at all certain that it should not also appear in the Newtonian equation (7). The value of k as determined by the astronomers is 666.07×10^{-10}. In the right member occur constants that have been independently determined by physicists. The following values are those in current use,

$m_o = .90 \times 10^{-27}$ grams (From Bucherer's value of e/cm_o and Millikan's value of e)

$m_H = 1.662 \times 10^{-24}$ grams. Millikan's value.

$h = 6.547 \times 10^{-27}$ Planck's constant. Millikan's value.

$c = 2.9987 \times 10^{-10}$ cm per sec. = velocity of light.

If with these values we use for e, 4.750×10^{-10}, we get the value of k exactly as determined by the astronomers above given by putting $p = 2$. This value of e is just *one half of one per cent* smaller than Millikan's value of e which is 4.774×10^{-10}. Values of e determined by other observers and by other methods vary from perhaps 4.65 to 4.81, and it is quite possible that 4.750 is a very accurate value of e. In fact, if it is admitted that equation (9) is a true relation, it will be a very accurate way to determine e itself, because the value of k is accurately known, and we would get a value of e^{10} with a fair degree of accuracy, and hence would have e with precision.

The fact that the number of electrons in the ring for the hydrogen atom, p_H, must be equal to 2 to satisfy the expression for k numerically is significant as indicating again that hydrogen has two electrons per atom.

The argument for this atomic theory, on account of the numerical agreement with the Newtonian constant, is of the same nature as the argument for the Bohr theory above mentioned, on account of the numerical agreement with the Rydberg constant; but it is doubly cogent, because both these constants are adduced from this theory.

i—Certain Features of Electromagnetic Theory Brought Into Question. It is for these good reasons that I have ventured to question the correctness of certain fundamental assumptions in the current form of electromagnetic theory. We are forced to question it or to abandon altogether the idea that the electrons describe circular orbits within the atoms. Of the two alternatives, I venture to think that the proper one has been chosen. As above stated, as we look back over the history of the development of electromagnetic theory from Maxwell's beginning to the present time, a process of gradual evolution has been at work. I had formerly supposed that the commonly accepted electromagnetic theory is complete as we have it, or finished, so to speak, but I have been led to believe that it is yet in its infancy. To avoid being misunderstood some further remarks on this subject seem to be required. The changes in the theory that seem to be demanded are not very great, and will probably not affect most of the applications of the theory. They relate more particularly to the inverse square of the distance terms, which only come into the account in such problems as we have been considering. It may seem impossible to you that any change can be made in the theory, because the equations already express

with complete satisfaction all the phenomena in the subjects of electricity and magnetism dealing with gross matter, with which we are acquainted. It was pointed out before that all these things represent very special cases, in fact, the older form of equations of Maxwell's day are just as good for these things. The applications to gross matter are not affected by the modern change in the introduction of the ideas of retarded potentials.

j—Direct Experimental Verification of Electromagnetic Equations Desirable, But Probably Impossible. We really need an experimental verification of the general electromagnetic equations. Prof. H. A. Rowland charged the circumference of a wheel and set it revolving at a rapid rate. He succeeded at best in detecting the magnetic force at the center, and showed that the magnitude agreed with the theory. What we really want, however, in order to test the theory is to have the charge at one point only of the wheel, and to measure the instantaneous, not the average, mechanical force upon a second electrical charge in its neighborhood both as to direction and magnitude. Needless to say this has never been accomplished and we may almost say never will be accomplished. We should need a recording apparatus that had no time lag, and that is capable of indicating a force in any direction equally well, and of a very insignificant amount. The conclusion is that we are compelled to resort to indirect methods of testing these equations. It seems that we may legitimately regard the process above described as one of these indirect methods. In such methods, if one assumption fails, we ought not to hesitate to try another, for the original theory itself rests upon assumptions that seem to us correct, but which can only be tested by the indirect method.

XII. The Results Obtained Above (XI) Agree With the Form of Atomic Theory Demanded by Electromagnetic Theory

The gravitational formula above is so simple, and has so many points in its favor that I have proceeded as though it were true, and have endeavored to find some of the consequences that it implies for atomic theory. We shall next consider some of these and you will note that they are in complete harmony with the conceptions of atomic properties that we have been considering. First, let us take the earth as one of the bodies, and a single atom on the earth's surface as the other body, and

write down the force of attraction according to the equation. We get the very simple expression

$$F = k \Sigma_A \beta^2. \tag{7}$$

where the constant,* k, involves the constant charge on the electron, the mass of it, and the summation of the velocities squared for the electrons in the earth, all of which does not vary when we substitute one atom for another.

a—The Weight of An Atom is Proportional to the Total Kinetic Energy of the Electrons it Contains, Relative to the Center of the Atom. Now this attraction is merely the atomic weight, and we, therefore, have the suggestion that the atomic weight is merely proportional to the sum of the squares of the velocities of all the electrons in the atom, that is, to the total kinetic energy of the electrons, and it should be added that the velocities must be those relative to the center of the atom. If the whole atom moves, this would add nothing to the weight because the effect of the positive nucleus would exactly annul that of the electrons. But, we know that atoms of the same kind, say hydrogen, have the same weight in a given locality. We may, therefore, conclude that the velocities of the electrons in the atom must always be the same in the different states of these atoms. This is in exact conformity with the theory of the atom above described. The velocity in any ring, according to electromagnetic theory, is a constant quantity dependent upon the number of electrons in the ring. So long as we leave the rings in an atom undisturbed they must always have the same velocity, but, when we change the number in the ring the velocity must suddenly change to a new value, and then the weight will change. We then really have a different atom. It has suddenly changed over to one of a different class.

The atomic theory above outlined also shows that any ring may change its radius without affecting the velocity of the electrons, the kinetic energy remains fixed independent of changes in the radius. The Bohr theory, on the contrary, demands a change in the kinetic energy with any change in the radius. On the one hand we keep a constant weight, and in the Bohr theory the weight must change, if the weight has anything to do with the kinetic energy. But it is a matter of measurement that the weight does not change. I may refer again to those crystals in which the same kind of atoms, sulphur for example, are re-

*This is not the same constant as the Newtonian constant above.

quired to have different radii in different circumstances, although the weight remains the same. This is in accord with the modified theory.

b—The Weight of any Ring of Electrons in an Atom is Fixed Independent of its Radius, and Always Contributes the Same Amount Toward the Total Weight of the Atom. According to these ideas, therefore, a ring of two, three, etc., electrons in any atom must always contribute the same amount to the weight of the atom whether its radius be large or small. Having received this suggestion from the theory, I have endeavored to ascertain whether it might not be possible to find out, from the known atomic weights of the different kinds of atoms, what the probable combinations of rings in such atoms are. The result of this has been more successful than I had reason to hope, knowing the very great irregularities that atomic weights exhibit. In the light of the theory it is possible to write down the approximate weights of any ring of electrons. I say approximate because it is not supposed that the speed is absolutely independent of the radius of the ring, but only so to a first approximation. Taking the speeds as strictly proportional to the number of electrons in the ring, the theory gives the weights of the rings as the squares of the number of electrons per ring, as the numbers, 1, 4, 9, 16, 25, for rings of 1, 2, 3, 4 and 5. Taking hydrogen as a ring of two and atomic weight 1.008, we get the numbers 2.268, 4.032, 6.300, and 9.072 as the weights of rings of 3, 4, 5 and 6 electrons. These are the strictly proportional figures, but I have found by trial that, if the following figures, differing very slightly from the above, are used, namely for a ring of two 1.008, three 2.269, four 3.99975, five 6.2898, and six 9.137, it is possible to get the atomic weights of all the elements from hydrogen to uranium within the probable error of experimental measurement in a large majority of cases. For most of the elements only the first three figures have to be used, corresponding to rings of 2, 3 and 4 electrons. Only a few elements require rings of five, namely the halogens, *Cl*, *Br*, and *I*, and the elements selenium and tellurium. The element iron is the only one based upon rings of six, with the possible exception of glucinum. In other words most of the atomic weights have been accurately obtained by the use of but three numbers, rings of 2, 3 and 4. Of the 70 elements calculated (the rare earths having been omitted) 52 of them fall very near to or within the experimental error of the chemist in measuring the weight, while 18 fall outside

of this accuracy. Of these 18, however, the weights of 15 of them come nearer than one part in 1000 to the measured weight, and most of them considerably nearer. Only three of these exceed a difference of one part in 1000. These are *L i*, 7.3 parts in 1000, *K* 1.2 parts, and *Y t* 2.26 parts. The largest difference by considerable margin is in the case of *L i*, which, however, is less than 1 per cent.

The system of rings as built up in this way seems to be based upon rings of four electrons, the number of rings of four increasing steadily with increasing atomic weight, the number of rings of two and of three being few. The cases of iron and the halogens are most interesting, as being built upon a different foundation. Rings of four will not fit these elements.

I would not give you an account of this matter here, were it not for the fact that the weights come out in this way remarkably close when based upon three numbers for the most part, and that these numbers agree so closely with the numbers arrived at by the theory above outlined.

XIII. The Number of Electrons per Atom According to the Above Satisfies Both Those Who Have Thought That the Number is Approximately Equal to the Atomic Number, and Those Who Have Taken it as the Same as the Atomic Weight

There are a few important matters which have been passed over without notice, that may now be mentioned. It has been an object among physicists for some time to discover just how many electrons the different kinds of atoms contain. This table of the elements made in the way just mentioned gives the number of electrons for each atom. Many have come to the conclusion that the number of electrons must be approximately equal to the atomic number, which is roughly one half the atomic weight. Others on different grounds have made the number approximately equal to the atomic weight. This table makes the number approximately the same as the atomic weight, but it will be observed that hydrogen has two and not a single electron. So the table satisfies both views of the matter, while it makes the number nearly the same as the atomic weight, the ratio of the number in an atom to the number in hydrogen is one half the atomic weight.

XIV. The Number of Electrons in the Hydrogen Atom

You have a right to be surprised that any question can be raised at this time as to the number of electrons in hydrogen. I can find, however, no experimental observation to prove there is but a single electron, and it seems that this view of the matter has been adopted along with the Bohr theory, which requires the single electron.

A few remarks upon this matter seem called for. If the velocities of electrons in rings are regulated as above described through the action of the tangential forces due to the other electrons in the ring upon the one according to electromagnetic theory, then we are deprived of this means of regulation when there is one isolated electron only. We do not have in such a system the perfectly symmetrical dynamic balance that obtains with two electrons.

Then, again, our present view of the matter does not neglect the radiation and loss of energy from the atoms. In electromagnetic theory the relative amounts of loss of energy through radiation for rings of 1, 2, 3, etc., using the speeds as above determined, diminishes very rapidly as the number in the ring increases. The loss from a ring of two is about 4000 times less than from the single electron, and from the ring of three about 40 million times less, and from four nearly a million million times less. For this reason it seems reasonable to suppose that the single isolated electron does not exist in the atoms. I shall not take the time here to answer the obvious objections to admitting loss of energy by radiation on account of the quantum theory, but may assure you that they can be met satisfactorily.

XI. The Order of Magnitude of the Orbits of the Electrons in Atoms

Another matter, which has been passed over relates to the order of magnitude of the orbits of the electrons. There are a number of things that have a very evident bearing upon this question. One of the things we know now with precision is the distance between the centers of the atoms in certain crystals. These distances in different crystals vary from perhaps 10^{-8} cm. up to 4 or 5 or possibly 6 times as much, and we may think of the order of magnitude as roughly 10^{-8} cm. Now, if the radii or the orbits of electrons in any of the atoms in the crystal approach close to the same order of magnitude, the electrons must necessarily approach each other much nearer than they

are from the nucleus to which they belong. Unless we are to assume that such electrons have no mutual influence upon each other, which seems uncalled for, there must occur in crystals an enormous perturbation with an ensuing radiation of energy. This could not possibly be a stable condition. Our exact knowledge of crystals alone ought to be considered a direct proof that the order of magnitude of the orbits of the electrons is much smaller than 10^{-8} cm.

Again, the kinetic theory of gases has been cited to show the order of magnitude of the radii. This theory has established the fact that the centers of the atoms in a gas approach each other on the average to within about 10^{-8} cm. before being deflected. If the orbits of the electrons were of this order too, they must at times come into very close proximity or even collision, and the result would be that some of the electrons would be separated from the nucleus, thus ionizing the gas under ordinary conditions. This does not occur. We may, I think, regard the kinetic theory as another direct proof that the orbits must be of a much smaller order. These matters, when taken in connection with the fact that the radii come out between 10^{-10} and 10^{-9} cm., when the forces due to the motion of the electrons is included, which was omitted from the Bohr theory, and with the fact that these radii agree with the values indicated by an application of the theory to crystals, do not leave much doubt as to the true sizes of the orbits of the electrons.

I can picture the thought that is probably occurring to many of you, while listening to this description of a difference of opinion affecting the radii of the orbits between ten and a hundred times. In what condition would astronomers find themselves so long as they were in doubt about the value of the radius of the earth's orbit by a factor of some ten or hundred times? Evidently, until this matter is determined with some precision, all attempts to place atomic theory upon a firm mechanical basis cannot be very successful. It will have to be acknowledged, however, that the physicist does not find any such single simple law applying to atoms at close range as governs the motions of the heavenly bodies, and the reasons why we are not further along are not difficult to understand.

Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, Feb. 16, 1918.

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THE POLYPHASE SHUNT MOTOR

BY W. C. KORTHALS ALTES

ABSTRACT OF PAPER

There exists a demand for a reliable, adjustable-speed, alternating-current motor, suitable for operation at a large number of speeds. The neutralized motor with shunt field control is analyzed and it is shown that it is not practical for commercial frequencies on account of the expensive control equipment required. The induction motor with commutator on the secondary side is discussed. It may find some application for the larger outputs; the control is, however, still too complicated to make this type of motor suitable for the smaller machine-tool drives. The induction motor with commutator on the primary side offers the best solution for machine-tool motors. Its theory is discussed in detail, and a complete description is given of the mechanism required to shift the brushes and the new type of armature winding used.

INTRODUCTION

AS FAR as generation, transmission, distribution and constant-speed motor drives are concerned, the alternating-current system is far superior to the direct-current system.

The development of modern tungsten lamps, has practically done away with arc lamps, so that both systems are equally suitable for lighting.

The development of speed-regulating sets has made it possible to regulate efficiently the speed of induction motors up to several thousand horse power.

The varying speed alternating-current brush-shifting motors, which have recently been put on the market, can be applied to variable speed pumps, blowers, exhausters and certain textile machines.

The single-phase crane motor opens up possibilities in regard to dynamic braking, simplicity of control and operation over a wide range of speed, which cannot be met by the induction motor with resistance control.

The adjustable-speed alternating-current motor makes it possible in plants requiring adjustable-speed motors for machine

tool, elevator service, etc., to use the alternating-current system throughout if desired, but due to the fact that the a-c. adjustable-speed motor is more costly than the d-c. adjustable-speed motor, t will be more economical to install machinery for changing over the alternating current to direct current, as long as a large number of motors is involved. However, the general tendency to standardization and centralization increases the number of processes which are carried on at constant speed and call for the simple squirrel-cage induction motor and the use of the a-c. adjustable-speed motor, in cases where only a limited number is required, will make it possible to extend the application of the squirrel-cage motor.

It is not intended to deal in this paper with all the different types of a-c. adjustable-speed motors, which have been proposed by the various engineers that have worked on the problem. The paper will be limited to the types that now seem most important. It is of value to the specialist to know the large number of possible combinations, so that he can use them when the constantly varying conditions make this possible, but engineers in general need interest themselves only in those schemes which are at present suitable for practical application.

Neither is it necessary to discuss the multi-speed induction motor, the speed of which is adjusted by connecting windings wound with different numbers of poles to the line. The field of application and the theory of this motor is well-known. As long as only a few definite speeds are required, it is very satisfactory. However, there are a number of cases for which either a large number of speeds is required, or the desired speeds cannot be obtained with the possible numbers of poles. In those cases, one of the following types of alternating-current adjustable-speed commutator motors can be used. (1) The neutralized polyphase commutator conduction motor, (2) the induction motor with commutator on the secondary side, (3) the induction motor with commutator on the primary side.

The Neutralized Polyphase Commutator Conduction Motor

This motor consists of a d-c. armature winding, on the commutator of which is arranged a polyphase system of brushes, connected to a neutralizing winding which neutralizes the magnetomotive force of the armature winding, and is connected to the line. The field winding is connected to the secondary of

a transformer, the primary of which is connected to the line. It can be arranged in the same slots as the neutralizing winding, in which case the leakage flux will induce in the field winding a voltage proportional to and in phase with the leakage reactance voltage in the neutralizing winding. The motor field flux in this case lags ninety degrees behind the vector difference of the line voltage and the reactance drop induced in the field winding by the primary leakage flux which is yielded by the ampere-turns of both the field and neutralizing winding. The characteristics of the motor can be determined by considering that the neutralizing winding has no reactance drop, but that a reactance equal to the reactance of the neutralizing winding has been connected between the line and the motor terminals to which both the neutralizing winding and the field winding are connected. The field winding excites a rotating field, so that at synchronous speed the armature voltage and the voltage in the armature coils short-circuited by the brushes is zero, while this voltage increases when running above or below synchronous speed.

The field winding can also be arranged in other slots than those of the neutralizing winding. The field flux in this case lags ninety degrees behind the applied line voltage and the reactance drop of the neutralizing winding should be added to the reactance drop of the armature winding, which means that the difference between the no-load and full-load speed will be greater than before. The neutralized motor with the field winding in the same slots as the neutralizing winding is to the one with the field winding in other slots, as the direct-current shunt motor with resistance in series with the entire motor is to the direct-current shunt motor with resistance in the armature winding. This can be seen from the vector diagrams.

Fig. 1 gives the vector diagram for the neutralized motor with the field winding in the same slots, as the neutralizing winding. $O\,M$ represents the current which flows through both the neutralizing and the armature winding, $N\,O$ the current in the field winding. Assuming that the field winding and neutralizing winding have the same number of turns w_1, then the primary leakage flux will be yielded by $N\,M \times w_1$ ampere-turns and the reactance drop $A\,B$ induced by the leakage flux in both the neutralizing and field winding will lag ninety degrees behind $N\,M$. The resistance drop of the field winding, $F\,B$ is opposite to the field current $N\,O$. $F\,O$ is the voltage induced in the field

winding by the alternation of the field flux. $A\,B$, $B\,F$ and $F\,O$ balance the applied line voltage $O\,A$. For the circuit consisting of the neutralizing and armature winding in series, we find that the applied line voltage $O\,A$ is balanced by the leakage reactance drop $A\,B$ of the neutralizing winding, the resistance drop $B\,E$ of the neutralizing winding which is opposite to the current $O\,M$, the reactance drop $E\,D$ of the armature winding lagging 90 deg. behind $O\,M$, the resistance and brush-drop of the armature winding $D\,C$ opposite to the current $O\,M$ and the counter e.m.f. $C\,O$, which is induced by the field flux yielded by the field winding in space quadrature to the phase under consideration, and is proportional to the speed, the number of armature turns and the field flux. Due to the pulsation of the field flux, a voltage $O\,F$ will be induced in both the neutralizing and the

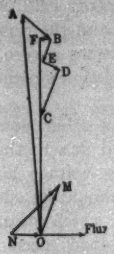

FIG. 1

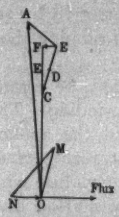

FIG. 2

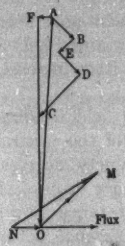

FIG. 3

armature winding. If the neutralizing and armature winding have the same number of turns, then the voltage in the armature winding will be neutralized by the voltage in the neutralizing winding and the resultant voltage appearing at the terminals will be zero. This voltage appears, however, when measuring the drop across the brushes and across the terminals of the neutralizing winding individually. The core loss has been neglected in the diagram. It causes the flux to lag behind the ampere turns. The output is proportional to $O\,C \times O\,M \times \cos\angle C\,O\,M$. Fig. 1 has been drawn for operation below synchronous speed.

Fig. 2 gives the same diagram when the motor is running at synchronous speed. It has been drawn under the assumption that the leakage reactance of the armature is proportional to the slip. In that case, the rotor reactance is zero at synchronous

speed and the points D and E come together. The voltage applied to the field winding is equal to the vector connecting A with a point H on $O\,F$, so that $O\,H = F\,C$. This vector is not shown in Fig. 2.

Fig. 3 covers the diagram for a motor in which the field winding is located in other slots than those of the neutralizing winding. $A\,B$ lags now 90 deg., behind OM instead of $N\,M$ and the leakage flux yielded by the neutralizing winding does not induce any voltage in the field winding.

Fig. 4 is similar to Fig. 3, except that the motor is running above synchronous speed in which case the rotor reactance appears negative. The voltage to be applied to the field winding has not been shown, but can be found by connecting A with a point H on $F\,O$, so that $F\,H = \frac{n_0}{n}\,O\,C$, if n is the speed of the motor and n_0 the synchronous speed.

In case we take the load off the motors covered by Fig. 1, and Fig. 3 the speed will change as $O\,C$ to $O\,F$. Due to the location of the drop $A\,B$, the variation in speed will be larger in the latter, than in the former case. Hence, as far as the variation between the no-load and the full-load speed is concerned, it is better to arrange the field winding in the same slots as the neutralizing winding. However, by locating the field winding in other slots than those of the neutralizing winding, we can build the field winding with definite neutrals like a direct-current motor, so that in the coils short-circuited by the brushes, no voltage is induced by rotation through the main field flux. Moreover, we can locate at these neutral points, windings that excite the "Commutating Flux" inducing in the armature coils short-circuited by the brushes, a voltage which both neutralizes the voltage induced by the alternation of the field flux and furnishes the e. m. f. required to reverse the current in the coils passing the brushes. Furthermore, it is possible to improve the speed-torque characteristics by using a series transformer, the secondary of which is connected to the field circuit and shifts the time phase of the flux depending on the load current drawn from the line. The characteristics of this motor are particularly favorable when running far above synchronous speed. The diagram of Fig. 4 shows how the reactance of the neutralizing winding can be completely compensated by a negative reactance drop of the armature winding, but when predeter-

mining the characteristics, it must be borne in mind that the voltage induced by the commutating pole shifts the point at which the rotor reactance becomes negative to a higher speed, than in a motor which has no commutating poles.[1]

With the most generally used frequency of 60 cycles, it is impossible to take advantage of these favorable operating characteristics above synchronous speed unless we connect the motor to the secondary of an induction motor, in which case we can supply the commutator motor with both a low frequency and a low voltage and use it to control the speed of the induction motor. This has recently been done with great success and has led to a very important development of a-c. commutator motors. A discussion of the various connections and possibilities of this

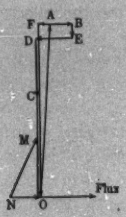

FIG. 4

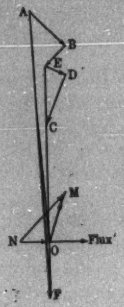

FIG. 5

application properly belongs to the subject of speed control of large induction motors and lies outside the scope of this paper.

The motor, with the field winding located in the same slots as the neutralizing winding, is more suitable for operation below synchronous speed and can be used directly on 60-cycle circuits. Instead of building it with a separate field winding, the neutralizing winding can serve at the same time as field winding which only slightly changes the diagram, as can be seen by comparing Fig. 1 with Fig. 5, which has been drawn for a motor in which both field and neutralizing winding have been combined in one The vector AB is equal to the leakage reactance drop in the neutralizing winding which lags 90 deg. behind $N\,M$, $B\,E$ is the resistance drop of the neutralizing winding, $E\,F$ is the voltage

1. See H. Meyer-Delius, *General Electric Review*, 1913, page 976.

induced in the neutralizing winding by the alternation of the field flux, $F E$ the voltage induced in the armature winding by the alternation of the field flux, $E D$ the reactance drop, $D C$ the resistance drop, $C O$ the rotation voltage induced in the armature winding. The measured voltage across the neutralizing winding is equal to $A F$, the one across the armature winding reduced to the phase-voltage of the equivalent Y connection $F O$. The resultant of $A F$ and $F O$ gives the line voltage $A O$. Thus, we see that below synchronous speed, the voltage across the neutralizing winding is higher than the line voltage per phase until F and O are at the same point which occurs slightly below synchronous speed.

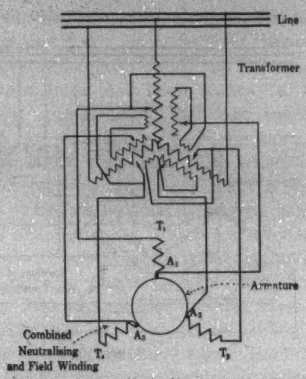

FIG. 6

The diagram of connections of a motor covered by the vector diagram of Fig. 5, has been shown in Fig. 6. Speed regulation can be obtained by connecting the terminals T_1, T_2, T_3 to different taps of the transformer which changes the terminal voltage applied to the motor, or by changing the points to which the armature terminals A_1, A_2, A_3 are connected, which changes the field flux, or by both. The principal disadvantage of this motor is that the transformer has to be designed for its full kv-a. capacity. In order to obtain satisfactory commutation, the armature must be built for a low voltage, which in general makes the ratio of the applied line voltage to the secondary voltage high, so that the size of the transformer is not materially reduced by building it

as a compensator. Instead of using the neutralizing winding also as field winding, the armature can serve for this purpose. Fig. 7 gives the diagram for this arrangement. In this case OM is the current in the neutralizing winding, NM the current in the armature winding. The volt-amperes required for exciting the field is equal to $FO \times NO$ of Fig. 7, instead of $AF \times NO$ of Fig. 5, as is the case when the field current flows through the neutralizing winding. As long as we are running at a lower speed than 50 per cent above synchronous speed, the field can be excited with less volt-amperes by using the armature, instead of the neutralizing winding, as field winding, which results in a small reduction in the size of the regulating transformer and an improvement of the power factor. It is also possible to excite

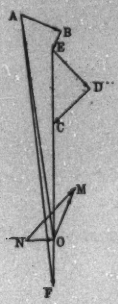

FIG. 7

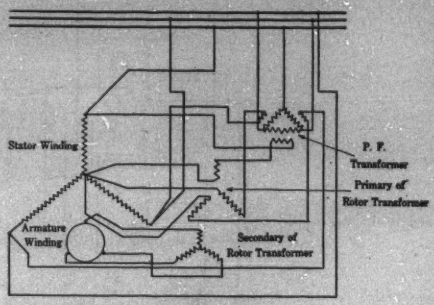

FIG. 8

the field by current flowing partly through the field winding and partly through the armature winding, which can be done by impressing voltage of the proper time-phase on both the neutralizing and the armature winding. However, all the above schemes, with the exception of the one where we connect the commutator motor to the secondary of an induction motor, have the disadvantage that a large transformer is required for the main circuit. The size of this transformer can be reduced by using the type of motor, which we will now describe.

INDUCTION MOTOR WITH COMMUTATOR ON THE SECONDARY

In this type of motor, the armature reaction is neutralized by induction instead of by conduction. It is clear that the operation of the motor covered by Fig. 5 and Fig. 6 will not be changed,

if we separate the neutralizing winding from the armature winding provided we impress on the neutralizing winding the voltage $A\,F$ and on the armature winding the voltage $F\,O$. We can then increase the number of turns of the neutralizing winding, so that it can be connected directly across the line while the armature is connected to a rotor-transformer which supplies the voltage $O\,F$. We can take a brush-shifting series motor, put the brushes in the neutral and connect the primary of the rotor-transformer to different points of the stator winding. In this way, we get a motor which has the same characteristics as a neutralized motor, operated with a constant field flux and a variable voltage applied to the terminals.

Fig. 8 gives the diagram of connections which is suitable for a motor having four poles, or a multiple thereof. Four poles have been connected in series and a tap is brought out at each pole. (The taps have not been shown in the diagram.) This gives four speeds below and four speeds above synchronous speed, in addition to the synchronous speed, and requires thirteen stator terminals. The transformer should be built with a relatively low impedance drop, in order not to spoil the regulation. This can be done as long as only a small variation in speed is required. For instance, if we desire to regulate the speed 25 per cent below and 25 per cent above the synchronous speed, the capacity of the transformer is equal to only 25 per cent of the kilovolt amperes flowing through the motor. The characteristics can be improved by adding a small transformer by means of which the time-phase of the voltage impressed on the primary of the rotor transformer, is shifted.

On small motors, the rotor transformer can be omitted and the stator can be equipped with both a main winding connected to the line and a regulating winding connected to the armature winding. This can be done in various ways.[2]

In all the motors thus far described, the frequency of the rotor currents is reduced to line frequency by means of a commutator connected to the rotor winding, which makes it possible to combine the voltage induced in the rotor with a voltage derived from the line. It is possible to build a satisfactory motor in this way. However, the complicated control which it requires makes it rather unsuitable for the American market. The motor covered by Fig. 8 may find a limited application in special

2. See F. Eichberg, *Elektrotechnische Zeitschrift*, 1910, page 749; E. Arnold, *Die Wechselstromtechnik*, Bd. V2.

cases, although too complicated for general application to machine tool work.

The machine tool builders require an a-c. motor which can be installed as easily as a d-c. motor and the speed of which can be changed in a simple manner. In this respect, the next type of motor is more promising.

The Induction Motor With Commutator on the Primary Side

Instead of changing over the rotor frequency to the line frequency, we can add to the motor a frequency changer, which makes available at every speed a voltage of the same frequency as the voltage induced in the secondary. This can be done by taking a rotary converter armature, running in a laminated field and driven by the motor of which the speed must be regulated. If we connect the slip rings of the rotary converter to the line and select the number of poles and the speed ratio between the shaft of the motor and frequency changer in the proper way, the frequency of the voltage appearing at the commutator will be proportional to the line frequency times the slip. The ratio of the voltage applied to the slip rings to the one appearing at the commutator is independent of the speed. If we use two movable brush yokes connected to the secondary of the induction motor, by changing the relative positions of these yokes, we can impress on the secondary, voltages of different time-phase and amount and the speed of the induction motor can be regulated, both below and above synchronous speed. If c is the ratio of the number of turns of the winding of the converter to the secondary winding and n_0 is the synchronous speed, then the approximate speed range at no load varies between $n_0\ (1 - c)$ and $n_0\ (1 + c)$.

It is possible to combine the frequency changer and the induction motor, if we locate the induction motor primary winding on the rotor and connect it over slip rings to the line. In that case the frequency changer, which we will denote as regulating winding, does not need to be connected to the line, but its coils can be located on the rotor in the same slots as the primary winding, so that the primary flux induces the voltage in this regulating winding. The secondary is located on the stator and can be built with any number of independent phases, provided each yoke is equipped with the proper arrangement of brushes to correspond to this number of phases. The diagram

of connections of a motor of this kind, having three independent secondary phases has been shown in Fig. 9.[3] The brushes A_1, A_2, A_3, are mounted on one yoke and the brushes B_1, B_2, B_3 on another yoke. The motor will operate as an induction motor with short-circuited secondary when the brushes A and B are on the same commutator bar. Both yokes are moved 90 electrical degrees each way from this middle position, the yokes moving in opposite directions. In this manner, regulation both above and below synchronous speed can be obtained. The voltage across every commutator bar is independent of the speed and is proportional to the flux, the frequency and the number of turns in series. If we want to build a motor with the highest possible flux, without exceeding the safe limits for the voltage per bar, we should use a single-turn multiple armature.

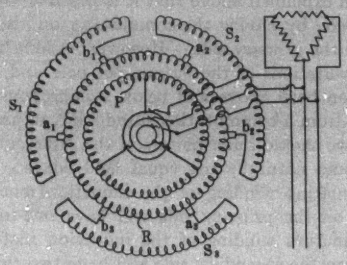

FIG. 9

But even with this winding, the maximum permissible flux is still low, so that this type of motor cannot be built for more than 5 h.p., per pole for 60 cycles and 12 h.p. per pole for 25 cycles, no matter what the speed range is, with the exception that the commutation below synchronous speed is sufficiently better than above, that if we do not make use of the above-synchronous speed range we can use a higher voltage per bar and a larger output per pole. In general, the small output per pole which can be obtained, is no serious draw-back, as most motors for machine tool and elevator work are geared and consequently built for low speed.

On the commutated induction motor with the commutator on the secondary side, the voltage across the commutator bars

3. This diagram is the one of the Schrage Patent 1,079,994.

is proportional to the flux, the number of turns and the frequency of slip. Therefore, if we build this motor for a small speed range, we can use a larger field flux and can obtain with 60 cycles and 20 per cent regulation above and below synchronous speed, 30 h.p. per pole and with 25 cycles, 75 h.p. per pole. This is only possible as long as the required starting torque is low, so that we can start the motor with reduced field flux to avoid excessive sparking.

On the commutated induction motor with commutator on the primary side, the voltage across the commutator bars at starting is the same as at running and high starting torque can be developed without any danger to the commutator. This fact makes this motor particularly suitable for reversible operation.

It has been explained above that it is possible to use a larger output per pole, by having the commutator on the secondary, instead of on the primary side. However, as long as we do not exceed the limitations of the output per pole and have 50 per cent regulation both above and below synchronous speed, it is much better, from a commutation standpoint, to use the second scheme. If we should build motors of the same output in both ways, we would naturally use equal field fluxes. This means that the regulating winding of the induction motor with the commutator on the primary side will have one-turn per coil, while the armature winding of the induction motor with the commutator on the secondary side will have two turns per coil. This will give the same sparking voltage at half speed and 50 per cent above synchronous speed for both motors.

It is clear that the commutation conditions, as far as the current commutation is concerned, are much better in the former case than in the latter, as we have a one-turn armature and a lower slot reactance, due to the fact that the regulating winding fills only part of the slots. The current commutation is the limiting feature above synchronous speed and for this reason, the induction motor with commutator on the primary side can be built for a larger speed range above synchronous speed than the one with commutator on the secondary side. This is a great advantage, because the speed-torque characteristics and the maximum output of the motor are much more advantageous above, than below synchronous speed. This partly offsets the limitation as to the maximum output per pole obtainable, as the reduction in speed resulting from the necessity of using a

large number of poles, is counteracted to a large extent by the possibility of raising the speed considerably more above synchronism.

The theory of the induction motor with commutator on the primary side can be explained with the aid of the same diagrams as have been given for the other motors. The following derivation of the diagrams made in a different way and some of the equations used in predetermining the characteristics, may be of interest.

In the approximate theory of the induction motor, the exciting

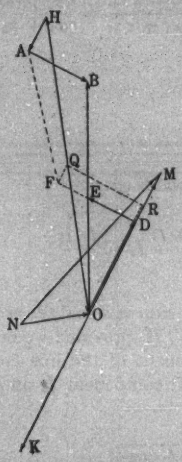

FIG. 10

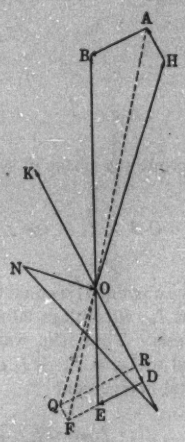

FIG. 11

current is considered constant[4], and as flowing through a separate exciting circuit. By doing this, a very slight error is made as we neglect the impedance drop of the exciting current, thus figuring with a flux which is slightly larger than the actual flux and we use, too high a value for the exciting current. The approximation simplifies the calculation considerably and will be used in the following. In Fig. 10, we have drawn the approximate vector diagram for the induction motor. We find for the primary circuit that the applied voltage $OH = E_1$ is balanced

4. See Steinmetz's Alternating Current Phenomena.

by the primary resistance and reactance drop $HA = I_0 r_0$ and $AB = I_0 x_0$ and the voltage $BO = E_0$ induced by the main flux. After having reduced secondary turns to the primary turns, we find that the same voltage $BO = E_1$ is induced in the secondary and that this voltage is balanced by the secondary reactance drop $DE = I_1 s x_1$ the secondary resistance drop $OD = I_1 r_1$, and the counter e.m.f., $EB = (L_1 - s) E_1$. If we draw $QR \parallel FD$, $EF \parallel BA$ and $QF \parallel HA$ then $OQ = s E_l$

Let $OQR = i$ then $\tan i = \dfrac{OR}{QR} = \dfrac{r_1 + s r_0}{s x_1 + s x_0} = \dfrac{\frac{r_1}{s} + r_0}{x_1 + x_0}$

and

$$I_1 = \frac{E_l \sin i}{\frac{r_1}{s} + r_0}$$

The torque per phase in synchronous watts equals

$$OB \times I_1 \times \cos \angle BOD = \frac{OD}{s} I_1 = \frac{I_1^2 r_1}{s}$$

The line current I_l is equal to the vector sum of I_1 and the exciting current I_m, which lags 90 deg. behind OH, provided we neglect hysteresis. Thus, the wattless component of the line current is equal to $I_{wl} = I_m + I_1 \cos i$ the watt component $I_l \sin i$ and the total line current

$$I_l = \sqrt{I_m^2 + I_1^2 + 2 I_m I_1 \cos i}$$

When we drive the induction motor above synchronous speed as an induction generator, the diagram assumes the form of Fig. 11. The counter e.m.f. EB is now larger than the induced voltage BO. We have to take into account besides that the rotating field in the rotor has changed its direction, so that the time axis for the rotor rotates counter clockwise, while the time axis for the stator continues to rotate clockwise. For the counter clockwise rotating time axis of the rotor, the secondary reactance drop DE lags 90 deg. behind the current. This secondary reactance drop appears, however, as a negative reactance drop when reduced to the primary circuit in which the

time axis rotates clockwise.[5] A complete calculation of an induction motor in accordance with the above method has been given in Table I. It will be noted that this method is the same in principle as the approximate "Steinmetz method."

The induction motor with commutator on the primary side can be calculated in a similar manner. If the primary and the regulating winding are located in the same slots, the leakage between these two windings is so small that it can be neglected. The vector sum of the ampere-turns resulting from the currents in both the primary and the regulating winding, will yield a leakage flux, which induces in the primary winding a voltage e_s and in

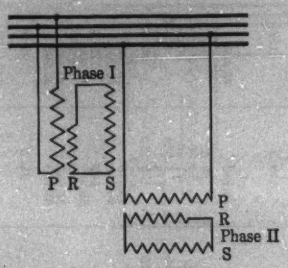

FIG. 12

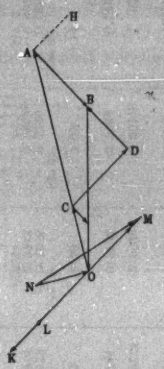

FIG. 13

the regulating winding a voltage $c\,e_s$ if c is the ratio of the number of turns of the regulating and the primary winding.

In order to find how the reactance should be taken into account, we can investigate the conditions at standstill and consider the motor as a quarter-phase stationary transformer having a primary winding, P, a regulating winding R and a secondary winding S. The windings P and R are fitted in between each other, so as to have only a small leakage between the two. R and S are connected together. Let P and S have the same number of turns, and the ratio of turns of R and S be c. If we assume as before that the magnetizing current flows in a separate

5. Dr. Rudenberg has called particular attention to this point in the *Elektrotechnische Zeitschrift*, 1910, page 1087.

TABLE I

s	0.01	0.0159	0.02118	−0.01	−0.0159	−0.02118
r_1	0.041575	0.041575	0.041575	0.041575	0.041575	0.041575
r_0	0.033778	0.03378	0.03378	0.03378	0.03378	0.03378
$\frac{r_1}{s}$	4.1575	2.619	1.968	−4.1575	−2.619	−1.966
$r_0 + \frac{r_1}{s}$	4.19275	2.6528	2.00178	−4.1238	−2.58522	−1.932
$x_0 + x_1$	0.2764	0.2764	0.2764	0.2764	0.2764	0.2764
$\tan i = \frac{r_0 + \frac{r_1}{s}}{x_0 + x_1}$	15.15	9.59	7.52	−14.9	−9.36	−6.98
i	86° 13′	84° 3′	82° 25′	273° 83′	263° 53′	261° 51′
$\cos i$	0.0658	0.1036	0.1320	0.06685	0.1067	0.1422
$\sin i$	0.9978	0.9946	0.9912	−0.9977	−0.9943	−0.9899
Et	317.5	317.5	317.5	317.5	317.5	317.5
$I_1 = \frac{Et \cos i}{x_0 + x_1}$	75.6	119.0	151.5	76.5	122.2	163.2
or $I_1 = \frac{Et \sin i}{r_0 + \frac{r_1}{s}}$	75.6	118.8	157.	76.8	122.	162.2
$D = \frac{I_1^2 r_1}{s}$	23,750	36,850	48,400	−24,400	−39,000	−51,800
I_w	59.6	59.6	59.6	59.6	59.6	59.6
$I_1 \cos i$	4.97	12.36	20.74	5.1	13.01	23.06
$I_{wl} = I_w + I_1 \cos i$	64.57	71.96	80.34	64.7	72.61	82.66
I_h	4.615	4.615	4.615	4.615	4.615	4.615
$I_1 \sin i$	75.4	118.1	155.8	−76.3	−121.1	−160.2
$I_{ip} = I_h + I_1 \sin i$	80.015	122.715	160.42	−71.685	−116.48	−155.585
I_{wl}^2	4,165	5,170	6,450	4,200	5,230	6,835
I_{ip}^2	6,400	15,010	25,750	5,150	14,700	24,190
$(I_{ip}^2 + I_{wl}^2)$	10,565	20,180	32,200	9,350	19,980	31,025
$It = \sqrt{I_{ip}^2 + I_{wl}^2}$	102.8	142	179.5	96.5	141.2	176.2
$P_1 = D\ (1 - s)$	23,500	36,250	47,400	−24,600	−39,600	−52,950
F_0	607	607	607	607	607	607
Mech. Output $= P = P_1 - F_0$	22,893	35,643	46,793	−25,207	−40,207	−53,557
Input $= I_w\ Et$	25,400	38,900	50,900	−22,760	−36,980	−49,300
Eff. $= \frac{P}{I_w\ Et}$	90.2	91.7	92.0	90.	91.9	92.1
P. F. $= \frac{I_w}{It}$	78.0	86.3	89.5	93.7	95.2	95.7
H. P. $= \frac{3\ P}{746}$	92.0	143.7	188.0	102.3	161.8	215.4

circuit, the ampere-turns yielded by P and R must be equal and opposite to the ampere-turns yielded by S. Hence, the current flowing in P of Fig. 12 is equal to $(1-c)\ I_1$. The primary leakage flux is yielded by the vector sum of the ampere turns of P and R, which is equal and opposite to the secondary ampere turns resulting from T_1 flowing through S. This primary leakage flux is interlinked with the turns of P and causes the reactance drop $A\ B = T_1\ x_0$, if x_0 is equal to the leakage reactance of P considered as the primary of an induction motor. Fig. 13 gives the vector diagram when neglecting the primary resistance drop. In the primary winding P, the line voltage $O\ A$ is balanced by the reactance drop $A\ B = I_1\ x_0$ and the induced voltage $B\ O$. The voltage across the regulating winding will be equal to $O\ C = c\ O\ A$ and we find that in the secondary circuit the induced voltage $B\ O$

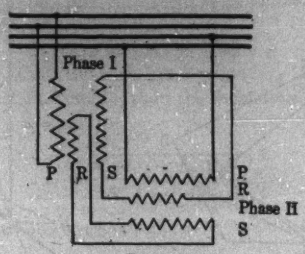

FIG. 14

is balanced by the reactance drop $D\ B$, the resistance drop $C\ D$ and the voltage $O\ C$ across the regulating winding. This voltage is the vector sum of the voltage induced by both the main and the leakage flux. The secondary current $O\ K = I_1$. The primary load current is equal to $-\ (O\ K - K\ L) = -(I_1 - c\ I_1) = O\ M$. We can add the exciting current $N\ O$ at right angles to $O\ A$ and find the line current $I_1 = N\ M$. The secondary resistance drop $C\ D$ should include the brush drop and the resistance of both the secondary and the regulating winding. If desired, the resistance drop of the primary load current which is equal to $(1 - c)\ I_1\ r_0$ can easily be added, as shown by the dotted line $A\ H$. If $c = 0.5$, the primary load current is equal to $0.5\ I_1$ and the ampere-turns of the primary are equal to those of the regulating winding. In that case, the minimum copper loss will be obtained if we make the copper-cross-sections of both windings equal. When running above synchronous speed, the ampere-turns of the primary

winding are equal to the sum of the ampere-turns of both the regulating and the secondary winding, hence in order not to have the primary winding overheat when running above synchronous speed, it is better to make the cross-section of the regulating winding smaller than those of the primary winding. As $A\,C = (1 - c)\,E_l$, we can find the reactance $(x_0 + x_1)$ by an impedance test applying E_l to the slip rings

$$x_0 + x_1 = \frac{E_l\,(1 - c)}{O\,M} \sin \angle\, A\,C\,D$$

or approximately:

$$x_0 + x_1 = \frac{E_l\,(1 - c)^2}{I_l} \sin \angle\, A\,C\,D$$

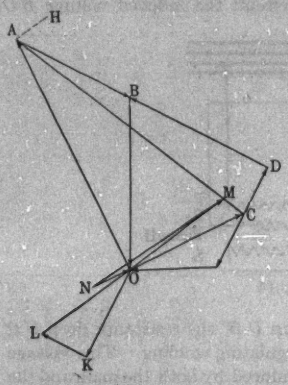

FIG. 15

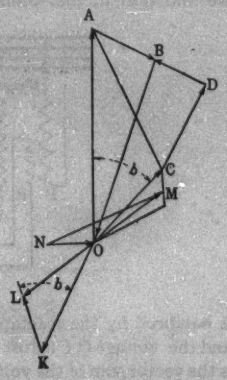

FIG. 16

It is clear that the condition described corresponds to the induction motor with the brushes in the neutral. We can also connect the secondary of phase I to the regulating winding of phase II and the secondary of phase II to the regulating winding of phase I, as has been shown in Fig. 14. The vector diagram of this arrangement has been shown in Fig. 15. In this case, the resultant voltage, which overcomes the impedance drop, is equal to

$$A\,C = E_l \sqrt{1 + c^2}$$

and

$$x_0 + x_1 = \frac{E_l\,(1 + c^2)}{I_l} \sin \angle\, A\,C\,D$$

This corresponds to the commutated induction motor with a brush-shift of 90 deg.

Fig. 16 gives a similar diagram for a brush-shift (b) in which case the resultant voltage is equal to

$$A\,C = E_l \sqrt{1 + c^2 - 2\,c \cos b}$$

and

$$x_0 + x_1 = \frac{E_l\,(1 + c^2 - 2\,c \cos b)}{I_l} \sin A\,C\,D.$$

All the above equations for $x_0 + x_1$ have been given without taking the primary resistance loss, the core loss and the magnetizing current into account. This is satisfactory as long as the impedance test is made at greatly reduced voltage. If necessary, these values can be taken into account as follows: Read primary watts (W) amp., (I_l) volts (E_l).

Determine $I_w = I_l \cos \varphi$ and $I_{wl} = I_l$ sine φ.

Determine at standstill with open secondary I_m for a voltage E_l and subtract I_m from I_{wl} then the secondary current reduced to the primary is equal to

$$I_1 = \frac{\sqrt{(I_{wl} - I_m)^2 + I_w^2}}{\sqrt{1 + c^2 - 2\,c \cos b}}$$

The secondary impedance is

$$z_l = \frac{E_l\,(1 + c^2 - 2\,c \cos b)}{\sqrt{(I_{wl} - I_m)^2 + I_w^2}}$$

Let the core loss with E_l volts applied to the slip rings be W_c watts and the primary resistance loss with a current I_l, be $3\,I_l^2\,r_0$ then

$$r_1 = \frac{W - W_c - 3\,I_l^2\,r_0}{3\,[(I_{wl} - I_m)^2 + I_w^2]}\,(1 + c^2 - 2\,c\,s \cos b)$$

and

$$x_0 + x_1 = \sqrt{z_l^2 - r_1^2}$$

A comparison of Fig. 16 and Fig. 13 shows that due to the shift b, the secondary current $I_1 = O\,K$ is shifted into the direction of the secondary induced voltage $B\,O$. This means that with the same flux and the same secondary current, we get a higher torque. The possibility of shifting the time-phase of the secondary current by means of the brush-position, can be utilized for obtaining a higher torque per ampere than is pos-

sible with the induction motor with resistance in the secondary. The fact that the secondary current at the same time flows through the regulating winding and thereby reduces the primary current, makes this condition still better.

Fig. 17 has also been drawn for a brush-shift b, only in this case the motor is running and a counter e.m.f. $E\,B$ is induced in the secondary. The resultant voltage in the secondary is equal to $E\,O = B\,O - E\,B$. If we draw $E\,F \parallel A\,B$ then

$$\frac{O\,F}{O\,A} = \frac{O\,E}{O\,B} = \frac{s}{1} \text{ or } O\,F = s\,E_1.$$

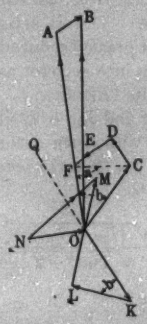

FIG. 17—RUNNING AS MOTOR BELOW SYNCHRONOUS SPEED—BRUSH SHIFT OPPOSITE TO DIRECTION OF ROTATION

Further $$F\,E = s\,A\,B = s \times x_0\,I_1.$$

$$F\,C = \sqrt{F\,O^2 + O\,C^2 - 2\,F\,O \times O\,C \cos \angle F\,O\,C}$$
$$= E_1 \sqrt{s^2 + c^2 - 2\,c\,s \cos b}$$

Let $$\angle\, O\,F\,C = a \text{ and } \angle\, C\,F\,D = i$$

then $$\frac{\sin a}{O\,C} = \frac{\sin b}{F\,C} \text{ or } \sin a = \frac{c \sin b}{\sqrt{s^2 + c^2 - 2\,c\,s \cos b}}$$

$$\tan i = \frac{D\,C}{F\,D} = \frac{r_1}{s\,(x_0 + x_1)}.$$

The secondary current is equal to

$$I_1 = \frac{F\,C \sin i}{r_1} = \frac{E_1 \sin i \sqrt{s^2 + c^2 - 2\,c\,s \cos b}}{r_1}$$

The torque per phase in synchronous watts is equal to

$$D = B\,O \times O\,K \cos \angle\, B\,O\,Q = O\,A \times O\,K \cos \angle\, A\,O\,Q$$

$$\angle\, A\,O\,Q = \angle\, O\,F\,C - (90^\circ - \angle\, C\,F\,D) = a + i - 90.,$$

or

$$\cos \angle\, A\,O\,Q = \sin\,(a + i)$$

and

$$D = E_1 \times I_1 \sin\,(a + i)$$

The electrical output is equal to $P_1 = D\,(1 - s)$
The mechanical output (P) is equal to the electrical output minus the friction and windage per phase, $P = P_1 -$ friction, windage. The primary current can be calculated by determining separately the component I_w in phase with the applied line voltage and the component I_{wl} which is 90 deg. out of time phase with the line voltage.

If I_h is the core loss current then:

$$I_w = I_h + O\,K \cos \angle\, Q\,O\,A - L\,K \cos\,(\angle\, Q\,O\,A + b)$$
$$I_w = I_h + I_1 \cos\,(a + i - 90) - c\,I_1 \cos\,(a + i + b - 90).$$
$$I_w = I_h + I_1\,[\sin\,(a + i) - c \sin\,(a + i + b)]$$

If I_m is the wattless component of the magnetizing current then,

$$I_{wl} = I_m + O\,K \sin \angle\, Q\,O\,A - L\,K \sin\,(\angle\, Q\,O\,A + b)$$
$$I_{wl} = I_m + I_1\,[\cos\,(a + i) - c \cos\,(a + i + b)]$$

The total line current is $I_l = \sqrt{I_w^2 + I_{wl}^2}$

The power factor $= \dfrac{I_w}{I_l}$

The input $= E_l I_w$

The efficiency $= \dfrac{P}{E_l\,I_w}$

When determining the angles from the sine and tangent, some care must be taken to get the angle in the proper quadrant. This can be done readily as long as the possible variation of these angles with the slip is taken into account, with the aid of the diagram. For instance, it follows from Fig. 17 that F moves over $A\,O$ from A to O when the slip changes from 1 to 0.

The angle $C F O = a$ changes from an acute to an obtuse angle. Fig. 18 gives the diagram for the same brush position as used in the diagram of Fig. 17, when the motor is driven above synchronous speed. In case the motor is rotating counter clockwise, the field in the rotor will rotate clockwise. Hence, as long as we operate below synchronous speed, the electrical field will rotate clockwise, in respect to both the stationary secondary winding and the stationary brushes connected to the commutator of the regulating winding. This means that if we want $O C$ to lag behind $O A$, as shown in Fig. 17, we have

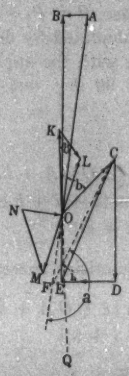

FIG. 18—DRIVEN AS GENERATOR ABOVE SYNCHRONOUS SPEED — BRUSH SHIFT FROM NEUTRAL AXIS OPPOSITE TO DIRECTION OF ROTATION

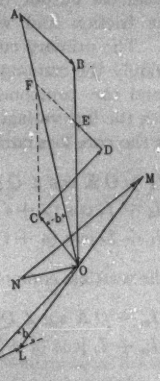

FIG. 19—RUNNING AS MOTOR BELOW SYNCHRONOUS SPEED — BRUSH SHIFT IN THE DIRECTION OF ROTATION

to shift the brushes in the direction of the rotation of the electrical field, *i.e.* opposite to the mechanical rotation of the rotor. When running above synchronous speed, the electrical field will rotate counter clockwise, with respect to both the stationary secondary winding and the brushes and if we leave the brush position unchanged $O C$ will lead $O A$. When reduced to the primary, for which the time axis rotates clockwise, $O C$ will lag behind $O A$ as in Fig. 17. This has been shown in Fig. 18. When running above synchronous speed, the secondary reactance

*Since this paper was first published, Mr. J. I. Hull has modified the rules so as to make them apply to the operation in all brush-positions and at all speeds.

7

$x_0 + x_1 = 0$
$r_1 = 0$
$E_1 =$

s	1.4	1.2	1.	0.8
s^2	1.96	1.44	1.	0.64
c^2	0.314	0.314	0.314	0.314
$s^2 + c^2$	2.274	1.754	1.314	0.954
$2\,c\,s \cos b$	1.563	1.34	1.119	0.894
$s^2 + c^2 - 2\,c\,s \cos b$	0.711	0.414	0.195	0.060
$\sqrt{s^2 + c^2 - 2\,c\,s \cos b}$	0.844	0.644	0.442	0.246
$E_{res} = E_1 \sqrt{s^2 + c^2 - 2\,c\,s \cos b}$	107.2	81.7	56.0	31.1
$\sin a = \dfrac{c \sin b}{\sqrt{s^2 + c^2 - 2\,c\,s \cos b}}$	0.0425	0.0557	0.0811	0.146
a	2° 27′	3° 12′	4° 40′	8° 25′
$\tan i = \dfrac{r_1}{s\,(x_0 + x_1)}$	0.475	0.554	0.667	0.836
i	25° 25′	29° 0′	33° 42′	39° 51′
$\sin i$	0.428	0.484	0.553	0.640
$I_1 = \dfrac{E_1 I_1 \sin i \sqrt{s^2 + c^2 - 2\,c\,s \cos b}}{r_1}$	153.2	132.1	103.2	66.3
$a + i$	27° 52′	32° 12′	38° 22′	48° 16′
$\sin (a + i)$	0.468	0.547	0.621	0.746
$D = E_1 I_1 \sin (a + i)$	9,090	9,180	8,140	6,270
$P_1 = D\,(1 - s)$	−3,636	−1,936	0	1,254
Friction per phase	107	53	0	53
Mech. Output $= P = P_1 -$ friction	−3,743	−1,989	0	1,201
$a + i + b$	31° 22′	35° 42′	41° 52′	51° 46′
$\sin (a + i + b)$	0.520	0.583	0.666	0.785
$I_1 b$	1.1	1.1	1.1	1.1
$I_1 \sin (a + i)$	71.7	72.3	64.1	49.4
$I_1 b + I_1 \sin (a + i)$	72.8	73.4	65.2	50.5
$I_1\, c \sin (a + i + b)$	44.5	43.1	38.4	29.0
$I_w = I_1 b + I_1 \sin (a + i) - I_1\, c \sin (a + i + b)$	28.3	30.3	26.8	21.5
$\cos (a + i)$	0.884	0.847	0.784	0.666
$\cos (a + i + b)$	0.854	0.812	0.745	0.619
I_m	20	20	20	20
$I_1 \cos (a + i)$	135.4	112.1	81	44.2
$I_m + I_1 \cos (a + i)$	155.4	132.1	101	64.2
$c\, I_1 \cos (a + i + b)$	73.3	60.0	43.15	22.96
$I_{\varphi} = I_m + I_1 \cos (a + i) - c\, I_1 \cos (a + i + b)$	84.1	72.1	57.85	41.24
I_w^2	800	917	718	452
I_{φ}^2	7070	5190	3343	1700
$I_w^2 + I_{\varphi}^2$	7870	6107	4061	2162
$I_1 = \sqrt{I_w^2 + I_{\varphi}^2}$	88.7	78.2	63.75	46.5
$P.F. = \dfrac{I_w}{I_1}$	31.9	38.79	42.1	46.35
Input $= E_1 I_w$	3590	3843	3402	2750
Eff. $= \dfrac{P}{E_1 I_w}$	Brake	Brake	0	44.

NOTE: *Definitions of Angles Etc.* Angle (a) is the angle between $F\,C$ (considering + from F to C) and $A\,O$ (consid
Angle (b) is the angle between $O\,A$ (considering + from O to A) and $O\,C$ (considering + from O to C) and (b) is + w
Angle (i) is the angle between $F\,D$ (considering + from F to D) and $F\,C$ (considering + from F to C) and (i) is + w

NOTE: As (s) varies from 0 to infinity, (a) varies from 180° − b if (b) is + (or − 180° − b if (b) is −) to 0.

As (s) varies from 0 to − infinity, (a) varies from 180° − b if (b) is + (or − 180° − b if (b) is −) to 180° if $\dfrac{(b) \text{ is } +}{\sqrt{s^2+c^2}}$

With positive (s); (a) is less than 90° when $c \cos b$ is positive and less than (s), or negative. (a) is greater than 90°
With negative (s); (a) is less than 90° when $c \cos b$ is negative and greater than (s). (a) is greater than 90° when c
(i) is always positive for positive (s), and varies from 90° to 0 as (s) varies from 0 to infinity.
(i) is always negative for negative (s), and varies from 90° to 0 as (s) varies from 0 to − infinity.

TABLE II

= 0.45 l_m = 20 c = 0.56
= 0.30 l/b = 1.1
127 b = 3.5°

0.6	0.4	0.2	−0.2	−0.4	−0.6	−0.8	−1.0
0.36	0.16	0.04	0.04	0.16	0.36	0.64	1.0
0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314
0.674	0.474	0.354	0.354	0.474	0.674	0.954	1.314
0.670	0.447	0.224	−0.224	−0.447	−0.670	−0.894	−1.119
0.004	0.027	0.130	0.578	0.921	1.344	1.848	2.433
0.063	0.164	0.36	0.759	0.959	1.16	1.36	1.56
8.03	20.84	45.8	96.4	122.0	147.2	172.7	198.0
0.565	0.218	0.0993	0.0471	0.0373	0.0308	0.0264	0.023
34° 24′	167° 23′	174° 17′	177° 18′	177° 51′	178° 14′	178° 29′	178° 41′
1.109	1.661	3.34	−3.34	−1.661	−1.109	0.835	−0.667
47° 57′	58° 58′	73° 20′	−73° 20′	−58° 58′	−47° 57′	−39° 51′	−33° 42′
0.741	0.857	0.958	−0.958	−0.857	−0.741	−0.640	−0.553
19.8	59.7	146.	−0.308	−348.1	−364.	−368.2	−365.2
83° 21′	226° 21′	247° 37′	103° 58′	118° 53′	130° 17′	138° 38′	144° 59′
0.991	−0.724	−0.925	0.970	0.876	0.763	0.661	0.574
2,494	−5,480	−17,100	−38,000	−38,720	−35,200	−30,900	−26,600
998	−3,288	−13,680	−45,600	−54,100	−56,300	−55,500	−53,200
107	160	214	320	373	427	480	532
891	−3,448	−13,894	−45,920	−54,473	−56,727	−55,980	−53,732
85° 51′	229° 51′	251° 7′	107° 28′	122° 23′	133° 47′	142° 8′	148° 29′
0.997	−0.765	−0.946	0.954	0.835	0.722	0.614	0.523
1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
19.6	−43.2	−135.0	−298.4	−305.2	−278.0	−243.8	−209.6
20.7	−42.1	−133.9	−297.3	−304.1	−276.9	−242.7	−208.5
11.4	−25.6	−77.2	−164.5	−162.7	−147.0	−126.3	−106.9
9.3	−16.5	−56.7	−132.8	−141.4	−129.9	−116.4	−101.6
0.133	−0.690	−0.378	−0.242	−0.483	−0.646	−0.751	−0.819
0.722	−0.645	−0.324	−0.301	−0.536	−0.692	−0.790	−0.853
20	20	20	20	20	20	20	20
2.63	−41.2	−55.2	74.5	168.2	235.4	276.4	299.0
22.63	−21.2	−35.2	94.5	188.2	255.4	296.4	319.0
0.8	−21.6	−26.46	51.8	104.7	141.0	163.0	174.2
21.83	+0.4	−8.74	42.7	83.5	114.4	133.4	144.8
86.5	272	3,215	17,600	20,000	16,820	13,560	10,300
447	0.16	76.5	1,822	6,980	13,060	17,800	20,900
563.5	272.16	3,291.5	19,422	26,980	29,880	31,360	31,200
23.88	16.5	57.4	139.4	164.2	173.0	177.8	176.8
39.00	100.0	98.75	95.3	86.0	75.0	65.5	57.5
1180	−2098	−7,200	16,820	17,950	16,480	14,780	12,890
75.5	60.7	52	36.6	33.0	29.0	26.4	24.0

nsidering + from A to O) and (a) is + when A O is rotated clockwise from F C with F as a center. [ALTES]

+ when O C is rotated clockwise from O A with O as a center.

when F C is rotated clockwise from F D.

+ (or − 180° if (b) is −),

$c - \Sigma\, z\, y \cos b$ is always +.

90° when $c \cos b$ is positive and greater than (s).

n $c \cos b$ is negative and less than (s) or positive.

drop appears negative when viewed from the primary, as explained for the usual induction generator with short-circuited secondary. Table II gives a complete calculation made in accordance with the above theory. It has been found that the theory is confirmed by test.

Fig. 19 gives the diagram for a motor operating below synchronous speed when the brushes are shifted in the wrong direction, *i.e.*, in the direction of rotation. It will be noted that this results in a low secondary power factor, which is always

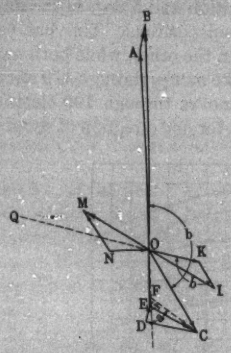

FIG. 20—RUNNING AS MOTOR ABOVE SYNCHRONOUS SPEED—BRUSH SHIFT FROM NEUTRAL AXIS MORE THAN 90 DEG. AND LESS THAN 180 DEG. OPPOSITE TO DIRECTION OF ROTATION. (BRUSH SHIFT IN DIRECTION OF ROTATION FROM REVERSED NEUTRAL AXIS)

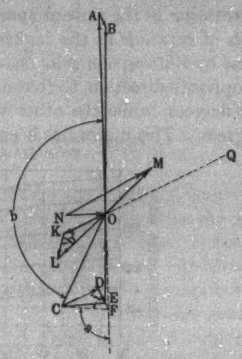

FIG. 21—RUNNING AS MOTOR ABOVE SYNCHRONOUS SPEED—BRUSH SHIFT FROM NEUTRAL AXIS MORE THAN 90 DEG. AND LESS THAN 180 DEG. IN THE DIRECTION OF ROTATION. (BRUSH SHIFT AGAINST ROTATION FROM REVERSED NEUTRAL AXIS)

combined with a reduction in maximum output and larger variation between the no-load and full-load speed.

Fig. 20 gives the diagram for operation above synchronous speed. The counter e.m.f., $E\,B$ is larger than the induced secondary voltage $B\,O$ and the voltage $O\,C$ of the regulating winding is impressed in opposite direction, being shifted over an angle b opposite to the direction of rotation from the neutral position.

Fig. 21 gives the same diagram, only the brushes are shifted in the direction of rotation which causes the secondary current to lead too much and results generally in inferior com-

mutation. Instead of calculating the characteristics by means of simple geometrical equations, circle diagrams can be used.[6] Both the derivation and the application of these diagrams are rather complicated and, therefore, will not be given in this paper.

Fig. 22 gives the tested speed-torque curves of a 440-volt, 3-phase, 1200 rev. per min., 60-cycle brush, shifting motor, built as an induction motor with the commutator on the primary side and on which the brushes are shifted 5 deg. opposite the direction of rotation in the slowest speed position and 5 deg. in the direction of rotation in the highest speed position. This has been done by shifting one yoke faster than the other, while both move in opposite direction, so that one yoke moves through 170 electrical degrees, while the other yoke moves through 190 electrical degrees. This motor has been built for one direction of rotation.

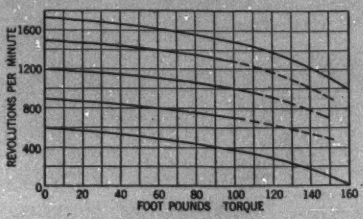

FIG. 22

If the motor has to operate with the same characteristics in both directions of rotation, the brush-shifting mechanism becomes more complicated, as will be understood from Figs. 23 and 24, which give the desired brush-position for maximum and minimum speed in both directions of rotation for a two-pole motor. However, a simple mechanical solution has been found and a model incorporating this scheme is now being tested. A photograph of it is shown in Fig. 25, while the brush-shifting mechanism is represented in Fig. 26. This motor has been built with four-poles and a six-phase secondary, each yoke having 12 studs.

The fast moving yoke *B* on which the brushes *b*1, *b*2, *b*3 have been mounted, is supported by the bearing-housing. The slow moving yoke on which the brushes *a*1, *a*2, *a*3 have been

6. See O. S. Braystad, *E. T. Z.*, 1903, page 368; E. Arnold, *Die Wechselstromtechnik*, Bd. V2; H. Meyer-Delius, *General Electric Review*, 1914, page 817; H. K. Schrage, *E. T. Z.*, 1914, page 81.

mounted, is supported by the end shield; it is moved by means of a stud which connects it to the disk C, which is supported on the bearing housing. The pinions D and E are keyed to a shaft which can rotate in a bearing supported by the disk C. The pinion D meshes with a gear F which is mounted on the bearing housing and the pinion E meshes with a gear G which is fastened to

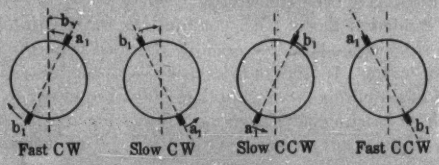

FIG. 23

b_1 moves through $360 - 2\,b$ deg. $C\,W$
a_1 moves through $360 + 2\,b$ deg. $C\,C\,W$. While b_1 moves through $2\,b$ deg. from slow $C\,W$ to slow $C\,C\,W$, a_1 should move $C\,W$ through 2 b degrees

yoke B. If the gear F is held stationary and the disk C is turned, both opinions D and E will rotate. Pinion E will drive the gear G fastened to yoke B. The gear ratio can be selected in such a manner that disk C and yoke A move in opposite direction to yoke B and at a slower rate. The gear F is held by a pinion H which is keyed to the same shaft as the intermittent gear I, while this shaft rotates in a bearing which is rigidly supported

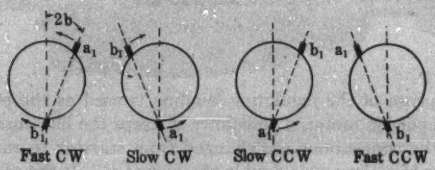

FIG. 24

b_1 moves through 360 deg. $C\,W$
a_1 moves through $(360 \text{ deg.} + \frac{1}{4}\,b)$ $C\,C\,W$. While b_1 moves through $4\,b$ degrees from slow $C\,W$ to slow $C\,C\,W$, a_1 should stand still

from the bearing housing. The intermittent gear I is moved through a small angle when it is struck by a pin, which is fastened to the disk C. After the pin has moved it, I is held stationary by sliding on the outside circumference of C. When I is moved by the pin, H will turn through the same angle and move the gear F. By properly selecting the gear ratios, it is possible, when we change from "Slow" in one direction to "Slow" in the other

direction, to have yoke *B* remain stationary or move in the same direction as *C*, while the intermittent gear is moved by the pin. The former arrangement gives a change of brush-positions in accordance with Fig. 24, the latter in accordance with Fig. 23. A control switch is connected to the shaft to which *I* is connected, and changes the phase-rotation of the lines to which the collector rings have been connected.

The induction motor with commutator on the primary side can be run single-phase, by opening one lead connecting to the collector rings. If only single-phase is available, it can be started like an ordinary single-phase induction motor by using a split-phase starting device, or as a repulsion motor, in which case a higher torque can be obtained. This connection is shown in Fig. 27, for a motor having a quarter-phase secondary.

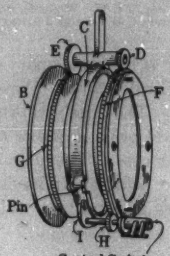

FIG. 26

One phase of the regulating winding is used as the primary of the repulsion motor, a resistance *R* keeps the line current down and the connections are changed from starting to running, by means of a three-pole double-throw switch. If the motor has a three-phase secondary, a four-pole double-throw switch is required.

In its original form, the induction motor with commutator on the primary side is wound with the primary winding in the bottom and the commutated regulating winding in the top of the armature slots. In this way, four coil ends have to be inserted in every slot, which takes up much room for the insulation. Moreover, if the primary winding has to be repaired, it is necessary to remove first part of the coils of the regulating winding. A winding is being developed which is arranged in such a manner

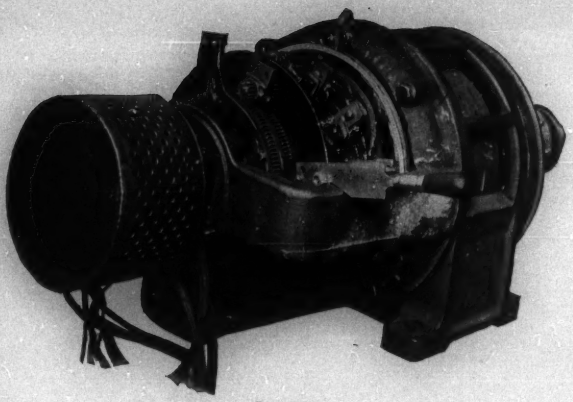

FIG. 25

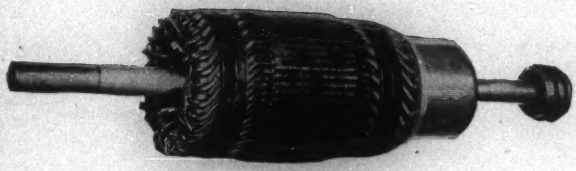

FIG. 28

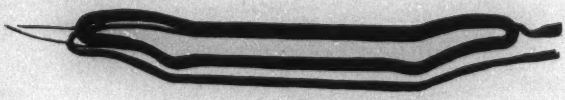

FIG. 29

[ALTES

that only two coil ends are located in one slot. If we have an even number of slots and use a coil pitch equal to an odd number, we can arrange the windings in such a way that we get a primary coil in the top of every odd and in the bottom of every even slot, and a coil of the regulating winding in the top of every even slot and in the bottom of every odd slot.

The leads of the primary winding can be brought out on the back end and the leads of the coils of the regulating winding on the commutator end. The insertion of such a winding is no more difficult than the insertion of an ordinary lap winding, it merely being necessary to have two kinds of coils. By doing

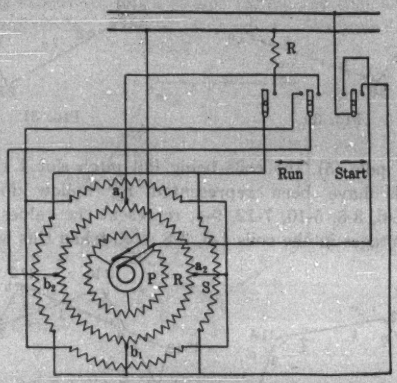

FIG. 27

this, we require half the number of coils and save room in the slots. Fig. 28 shows a photograph of a completed armature and Fig. 29 one coil of the primary and one coil of the regulating winding of a 220-volt, 750-rev. per min., 25-cycle motor with this new type of winding. The coils are inserted into open slots which are closed by a magnetic wedge. It will be noted that the coil of the regulating winding is lighter than the one of the primary winding, and that a slight bend has been added to each end of the lower straight part of the primary coil, in order to make room for the upper half of the next primary coil.

The regulating winding is a closed commutated winding with one coil per slot. The coils should always be connected in such

a manner, that we get a balanced mesh connection without internal short-circuiting current. We can obtain this condition, by making the coil pitch equal to an odd number of slots. This has been shown in Fig. 30, in which we have an odd number of

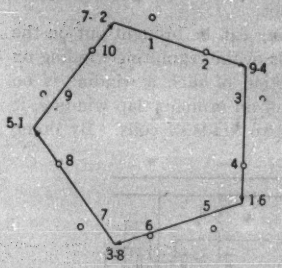

FIG. 30

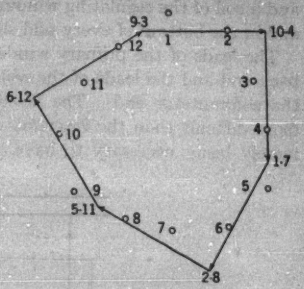

FIG. 31

slots per pole (5), the coils being full pitch slot 1 to 6. The coil ends have been represented by hollow dots. The vectors 1-6, 3-8, 5-10, 7-12, 9-4, represent the values of the induced voltages in the coils 1-6, 3-8, etc., when this armature is

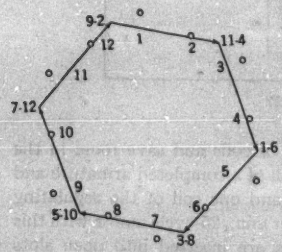

FIG. 32

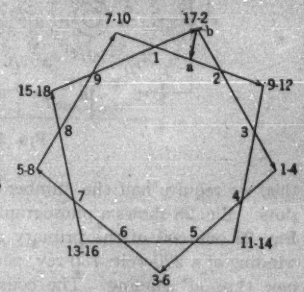

FIG. 33

surrounded by a rotating field. The resultant voltage of all the coils is zero, which means that we will have no circulating current. The same result could have been obtained with a coil pitch 1-4, 1-2, 1-8, 1-10.

In Fig. 31, we have an even number of slots per pole (6) and a

coil pitch equal to an even number of slots. This results in a balanced winding, since the coils have been arranged in groups of two, so that we get the same effect as though we had an odd number of slots per pole (3). This arrangement is possible, if the number of slots per pole is equal to an odd number times an integer.

In Fig. 32, we have an even number of slots per pole. This gives also a balanced winding, as long as the coils span an odd number of slots (5). We could also have used a coil pitch 1-4, 1-2, etc.

Fig. 33 gives the vector diagram for a four-pole armature having (18) slots, *i. e.*, $4\frac{1}{2}$ slots per pole. Slot 1 is in the same time-phase as slot 10, slot 2 as slot 11, etc., so that we can use one potential circle. The coil pitch is 1-4. We have $4\frac{1}{2}$ coils for

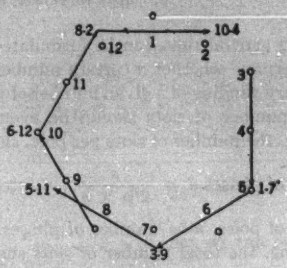

FIG. 34

every two poles. Even if we have two commutator bars per coil, we would not get zero voltage between the beginning of coil (1-4) and the middle point of the 5th coil (9-12), but the vector difference would be equal to *a-b*. However, all nine coils together give zero voltage, so that we can use this arrangement either as a four-pole series armature, or as a four-pole multiple armature, without equalizers. If we put on equalizers, a voltage equal to *a-b* would be short-circuited. Neither is it advisable to use bus rings in this case, and we should wind the secondary which connects to the brush-studs, with two separate circuits per phase

Fig. 34 is an arrangement which would tend to a large circulating current, and in which we have an even number of slots per pole and a full pitch coil. If we use (10) commutator bars

and (2) conductors per slot in all the slots with the exception of slot 5, 11, 6 and 12, in which one conductor per slot should be used, we get a closed polygon and can use this winding.

From Figs. 30, 31, 32, 33 and 34, it follows that the regulating winding can be wound with one coil per slot, as long as certain conditions are fulfilled.

We may now investigate which numbers of slots give a satisfactory primary winding. In order to get a balanced winding without excessive local leakage, it is necessary to have one coil end of the primary winding in every slot. This is possible only when the total number of primary slots is even. The coil ends belonging to the different phases, should be inserted in the same order; otherwise the phase rotation will be reversed on some parts of the circumference. This limits the use of fractional pitch coils for the primary to certain special cases, which will be described below.

The following general rules can be formulated which make it possible to determine whether a certain number of slots is suitable for a primary winding of a given number of poles and phases. (*a*) The total number of slots should be even. (*b*) To get a regular winding, the number of slots per pole, should be divisible by the number of phases $d = \frac{s}{2p \times n}$ = integer. (In which $2p$ = number of poles, n = number of phases.) (*c*) To get a balanced winding, the total number of slots should be divisible by two times the number of phases $b = \frac{s}{2n}$ = integer.

If we want to find whether a satisfactory winding can be obtained, we can proceed as follows:

Determine $e = \frac{s}{2p}$ and $b = \frac{s}{2n}$

Case 1. If e = odd number, make the coil pitch equal to e and connect primary so as to get b slots per phase and an equal number of slots per pole. (For regulating winding, see Fig. 30.)

Case 2. If e = even number, see whether e has an odd factor c and if so, make coil pitch equal to e and treat $\frac{e}{c}$ coils like under 1. (For regulating winding, see Fig. 31.)

Case 3. If e = even number, determine $\frac{s}{2\,p\,n} = d$ = number of slots per pole per phase. If d is even, then make coil pitch equal to $\frac{s}{2\,p} \pm 1$ and connect as under 1. (For regulating winding, see Fig. 32.) If d is odd, it is impossible to get a balanced winding.

Case 4. If e is a mixed number, see whether $\frac{s \times f}{2\,p}$ gives an integer in which $f = p$, or $f < p$ and $\frac{p}{f}$ = integer. This gives an irregular winding, while the regulating winding, if connected as a multiple winding, can have equalizers which span $2\,f$ poles,

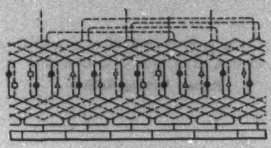

FIG. 35

instead of two-poles. If $p = f$, then we can have no equalizers. (For regulating winding, see Fig. 33.)

In all the above cases, we will get a balanced winding only when $b = \frac{s}{2\,n}$ = integer.

A few examples may make it clear how the above rules can be applied.

Case 1. The armature shown in Fig. 28 may be mentioned as an example; it has four poles, three phases and sixty slots.

$$e = \frac{60}{2 \times 2} = 15;\ b = \frac{60}{2 \times 3} = 10 \text{ and } d = \frac{60}{2 \times 2 \times 3} = 5.$$

Case 2. As an example may be mentioned the 60-slot, 6-pole, 3-phase winding used on the 10-h.p. motor, the speed-torque curves of which are covered by Fig. 25. In this case we have: $e = \frac{60}{2 \times 3} = 10$; e has an odd factor $c = 5$, while $\frac{e}{c} = \frac{10}{5} =$

2. The pitch of the primary coil is equal to 10 slots, $\frac{e}{c} = 2$, coils are treated as a single-coil.

Case 3. A 48-slot, single-pole, quarter-phase armature can be wound in this way,

$e = \frac{48}{2 \times 3} = 8$; $d = \frac{48}{2 \times 3 \times 2} = 4$. The coil pitch is equal to $e - 1 = 7$. As another example

Fig. 35 represents the diagram of both the regulating and the primary winding wound with two poles, three-phases and twelve slots. $e = \frac{12}{2} = 6$; $b = \frac{12}{2 \times 3} = 2$, and $d = \frac{12}{2 \times 3} = 2$.

Case 4. The scheme covered by this case is suitable for a 60-slot, 8-pole, 3-phase armature.

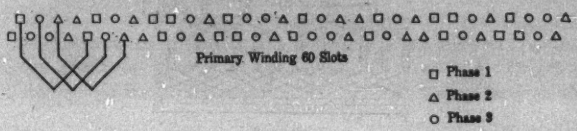

FIG. 36

$e = \frac{60}{2 \times 4} = 7.5$; $d = \frac{60}{2 \times 4 \times 3} = 2.5$, *i. e.*, the winding is irregular, so that we have to use alternately 2 and 3 slots per pole.

$b = \frac{60}{2 \times 3} = 10$, *i. e.*, the phases are balanced.

By making $f = 2$, we get $\frac{s\,f}{2\,p} = \frac{60 \times 2}{8} = 15$.

The equalizers should span $2f = 4$ poles and each phase of the secondary should have two independent circuits. An examination of Fig. 36, which covers the arrangement of the phases of this winding, will show that this arrangement repeats itself every 30 slots, which means that equalizers spanning 30 slots, *i. e.*, four poles will connect slots of equal potential, with respect to the primary winding. From Fig. 33, it follows that these

equalizers are also permissible, as far as the regulating winding is concerned.

A 60-slot armature is particularly suitable for this type of winding, as it can be wound with 2, 4, 6, 8, 10 and 12 poles. It is possible to use the same commutator and brush rigging for motors having 4, 6, 8 and 12 poles, by building the secondary with 6, 4, 3, and 2 phases, respectively. This leads to a reduction in the number of required mechanical parts for motors of different frequencies and speeds, which is very advantageous from a manufacturing point of view, and will make this type of motor less costly as soon as there is a sufficient demand for it.

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THE SECOMOR

A Kinematic Device Which Imitates the Performance of a Series-Wound Polyphase Commutator Motor

BY V. KARAPETOFF

Abstract of Paper

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

List of Symbols Used

E total useful voltage (or counter e. m. f.) of the machine.
E_1 stator e. m. f.
E_2 rotor e. m. f. at standstill.
I current in the machine without the transformer.
I_1 stator current with the transformer (Fig. 11).
I_2 rotor current with the transformer (Fig. 11)
I_m exciting current of the machine (Fig. 6)
I_0 exciting current of the series transformer (Fig. 11).
M exciting m. m. f. of the machine (Fig. 6).
M_1 stator m. m. f.
M_2 rotor m. m. f.
n_1 effective stator turns
n_2 effective rotor turns.
P line voltage.
r resistance of the motor circuit
s slip expressed as a fraction.
u ratio n_2/n_1 of the effective rotor turns to the effective stator turns.

x leakage reactance of the machine.
Z_1 primary leakage impedance of the series transformer.
Z_2 secondary leakage impedance of the series transformer.
z leakage impedance of the machine.
α brush shift angle measured from the opposition setting.
α' same, modified by the transformer magnetizing current, Fig. 12.
β, γ_0 angles defined in Fig. 6; an angle defined in Fig. 12.
Φ useful flux in the air-gap.
ϕ phase angle of the machine.

A. Introduction

The Meaning of the Name Secomor: An abbreviation of the words "series commutator motor".

What the Secomor is. A combination of movable and adjustable bars (Fig. 9) which can be set to represent a vector diagram of voltages, currents, m.m.f's and fluxes in a series-wound polyphase commutator motor with any desired constants.

The Purposes of the Device. (1) To enable a designer to select the best electrical constants and to "test" a motor before it has been actually built; (2) To take the place of a complicated circle diagram which does not hold true anyway when the iron is saturated; (3) To do away with an involved analytical theory because it is often difficult to see the effect of separate factors upon the performance characteristics, and because it takes considerable mathematical skill to deduce the equations of the various loci; (4) To add the judgment of the eye and the skill of the hands to the purely mental ability in selecting the constants of a motor for a desired performance, or in judging the characteristics for assumed constants; (5) To enable an investigator or a student to familiarize himself with the motor as if he had one available for tests.

The performance curves that the secomor enables one to draw: Current, torque, speed, input, output, efficiency, power factor, magnetizing current. These may be obtained just as easily at a constant applied voltage as at a constant current, or under any variable conditions of service.

The factors which may be taken into account and varied at will in the secomor: The ratio of the primary to the secondary ampere turns; the ratio of either one to the exciting m.m.f.; the angle of brush shift; the ohmic drop and the reactive drop;

saturation in iron; variable core-loss and friction; reaction of short-circuited armature coils upon the exciting current.

B. General Properties of the Motor

A detailed description of the motor, its field of application, its performance characteristics and a complete theory, both graphical and analytical, will be found in E. Arnold's "Die Wechselstromtechnik", (1912) Vol. V, Part 2, Chap. 2. See also the bibliography below.

Diagrams of connections are shown in Figs. 1, 2 and 3. The stator is phase-wound and is similar to that of an induction motor. The rotor is like a d-c. armature with a commutator. With a multiple armature winding the number of brushes per

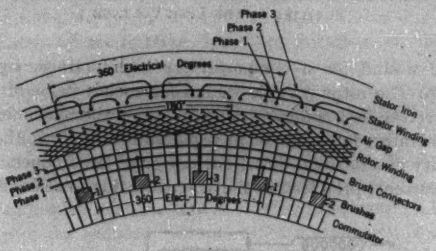

Fig. 1—The Stator, the Armature Winding and the Brushes in a Three-Phase Commutator Motor

pair of poles is equal to the number of phases; with a two-circuit winding it is considerably less.

The rotor is connected in series with the stator, either directly (Fig. 2), or through a current transformer (Fig. 3). In the latter case the stator may be either star or mesh connected.

Characteristics and the field of application. The motor has a speed-torque characteristic similar to that of a d-c. or single-phase series motor. As the load increases the speed decreases. At no-load there is a tendency to run away. Roughly speaking, one may consider the motor as a combination of three single-phase series-connected motors built on the same frame. On account of the mutual action between the phases such a view is not quantitatively correct. The motor may be used for crane service and in other applications in which it may replace a series-wound d-c. motor. A peculiar field of its own is in cascade

connection with an induction motor, as a counter e.m.f. arrangement for speed control. This is useful in mine-hoist work and in rolling mills; see Arnold, p. 266. A shunt-connected polyphase commutator motor is also suitable for cascade connection.*

Advantages of a series transformer (Fig. 3): (a) The stator

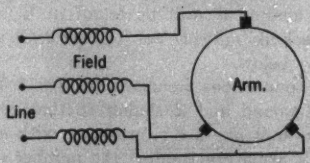

FIG. 2—THREE-PHASE SERIES COMMUTATOR MOTOR—THE ARMATURE SUPPLIED AT THE LINE VOLTAGE

may be wound for a comparatively high voltage, *e.g.*, 2200 volts; (b) The rotor may be wound for such a value of current as to give the best commutation; (c) When the machine is used as a generator, for example in regenerative braking, there is a possibility of a harmful direct current or of low-frequency alternating currents produced in the armature, due to self-excitation.

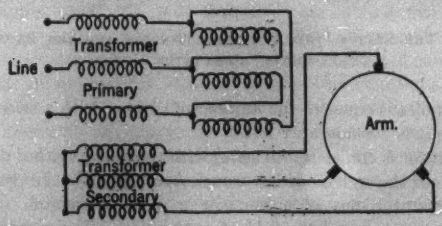

FIG. 3—THREE-PHASE SERIES COMMUTATOR MOTOR—THE ARMATURE SUPPLIED THROUGH A TRANSFORMER

By saturating the current transformer this tendency is counteracted; see Arnold, p. 64. (d) By properly selecting the saturation and the magnetizing current of the transformer the speed above synchronism may be controlled to some extent, so that the motor does not run away when the load is removed.

*See J. D. Wright, *General Electric Review*, 1916, p. 104.

Four fundamental properties of an armature connected to a polyphase a-c. circuit through a commutator and brushes, Figs. 1 and 2.

(a) The *currents* in the armature conductors are of the same frequency as the line currents, independent of the speed of rotation. Call the armature coils (Fig. 1) between two adjacent brushes "a group". The function of the commutator is to transfer the coils in succession from one group to the next. But a new coil is always substituted in place of the one transferred, so that *the group persists* and forms a steady path for the line currents. This, of course, does not apply to the coils undergoing commutation, in which high-frequency currents are induced.

(b) The *voltages* induced in the armature conductors are always of the same frequency as that of the stator flux which induces them. The relative speed of rotation of the flux and armature influences only the magnitude of the induced voltage but not its frequency. Consider a d-c. machine in which the field current and the flux are periodically varied. The e.m.f. induced between the brushes undergoes corresponding fluctuations. If the field be varied harmonically, the induced e.m.f. will also vary at the same frequency according to a sine law. Speeding up the armature or slowing it down will influence the value of the induced voltage, but its frequency is always that of the flux. With a constant flux the voltage is also constant. Similarly, in an a-c. motor the instantaneous voltage induced in a group of coils between adjacent brushes is proportional to the instantaneous value of the stator flux and to the speed of rotation. The stator flux varies at the impressed frequency, and so does the induced voltage. This simple property of armature currents and voltages makes it possible to use vector diagrams at any speed of rotation. It will be remembered that in an induction motor (without commutator) the frequency of the secondary currents depends upon the speed of the rotor.

(c) The *magnetomotive force* and the *flux* due to the armature currents are nearly the same whether the armature is revolving or is at rest, provided that the armature current is kept constant. We have seen above that the groups of armature coils between the brushes are always stationary, and the currents that flow through these groups are of the same frequency as the line currents. Hence, the *m.m.fs. of the individual groups* are sta-

tionary in space at all speeds of the armature. The combined action of several groups of coils produces a revolving field, same as in the stator winding, and this field glides synchronously in the air-gap with respect to the brushes, no matter what the speed of the armature might be. The m.m.f. due to high-frequency currents in the coils undergoing commutation is a disturbing factor, being a function of speed, current, stator flux, etc., but its effect is usually small. For its computation see Arnold, pp. 23 and 56.

(d) The *leakage reactance* of a commutated armature is a function of its speed, unlike that of ordinary a-c. windings. Two kinds of leakage fluxes may be distinguished, (1) those linking with individual armature conductors or with groups of conductors belonging to the same phase, and (2) those linking with conductors belonging to different phases. The first named fluxes (phase leakage) are carried around with the conductors and cause a reactance drop which is independent of the speed of rotation of the armature. The latter fluxes (interlinked leakage) in combination form a true revolving flux which glides synchronously in the air gap whether the armature is standing still or revolving at any speed.

The effect of the interlinked leakage flux depends upon the slip of the armature. At synchronous speed the armature conductors do not cut this flux at all, and the only reactance is that due to the phase fluxes which travel with the conductors. At standstill the interlinked leakage flux exerts its full effect. At speeds above synchronism the armature conductors cut this flux in the opposite direction, so that it partly compensates for the effect of local or phase fluxes. At a certain synchronous speed the total armature reactance becomes zero, and beyond that speed it becomes negative. Whatever the theory of the phenomenon, actual experiments show that the reactance of a commutated armature winding decreases with increasing speed. See Arnold, p. 20.

Brush shift and the m.m.fs. The alternating currents in the three stator phases together produce an m.m.f. M_1 (Fig. 4) of constant amplitude. This m.m.f. is distributed in space approximately according to a sine law, and it glides synchronously along the air gap. The action is the same as in the stator of an ordinary polyphase induction motor; see for example the theory as treated in the author's "Magnetic Circuit", p. 128. The three rotor currents together produce a similar gliding magneto-

motive force M_2, whether the armature is revolving or stationary. Its position in space with respect to M_1 depends only upon the angle of brush shift and is independent of the value of the

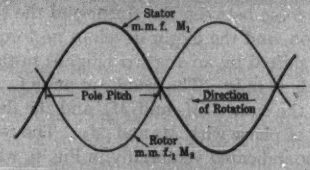

FIG. 4—STATOR AND ROTOR M. M. F'S IN SPACE—THE BRUSHES ARE IN THE NEUTRAL POSITION

current or of the speed of rotation. This is because the m.m.fs. in a commutated armature are the same as if the armature were at rest (see proof above). In a certain position of the brushes

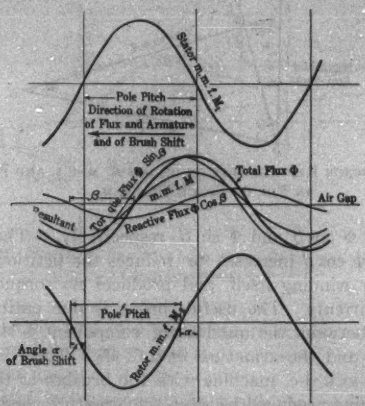

FIG. 5—STATOR AND ROTOR M. M. F'S. AND THE RESULTANT FLUX IN SPACE—THE BRUSHES ARE SHIFTED BY AN ANGLE α IN THE DIRECTION OF ROTATION

called *neutral* the stator and the rotor m.m.fs. are in phase opposition in space (Fig. 4). In this paper the angle of brush shift α is measured from this position. Arnold measures the brush shift angle ρ from the position at which M_1 and M_2 are

in phase coincidence. Hence, his $\rho = 180$ deg. $- \alpha$. In the secomor it is more convenient to deal with an acute angle α than with an obtuse angle ρ.

The Torque. In the neutral position of the brushes (Fig. 4) the mechanical force between the stator and the rotor is simply a radial repulsion, and the motor develops no torque. Let now the brushes be shifted by an angle α (Fig. 5) in the direction of rotation of both m.m.fs. The resultant m.m.f. M in the air-gap is equal to the sum of M_1 and M_2, and the flux Φ which M produces is out of phase with M_1 and M_2. The relations shown in Fig. 5 are also indicated vectorially in Fig. 6, which is a *space* (not time) diagram. The actual air-gap flux Φ may be resolved into two space components, one in phase (or phase opposition) with M_2, the other in quadrature with it. These components

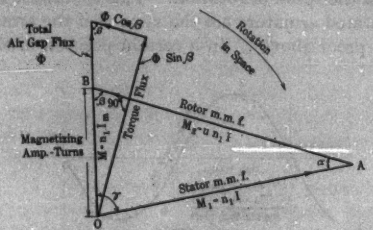

FIG. 6—A VECTOR REPRESENTATION OF THE M. M F'S. AND FLUX SHOWN IN FIG. 5—SPACE DIAGRAM

are marked $\Phi \cos\beta$ and $\Phi \sin\beta$ respectively. The in-phase component $\Phi \cos\beta$ increases or reduces the fictitious flux due to the rotor winding itself, and produces no torque with the armature currents. The useful torque is due entirely to the interaction between the quadrature component $\Phi \sin\beta$ of the air-gap flux and the armature m.m.f. M_2. There is a similar relationship in a d-c. machine with the brushes in the neutral, no torque being produced between the armature conductors and the flux of armature reaction. The useful flux must be in space quadrature with that which the armature windings themselves excite.

Direction of Rotation of the Armature. By marking the directions of the currents and fluxes in Fig. 5 and applying the familiar right-hand screw rule one finds that the torque is exerted upon the armature conductors in the direction in which the

brushes are shifted from the neutral. A simple way to check this fact is to consider the fictitious fluxes due to M_1 and M_2 acting alone. Since the m.m.fs. are nearly in phase opposition, these fluxes repel each other, and the armature tends to carry its m.m.f. away from M_1, by moving to the left. The stationary brushes fix definite groups of armature conductors in space, so that the armature moves away steadily without coming into a position of stable equilibrium. If the brushes were shifted in the opposite direction, the direction of rotation would be reversed. It is of importance to note that the characteristics of the motor are entirely different for the two directions of

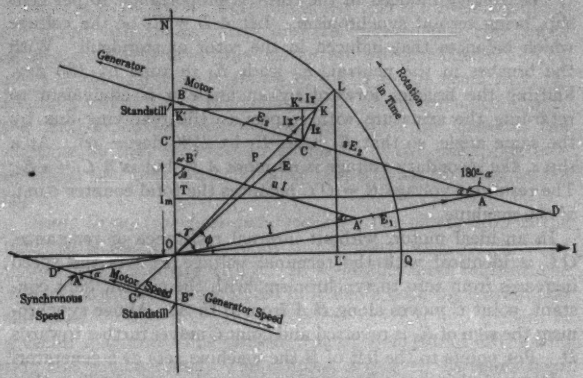

FIG. 7—TIME DIAGRAM OF VOLTAGES AND DATA FOR PERFORMANCE CHARACTERISTICS

rotation. In the first case the armature conductors revolve in the direction of the flux, so that the rotor e.m.f. at synchronism becomes zero, and then its sign is reversed. In the second case the rotor e.m.f. increases indefinitely with the speed of rotation. In practise the first combination has been used so far, and is the only one considered in this paper.

C. TIME DIAGRAM OF E.M.FS.

Consider the motor running at a given current, which is kept constant and let the speed vary, the terminal voltage being adjusted for each speed. The e.m.fs. M_1 and M_2 are constant and

so is the resultant flux Φ, so that Fig. 6 applies at all speeds. The flux Φ induces certain e.m.fs. in the stator and rotor windings, and these e.m.fs. are shown in Fig. 7.

Fig. 7 is a time diagram, and it shows the applied voltage $OK = P$ consumed, in three parts: (1) $OA = E_1$ is that part of the applied terminal voltage which is equal and opposite to the e.m.f. induced by flux Φ in the stator windings. For the sake of brevity E_1 is further referred to as the stator voltage. (2) $AC = sE_2$ is the voltage consumed in the rotor, in opposition to the e.m.f. induced by flux Φ. (3) $CK = Iz$ is the impedance drop in the windings of the machine and in the brushes.

The voltage induced in the rotor is proportional to per cent slip, being zero at synchronism. Let $AB = E_2$ be the voltage which balances that induced in the rotor at standstill. With the brushes in the neutral, E_2 leads E_1 in time by 180 deg. Shifting the brushes forward by an angle α is equivalent to retarding the armature with respect to the revolving flux by the same angle, so that E_2 leads E_1 by (180 deg. $- \alpha$). At a slip s, the secondary voltage is no more AB, but is $AC = sE_2$. The resultant voltage $E = OC$ balances the total counter e.m.f. of the machine.

In an ideal motor, without internal resistance or reactance, OC is identical with the terminal voltage P. As the speed increases from zero to synchronism, with the current kept constant, point C moves along BA from B to A. Above synchronism the sign of E_2 is reversed and point C moves further towards D. For points to the left of B the machine acts as a generator.

In an ideal motor no power is consumed at standstill so that the applied voltage OB at the speed zero must be in leading quadrature with the horizontal current vector I, which is the reference vector. Thus OB in the diagram must be vertical. As the speed increases the phase angle between OC and I decreases. By continuing BD further to the right to its intersection with I, a super-synchronous speed is found at which the applied voltage is in phase with the current. Beyond that speed, the motor takes in a leading current.

The Triangles of Voltages and of m.m.fs. are Similar. In the construction of the secomor use is made of the fact that the triangles OAB in Figs. 6 and 7 are similar. The two m.m.fs., M_1 and M_2, being produced by the same current are displaced *in space* by 180 deg. $- \alpha$, where α is the brush shift angle measured from the opposition point (Fig. 5). Similarly, the two voltages,

E_1 and E_2, being induced by the same flux Φ, are displaced *in time* by 180 deg.$-\alpha$. The ratio of M_1 to M_2 is equal to that of the effective numbers of turns in the stator and in the rotor; the ratio of E_1 to E_2 is equal to the same ratio of turns. Thus, the two triangles have an equal angle between two proportional sides, and therefore are similar. This important relationship permits to incorporate the m.m.f. triangle $O\,A'B'$ in the time diagram (Fig. 7) and to do away with a separate space diagram. To take into account the m.m.f. of the short-circuited coils undergoing commutation a slight correction is necessary; see Arnold, p. 56.

Leakage Inductance. Both the stator and the rotor windings are linked with leakage fluxes, and the corresponding reactances cause a voltage drop in time quadrature with the current. The effect of the reactance is shown in Fig. 7 in the usual way, by adding a vector $I\,x$ to the voltage $O\,C$. As is explained above, the total armature reactance depends upon the speed of the machine, so that the vector $I\,x$ consists of two parts, one proportional to I only, and the other proportional to I and to the slip s.

Ohmic Drop, Iron Loss and Friction. The ohmic drop in the machine is taken into account in the usual way by means of the vector $I\,r$ in phase with I. In a variable-speed machine the iron loss is rather a complicated function of both the current and the speed, so that it is hardly feasible to take it into account in a vector diagram. The same is true to some extent of the friction and windage loss. The simplest way is to disregard them in the vector diagram and to correct the obtained values of input and output afterwards. One may also increase the actual ohmic resistance of the machine so as to include the iron loss in the term I^2r, and consequently in the vector $I\,r$, but the procedure is of doubtful accuracy and value.

The Saturation Curve. In a series-wound machine it is hardly permissible to disregard the effect of saturation in the magnetic circuit upon the performance at large values of current. The starting torque and the characteristics at low speeds are appreciably affected thereby. Let Fig. 8 represent the a-c. saturation curve of the magnetic circuit of the machine. To obtain it experimentally, the brushes are raised from the commutator, and the armature is run at the synchronous speed by some external power. The stator winding is connected to a source of variable a-c. voltage, and a series of readings is taken of the

voltage E_1 and of the corresponding magnetizing current I_m. The values of E_1 are corrected for the ohmic drop and those of I_m for the core loss. The corrected values give the curve shown in Fig. 8. This curve is used in connection with the diagram in Fig. 7, and also in connection with the secomor.

D. Performance Characteristics of the Motor

The brush shift angle α and the ratio $u = M_2/M_1$ of the effective rotor turns to the effective stator turns are the data to begin with. These determine the shape of the triangle $O\,A\,B$ (Fig. 7). Its position with respect to I is also determined since $O\,B$ is perpendicular to I. Let $O\,A'B'$ be the m.m.f. triangle transferred from Fig. 6. Then if we make $O\,A'$ equal to I (in magnitude only, but not in phase), $O\,B'$ will represent the magnetizing current I_m. This is because in Fig. 6 all the three

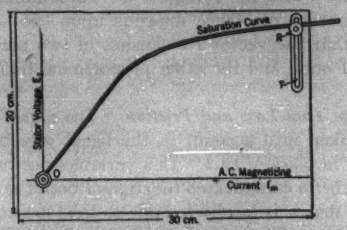

Fig. 8—An Adjustable Saturation Curve Made of Soft Wire

sides of triangle $O\,A\,B$ have a common factor n_1, that is, the number of stator turns.

From the saturation curve (Fig. 8) we get the value of E_1 corresponding to this magnetizing current and lay it off as $O\,A$. This determines the direction $B\,D$. The rest of the problem is to find such a point C that the impedance triangle $I\,x$, $I\,r$, constructed at it, would give the desired terminal voltage $O\,K = P$ in magnitude. This is purely a geometrical problem which can be solved either analytically or graphically. In the secomor the point C is found mechanically, by means of a few simple trials, shifting one part of the device relatively to another. Herein lies one of the principal advantages of the secomor over the analytical or the graphical method.

Speed. In Fig. 7, $B\,C = (1 - s)\,E_2$, so that the speed of the machine $(1 - s) = B\,C/B\,A$, expressed as a fraction of the

synchronous speed. Instead of measuring every time two lengths and taking their ratio, it is convenient to draw an arbitrary straight line $A''B''$ parallel to $A\ B$. Produce $B\ O$ and $A\ O$ so as to get points B'' and A'', and divide $A''B''$ into 100 equal parts, beginning with zero at B''. Extend the divisions both ways to cover the operation above the synchronous speed, and also with the machine running backward as a generator. For any operating point, such as C, extend $O\ C$, and read per cent speed at C''. A proof for this construction follows directly from the similar triangles $O\ A\ B$ and $O\ A''\ B''$.

Input, Output and Efficiency. The vector $K\ K'$ is the component of the applied voltage P in phase with the current. Therefore $K\ K' \cdot I$ represents the input into the motor per phase. To it should be added the estimated core loss in the stator. The mechanical output is proportional to the component of the counter-e.m.f., E, in phase with the current; hence the output is equal to $C\ C' \cdot I$. The friction loss and the core loss in the armature should be subtracted from this value. The efficiency is found as the ratio of the true output to the true input.

Power Factor. The power factor, $\cos \phi$, is equal to the ratio of $K\ K'$ to $O\ K$. A simple way to read it directly is to draw the quadrant $N\ L\ Q$ with O as a center, using 100 convenient divisions, for example 100 mm., as the radius. The intersection L of $O\ K$ with the quadrant determines point L'. Radius $O\ Q$ is marked with a uniform scale, zero being at O to 100 at Q. Thus, the power factor is read at L' directly in per cent.

Torque. In a d-c. series-wound motor the torque is a function of the current and is practically independent of the speed. In the motor under consideration the torque is also independent of the speed, except for the disturbing effect of the armature coils undergoing commutation. The torque depends essentially upon the current and upon the brush shift. With the given I and α the torque is nearly the same at standstill as at any other speed. But at standstill $A\ T$ represents the energy component of both the stator voltage E_1 and the rotor voltage E_2. The amount of power $A\ T \cdot I$ is absorbed in the stator and an identical amount is returned to the line from the rotor. Thus, $A\ T \cdot I$ is the torque in synchronous watts, corresponding to I at any speed.

Another proof is as follows: In Arnold, on bottom of p. 41 the torque in synchronous watts is expressed as $I^2\ x_a\ u \sin \alpha$. Here $x_a = E_1/I_u$ is the so-called exciting reactance of the machine. Multiplying and dividing the foregoing expression

by I_m we get, torque $= I^2 x_s u \sin \alpha = E_1 I (u I \sin \alpha / I_m)$. But from the triangle $O A' B'$ (Fig. 7) we have that $u I / \sin \gamma = I_m / \sin \alpha$. Substituting in the preceding equation we find that the torque $= E_1 I \sin \gamma = A T \cdot I$.

Recapitulation. The motor characteristics (Fig. 7) are essentially determined by the ratio $O A$ to $A B$ of the effective stator and rotor ampere-turns, and by the brush shift angle α. The direction of the vector of current I is always along the axis of abscissas, that of the terminal voltage $O K$ approaches I as the speed increases. Select a value of I and plot $O A' = I$, remembering that $O A'$ does not represent the true direction of I in the time diagram, but only its magnitude. Then the following values are obtained:

Magnetizing current $O B' = I_m$

Stator voltage $O A = E_1$. It is found from Fig. 8 as the ordinate corresponding to abscissa I_m

Rotor voltage at standstill $A B = E_2$

Rotor voltage at slip s, $A C = s E_2$

Total useful voltage at slip s, $O C = E = \underset{\cdot}{E_1} + s \underset{\cdot}{E_2}$ (geometric addition)

Reactive drop $C K'' = Ix$

Ohmic drop (which may be increased to cover core loss) $K'' K = I r$

Terminal voltage $O K = \underset{\cdot}{P} = \underset{\cdot}{E} + \underset{\cdot}{I} x + \underset{\cdot}{I} r$

Slip $A'' C'' = s$, where $A'' B'' = 100$ per cent slip

Speed $C'' B'' = 1 - s$, where $A'' B'' = 100$ per cent synchronous speed.

Input $K K' \cdot I$

Output $C C' \cdot I$

Efficiency $C C' / K K' = 1 - (K K'' / K K')$

Power factor $\cos \phi = K K' / O K = O L'$, where $O Q = O L = 100$ per cent

Torque in synchronous watts $A T \cdot I$

E. The Secomor and its Use

The secomor (Fig. 9) is built out of four flat iron bars; these bars represent the principal vectors in Fig. 7. Each bar is provided with a centimeter scale, and they are mounted on an ordinary drafting board. A sheet of cross-section paper is tacked to the board and serves as a universal scale. The four bars and their functions are as follows:

(a) *The stator bar*, $O\,A$, determines the stator voltage E_1 and the current $O\,A' = I$ (Fig. 7)

(b) *The rotor bar*, $D\,B$, determines the rotor voltages E_2 and $s\,E_2$.

(c) *The speed bar*, $D''B''$, determines the slip s or the speed of the motor.

(d) *The setting bar*, $C''L$ determines the point C for which the useful voltage E plus the impedance drop $C\,K$ gives the terminal voltage $P = O\,K$.

The setting bar is pivoted at O next to the cross-section paper, the stator bar is pivoted on top of it, and the other two bars slide on top of the stator bar. On each bar one edge is called the "reading" edge; on the pivoted bars the reading edge passes through the geometric center of the pivot O.

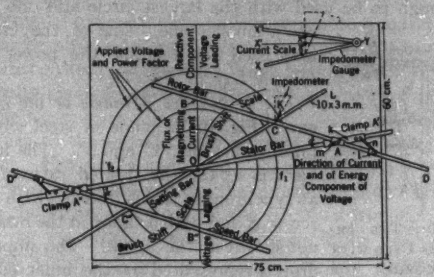

FIG. 9—THE SECOMOR—THE NOTATION CORRESPONDS TO FIG. 7

The clamps A' and A'' have to be rather carefully made, to permit of an accurate setting and not to have too much lost motion. The long sleeve k of the clamp slides on the stator bar; the sleeves l and m guide the rotor bar as it slides in them. The sleeve m is pivoted on top of sleeve k; l is connected to k by means of a pivoted rod n. By tightening the set screw on k the rotor bar may be fixed at any desired point of the stator bar. By loosening the set screws on l and m the angle α can be adjusted, and after the screws have been tightened the angle remains constant. Part of each sleeve is cut out to enable one to see the scale and the reading edges of the bars.

It would be inconvenient to use a protractor for setting the rotor bar and the speed bar at a desired brush-shift angle α. Therefore, two large circular scales are provided on the base

of the apparatus, with centers on the axis of abscissas, at f_1 and f_2. These scales are marked in degrees and each is labeled in Fig. 9 "brush shift scale". To use the scale f_1 the stator bar is so turned that its reading edge coincides with the axis of abscissas; the clamp A' is shifted until the reading edge of the rotor bar passes through f_1. Then the rotor bar may be set at any desired angle. After the set screws on l and m have been tightened the angle remains unchanged when the clamp is shifted. The speed bar must be set at the same angle at f_2, and the two bars must always be parallel to each other when the secomor is in use.

The four concentric circles with the centers at O are used for the applied voltage. Any of them can be selected as a locus of point K. With some motor characteristics the bars are not long enough to be used with the largest circle and one has to use a smaller one. One of the circles is also used for reading the power factor (arc $N L Q$ in Fig. 7).

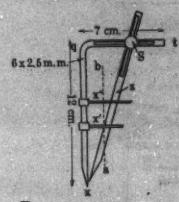

FIG. 10—THE IMPEDOMETER USED WITH THE SECOMOR FOR TAKING INTO ACCOUNT THE IMPEDANCE DROP

The Impedance Triangle or the Impedometer (Fig. 10) is made of steel with two brass indicators, x' and x''. The inclined bar z is set for the desired ratio of r to x at synchronism and is fastened with the set screw S and with a clamp at the other end, not shown in the figure. The impedance drop is represented by the inner edges of the three bars. The ower indicator, x', is set for the desired value of $I\,x$ at synchronous speed, the upper indicator x'' is set for the value of $I\,x$ for standstill at the same current. The value of $I\,x$ for any sub-synchronous speed lies between x' and x'', and for super-synchronous speeds it lies below x'. These values can be readily estimated by the eye, the whole correction for the impedance drop being small. Point K (Fig. 7) lies somewhere on $a\,b$ (Fig. 10) but it is not necessary to have this line marked on the impedometer, because the device, when in use is lying on the cross-section paper with $x\,q$ parallel to one of the rulings (Fig. 9), and the eye easily follows the direction $a\,b$.

The two diverging straight edges X and X'' (Fig. 9) shown in the upper right-hand corner of the secomor serve as a guage for a quick setting of the impedometer. The horizontal line X' is one of the rulings on the cross-section paper, and serves as a current scale and the locus of the point x' of the impedo-

meter. Bar X is the locus for point x, and bar X'' is the locus for x''. The set screw Y fixes the bars in any desired position.

The saturation curve (Fig. 8) used with the secomor, is made out of a piece of ordinary solder wire, and can be readily bent and adjusted by hand to any desired shape. The lower end of the wire is held in a swivel clamp at O, and the upper end is clamped at R. The clamp R can be loosened and moved up and down in the slot T. The whole is mounted on a piece of board; a sheet of cross-section paper tacked to it serves as a universal scale.

The method of using the secomor follows directly from Fig. 7, the setting for different constants and for different loads being accomplished by shifting or turning the four bars. The reader can simply follow the "Recapitulation" above. As to the selection of scales for volts and amperes, the simplest method seems to be always to read everything directly in centimeters and to use constants afterwards. When testing an actual machine one usually reads the meters without regard to their constants, and later recomputes the data.

The use of the impedometer requires no particular skill or precision. Having located point C approximately near the chosen circle of applied voltage, the impedometer and the setting bar are shifted to and fro until the electrical condition for the sum of the voltages is fulfilled. This condition depends upon the speed which is simultaneously read at C''. Point K must always lie on $a\,b$ (Fig. 10), but its exact position on $a\,b$ is determined by the speed of the machine. At standstill it must lie on the intersection of $a\,b$ with the indicator x'', at synchronism it lies on the intersection of $a\,b$ with x'. At any other speed K divides $x'x''$ in the same proportion in which C'' divides $A''B''$. The length $x'x''$ being small as compared to $O\,C$ or $O\,K$, the judgment of the eye is amply sufficient. An engineer who uses the device regularly will soon find several short cuts which it is not necessary to mention here.

The Effect of a Series Transformer. When the armature of the motor is fed through a series transformer (Fig. 3) the vector diagrams (Figs. 6 and 7) become somewhat more involved, and the use of the secomor is accordingly modified.* A series transformer introduces the following new factors:

(a) A magnetizing current and an appreciable saturation in

*For a detailed graphical and analytical treatment of this case see an article by Dreyfus and Hillebrand in the *Elektrotechnik und Maschinenbau*, Vol. 30 (1912), p. 389.

the transformer core at speeds remote from the synchronous speed, when the armature voltage is high.

(b) A primary and a secondary leakage impedance, due to the transformer windings.

(c) An iron loss in the transformer core.

(d) A new vectorial difference in time phase between the armature and the field currents, and consequently a new space angle between the primary and secondary m. m. f's, not determined by the brush shift alone.

The equivalent electric diagram which takes the series transformer into account is shown in Fig. 11, for one phase only. First, the actual transformer is replaced by a one-to-one transformer, with the corresponding changes in the constants of the

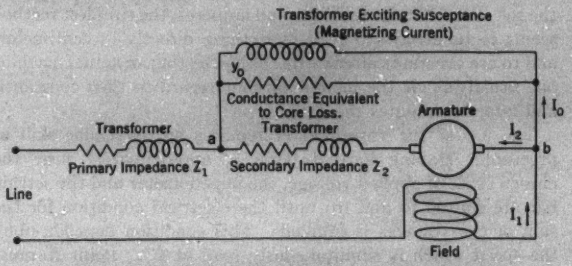

FIG. 11—THE EQUIVALENT DIAGRAM OF CONNECTIONS WHEN THE ARMATURE IS SUPPLIED WITH POWER THROUGH A TRANSFORMER, ACCORDING TO FIG. 3

armature circuit. Then this equivalent transformer is removed and an exciting admittance γ_0 is shunted around the armature to provide a path for the magnetizing current of the actual transformer and for its core loss. The resistance of the transformer windings and the effect of the magnetic leakage are taken into account by means of two equivalent impedances, Z_1 and Z_2, in the primary and armature circuits respectively. The combination of γ_0, Z_1 and Z_2 is equivalent to the actual transformer in its effect upon the performance of the motor.†

The magnetizing current of the transformer is a function of the voltage between the points *a* and *b* (Fig. 11), and is supposed

†For the theory of the replacement of a transformer by equivalent mpedances see for example the author's *Electric Circuit*, Chap. XI.

to be given in the form of an a-c. saturation curve similar to the saturation curve of the motor itself, shown in Fig. 8. The voltage E_{ab} between a and b is somewhat different from the e. m. f. $s\,E_2$ induced in the armature, the difference being due to the impedance drop between a and b. But it is only at speeds far remote from synchronism that E_{ab}, and consequently the magnetizing current of the transformer, are of importance. For such speeds the difference between E_{ab} and $s\,E_2$ may be neglected in the first approximation, and the magnetizing current assumed to be a function of the counter e. m. f. $s\,E_2$ of the machine. A

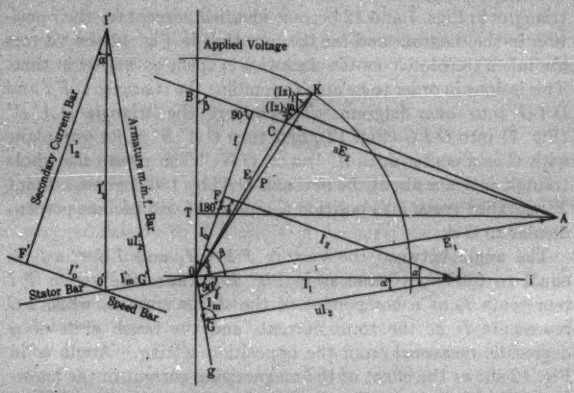

FIG. 12—VECTOR DIAGRAM OF CURRENTS AND VOLTAGES, TAKING INTO ACCOUNT THE MAGNETIZING CURRENT OF THE TRANSFORMER AND ITS IMPEDANCE

correction may be made later, using the approximate values first obtained.

Vector Diagram with Transformer. In Fig. 12, let the triangle $O\,A\,B$ represent the same voltages as in Fig. 7, and let $O\,I$ be the vector of primary current I_1 which flows through the stator windings. The secondary or armature current I_2 differs from I_1 by the amount I_0 of the magnetizing current in the transformer, as shown in triangle $O\,F\,I$. The transformer is supposed to have a one-to-one ratio. According to the approximate assumption made above, I_0 is a function of the armature e. m. f., a-c., and lags behind it by 90 degrees (neglecting

the core loss in the transformer). The corresponding corrections will be considered later.

The m. m. f. space triangle which is shown in Fig. 7 in the position $O A' B'$, is shown in Fig. 12 in the position $O I G$. One should think of its three sides as being multiplied by the number n_1 of the effective stator turns (Fig. 6). It expresses the fact that the stator ampere-turns $n_1 I_1$ and the armature ampere-turns $n_2 I_2 = u n_1 I_2$ together give the magnetizing ampere-turns $n_1 I_m$ of the machine. The magnetizing current I_m is drawn in lagging quadrature with the stator e. m. f. E_1. When the magnetizing current of the transformer $I_0 = 0$ the m. m. f. triangles in Figs. 7 and 12 become identical, except for their position in the diagram and for the fact that in Fig. 12 the vectors are taken to rotate counter-clockwise in space as well as in time. This is done in order to be able to combine the triangles $O F I$ and $O I G$ into one diagram. To convert the triangle $O A' B'$ (Fig. 7) into $O I G$ (Fig. 12), first turn $O A' B'$ in its own plane with O as a center until A' lies on $O I$. Then rotate the whole triangle in space about the new axis $O A'$ by 180 degrees. Point B' will then come into position G, and $O B'$ will become perpendicular to $O A$.

The angle between the vectors $F I = I_2$ and $I G = u I_2$ is equal to $180° - \alpha$, same as in Fig. 7, because the vector $F I$ represents I_2 as a component of the stator current, while $I G$ represents I_2 as the rotor current, and the brush shift of α degrees is measured from the opposition setting. Angle α' in Fig. 12 shows the effect of the magnetizing current in the transformer upon the space phase-angle between the stator and rotor m. m. f's. When $I_0 = 0$, $\alpha' = \alpha$.

Since the directions $O A$ and $A B$ are fixed, the directions $O f$ and $O g$ of the vectors of the two magnetizing currents are also fixed, as well as the direction $O I$. When the load varies, points G, F, and I move on their respective loci in such a manner that the angle $F I G$ is always equal to α. When this condition is fulfilled the ratio of $I G$ to $I F$ remains equal to u. This property of the quadrilateral $O F I G$ is of sufficient importance to warrant a proof.

Lemma. In a variable quadrilateral such as $O F I G$ (Fig. 12) in which the sum of the two opposite angles remains equal to 180°, and the angles α and $180° - \alpha$ are constant, the ratio of $I G$ to $I F$ remains constant when the three vertices I, F, G, move in an arbitrary manner on the three loci $O i$, $O f$, and $O g$.

Proof: If the sum of the angles at O and I is equal to π, then the sum of the angles at G and F must also be equal to π, and we have:

$$\frac{I_1}{\sin \delta} = \frac{u\, I_2}{\sin \gamma} = \frac{I_2}{\sin \beta}$$

from which

$$u = \frac{\sin \gamma}{\sin \beta}$$

With the motion of the three vertices as specified above, the angles β and γ remain constant, so that u also remains constant, and this proves the theorem.

This value of u is the same as that used in the construction of the voltage triangle $O\,A\,B$, because from this triangle we have

$$\frac{\sin \gamma}{\sin \beta} = \frac{A\,B}{O\,A} = \frac{u\,E_1}{E_1} = u.$$

Transformer Attachment. In the secomor, it would be rather inconvenient to use the quadrilateral of currents $O\,F\,I\,G$ in its true position, because the rods $O\,F$ and $O\,G$ would have to move perpendicularly to $A\,B$ and $O\,A$ respectively. For this reason he quadrilateral is turned by 90 degrees into the position $O'\,G'\,I'\,F'$, in order that the two magnetizing currents, I'_m and I'_0 may be represented by certain lengths $O'\,G'$ and $O'\,F'$ on the stator bar and the speed bar. The cross-section paper provides the direction $O'\,I'$, so that only two new members or rods are needed, viz., to represent I'_2 and $u\,I'_2$.

The actual construction of the *transformer attachment* is shown in Fig. 13; it is fastened to the secomor board (Fig. 9) in the upper left-hand corner. Two strips of brass are used to represent the sides $I'\,F'$ and $I'\,G'$ of the quadrilateral. They will be referred to as the *secondary current bar* and the *armature m. m. f. bar* respectively. These bars are pivoted together on a setscrew I' and the directions of their inner edges pass through the center of I'. The bars may be set at any desired angle by means of the graduated scale G and setscrew S'. The position of I' on the board may be varied by moving the iron rod R in the clamp C, and by moving the clamp itself along the edge of the board. S is a setscrew which holds R on C.

To illustrate the use of the transformer attachment, let us assume that the performance characteristics are desired at a given total current I_1 with a given terminal voltage. Let the

impedance drop (Fig. 12) be at first neglected so that the counter e. m. f. $OC = E$ is equal to the applied voltage $OK = P$. In the transformer attachment to the left, set the pivot I' at the proper point corresponding to the given length $O'I' = I_1$, and spread the rods $I'F'$ and $I'G'$ to the proper angle α. Select a reasonable arbitrary position of point K on the quadrant of applied voltage. This will give some tentative values of the stator voltage $E_1 = OA$ and of the armature voltage $CA = KA = sE_2$. From the saturation curve of the motor (Fig. 8) the

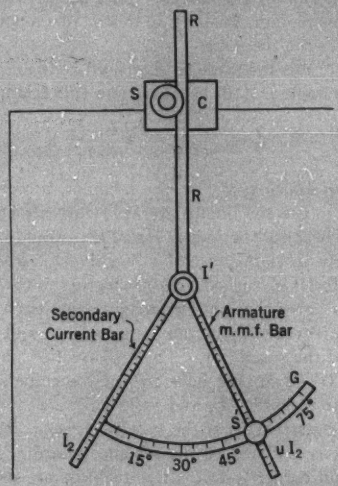

FIG. 13—THE TRANSFORMER ATTACHMENT TO THE SECOMOR

magnetizing current I_m is found, and its value is marked on the stator bar as $O'G'$ by bringing the armature m. m. f. bar $I'G'$ to point G'. Similarly, the transformer magnetizing current I_0 corresponding to sE_2 is found from its saturation curve; it must check with the value OF' indicated by the secondary current bar at F'. If it does not check, the point K was guessed at wrong and a new trial is necessary. After a few trials, the correct position of C or K is found, and this enables one to find the slip, the power factor, the torque and the output of the machine, in the same manner as without the transformer.

The Torque. The expression for the torque is not modified by the magnetizing current of the transformer, if we neglect the core loss. Since I_0 is in quadrature with E_2, the components of I_1 and of I_2 in phase with E_2 are equal to each other. Thus, either with or without the transformer the true input into the field *at standstill* is equal to the armature output. Both are a measure for the torque in synchronous watts, and the expression $A\,T \cdot I_1$, deduced above, holds true in this case, with the machine running or standing still. In the language of the Vector Analysis, using the dot for the inner or scalar product, we have:

$$I_1 = I_0 + I_2$$

$$(E_1 + E_2) \cdot I_1 = O$$

Multiplying the first equation by $E_2 \cdot$ we get

$$E_2 \cdot I_1 = E_2 \cdot I_0 + E_2 \cdot I_2$$

But $E_2 \cdot I_0 = 0$, and combining the second and the third equations we get

$$E_1 \cdot I_1 + E_2 \cdot I_2 = 0.$$

This means that the two ideal wattmeter readings are equal to each other, and either one is a measure for the torque in synchronous watts.

Correction for Impedance. The impedance drop in the motor and in the transformer (Fig. 11) may be divided into two parts:

(a) That due to current I_1 in the stator winding and in the transformer primary;

(b) That due to the current I_2 in the motor armature and in the transformer secondary.

Consequently, two impedance triangles are necessary, as is shown in Fig. 12 between points C and K. The long side of one is parallel to I'_1, that of the other to I'_2. In many cases it will be permissible to neglect the difference between I_1 and I_2 and to use a single impedometer (Fig. 10) for an intermediate value of the current, but if a greater accuracy is required, a double impedometer could be constructed, or a single impedometer applied twice.

We shall assume now that the point K has been tentatively assumed on the quadrant of applied voltage, and then the impedometer applied to find the point C. This gives the position of point A and consequently the speed and the magnetizing current I_m of the motor. The magnetizing current I_0 of the transformer has been assumed above to be a function of the induced voltage

$CA = sE_2$ in the armature. In reality it is a function of the voltage between points a and b (Fig. 11). This voltage is equal to A_m (Fig. 12), being a sum of the e. m. f. $A\ C$ and the primary impedance drop $C\ m$. Thus, a more accurate value of I_0 may be taken from the saturation curve. Strictly speaking the direction of the new I'_0 in the transformer attachment should be parallel to $A\ m$ and not along the speed bar which is parallel to $A\ C$. This small correction is best made by the judgment of the eye, without disturbing the bars.

If desired, a correction may also be made for the core loss in the transformer, as is indicated in Fig. 11 by an "exciting conductance". This means the addition to I_0 of a component in phase with $A\ C$, or more accurately, in phase with $A\ m$. Similarly, the current I_m may be corrected to account for the stator core loss.

It would complicate the mathematical theory of the motor very much if one attempted to take all these refinements into account accurately. A kinematic attachment for this purpose would also be unnecessarily involved for most practical purposes. *The device as described above gives a nearly accurate solution with a simple setting; small corrections are best applied by the judgment of the eye from this setting.* The transformer attachment has a valuable property expressed by the lemma proven above, as long as the sum of its opposite angles is kept equal to 180°. The exact values for the transformer magnetizing current and for the core losses would destroy this valuable property without an adequate advantage in the accuracy of the results achieved.

Conclusion

In presenting the secomor to the electrical profession the author wishes to point out the possibility of predicting the performance of the polyphase series commutator motor by means of a kinematic device. He also hopes to arouse interest in the use of similar kinematic devices for the prediction and analysis of performance of other types of electrical machinery. Besides the secomor, he has designed the "shucomor" or a kinematic device which imitates the performance of a shunt-wound polyphase commutator motor, and as a by-product has obtained a device which gives the performance of the ordinary induction motor. Alternator characteristics can probably be imitated in a similar manner as well as those of single-phase commutating machines.

The usefulness of such devices is not limited to a-c. machinery,

but embraces special cases of d-c. machinery as well. Sometime ago the author became interested in the operation of the Entz electromagnetic clutch and transmission used in some gasoline motor cars. He has built a kinematic device which imitates the performance of this ingenious clutch at different engine speeds and with any setting of the regulating resistances. The saturation curves of the two d-c. machines are also incorporated in the device. The device was demonstrated at the A. I. E. E. meeting in Syracuse, N. Y., in May 1917, and has been presented by the author to the Electrical Engineering Department of Syracuse University.

A kinematic device can no more replace human intelligence than a formula or a vector diagram can. But a mechanical device helps and guides an engineer's judgment, and makes it possible to achieve results with less time and trouble. One working on the design of the same type of electrical devices year after year finally attains some proficiency and needs but little outside help. But industrial efficiency demands that younger men do at least routine designing without much previous experience. A mechanical device makes it safer to entrust them with the determination of dimensions, because it enables them readily to check the performance. The effort of more gifted and mature engineers may thus be devoted to new developments and to large important problems, and less of their time need be occupied by the supervision over younger men. A wide use of mechanical devices that imitate the performance of electrical machinery is thus a step in the desired direction.

BIBLIOGRAPHY

The subject is covered quite thoroughly up to 1910 in E. Arnold's "Die Wechselstromtechnik," Vol. 5, part 2. The theory of both the series and the shunt-wound polyphase commutator motor has been principally studied in Germany, Austria, Switzerland and Sweden, and a few electric manufacturing companies in those countries have put such motors on the market. In France M. Latour has built some 10-pole motors as large as 500 h.p., with the armature connected 12-phase, and with only one stud of brushes per pole, using a two-circuit winding. A brief note on these motors will be found in *La Lumière Electrique*, Vol. 29, p. 289.

A person interested in some phase of the subject should refer to the last volumes of the principal German and French electrical periodicals and search through the back volumes "counter-clockwise," until he has found the particular bit of information sought. For such a search the entry words in the indexes are probably much more valuable than a specific reference to a few articles. An entry word will enable the future

investigator to locate in time articles yet unwritten. The magazines below are arranged in the order of their importance for the particular topic under discussion. The first two periodicals contain by far the largest amount and the best information. Of the American periodicals, *The General Electric Review* so far has been the only one in which original articles on the polyphase commutator motor have been published. *Science Abstracts* (London), part B, Electrical Engineering, is a most useful help in literature search on most any electrical subject.

Name of Periodical	Entry Word in the Index
Elektrotechnische Zeitschrift	Elektromotoren
Elektrotechnik und Maschinenbau	Kollektor, Kommutator, Mehrphasen
Archiv für Elektrotechnik	Name index, see Glossary below
General Electric Review	Motors
*La Lumière Electrique**	Machines, Moteurs

Glossary of German and French Terms for Facilitating Literature Search

English	*French*	*German*
machine	machine	Maschine
motor	moteur	Motor or Elektromotor
single-phase	monophasé	Einphasen or Wechselstrom
two-phase	diphasé	Zweiphasen
three-phase	triphasé	Drehstrom
commutator	collecteur	Kommutator or Kollektor
series-wound	série	Reihenschluss or Serien
shunt-wound	shunt	Nebenschluss

*Continued since 1917 as *La Revue Générale de l'Electricité.*

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COMMUTATION IN ALTERNATING-CURRENT MACHINERY

BY MARIUS C. A. LATOUR

ABSTRACT OF PAPER

As is now well known, the inductive reactive effect of a commuted winding in a revolving magnetic field decreases directly with the increase in speed from standstill to synchronism, when its value becomes zero. As first pointed out by the author some years ago, it becomes negative at speeds above synchronism, under which condition the rotor of a motor operates as a capacity.

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

In a motor built in accordance with the principles set forth, the commutator difficulties are not serious, the overload range is in excess of that of an induction motor, and the machine can act as a condenser on the system.

THE writer has already had the honor of presenting two communications on the use of commutators with alternating currents before the American engineering public, the first being in June, 1903, in co-operation with Mr. A. S. Garfield (see *The Compounding of Self-Excited Alternating Current Generators, for Variation in Load-Factor*, in A. I. E. E. TRANSACTIONS, Vol. XXI, pages 569-577, and discussion, pages 583-585); the other being a paper presented at the St. Louis Congress, 1904, ("*Alternating Current Machines with Gramme Commutators,*" published in Transactions of the International Electrical Congress of St. Louis, 1904, Vol. III, pages 149-154).

The purpose of the present paper is to enter more deeply into the theoretical considerations which were outlined by the writer in his earlier publications, and which have since been verified by practical experience. A distinction will be made between the use of commutators for polyphase and for single-phase alternating currents.

I—Commutator in Polyphase Machinery

Let us consider a direct-current bipolar armature, A, Fig. 1, placed in a uniform air gap inside the laminated stator, B, with regularly spaced slots and receiving sinusoidal currents of p phases, of frequency equal to $\omega/2\pi$ through p brushes located at points situated $2\pi/p$ in angular distance from each other.

Fig. 1 shows diagrammatically a smooth core armature, on which four brushes, a, b, c, d, make contacts through which two-phase (more properly four-phase) currents are supplied. The brushes, a, c, receive a current $I \sin \omega t$, and the brushes b, d, receive a current $I \cos \omega t$. Suppose the armature to be at rest and let $L\omega$ be its inductance per phase. If the armature is made to turn at the angular velocity ω_1, measured in the direction of the rotating field developed by the polyphase currents, it will be found that the inductance of the armature becomes $L(\omega - \omega_1)$; consequently, it will vanish at synchronism and, *above synchronism, the arrangement will operate as a capacity* (condenser).

When this property was first made known by the writer, in his patents, in 1900-1901, its correctness was questioned by several prominent electricians. Among these, M. Maurice Leblanc devoted a long article (see *Eclairage Electrique*, October 26th, 1901, pages 113 *et seq*) to the refutation of the writer's conclusions. Many electricians have, since then, accepted as satisfactory the theory which introduces the relative velocity $(\omega - \omega_1)$ of the revolving field developed by the armature with respect to the armature itself. Mr. Leblanc, however, maintained, in his article, that since the current, i, in each turn of the armature winding, preserved its variable character, no matter what the velocity ω_1 might be, at any instant i was subject to the same rate of variation, $\frac{di}{dt}$, and consequently the e. m. f. of self-induction between the brushes should persist at all speeds

As was shown by the writer, in a detailed article, published in November, 1901, (see *Eclairage Electrique*, 1901, pages 294

et seq), it is quite true that the e. m. f. of self-induction of the sections included in the circuit between the brushes persists independently of the velocity ω_1; but, in consequence of the phenomenon of commutation, there appears between the brushes an e. m. f. of opposite sign which is proportional to the velocity ω_1, and which is due to the mutual induction between the sections which are short-circuited and those which are in the circuit.

In this connection, let us consider Fig. 1. Let M be the coefficient of mutual induction between the sections which are short-circuited by the brushes, b, d, and the sections that remain in circuit between the brushes a, c. Let n be the number of sections of the armature. With brushes covering the width of a commutator segment, the duration of commutation for the sec-

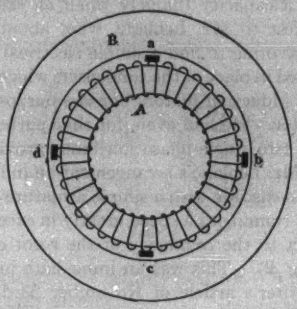

FIG. 1

tions of winding passing under the brushes b, d, at the velocity ω_1, will be $T = \dfrac{2\pi}{\omega_1 n}$. The e. m. f. E_M developed by mutual induction between the short-circuited sections passing under the brushes b, d, (where the reversal of a current $\frac{1}{2} I \cos \omega t$ is taking place during the time T) and the sections which are in circuit between the brushes a, c, (through which the current $I \sin \omega t$ passes)will be $\dfrac{M}{T} I \cos \omega t = \dfrac{Mn}{2\pi} \omega_1 I \cos \omega t$.

We first ascertain that E_M is opposed in polarity to the e. m. f. of self-induction $E_L = L \omega I \cos \omega t$, due to the self-induction of the armature; and we then can show that, in the case where

each section of the armature is supposed to produce a sinusoidal flux at the armature periphery, we will have $\frac{Mn}{2\pi} = L$ and consequently $E_L - E_M = L(\omega - \omega_1)$.

It is to be noted that the same expression may be obtained for the e. m. f. (E_M) by supposing it to be induced by the rotation of the armature in its own field produced by the current sent through brushes a, c; but this explanation of the appearance of the e. m. f. E_M, though it may always be correct from the mathematical point of view, is not quite so near the physical reality, since, as a matter of fact, the field of the armature itself must follow the armature in its rotation.

The fact that the armature in the arrangement shown in Fig. 1 can operate as a capacity (namely when ω_1 *exceeds* ω) was of particular interest to Mr. Leblanc, who, about that time (in 1902) was endeavoring to find a simple electrical system susceptible of being used in place of the condensers which he was placing in the rotors of induction motors, for the purpose of improving their power-factor. He had even, for that purpose, devised the combination of two single-phase machines provided with commutators. In Mr. Leblanc's arrangement an induction motor C was connected in cascade with a simple armature A, such as that shown in Fig. 1, running at a speed greatly in excess of synchronism with respect to the frequency of the rotor currents of machine C (see Fig. 2). This was an immediate indication of the value of the writer's article of November 23rd, 1901, as Mr. Leblanc himself has since expressly acknowledged, (see *La Lumière Electrique*, July 12th, 1913, page 60, and the *Electricien*, July 25th, 1913, page 658).*

The Swiss firm of Brown-Boveri, utilizing the methods patented by M. Leblanc for improving the power-factor of induction motors, has put on the market, in the last few years, under the name of phase-compensator, armatures with commutators rotating above synchronous speed. Fig. 3, made from a photograph, shows one of these arrangements, designed for a

*The writer had formally proposed that combination in a letter addressed to *Industrie Electrique* (in 1902) which was not published; but the writer, in any case, considers that M. Leblanc should be credited with the idea of placing condensers or any dynamic apparatus equivalent to them in the rotor of induction motors for the purpose of improving their power-factor. It is by mistake that the arrangement shown in Fig. 2 was, during a certain time, attributed to Mr. Scherbius.

500 h.p. motor, operated by a motor of 0.8 h.p., running at 1000 rev. per min. This outfit relieves the supply mains of the necessity of furnishing 180 kv-a. of reactive current.

With respect to the introduction of a commutator to bring the power-factor of an alternating current system up to unity, the question has been asked whether it would not be possible to raise the power-factor to unity in an alternating current system in some other way than by the introduction of a commutator. The answer to that question is known to be negative.

If we consider a system of non-deformable electric circuits composed of any number p whatever, and at rest, the writer has shown (see *Lumière Electrique*, 1907, page 5) that *whatever may be the complexity of the mutual inductions between the circuits considered by pairs*, the system can only absorb magnetizing power, and cannot supply any. The writer demonstrated, in fact, that the determinant which can be constituted with all the

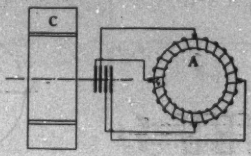

FIG. 2

induction coefficients is necessarily positive. A particularly well known case for two circuits is

$$\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix} > 0$$

Finally, the late Prof. Henry Poincaré—to whose attention the writer had brought the more general statement of the case, to the effect that the appearance and maintenance of any currents whatever in a system of stationary or moving circuits is impossible without the existence either of batteries or permanent magnets, or else of capacities or of means capable of modifying the internal connections of the circuits (such as by commutators)—demonstrated that a theoretical necessity was really involved (see *Lumière Electrique*, March, 1907, page 293).

The possibility of raising the power-factor to unity without the use of a commutator would involve the negation of this necessity, and, therefore, could not exist.

In accordance with what has been said previously, with reference to the expression of the armature inductance in Fig. 1, under the form $L\ (\omega - \omega_1)$, it can be supposed, in order to obtain a first classical approximation, that the flux per phase on the armature-periphery is sinusoidal. But the distribution here considered is a theoretical one, which does not correspond to the real distribution. Let us begin by noting on what fictitious surface this distribution may be defined.

It is the general practise to consider the surface of the air gap, but that is not the point of view which the writer has adopted in his different studies. It has seemed to him that the distribution of flux which it was desirable to determine was that which is related to the geometrical surface containing the axes of the conductors subjected to inductive effects. Reference to Fig. 4, which shows a two-layer drum-winding, shows two cylindrical surfaces, S' and S'', which contain the axes of the conductors

FIG. 4

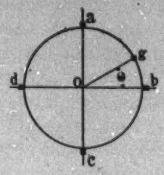

FIG. 5

subjected to induction. We ought, therefore, to consider the flux on the surfaces S' and S''. But the writer proposed previously (see *Lumière Electrique*, January, 1907, page 6) to make an approximation which consists in considering only the midway surface S, situated between S' and S'', passing through points half-way down across the slots.

It will be noted that the magnetic flux referred to comprises the lines of force which other writers consider as constituting local leakage fluxes around the conductors; and that if we extend the surface S between the lateral connections of the armature, it comprises also the external leakage fluxes produced by the heads of the coils. We may imagine that the discontinuity caused by the teeth and slots gives rise to a special supplementary harmonic field. The result is that leakages can be represented by harmonics.

We have already given the name *magnetic periphery* to the

fictitious surface S of the armature. Diagrammatically, an armature having a commutator will be represented by a circle, whose circumference will be the magnetic periphery (see Fig. 5). It is on this magnetic periphery that the positions defined by the brushes a, b, c, d, will be indicated. The position of a radial line to any point, g, on the periphery, is defined by the angle $g\,o\,b = \theta$. Let $f(\theta)$ be the periodic function which represents the distribution per phase of the magnetic field normal to the surface S.

Simple calculations have already enabled the writer to establish (*see Lumière Electrique*, January, 1907, page 7), that the apparent inductance of the armature in Fig. 1, when supplied with two-phase currents, and when assumed to have an even number of sections, may be expressed at the angular velocity ω_1, as follows:

$$L\left(\omega - \omega_1 \frac{\int_0^{\pi/2} f(\theta)\, d\,\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\,\theta}\right)$$

It should be carefully noted that, for a sinusoidal distribution, $f(\theta) = a \sin \theta$, the second term reduces to unity, whereas for a triangular distribution $f(\theta) = a\,\theta$, the quotient

$$\frac{\int_0^{\pi/2} f(\theta)\, d\,\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\,\theta}$$

assumes the value $3/\pi$. In that case, the condition of zero inductance is obtained only at the velocity $\omega_1 = \pi/3\omega$.

It is proper to note, however, that a fractional pitch winding enables the velocity at which the inductance vanishes to be brought to a value nearer ω, and that, finally and especially, whatever may be the flux distribution on the magnetic periphery, the approximation to the theoretical expression $L(\omega - \omega_1)$ will become closer in proportion as the number of phases employed for supplying the armature in Fig. 1 is increased.

The very important question which remains to be considered in connection with the operation of the armature in Fig. 1 is that of commutation. We may here recall the manner in which that question was approached in the writer's study in 1901 (see *Eclairage Electrique*, page 294). Let λ be the full coefficient of self-induction of the sections of armature winding undergoing commutation under the brushes, a, c, through which is passing the current $I \sin \omega t$. This coefficient (λ), it should be noted, includes, in the case of windings of the drum type, the effect due to mutual induction between two sections in process of commutation under the brushes a, c. of opposite polarity. Now, let T represent the time that is consumed in the commutation of one section of winding. The condition which is *necessary* and *sufficient* in order to obtain perfect or "linear" commutation (which is gone into more fully in the article referred to), is that there should be available, in the winding sections which are in short-circuit, a reversal e. m. f. equal to $\frac{\lambda}{T} I \sin \omega t$.

Now the flux developed along the rectangular axis b, d is proportional to $\cos \omega t$. Consequently, the induction resulting from its periodic variation produces an e.m.f. in the sections short-circuited by the brushes a, c, which is proportional to $-\frac{d}{dt} \cos \omega t$ and, therefore, to $\sin \omega t$. This e.m.f. is constant for a given current $I \cos \omega t$. There is, therefore, a certain velocity ω_1, for which there is produced, exactly, in the commutated sections, the reversal e.m.f. necessary and sufficient, $\frac{\lambda}{T} I \sin \omega t$, to produce a perfect or linear commutation.

Below that velocity the reversal e.m.f. will be too high and commutating conditions will be produced which are analogous to those existing in a continuous-current dynamo when the brushes are set too far forward in the direction of rotation. Above that velocity, on the other hand, the reversal e.m.f. produced will be too weak, and the commutating conditions produced will be analogous to those which exist in a continuous-current dynamo when the brushes are not set sufficiently far forward in the direction of rotation, or even when they are set in the opposite direction. It will be pointed out later, that precisely this same condition of operation exists (namely

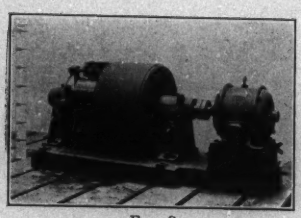

FIG. 3

FIG. 9

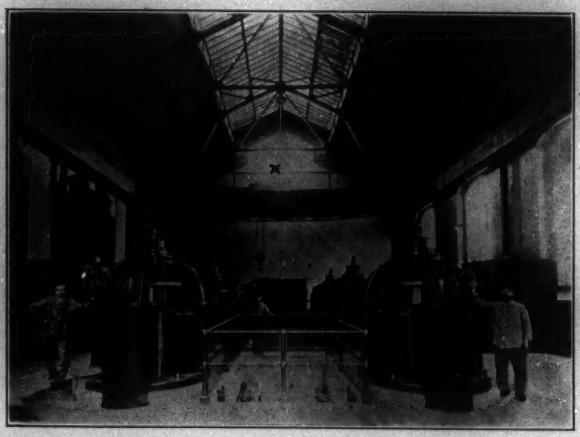

FIG. 8 [LATOUR]

insufficient reversal e.m.f.) when the armature in Fig. 1 is operating as a capacity, and it will be interesting to note that in the production of reactive current by a commutator we encounter the same commutating difficulties as the reduction of the normal armature reaction or the production of an armature reaction which assists the field excitation in a continuous-current dynamo.

It was shown by the writer (see *Bulletin of the Société Internationale des Electriciens*, June, 1910, page 392) that in the case of an armature placed in a uniform air-gap, inside a laminated stator having evenly spaced slots, the e.m.f. necessary and sufficient to produce the perfect commutation of a current I is $\lambda\, I/T$, no matter what may be the distribution of the flux produced by the armature. This e.m.f. is equal rigorously to the e.m.f. which may be imagined to be induced in the winding sections under short circuit, by reason of the rotation of the

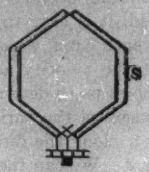

FIG. 6

armature in the flux developed by the armature as a whole. It should be well understood that the flux under consideration is that measured on the magnetic periphery of the armature, and on the portion of the surface comprised between the axes of the conductors of two contiguous sections in the region where commutation is taking place, as is shown in Fig. 6 (see the cross-hatched portion of the figure). There exists, in fact, a close relationship, in a continuous-current armature, between the coefficient (λ) of the winding sections under commutation and the flux produced by the entire armature through the shaded surface represented in Fig. 6, that is to say, in the interval comprised between two consecutive winding sections in the region in which commutation is taking place when unit current is passing through the armature. It is, moreover, by virtue of this close relationship that the writer has, since 1902, in the course of numerous controversies (see, for example, E. T. Z.,

1906, page 781; *Lumière Electrique*, 1902, page 53; *Electrician*, 1913, pages 105 and 325; *Elektrotechnik und Maschinenbau*, 1913, page 633), defended the very simple point of view according to which the matter of obtaining perfect commutation in a continuous-current dynamo depends substantially on the neutralization of the magnetic field of the armature, or, more properly, on the production of a mean resultant neutral field in the region where commutation is taking place. It is understood that the field is measured on the portion shown shaded in Fig. 6, on a surface S which passes about half-way across the armature slots, and not through a surface situated in the air gap. The exactness of this view seems to be recognized more or less explicitly at the present time.

Having shown that the determination of $\frac{\lambda}{T} I \sin \omega t$ in the case of Fig. 1 amounts to the evaluation of the e.m.f. induced in the short-circuited winding sections by the rotation of the armature in its own field, it is easy to determine, for a given flux distribution, the reversal e.m.f. which is necessary to obtain perfect commutation at the velocity ω_1. This knowledge of the flux distribution enables us to determine the reversal e.m.f. which is available by reason of the variation of the flux which is proportional to $\cos \omega t$, along the direction perpendicular to the axis of the brushes a, c. We can then finally determine the difference between these two e.m.f's. and this is what interferes with commutation.

Let $f(\theta)$ be the flux distribution. It will be found by a simple reasoning that in an armature wound with a full pitch winding the e.m.f. under the brushes resulting from an excess or lack of reversal e.m.f. is proportional to

$$\left(\omega - \omega_1 \frac{f(\theta)_{\theta=\pi/2}}{\int^{\pi/2} f(\theta)\, d\theta} \right)$$

If we make the theoretical assumption that $f(\theta) = a \sin \theta$, it will be readily seen that the quotient $\frac{f(\theta)_{\theta=\pi/2}}{\int_0^{\pi/2} f(\theta)\, d\theta}$ reduces to

unity, and that perfect commutation is obtained at synchronism for the condition $\omega_1 = \omega$.

If we suppose that $f(\theta)$ is of the form $a\,\theta$, which is nearer the actual condition, it will be found that perfect commutation is obtained at the velocity $\omega_1 = \frac{\pi}{4}\,\omega = 0.79\,\omega$. However, the conditions are changed if the number of phases employed for supplying current to the armature in Fig. 1 is increased. With six phases, we find, for the velocity of perfect commutation

$$\omega_1 = \frac{\sqrt{3}}{2}\,\omega = 0.86\,\omega$$

and finally, for twelve phases, we find $\omega_1 = \omega$. By adopting a fractional pitch for the winding of the armature it is possible to approach more rapidly the condition $\omega_1 = \omega$. It is important to note that if the commutation is not perfect at synchronism in all cases, it is because we are not dealing, in general, with a uniform revolving field, in consequence of a non-sinusoidal distribution of flux on the magnetic periphery of the armature. The simple reasoning which the writer followed in a first approximation (1900–1901), and in which he asserted that commutation must always be perfect at synchronism when there is a real revolving field, is exact, for the very reason that the variation of flux in the short-circuited sections vanishes at synchronism.

Everything that can be done to obtain a perfect revolving field (such as increasing the number of phases and using fractional pitch winding) unquestionably assists in insuring perfect commutation at exact synchronism.

When the revolving field produced by a rotor having a commutator is imperfect, the commutation is disturbed at synchronism under conditions equivalent to those wherein it would be disturbed by an external imperfect revolving field produced by the stator surrounding it.

In reality, the increase in the number of phases for the supply of current to the rotor of a commutator machine, as set forth in the writer's German patent No. 145,433 of 1901, is related rather to a specific purpose, and it has less immediate relation to commutation. It may be interesting to note the principal reasons for this.

In the first place, in increasing the number of phases for

supplying current to the rotor, the current to be commutated per phase for a given total current in the armature is materially diminished. The frictional surface on the periphery of the commutator remains the same in the two cases, but it is differently distributed. The result of reducing the current per phase is that the resistance of a single line of brushes in the case of a bipolar armature, or of the assemblage of lines of brushes coupled in parallel in the case of a multipolar armature, increases with the number of phases. Now, the increase of this resistance, as is well known, is favorable to good commutation, so long as the machine is not working at a theoretical load corresponding to perfect commutation.

In a load which does not correspond to perfect commutation, there exists, so to speak, in the short-circuited sections, a disturbing e.m.f. which is equal to the difference between the e.m.f. necessary to produce perfect commutation and that which

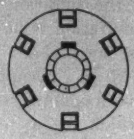

FIG. 7

is actually induced; it is this parasitic e.m.f. which is the real cause of the excessive heating and of the sparking at the brushes.

With reference to sparking, Mr. F. Carter and the writer (see the *Electrical World*, March 31st, 1910, page 804, and *Bulletin de la Société Internationale des Electriciens*, April 6th, 1910, page 276) have advanced the opinion that the effects of a given parasitic e.m.f. increase, other things being equal, with the value of the quotient l of certain determinants composed of the coefficients of self and mutual induction of all the closed circuits in the machine. This quotient l itself has the dimensions of a coefficient of self-induction, and in a continuous current machine it is generally but slightly lower than the coefficient of leakage self-induction of two consecutive winding sections *placed in contiguous slots*. In an armature supplied with polyphase currents, however, the quotient l may fall much below that value.

In this connection reference may be made to Fig. 7, which is the reproduction of a diagram already published by the writer

(see A. I. E. E. TRANSACTIONS, 1903, page 583). The diagram represents an armature with a drum winding composed of twelve sections, supplied by three-phase currents. It will be easily seen that, as the width of the brushes exceeds the width of the commutator segments, there is always, at any instant, at least one short-circuited section per slot. Armatures having as few as three slots per pole are seldom used, but if we adopt a higher number of phases, preferably an odd number (7 or 9, for example, as indicated in the writer's A. I. E. E. communication of 1903) the remarks relative to the conditions shown in Fig. 7 can be extended to the case of an armature having a number of slots actually used in current practise. Under those conditions, the quotient l becomes lower than the coefficient of self-induction due to leakage between two sections *placed in the same slots*. The parasitic e.m.f. will then produce practically a simple short-circuit current as if the armature was at rest or revolving slightly. Under these conditions, we will have the sparkless commutation which I named "squirrel-cage commutation."

The three types of a-c. polyphase commutator machines which can be rationally constituted by taking synchronism as the mean working condition, are analogous to continuous-current machines, namely, the shunt machine, the series machine, and the compound-wound machine. The theory of these machines was published by the writer in 1902 (see *Eclairage Electrique*, 1902, pages 50 and 358), and different authors have gone over the subject since in a more detailed manner. When these machines are working above or below synchronism, then, from the point of view of commutation, it is necessary to take into consideration the e.m.f. induced by the *resultant* field of the stator and rotor in the short-circuited winding sections, and to judge the commutation accordingly. If a machine is intended to have the greatest possible range of speed, then it is immediately obvious that it is important to arrange matters in such way that synchronism shall correspond to a speed of perfect operation so far as commutation is concerned. Allowance can then be made for equal values of slip above and below synchronism, and in this way the allowable variation of speed becomes greater. With an absolute slip of 20 per cent, for example, it is possible to obtain speed variations in the ratio of two to three.

When the slip is to be materially increased, the width of the brushes should be reduced as much as possible, in order that

the e.m.f. induced between the opposite edges of the brush may be as low as possible; but for mechanical reasons it is scarcely practical to reduce the thickness of the brushes to less than 9 to 10 mm.

This minimum width of brushes being a limitation, it is desirable (for the same e.m.f. developed between the two edges of the brush) to divide this e.m.f. in some way by adopting a type of winding which allows the commutator segments to have a width of half that of the brush (see the writer's article in the *Electrical World* of December 3rd, 1904). As a rule, commutator segments of 4.5 to 5.0 mm. will be used.

With brushes of good quality the e.m.f. allowable between the segments under those conditions will be 1.5 volts. This e.m.f. allows high values of slip to be attained without necessitating a commutator of excessive size.

The writer has devoted much attention in the last ten years to a number of installations in which polyphase machines with commutators have been used. Figs. 8 and 9 represent an installation made about five years ago at the pumping station of the City of Paris. The machines are 500-h.p. motors supplied with three-phase 50-cycle current and operating at speeds ranging between 450 and 700 rev. per min., the synchronous speed being 600 rev. per min. The speed regulation is obtained by shifting the brushes. The rotor is supplied with twelve phases by means of twelve equidistant brush-holders. The total width of the commutator is 40 cm. The operation of the machines is perfect in every respect.

In concluding, the writer believes that the following historical résumé of his part in development of polyphase commutator machines may be of interest.

The English patent to Wilson (No. 18,525, of 1888) describes a polyphase commutator motor having the stator and the rotor connected directly in multiple without any transformer. According to the inventor, a squirrel-cage winding or a short-circuited winding might, if necessary, be placed on the rotor or on the stator or else on both of them at the same time if it were desired to operate the motor at or very near synchronous speed. The writer took up this same mode of connection later (French patent No. 306,229 of 1900) from a more modern point of view. His analysis led him to foresee the importance of lowering as much as possible the e.m.f. in action at the commutator by employing either transformers, a special tension-lower-

ing device, or else sectional windings placed on the stator as means of supplying current to the rotor. In a German patent of 1891 (No. 61,951 to Georges) a series polyphase commutator motor is described. The writer has also introduced in that arrangement a series transformer between the stator and the rotor. Later, the writer adopted the plan of constructing these transformers with an air-gap, in order to improve the characteristics of the motor; and the connections employed are of the star-delta type, as shown in Fig. 10 (German Patent, No. 237,849).

The star-delta connection enables one to regulate the speed in a practical way, within wide limits, by simply shifting the brushes. The star connection is used for starting and for running at lower speeds; the delta connection is used at higher speeds.

Besides the shunt and series connections, mentioned in the two patents just referred to, the writer has also indicated how a compound connection could be used between the stator and the rotor (German patent No. 154,509 of 1901) and has favored the provision of a rotor with a greater number of phases than are obtainable directly from the supply mains (German patent No. 145,433 of 1901). This greater number of phases is obtained by means of a transformer, or else, in the case of a shunt machine, by means of auxiliary windings placed on the stator.

Finally, the writer has also proposed the use of all these kinds of machines as generators, and has called attention to their advantages in regard to their coupling in multiple and their compounding. Before the war, the writer had begun the construction of 100 kv-a. three-phase generators, which were to operate as boosters for mains supplying three-phase currents of 50 cycles. The writer is confident that this question of a-c. commutator generators, *in which, by the way, the commutator is relatively very small*, is a development which is sure to command attention in the near future.

II—Commutator in Single-Phase Machinery

Let us suppose that the armature in Fig. 1 is supplied with a simple alternating current through two brushes a, b, placed 180 deg. apart, instead of being supplied with two-phase currents (see Fig. 11). The armature-reactance then retains the same value $L\omega$, whatever may be the angular velocity ω_1. But, if we place, 90 deg. from the brushes a, b, some supplemental brushes c, d, connected together by a short-circuit connection (see Fig. 12), the armature-reactance then takes, as a first ap-

proximation, the value $L\left(\omega - \frac{\omega_1^2}{\omega}\right)$. It therefore vanishes when synchronism is attained and becomes negative above synchronism.

Thus, when a current $I \sin \omega t$ is sent through the armature by means of the brushes a, b, there is, in consequence of the arrangement of the short-circuited brushes c, d, an e. m. f., $-L \frac{\omega_1^2}{\omega} I \cos \omega t$ which is in opposition to the e. m. f. of self-induction $L \omega I \cos \omega t$, and, consequently, balances it at synchronous speed. The result is the same as if two-phase currents were sent into the armature. This phenomenon was discovered by the writer in 1901. This e. m. f., $L \frac{\omega_1^2}{\omega} I$ is produced by the fol-

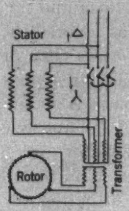

FIG. 10

FIG. 11

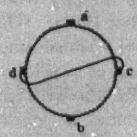

FIG. 12

lowing instrumentality. Let M, as before, represent the coefficient of mutual induction between the sections of winding which are in process of commutation under the brushes a, b, in which there is a reversing current $\frac{I}{2} \sin \omega t$, and the sections in circuit between the brushes c, d. Let again n be the number of sections of armature winding. At the speed ω_1 the duration T of the short circuit in any section short-circuited by the brushes will be, $T = \frac{2\pi}{\omega_1 n}$. The induced e. m. f. between the brushes c, d, due to the mutual induction between the sections undergoing commutation under the brushes a, b, and the sections which are in circuit between the brushes c, d, will be,

$$\frac{M I \sin \omega t}{T} = \frac{M}{2\pi} n \omega_1 I \sin \omega t$$

This e. m. f. will produce, in the conductor connecting the brushes c, d, a short-circuit current which is $\frac{\pi}{2}$ out of phase with respect to that e. m. f. $\frac{M}{2\pi}\frac{\omega_1}{L\omega} I \cos \omega t$. This current, in phase with $\cos \omega t$ is, in turn, commutated under the brushes c, d, and the mutual induction between the armature windings short-circuited by the brushes c, d, and the windings short-circuited by the brushes a, b, develops, between the brushes a, b, an e. m. f.,

$$\frac{Mn}{2\pi}\frac{\omega_1}{L\omega} \cdot \frac{Mn}{2\pi}\omega_1 I \cos \omega t = \left(\frac{Mn}{2\pi}\right)^2 \frac{1}{L}\frac{\omega_1^2}{\omega} I \cos \omega t$$

If we suppose the flux to be distributed sinusoidally we find that $\frac{Mn}{2\pi} = L$, as we have already noted; and the e. m. f. induced in consequence of the rotation of the armature becomes equal to $L\frac{\omega_1^2}{\omega} I \cos \omega t$. In reality, the distribution of fluxes on the magnetic periphery is not sinusoidal.

Let $f(\theta)$ express the periodic function which represents the actual distribution of flux, and let r represent the resistance of the armature between the brushes. The writer has already shown, by simple calculations (see *l'Eclairage Electrique*, January, 1907, page 811) that the armature-reactance at the velocity ω_1 is, in reality equal to,

$$L\left[\omega - \left(\frac{\int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta}\right)^2 \frac{1}{1 + \frac{r^2}{L^2\omega^2}}\frac{\omega_1^2}{\omega}\right]$$

On the assumption that the flux has a triangular distribution, $f(\theta) = a(\theta)$, it will be found that the quotient

$$\frac{\int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta}$$

is equal to $\frac{3}{\pi}$ and that the armature-reactance may be expressed as follows:

$$L\left(\omega - \frac{9}{\pi^2}\ \frac{1}{1+\frac{r^2}{L^2\,\omega^2}}\ \frac{\omega_1^2}{\omega}\right)$$

The armature-reactance vanishes, under those conditions, when the velocity attains the value $\omega_1 = \frac{\pi}{3}\sqrt{1+\frac{r^2}{L^2\,\omega^2}}$.

Generally, $\frac{r}{L\,\omega}$ is very small, and it might be said that the reactance vanishes when $\omega_1 = \frac{\pi}{3}\,\omega$, exactly as in the case when the machine is supplied by two-phase currents.

Let us consider now the important question of commutation. The reversal e. m. f. necessary for commutating the current $I \sin \omega t$ under the brushes a, b, in Fig. 12, is induced by variation of the flux in phase with $\cos \omega t$ which exists along, c, d.

If we imagine the distribution of flux to be represented by the function $f(\theta)$ it will be seen, from what precedes, that the commutation under the brushes a, b, requires, in the case of an armature of even (full) winding pitch, a reversal e. m. f. which is proportional to $f(\theta)_{\theta=\pi/2}\ \omega_1$.

On the other hand, it is easily shown, after calculating the current in the short circuit, c, d, that the e. m. f. induced by the variation of flux along c, d, in the section short-circuited by the brushes a, b, is proportional to

$$\frac{\left(\int_0^{\pi/2} f(\theta)\,d\,\theta\right)^2}{\int_0^{\pi/2} f(\theta)\,\theta\,d\,\theta}\ .\ \omega_1$$

The ratio of the two e. m. f. considered, is therefore equal to

$$\frac{\left(\int_0^{\pi/2} f(\theta)\,d\,\theta\right)^2}{f(\theta)_{\theta=\pi/2} \times \int_0^{\pi/2} f(\theta)\,\theta\,d\,\theta}$$

Assuming a sinusoidal flux-distribution ($f(\theta) = a \sin \theta$), it is actually found that this ratio is equal to unity, so that the e. m. f. necessary to insure perfect commutation under the brushes a, b, is always equal to the e. m. f. actually induced by the field variation along c, d. But, if we assume that the flux-distribution is triangular $f(\theta) = a\theta$, it will be found that the ratio, above mentioned, becomes equal to $\frac{3}{4}$ simply.

Under those conditions the e. m. f. actually induced in the armature sections which are short-circuited by the brushes a, b, is not sufficient to insure perfect commutation of the current $I \sin \omega t$, and 25 per cent of the volume of that current will have to be commutated by brush-resistance.

Let us consider the commutation obtained under the brushes c, d. The e. m. f. necessary to obtain perfect commutation is proportional to the current passing through the short-circuit connection between the brushes c, d, and proportional to the velocity ω_1.

As the current in the short-circuit connection between the brushes c, d, is already proportional to the velocity ω_1, the e.m.f. necessary is proportional to ω_1^2; and, taking into account the flux distribution, it will be found to be proportional to

$$\frac{f(\theta)_{\theta=\pi/2} \int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta} \frac{\omega_1^2}{\omega}$$

On the other hand the e.m.f. induced by the variation of flux along a, b is proportional to

$$\int_0^{\pi/2} f(\theta)\, d\theta .\ \omega$$

Consequently the e.m.f. causing disturbance in the armature-sections which are short-circuited under the brushes c, d, which is the result of either excess or insufficiency of the e.m.f. of reversal, will be proportional to

$$\left(\omega - \frac{\omega_1^2}{\omega} \frac{f(\theta)_{\theta=\pi/2}}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta} \right)$$

Assuming a sinusoidal flux distribution $f(\theta) = a \sin \theta$, perfect commutation will actually take place when, $\omega_1 = \omega$.

In the case of triangular flux distribution $f(\theta) = a\,\theta$, we have

$$\frac{f(\theta)_{\theta=\pi/2}}{\int_0^{\pi/2} f(\theta)\,\theta\,d\,\theta} = \frac{12}{\pi^2}$$

and perfect commutation will be obtained when $\omega_1 = \frac{\pi}{\sqrt{12}}\,\omega$.

In the case of an armature wound with fractional pitch winding in which each section corresponds to an arc equal to $\pi - \alpha$ instead of a half circumference, π, if we assume that a winding of this character produces a trapezoidal flux-distribution, the results obtained are as will be described.

With regard to the commutation obtained under the brushes a, b, the expression of the ratio of the e.m.f. actually induced to the e.m.f. necessary to obtain perfect commutation, will now be as follows:

$$\frac{3\left(\frac{\pi}{2} - \alpha\right)^2 \left(\frac{\pi}{2} + \alpha\right)^2}{(\pi - \alpha)^3 (\pi + 2\,\alpha)}$$

A numerical application, such as for instance, when $\alpha = \frac{\pi}{3}$, gives 0.92 for the value of this ratio, which is quite near unity, and, consequently, there is a perceptible improvement as compared with full pitch winding, which gives 0.75 for that ratio. With regard to the commutation obtained under brushes c, d, the disturbing e.m.f. in the sections which are short-circuited by the brushes c, d, and which results from either excess or insufficiency of the e.m.f. of reversal, becomes proportional to

$$\left(\omega - \frac{\omega_1^2}{\omega}\,\frac{12}{(\pi - \alpha)(\pi + 2\,\alpha)}\right)$$

A numerical application, such as that corresponding to $\alpha = \frac{\pi}{3}$ leads to the conclusion that perfect commutation will

be obtained for a velocity $\omega_1 = \frac{\pi}{\sqrt{10.8}}\,\omega$, which is very near that of synchronism.

In addition to the use of armatures with fractional pitch windings, (recommended by the writer in 1905; See *Eclairage Electrique*, page 125) the writer had proposed, in January 1903, the arrangements shown in Figs. 13, 14 and 15, which have a certain analogy with the phase-multiplications previously proposed by the writer for supplying current to the rotor in the case of polyphase machines.

It will be seen at once that, in consequence of the circulation of a current which is 90 deg. out of phase in the short-circuit connections between the brushes, a rotating field will be available at synchronism; and, by analogy with what has previously been said in regard to armatures provided with commutators and supplied by polyphase currents, it may be said that the

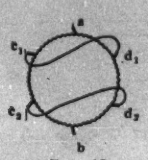

FIG. 13

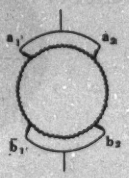

FIG. 14

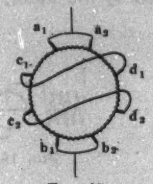

FIG. 15

problem of securing perfect commutation at synchronism comes back to the realization of a perfect rotating field. This result is attained by utilizing, conjointly, the arrangements shown in Figs. 13 or 15 and a suitable fractional pitch winding.

From a clear understanding of the operation of the armature shown in Fig. 12, it will readily be seen how single-phase commutator machines similar to polyphase commutator machines, may be made with all kinds of windings and connections, shunt, series or compound, having a power factor equal to unity. Figs. 16 and 17, show respectively the shunt and series arrangements.

The machines which have been put on the market are the shunt and series motors, such as for example, those described in U. S. Patent No. 1016021. The writer has devoted attention to the construction of a large number of these motors which have, however, been used more for continuous operation on

ordinary power circuits than for electric traction by single-phase current.

Fig. 18 is taken from a photograph of a type of motor of 200 h.p., 600 rev. per min., 50 cycles designed for continuous service, of which a considerable number have been constructed in the last 10 years. The number of sets of brushes and their connections correspond to the arrangements shown in Fig. 15. The speed-regulation, between 400 and 750 rev. per min., is obtained simply by shifting the brushes in accordance with the method of regulation described by the writer in 1903.

In motors which are intended to be regulated by shifting the brushes, it is important to take into account the action of the field produced by the stator, considered by itself, on the commutation of the rotor. In reality, in order that this action may not produce any special disturbance when a given shift is made in the brushes, it is necessary and sufficient that the field produced by the stator, considered by itself, should correspond to

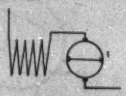

FIG. 16

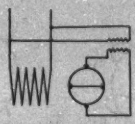

FIG. 17

a sinusoidal distribution on the magnetic periphery of the rotor. This result will be obtained, with a close degree of approximation, by winding the stator partially on an arc equal to about $\frac{2\pi}{3}$ per pole, and it will be obtained more perfectly in motors of large power by the plan, already mentioned, of arranging for a sinusoidal variation in the number of conductors placed in the slots of the stator, which are supposed to be evenly spaced.

The term "*compensated repulsion-motor*" was applied, by the writer, to the motor shown in Fig. 17 many years ago, because he appreciated, at that early date, the close relationship which is recognized generally today, between that motor and the repulsion-motor. It is proper, however, to consider the repulsion-motor, not in the primitive form given to it originally by Elihu Thomson, but in the form which it assumes when the stator of an induction motor is adopted in its construction.

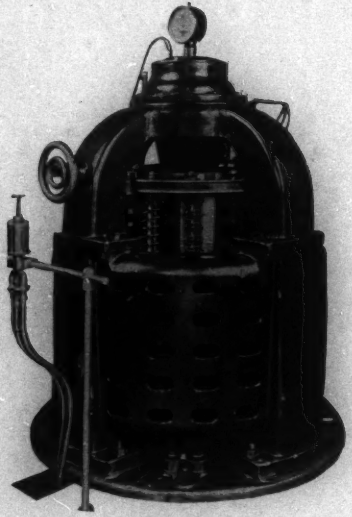

FIG. 18

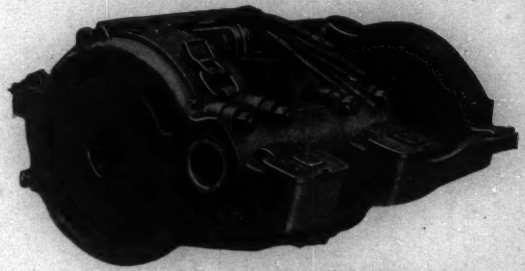

FIG. 21

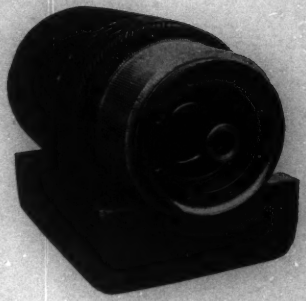

FIG. 22 [LATOUR]

The writer published his first theory of this form of repulsion-motor (see *E. T. Z.*, June 11, 1903) before entering upon a detailed consideration of the compensated repulsion-motor. The great peculiarity of the repulsion-motor when constructed with the stator of an induction motor, is that a rotating field is produced in that type of motor at synchronism, and that, at that speed, the existence of that rotating field insures perfect commutation. Before the publication, by the writer, of the fundamental theory, the only theory known was that of Steinmetz, which related to the repulsion-motor constructed with pole-pieces, and which consequently did not foresee the formation of a rotating field and its consequences.

Of course, in order to obtain a perfect rotating field, it is necessary to resort to the precautions which have just been mentioned and discussed with respect to the compensated repulsion- motor, namely, the rotor must have fractional pitch winding and multiple short circuits; and the stator must have a sinusoidal distribution of conductors.

With regard to the motors having a high power-factor, shown diagrammatically in Fig. 16, it is the shunt type of motor which has found most numerous commercial applications. The supply of current to the rotor is effected, as already indicated in the case of polyphase motors, either by a transformer or by an auxiliary winding placed on the stator.

The detailed theory of this motor, published ten years ago by the writer, (see *Eclairage Electrique*, January 1907, page 8) showed the great importance of flux distribution in that motor from the point of view of the torque produced, and also showed that great overload capacity may be given to this motor.

It is interesting to note that this motor can be constructed for very large powers with a commutator of very small dimensions when a heavy torque is not necessary in starting. If proper care has been taken in the design of the winding and in the distribution of the brushes on the commutator, this motor can operate at full load *with extraordinary commutation*. The drawback of a commutator is therefore, not as serious as might be supposed. On the other hand this motor has in its favor the following advantages:

1. It can supply a reactive current to the electrical mains,
2. Its overload capacity is much higher than that of the induction motor.

These advantages are sufficiently real, in the opinion of the

writer, to warrant his considering quite practicable an electrical traction system based upon the use of single-phase current of 50 or 60 cycles involving the conversion, on the locomotive itself, of the single-phase current into a continous current by a motor-generator set comprising a commutating shunt motor. By employing high speeds, these motor-generator sets can be made quite light.

The fact that regulating resistances and all accessory apparatus are eliminated, and also the possibility of recovering energy at all speeds without any special complication, render this system more attractive than it might seem to be at first glance. The writer expects to return to this subject later with comparative figures.

With regard to electric traction by the direct use of single-phase alternating current, although that system does not appear

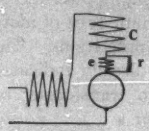

FIG. 19

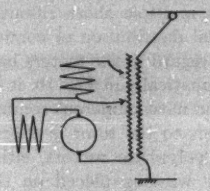

FIG. 20

to remain in high favor in the United States in recent years, it may be worth while to refer to the two types of single-phase motors to which the writer has given attention.

The electrical world is quite familiar today with the motor having a local commutating field (see Fig. 19) which has, outside of the compensating winding *c*, a local winding *e*, shunted by a resistance *r* in such a way that this local winding produces a field that is out of phase above the sections which are undergoing commutation. With a frequency of 15 or 16 cycles, the loss of power in the resistance *r* can then be rendered negligible. This motor is then, undoubtedly, as the writer has shown (see *E. T. Z.*, Nov., 1912, page 1231) the best variable speed motor so far as commutation is concerned. That motor has been constructed especially in Switzerland by the Maschinenfabrik Oerlikon, but motors of similar type have been constructed in France by M. Perret, in co-operation with the writer.

At a frequency of 25 cycles the introduction of the resistance r would occasion too high a loss of energy, and the writer then gives preference to a motor which is designated by him the "elliptical field series motor," which is represented diagrammatically in Fig. 20 (see also U. S. Patent 841257 and *E. T. Z.*, 1906, volume 27, page 89). As the diagram indicates, the arrangement of connections of that motor implies the presence of a transformer T for supplying current to the motor. The dephased commutating field is produced by the compensating winding itself. The resemblance of this motor with the motor constructed by Mr. Alexanderson in America, will be readily noted.

Subsequently to the introduction of this type of motor by the writer in German technical publications, under the name of "Series-motor with Elliptical Field" (see article by the writer in *E. T. Z.* 1906, volume 27 page 89) German authors gave it the name of "doppel gepeist" ("double fed"), which seems to have

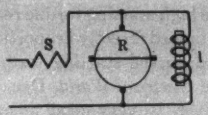

FIG. 23

become more or less current to-day in America. Nevertheless, it seems preferable to the writer to retain in the designation of the motor, the peculiarity which gives it good commutation insomuch as good commutation is the matter of utmost importance in commutating motors. It seems proper to the writer also that this designation should suggest a trace of the difficulty which inspired the very conception of the motor itself.

Figs. 21 and 22 are reproduced from photographs of a 75-h.p. motor for an electric railway of narrow gauge (1 meter) constructed in accordance with the writer's inventions, and of which a considerable number have been in operation for several years. These motors have eight poles and run at 750 rev. per min. with 320 volts. The width of the commutatoris 240 mm. The writer has also constructed, for 25-cycle electric traction lines, some compensated repulsion motors with a reactance coil l in parallel with the exciting brushes (see Fig. 23), this arrangement being one proposed by the writer in 1903.

The reactance coil is useful at the time of starting because it allows a greater excess of current in the induced circuit (stator) without requiring an excessive strength of inducing field (rotor), which latter is shunted at low speed. The result is that, *automatically*, as foreseen by the writer, there is, at the time of starting, a low flux with a heavy current, so that the commutator suffers less. Moreover, the reactance coil limits the hyper-synchronous speed of the motor, which is always to be feared from the point of view of commutation. The limiting hyper-synchronous speed is exactly that for which the reactance of the coil l comes into resonance with the capacitance of the rotor. The adjustments are made in such a way that this hyper-synchronous speed cannot exceed a rise of 50 per cent above synchronism. Motors of this type, which have been in practical operation about 10 years under particularly severe conditions, have given entire satisfaction.

It is well known that the drawback of a single-phase traction motor designed for 25 cycles, is the necessity of a very large commutator, owing to the high starting torque required. The volume of the commutator is indeed directly proportional to the frequency of current used, and to the starting torque required (see the writer's article in the *Electrical World*, Dec. 3, 1904). The commutation may, however, be very good at normal load, and it is the belief of the writer that opinions have been much too pessimistic on that point. At the same time, it is not our intention at this time to defend single-phase traction, which was only an incidental object of this paper.

DISCUSSION ON "THE POLYPHASE SHUNT MOTOR (ALTES), "THE SECOMOR—A KINEMATIC DEVICE WHICH IMITATES THE PERFORMANCE OF A SERIES WOUND POLYPHASE COMMUTATOR MOTOR" (KARAPETOFF) AND "COMMUTATION IN ALTERNATING-CURRENT MACHINERY" (LATOUR). NEW YORK FEBRUARY 16, 1918.

A. M. Gray: It might be worth while to spend a few minutes in showing how Prof. Karapetoff's diagram is derived.

The a-c. series commutator motor consists essentially of an induction motor stator and a d-c. armature supplied, in the case of an n-phase machine with n sets of brushes per pair of poles. These brushes make the armature equivalent to a stationary n-phase armature, because in any one phase, as one coil leaves the coil group another moves in to take its place. Since the stator and rotor are connected in series, two revolving fields are produced, which rotate at synchronous speed no matter what the actual speed of the rotor may be. The connections are such that these fields rotate in the same direction and a torque is produced whose value depends on the strengths of the fields and on the distance between them. If the brushes are moved backwards against the direction of motion of the fields the torque will tend to make the armature rotate in the same direction as the fields.

In discussing the vector diagram of such machines the teacher is liable to get space and time diagrams mixed up together. This I believe can be avoided as follows: In Fig. 1.

I is the current in phase one of both stator and rotor.

ϕ_s is the flux that would thread phase one of the stator if the stator m.m.f. were acting alone.

ϕ_r is the flux that would thread phase one of the stator if the rotor m. m. f. were acting alone. The rotor revolving field lags that of the stator by $(180 - \alpha)$ electrical degrees.

ϕ is the actual flux threading phase one of the stator.

E_{sb} is the e.m.f. generated in phase one of the stator by the resultant flux.

E_{rb} is the e.m.f. generated in phase one of the rotor by the resultant flux. This e.m.f. is proportional to the slip.

E_s is the component of applied voltage to overcome E_{sb}.

E_r is the component of applied voltage to overcome E_{rb}.

E_a is the applied voltage neglecting resistance and reactance drop.

At standstill the resultant voltage is 90 deg. out of phase with the current, since the output is zero. At half speed the rotor voltage for the same current will have only half the standstill value and the applied voltage will then be as represented by the dotted line. The locus of the applied voltage is the line a b for the case of constant-current operation, and this voltage diagram can readily be changed into a current circle diagram by inversion. Such a circle diagram, however, is not very accurate because it

does not take account of saturation and change in the rotor reactance. Because of this difficulty Prof. Karapetoff has suggested that the "Secomor", which is nothing more or less than an adjustable voltage diagram, be used. Further discussion of this diagram would be out of place because the various possi-

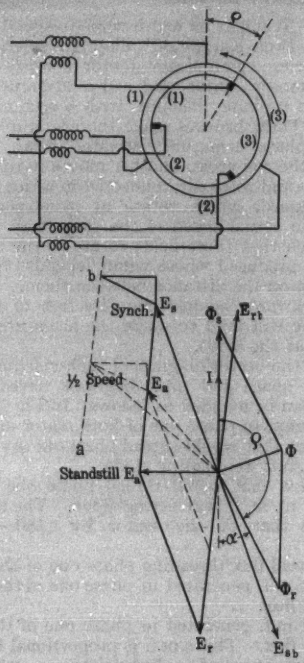

FIG. 1

bilities have already been pointed out in Prof. Karapetoff's paper.

Paul M. Lincoln: Mr. Latour proposes for trunk-line electrification to use a single-phase motor upon the locomotive driving a direct-current generator, which in turn will drive the locomotive. It seems to me if that is a proposition which comes from so distinguished a man as Mr. Latour, it is one which deserves a considerable amount of attention from those who are studying this question of the electrification of railroads.

Charles F. Scott: I was struck with one of Prof. Karapetoff's first sentences in which he says: "A combination of movable and adjustable bars which can be set to represent a vector diagram." What is a vector diagram? I would say that a vector diagram is itself a representation of certain physical things, and those physical things are the coils on an armature. If you take the simplest form of an alternator, one having two poles and a simple diametral coil on a rotating armature, and look at the end of the machine, you see a straight line which is the end of a coil which produces a complete cycle during a revolution. The angular position of the coil at a given time determines its relation to the field poles, and therefore represents the phase which the coil is producing. A second coil, placed at right angles gives an electromotive force 90 deg. from the first. Two lines at right angles, representing the physical angle between these coils, can be used in a diagram as a picture of the coils and are called vectors.

Go a step further, and make the length of a line correspond to the number of turns, and it follows that a straight line of proper length and direction (a vector) is simply a physical picture of a coil with a certain number of turns. If two coils are connected in series physically, we simply on the diagram join the lines in series with their proper angular relations and get the resultant. The vector diagram is a simple representation of the physical structure, as to angular position, number of turns and order of connection of the coils. Prof. Karapetoff has reversed the order, and starting from the vector diagram obtains a physical representation by an admirable structure using adjustable bars.

It has been suggested that I take part in the discussion of the single-phase motor as a factor in the railway situation and it has been pointed out now that the single-phase motor has not preempted the whole field of railway work, as some thought it might a number of years ago.

It is easy to look back and comment from the present standpoint. What was the standpoint in 1902, some fifteen years ago when the single-phase motor for railway work was first presented? That subject, by the way, was presented at a meeting of the Institute in the fall of that year. The previous session of the Institute had been a convention at Great Barrington, at which railway motors were considered. There were several railway papers and many pages of discussion. The discussion looked forward to the future of railway work, and the great cry was for a higher transmission voltage, and the great lament was that there was no way known to use such a voltage. Four or five different schemes were presented and discussed, but they were discussed merely as schemes, and nothing seemed practicable.

Then the single-phase system was presented, it was adopted and used on a number of roads, as the one thing which was commercially practicable under a good many of the situations where 600-volt direct-current apparatus was inapplicable.

But great developments have occurred since that time, which was fifteen years ago, and that is in railway history a very long time. If you go back another fifteen years, that is, go back from 1902 to 1887, you will find the electric motor was beginning to prove itself a success. I refer to the Richmond road. This whole development has taken place within thirty years. And then came the single-phase system fifteen years ago, when the operation of railways at high voltage was begun, and in that time have come other developments, the combination of the single-phase transmission and polyphase motors, and the direct -current motor has gone up to voltages which fifteen years ago were presumed to be impossible.

Consequently, in the light of development, the single-phase motor had its place, and is still occupying a place. The single-phase system is doing some of the heaviest work, and while it would be desirable to have one universal system in use everywhere, who knows what that system should be? If it had been adopted fifteen years ago, it would have been one thing; if it were to be chosen today, it might be the same thing or something else and possibly in five or fifteen years from now there may be other things which would make the effect of any final decision at this time a mistake.

W. C. Korthals Altes: I think Mr. Latour is pointing in absolutely the right direction so far as polyphase series motors are concerned. He has distinguished himself by opposing the multiplication of phases, and also he has brought out the point about the squirrel-cage commutation.

We have tried such motors, having squirrel-cage commutators, and we can say they operate very satisfactorily. I could show a d-c. motor and an a-c. motor and a three-phase a-c. motor, and put these motors through their stunts. You would see that the d-c. motor will flash and the a-c. motor will not show any sparking, so that the experiments made some years ago would not agree with the results which are obtained. Different conclusions have been reached. There is nothing remarkable about that, because we know the a-c. motor being completely compensated, and the current at all times passing through zero, is in a better condition for commutation. The unfortunate thing is we started out with single-phase motors and not three-phase motors. The three-phase motor is very superior to the single-phase motor, because we have there, as Mr. Latour has pointed out in a very interesting manner, the real sine distribution of the flux, and in all the work we have done we have always tried for that because with those conditions we get satisfactory commutation.

Another motor, which Mr. Latour has referred to, is what he calls the squirrel-cage commutation motor. I must take exception to some features of Mr. Latour's paper with respect to that. In Fig. 7 the author represents a two-pole three-phase commutator motor having six slots and an ordinary drum-wound armature.

Now, if there is a brush which short-circuits two commutator bars, there will be a short-circuited coil in two slots. If there are twelve commutator bars, a brush spanning two commutator bars, no matter in what position the armature is, there will at all times be at least a slot and coil which are short-circuited. You know if you take a direct-current motor with another winding in the links of the field winding, and you suddenly break the field, you will not have a series circuit, because the stored energy of the magnetic field would be equal in the circuit, which is formed.

On a single-phase machine, when the coil passes under the brush, we have to reverse the current. If we have a three-phase armature, we have to pass that current through 120 deg. You see immediately the inductive circuit by shifting through 120 deg. is much reduced on the 6-phase; on the 9-phase it is reduced to 40 deg. By shifting the current through a smaller number of degrees we get a more satisfactory operation, and that is the important thing Mr. Latour brought out. We get ideal current commutation.

We have only built a few motors so far, and I can mention as an example, a 60-cycle, 12-pole, 250-h. p. motor, and it cannot be made to spark. That is the very important work which Mr. Latour has done on three-phase motors.

As far as the single-phase motor is concerned, he has also distinguished himself by his so-called Latour connections. We have built quite a few motors with Latour connections, or something very similar. The two-circuit repulsion motor is not entirely new, but Mr. Latour has combined this with rotor excitation. The single-phase motor, built with many rheostats, as proposed by Mr. Latour, having, moreover, a series transformer to limit the possible voltage which can appear at the brushes, is an ideal motor from a commutation standpoint. But what Mr. Latour has said, in regard to the possibility of building large single-phase motors of high frequency, is absolutely true, although it would be practicable to build them three phase, and in that case they would be less sensitive. That it is absolutely impossible to build large single-phase motors of the commercial type of repulsion motors built in this country is not true. We have no demand for large single-phase motors and so have built none, but we could at any time build the largest. There is no question that motors can be built which operate satisfactorily but whether the motor works well in connection with the railway telephones and transmission and distribution systems, sub-stations, etc., is entirely a railway problem.

Referring to the characteristics of the series motor Prof. Karapetoff has succeeded in taking into account the saturation of the motor itself, which is most important, but of equal if not greater importance, is the saturation characteristic of the transformer.

If we should build a motor in which we had no increase of

magnetizing current in the transformer, we would have an absolutely unstable motor. The motor has a characteristic which is very undesirable and the only way of improving that characteristic is to have a transformer which saturates and gives increased stability. I have attempted to accomplish this but have never succeeded because I did not know how to saturate a transformer and obtain that characteristic. If Prof. Karapetoff can show us the way he will do a great thing for the technical engineer.

H. M. Hobart: Some fifteen years ago the designers of electric motors were filled with enthusiasm to do their best to improve the single-phase motor. They entertained probably too optimistic an opinion of the possibilities of single-phase motors when applied to railway work. Wholly unintentionally they delayed the progress of railway electrification by perhaps many years. I speak of this, because I think Mr. Altes is wrong in thinking it his function to simply design motors when the railway engineers come to him and ask for them. He knows better than most engineers the difficulties with which he contends in the design of such motors. Notwithstanding these difficulties, by the skill which he has acquired, he designs excellent three-phase commutating motors for certain work. He knows they are heavy, he knows they are more expensive, but he does his best, and when the practical application is of such a nature that, notwithstanding the increased cost and weight, this polyphase motor is the right thing, he gives them an excellent motor. That is entirely right and proper, but if instead of merely designing the motor when he is asked to do so, he were to be less modest, and go to the railway engineers and say—"Look here, there are enormous difficulties in this kind of motor and disadvantages that you may not realize. I see that you are beginning to entertain the thought that they will be suitable for railway work, and with all due deference to your experience in railway work, I will counsel you that they have all sorts of faults. We will, however, do our best to overcome these faults." Were he to do this, he would be serving a useful purpose and helping progress in railway electrification. I felt quite strongly that that was a point that ought to be made, when I heard Mr. Altes state that his conception of his duty was merely to design what he was asked to design.

If we look back fifteen years, I think we can see that if those who had become so enthusiastic over single-phase railway motors had endeavored to curb their enthusiasm, and to be more frank regarding the difficulties, the situation would not have got out of hand to the extent of wasting so many million dollars as we know now were wasted. This assessment of the waste is not an uncorroborated assertion coming from myself alone. We now all realize that. The admission is again today implied quite distinctly in Latour's paper, where we can read between the lines that he would not advocate single-phase equipment of the

rolling stock. He would put in a motor generator and use d-c. motors. Mr. Murray some years ago came out with the same admission when he said that by putting a rectifier on his locomotive he would be able to rate up the motors to one and half times the load of which they were capable when operating directly from a single-phase circuit.

Furthermore, I do not think those who did so much in trying to develop the single-phase railway system should feel so disheartened, as I gather they are inclined to feel from some of the remarks which have been made. They did admirably and developed a really substantial working system in several instances. Large values in train mileage have been performed by locomotives and cars operated by single-phase motors, and considering the enormous difficulty with which these single-phase advocates were contending, they did better than could reasonably have been supposed possible. They only produced these relatively satisfactory results by dint of the ablest sort of engineering.

B. A. Behrend: Mr. Latour refers most suggestively to the transformation, in the locomotive itself, of single-phase current into direct current. In the powerful electric engines of the Norfolk & Western Railway, single-phase current is being transformed into three-phase current. Thus we note broadly the tendency to utilize single-phase alternating current on the trolley with motors of a type other than single-phase. There are many in this audience here to-day who, sixteen years ago, discussed with great enthusiasm the promise of the application of single-phase currents to railroad work, while others, like Mr. Hobart and myself, showed greater skepticism. These remarks are made, not to deprecate the great credit due those who developed the single-phase railway system, but to point out that there must be reached an agreement among the engineers of the country in regard to the system of electrification, if the great work of operating the trunk lines of the country electrically is to be accomplished with the least opposition and the maximum amount of co-operation.

It is time that such a decision be reached. Almost twenty years ago the turbo-generator began its evolution. Numerous types were developed which have now all been superseded by the radial-slot type, upon which those who started with this type look back with satisfaction. There is likely to be the same result in the field of electrification. It would seem as though it behooved the engineers of the country to understand the issue of common aim and to unite upon one system after weighing with care and understanding the advantages and disadvantages of the available methods of electric propulsion. Thus the directors of our great transportation system can be met by a united front, advocating for each problem in electrification as it may come up the same system, thus infusing confidence into the great problem before us, viz., that of bringing the railroad system of our

country up to the utmost demands that may at any time be placed upon it.

William B. Jackson: In considering these papers I naturally think of the large practical suggestion that is made by them, which has been referred to by several of the speakers, namely, the work that must still be done, in developing the most effective and satisfactory motor for operation upon a-c. systems.

It has been stated that we are considering merely a railroad problem, that is, the electrification of our trunk line railways. We are doing this and are doing considerably more, for the reason that we need very little vision to see in the not distant future a comprehensive a-c. electric system covering almost every square mile of the now populated territory of the United States. This means the development and regular use of electric current in ways that are now unusual. It means that we need to arrive at the most efficient and most effective a-c. motor for general use, having good speed-regulation characteristics.

There is another phase of the situation which is illustrative of the need for simplicity with flexibility in a-c. motors; the question in the equipment of our large power stations, whether to have alternating current alone for operating auxiliaries or direct current alone, or must we have two systems throughout the power station; one of which to serve constant-current motors and the other to serve variable-speed motors? This seems a very unnecessary situation when we come to the final analysis, since if we need motors operating at both constant speed and variable speeds, motors should and will be developed so as to eliminate even a question as to the installation of a dual system.

It thus seems to me that the problem set out by these papers is a very broad one, taking in the railways as one of the important features, but also involving the general distribution and utilization of power throughout the United States to an extent to which it has not yet been developed but such as our vision readily shows us must be the case in the not distant future.

V. Karapetoff: Prof. Gray combines the time diagram of the e.m.fs. with the space diagram of m.m.fs., and this is exactly what I have done in both the device and Fig. 7. Fig. 6, shows only the space diagram, and then I incorporate that diagram into a figure showing the e.m.f. diagram, and I point out that the stator bar is used both for currents and voltages, which is equivalent to saying it is used for the space diagram and time diagram as well. If the time diagram is in the usual counter-clockwise representation, then the space diagram comes in the clockwise representation. While I am not sure that this is always so, I was careful to point the clockwise representation in Fig. 6 and counter-clockwise for the time diagram in Fig. 7.

I am glad to know that at least one manufacturing company in this country has undertaken to further develop the Schrage motor, to which Mr. Altes makes a detailed reference. When I first

saw a description of the Schrage motor I was full of hope that that was really the most satisfactory coming type. I could see it, from the principle incorporated in the motor. It is really a combination of an induction motor with the commutator motor incorporated in it, only for the fraction of the output differing from synchronous speed. For that part of the output which corresponds to the synchronous speed we need no commutator, and the ordinary induction motor is satisfactory. What you need is the counter e.m.f. of the proper frequency to be applied at the terminals of the rotor to force the induction motor to operate without $I^2 R$ losses in the resistance at any designed speed, either above or below synchronism, and that is just what the additional circuit in the Scharge motor does.

W. C. K. Altes: I find that I did not express myself properly when I spoke about single-phase motors. What I meant to say was that the single-phase motor has low starting torque referring to the generator set, referred to by Mr. Latour. I will leave it to the railway engineers to judge if it is a practical scheme.

We would like to provide a motor which will make it unnecessary in many cases to obtain direct-current, to change from alternating-current to direct-current. It has great advantage in simplicity and efficiency, the only reason why we do propose to build two kinds of motors, a series motor and a shunt motor, instead of building one shunt motor, is that the series motor is so much cheaper than the shunt motor. In cases where we do not require the shunt characteristics, as with fans and pumps and similar applications, we use a series motor, and in cases where we do require the shunt-motor characteristic, as on tools, we will have to use the shunt motor.

ADDRESS BY PRESIDENT E. W. RICE, JR. AT A. I. E. E. MIDWINTER CONVENTION FEBRUARY, 1918.

MEMBERS of the electrical profession and industry have reason to be pleased with the contributions which they have made for the benefit of the world. While we are glad to think that our science and our industry are fundamentally devoted to the products and conditions of peace, we realize that in the electric light, searchlights, the X-ray, telephones, telegraph, wireless apparatus, electric motors, etc., electricity plays an important part in the grim business of war.

We are in the midst of an extraordinary coal famine, due to causes which it is perhaps undesirable for us to attempt to outline. However, I would like to point out how much worse the situation might have been were it not for the contributions of the electrical engineer; and also how much better our condition might have been if our contributions had been more extensively utilized.

Suppose we assume that the present serious situation is due to a lack of production of coal. It is comforting to consider to what extent conditions surrounding such production have been improved and how the output of our coal mines has been already increased by the use of electrical devices in connection with coal mining—such for example as the electric light, electric coal cutters, electric drills and electric mining and hauling locomotives. I have no figures before me but I think it is a fair assumption that the output of coal mines should have been increased at least 25 per cent on the average by the employment of such electrical devices. If this estimate were cut down to 10 per cent it would still leave a possible increase in the tonnage of coal produced of something like 50,000,000 tons during the past year.

If on the other hand, our situation is not due to a shortage in the production of coal, but rather to the failure of the distributive agencies of the country, which is more probable, it is interesting to see how this difficulty would have been largely removed if the railroads of the country were operated by electricity instead of steam.

Where electricity has been substituted for steam in the operation of railroads, fully 50 per cent increase in available capacity of existing tracks and other facilities has been demonstrated. This increased capacity has been due to a variety of causes, but largely to the increased reliability and capacity, under all conditions of service, of electric locomotives, thus permitting a speeding up of train schedules by some 25 per cent, under average conditions. Of course under the paralyzing conditions which prevail in extremely cold weather, when the steam locomotives practically go out of business, the electric locomotives make an even better showing. It is well known that extreme cold (aside from the physical condition of the traffic rail) does not hinder the operation of the electric locomotive but actually increases its hauling capacity. At a time when the steam locomotive is using up all its energy by radiation from its boiler and engine into the atmosphere, with the result that practically no useful power is available to move the train, the electric locomotive is operating under its most efficient conditions and may even work at a greater load than in warm weather. It may therefore be said that cold weather offers no terrors to an electrified road, but on the contrary, it is a stimulant to better performance instead of a cause of prostration and paralysis.

But this is not all. It is estimated that something like 150,000,000 tons of coal were consumed by the railroads in the year 1917. Now we know from the results obtained from such electrical operation of railroads as we already have in this country that it would be possible to save at least two-thirds of this coal, if electric locomotives were substituted for the present steam locomotives. On this basis there would be a saving of over 100,000,000 tons of coal, in one year.

This is an amount three times as large as the total coal exported from the United States during 1917.

The carrying capacity of our steam roads is also seriously restricted by the movement of coal required for haulage of the trains themselves. It is estimated that fully 16 per cent of the total ton-mileage movement behind the engine drawbar is made up of company coal and coal cars, including in this connection the steam engine tender and its contents. In other words, the useful or revenue carrying capacity of our steam roads could be increased about 10 per cent with existing track facilities by eliminating the entire company coal movement.

I have not mentioned the consumption of oil by the railroads

which we are told amounted in 1915 to something like 40,000,000 barrels, nearly 15 per cent of the total oil produced. This fuel is entirely too valuable to be used in a wasteful manner. It is important for many reasons that such a wonderful fuel as oil should be most economically used, if for no other reason than that it will be needed for the ships of our forthcoming merchant marine, for the tractors that till our fields, and the motor trucks that serve as feeders to our railways.

The possible use of water power should also be considered in this connection. It is estimated that there is not less than 25,000,000 h. p. of water power available in the United States, and if this were developed and could be used in driving our railroads, each horse power so used would save at least six pounds of coal per horse power-hour now burned under the boilers of our steam locomotives. It is true that this water power is not uniformly distributed in the districts where the railroad requirements are greatest but the possibilities indicated by the figures are so impressive as to justify careful examination as to the extent to which water power could be so employed and the amount of coal which could be saved by its use. There is no doubt that a very considerable portion of the coal now wastefully used by the railroads could be released to the great and lasting advantage of the country.

The terrors of these "heatless days" will not have been without benefit if they direct the attention of the people and of our law makers to the frightful waste of two of our country's most valuable assets—our potential water power and our wonderful coal reserves. The first, potential water power, is being largely lost because most of it is allowed to run to waste, undeveloped, unused. The second asset, coal, is wasted for exactly the opposite reason. It is being used but in an extravagant and inefficient manner.

Our water falls constitute potential wealth which can only be truly conserved by development and use—millions of horsepower are running to waste every day, which once harnessed for the benefit of mankind become a perpetual source of wealth and prosperity.

While the amount of coal in our country is enormous, it is definitely limited. While Providence has blessed us with a princely amount of potential riches in our coal beds, it is known that there is a finite limit to the amount of coal so stored and when this coal is once exhausted, it is gone forever. It is really

terrifying to realize that 25 per cent of the total amount of coal which we are digging from the earth each year is burned to operate our railroads under such inefficient conditions that an average of at least six pounds of coal is required per horse power-hour of work performed.

The same amount of coal burned in a modern central power station would produce an equivalent of three times that amount of power in the motors of an electric locomotive, even including all the losses of generation and transmission from the source of power to the locomotive. Where water power may be utilized, as in our mountainous districts in the West, all of the coal used for steam locomotives can be saved. In the Middle and Eastern states, however, water powers are not sufficient and it will be necessary in a universal scheme of electrification that the locomotives be operated from steam turbine stations, but as I have already stated, the operation of the electrified railroads from steam turbine stations will result in the saving of two-thirds of the coal now employed for equivalent tonnage movement by steam locomotives.

It is therefore not too much to say that if the roads of the country were now electrified that no breakdown of our coal supply, due to failure or distribution, would exist. What this would mean for the comfort of the people and the vigorous prosecution of the war, I will leave for you to imagine.

Of course this picture which I have briefly and inadequately sketched of the great benefits which our country would have received if the roads had been electrified does not improve our present situation and it may be claimed that any discussion of such a subject at this time is of an academic nature. This point of view is, in a sense true, but I think that we can properly take time to consider it because of the effect which it may have upon our future efforts. This picture is not merely an inventors dream but is based upon the solid foundation of actual achievement. We have had enough experience upon which to base a fairly accurate determination of the stupendous advantages and savings which will surely follow the general electrification of the railroads; in fact, I think we can demonstrate that there is no other way known to us by which the railroad problem facing the country can be as quickly and as cheaply solved as by electrification.

The solution of the railroad problem would also "kill two birds with one stone" by solving the fuel problem at the same time.

If it is a fact, as has been stated, that the steam railroads of the country have failed to keep pace with the country's productive capacity—the increased output of manufacturing industries, the extension of agriculture and other demands for transportation —it is obvious that if the country is to go ahead, the railroad transportation problem must be solved and it must be solved at the earliest possible date. It becomes a matter of national importance that the best solution should be reached in the shortest possible time. That solution is best which will give the greatest amount of transportation over existing tracks, in the most reliable manner, and if possible, at the lowest operating cost. We electrical engineers are confident that we can make good our claim that the best solution is to be found in a general electrification of the railroads. That such a solution would be of great advantage to our profession and to our industry is important, although not as important as the great advantage which it would be to our country, freeing it as it would from the present threatened paralysis of business, possibility of untold human suffering and incalculable financial loss. It should give us courage and optimism for the future of our profession to contemplate the service which we may render in this direction, and which it seems to me is immediately at hand. It should arouse in all of us, and particularly in the younger engineers, an enthusiastic confidence in the present and future stability and value of our profession and of the electrical industry. It should satisfy the young engineer that the opportunity for him to render important service is as real and great to-day as it has been in the past for those of us who have seen and participated in the marvelous growth of the industry up to the present time.

We would not be justified in being so confident of the benefits of electrification of railroads if every element in the problem had not been solved in a thoroughly practical manner. The electric generating power stations, operated either by water or by steam turbines, have reached the highest degree of perfection, efficiency and reliability, while the transmission of electricity over long distances, with reliability, has become a commonplace. Electric locomotives capable of hauling the heaviest trains at the highest speeds, up and down the heaviest grades, have been built and found in practical operation to meet every requirement of an exacting service. There is, therefore, no element of uncertainty nothing experimental or problematical, which should cause us to hesitate in pressing our claims upon the attention of the country.

Electrification of railroads has progressed with relative slowness during these many years, waiting upon the development and perfection of all of the processes of generation and transmission and of the perfection of the electric locomotive itself. When all these elements had been perfected, as they now have been for several years, the railroads found themselves without the necessary capital to make the investment.

I realize that the task of electrifying all of the steam railroads of the country is one of tremendous proportions. It would require under the best of conditions many years to complete and demand the expenditure of billions of dollars.

The country, however, has clearly outgrown its railway facilities and it would require, in any event, the expenditure of billions of dollars and many years of time to bring the transportation facilities up to the country's requirements.

It is not necessary that electrification should be universal in order to obtain much of its benefits. It is probable that the most serious limitations of our transportation system, at least in so far as the supply of coal is concerned, is to be found in the mountainous districts and it is precisely in such situations that electrification has demonstrated its greatest value. Electrification of a railroad in a mountainous district will in the worst cases enable double the amount of traffic to be moved over existing tracks and grades.

If a general scheme of electrification were decided upon, the natural procedure would be, therefore, to electrify those portions of the steam railroads which will show the greatest results and give the greatest relief from existing conjestion. Electrification of such sections of the steam railroads would have an immediate and beneficial effect upon the entire transportation system of the country and it is our belief that electrification offers the quickest, best and most efficient solution that is to be obtained.

It may be said that the present is not a propitious time in which to deflect any of the country's money into railroad electrification. I think that in spite of the enormous advantages of which I have spoken, we would be inclined to agree with such a point of view if it were not for the recent unpleasant demonstration of the failure of our railroad transportation systems to meet the demands which have been placed upon them by the industries, aggravated it is true by the war conditions and also by the unkindness of the weather.

After all, the question for the country to decide is whether we

dare to limp along with the present conditions of restricted production, due to limited transportation, at a time when the world demands and expects from us the greatest possible increase in our efficiency and total production.

What assurance have we that the present conditions are temporary, and even if they improve as they surely shall with the coming of warm weather, what are we going to do next winter? Of course, even if we should start electrification at once, we could not have all our railroads electrified by next winter but we could have a good start, and as Sherman said about the resumption of specie payments—"The way to resume is to resume," so "The way to electrify is to electrify."

Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.

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DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

BY G. J. NEWTON

ABSTRACT OF PAPER

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author made some suggestions, based on many years experience in designing underground distribution systems, for the guidance of engineers interested in this class of work.

The previous article treated the subject only in a general manner; the object of this article is to show each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

No two systems are entirely alike. The operating conditions and municipal requirements are different in every locality, it is impossible, therefore, to make any definite rules that will apply under all conditions.

By assuming the average conditions met with in the smaller cities it is possible to show the fundamental principles of handling this work in a systematic manner. While systems may, and do, differ, still it is possible to design a system for any type of distribution if these principles are followed.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison however and will be of value in that manner only, and if treated as percentage difference will apply under any reasonable conditions.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

IN ORDER to show each step in the design of an underground distribution system in the business section of a medium size city, it will be assumed that the present system consists of 110-220-volt single-phase lighting, 220-volt three-phase power and a street lighting system, as this, or some similar arrangement is commonly met with.

In order intelligently to design a system of underground distribution it is necessary to have the following information:

1. The lighting and power load in each building and the most suitable place to terminate the service cables, due regard being given to the present feeding point, so that the new system of

secondary mains can be located to the best advantage with the least changing of the present inside wiring.

2. Location of all street lights that are to be connected to the new system of distribution.

3. Voltage of the lights and motors that are to be supplied from the new system.

4. Location of pipes, sewers, foreign conduit systems and all other sub-surface obstructions in the streets where it is proposed to locate the new conduit system.

5. Kind of pavement on each street.

6. An accurate map of the proposed underground district drawn to a scale that will permit showing the details of property lines, curbs etc.

7. A copy of all city ordinances governing this class of work.

8. Cost of labor, teams, paving, material and equipment required to install the system.

The design of an underground system is not a difficult matter if handled in a systematic manner; there are however, a few general rules that should be adhered to, as far as possible, to get the best results.

1. Design the cable system to give the most economical, efficient and reliable service and then lay out the conduit to serve that arrangement.

2. Try to foresee, and provide facilities for meeting every kind of trouble that is liable to occur. As far as possible provide two sources of supply for every distribution center, particularly those located in important districts.

3. Remember that the ultimate value of the system is not its first cost; an improperly designed and installed system will never give satisfaction and will cost more for changes and additions in the end.

4. Use the best material and experienced workmen; there is no branch of an electrical undertaking where inferior material and poor work will give greater trouble than in an underground system.

Before taking up the actual design it will be well to consider the selection of cables to be used for the various systems as this must be kept in mind while making the design.

Feeders. Assuming that the station is equipped with three-phase generators and that the lighting load is balanced on all three phases, it will be advisable to select uniform sizes of cables for the light and power feeders, in order to reduce the amount of

emergency cable required and also to standardize the switches and other equipment.

Primary Feeders run direct from the station to centers of distribution with few or no taps on them; they are installed in the lower or trunk ducts and are not liable to mechanical injury after being installed. These feeders receive the greatest benefit from the diversity factor, therefore it is not necessary to provide a great amount of reserve capacity as other feeders can be added as the demand increases.

Secondary Feeders run from the centers of distribution to the various transformers and are also in the lower ducts; these cables receive less benefit from the diversity factor than the primary feeders therefore they should have a little more reserve capacity.

For the system under consideration both of these classes of cables should be three-conductor cables. They can be either paper or varnished cambric insulated. If paper is used it will be necessary to splice rubber or varnished cambric insulated cable tails on it wherever they enter equipment. Personally I prefer varnished cambric or varnished cloth insulated cables for all distribution work, particularly in the smaller cities where usually the cable department is very limited.

Secondary Mains. The secondary mains for the power system will be three-conductor cables as they supply only three-phase power and will have three-conductor services spliced to them.

The secondary mains for the lighting system will have to be large, as that load is single-phase, as far as the secondary distribution is concerned, and next to the service cables they receive the least benefit from the diversity factor.

If three-conductor cable was used there would be a considerable reduction in the carrying capacity over single-conductor cables (25 per cent) also it would not be possible to take more than about two services out of each splice; this would increase the number of service boxes required and add considerably to the cost of the system.

The neutral of the 110-220-volt lighting system should be grounded at every transformer vault, and therefore unless there are some exceptional conditions, such as excessive electrolysis, there is no reason why the neutral can not be a bare copper stranded cable, and by using two single-conductor cables for the "outers" it will be possible to get the greatest benefit from the carrying capacity of the cables installed.

Considering the fact that there is no way of providing any

emergency facilities for the secondary mains and that they receive the least benefit from the diversity factor, and are cut at frequent intervals for splicing the services, it is evident that they are the most important link in the distribution system. Trouble on a secondary main is very liable to interrupt service to every consumer spliced to it, and requires considerable time and expense to repair; therefore these cables should be given considerable excess carrying capacity to provide for future growth, and have varnished cloth or cambric insulation. (Except in special cases it is unnecessary to use rubber insulated cable on any part of a distribution system, except to enter equipment as mentioned previously.)

The single-conductor secondary mains will permit four service cables being taken out of each splice if necessary.

It is a good plan when installing secondary mains, particularly when there is a heavy load about the center of a block, to make a loop service of the main, instead of running service cables into the building, as this permits sectionalizing the main in case of trouble and reducing the number of consumers out of service. This arrangement also reduces the time required to locate trouble on a main.

Service Cables. The service cables for the power system should be three-conductor, but those for the lighting system should be single-conductor. It is advisable to install three service cables for every consumer where there is any liability of the load being over one kw. and if three cables are taken to all consumers it will be an easy matter to keep the 110-220 volt load balanced without the necessity of opening splices to make changes.

Laterals. The lighting and power services should be installed in separate pipes, and in the business district of a city it is a good plan to install two laterals to all consumers when the system is installed. When fibre conduit is used for the laterals, as is generally the case, the cost of installing the additional duct is very small as there is practically no additional excavation or paving required and very little additional concrete.

Service Boxes. Service boxes should be about three by four feet, and located at the most convenient points to permit the services entering the buildings, they should be spaced so that the services are not unreasonably long and that not more than four services are taken out of each box.

Keeping the above facts in mind we can now proceed with the design of the system.

Load Map

Fig. 1 shows the load map. Each block is numbered for future reference and the building lines are shown (it is not necessary to show the individual stores in a building as all of the consumers in a building should be supplied from one service in order to save cable and get the benefit of the diversity factor between consumers). The house numbers should be given but are omitted from this plan to avoid confusion.

The point where the service is to enter the building is shown,

TABLE I
LOAD RECORD BY STREETS

Street	From	To	Light	Power	Total
Ave. A.	First St.	Second St.	88.0	98.0	186.0
" "	Second "	Third "	93.0	57.0	150.0
" "	Third "	Fourth "	53.0	33.0	86.0
" "	Fourth "	Fifth "	110.0	41.0	151.0
" "	Fifth "	Sixth "	98.0	23.5	121.5
Ave. B.	First St.	Second St.	69.0	36.5	105.5
" "	Second "	Third "	93.5	45.0	138.5
" "	Third "	Fourth "	197.0	102.0	299.0
" "	Fourth "	Fifth "	178.0	83.0	261.0
" "	Fifth "	Sixth "	93.0	58.0	151.0
Ave. C.	First St.	Second St.	44.0	32.0	76.0
" "	Second "	Third "	48.5	20.5	69.0
" "	Third "	Fourth "	57.5	20.0	77.5
" "	Fourth "	Fifth "	51.0	14.5	65.5
" "	Fifth "	Sixth "	40.0	32.0	72.0
1st St.	River	Ave. C	138.5	76.0	214.5
2nd "	"	" "	139.0	77.0	216.0
3rd "	"	" "	120.5	62.0	182.5
4th "	"	" "	214.0	62.0	276.0
5th "	"	" "	128.0	31.0	159.0
6th "	"	" "	109.0	25.5	134.5
Big Store			65.0	150.0	215.0
			2227.5	1179.5	3407.0

Streets 12,000 ft., or an average of 284 kw. per 1000 ft.

and the loads are marked on the plan facing the street from which they are to be supplied. While it is advisable to mark the most desirable location for the laterals still it is frequently necessary to select a less desirable location in order to reduce the amount of conduit or cable; as far as possible however, it is advisable to adopt a uniform method of installing the laterals as this permits standard construction and reduces the cost.

Having completed the load map the next step is to tabulate it by streets and blocks, as shown in Table 1. Referring to this

table, it is seen that the lighting load is 2227.5 kw. and the power load is 1179.5, or a total of 3407 kw. to be supplied as follows:

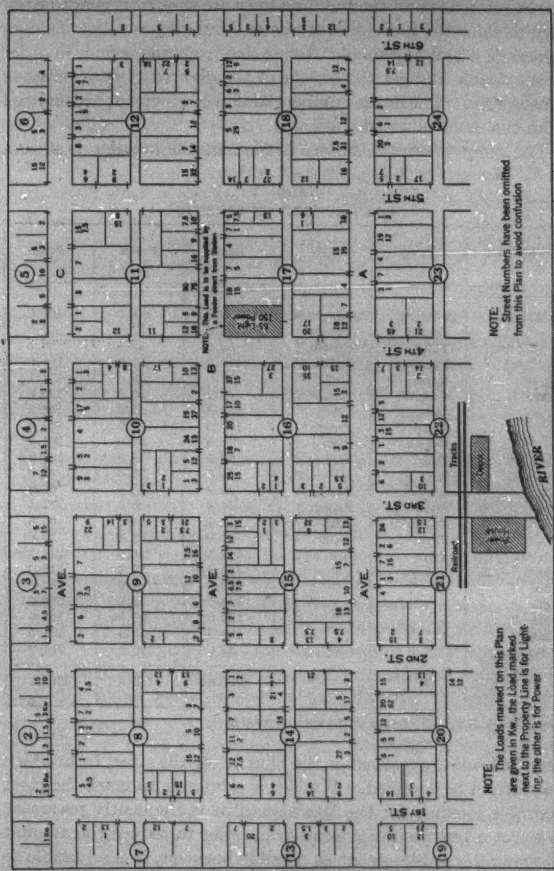


FIG. 1—LOAD MAP

Lighting system a-c. 110-220 volts single-phase three-wire.
Power load a-c. 220 volts, three-phase.
Street light system, a-c. 5.5 amperes, series.

It is always advisable to show the location of the street lights on the load map for, assuming that *all* wires in the district must be placed underground, it is evident that there must be conduit to every lamp and by laying out a rough plan of that system, or circuit, a general idea of the conduit arrangement can be determined. As the location of the street lights is determined by the city authorities it is seldom possible to change their location and by knowing where the conduit must be placed for street lights it will frequently permit a saving in conduit in other localities by changing the secondary arrangement of some of the consumers.

Owing to the fact that conduit systems are very expensive there is a great tendency to select streets having the cheapest pavement in which to install the conduit. While it is desirable to do work as cheaply as possible, still there is more than the first cost to be considered. Where a conduit system is used for feeders or high-tension systems exclusively, and there is no distribution from it, there is no objection to selecting such routes as will permit the conduit being installed as cheaply as possible.

Where a conduit system is installed for distribution to consumers the location should be governed entirely by the best method of reaching the various buildings regardless of the kind of pavement.

Series Street Light System

Fig. 2 shows the series street light system. There are 51 72-volt arc lights and 15 20-volt incandescent lights on the circuit in this district, the approximate voltage of the system being about 4000. As No. 6 6000-volt cable is standard for this class of service it is advisable to use it for all circuits of this system.

As the location of the lamps is determined by the city authorities, and the circuit must reach every lamp, there are only two things to consider; first to use as little cable as possible and second to arrange the circuit so that it will be possible to sectionalize it in case of necessity. The arrangement, shown in Fig. 2, permits sectionalizing the circuit at three points, in case of trouble, without any unnecessary cable being used.

It is doubtful if a circuit as small as this one would require any sectionalizing facilities; however the arrangement is shown here for illustration.

Assuming that the arc lights on the street are to be placed on new iron poles and the incandescent lights in the alleys on wooden poles the best arrangement for the cable would be to use either

paper- or varnished-cloth-insulated cable for the main cable and rubber-insulated cable for the pole ends.

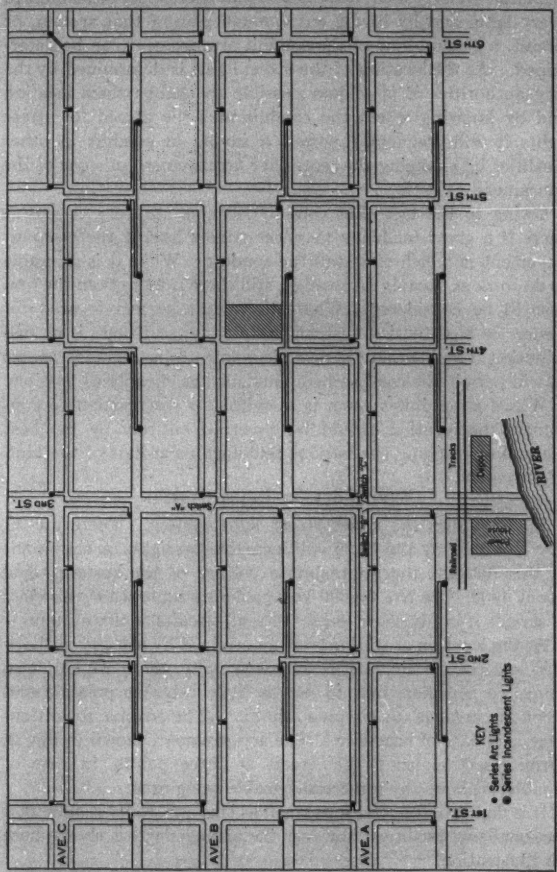


FIG. 2—SERIES STREET LIGHT SYSTEM

Frequently the pole end cable is run from the manhole, or handhole, up the pole to the lamp, this is a mistake, and a better plan is to terminate the pole end cable in series cut-outs mounted

in the base of the pole and run insulated wire from this point up to the lamp, this permits the lamps being cut off from the circuit easily in case it is necessary to test the cable for trouble at any time and also saves considerable cable.

Referring to the figure, it is seen that there are many places where two-conductor cable could be used to advantage and in such cases the engineer must decide whether to use all single-conductor cable or two-conductor where possible. The difference in cost will be very small and undoubtedly the single-conductor cable would give the most reliable service, and as two cables can easily be drawn in one duct there is no saving in duct space by using the two-conductor cable.

If two-conductor cable is used it should be made in the round form and not figure "8" or flat, as it is practically impossible to train the flat cable without kinking it, and this will cause damage, particularly with paper-insulated cable as small as this type of cable would be.

Assuming that the cable has to reach the 15 lights in the alleys and that the alleys are paved, and that, with the exception of Block 14, it is unnecessary to run any secondary mains in the alleys, the question may arise as to the advisability of installing steel-armored cable laid directly in the ground for these extensions.

If the alleys are paved there will be very little difference in cost between the two systems, possibly a saving of 25 per cent. This is too small a saving to warrant purchasing and keeping in stock the extra type of cable. The best plan would be to install a single fibre duct to all of the lamp poles in the alleys. Armored cable should never be installed under permanent pavement, particularly in the business section of a city.

In making the estimate for this system it will be assumed that No. 6 varnished-cambric-insulated cable, for 6000 volts, is used, terminating in cutouts in the base of the iron poles and in cutouts mounted on the wooden poles above the lateral pipe.

Owing to the location of the street lights the cable for this system will have to be installed in one of the upper, or distribution, ducts so that it can be taken out at any handhole for future lights that may be installed. If there were other arc circuits, feeding other districts, they would of course be installed in the lower or trunk ducts, such cables could be paper insulated as they would have no taps on them until brought out at the pole from which they were to run aerial.

While a series arc circuit has been shown in this system it is practically certain that in this district the streets, or the principal ones at least, would have an ornamental system, these systems vary so greatly that it was not thought advisable to show one on these plans. If an ornamental system is required in this district it will be found that there is a spare duct in the upper tier of conduit that is available.

While the series arc circuit has been designed first, in this case it does not particularly aid the design of the secondary layout as the load in the district is so heavy and so well distributed along the streets that it is evident that mains will have to be placed along each street; still this method is usually advisable.

Center of Distribution

From a distribution point of view, the ideal location for a station would be at the center of the load that it was to supply, but unfortunately such a location is seldom possible or advisable owing to the value of real estate and lack of facilities for receiving and storing fuel. In the case under consideration the power house is already located and the distribution must be designed from that point. It is, therefore, necessary to find where the center of the load is, or if it is too large for one feeder to supply, the load must be divided and separate centers located.

Where the load is large enough to require two or more feeders it should be divided as equally as possible so as to permit using a uniform size of cable for all the feeders. If there is any section where the load is liable to increase rapidly this fact should be considered in locating the center of distribution and assumed to exist, and feeder capacity allowed for it.

Fig. 3 shows the three-phase power arranged for locating the centers of distribution. Ordinarily it is sufficient to assume that the entire load in a block is concentrated at the center of the block, for the purpose of illustration in this case the load has been shown on each side of each street, and the loads are marked facing the streets from which it will be supplied. (This system of marking is carried out on all of the illustrations.)

Referring to Table 1, the power load is given as 1179.5 kw., this includes the 150 kw. that is in the store at 4th Street and Ave. B. which is to be supplied by a separate feeder from the station, leaving 1029.5 kw. (say 1030 kw.) to be supplied by the regular feeders. This is too large a load to supply with one feeder, and as 2/0 three-conductor cable will supply 558 kw.

at 2200 volts, it is advisable to use two feeders of that size and locate two centers of distribution, each having about 515 kw.

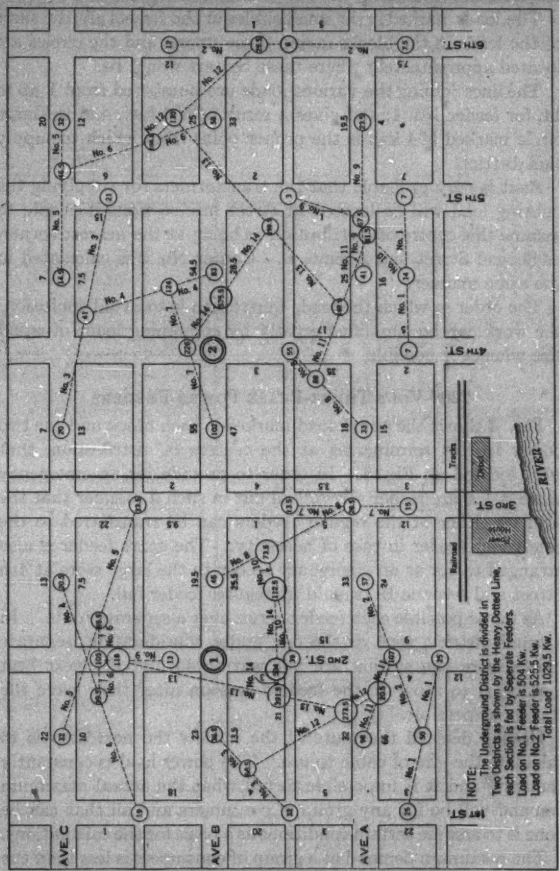


FIG. 3—CENTER OF DISTRIBUTION

or half the total load. Later the actual size of feeders can be calculated, considering the power and diversity factors.

Referring to Fig. 3, the heavy broken line running north and

south divides the load as follows: Feeder No. 1, 504 kw., feeder No. 2, 526 kw. (It is always best to try several ways of dividing the load and then select the one giving the best results.)

The loads marked in the small circles in the streets are the sums of the loads in the blocks facing those streets and the circles are located approximately where these centers would be.

The lines joining the various loads are numbered from 1 up to 14, for feeder No. 1, and gives a result of 504 kw., and the large circle marked 504 kw. is the proper point from which to supply this district.

As it is very probable that the transformers for supplying the lighting load will be located at street intersections it is safe to assume this centre of distribution as being at the nearest corner, or Second Street and Avenue B. Center No. 2 is calculated in the same manner.

The order in which the loads were taken is to avoid confusion; the work can be simplified greatly by combining loads of equal size whenever possible.

2200-Volt Three-Phase Power Feeders

Fig. 4 shows the power load marked in each block and the two power feeders terminating at the centers of distribution, that were located on Fig. 3. In order to provide for an emergency there is a spare feeder shown laid out in such a manner that the load on either of the regular feeders can be transferred to the emergency feeder in case of necessity. The spare feeder is also arranged to act as an emergency feeder to the large store at 4th Street and Avenue B., should its regular feeder fail.

As far as possible each feeder is run over a separate route. In a small district, such as the one under consideration, separate routes from the station are not warranted and the next best method is to separate the feeders as soon after they leave the station as possible.

Having decided the route of the feeders the next step is to calculate the size of cable to use. The power load is constantly changing and it is impossible to tell what the actual maximum demand will be for any group of consumers and all that can be done is to assume certain conditions as a basis for the calculations.

The maximum demand of a group of consumers is less than the sum of the several maxima, likewise the sum of several groups is less than the sum of their maxima, therefore, in a distribution system the primary feeders will receive the greatest benefit from

the diversity factor, the secondary feeders less benefit, and the secondary mains the least benefit.

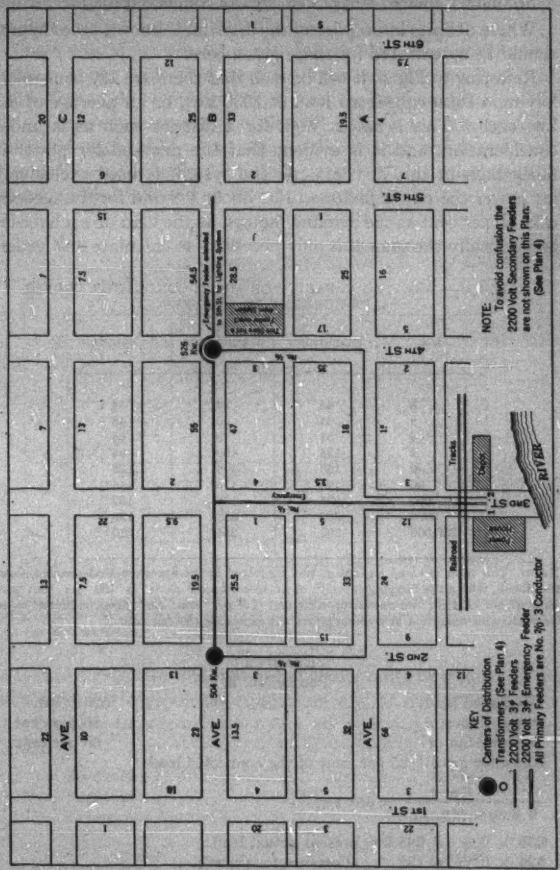


FIG. 4—PRIMARY POWER FEEDERS—2200-VOLT—THREE-PHASE

Assuming that the maximum demand will vary from 60 to 80 per cent of the connected load on various parts of the system, the feeders and mains can be calculated as follows:

Primary feeders, 60 per cent of the connected load.
Secondary feeders, 70 per cent of the connected load.
Secondary mains, 80 per cent of the connected load.

Where definite knowledge of the demand is known other figures should be substituted for those given here.

Referring to Fig. 5, it will be seen that there are 127 consumers having a total connected load of 1030 kw., or an average of 8.1 kw. each. This is rather high for a district such as is under consideration, and it is evident that the demand for power is fairly high in this district. As the system is used exclusively for power the power factor will be about 0.8 and for the feeders, take 60 per cent as the demand factors, as the load on each feeder is practically the same it is only necessary to calculate one feeder.

TABLE II—CARRYING CAPACITY OF MULTI-CONDUCTOR CABLES. SAFE CURRENT IN AMPERES

Size	1-Conductor	2-Conductor	3-Conductor
8	45	39	34
6	64	56	48
4	91	79	68
2	125	109	94
0	168	146	126
00	195	170	146
000	225	196	169
0000	260	226	195
250,000	293	255	220

NOTE: This table is based on information contained in the Standard Underground Co's. Handbook, taking the carrying capacity of single-conductor cable as 100 per cent and using 87 per cent for two-conductor cable and 75 per cent for three-conductor cable for continuous operation at a temperature not exceeding 150 deg. fahr..

EXAMPLE.

Feeder No. 2

Connected load	526.0 kw.
Power factor	80 per cent
Demand factor	60 per cent

Allow for growth 25 per cent of the connected load.

$$\frac{526 \text{ kw.}}{0.8 \text{ power factor}} = 658 \text{ kw.}$$

658 × 0.6 = 395 kw. present actual load.
526 × 0.25 = 132 " allowance for growth.

527 " total load for which to provide feeder capacity.

$$I = \frac{\text{kw.} \times 1000}{E \times 1.73} = \frac{527 \times 1000}{2200 \times 1.73} = 138 \text{ amperes}$$

Referring to Table II under three-conductor cable, it is seen that 2/0 cable will carry 146 amperes, therefore this is the proper size to use, and this checks the first selection of feeder size in calculating the centre of distribution.

Three-conductor 2/0 cable will deliver 558 kw. at 2200 volts, which is about 6 per cent more than the present connected load on the feeder, or 41 per cent more than the estimated load of 395 kw. Three-conductor 1/0 cable will deliver 481 kw. at 2200 volts and while this would supply the present estimated load, it would leave a very small reserve capacity, particularly as the estimated load is based on assumed conditions.

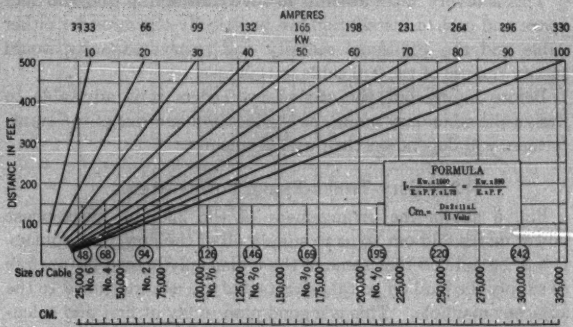


CHART I—LOAD IN KW.-THREE-PHASE-220 VOLTS—80 PER CENT POWER FACTOR—5 PER CENT LOST = 11 VOLTS

This chart is for use in determining the size of the three-phase power mains to save calculating each one—in using it care must be taken to see that the cable will carry the amperes that the load requires

It is in a case like this that the engineer must be guided by his personal knowledge of the conditions, and as cost is frequently the deciding factor we can make an estimate for each size of feeder.

Feeder No. 1	1000 ft.
Feeder No. 2	1000 ft.
Emergency Feeder	1400 ft.
Total	3400 ft.

The following prices are approximately correct for three-conductor, varnished-cambric-insulated, lead-covered cables for 2300 volts working pressure, with copper at 16 cents per lb.

Size	Price per 1000 ft.
No. 4 cable	$220.00
" 2 "	300.00
" 1/0 "	390.00
" 2/0 "	450.00
" 4/0 "	635.00

No. 2/0 cable 1400 ft. at $450.00	$1530.00
" 1/0 " 1400 " " 390.00	1326.00
	$ 204.00

This is a very small amount to save considering that the duct space and cost of installation are practically the same for either cable and the additional security and reserve capacity would justify the use of the 2/0 cable in this case.

Before laying out the secondary feeders it is advisable to design the secondary mains, as the location and size of the transformers will depend on their arrangement.

220-Volt Three-Phase Power Mains

Fig. 5 shows the arrangement of the 220-volt three-phase power mains laid out so as to supply all of the power consumers. It will be noticed that some of the mains only extend far enough to supply the load on certain streets and are not continued to the adjacent manhole. This is a good plan to try at first and calculate the size of cable required for each main, and where the cable will be too large the main can then be extended to the next manhole so that it will complete the network and be supplied from each end.

The two feeder districts are kept separate, but where mains from one feeder extend to a manhole of the other district (as at Ave. B. and Third Street) they can be tied together in a suitable junction box so that they are supplied from both ends. The main will form a loop in such cases.

In calculating the size of cable to use for the mains it must be remembered that all of the services are spliced directly to the mains, and the replacement of a main is a very expensive matter, the old cable removed, being in short lengths, is of little value except for junk.

While it is certain that all of the motors will not be operated at the same time still the mains must be of sufficient size to maintain the voltage without excessive drop under the worst

condition. Where there is a reasonable prospect that the load will increase it is advisable to install mains large enough to provide for this increase.

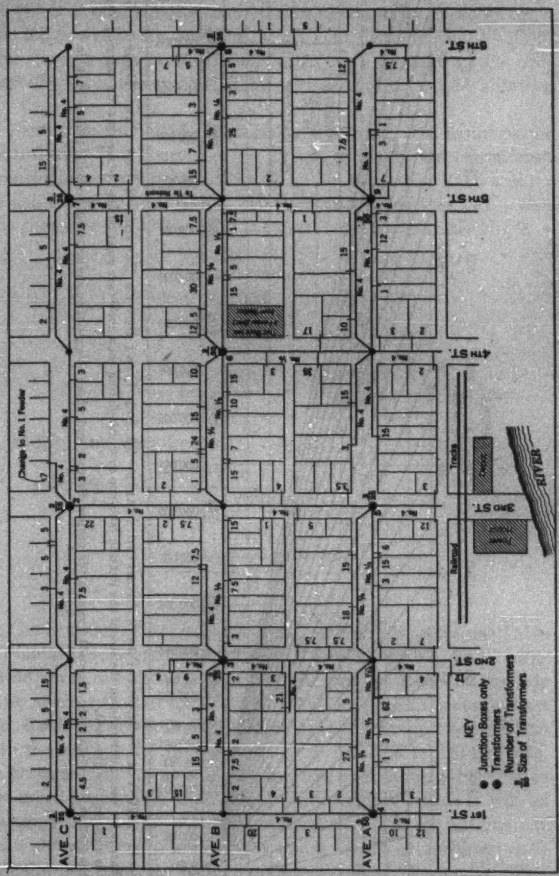


FIG. 5—220-VOLT—THREE-PHASE—POWER MAINS

To illustrate the method of calculating the size of mains, in Fig. 5, take the two blocks on Avenue C, from First to Third

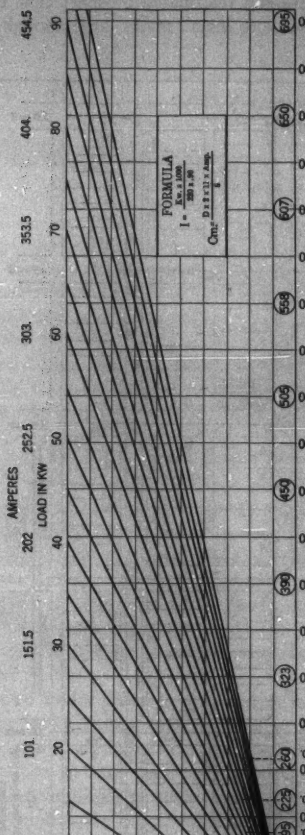


CHART II—220-VOLT—SINGLE-PHASE LIGHTING MAINS—90 PER CENT POWER FACTOR—2 PER CENT DROP = 5 VOLTS

This chart is for use in determining the size of the lighting mains to save calculating each one—care must be used to see that the carrying capacity of the cable given is sufficient for the load.

Take the load in kw.—follow the diagonal down to the proper distance and under this point on the scale will be found the size of cable.

Street, which has mains on both sides of the street and is supplied from transformers at each end.

Allowing 5 per cent drop, 0.8 power factor and assuming that 80 per cent of the motor load is to be operated at the same time, the 0.8 power factor, and the 80 per cent demand factor will, in this case, counterbalance each other and by taking the connected load as the amount to be provided for we will be allowing for these conditions.

First take the load in each block separately and assume that transformer No. 1 supplies the 22 kw. in block No. 1, and that transformer No. 2 supplies the 13 kw. in block No. 2; the center of the 22-kw. load is about 310 ft. from transformer No. 1 and the center of the 13-kw. load is 165 ft. from transformer No. 2.

EXAMPLE

$$I = \frac{\text{kw.} \times 1000}{E \times 1.73} = \frac{\text{kw.} \times 580}{E}$$

$$I = \frac{22 \times 580}{220} = \frac{12{,}760}{220} = 58 \text{ amperes.}$$

And for the size of cable,

$$\text{Cir. Mils.} = \frac{\text{ft.} \times 2 \times 11 \times \text{amperes}}{\text{volts lost}}$$

$$= \frac{310 \times 2 \times 11 \times 58}{11} = 35{,}960.$$

The next larger size is No. 4 or 41,740 cir. mils, and from Table II, under three-conductor cable, it is found that No. 4 will carry 68 amperes so this size will carry the present load but provide little spare capacity for growth. For the present, assume that No. 4 will be used here and proceed in the same manner to calculate the size for the 13-kw. load. The result is 11,220 cir. mils, or between No. 9 and No. 10 wire, but as it is not advisable to use anything smaller than No. 4 for secondary mains it will be assumed that this section will be No. 4 also. So far nothing has been allowed for growth but the mains would extend from each transformer bank far enough to supply all of the loads. This arrangement could be used at first and later when the load demanded the two mains could be tied together with the following result.

The total load in the two blocks will be 22 + 13 = 35 kw. which is assumed to be located at the center, or 400 ft. from each end of the main.

Total load, 35 kw.

Supplied from each end, 17.5 kw. (say 18 kw.).

$$I = \frac{18 \times 580}{220} = 48 \text{ amperes.}$$

$$\text{Cir. mils} = \frac{400 \text{ ft.} \times 2 \times 11 \times 48}{11} = 38{,}400 \text{ (Use No. 4).}$$

This will give a continuous main and would provide for a very considerable increase in load as the entire load is not concentrated at the center of the main, as assumed.

The assumption of 80 per cent demand factor is undoubtedly high, for this class of service and is only used here as an example of the method of calculation. All of the other mains are calculated in the same manner.

The mains should all be calculated and marked in pencil on the plan temporarily, after which it is advisable to select a few standard sizes and use them throughout the system. If the power and lighting cables are all marked temporarily at first it is frequently possible to standardize both systems with a few different sizes of cable.

In calculating the size of mains on Avenue A, from First to Third Street, it was found advisable to extend two of the mains as shown by the dotted lines to permit using No. 1/0 cable for all of the mains in those two blocks and assuming that the total load was concentrated at the manhole at Avenue A and Second Street, which is the worst condition that could occur in this section, the four No. 1/0 mains terminating there are ample to supply it.

The mains as laid out in Fig. 5 are of two sizes, No. 4 and No. 1/0, and these sizes are very suitable as probably with the addition of some three-conductor No. 6 cable, for small services, all of the mains and services can be installed with three sizes of cable.

The consumer service cables will average about 50 ft. each and the loads vary from 1 to 60 kw. The drop in the services should not be more than 1 per cent, or 2 volts, and the full connected load should be provided for to take care of the extra current required for starting the motors.

The services can be divided into three groups as follows:

0 to 10 kw.	Use No.	6,	three-conductor	cable.		
11 " 20	"	"	"	4,	"	"
21 " 40	"	"	"	1/0,	"	"

Where the load is over 40 kw. two service laterals can be installed if the operating conditions of the load demand it, and frequently the location of the motors is such that two services are desirable; a case of this kind is shown at Avenue A. and Second Street where two services are required and they are taken from separate mains.

The next operation is to determine the size of the transformers to install at the feeding points. Owing to the fact that the motor load is constantly changing it is a very difficult matter to determine the most economical size of transformers to install, while there are occasional heavy loads still the average load is considerably less.

It is not advisable to have a large variety of sizes as this makes it necessary to keep a large number of transformers on hand for emergency purposes. The best method is to decide on as few sizes as possible and use them in open delta where necessary on small loads.

It will be noticed that the transformers are spaced, and the mains laid out, so that a large increase in load can be taken care of by simply installing transformers at the intermediate manholes without the necessity of changing any of the mains; also at some points the mains can be extended to the next manhole to complete the network.

Following is a summary of the loads to be supplied at the various feeding points. Transformers No. 1 to No. 5 are supplied by feeder No. 1 and transformers No. 6 to No. 9 are supplied by feeder No. 2.

Location	Connected load	75 % of load	Number of transformers	Size of trans.	Total capacity
No. 1	51 kw.	38 kw.	2	25 kw.	50 kw.
" 2	61 "	46 "	2	25 "	50 "
" 3	118 "	88 "	3	25 "	75 "
" 4	175 "	131 "	3	50 "	150 "
" 5	105 "	79 "	3	25 "	75 "
" 6	212 "	159 "	3	50 "	150 "
" 7	84 "	63 "	3	25 "	75 "
" 8	76 "	57 "	3	25 "	75 "
" 9	150 "	112 "	3	50 "	150 "
			Total transformer capacity.....		850 kw.

The total power load was 1030 kw. which is equivalent to 1380 h. p., and as 60 per cent of the connected load was taken as

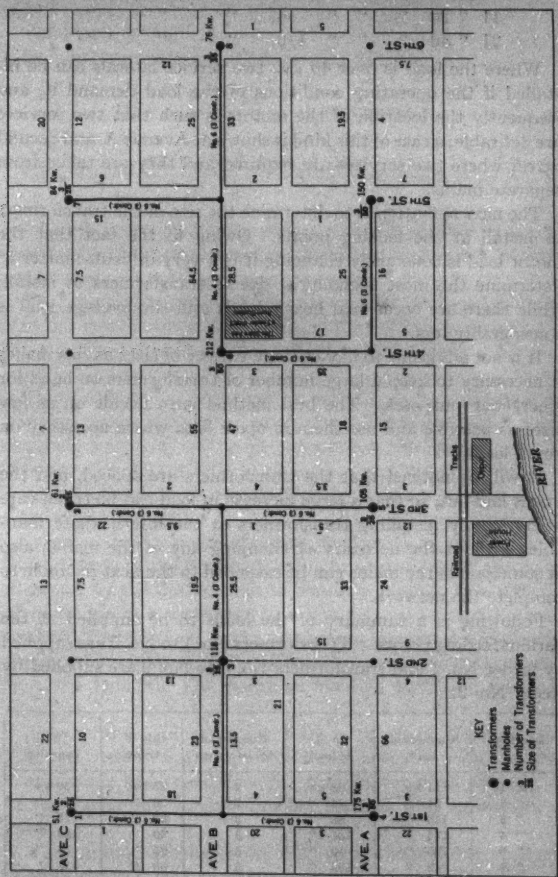


FIG. 6—SECONDARY POWER FEEDERS—2200-VOLT—THREE-PHASE

the maximum demand, or 828 h. p. the transformer capacity is a little over one kw. per h. p. which is the usual practise in power installations.

At feeding point No. 8, it would no doubt be advisable to start with two 25-kw. transformers in open delta as the load is only 57 kw. but for the purpose of this estimate it was considered best to be on the safe side.

It must be remembered that in designing a system and making an estimate of the cost, considerable allowance must be made for future conditions, as considerable time will undoubtedly elapse between the time that the designs are prepared and the installation of the system. The engineer also must not underestimate the cost.

2200-Volt Three-Phase Secondary Power Feeders

Fig. 6 shows the 2200-volt three-phase secondary power feeders that run from the centers of distribution to the various feeding points to supply the transformers and the low-tension network as shown in Fig. 5.

The total connected load and the number and size of transformers is marked at each feeding point. In the original estimate for this system it was assumed that 70 per cent of the connected load would be the maximum demand on these feeders; the cable has been calculated on this basis, and there are only two sizes of cable used in this system.

From the manner in which the cables are installed it is evident that the capacity of the system can be greatly increased by simply running a few sections of No. 6 cable and installing additional transformers at intermediate vaults. This would make it necessary to alter the secondary network so as to maintain a balance but this would be simply a matter of changing connections in the junction boxes.

The largest single load at any point is at Avenue A. and First Street, 175 kw. and as 70 per cent of this, or 122.5 kw., is assumed to be the maximum demand and the three-conductor No. 6 cable will supply this.

On Avenue B., from Fifth to Sixth Street, No. 6 cable is specified, and while this is large enough for the present load it might be a good plan to use No. 4 here as this is on the main feeder that would have to be extended in case the district was enlarged later.

This completes the power system and the next step is to consider the lighting system.

2200-Volt Three-Phase Lighting Feeders

Fig. 7 shows the 2200-volt three-phase lighting feeders. By reference to Table No. I the lighting load, exclusive of the big store, at Avenue B. and Fourth Street, is given as 2162 kw. This is the connected load in the district.

The demand of a group of residence consumers will vary from 15 per cent to 30 per cent of the connected load, and the average will probably be between 20 and 25 per cent. In commercial lighting the demand is much higher, as sign and window lights, as well as most of the store lights, are used at the same time. To offset this demand it is seldom that the lights in offices over stores are used when the store demand is at a maximum. The demand for commercial lighting will vary from 40 to 70 per cent of the connected load, depending on the nature of the district and the class of service, some nights the demand being much greater than other nights. The average demand for commercial lighting will be from 50 to 60 per cent of the connected load.

The district that is under consideration is the business district of a small city where practically all of the consumers are in the commercial class, it will therefore be assumed that the demand factor will be 60 per cent of the connected load.

The total connected load is 2162 kw., and 60 per cent is 1297 kw. (say 1300 kw.) and the first thing to decide is the proper size of feeders to use for this load. No. 1/0 will carry 481 kw. at 2200 volts, and it would require three such cables to carry the load and leave about 140 kw. spare capacity. This is about 11 per cent but as it is doubtful if the load could, in practise, be evenly divided between the three feeders this would be a small margin; also if possible it is a good plan to use the same size cables as was used for the power feeders so as to reduce the number of different sizes that must be kept on hand for emergency, and also permit using uniform equipment on the two systems.

The total amount of cable required for this system is about 3800 ft. and it would cost approximately as follows:

No. 2/0 3800 ft. at	$446.00 per M.	$1694.80
No. 1/0 3800 ft. at	386.00 per M.	1466.80
		$ 228.00

The saving in cost, by using No. 1/0 would not be enough to warrant carrying the additional size in stock, and considering that the duct space and cost of installation is the same for both

sizes, it is advisable to select No. 2/0 for all primary feeders for both systems.

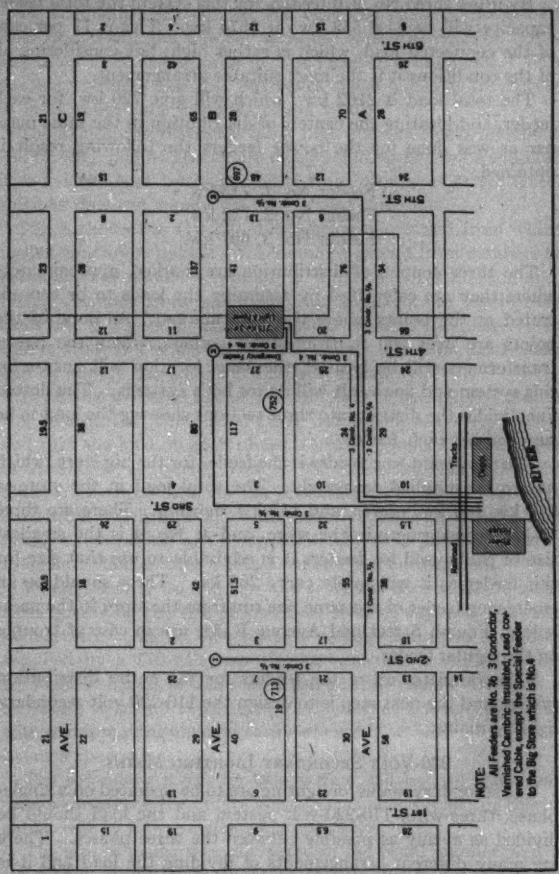


FIG. 7—2200-VOLT—THREE-PHASE LIGHTING FEEDERS

There is another point in favor of using No. 2/0 cable for this system, and that is, that the emergency feeder was of this size and by extending it from Fourth to Fifth Street on Avenue B.

(as shown by the dotted line in Fig. 4) it can be used as an emergency feeder for both systems.

By using three No. 2/0 feeders for this system the total feeder capacity will be 3 × 558 kw. or 1674 kw. This is 77 per cent of the connected load, which is rather high, but considering all of the conditions it is the most suitable arrangement.

The total load is 2162 kw., which will give 720 kw. for each feeder, and locating the centers of distribution in the same manner as was done for the power feeders the following result is obtained:

Feeder No. 1, 713 kw.
Feeder No. 2, 752 kw.
Feeder No. 3, 697 kw.

The three centers of distribution are marked approximately where they are calculated by assuming the loads to be concentrated at the points where they are marked. As none of the points are over 200 ft. from street corners, where the power transformers will be located, the same location will answer for this system and one vault will do for both systems. The dotted line divides the district into three sections showing the load to be supplied by each feeder.

The next point to consider is the feeder for the big store, which has to be supplied separately. The total load in the store is 215 kw. 100 kw. power, and 115 kw. lighting. There are three 75-kw. transformers in the store, and as No. 4 is the smallest size of cable used for feeders it is advisable to use that size for this feeder as it will safely carry 260 kw. There should be an emergency feeder of the same size run from the store to the manhole at Fourth Street and Avenue B. for use in case of trouble on the regular feeder.

This completes all of the feeders for the entire distribution system and the next step is to design the 110-220 volt secondary lighting mains.

220-Volt Secondary Lighting Mains

The secondary mains for lighting are to be operated on a single-phase, three-wire, 110-220-volt system and the load should be divided as evenly as possible between the three phases. There are many different arrangements of dividing the load and it is advisable to try several methods before deciding which is the most suitable.

In designing this system it is desirable to arrange the circuits

so that as few transformers as possible will be required, in order to save space in the vaults, save transformer investment, and to get the benefit of operating as large units as possible.

By arranging the mains so that only one phase is distributed from each vault the number of fuses, switches, junction boxes and other electrical equipment is reduced to a minimum. This not only reduces the cost but saves space which is frequently of vital importance in underground vaults.

The load in this district is fairly high and the blocks are about 350 ft. long, and as practically every building has to be supplied it is evident that by locating the transformers at street intersections they can supply the load in four directions.

Fig. 8 shows the 110-220-volt lighting mains and from which feeder they are supplied. Where mains extend from manhole to manhole they should terminate in junction boxes at each end, fusing the ends from which they are supplied and leaving the fuses out at the other end, to prevent crossing the phases. In case of trouble it is then possible to supply any main from either end by simply changing the direction of supply.

Referring to Table III in the summary of the lighting load it will be seen that the load is fairly well balanced on the feeders and that the load on each phase is nearly equal. This division of the load is, of course, based on the connected load and in the absence of actual data on the operating conditions is the only basis from which to work. After the system is in operation, or rather as the loads are being connected it is possible to alter these first plans so that a better balance can be obtained.

When installing an underground system it is advisable to make rules governing the size of load that is to be supplied by a two wire service. About 1300 watts is as large a load as should be supplied by a 110-volt two-wire service in a business district. All loads in excess of this should have three-wire services. It is advisable to install three service cables to all consumers as this will permit making changes easily when it is necessary to do so to balance the load.

The secondary mains for this system will be two single-conductor cables for the outers and a bare neutral, and the service cables will be three single-conductor cables.

The size of mains was calculated in the usual manner, and in practically every place where 200,000-cir. mil mains are shown on the plan, it will be found that No. 2/0 cable would carry the present load, but in a district such as this one it is not advisable to install any main of less than 200,000 cir. mils.

Where the mains on both sides of a street are supplied from the same phase and are tied together at the next manhole it is

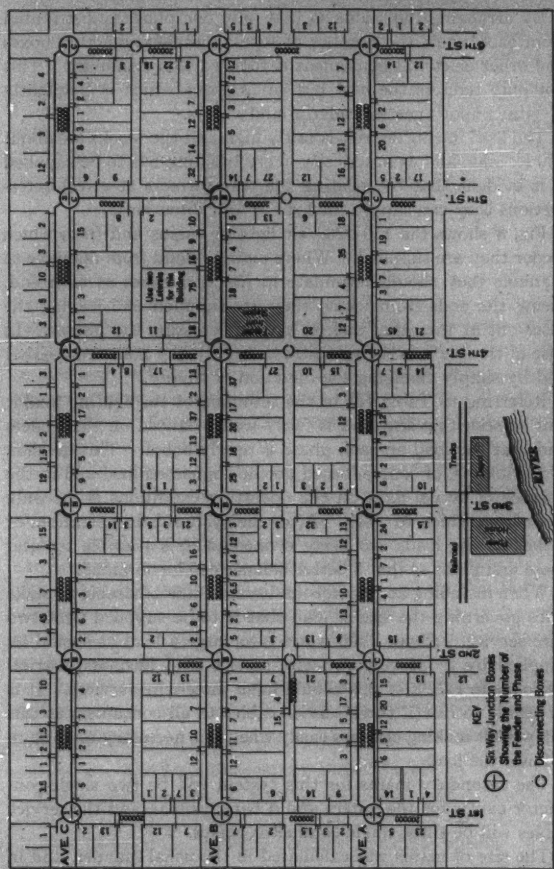


FIG. 8—220-VOLT SECONDARY MAINS

advisable to use the same size cable for both, regardless of the load on each, so that the main can be supplied from either end in case of necessity.

TABLE III—SUMMARY OF LIGHTING LOAD

Feeder No. 1 Block	Phase A	Phase B	Phase C	
No. 1	1.			
2	21.			
3		30.		
7	34.			
8	64.	25.		
9		62.		
13	15.5			
14	59.	66.	21.	
15		54.5	72.	
19	28.			
20	19.		83.	
21			56.	
	241.5	238.	232.	
Feeder No. 2				
No. 4		19.5		
9		55.		
10	67.	84.		
11	23.			
15		37.		
16	144.	37.	25.	
17			96.	
21		1.5		
22		10.	53.	
23			100.	
	234.	244.	274.	
Feeder No. 3				
No. 5			23.	
6			21.	
11		137.	38.	
12	107.		37.	
17		54.	6.	
18	28.	48.	82.	
23				
24	54.		24.	
6th St.	36.		2.	
	225.	239.	233.	

SUMMARY

Feeder	Phase A.	Phase B.	Phase C	Total
No. 1	241.5	238.	232.	711.5
No. 2	234.	244.	274.	752.
No. 3	225.	239.	233.	697.
	700.5	721.0	739.0	2160.0

The phases have been referred to as *A*, *B* and *C* and while this is the common practise a better method is to distinguish the

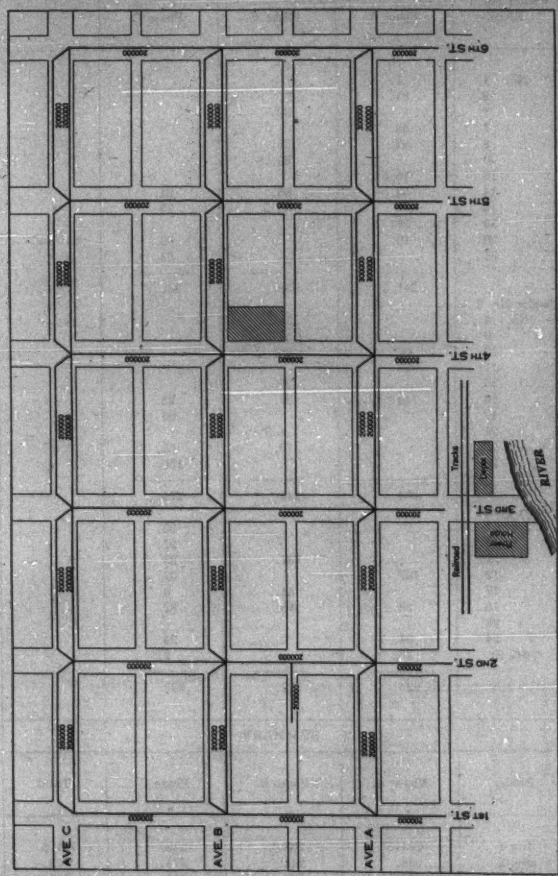


FIG. 9—BARE NEUTRAL MAINS.

phases by colors, and paint the cables of each phase a different color, this method of marking the phases will prevent mistakes

in making connections as only cables of like color should be connected together.

The bare-neutral network is shown in Fig. 9 and is the same size as the outers wires shown in Fig. 8. Bare cable has been selected for the neutral in this system, but this is a matter that should be decided by the engineer for each particular system.

2200-Volt Secondary Lighting Feeders

The cables for this system can be either two- or three-conductor. While the lighting load is single-phase, still it is necessary to divide it between the three phases, therefore, the three-conductor cable has been selected so that all three phases are available at each transformer vault.

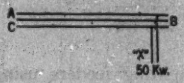


Fig. 10

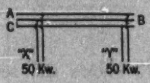


Fig. 11

The following rules for calculating the temperature rise in three-phase feeders supplying one or more single-phase loads were devised by Mr. R. W. Atkinson and are given here with his permission.

Rule 1. A single-phase load applied between two of the phases of a three-phase circuit (Fig. 10) is equivalent, insofar as the maximum temperature rise is concerned, to a balanced three-phase load 50 per cent greater.

Fig. 12

Fig. 13

Rule 2. If two single-phase loads, of equal magnitude, are applied, one between phases A and B and one between A and C, (Fig. 11) the maximum temperature rise is the same as for a balanced three-phase load 25 per cent greater.

Rule 3. If various single-phase loads are applied between the various phases of a three-phase circuit (Fig. 12), the resulting load can be considered to be a combination of the two conditions just mentioned and a balanced three-phase load.

Referring to Fig. 13, and applying these rules the following result is obtained:

Phase	*A-B*	*B-C.*	*A-C.*		
Load....	50 kw.	100 kw.	80 kw.	= 230 kw.	
Subtract.	50 "	50 "	50 "	= 150 "	three-phase load at *X*
		50 "	30 "		
Subtract.		30 "	30 "	= 75 "	(60 × 1.25)
		20 "		= 30 "	(20 × 1.50)
				255 kw.	

Fig. 14 shows the secondary lighting feeders using three-conductor cable. The load, size of transformers and the phase from which they are supplied are marked at each feeding point.

Fig. 15 shows two-conductor single-phase feeders supplying this same system and the sizes shown on the plan are based on 70 per cent demand and as this system is used exclusively for lighting the power factor has been taken at 100 per cent. It will be seen that this system requires approximately 2000 ft. more conduit than the system shown in Fig. 14, and the approximate difference in cost of the two cable systems is as follows:

No. 6 three-conductor cable	4800 ft.	at $160.00...	$ 768.00
No. 2 " " "	1200 " "	340.00...	408.00
			$1176.00

No. 6 two-conductor cable	7200 ft.	at $100.00...	$ 720.00
No. 2 " " "	800 " "	204.00...	163.00
			$ 883.00

This is a difference of $293.00 in favor of the two-conductor cables on the cable alone, but if the additional conduit is considered it will be seen that the three-conductor cable is not only the cheapest but the better system to install.

There is another point to be considered in selecting the three-conductor cable; the sections will be in comparatively long lengths and if replaced by larger cable at any time the old cable can be

used in other places, even on the power system, while if two-conductor cable were used it would be available only for the

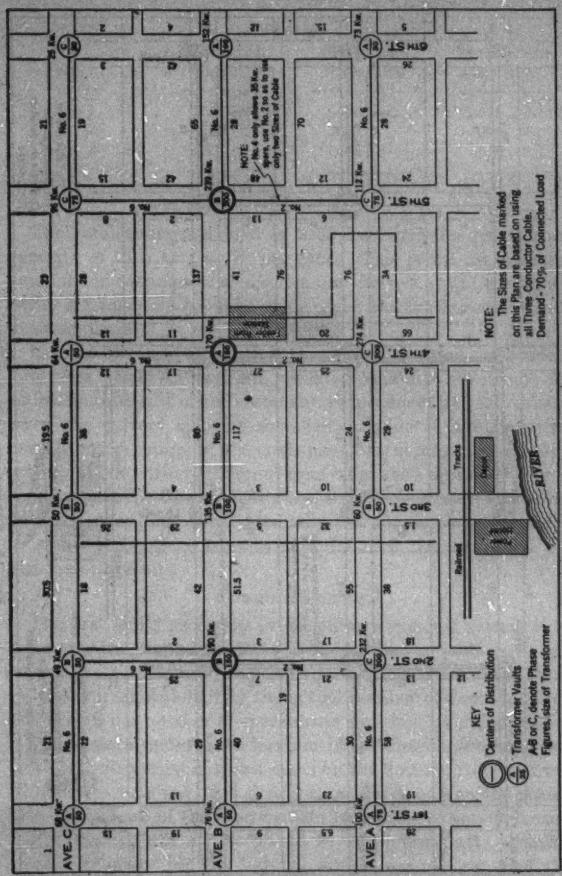


FIG. 14—SECONDARY LIGHTING FEEDERS

lighting system and would be an additional type of cable to keep on hand.

As the three-conductor cable will require less conduit, and be available for use on either system it is advisable to select it for

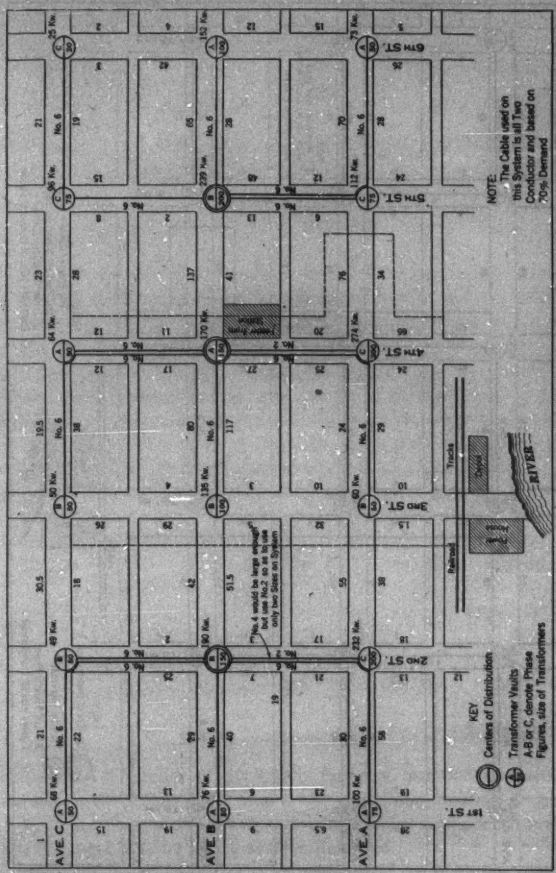


FIG. 15—SECONDARY LIGHTING FEEDERS

these feeders, as the system will be more flexible than if two-conductor cable were used.

The size of transformers is shown at each feeding point and is based on 70 per cent demand factor as follows:

SUMMARY.

Feeder	Connected load	70 per cent demand	Transformer capacity
No. 1	715 kw.	500 kw.	575 kw.
" 2	753 "	527	600 "
" 3	697 "	488 "	520 "
	2165 "	1515 "	1695 "

The total connected load is 2165 kw. and the transformer capacity is 1695 kw., or 78 per cent. This is not an excessive amount for a commercial district and it is doubtful if a much better arrangement could be made under the conditions assumed in these plans.

It must be remembered that plans, such as these, are made a long time before the work is actually done and are simply for estimating the cost of the system and are subject to final revision when the system is being installed. Owing to the fact that usually the grouping of the consumers on the new underground system will be entirely different from what they were on the overhead system, the demand factor can only be estimated, and as the new system is "cut over" gradually there is ample opportunity to select the size of transformers, balance the loads and get satisfactory results.

CONDUIT SYSTEM

The first thing necessary in designing a conduit system is to determine the number of ducts to be installed on each street and this can best be done by making a tabulation as shown in Table IV this will show definitely the actual number of ducts required and the proper number of spare ducts can then be added.

The system under consideration is for distribution purposes only and will consist of trunk ducts on the bottom, running from manhole to manhole, and distribution ducts on the top which pass through all of the intermediate handholes or service boxes.

The feeders will all be placed in the lower ducts and the secondary mains for the lighting and power system, and the street light cable will be in the distribution ducts.

Provision must be made for the following cables in the distribution ducts.

1 Three-conductor power main.

2 Single-conductor lighting mains (Outers).

1 Bare neutral for lighting system.

1 Street light cable (Two in some places, but they will be in one duct).

If separate ducts were used for each cable it would require 5 ducts on the top tier, this is a bad arrangement because it would increase the cost of the system, as the width of the trench would be about 29 in. using 3.5 in. conduit. The best arrangement would be to place one of the lighting mains and the neutral in one duct and install the conduit four ducts wide, which would require a trench about 24 in. wide.

By using four ducts for distribution it is possible to lay all of the conduit in multiples of four which is a very good arrangement. On streets having conduit on both sides it is only necessary to lay four ducts on one side, for distribution, keeping the main conduit on the other side of the street.

In systems where only three ducts are required for distribution it is sometimes advisable to use multiples of three in laying the conduit and thus make considerable saving in first cost. On all distribution systems the number of ducts wide should be determined by the number of ducts required for the distribution cables.

In laying four ducts they should, if the conditions will permit, be laid two wide and two high, in fact on all conduit systems, except for high-tension cables, it is advisable to have the conduit form as near a square section as possible as this makes a stronger structure. As far as the subsurface conditions will permit it is advisable to use a multiple of the distribution ducts as the number to install in one trench.

The conduit plan must show the point at which all laterals enter the buildings and the location of the street lights so that the service boxes can be located to the best advantage, the exact location, of course, depending on the subsurface conditions.

Fig. 16 shows the conduit arrangement for this system of distribution and is based on the tabulation shown in Table No. IV. To avoid confusion there is only one lateral shown entering each building but there should be two wherever power is to be supplied.

The size and location of the service boxes should be determined by the number of mains passing through them, the number of services that it is permissible to splice on a main at one service box, and the available space for locating them. Long laterals are expensive and require larger service cables, and it should be

remembered that to make a neat splice not more than two three-conductor services can be taken from a main in one splice.

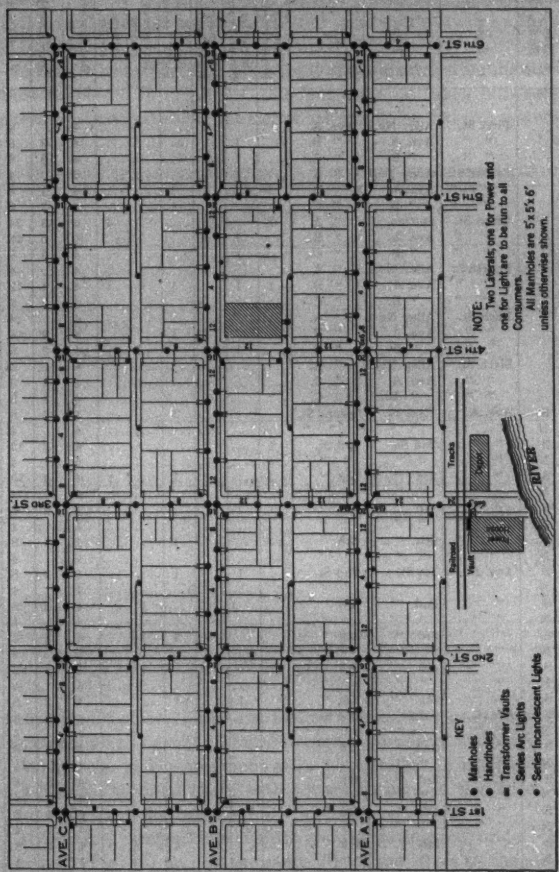


FIG. 16—CONDUIT SYSTEM

Where the mains are single conductor it is possible to take out four service cables from one splice.

On a system such as is considered here the service boxes should

TABLE IV.—SUMMARY OF CONDUIT

Side of Street	Street	From	To	Primary Power Feeder	Secondary Power Feeder	Primary Light Feeder	Secondary Light Feeder	Emergency Feeder	Private Feeder	Distribution Ducts	Spare Ducts	Total Duct
	First St.	Alley No.1	Ave. A.	....	....	....	....	....	....	4	....	4
	" "	Ave. A.	" B.	....	1	....	....	....	....	4	3	8
	" "	" B.	" C.	....	1	....	....	....	....	4	3	8
	Second St.	Alley No.1	" A.	....	....	....	....	....	....	4	....	4
	" "	Ave. A.	" B.	1	....	1	1	....	....	4	1	8
	" "	" B.	" C.	....	....	....	1	....	....	4	3	8
	Third St.	Station	" A.	2	....	3	....	1	1	4	13	24
	" "	Ave. A.	" B.	....	1	1	....	1	....	4	5	12
	" "	" B.	" C.	....	1	....	....	....	....	4	3	8
	Fourth St.	Alley No.1	" A.	....	....	....	....	....	....	4	....	4
	" "	Ave. A.	" B.	1	1	....	1	..	1	4	4	12
	" "	" B.	" C.	....	1	....	....	....	....	4	3	8
	Fifth St.	Alley No.1	" A.	....	....	....	....	....	....	4	....	4
	" "	Ave. A.	" B.	....	....	1	1	....	....	4	2	8
	" "	" B.	" C.	....	1	....	1	....	....	4	2	8
	Sixth St.	Alley No.1	" A.	....	....	....	....	....	....	4	....	4
	" "	Ave. A.	" B.	....	....	....	....	....	....	4	4	8
	" "	" B.	" C.	....	....	....	....	....	....	4	4	8
S	Ave. A.	First St.	Second St.	....	....	....	1	....	....	4	3	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	2nd St.	3rd St.	1	....	1	....	....	....	4	6	12
N	" A	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	3rd "	4th "	1	....	1	1	....	1	4	4	12
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	4th "	5th "	....	1	1	....	....	....	4	2	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	5th "	6th "	....	....	....	1	....	....	4	3	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	Ave. B.	1st St.	2nd St.	....	1	....	1	....	....	4	2	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	2nd "	3rd "	....	1	....	....	1	....	4	2	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	3rd "	4th "	....	....	1	1	1	....	4	5	12
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	4th "	5th "	....	1	....	....	1	....	4	6	12
N	" "	" "	" "	....	....	....	....	....	....	4	..	4
S	" "	5th "	6th "	....	1	....	1	....	....	4	2	8
S	Ave. C.	1st St.	2nd St.	....	....	....	1	....	....	4	3	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	2nd "	3rd St.	....	....	....	....	....	....	4	4	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	3rd "	4th "	....	....	....	1	....	....	4	3	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	4th "	5th "	....	....	....	....	....	....	4	4	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
S	" "	5th "	6th "	....	....	....	1	....	....	4	3	8
N	" "	" "	" "	....	....	....	....	....	....	4	....	4
	Alley No. 2	1st "	2nd "	....	....	....	....	....	....	4	....	4
	Alley No. 2	4th "	Big Store	....	....	....	....	....	....	4	....	4
	Station	To Manhole No. 1.		2	....	3	....	1	1	....	13	20

be about 3 ft. by 4 ft. and the depth will depend on the grade of the conduit, as the top tier of ducts must enter the service box. For the purpose of estimating, it may be stated that on a system of this kind that the average depth of service boxes will be from 36 to 40 inches.

Where subway junction boxes and other electrical equipment must be placed in manholes, this fact must be taken into consideration in determining the size of the manholes, but in a system

TABLE V—SIZE OF CABLE FOR LATERALS. INDIVIDUAL CONSUMERS LOADS

Load	Number of Consumers	Size of Cable
For Power		
0 to 10 kw.	92	3 Conductor No. 6
11 " 20 "	29	" " " 4
21 " 40 "	8	" " " 1/0
	129	
For Lighting		
0 to 10 kw.	168	3 Single Conductor No. 6
11 " 15 "	48	" " " " 4
16 " 35 "	30	" " " " 1/0
36 " 65 "	4	" " " " 4/0
	250	

Assume that laterals will average 60 ft. long each and that three-conductor cable is used for all power laterals and three single-conductor cables for the lighting consumers, therefore each lighting lateral will require 180 ft. of cable.

where transformer vaults are built the manholes need only be large enough to permit training the cables properly.

Where electrical equipment is to be installed in manholes it is advisable to build them with square corners as this space is frequently very desirable.

If possible, transformer vaults should be located under the sidewalk (Fig. 17) and on the same side of the street as the main conduit. By locating the vaults under the sidewalk there is less liability of damage from being flooded, also they can be more easily entered in the winter when the ground is covered with ice

and snow. All covers to manholes, or vaults, under the sidewalk should be filled with cement to match the color of the walk and also prevent pedestrians from slipping on them.

In many of the old systems, where a large number of ducts were laid in one trench, the manholes were entirely too small to permit the cables being properly trained in them. In a small system, such as this one, large manholes are not required, as it is proposed to locate the transformers in vaults and have only the cables and secondary junction boxes in the manholes.

By locating the transformers and the 2300-volt equipment in the vaults and the low-tension junction boxes in the manholes there is less liability of a burnout damaging the secondary net-

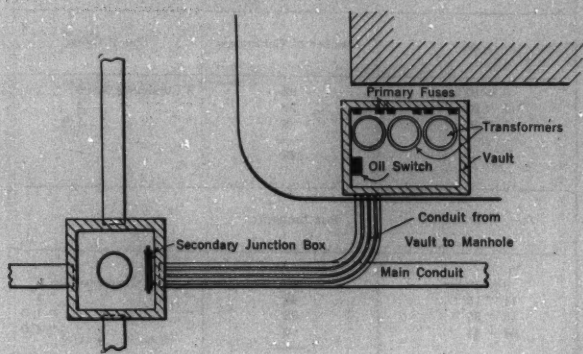


FIG. 17

work, also the mains are shorter, and less conduit is required between the vaults and manholes.

Fig. 18 shows a typical arrangement of conduit and manholes at a street intersection, each duct between manhole *A* and *B* is numbered to correspond with the ducts leaving manhole *B*.

Owing to the obstructions usually encountered at street intersections, it is generally necessary to build two manholes, but where the grade of the conduit can be maintained across the street it is only necessary to have a service box on one side, deep enough to take the two top tiers of ducts.

If the secondary junction boxes are installed in the vaults it will be necessary to have 18 ducts between the manholes and vaults as follows:

1 Power feeder.
1 Lighting feeder.
1 Emergency feeder.
6 Power mains.
6 Lighting mains.
1 Ornamental street light (may be required).
2 Spare ducts.

While it will not require six ducts each for power and lighting mains in all cases at present, still provision should be made for this number eventually.

In making the estimate for this system the following figures will be taken as being fairly accurate.

All service laterals will consist of two ducts and will average

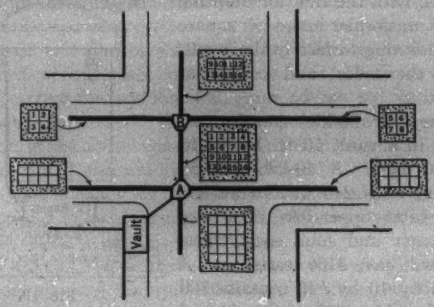


FIG. 18—ARRANGEMENT OF CONDUIT AND MANHOLES AT STREET INTERSECTION

20 ft. long from the trench to the property line, on streets where there are two lines of conduit, and 30 ft. long on streets having but one line of conduit.

Street light laterals 10 ft. from trench to lamp pole, the lights in the alleys having a single fibre-duct lateral 180 ft. long.

Street crossings and the distance from manholes to vaults are 30 ft. each.

A vault is shown at the station and is frequently required in large systems, but in a system of this size it is very probable that the conduit could enter the basement of the station at the most convenient point for the various systems.

The size of the transformer vaults will depend on the amount of equipment that is to be installed in them. For this estimate

it will be assumed that the three vaults at the centers of distribution are 10 by 15 by 8 ft. and all other vaults 10 by 12 by 8 ft. As this district is to be extended later and will probably require larger transformers when the load increases, it is advisable to make the manholes as small as possible and allow sufficient room in the vaults to permit installing the equipment in a neat and workmanlike manner and have sufficient room for operating it safely.

The transformer capacity that can be installed in a vault without providing special ventilating facilities will depend on the operating conditions. For transformer capacities not exceeding 200 kw., under favorable conditions, 3.5 to 4.0 cubic feet of vault space per kw. will permit safe operation; these figures correspond fairly well with the rule for allowing 8 watts transformer losses per square foot of radiating surface in the vault, figuring the sides and ceiling as follows:

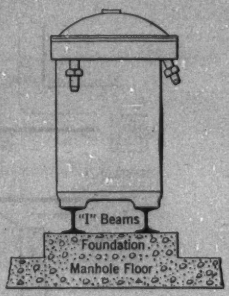


FIG. 19

Assume four 50-kw. transformers installed in a vault 10 ft. by 10 ft. and 7 ft. high. A 50-kw. subway transformer has 240 watts core loss and 550 watts copper loss, or a total of 790 watts and four such transformers will have 3160 watts loss. A vault 10 by 10 by 7 ft. contains 700 cubic feet and has 380 square feet of radiating surface, counting the walls and ceiling.

Allowing 3.5 cubic feet per kw. this would give exactly 700 cubic feet required, which is the size of the assumed vault. Applying the rule of 8 watts loss per ft. of radiating surface the result is 8×380 or 3040 watts, which corresponds very closely with the first estimate.

The above losses are based on the assumption that the transformers are fully loaded but in a large distirbution system where the transformers serve both lighting and power loads this is seldom the case as the maximum lighting and power loads rarely occur at the same time, and the overlap of these demands is usually of such short duration that no serious rise in temperature results from this cause.

The above method can be used to calculate the size of vaults for a given transformer capacity not exceeding 200 kw., still it is

always necessary to allow sufficient space to properly install and operate the equipment in the vault.

In installing transformers in vaults it is advisable to have them raised above the floor, and if possible mounted on two or three I beams as this permits the air to circulate all around them. (See Fig. 19.) It is a good plan to have a thermometer in each vault and keep a record of the temperature as a guide to the actual conditions. Where this is done the thermometer should be noted immediately on entering the vault before the air has a chance to cool; in taking the temperature of the transformers the thermometer should be placed at the oil level of the transformers.

Having determined the general arrangement of the conduit system the next step is to select the most suitable location in the streets for installing it. In order to plot the location of sewers, pipes and other obstructions it is necessary to prepare a map of each street and alley on a scale of about 20 ft. to the inch.

Records of subsurface conditions are usually far from accurate, but by getting the location of all gates, covers, etc. visible on the surface of the streets it is frequently possible to prepare a very fair plan of the actual conditions. All existing manholes should be entered and measured and they should be laid out accurately on the plan, allowing for the thickness of the walls, as getting the new conduit by existing manholes is frequently one of the most difficult points in conduit construction.

It is a good plan to make two sets of these large scale street maps, one showing the obstructions and the other one to show the new system exactly as it is installed for future reference. These latter drawings should be made on cloth backed drawing paper and can be kept in rolls, and all additions and changes should be entered on them when made, giving the date and all details of the work.

Manholes are built of brick or concrete, except in special cases there is little difference in the cost. In streets where the space is limited and many obstructions are encountered it is cheaper to build brick manholes, but in residential sections, or for high-tension systems, it is frequently cheaper to build concrete manholes when a standard form can be used.

In locating and building manholes every effort should be made to avoid any pipes passing through them, it is frequently possible to change pipes or cut them around the manhole and this should be done if permitted.

The service boxes should be built of brick as owing to their difference in depth a standard form can not be used to advantage.

Vaults, where built under the sidewalk, should be of concrete and have suitable ventilating pipes extending up the side of buildings or adjacent poles. There should be one pipe located

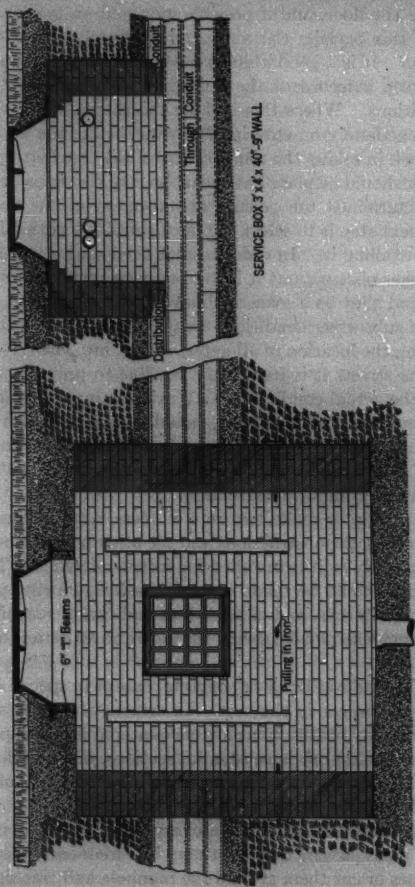


FIG. 20—MANHOLE AND HANDHOLE

low at one end of the vault and one located high at the other end and care should be taken not to locate transformers directly under the ventilating pipes.

Fig. 20 shows the usual arrangement of manholes and vaults.

Owing to the increase in weight of automobile trucks it is very important that all casting be made much heavier than was done formerly. The castings should be inspected at the foundry, or before being placed, to see that the covers set firm and solid in the seat of the casting. No loose or uneven covers should be permitted for no amount of chipping or grinding will make them satisfactory.

Single cover castings are advisable and they should have ventilating holes in the covers for the manholes but the service box

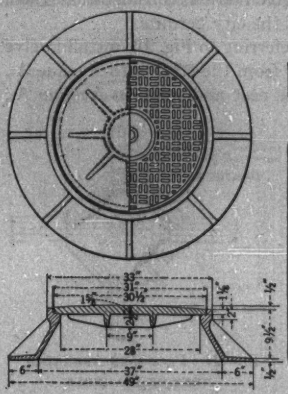


FIG. 21—MANHOLE CASTING

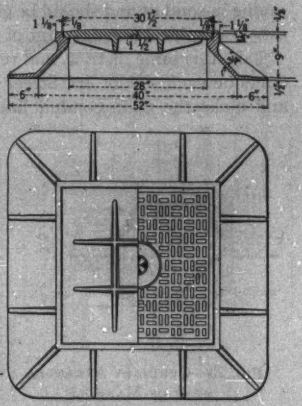


FIG. 22—HANDHOLE CASTING

castings should be unventilated, Fig. 21 and 22 show castings for manhole and service box respectively.

If possible, it is advisable to locate the conduit about five feet from the curb so as to keep the service box covers out of the gutter and if possible locate the conduit outside of the gas and water main as it will not then be necessary to sandwich the conduit around the service pipes for those systems.

All vaults and manholes having electrical equipment in them should be connected with the sewer and the floors should be sloped in all directions to the sewer outlet.

Cable racks should not be placed until the cable is being installed, except on high-tension systems where the arrangement of the cables will be uniform in all manholes.

When building vaults, or manholes where transformers are to be installed, it is advisable to install a suitable ground connection at the time of construction, a ground cone, or one-in. galvanized pipe should be used if it is not possible to connect to the water pipe system, but whatever kind of ground is decided on care must be taken to make it permanent and reliable, for large vaults it is advisable to install two ground connections.

The conduit as laid out in Fig. 16 is the usual method adopted in systems of this kind. There is, however, another method that can be adopted in some cases which will permit a very material saving in conduit and should be given serious consideration where the geographical arrangement of the city permits.

Take this same district, and referring to Fig. 16, install twelve ducts on each side of 3rd Street from the station to Avenue A., and make the main conduit line east and west on Avenue A.,

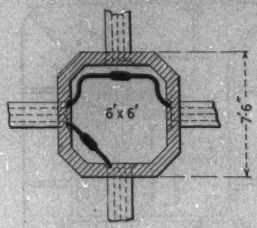


FIG. 23—ORDINARY ARRANGEMENT OF MANHOLE

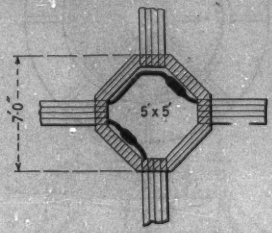


FIG. 24—PROPOSED ARRANGEMENT OF MANHOLE

and run all feeders over this route branching them off at the north and south streets, as 1st, 2nd, etc., this would permit reaching every distribution point.

On such streets as Avenue B. and C. it would only be necessary to install four ducts on each side of the street for distribution this would make a saving of approximately 17,000 ft. of conduit on those streets but it would be advisable to add more conduit on Avenue A.

Possibly a modification of this plan would be better, that is, to make the main conduit lines on Avenue A. and on 3rd Street, the saving would probably be about 10,000 ft. If later a substation were to be installed to feed other sections of the city this plan would be very desirable as it is only necessary to have ducts on the avenues sufficient to provide space for the distribution cables and street light system.

Subway Junction Boxes

When the load in a district is supplied by a low-tension network it is necessary to supply facilities for fusing and disconnecting the mains. When the low-tension network is supplied by low-tension alternating- or direct-current feeders this is a simple matter but where a low-tension network is supplied by subway transformers the conditions are more complicated and additional equipment is necessary to insure reliable service, as both the primary and secondary systems must have protective, sectionalizing and switching facilities. Where the power and lighting load are supplied separately this practically doubles the equipment and space required, and greatly increases the cost of the system.

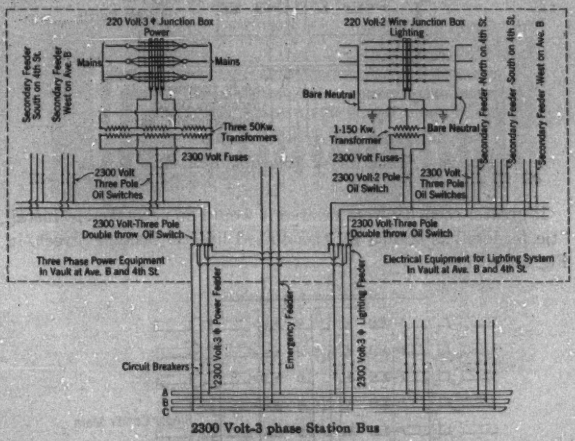


Fig. 25—Diagram of Equipment in Vault

Fig. 25 shows the method of protecting, sectionalizing and switching used on the lighting and power systems; it is a diagram of what would be necessary in one of the vaults, the one chosen for illustration being the vault at Avenue B. and Fourth Street.

It will be seen in case of trouble on one of the regular lighting or power feeders that the load can be switched over to the emergency feeder. Should one of the secondary feeders fail it would cut off the supply to all of the transformers connected to it, but by disconnecting these transformers from the network and con-

necting this portion of the network to other phases (by putting the fuses in at the several junction boxes) the system could be operated temporarily even though greatly unbalanced.

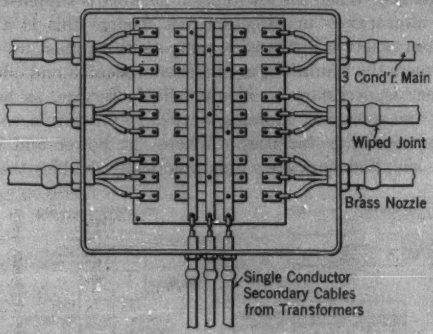


FIG. 26—THREE-PHASE JUNCTION BOX—FOR POWER

If greater security were desired it would be necessary to install "tie feeders" as shown by the dotted line on Second Street, in Fig. 6.

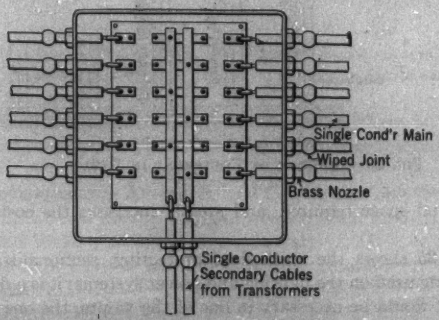


FIG. 27—TWO-WIRE—220-VOLT JUNCTION BOX—FOR LIGHTING

The mains are protected by fuses at each end and in case of trouble on them, or the house services, would be cut off automatically; where there are mains on both sides of the street the

number of consumers affected in case of trouble is considerably reduced. The fuses on the mains should be heavy enough to carry at least 50 per cent more current than the normal carrying capacity of the mains, as it is not desirable to have the fuses blow out except in case of serious trouble.

It will be noticed that in Fig. 25 only three fuses are shown on the primary side of the power transformers. In actual practise, where fuses are used on the primary side, it is better to install two fuses for each transformer. There is considerable difference of opinion among engineers as to the advisability of installing any fuses on the primary side of subway transformers as they gener-

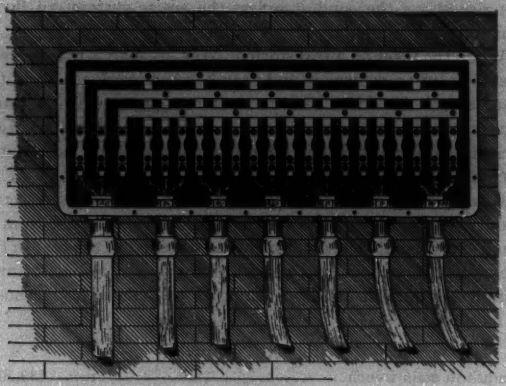


Fig. 28

ally cause more trouble than their protection is worth, but as it is advisable to have some point where the transformers can be easily disconnected from the feeders it is a good plan to install the fuse boxes and use fuses having 100 per cent greater carrying capacity than is required to protect the transformer and then trust to the station circuit breaker to disconnect the feeder in case of serious trouble.

Assuming that the system is to be installed as shown in Fig. 25, the subway junction boxes shown would provide facilities for disconnecting and fusing the various mains. The junction boxes for the lighting system would be fitted for single-conductor cable and those for the power system for three-conductor cable.

Owing to the fact that there are so many different systems in use there is no standard type of junction box that can be specified, and it is generally necessary to design boxes for each system to meet the conditions. In underground work there is probably no part of the system that is more liable to cause trouble than are the junction boxes; this is not always due to defects in the box but to careless work in installing it or not properly closing the cover.

Fig. 28 shows a type of junction box that is well suited for subway installation, it is placed near the top of the manhole and the cables all enter it from the bottom, it is practically certain that a box of this type will never get flooded as the water would not rise above the conduit unless the entire system got flooded which is very unlikely.

When the fuses on a transformer blow, its load is transferred to the adjacent transformers and they become overloaded and their fuses blow. This is a serious matter as this condition usually occurs at the most inconvenient time, and where additional security is warranted it is advisable to install the "alternating-current network protector."

Having completed the designs for all of the various systems it is only necessary to scale off the amount of material from the plans and tabulate the equipment, as specified in the text or from the plans. Space will not permit giving these various estimates completely but the following samples will show the method to be adopted.

Conduit System

Conduit in main trench	139,500 ft.
Trench feet	17,145 "
Conduit for laterals in main trench	4,110 "
Conduit for laterals run separate	10,650 "
All conduit used for laterals to be 3-in. fibre.	
Manholes	37 "
Handholes	108 "
Vaults 10 by 12 by 7 ft.	15 "
Vaults 10 by 15 by 7 ft.	3 "
Sewer Connections	18 "

Three-Phase Power System

2300-volt three-condr. Cable No. 2/0	3,700 ft.	primary feeders	
2300- " " " " " 4	1,100 "	"	"
2300- " " " " " 6	2,600 "	"	"
220- " " " " " 1/0	4,600 "	secondary mains	
220- " " " " " 4	12,700 "	"	"

220-	"	"	"	"	"	6	.....	5,520 "	laterals
220-	"	"	"	"	"	4	.....	1,740 "	"
220-	"	"	"	"	"	1/0	.....	480 "	"
25-kw. transformers								16	
50- "	"							9	
2300-volt subway fuse boxes								50	
3-pole single-throw oil switches								6	
3-pole double-throw oil switches								3	
6-way junction boxes								18	
0	30-ampere consumers fuse boxes							70	
31	60-	"	"	"	"			23	
61	100-	"	"	"	"			27	
101	200-	"	"	"	"			9	

The other systems should be handled in the same manner, after which the total cost can be calculated and the specifications prepared.

DISCUSSION ON "DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS" (NEWTON), CLEVELAND, OHIO, MARCH 8, 1918.

A. A. Meyer: The author's paper covers only underground a-c. distribution, but I would like to ask him if he has any data covering d-c. distribution and the relative merits of the two systems. In looking over the distribution systems of some of the largest operating companies we usually find direct current in the down-town or business district where the "load density" is quite high and alternating current in the surrounding territory where the density is appreciably less.

In Detroit we have both d-c. and a-c. distribution systems and the load density has been a good reason for continuing the d-c. system and expanding it to take over the a-c. territory on its border. But of course there have been some other factors besides density determining a change from either one to the other system.

A comparison of load density was made for parts of the Detroit distribution systems. Comparing the load per sq. ft. in the territory served by a d-c. substation with that in a territory served by an a-c. substation, it was found that the load density in the former was about twice as great as in the latter. This comparison includes both power and lighting load. A similar comparison of lighting load only, showed the demand of the d-c. system to be about three times as great as the a-c. system in comparable districts. These ratios are not applicable in all comparisons between d-c. and a-c. systems, but they bring out a point of consideration in designing systems of distribution.

You might be interested to hear something about our remote controlled substations and the purpose they serve. These substations are located in our d-c. territory and are automatic in the sense that they have no attendant and are controlled entirely by the operator in a nearby and regular type substation. They serve a territory comparatively remote from the regular substation and are usually connected to the system only during the peak load. There are four such substations with a fifth going in, each having a 500-kw. rotary converter taking energy from a 4600-volt feeder and delivering 250 volts to the feeders tying in with the street mains in the immediate vicinity. Without these remote controlled stations the cost of copper for long and sufficient size feeders from the ordinary substation (with attending operators) would be excessive and considerable beyond the cost of a remote control station with complete equipment. With the growth of a city and the expansion of the d-c. network, some of these substations will probably be discontinued and replaced by the regular type substation. Others, however, will continue for a long time, for they are well adapted to handle large concentrated blocks of power such as required by hotels, large department stores or large shops located in a d-c. network.

C. W. Rakestraw: The author seems to have a decided preference throughout his paper for the use of varnished cambric or cloth, and he refers to varnished cambric or cloth all the way through his paper. He seems in taking this position to consider that the cost of installation has nothing to do with it, that it is a question of safety and maintenance, and he goes to varnished cambric immediately and scarcely considers paper. In my opinion the cambric is all right, except on the question of cost, and where any of us are held down to a question of cost, it is practically out of the question for general cable systems.

The author says,—"If three-conductor cable was used there would be a considerable reduction in the carrying capacity over single-conductor cables (25 per cent) also it would not be possible to take more than about two services out of each splice." For that reason, he seems to favor single-conductor cable for his secondary mains. We have found in our practise, that is not so. I know in many cases where we have had more service than that. We have taken four or five services out of one splice. While it is not good practise, it can be done if care is taken in the insulation.

The author speaks of a "bare neutral" all through his system. Have any of the underground men here ever had experience with bare neutrals on their systems? If so, I would like to know how that works out, using the bare neutral, instead of the regular insulated cable.

The author says in carrying the cable into the various appliances—into the manholes, I presume he means—transformers, switches or service boxes, that varnished cambric should be spliced on to the cable ends. I take issue on that. Certainly in going into the oil switches, which he must have to do frequently, it would be unwise to use rubber-covered cable. It will not last. There is no objection to using varnished-cambric cable in that case, whereas if you use ordinary varnished paper it will not serve the purpose.

On our local systems we have never followed that practise, but although it certainly would not do any harm to supply some varnished cambric, it again becomes a matter of cost.

The author says several times that he uses single-conductor cable for his services on all his two-voltage system, his lighting system. I would like very much to ask the author just what the idea is in that, as long as he is taking off alternating-current from his mains, why he would split that into three single-conductor cables, and whether he has any trouble in doing that. There is no objection to using three-conductor cable, and there is certainly some advantage in using it on the score of cost.

The author has given some figures here as to the assumed capacity, that is the assumed capacity of what he calls the primary feeders and secondary main. He says that primary feeders should be figured for 60 per cent of the connected load, and later on he figures the size of the transformers, and arrives at a certain size of transformers which he connects on the system.

I endeavored to check up in the short time at my disposal a couple of the figures presented by the author, and I found that those figures do check fairly well with a similar practise we have here. We have two feeders, practically the same kind of system which the author is endeavoring to portray, that is an a-c. distribution system, in which we have the feeders leading from the substation to the feeding point, and then sub-feeders, or secondary feeders, as he calls them, leading along the street to the transformers and secondary mains, for the distribution of the current. On one of these feeders I find that with a maximum demand, that is an average maximum demand, covering the average of ten or eleven maximum readings, the average kv-a. on this one feeder was 196, and the transformer connected to that was 242. The transformers connected are 123 per cent of the demand on the feeders, and the author in his assumed case has his transformers at about 133 per cent, which seems to check very well with the practise we have found here. That would seem to lead to the assumption, although I have not been able to check that, that the 60 per cent he assumes is quite close to being correct.

In his paper, where the author speaks about transformer connections, he says: "There is considerable difference of opinion among engineers as to the advisability of installing any fuses on the primary side of subway transformers, as they generally cause more trouble than their protection is worth, but as it is advisable to have some point where the transformers can be easily disconnected from the feeders it is a good plan to install the fuse boxes and use fuses having 100 per cent greater carrying capacity than is required to protect the transformer, and then trust to the station circuit breaker to disconnect the feeder in case of serious trouble." On our system we have primary fuses on all transformers. We have adopted a standard bank consisting of one 25-kw. transformer connected 110-220 and a 10-kw. single-phase transformer banked as a teaser, to give three phases on our power work. The light and power are taken from the same bank in that way on this system and that makes a standard bank of 35 kw. We have on the primary side a 15-ampere, 2300-volt fuse, and have very little trouble. Occasionally the primary fuse will blow, but that is generally due to some trouble on the distribution system.

The secondary mains from each transformer are normally connected separately, and there is a box between each set of mains in which the connecting bars are left out to connect up to in order to carry the mains from one bank to the next bank, or to the bank on two sides, in case of trouble in any given bank. That system has been in operation for about four years, and we have had very little trouble, indeed. I think it would be safe to say that since we have put the system into effect we have not had $100 maintenance cost on the system—I mean from added work caused by trouble, and aside from the ordinary inspection.

H. L. Wallau: I want to supplement Mr. Rakestraw's remarks in regard to this system by stating that in the TRANSACTIONS of the American Institute of Electrical Engineers, Volume 34, page 746, there is a discussion on this particular system, with a diagram of the connections and type of boxes used, which may be of interest.

E. Friedlaender: Operating men in the steel industry now have up the question, whether it wouldn't be advisable to put transmission lines in the plants underground.

The main advantage, probably, would be absolute safety against shocks through coming in contact with the conductors. Another great advantage would be avoiding lightning troubles. Would these benefits justify the additional expense?

There are a few points, which we must not overlook when installing underground systems. We cannot overload underground cables as readily as bare overhead lines. Another point of importance is continuity of service especially when voltages of 22,000 volts or over are used. It would require more time to locate and repair faulty underground than overhead lines.

Another source of trouble in underground systems is the liability of cable trouble in a manhole spreading over the whole system. On overhead lines, shorts or grounds are easily confined to one line, as lines are usually spread far enough apart.

The question of insulation of underground cable is a vital one. Paper is the cheapest and probably is good enough for dry and cool ducts, and lower voltages. Where cables are subjected to heat and moisture, as is often the case in mill work, would not rubber or cambric insulation be better?

Where large currents are handled, great care must be taken to guard against electrolysis.

We cannot overlook the fact that the carrying capacity of underground cable is considerably smaller for the same size cable, than that of overhead cable. The proper construction and ventilation of ducts is therefore of great importance. In many cases in the steel industry, the ground surrounding the ducts is made of cinder, or gravel, and is very porous and dry, and would not carry off the heat.

Should we entirely close up all manholes in our mills, or should we ventilate same? There is danger of gases accumulating in an entirely enclosed manhole, but on the other hand there is danger in the mill of having either hot cinder or metal spilled in the manhole if of the ventilated type.

It probably would be advisable to bring out cables, only, in every other manhole, so that in case of trouble in a manhole, not all cables are affected.

E. B. Meyer: The author as noted in his paper undoubtedly favors the use of varnished-cloth or cambric-insulated cable, whereas the writer's experience with large systems of distribution over a period of fifteen years indicates that paper cable can be used almost exclusively for this class of work without

fear of giving trouble, the primary feature being, as in all kinds of electrical construction, care in installation.

Service or distribution holes where the space in the street allows should be built on the side of the main conduit so as to permit ample room for the proper racking of cable and subway boxes. In congested streets, however, this plan is not always feasible and under such conditions the hole may be placed on top of the main conduit and sufficient ducts run therein for distribution work. See Fig. 1.

In many systems subway branch boxes are used on the main

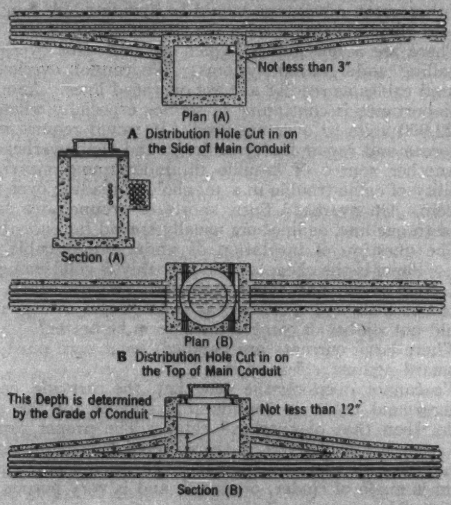
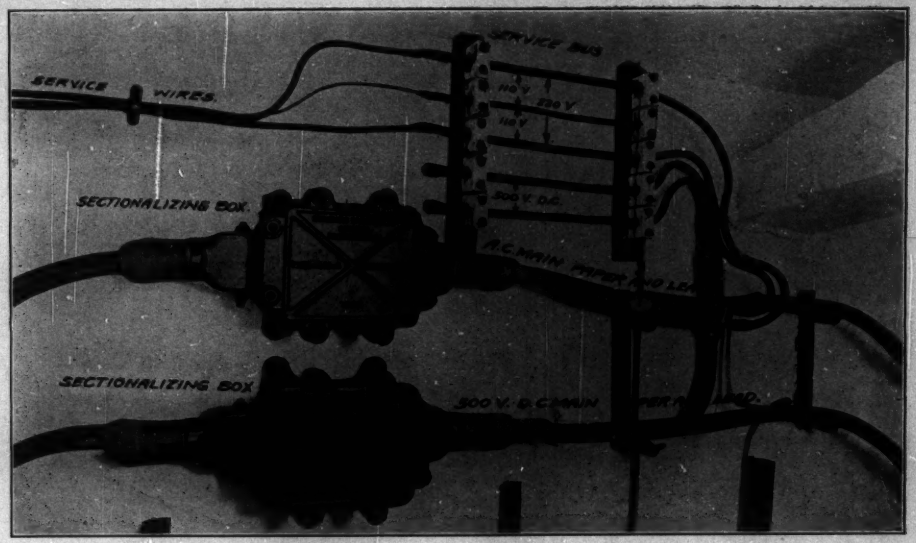


FIG. 1—METHODS OF BUILDING DISTRIBUTION HOLE IN MAIN CONDUIT LINE

secondary cable for taking care of service connections to consumers. These boxes add considerably to the cost of installation and it is frequently found necessary to install additional boxes in cases where the number of outlets in the original box installation is not sufficient to take care of the ultimate number of service connections.

A novel method of taking care of such service connections is by the use of rubber insulated service bus mounted on the wall of the manhole or distribution hole as shown in Fig. 2. In this type of construction the usual splice is made to the main paper-insulated lead-covered cable with a rubber-insu-



[MEYER]

INTERIOR OF MANHOLE SHOWING SECTIONALIZING BOXES ON THE SECONDARY A-C. MAIN AND THE 500-VOLT POWER MAIN. SERVICE BUS MOUNTED ON THE WALL IN THE UPPER CORNER OF THE MANHOLE TO WHICH SERVICES MAY BE TAPPED AND RUN TO THE VARIOUS CONSUMERS

lated lead-covered cable. The lead sheath on the branch cable terminates a short distance below the bus rack. Service connections are made to the bus with rubber-insulated cable covered with weather-proof braid.

Installations of this character have proved very successful and have been in operation on 220-volt 3-wire a-c. systems for a period of over ten years without a failure. The principal advantage with this form of construction is that the service of a lead jointer are not required to make the connections to the service bus and any number of connections up to the capacity of the bus may be installed as the occasion requires. In a system as just described, the secondary mains may be either single- or 2-conductor paper-insulated lead-covered and the

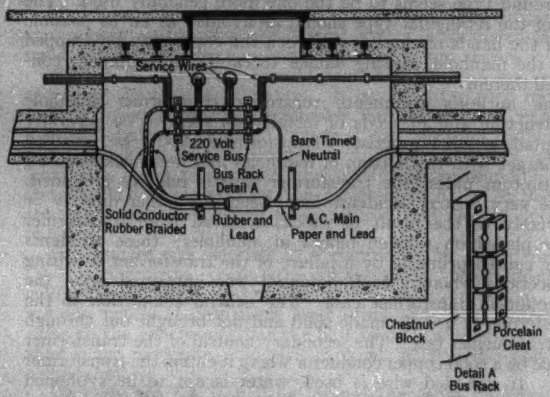


FIG. 2

neutral of bare tinned copper wire. The lateral connections from the service holes to the customer's premises are usually installed in iron pipes thoroughly coated with asphaltum or other compound to prevent corrosion, the entire system being water-proof from the manhole to the customer.

With reference to series street lighting systems the writer has found No. 8 wire is usually a sufficient size for all classes of street lighting work, a single-conductor rubber-insulated lead-covered cable apparently being best adapted for this particular use as the lead can be removed from the rubber cable making it easy to loop the cable into the bases of the ornamental iron posts along the underground system.

While the author does not specifically state the kind of conduit material recommended, recent practise indicates that

fibre conduit has come into very general use for all classes of underground electrical work. In the laying of fibre conduit a concrete base is usually provided. There is also provided a side and top cover of concrete with one-inch concrete separation between the adjacent ducts. Experience has shown that this form of construction resists the heat of an arc from burning cables, preventing trouble communicating from one duct to another. Multiple-tile duct affords the least protection to cables, as it is impossible to prevent the communication of trouble between the ducts at the joints. Multiple duct is, therefore, better adapted for telephone, telegraph and other similar work.

Regarding the several types of manhole heads or covers, the round type seems to be the one more generally used. The use of the retangular type should be avoided as far as possible as in the hands of careless workmen the cover may be dropped into the manhole, causing danger to cables or equipment contained therein.

The author's comments regarding transformer manhole construction and methods of installation seem to be generally accepted as good practise. Transformers should be installed in contact with the bottom of manholes and not blocked up off the bottom unless the transformer case is reliably grounded. Some very serious accidents have been due either directly or indirectly to the result of shocks received from transformer cases placed on wooden blocks in manholes; these accidents being caused primarly by a failure of the transformer or wiring connections whereby high potential was impressed upon the ungrounded transformer case. The neutral connection to the transformer is usually made solid and not brought out through the transformer box. The secondary neutral of the transformer should be a solid copper conductor where it enters the transformer case. If stranded wire is used, water is apt to be syphoned into the transformer when manholes are flooded and special precaution should, therefore, be taken to see that this connection is made water-tight.

The importance of maintaining the oil in underground transformers in perfect condition free from moisture or sediment cannot be too strongly emphasized as the life of the transformers depend on the maintenance of these conditions. Precaution should be taken by operating companies to insure proper installation and operation of the transformer and in addition to an inspection of the oil at least once a year, it is desirable that air pressure be applied to the transformer cases after installation to detect leaks.

For the purpose of checking the load on transformers, the use of a split-core current-testing transformer will be found convenient.

Transformers should be provided with cut-out subway boxes on both the primary and secondary sides if they feed an under-

ground distribution net work. If they feed only isolated sections the cut-out on the secondary side may be omitted. These boxes need not necessarily be fused, as a number of companies believe that fuses give more or less trouble. In most cases the omission of fuses is recommended, the boxes being provided simply as disconnectors in the event of trouble with the transformer. Where solid connections are provided, it is necessary to depend for protection entirely upon the automatic devices in the station.

In general it may be said that an underground system of distribution depends to a large extent upon local conditions and in many cases it will follow the existing plan of the overhead lines.

Mr. Newton's paper, however, has covered the subject in such a way as to assist the central station engineer in laying out a cable system for supplying light and power service.

Wilfred Sykes: Mr. Friedlaender has raised some points regarding the type of insulation. It has been my good fortune to be associated with a considerable number of cable experts, in work for the Government where it was necessary to find the best cable that could possibly be designed to carry power from one point to another, the conditions surrounding this installation being such that the cables were likely to be subjected to particularly high temperatures, and also to flooding at times.

This matter was referred to a committee for consideration. On this committee there were representatives of the principal cable manufacturers, and we had a very frank discussion. The various types of insulation were discussed, and everybody came to the conclusion that the best insulation they could get, that is for moderate potentials, around 5000 volts, was varnished cambric.

After we got the insulation around the cable, the question of protecting the cable created a good deal of controversy. Some thought that we ought to have a lead sheath around the cable, and others pointed out that lead sheaths were likely to crack from unknown causes, and that they were pretty sure to crack if there was any vibration, and for that reason another type of moisture protection was advocated and definitely adopted. We know that plain rubber has short life, especially where the temperature is high. Some experiments carried on over a period of years in the New York Edison Company has shown that an insulation made up of reinforced rubber, as it is called, will stand high temperatures, apparently without deterioration, for quite a long time. This reinforced rubber is nothing less than rubberized tapes laid over one another, and held together by cement, and the whole vulcanized. As a moisture protector that is quite effective, but it is not used as an insulator. Laboratory tests made over periods of years, where it has been subjected to temperatures of 100 deg., show it is apparently un-

changed. The paper-insulated cables will stand a little higher temperature than the cambric-insulated cables without deterioration, but not much. The upper limit of temperature varies somewhat with the voltage to be used. The paper cable was not suitable for this work, because we had to make a number of rather sharp bends in it, and nobody would contend that the paper cable was as good as the cambric cable for that work.

One other point was the question of grounding the sheaths. In one case we had to work with three-phase cable, where the lead sheaths were used, and there was no question of being able to ground the sheath of that cable, and not having any circulating current appreciable. In the case of single-phase cable, with lead sheaths, we made an investigation and found that the circulating current was such that if you grounded the lead sheath you would be likely to burn it off, or you would have such temperatures that the current carrying capacity of the cable was cut down, so that it was not a practicable proposition. The Committee finally came to a decision, that where the cost was not the main consideration, and where the cable would have pretty rough handling, that the varnished-cambric cable was the best type.

F. M. Hibben: I would like to ask the members if they have used multiple-conductor arc cable. The author advocates single-conductor cable No. 6, he advocates No. 6 for mechanical reasons only, the current being 5 or 6 amperes. We are contemplating using 4, possibly 6, or a larger number of conductors, perhaps we may reach to ten, all in one sheath for different arc circuits, and I would like to find out if any of the companies have tried anything like that, and with what success. With two-conductor arc cable, you have little chance of one arcing and causing short circuits in adjacent cables. On the other hand, the smaller cost of subway acts as a point in favor of the use of multiple-conductor arc cable.

In one case the author advocated a bare neutral, and in another case he said pull the bare neutral into a duct with a lead-sheath cable. With a slight amount of voltage in the ground from the street railway return current, I think the current from the bare neutral to the rails would eat holes in the lead sheath. In Cleveland we had a d-c. distribution something similar to that, where we pulled out all the bare neutral and put in regular insulated lead cable. We found it impossible to maintain the bare neutral, even when it was in a separate duct—we had more or less electrolytic trouble with it.

G. J. Newton: In reply to the questions that were brought out in the discussion of this paper I desire to make the following explanation, after which the various questions will be replied to.

In the first place the paper as presented was very much condensed from a manuscript prepared for book publication, it was not possible, therefore, in the space available to give

more than a general outline of the method of proceeding to design a system of underground distribution and a simple system, common to the average cities of moderate size, was selected as being representative of this class of work. It was clearly stated that an a-c. system would be considered as, with the exception of d-c. power, there would be little or none of the d-c. system placed underground in a city of the size considered where a new underground system was installed.

In the second place the conclusions arrived at and stated in the paper do not apply to any particular locality but were intended to represent average conditions and were based on over twenty years experience in designing, building and operating underground systems; most of the questions brought out in the discussion were based on the experience of the various gentlemen in their particular locality. With the conditions assumed in the paper, and the further assumption that the engineer has not had previous experience in this class of work I feel certain that the methods suggested are satisfactory for guidance.

Large existing systems are in charge of competent engineers who are familiar with the most suitable methods and apparatus to use in their particular system, the author had no intention of criticizing their methods but simply desired to give useful information to those engineers who were confronted with this class of work and lacked previous experience in handling it.

Replying to Mr. A. A. Meyer: I believe that both the a-c. and d-c. systems have their particular place in distribution problems but in a place similar to that assumed I doubt if much d-c. distribution would be placed underground, the chances being that the d-c. system would be changed to a-c. in this district with the ultimate intention of doing away with it entirely as the underground district was enlarged.

The methods suggested for designing the a-c. system would apply for d-c. work were it necessary to install that system, as the much less complicated equipment makes that class of design comparatively a very simple matter as this type of service is fairly well standardized in the large cities, and undoubtedly the new system installed in a small place would follow existing practise.

Replying to Mr. C. W. Rakestraw: I stated that "Personally I prefer varnished cambric or cloth insulation in all distribution work, particularly in the smaller cities where usually the cable department is very limited." I see no reason to change my opinion on that statement and refer to Mr. Wilfred Sykes discussion of the paper in which he states that varnished-cambric cable under the conditions assumed would be the best. In preparing the paper I felt that it was proper to suggest what my experience had shown to be the best, if, however, the cost would prevent the use of this class of cable then the engineer must use paper.

In small cities it is foolish to install anything but the best as the system once installed is seldom inspected or taken care of properly and it is better to do a little at a time and do it right and give good reliable service.

I said that if paper cable was used it would be necessary to splice *either* varnished-cambric or rubber tails on it where it entered equipment. Naturally, with the knowledge of modern engineers they would not use rubber insulation in oil-insulated equipment.

The system under discussion was stated to have single-conductor mains for the lighting system as that type of cable gave the greatest capacity for those mains and, therefore, single-conductor services were stipulated. The power system was stated to have three-conductor cable and three-conductor services.

As stated, there is a difference of opinion as to the advisability of installing fuses on the primary side of underground transformers. The fact that fuses are considered advisable on Mr. Rakestraw's systems does not make my statement incorrect.

The author installed a bare neutral on an a-c. distribution system in Peoria, Ill., and as far as I know there has never been any trouble from this neutral, however, at the time that I tested the system it was exceptionally free from electrolysis. In this connection I might state that I have heard of the sheath of the cable being used as the neutral in some three-phase work and would refer you to a paper by Mr. Hood of one of the Canadian companies.

Replying to Mr. Friedlaender: In my paper I pointed out the necessity of providing emergency facilities to take care of every case of trouble that could be reasonably foreseen, I am confident even under the exacting conditions required by a steel mill that with proper study a satisfactory system could be designed and installed, which would fully warrant the cost of the work.

If the entire power and lighting system of the plant were changed to underground or at least the design made with that end in view, it would be possible to provide ample facilities for meeting the most difficult conditions.

Replying to Mr. E. B. Meyers: In the original manuscript it was suggested that service boxes could be built on the side of the main run of conduit but the statement was omitted in the revision for this paper, however, conditions govern this part of the work.

The particular method of taking care of service in Mr. Meyers company is known to the author and is a special case and certainly not suitable for a system in a small city where the cable department is limited.

Undoubtedly fibre conduit can be installed for less money than clay conduit and will give equally good service. This subject was also treated in the original manuscript. Multiple

clay duct should not be used on lighting and power systems, either single clay or fiber is better.

By installing the transformers on I beams there is a chance for the air to circulate around and under them and under heavy load they would have more cooling surface. Naturally, these I beams should be connected securely to the transformers and grounded.

Replying to Mr. F. M. Hibben: There is no objection to using No. 8 conductor for series arc light systems and it is quite common practise to have several circuits in the same sheath provided that they all terminate at the same point as is frequently the case where several pass through the underground district in order to supply outlaying sections of the city.

Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.

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SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

BY ROBERT F. HAMILTON

ABSTRACT OF PAPER

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

FOR several years past the attention of engineers in Great Britain has been directed to the electric driving of rolling mills. This is a natural result of the constantly increasing use of electric power for auxiliary apparatus in steel works, and the economies which accompany generation of power in large units. When blast furnace gas may be utilized under boilers for turbo-generators or for driving gas engine units the prospects of electric motor application throughout an entire works are very favorable. The present demand for steel has accelerated the application in that it has caused many manufacturers to scrap existing plant and adopt more modern methods. In a very few months the steel production of the nation has changed from what might have been called a decaying industry into a vital necessity.

GENERAL CLASSIFICATION

In any rolling-mill electrification the paramount problem is the delivery of a specified tonnage of steel, of which the grade and section are known, at a minimum total cost. Each case must therefore be analyzed carefully and the most suitable equipment chosen. In general, the drives may be classified as follows:

1. Continuous running mills.
   a. Constant-speed motors.
   b. Adjustable-speed motors.
2. Reversing Mills.

Item 1 may include two-high or three-high mills, of nearly all types, *i. e.*, plate mills, sheet mills, cogging or blooming mills,

merchant mills, rod mills, etc. By far the greater number is driven by constant-speed motors. Adjustable-speed motors are found where it is desirable to roll a variety of sections in the same mill.

One of the most efficient drives is by means of a high-speed induction motor with ropes or gearing. If speed variation is necessary this may be accomplished in the case of an induction motor by controlling the rotor frequency with a synchronous converter or alternating-current commutating-motor. Instead of the adjustable-speed induction motor it is often found advisable to use adjustable-speed direct-current motors and synchronous converters, assuming of course, that the supply is an alternating-current system. If the first mentioned scheme includes an auxiliary motor on the shaft of the main motor, constant power output may be obtained at the rolls throughout the range of speed. When it is necessary to increase the speed of rolling, the torque required is usually less, which condition is also met by a direct-current motor with shunt-field control. In a few special cases cascade motors or Ward-Leonard motors have been used to drive continuous mills.

The reversing mill is best suited to the work of cogging ingots to billets. The speed of rolling is changed from pass to pass as the billet lengthens and the reversing does away with the lifting tables, which are generally necessary in a three-high mill. Electric motors which are to meet such service must be capable of withstanding large overloads, since no flywheel is interposed between the motor and mill. Ward-Leonard control is used over the range of speed in which the torque is a maximum, and shunt-field control at the top speeds when the torque is reduced. An Ilgner set supplies the energy for the motor.

Selection of Mill

The type of mill is determined by the tonnage and sections rolled; the temperature at which the rolling takes place; the quality and composition of the product; the general layout of the steel works and the labor conditions existing in the particular district. The latter is emphasized in the consideration of automatic features and skilled attendance.

This discussion presumes that the following facts have been determined:

1. Type of mill including layout.
2. Weight, shape, material and temperature of ingot or billet.

3. Section and tonnage of finished product.
4. Diameter and profile of the rolls.
5. Time required to handle material between passes.

Energy Required for Rolling

Opinions vary as to the methods employed in calculating the energy required for rolling. Many engineers use a formula which includes the logarithm of the elongation. That is, the torque in any pass is proportional to the logarithm of the elongation of the

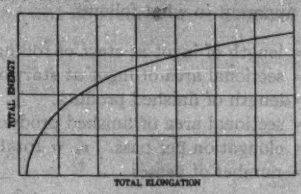

Fig. 1

material during the pass. A basis for this method may be found by plotting a curve of energy versus total elongation as shown in Fig. 1. Such a curve agrees very closely with a logarithmic function.

A second method is based upon the energy per unit volume displaced or upon its reciprocal, the volume displaced per unit energy. Referring to Fig. 2, $a\,b\,c\,d$ is considered as the volume of metal displaced in rolling from length l_1 to l_2, that is, the volume

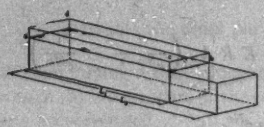

Fig. 2

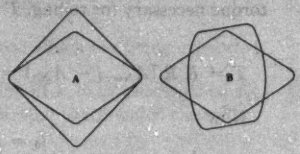

Fig. 3

of metal displaced is the product of the reduction in area and the original length.

The method used by the author consists in the application of a certain tangential force at the circumference of the rolls per unit reduction or transposition of sectional area. The following analysis will make this clear:

Referring to Fig. 3, two distinct kinds of rolling are represented

by A and B respectively. In case A, the reduction in area is the arithmetical difference of the two areas involved, while in case B, the transposition in area may be considered equal to that area of the one section which is not included within the outline of the other. Case A is by far the more common in rolling work. A combination of A and B may occur in certain forming passes where there is a slight reduction in area. Case B does not cause an elongation in the material but may be reduced to terms of an equivalent elongation when considering the total energy. This will become apparent in what follows.

Let l_1 = length of ingot at start of rolling
A_1 = sectional area of ingot at start of rolling.
l_2 = length of finished product.
A_2 = sectional area of finished product.
e = elongation per pass. e_s = total elongation.
N = number of passes.

Then

$$e_s = \frac{l_2}{l_1} = \frac{A_1}{A_2} \text{ and } e = \sqrt[N]{e_s} = \sqrt[N]{A_1/A_2}$$

assuming for the purpose in hand that there is no increase in density, nor loss of material and that the elongations in each pass are equal, which assumption will do for preliminary considerations.

Now if K be the force per unit reduction in area for any one pass, say the N_oth, R the effective radius of the rolls, and T the torque necessary for rolling, T will be given by

$$T = K\,R\,(A_{No-1} - A_{No}) = K\,R\,A_1 \left(\frac{1}{e^{No-1}} - \frac{1}{e^{No}}\right) \quad (1)$$

Now

$$l_0 = l_1\,e^{No}$$

and since energy is the product of force and distance through which the force acts, the energy for the N_oth pass is

$$E_o = K\,l_1\,A_1\,(e - 1) \quad (2)$$

and the total energy for rolling is equal to

$$E = (\Sigma\,K)\,[l_1\,A_1\,(e - 1)] \quad (3)$$

since (2) is dependent upon the number of the pass only in so far as K is dependent. If K_o be taken as an average value of K over the conditions of ordinary rolling, W the weight of the ingot which is proportional to $l_1 A_1$, and E the total energy for rolling, then

$$E = N K_o W (e - 1) \tag{4}$$

From large numbers of tests the h. p.-sec. per ton $\cdot \frac{E}{W}$, for different values of e, have been determined for various sections.

The factor K_o varies with the temperature of rolling and diameter of the rolls as well as with the kind of steel and shape of the section rolled. Within the limits of practise, however, K_o as calculated from the above; *i. e.*

$$K_o = \frac{E}{W} \times \frac{1}{N(e-1)} \tag{5}$$

changes very little owing to the first three causes mentioned and rather less than might be expected due to the fourth. For example in a certain reversing-cogging-mill rolling 4-in. by 4-in. billets from three-ton ingots, K_o has a relative value of 3.2 compared with a value of 4.0 for a merchant mill rolling 3-in. channels from 6-in. by 6-in. billets. In the former case 22 passes are employed; in the latter 15. The two examples present totally different conditions of rolling. Of course in finding the total energy consumption per ton, the mill and motor losses must be added to the energy required for rolling.

If $\frac{E}{W}$ represents the units per ton which are being expended to roll steel having a total elongation of e, in N passes, then K_o as given in (5) is a measure of the efficiency with which the rolling is being accomplished.

Table I gives a few typical values of K_o.

From the above it will be noted that the change in area in any pass is

$$A_1 \left(\frac{1}{e^{N_o - 1}} - \frac{1}{e^{N_o}} \right)$$

and the length of the entering billet or ingot is given by $l_1 e^{N_o - 1}$

TABLE I

Type of Rolling	Original section	Final section	Elongation	No. of passes	h. p.-sec. per ton for rolling	Av. K_o
Ingots to Billets	20 in. x 20 in.	6 in. x 6 in.	11.1	15	60,500	23.2×10^6
Ingots to Billets	20 in. x 22 in.	4 in. x 4 in.	27.5	23	85,000	23.8 "
Billets to Bars	5 in. x 5 in.	1.5 in. x 1.5 in.	11.1	12	89,000	33.4 "
Ingots to 70-lb. Rails	16 in. x 16 in.	7.2 sq. in.	35.6	23	120,500	31.2 "
Billets to channels	5 in. x 5 in.	$1\frac{1}{4}$ in. x $\frac{5}{8}$ in. x $\frac{1}{4}$ in.	40	17	164,500	39.9 "
Billets to *T* bars	5 in. x 5 in.	2 in. x 2 in. x $\frac{1}{4}$ in.	24.6	13	141,000	38.8 "
Sheet Mill	3 in. x 28 in.	45-in. sheets, 0.11 in. thick.	27.25*	17	144,000	39.5 "

*For simplicity in this case s has been taken as the ratio of the original to the final thickness of the sheets.

It should be noted that the values of K_o as given have not been reduced to a common temperature but represent typical conditions as found in practise.

Therefore, the volume of metal displaced in the N_oth pass is

$$l_1 e^{N_o-1} A_1 \left(\frac{1}{e^{N_o-1}} - \frac{1}{e^{N_o}} \right) = V \left(1 - \frac{1}{e} \right)$$

where V is the original volume.

This would show that under the assumed conditions the volume displaced per pass (see Fig. 2) is a constant value, and if K_1 is the energy per unit volume displaced and $\frac{E}{W}$ the energy per unit mass of the material rolled,

$$K_1 = \frac{E}{W} \times \frac{\rho}{N\left(1 - \frac{1}{e}\right)}$$

where ρ is the average density of the metal.

From the foregoing it follows that

$$\frac{K_o}{K_1} = \frac{1}{\rho e} \tag{6}$$

Since e varies slightly for different conditions, equation (6) gives an idea of the discrepancies which will occur between a calculation in which K_o is taken as a given value, and one in which K_1 is the factor assumed.

K_o conforms to the method of "force per unit area" while K_1 applies to the method of "energy per unit volume displaced" under the same conditions.

With an average value of e determined upon, the number of passes required to roll any given section would be

$$N = \frac{\log A_1 - \log A_2}{\log e}$$

$$= \frac{\log e_o}{\log e}$$

Upon laying out the individual passes it may be necessary to modify e somewhat. Then too, if there is a large temperature drop from start to finish, K_o should vary from pass to pass ac-

cordingly. For changes in temperature the writer has derived the following empirical expression, which although not exact is very useful when used within ordinary temperatures for rolling steel.

in which $$K_o = C \log \frac{\theta_m}{\theta}$$

θ_m = temperature at which the metal liquefies in deg. cent., generally about 1380 deg.
θ = temperature of rolling in deg. cent.

For cold rolling the energy required for various materials may be taken as about proportional to the moduli of elasticity of the materials.

As previously pointed out K_o must also be modified to correspond to the shape of the section. A good idea of the values to apply in the individual passes can be obtained from the average value of K_o for rolling different sections.

In this connection it should be remembered that in certain forming passes there will be a transposition or shifting of sectional area without a corresponding volume displacement or elongation and K_o will be altered accordingly. However, what interests the electrical engineer is the aggregate result rather than what may happen in any one pass and this aggregate may be very accurately determined from the average value of K_o for any particular type of section rolled.

Should there be considerable transposition of section K_o as found in the ordinary way will be larger than usual. In such a case transposition may be considered as a certain hypothetical elongation.

With this method all types of rolling may be compared on the same basis.

Relation of Mill Speed to Tonnage

Let W = weight of each ingot in tons
S = tonnage per hour required.
t_i = total time of intervals; seconds } for one ingot.
t_r = total time of passes; seconds

Then

$$t_r = \frac{3600\ W}{S} - t_i$$

Now if l_1 = original length of ingot
N_o = number of the pass
d = effective diameter of the rolls
n = average rev. per min. during a pass
t_p = time of the pass,

then

$$t_p = \frac{60\, l_1\, e^{N_o}}{\pi\, d\, n}$$

and

$$t_r = \frac{60\, l_1}{\pi\, d\, n}\, [e + e^2 + e^3 + e^4 + \ldots\ldots e^N]$$

from which

$$n = \frac{60\, l_1\, [e^{N+1} - e]}{\pi\, d\, [e - 1] \left[\dfrac{3600\, W}{S} - t_i\right]}$$

n is generally limited by the speed at which the material may be handled. In this case with n known, S is found from

$$S = \frac{3600\, W\, n}{\dfrac{60}{\pi} \times \dfrac{l_1}{d} \left[\dfrac{e^{N+1} - e}{e - 1}\right] + t_i\, n}$$

or with n and S both known, an insight into the rapidity with which the material must be handled may be found.

Fig. 4 shows a graphic application of the formula which is useful in obtaining an idea of the tonnage which any continuous-running mill may deliver.

The results obtained with a reversing mill are quite different. The calculated torque-time diagram for a typical reversing mill is given in Fig. 5. In this case the accelerating time in the pass is about one-half the time of the pass and the material is discharged from the rolls at top speed. The time of reversing and attaining the "nipping" speed is here included in the time of the interval.

Let T_p = turns in pass.
n_1 = nipping speed in pass
N_o = number of the pass
n = Max. speed in pass
t_p = time in pass seconds
t_a = acceleration time in pass.

Now it is good practise to make $n_1 = \frac{1}{3}\, n$ and $t_a = \frac{1}{2}\, t_p$.

Then

$$n = \frac{72\,T}{t_p}$$

$$= \frac{72}{\pi} \times \frac{l_1}{d} \times \frac{e^{No}}{t_p}$$

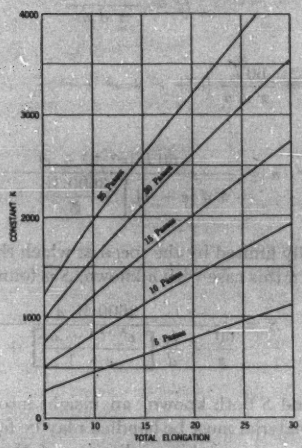

FIG. 4

NOTE: To find the total time in seconds for billet to pass through the rolls add to the time of the intervals the time T, when

$$T = \frac{L_1}{D} \times \frac{K}{n}$$

$\frac{L_1}{D}$ the ratio of the original length of the billet to the effective diameter of the rolls.

$\frac{K}{n}$ the constant from the curve divided by the average rev. per min. of the rolls.

Let the time of first pass $= t_1$
Let the time of final pass $= t_f$
Nipping speed in first pass $= n_1$
Nipping speed in final pass $= n_f$
Then

$$t_f = t_1 \times \frac{n_1}{n_f} \times e_s = K_2\, t_1 \text{ where } K_2 = \frac{n_1}{n_f} \times e_s$$

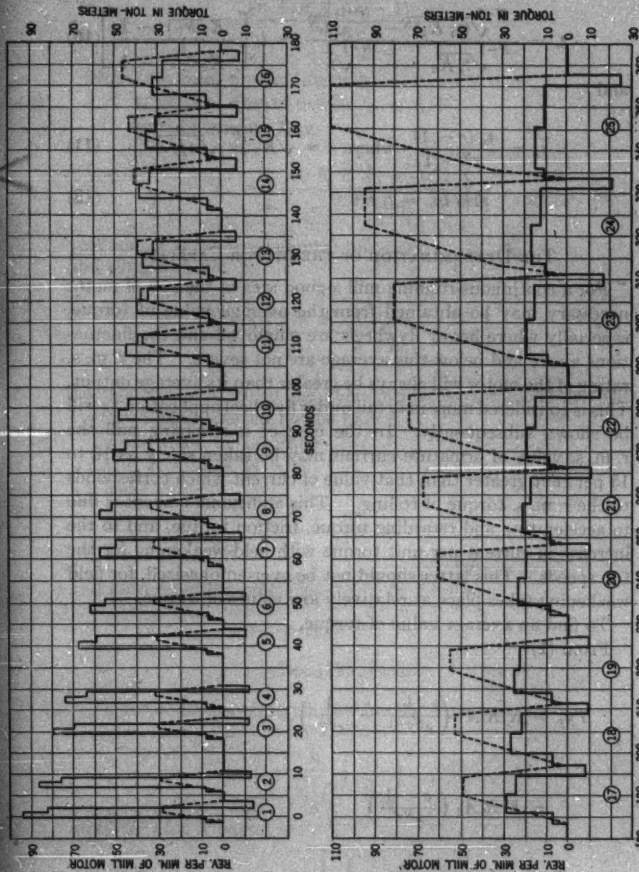
TORQUE IN TON-METERS
REV. PER MIN. OF MILL MOTOR
SECONDS
TORQUE IN TON-METERS
REV. PER MIN. OF MILL MOTOR
SECONDS

FIG. 5

and assuming that the times are in geometrical progression the total time of the N passes, t_r, is given by

$$t_r = \frac{\sqrt[N-1]{(t_1 K_2)^N} - \sqrt[N-1]{t_1^N}}{\sqrt[N-1]{t_1 K_2} - \sqrt[N-1]{t_1}} = t_1 \left[\frac{K_2^{\frac{N}{N-1}} - 1}{K_2^{\frac{1}{N-1}} - 1} \right] \quad (10)$$

and

$$t_1 = \frac{t_r (r-1)}{r^N - 1} \text{ where } r = \sqrt[N-1]{K_2} = \sqrt[N-1]{tf/t_1} \quad (11)$$

$$\text{also } t_{No} = t_1 r^{No-1} \quad (12)$$

The Determination of the Motor Capacity

For a continuous-running mill a good idea of the size of motor necessary may be obtained from the average value of torque, especially where heavy flywheels are employed and the fluctuations above and below this average are not severe. The r. m. s. rating of the motor will always be greater than its average output. This is considered more in detail under flywheel application as will be shown subsequently. In the case of a reversing mill the r. m. s. value of armature current may be taken safely as 10 to 15 per cent greater than that value of current which corresponds to the r. m. s. torque of rolling. This additional current is due to accelerating and retarding torque, friction torque, and to the increase in current per unit torque with field-weakening for the top speeds. This latter should not be over-emphasized, for field weakening takes place at relatively low values of torque.

To find an average value of torque,

From (1)

$$T_{No} = K R A_1 \left(\frac{1}{e^{No-1}} - \frac{1}{e^{No}} \right)$$

$$= K R A_1 \left(\frac{e-1}{e^{No}} \right)$$

Also $t_p = \dfrac{60\, l_1\, e^{No}}{\pi\, d\, n}$ (for continuous mill)

Therefore, torque × time for N_oth pass is

$$\frac{60\,K\,R\,A_1\,(e-1)}{\pi\,d\,n}$$

Now t_i = total time of intervals
t_r = total time of passes, $t_\Sigma = t_i + t_r$
∴ Average torque, T_a

$$T_a = \left[\frac{60\,K\,R\,A_1\,(e-1)}{\pi\,d\,n\,t_\Sigma}\right] N$$

Where N is the total number of passes, and since $d = 2R$

$$T_a = \left[\frac{30\,K_o\,A_1\,(e-1)\,N}{\pi\,n\,t_\Sigma}\right] \tag{13}$$

To this torque is added the average friction torque to find the average torque exerted by the motor.

To find the r. m. s. torque,

Let $K\,R\,A_1\,(e-1) = c_1$

Then

$$T^2 = \frac{c_1^2}{e^{2\,No}}$$

and for a reversing mill,

$$t_{No} = t_1\,r^{No-1}$$

[see (**12**) above.]

Therefore torque-squared-time for the N_oth pass is

$$\frac{c_1^2\,t_1}{e^2}\left(\frac{r_o}{e^2}\right)^{No-1}$$

and $\Sigma\,T^2\,t$ for N passes is

$$\frac{\Sigma\,c_1^2\,t_1}{e^2}\left[\frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e} - 1}\right]$$

which gives

$$\text{r. m. s. } T = \frac{\Sigma c_1}{Ne} \sqrt{\frac{t_1}{t_\Sigma} \left[\frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e^2} - 1} \right]}$$

If torque is in kg-m. then

$$\text{r. m. s. kg-m.} = 38 \frac{d}{l_1} W K_o \frac{e-1}{e} \sqrt{\frac{t_1}{t_\Sigma} \left[\frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e^2} - 1} \right]}$$

Where d = diameter of rolls

l_1 = original length of ingot.
W = weight of ingot in tons.
K_o = constant for rolling (force per unit reduction.)
t_1 = time of 1st pass.
t_Σ = time of cycle.
$r = \sqrt[N-1]{K_2}$
$e = \sqrt[N]{e_s}$

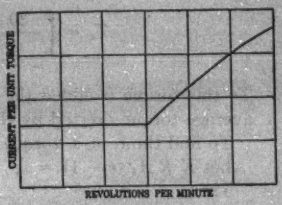

FIG. 6

In applying the above formulas it will be found that the torques in the first passes will be excessive and those in the last smaller than actually occur in practise. When a diagram is constructed the necessary corrections should be applied. As previously stated however, the aggregate result is what is most desired and within ordinary limits that is not greatly affected by what may happen in any one pass.

After torque-time and speed-time curves have been prepared for the mill-motor a current-time curve is drawn. This is determined from a motor characteristic curve which gives current per unit torque versus speed, as shown by Fig. 6. The output of the mill-motor is given for every instant by the product of the torque and speed curves, and to this output are added the losses which are easily determined from the above curves.

In this manner are found the energy diagrams for the generators of the Ilgner sets, as well as the exciter sets. The rating of these

machines is obtained by finding the r. m. s. value of current for armature and field circuits and in some cases the average iron

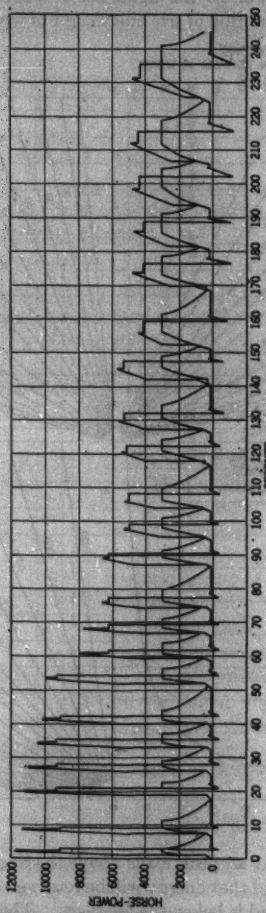

FIG. 7

losses throughout the cycle of operation. A typical "h. p.-time" diagram for a reversing mill is given in Fig. 7.

Flywheel Application

After a power-time diagram has been determined upon for a continuous mill or for the Ilgner set of a reversing mill the next step is the calculation of size and weight of wheel. In this it may

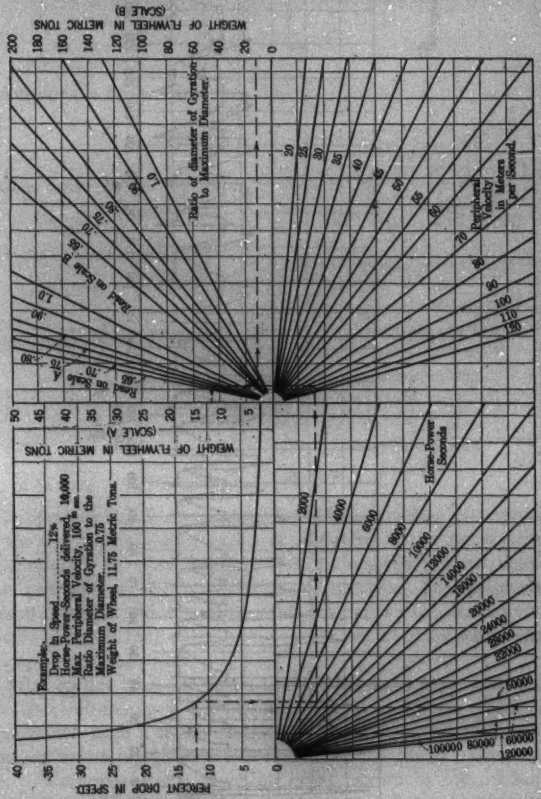

Fig. 8—Chart for Determining Weight of Fly Wheel

be assumed that all peak loads above the average will be carried by the wheel and that energy will be returned during the intervals in which the load is below the average. The amounts of energy which the flywheel must supply (both positive and negative) are

added algebraically one at a time for a complete cycle of retardation and acceleration to the original speed. The largest numerical value is taken as that which the flywheel must supply with a given speed drop.

The necessary weight of wheel may be obtained from

$$W = \frac{C\,E}{K_3^2\,V^2\,(2-d)\,d} \qquad (1)$$

where

W = weight of wheel.
E = energy delivered with total drop in speed.
K_3 = ratio of diameter of gyration to maximum diameter
V = maximum peripheral velocity of rim.
d = fractional drop in speed.

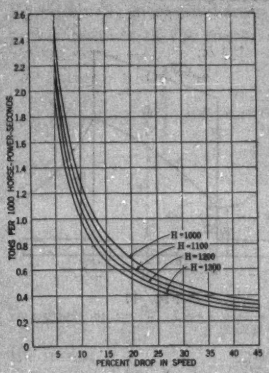

FIG. 9

Thus if E is in horse-power-seconds, V in meters per second and d in per cent drop in speed divided by 100, W will be in metric tons for a value of C equal to 1.491. A graphical application of the formula is shown in Fig. 8, by means of which an idea of fly-wheel weight may readily be obtained.

A very useful way of expressing flywheel efficiency is by the vertical height to which the intrinsic energy of the wheel would lift its weight when running at maximum speed. Fig. 9 gives a ready method of obtaining the weight of the wheel necessary for various designs.

The equation just given shows something which is often not apparent, namely that the speed of the flywheel shaft does not enter into the weight of wheel necessary, provided that the wheel be so designed as to get the maximum allowable stresses in the rim.

At this point it should be stated that there are many other factors which might influence the size of wheel selected. For example if an induction motor with slip-regulator is used, increasing the weight of the wheel means not only an increase in capital outlay, but also in constant losses. To offset this there

may be a reduction in size of the motor due to a more constant output; less loss in the slip-regulator and consequently a smaller one. The generators may be designed for a smaller variation in speed and the maximum demand from the line may be reduced. In each problem the total cost of these factors in pounds sterling (or dollars) per annum should be made a minimum. Fig. 10 illustrates the effect upon the motor peak load caused by a change of flywheel weight or the speed characteristics of the motor.

The opinion is often expressed that a mill cannot have too large a wheel. The writer has in mind a merchant mill where half the output of the motor is used continuously to keep an excessively large flywheel running with the result, incidentally, that the full-load output has never been reached.

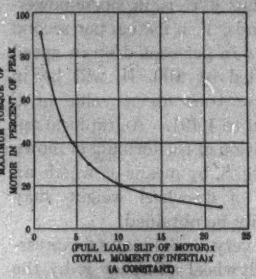

FIG. 10

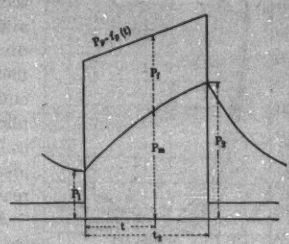

FIG. 11

Referring to Fig. 11

Let P_p = power of the peak load.
P_m = power from the driving motor.
P_f = power from the flywheel.

In general the above quantities can be expressed as follows:

$$P_p = f_p(t)$$
$$P_m = f_m(n)$$
$$P_f = I n \frac{d n}{d t}$$

where t stands for time and n for the speed of the flywheel set. The function $f_p(t)$ is obtained from the power-time curve to which the flywheel set is to be applied and $f_m(n)$ from the speed characteristics of the driving motor.

Then the differential equation which shows that the instantaneous rate of transfer of energy to the generators (or to the mill) is equal to the algebraic sum of the power from the motor and that from the flywheel is given by

$$P_p = P_m \pm P_f \tag{2}$$

Now a very common application is that in which $f_p(t)$ is a constant value for a given interval of time and in which the torque of the motor is proportional to its drop in speed (or slip in the case of an induction motor), that is

$$P = K(n_0 - n)\,n \pm I\,n\,\frac{d\,n}{d\,t} \tag{3}$$

Equation (3) integrated from speeds n_1 to n_2 corresponding to power outputs of the motor P_1 and P_2, gives

$$t = \pm \frac{I}{2K}\left[\log\frac{P - P_1}{P - P_2} + \frac{1}{m}\log\left(\frac{1 - \dfrac{2\,n_1 - n_0}{n_0/2\,(m-1) + n_1}}{1 + \dfrac{2\,n_2 - n_0}{n_0/2\,(m-1) + n_2}}\right)\right] \tag{4}$$

in which $m = \sqrt{1 - \dfrac{4\,P}{K\,n_0^2}}$ and n_0 is the synchronous speed of the motor.

For all ordinary calculations the second term in the above equation may be taken as equal to the first. Making this assumption, we have

$$P_2 = P - \frac{P - P_1}{e^{\frac{K}{I}t_1}} \tag{5}$$

when the flywheel is decelerating, and

$$P_2 = P + \frac{P_1 - P}{e^{\frac{K}{I}t_1}} \tag{6}$$

when the wheel is accelerating.

From the differential equation it will be seen that all terms are expressed as power. Therefore in the case of an induction motor K will be given as $K = \frac{P}{n \times s}$ where P is the horse-power output of the motor at speed n and slip s (both in rev. per min.). I is a constant times the moment of inertia of the fly-wheel. Its value may be obtained from $I = \frac{2E}{n^2}$ in which E is the kinetic energy of the wheel in horse-power seconds when the speed is in revolutions per minute.

From this the time constant

$$I/K = 2\left(\frac{E \times s}{n \times P}\right) \tag{7}$$

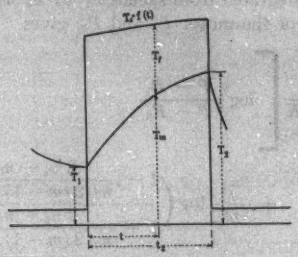

FIG. 12

in which as before E is the kinetic energy of the flywheel at n rev. per min. and P is the power output of the motor with slip s and speed n. Where there are heavy peaks of long duration $\frac{I}{K}$ will have a value of many seconds, depending also of course, upon the allowable drop in speed. On the other hand $\frac{I}{K}$ may be only a fraction of a second when the peaks are of short duration and infrequent.

Very often the torque instead of the power which is supplied by the motor and flywheel may be expressed as a function of the time or a constant value. Thus in the example illustrated in Fig. 12

$$c_1 + c_2 t = T_m + T_f$$

$$= (n_0 - n) K' \pm I' \frac{dn}{dt} \tag{8}$$

or

$$T = c_1 + c_2 t - c_2 \frac{I'}{K'} + \left(T_1 + c_2 \frac{I'}{K'} - c_1\right) e^{-\frac{K'}{I'} t} \tag{9}$$

or in general if $T_p = f(t)$

$$T = e^{-\frac{K'}{I'} t} \left[\int \frac{K'}{I'} f(t)\, e^{\frac{K'}{I'} t} \, dt + K_1 \right] \tag{10}$$

If T_p is a constant value, then

$$T = T_p - \frac{T_p - T_1}{e^{+\frac{K'}{I'} t}} \tag{11}$$

when the wheel is accelerating, and

$$T = T_p + \frac{T_1 - T_p}{e^{+\frac{K'}{I'} t}} \tag{12}$$

when the wheel is decelerating.

The time constant $\frac{I'}{K'}$ in this case corresponds to $\frac{I}{K}$ in the previous discussion. K' is the torque per rev. per min. drop in speed and I' is the torque necessary to accelerate the wheel one rev. per min. per unit time.

In other words $\frac{I'}{K'}$ is the time required for the motor when exerting a constant torque, (assuming that this be possible) to accelerate the wheel over a range of speed which is equal to the drop below synchronous speed necessary to produce this torque in the motor.

When an induction motor is used in conjunction with a slip regulator the current to the motor is limited to a fixed maximum value. Accordingly when the maximum torque of the motor has been reached equation (3) is modified to read

$$P = K'' n \pm I n \frac{dn}{dt} \tag{13}$$

the solution of which gives

$$t = \frac{I}{K''}(n_1 - n_2) - \frac{I}{(K'')^2} P \times \left(\log \frac{P - K'' n_1}{P - K'' n_2} \right) \quad (14)$$

K'' is the ratio of the h.p. output of the motor to its speed in rev. per min. when the slip regulator comes into action.

If we are working with a torque-time diagram, equation (8) becomes

$$T_m \pm I' \frac{dn}{dt} = c_1 + c_2 t \quad (15)$$

from which

$$\pm (n_1 - n_2) I' = (c_1 - T_m) t + \tfrac{1}{2} c_2 t^2 \quad (16)$$

When a flywheel curve has been plotted, the losses in the slip regulator may be obtained, the average value of which determines the size of the regulator. The losses in the driving motor are added to the ordinates which give the output of this motor and from the area of the resulting curve are found the required units per ton. In plotting flywheel curves, or in solving any of the equations given in which exponentials occur, the use of a log-log-slide-rule is recommended.

DISCUSSION ON "SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING MILL MOTORS, (HAMILTON), CLEVELAND, OHIO, MARCH 8, 1918.

A. M. Dudley: One thing that occurs to me in connection with this paper is the effect of the temperature of the material in the roll upon the power required to roll it. It is evident that the layout of some mills allows handling the material more rapidly from the furnace to the finishing pass, and hence operates with minimum power consumption so far as temperature is concerned. There are enough similar items to considerably modify an estimate of the power requirements, as determined in this paper.

Also, the personal characteristics of some operators demand a greater margin of available power than others. Ever since iron has been smelted and forged and worked by man, the processes have been handed down from father to son, and the successful mill application has been regarded as more a question of experience than scientific investigation. Nevertheless, it is entirely possible to make a fair preliminary analysis of the power requirement by some such method as that given by the author, and there is need for further data along the same line. The only work that I can recall on this subject is the one by Dr. Puppe published several years ago.

It is not expected that a new mill will be completely laid out entirely along theoretical lines as to power consumption, but a fair estimate of the total power required is very useful for other problems in the installation. There are two considerations that should be always kept in the foreground in applying electric motors to mill drive. The first of these is that *reliability is the prime consideration*. No chance must be taken of a shut down caused by an overloaded motor. In the nature of the work, a shut down in an iron or steel mill is more serious than in almost any other industry. The cost of power in making steel cannot in any case exceed 25 per cent of the total cost of production, and is lower rather than higher. If this is the case, a loss in all-day efficiency of 2 per cent or 3 per cent, due to the main driving motor being somewhat under loaded, is negligible, if it makes for greater reliability of operation. This consideration alone demands a liberal factor of safety in selecting the motor. The second point is that the electrification of a mill usually increases the output considerably, and the increased output in turn demands more power in the motor to take care of it.

I have in mind a mill laid out for ten thousand tons of steel a month where fifteen thousand tons is regularly rolled, and a peak has been reached where twenty thousand tons were rolled in a single month. The original motor was applied with this possibility in mind and has proven adequate. When orders are in hand and production is rushed, the only limit is the mill itself, and the maximum tonnage that can be crowded through the mill is the load the motor will be called upon to handle. Both

of these considerations call for great liberality in the selection of the motor after the power requirements for the nominal tonnage are determined.

C. A. Menk: I agree heartily with what Mr. Dudley has just said. About a year ago we were called on, with about two hours' notice, to determine on the size of a motor to do a certain amount of work in a mill which we expected to build in four or five months. The time was short, and we did not have the data at hand which would tell us just the size of the motor we should adopt.

We went to the motor manufacturing companies, in a hurried way, and got their ideas, and those we put with what data we had, it was very little, largely being from steam-driven mills. One man would say—"You can do that" and would show us where we could do the work with a 2000-h.p. motor. Another man would then come along and undertake to show us that it would take a 3000-h.p. motor to do the same work. Then still another would come along, and he would say—"It will take a 4000-h. p. motor to do that work" There must be something lacking in the engineering departments of the manufacturing companies, when they figure on this class of work.

Probably a great deal of the trouble is due to the fact that they do not take into consideration the material you have to roll, the temperatures, etc.

To be safe, we decided we would build a mill that would roll 15,000 tons a month. That was settled on, that was what they wanted. You see, when they made up their minds, that that was what they wanted to roll, we had to roll it. We selected a motor of 4000-h.p. The mill has been operating for about four months, with an output of about 15,000 tons, and then they increased the output of the mill about 25 per cent, so you can see where we would have been if we had taken the 2000-h.p. motor.

I think we should rate these motors liberally, so you can always be safe in doing just about 50 per cent more than you ever expected to, because you will be called on some time to do that extra work. You cannot replace the motor, but you can replace the mill. You can tear out the foundations and go ahead and build in a larger foundation. If you break a couple of brackets made of cast iron, just turn around and make them out of steel. So you see, I think, when you start on the power end, you do not make a mistake in any way, shape or form by putting on plenty of capacity, because you will need it. If four furnaces do not do the work and keep the mill operating fully, build another furnace, so it is strictly up to the driving unit, because that mill is sure to operate, and turn out steel, and if you try to save on the motor you are trying to save on the wrong end.

N. W. Storer: I believe it is wise to put in motors amply large for the work, especially if it is to go into steel mill work

where so little reliable calculation can be made as to requirements.

There was one point in the remarks of the gentleman who has just spoken, Mr. Menk, in regard to the 4000-h. p. power motor, that I desire to refer to. He says that they increased the output 25 per cent, which is undoubted, but the point is, how much of a load is the motor taking now with that increase? It might only have been taking 2000 h. p. previously, and is now getting that 25 per cent more. It would be interesting to know just how much power is taken by the motor and what factor of safety remains.

C. A. Menk: In the steel mill it is a question of doing the work. In a plate mill you heat the slab and bring the slab up to the mill. The first time the slab goes through the mill is when you get big load, and from that time on the peaks go down. At the first big load, or maybe the second pass, or the third pass, the motor may run up to 4500 h. p., but by the time the slab is finished it runs down to 2000, and that is where the large motor comes in, with a good big liberal figure, to allow for these variations.

Wilfred Sykes: I wish to take issue with the author on the same line as some of the other speakers. First, on the general classification he says: "In any rolling-mill electrification the paramount problem is the delivery of a specified tonnage of steel, of which the grade and section are known, at a minimum total cost." I have yet to meet the case where we really know the requirements. We always start off with certain ideas as to what is desirable to roll, and these ideas, as a rule, do not match up at all with what we actually get when the installation is made.

Now, the basis of calculating these things is all right. You have got to have a start somewhere, and for many years I have been conducting experiments and tests to determine a basis for calculations of this kind. We started in about nine years ago, and we are still testing and still have a great deal to learn.

After all, these tests give you certain unit values, but you have to use judgment in applying these unit values to your problem. I do not believe it makes a great deal of difference, what particular method of calculation you use, so long as you talk in the same terms all the time.

Now, this question of using logarithmic elongation and all that sort of thing has been discussed quite a lot amongst steel engineers, and I have not yet been able to see that it made any particular difference, so long as you plot your tests in the units that you use.

I do not believe anybody who has been through this business would attempt to use the data gotten from, say, a blooming mill and apply it to a rail mill. The only way I ever found that I could get any results that were at all consistent with actual operating figures was to get tests on a blooming mill, if I was dealing with a blooming mill, and not get tests on some

other mill, such as a plate mill or rail mill, and use them as a basis for my calculations on some other mill operation.

Mr. Menk brought out one point very well, and I would like to enlarge upon it a little bit. We start off with some ideas as to what the demand is to be. Take the case of this plate mill, we want to roll 15,000 tons a month. That 15,000 tons a month is absolutely nominal, and not a real figure. What they will roll is all they can put through the mill, if they have the business, and if we are to get anywhere, and successfully apply motors to mill drive, we must study not only the figures given us by the superintendent of the mill, but all other data that are available. The difficulty has been, however, to get steel mill people to put these things down on paper. You can get all kinds of statements based on some plant and the number of tons of steel they roll. We are putting in a better mill than they have and we should get much more, but to get that down to a time study, or something of that kind, is extremely difficult. The electrical engineer has been forced to come in and study this problem.

The only use all this calculation would be is as a guide to judgment. These conditions in the steel mills are not subject to definite schedules, etc., as outlined in a paper of this kind. Take the blooming mill, there the passes are not fixed, you are in the hands of the operator, and the operator can run the mill pretty well any way he wants to, and usually it differs from some other mill, being possibly an advance in the art of mill building—and then you have to face new conditions, and your success depends on the value of your judgment. We find sometimes that judgment in these cases is on a par with a pure guess, but the thing you are up against is they will put through, if they have the business, everything that mill can handle. If the limit of it is in some mechanical part of the mill, that will be strengthened and eventually it will be put back on the motor or engine.

In the development of engines the work has been empirical. They find a certain blooming mill had a 48 in. by 56 in. engine, and the mill stood up, but the engine let go. Next they put in a 55 in. by 60 in. engine, and when they get to a point when everything lets go simultaneously, and the parts cannot be strengthened up any more, then the engine was all right.

That has been going on ever since we began rolling steel, and the mill engineers in this country have reached a point where, if they have a blooming mill, they can pretty well tell what kind of engine it should be, without any calculation at all. They know with a certain size blooming mill it takes a certain size engine to push the metal through. As the art progressed they made the parts larger, and got to a point where the engine would hold together. That, however, is not absolutely true, because in the case of some of the largest engines which were put in, it came to a point where the crank shafts could not be made larger on account of forging facilities.

When we came to motors, starting in that subject anew, we had to go through more or less of that same experience. If we are good engineers, we try to gather all the data we can and utilize the information given us by the steel mills, and are enabled thereby to form a fairly good judgment as to what the equipment should be.

But the point I want to bring out is that the calculations are only the first step. These figures must not be accepted at their face value. The engineer who believes in figures will not get very far. They are a guide to judgment only, and experience must be gathered in different plants to determine the actual size of the motor to be used, and in a case such as Mr. Menk mentioned, where the capacity increased 25 per cent, I am willing to predict when the plant has run another year, it will be another 25 per cent, and it may be five years from now the motor will be the limiting feature. This is our normal experience, and if for instance the tables are slow they will be speeded up and they will change all the small parts, and eventually get to a point where the mill will break or the motor will reach its limit.

That is the condition met with in steel mills.

There is one other condition we meet, and that is when we change over from steam to electric drive we introduce different characteristics in the operation of the mill, which people find out and utilize, so if the motor has more favorable characteristics, which make a larger output possible, they will forget what the engine did and establish a new standard for the motor.

The question of rating has caused a good deal of confusion in the past. We started out, frankly not knowing anything about the subject, and rated the motors 35 deg. continuous, and 25 per cent overload continuous, with 50 deg. temperature rise, and I think what we want to do is to stop fooling ourselves on the question of ratings. We have gained experience in the meantime, and we ought to face the fact that if it takes 1250 h. p. to drive the mill, we ought to have a 1250-h. p. motor to drive it, and not call it 1000-h. p., because it is easier to get through the purchase of a 1000-h. p. motor, because it does not sound so much. We ought to face the situation and try to arrive at what is the right equipment, taking into consideration not only the problem as presented by the builder at the time, but also what it is possible to put through the mill, because it is the business that will actually fix the character of the equipment sooner or later, and not the nominal figures that are so common in many calculations of this kind.

Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.

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SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

BY J. D. WRIGHT

ABSTRACT OF PAPER

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

USUALLY the first questions which one is likely to ask when considering the proper type of motor and control for any steel mill auxiliary machine, are, "What is its function; what is the mechanical layout, and how does the device operate?" Without a thorough knowledge of the answers to these questions one is unable to make an intelligent recommendation on the application of either motors or control.

When discussing Standardization of motors and control at a meeting of the A. I. and S. E. E. in Pittsburgh about a year ago, Mr. Friedlander, of the Carnegie Steel Co. said, "What we need is more papers descriptive of installations and operation of our mills. In such a way we would gradually get uniform ideas about general requirements at all our mills and thereby drift toward a stable system of standardization." The PROCEEDINGS of the A. I. E. E. and A. I. and S. E. E. contain very few papers describing installations and operation of steel mill auxiliary apparatus and the writer heartily agrees with Mr. Friedlander's remarks regarding the need of such descriptive articles.

It is, therefore, the purpose of this paper briefly to describe a few steel mill auxiliary machines with particular reference to their mechanical details.

Manipulators for Blooming Mills

Manipulators for blooming mills consist of side guards and lifting fingers. The function of the former is to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over.

A reversing mill is usually provided with four moving side guards two on each side of the mill, one guard on the side of the mill with the operators pulpit being provided with the lifting

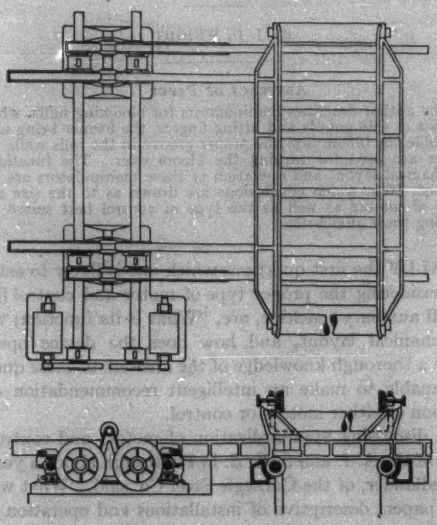
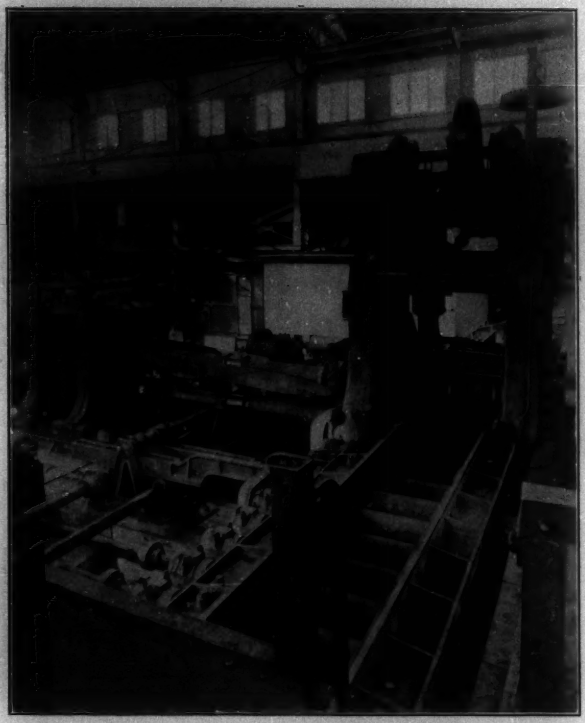


Fig. 2—Operating Mechanism for Manipulator Side Guards

fingers. Occasionally a set of lifting fingers is provided on each side of the mill. but as the bloom is usually turned only on the pulpit side, fingers are more commonly provided only on that side.

Fig. 1 is a general view of the entering side of a reversing blooming mill equipped with electrically operated manipulators. The two side guards are clearly shown and, on close inspection, the lifting fingers may be seen just inside the left hand guard.

Each side guard on the entering side of the mill is connected



[WRIGHT]

FIG. 1—BLOOMING MILL WITH ELECTRICALLY OPERATED MANIPULATORS

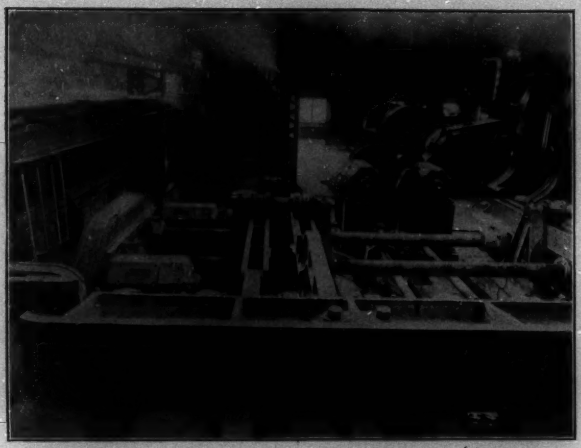
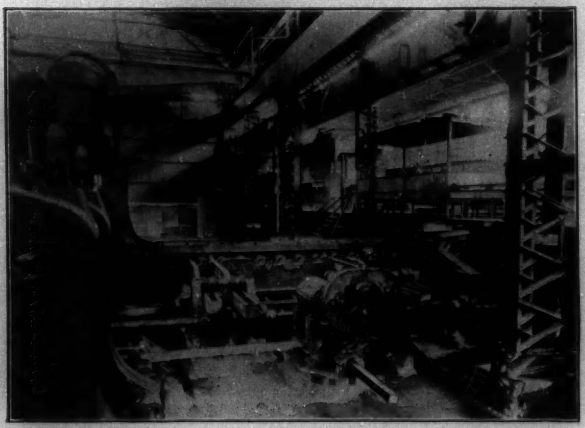


FIG. 3—GENERAL VIEW OF OPERATING MECHANISM FOR MANIPULATOR FINGERS AND SIDE GUARDS



[WRIGHT]

FIG. 5—VIEW OF BLOOMING MILL WITH ELECTRICALLY OPERATED MANIPULATOR FINGERS AND SIDE GUARDS

by suitable mechanical means to the corresponding guard on the delivery side, so that the right hand and left hand guards on opposite sides of the mill always have the same movement and are always in line.

Fig. 2 shows one method of mechanical connection between the motors and the side guards. Two motors, one on each side of the mill, are geared through slip clutches to a common shaft on which are mounted four pinions which engage with four racks. Two of the racks are attached to one guard on the entering side and the other two to the corresponding guard on the delivery side of the mill.

The remaining two guards on opposite sides of the mill are similarly operated by a second pair of motors. The racks are supported by rollers and mechanical stops are provided to limit their outward movement.

Fig. 3 is a view of the same mill shown in Fig. 1 but gives a more detailed view of the side guard mechanism.

As previously stated, the function of the side guards is to guide the bloom into the proper groove in the rolls. The operation of the guards during the process of rolling an ingot consists, therefore, of many fast, short movements back and forth across the table. Each guard on one side of the mill can move independently of the other and can make a stroke equal to the width of the table. It is possible therefore for the guards when moving inward to come together or against the ingot at full speed. It is the function of the slip clutch to prevent damage as a result of such action should the operator be careless enough to allow it to occur.

The motor equipment for the side guards illustrated in Figs. 1 and 3 consists of four 80-h. p. series motors, two connected in series driving each pair of guards. Satisfactory operation of the guards requires rapid acceleration and retardation, and for such service series motors are most desirable. Some installations, however, use compound wound motors, primarily because dynamic braking for stopping was desired. Furthermore, solenoid brakes are sometimes used to assist the dynamic braking. In the equipment illustrated, neither dynamic braking nor solenoid brakes are used. When a quick stop is desired the motors are plugged. The equipment is therefore extremely simple.

The control for each pair of motors is of course reversible and is arranged so that a bank of resistance is permanently connected in series with the motor armatures. This acts as a buffer and

reduces the shock when the guards jam together or against the ingot. A one-point reversible master controller is used for each pair of motors. Limit switches are provided for cutting off power when the side guards are moved to the edge of the table. These limit switches are usually of the track type and are operated by movement of the rack. A geared switch might be used but the track type switch is much simpler and more positive in action.

The lifting fingers, which are used for turning the bloom over, are carried by a frame attached to the two inside racks connected to one of the side guards. They therefore move back and forth with the guard. See Fig. 4. The fingers are located just inside the guard and their lower ends are pinned to a yoke which

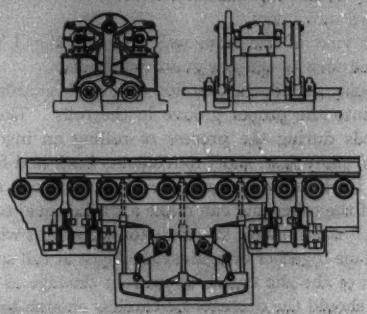


FIG. 4—OPERATING MECHANISM FOR MANIPULATOR FINGERS

is suspended from two horizontal shafts by means of two links and levers. The two horizontal shafts, which are supported by bearings in the finger frame, are connected through cranks to a common driving shaft. On this shaft is mounted a large gear to which the two driving motors are geared. It is evident that one-half revolution of the main driving shaft in either direction from the position shown in Fig. 4 will produce a complete up stroke of the lifting fingers. Another one-half revolution would produce a complete down stroke and return the fingers to their original position.

The motors may therefore be either reversible or non-reversible. Successful installations of each have been made. When a non-reversing equipment is used, one complete cycle

consists of an up and down stroke of the fingers. When a reversing equipment is used forward direction of rotation may produce the up stroke and reverse rotation the down stroke.

The motor equipment illustrated in Figs. 3 and 5 consists of two 50-h. p. series motors connected in series, operating non-reversing. For each throw of the operator's master controller a complete up and down stroke of the fingers is produced. The motors are automatically stopped at the end of the down stroke by a limit switch which cuts off power and completes a dynamic braking circuit.

The dynamic braking connections for a series motor operating non-reversing are comparatively simple. Where reversible control is used, a compound wound motor is preferable if dynamic braking is desired, as the control connections are simplified. Solenoid brakes are required if the equipment is reversible, whereas if non-reversible, no brakes are necessary. It has at times been stated that the operator should be able to control the stroke of the fingers. This of course necessitates the use of reversible control, but the movement of the fingers is so rapid that it very frequently happens that the fingers make a complete up stroke before the operator throws his master controller to the reverse position to lower them.

It has been shown that for the side guards shown diagrammatically in Fig. 2 a reversible equipment must always be provided. The motors may, however, be either series or compound wound. Dynamic braking and solenoid brakes may or may not be used. The operator's master controller may be one-point reversible or multi-point reversible. Track-type limit switches or geared limit switches to limit the movement of the side guards may be used.

For the lifting fingers in Fig. 4 it has been shown that the equipment may be either reversible or non-reversible. The motors may be either series or compound wound. Dynamic braking and solenoid brakes may or may not be used. The operators master controller may be one-point or multi-point reversible or non-reversible. The limit switch may be arranged to stop the motors when the fingers are at the end of their down stroke if the control is non-reversible, and if reversible the limit switch may be used to stop the motors with the fingers in both up and down positions.

There must, however, be one method of operating these devices which is superior to all others. What this method may be can

be determined only from actual operating experience, and it is to the steel mill electrical engineers that we must look for the results of such experience The writer sincerely hopes that Mr. Friedlander's suggestion may be followed and that we may have more papers on installations and operation of auxiliary drives.

DISCUSSION ON "SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE" (WRIGHT), CLEVELAND, OHIO, MARCH 8, 1918.

E. Friedlaender: I prefer using two standard mill-type motors, connected in series for quick reversing work, in place of one special low-speed motor which is hard to get during these times and also quite expensive.

To be able to raise and lower these heavy lifting tables quickly and accurately, it is important that they are properly counterbalanced. Our table operators demand that the controller allows them to stop and reverse machine at will in any position; this in order to gain time and avoid wrecks.

C. A. Menk: I was very much interested in Mr. Wright's presentation especially where he showed all the mechanism, with a 150-horse power motor operating the side guards and the lifting arrangement. It all looked to me as if they were going to extremes.

I spoke this afternoon on large mill motors. We have that very same mill equipped and doing that same work, and we have practically the same problems, the tilting, etc., and we do it all hydraulically, using one 40-h. p. motor, and getting all the service we want out of the mill. It looks to me as if in installing all of that lifting mechanism, we are going to extremes in what we are trying to accomplish. I may be wrong, but I would imagine the expense of upkeep would be considerable.

As to the tilting arrangement for unloading the tables, I cannot see any particular advantage in that, especially in connection with the plate, for in turning out plate you are probably rolling on a thousand different orders a week. Now, you have got to sort that product out, so you can make shipment in carload lots and you must pile it up all over the floor until you get a carload. The mill operator may have nine different places where he is going to handle that table. He would have to start right away and assemble and sort out right from those different piles.

I cannot just see the possibility of using nine different places to assemble these plates and get them ready for shipment. It may be done where you roll one class of product for cars, or in case of ship plate, to continually roll that one class of material. But I see every day, one plate rolled for a ship and the next day one rolled for a boiler, and the next plate rolled for a car, and so on. You look over the shop floor and you find it covered continually with small piles of plates, and some of it would be there for a week before you can ship it, and some longer than that. So it seems to me the unloading arrangement might be all right, in some cases, but for a mill where you expect to cater to the trade and furnish everything you get an order for, it would be almost impossible.

Also, in regard to the tilting table, I cannot see why they want to lift that whole table from the one end. There is no reason

why that table, as far as I know, cannot be balanced in the middle and could balanced with a very small amount of power. Some of the slides showed the double hinge at one end, if I am not mistaken, the motor at the other end, practically lifting half of the table, whereas if it was balanced in the middle, all you would need to do would be to over balance it to get the desired results.

There are many of these things which I believe can be brought out by a very careful study, and especially in the elimination of excessive mechanism in order to try to accomplish something that you can do with a very small amount of power, even if you have to go back to the motor-driven hydraulic pump.

T. E. Tynes: I want to bear out what Mr. Friedlaender mentioned about standard motors in series rather than one special motor of the same speed. I had occasion to look into a problem that required a 200-h. p. motor to drive a table. We found by using two standard-speed motors connected in series that the cost of the two was little more than the cost of the special motor, and we had the benefit of two standard motors in series, for which we had all the necessary spare parts, and were not obliged to buy spares for that drive.

H. S. Richardson: I was interested in Mr. Wright's citation of the case of the controller with the manipulator sideguards. In most of the cases I know of, brakes have been used, as there is a great deal of momentum in these side guards, and it seems to me that if brakes are omitted it means that every motion of the side guards has got to be stopped by plugging. That is perfectly safe and is used a great deal, but on account of the very frequent and short moves of these side guards it is putting on the operator additional work that can be eliminated by putting on larger brakes. It does not necessarily complicate the mechanism, except by the addition of the brakes themselves which, of course, are series wound and are set electrically.

Mr. Wright also mentions in his paper a permanent resistance in the side guard motors, which act as a buffer. It seems to me it is preferable to get that result by the use of a torque-limit, or jam relay, unless the mill is laid out and the motor gearing designed with that idea in view. I know of one case where the permanent resistance did not work satisfactorily, because they could not get the torque and the speed. The torque-limit relay, simply as an over-load relay, inserts the resistance permanently in the circuit when the torque exceeds a certain amount.

With regard to the manipulator fingers, Mr. Wright made mention of the fact that they could be either non-reversing or reversing. In the case where they are reversing, the up motion is secured in one direction—the up motion of the fingers—and the down motion in the opposite direction to the motor. It is a very simple matter to get the up and down motion in each direction of the motor, by simply allowing the crank to make a com-

plete return for one up and down motion, and the next cycle is in the opposite direction. In a few cases that has simplified the equipment from the operator's standpoint and it is not at all necessary to use a brake for that reversing operation. In fact, it is much more satisfactory without the brake, providing the finger mechanism is not counter-balanced, but has the natural position of the fingers down, and simply make one complete turn in one direction, and the next turn in the other direction.

Another point in connection with the fingers. There is scarcely any installation, I believe, where it is not sometimes necessary to reverse the finger motor. Very frequently the fingers stick in their bearings, sometimes from expansion by heat or because of shale, and it is desirable to be able to reverse these fingers.

I think the problems that have confronted electrical engineers on the manipulator fingers and side guards, at least in the earlier designs, were made considerably harder by the fact that so little gear reduction has been put between the motor and the load. I know of one case where the travel of the side guard is 72 inches, and the motor made 6.7 revolutions for that complete travel. Also, one of the hardest problems on the side guards is the stopping of the motor, because it is, of course, necessary to clear the pass, and there is generally less than an inch or two, sometimes only a fraction of an inch, clearance between the position where the side guard clears the pass and the final positive stop

If the mechanical engineers would only appreciate these points and give us a little more clearance, it would simplify matters very much. It is a hard thing to get the extra clearance on the side guards, particularly if the side guards have to extend inside the mill housing. I have met a number of cases where, if the electrical engineer had only pointed out to the mechanical engineer what the problem was going to be, he could have very easily had an extra clearance.

B. G. Beck: I would like to ask Mr. Menk if he has charge of all of the hydraulic machinery in the winter time? I have charge of that, and it is quite a job. I am a great believer in the idea that if a thing can be done at all it can be done electrically better than any other way.

G. E. Stoltz: In reference to the high-speed and low-speed motor, for the various mill auxiliaries—the mill-type motor, as applied, with small diameter and a low inertia, does not leave as much to be gained by going to the low-speed motor as would be expected. In fact not more than 10 or 15 per cent is gained in the current consumed to perform the same operation by installing a special low-speed motor, which is not a duplicate of other motors in the mill, and which, naturally, is harder to handle in case of repairs.

Mr. Wright has mentioned several cases where motors may be used in either parallel or series. I am sorry he did not give

the results attained in using the motors according to the two different schemes. It is simply a matter of whether the application is such that the inertia of the motor is a large percentage of the work performed. On a screw-down, the inertia of the motor is a large part, and in order to reduce that part of the load, the motors are placed in series to reduce their speed to one-half. On drives such as main tables or heavy side-guards, this is not the case, and it would be better to place the motors in parallel, if it is desired to obtain the highest rates of acceleration and operation. However, if no effort is made to obtain this quick acceleration, it is naturally easier on the motors to have them placed in series.

I have in mind a case where an effort was made to bring a large mill table up to speed in one and one-quarter seconds, by placing the motors in parallel. A speed of 400 ft. per min. on the main table of a 40-in. mill was obtained in this time with two mill-type motors in parallel. On the side guards a speed of 200 ft. per min. was obtained in one second. We should not overlook the fact that on many applications, the inertia of the motor is rather a small part of the load and a much faster mill will be obtained by placing the motors in parallel.

The question has been brought up as to whether brakes should be placed on side guards. Most of these applications of side guards and screw-downs are inching operations, where it is important to have a quick and accurate stop, particularly when a man is using the side-guard to line a piece up with some particular groove in the mill. This can be obtained to a much greater degree of accuracy by having compound motors and dynamic braking. I think most of these motors are almost always brought to rest before the brake sets, although it is an advantage to have a brake take effect just about the time the motor stops, when the dynamic braking is becoming very weak, so that probably the last operation is obtained by a combination of dynamic braking and shoe brakes.

Another man has mentioned the fact that a jamming relay will be of some advantage on side-guard motion. The operation of a side-guard is so quick that a jamming relay will not have time to act. A better plan is to place permanent resistance in series with the motors, to limit the current to approximately 200 per cent, with the motor at stand-still. With this amount of resistance, the speed of the motor is reduced so that in laying out the mill this should be taken into account.

A number of gentlemen have asked whether or not it is feasible to use reversible control on the fingers. Ordinarily it is not required, but as the last speaker mentioned, very often operators have trouble with the fingers, in which case they want to reverse the fingers, and for that reason reversing control is usually applied. I know of one mill where the operator became so expert in manipulating the fingers that he simply throws on the switch to the forward position, allows the fingers to turn

partly over, and turns them back quickly, without allowing them to make a complete cycle.

J. D. Wright: Regarding the use of jamming relays on side guards, I believe the installation I showed was equipped with a jamming relay to start with, but the electrical engineer in that particular plant found that unsatisfactory, and put a permanent resistance in series with the motor armature, cutting out the jamming relay, and more satisfactory operation was obtained.

The question of gearing is a very important one which I think electrical engineers overlook. In one particular instance I know of some side guards that were geared for a running speed of 600 ft. per min., which is ten ft. per sec. Now, the total travel of the guard is not more in ordinary cases than 6 ft., so you can see how absurd it is to arrange your gearing for such a high speed as that.

I think what the electrical men ought to do is to get together with the mechanical men more and study these problems from the mechanical standpoint, and in that way the mechanical men can get the benefit of the electrical men's experience in working out the problems satisfactorily.

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INDUCTIVE EFFECTS OF ALTERNATING-CURRENT RAILROADS ON COMMUNICATION CIRCUITS

BY H. S. WARREN

ABSTRACT OF PAPER

A brief discussion of inductive interference in general is first given, including reference to the work of the Joint Committee on Inductive Interference in California. Inductive interference due to electrified railroads is then taken up and various possible means for reducing such interference considered. A description is given of four important installations of railroad electrification and the specific means adopted in each case for preventing interference, with the degree of success which has been met with.

INTRODUCTION

TELEPHONE and telegraph companies in the conduct of their business not only have to maintain their lines and service against ordinary forms of interruption, such as lightning disturbances, mechanical failures, etc., to which all overhead electrical lines are inherently subject, but also, they have to see to it that their lines are protected against interference by electric power lines. Such interference may be due to actual or threatened physical contact between wires of the two systems, to passage of current from one system to the other by leakage, or to the class of disturbances known as "inductive."

One important kind of inductive disturbances on telephone and telegraph lines is that arising from installations of alternating-current railroad electrification, such installations being principally employed in connection with trunk line railroads carrying heavy traffic.

In approaching the subject of interference to communication systems by such electrified railroads, it seems desirable first to consider some aspects of the general subject of inductive interference from power lines, of which interference from electrified railroads is a special case. In this general discussion of the subject it will be convenient to borrow freely from the work of the Joint Committee on Inductive Interference in California.

Historical

The induction of voltages in an electric circuit by current changes in a parallel circuit was discovered by Faraday in 1831. The property of self induction of an electric circuit was discovered by Joseph Henry at about the same time. The phenomenon of induced electrostatic charges was known already through various experiments. In 1838 Henry, in the course of his researches, observed that a current was induced in an electric circuit when a Leyden jar was discharged through a parallel circuit.. This seems to be the first case on record of electric induction between circuits.

Since the time of Faraday and Henry a stupendous amount of electrical research and experimentation has been conducted and as a result of this and of brilliant theoretical work by Maxwell and others, the fundamental laws of electrostatics and electrodynamics has been very fully worked out. The general equations expressing the laws of induction are, however, not suitable for us in the solution of practical problems. On account of the large number of factors involved, many of which cannot readily be evaluated, it is generally necessary in specific cases to resort to simplifying assumptions and approximations.

As to what actually happens when a voltage is set up in an electric circuit by induction we know very little. It is well to bear in mind that we do not even know what voltage really is, or current, or electricity. We use these terms merely to express certain phenomena and relations found by observation, whereby we are able to derive results of much practical value.

Characteristics Affecting Disturbances. Telephonic currents consist of numerous component simple currents varying in frequency from about 200 to 4000 or more periods per second. The signaling currents employed on telegraph lines are also complex, but their most important components are of less than 300 periods per second.

Power circuits are commonly of either 25 or 60 periods per second but, in addition to the fundamental periodicity, harmonics are usually present to a greater or less extent and it is due almost wholly to the harmonics which come within the range of the most important voice wave components, that noise is produced in telephone circuits by induction. By care in designing electric power machinery so as to reduce the proportion of such harmonics, it is probably feasible to avoid much of the noise disturbance to telephone circuits which otherwise would result.

Thus a small expense in eliminating this trouble at its source may obviate a much larger expense later.

Abnormal Conditions. It is important to recognize that the inductive effects of power circuits are liable to be greatly magnified at times of abnormal conditions. The violent changes which occur in the electric and magnetic fields surrounding a power circuit, when one of its conductors breaks or becomes grounded, set up in neighboring communication circuits surges which may represent relatively large amounts of power. A parallel which, under normal operating conditions, causes no disturbance may produce very serious interference under abnormal conditions.

Balanced and Residual Voltages and Currents. In analyzing inductive phenomena it is advantageous to classify power circuit voltages and currents under two general heads: (1) Balanced voltages and currents, that is, those which are balanced or symmetrical with reference to the earth; (2) residual voltages and currents, that is, those which are wholly unbalanced with reference to the earth. The circuit of the residual voltages and currents is comprised of the metallic circuit conductors as a group, constituting one side, and the earth (including earthed conductors), constituting the other side.

At every instant the algebraic sum of the balanced currents, and likewise of the balanced voltages, is zero. Hence, at any instant the algebraic sum of the currents in the line conductors is the residual current and, similarly, the algebraic sum of the voltages to earth of the line conductors is the residual voltage. For example, in an ordinary railway circuit, consisting of overhead trolley wire and grounded rail return, all the voltage and current are residual.

There is no definite relation between these two classes of voltages and currents and entirely different means usually have to be taken in counteracting their respective induction effects. In general, the residuals cause more inductive disturbance than the balanced components as the residuals are all in phase and their effects are cumulative whereas the induction from the balanced voltages and currents in one conductor is partially neutralized by the induction from balanced components in each other conductor of the circuit. Also the residuals usually contain a larger proportion of harmonics than the balanced components and this is another reason why they are likely to cause more disturbance, particularly to telephone circuits.

Causes and Preventive Measures for Residuals. As residual

voltages and currents are largely responsible for inductive interference, it is of great importance in the prevention of such interference that they be suppressed or reduced by all reasonable means available.

Star-connected, three-phase transformer banks with the neutral grounded will set up triple harmonic residuals due to variation of permeability of the iron with varying magnetic density and may also cause residuals by reason of inequalities of the transformer impedances. In general, the most effective measure against this triple harmonic effect is the use of a delta-connected secondary or tertiary winding, thus providing a shunt path for the triple harmonic currents. The magnetic density also should be kept as low as practicable. Residuals due to inequalities of impedances in transformer banks can, of course, be eliminated by equalizing these impedances.

Another condition which may produce large residuals is the use of generators with star-connected armature windings. When such a generator with its neutral grounded is connected to a power line, either directly or by auto-transformers, residuals are set up in the line. When such a generator, with its neutral not grounded, is connected to the line through standard transformers, residuals will be impressed on the line if the transformer bank is connected star-star, the line side neutral grounded, and the station side neutral connected to the generator neutral.

Grounding of transformer or generator windings at any points not normally at zero potential, unbalances an electrically connected circuit and thereby causes residual voltages and currents.

Another important cause of residuals, which however may not be so obvious, is the unbalance of a power circuit due to inquality of the capacitances to ground of its several conductors. If the three line conductors of a three-phase circuit are carried throughout in the same relative positions, that is, if the circuit is not transposed, the capacitances to ground will be unequal and a part of what would otherwise be balanced voltage and current becomes residual. These inequalities may be overcome by transposing the power circuits throughout their entire length. To make such transposition effective with respect to interference to telephone circuits it is necessary that the power circuits be transposed at intervals which are short in comparison with the wave lengths of the higher harmonics present. The frequency of transpositions depends somewhat, however, on the inherent unbalance of the conductor configuration. As an indication of

the number of transpositions required for a reasonable degree of balance to ground, it may be said that barrels of 6 to 12 miles, the latter applying to a triangular configuration, are usually adequate.

It is evident that the symmetry of a line which has been thoroughly transposed may be destroyed by connecting to it branches or taps which unbalance the capacitances to earth of the line conductors. Of course, if such a tap is grounded, the residuals resulting may be very large.

At times of accident, when a power circuit is in an abnormal condition, residuals of relatively enormous values are liable to be created. These set up correspondingly large induced voltages in parallel communication circuits. This emphasizes the importance of high grade construction and maintenance of power lines involved in parallels so as to minimize the frequency of such occurrences.

Unbalance of Communication Circuits. Not only are there these two kinds of disturbing voltages and currents on power circuits, namely, balanced and residual, but each of these components may set up, in a neighboring metallic communication circuit, two different effects; (1) an induced voltage between the two conductors of the communication circuit, which directly tends to cause currents through the signaling instruments; (2) an induced voltage between the conductors of the communication circuit and ground, which by reason of unbalances in the communication circuit indirectly causes currents through the signaling instruments. Theoretically, assuming a telephone circuit and all its connected apparatus absolutely symmetrical, electrically, with respect to earth and always so maintained, voltages induced equally in the two sides of such a circuit would not cause noise in the telephone. In practise it is not possible to attain absolute symmetry although in well constructed and well maintained telephone circuits the degree of balance is very high indeed. The telephone is, however, such a very sensitive instrument that no attainable degree of balance can avoid noise when relatively very high voltages are induced between the telephone wires and ground, as is done in many cases by parallel power circuits. It is therefore essential that induced voltages to ground be limited to values which are permissible on communication circuits so maintained.

Transpositions Within Parallels. Interference by induction from balanced currents and voltages can most readily be pre-

vented by means of a coordinated system of transpositions applied to both power and communication circuits within the limits of parallels, the term "parallel" being understood to mean the region within which the two classes of line are in sufficiently close proximity for inductive disturbances to be set up in the communication circuits by the power circuits. It is to be noted that transpositions for this purpose to be applied to power circuits within the limits of parallels, are quite distinct from the transpositions previously referred to as being necessary throughout the entire length of a power circuit in order to equalize the capacitances to ground of the several conductors.

Principal Factors in Determining Interference. Before leaving this general part of the subject I will enumerate the principal factors which determine the amount of induction and whether it is sufficient to constitute interference.

1. *The length of the parallel.*

Other things being equal, the longer the parallel, the greater the induced voltage.

2. *The separation of the two classes of lines.*

In general, other things being equal, the less the separation of the power line and communication line, the greater the induced voltage.

3. *Configuration of the power line.*

The investigations of the Joint Committee on Inductive Interference show that the configuration of the power line has an important bearing on inductive effects, the relative merits of different configurations varying with the separation of the power and communication lines, the spacing of the power conductors, and the relative importance of balanced voltages and currents. While it is not possible to draw a simple general rule for determining the most advantageous configuration the differences in particular cases are marked and deserve special attention as oftentimes substantial benefit can be secured in this way at small additional cost. This is particularly true of multiple circuit lines, the resultant induction depending largely on the relative poling of the power circuits.

4. *The magnitudes and fundamental frequency of the normal operating voltages and currents of the power circuits.*

The effect of electric induction, of course, is proportional to the voltage of the power line, and of magnetic induction to the current on the power circuit.

5. *The magnitudes of residual voltages and currents.*

It has already been explained that residual voltages and currents are a principal cause of inductive interference. Hence, while the amount of residuals on metallic circuits is usually small as compared to the balanced components, the inductive effects of the former are liable to preponderate.

6. *The wave shapes of both balanced and residual voltages and currents, involving the magnitudes and frequencies of all harmonics.*

The effect of wave shape on interference, to telephone circuits particularly, is exceedingly important. Wave shapes in practise on different power systems are found to have extremely wide variations. An unfavorable wave shape, *i. e.*, one having a large proportion of high harmonics, may produce a hundred times as much noise as a pure sinusoidal wave of fundamental frequency.

7. *The unbalances of the communication circuits, their magnitude, character and location.*

Such unbalances are caused by inequalities in resistance, inductance, insulation or capacitance to ground. The last mentioned quantity is balanced approximately by transposing the conductors. The other elements enumerated require proper design, construction and maintenance of these lines, whether in open wire or in cable, together with their connected apparatus.

8. *Terminal apparatus of the communication circuits and the distance of such apparatus from the parallel.*

The sensitiveness of the terminal apparatus is, of course, an important factor in determining the allowable amount of induced voltage. Also if the parallel is at a considerable distance from either terminal of the communication circuit, the induced voltages and currents may become considerably attenuated before reaching the receiving instruments.

9. *The voltages and currents of the power circuits under abnormal conditions.*

It has already been stated that the voltages and currents of power circuits under abnormal conditions, which are liable to be largely residual in character, produce the most severe inductive effects. The values of these quantities under abnormal conditions, in relation to corresponding values under normal conditions, vary a great deal on different power systems.

10. *The number of parallels which may effect cumulatively the same communication circuit.*

In many cases the same communication circuit, especially if

it be on a long trunk line, may be involved in a considerable number of different parallels. In such cases the induction contributed by each parallel must be sufficiently restricted so that the cumulative results from all will not produce disturbances which cannot be endured.

11. *The importance and character of use of the communication circuit.*

It is obvious that the more important circuits, on which interference is most serious, should be afforded a higher degree of immunity from disturbance than circuits of less importance. Also, of course, the character of the communication circuit as, for example, whether it is a telephone or telegraph circuit, is of fundamental importance in considering the question of inductive interference.

12. *The volume of transmission on the communication circuit.*

In case of a long distance telephone circuit, where the volume of transmission is small, a less amount of extraneously induced current will interfere with receiving than on circuits of less length where there is a large volume of transmission.

13. *The relative cost of preventing interference.*

While in all cases means should be employed which will allow adequate communication service to be given, still it is not expected that complete freedom from inductive disturbances can be attained. Any induced voltage, no matter how small, will generally cause some impairment of service. The amount of induced voltage which it is justifiable to allow, depends to some extent on the difficulty and expense involved in further reducing such voltage. After the foreign voltage has been reduced to an amount which can, if necessary, be tolerated, it becomes simply a problem of balancing the value of further improvement against its cost.

It will be seen that the number of elements affecting inductive interference is quite large. Moreover, some of these elements, as for example, the wave shapes of the power circuit voltages and currents, are not ordinarily known, and have induction-producing values varying enormously in different cases. Hence the difficulty of formulating any simple method of determining in advance whether a given construction will or will not produce interference.

The foregoing discussion, while general in its application, is in many respects concerned with induction from power transmission lines. We will next consider specifically some of the inductive effects of alternating-current railroad installations.

Alternating-Current Railroad Electrification

The reasons why alternating-current railroad electrifications cause large disturbances to neighboring communication lines, principally by electromagnetic induction, will now, I think, be apparent when it is considered that, (1) the railroad trolley current is large, (2) it is all residual current, (3) the railroad circuit, from its nature and use, is more subject to abnormal conditions, such as short circuits, than ordinary power transmission lines.

Classes of Interference. Some of the different ways in which disturbances due to alternating-current electrified railroads manifest themselves in the telephone and telegraph plant, may be classified as follows:

1. *Interference with operation.*
   a. Interruption of service
   b. False bell ringing
   c. Noise
   d. Interference with telegraph signals
2. *Physical injury to plant.*
   a. Fire hazard
   b. Magnetization of loading coils
3. *Hazard to employees and to telephone using public.*
   a. Electric shock
   b. Acoustic shock

These various disturbances may be of a most serious nature and telephone and telegraph companies are unable by themselves to cope with the problem of protecting their lines and service against them. In order to make this more clear, we will review briefly some of the fundamental characteristics of telephone service and point out some of the distinctive features of the plant required to make this service possible.

The Telephone System. The fundamental electrical problem of telephony is three-fold:

1. The production of an electrical wave which is a faithful copy of the spoken word.

2. The transfer of this wave without appreciable delay over distances which may amount to hundreds or thousands of miles, without excessive change of form by distortion, without the accession of foreign disturbances, and without undue loss of intensity.

3. The production at the receiver of an audible sound wave

which is an adequate counterpart of the electrical wave and, therefore, of the original spoken word.

As speech is carried on telephonically by means of an extremely small amount of energy, it is necessary that a large part of the telephone plant be of a sensitive and delicate construction. This includes the subscribers' sets where occur the delicate transformations from air wave to electrical wave and *vice versa*.

These substation instruments cannot be located at central offices where they would be under the immediate supervision of a trained staff but they must be placed in the subscriber's office, factory, home or wherever they will be most available and convenient for his instant use. There are now over ten million telephone stations in the Bell System. These sensitive nerve ends of the telephone system are distributed throughout the entire country in every conceivable variety of location.

In addition to the delicate substation apparatus, each telephone conversation requires the exclusive use of a connecting circuit. Even though the circuit be hundreds of miles in length it cannot be used for any other telephonic purpose. This exclusive circuit must be low in resistance, capacity and leakage so as not unduly to attenuate the telephone wave. It must be so transposed, balanced and protected that so far as possible it will not pick up electrical disturbances from earth currents telegraph lines or other telephone circuits or itself constitute a source of disturbance to the latter. The network of telephone circuits now comprises more than twenty-two million miles of wires.

In addition to meeting the above basic requirements, the telephone system, in order to realize its potentialities as a utility of the greatest benefit to the public, must include facilities such that at any time on request of a subscriber connections can be made between any two points, without delay or other inconvenience, and the charges for the service must be as low as possible. At present about thirty-two million such telephone connections are made per day in the Bell System.

Prompt, efficient and economical service on the existing scale requires that an immense number of separate circuits be brought together into common central offices and provided with every device and attendance which will facilitate traffic over the system. It requires, for example, that hundreds of wires be crowded into cables, the latest types of which have 2400 conductors within a sheath whose outside diameter is 2 5/8 inches. It requires

great congestion of wires and apparatus in switchboards in order that many thousands of lines may be brought within reach of a single operator. It requires elaborate and reliable signaling arrangements to economize time and circuits. It requires uniformity in plant and methods throughout the entire system so as to make possible prompt connection between any two points. While it has been found practicable to devise means for transmitting the required signaling currents over the telephone plant safely, the danger of fires from the currents and voltages employed for signaling has been avoided only by the exercise of extreme care, although these currents and voltages are very small compared with the currents and voltages on power lines.

From this brief consideration of the telephone problem, showing that a large portion of the telephone system is inherently of a delicate nature and susceptible to interference, it is clear that telephone apparatus and circuits would be destroyed if but a small fraction of the powerful currents and voltages used by other electric utilities were permitted to enter into the telephone system.

Values of Induced Voltage. In studying the inductive effects of electrified railroads, it has been found advisable to determine approximately the amount of induced voltage in a communication circuit, per mile, per 100 amperes in the trolley, for different horizontal separations between the trolley and the communication circuit, and with different percentages of the trolley current in the rails. For example, it has been determined that, with 60 per cent. rail current (that is, 40 per cent of the trolley current return flowing in the earth as stray current,) the induced voltages per mile, per 100 amperes in the trolley, are in general about 10 volts, 5 volts and 1 volt, at 50 feet, 300 feet and 4000 feet separation respectively. Thus at 50 feet separation, with 1000 amperes in the trolley, a ten mile exposure would result in 1000 volts induced. These are maximum figures in that they are based on the assumption that power is supplied in one direction only. It should be understood that the induction varies considerably in different cases since the induced voltages are affected by all the various conditions which go to determine the course that the stray current takes. Some parallelisms may extend more than ten miles and at times of short-circuit the current may amount to many thousands of amperes, and, in such cases, the induction is liable to be correspondingly more severe unless preventive measures are taken.

The specific effects of these induced voltages will now be touched upon briefly.

Interruption of Service. Induced voltages may be high enough to operate the telephone protective devices and, if the current across the protector is sufficient, the line will become permanently grounded and the telephone service interrupted until the protector is restored to normal condition. If the protector is located in a central office, the time required to make repairs is relatively short, but if it is at a subscriber's premises, considerable time may be required for a repair man to reach the station. In cases where the operation of the protector does not actually ground the line, it may lower the insulation resistance, sufficiently to make the line noisy.

It may also sometimes happen that foreign voltage of a value below that required to break down the protector spark gap will yet be sufficient to puncture the insulation of the wiring at some point.

False Bell Ringing. Voltages of about 8, 20 and 200 volts, depending somewhat on the prevailing earth potentials, are sufficient to ring ordinary grounded bells, standard biased bells, and (by breaking down protector spark gaps) metallic circuit bells, respectively.

An accidental trolley ground on a 25-cycle single-phase electrification through a thickly settled community may ring scores or even hundreds of subscribers' bells, some of which may be located a mile or more from the railroad. Such false bell ringing is apt to be a source of serious complaint by subscribers, and is particularly annoying when it occurs at an unseasonable hour as, for example, 5 o'clock in the morning.

Noise. In order to appreciate the effect of small currents in producing noise in telephone circuits, it must be considered that a very small fraction of a microwatt of power at voice current frequencies will produce an audible sound in a telephone receiver and a few microwatts are sufficient for a telephone conversation in a quiet place. When the current in the telephone receiver caused by induction from outside circuits is large enough to produce an audible sound, it has an important effect on the efficiency of the circuit for transmitting speech, particularly when the circuit is used for talking over long connections so that the energy of the voice currents approaches the minimum which will give a satisfactory conversation. An extraneous sound which is scarcely more than audible to an

untrained ear and might be thought to be of negligible consequence, has in reality, the effect of impairing a telephone circuit by a large percentage, or otherwise expressed, of destroying a material part of the circuit's value for service purposes.

The interfering effect of foreign current of a given magnitude depends very greatly, however, upon the frequency. The maximum effect is for current having a frequency of about 1100 cycles per second. At lower frequencies the effect falls off rapidly, and at 25 cycles is probably only about one two-thousandth as great as at 1100 cycles. This fact explains why the inductive interference to telephone circuits from 25-cycle railway systems is not predominantly noise. Twenty-five-cycle current normally has relatively very small components in the telephone-frequency range and the effect of these high-frequency components is damped out much more rapidly than that of the fundamental by separation of telephone and railroad circuits. Noise from such railways is, however, present to some extent, and is liable to become serious under any conditions which produce a bad wave shape in the power circuit.

Interference with Telegraph Signals. At 25 cycles, an induced current of one milliampere is liable, under some conditions, to interfere with ordinary Morse transmission, while rapid telegraph systems, printers, etc., are more or less impaired by extraneous currents of any value.

Fire Hazard. The use of heavy insulating coverings for wires in telephone switchboards is impossible on account of the necessity of bringing many lines within a limited space. It is not feasible to employ for this purpose such insulation as is considered good practise for electric light and power wires. Thus it is unavoidable that the dielectric strength of the telephone wiring be relatively low.

Investigations of the fire hazard due to foreign voltages impressed on telephone lines indicate that voltages of 200, or even less when backed by considerable power as in the case of induced voltages from alternating-current railways, create a distinct fire hazard.

Although the fire hazard brought about by railroad electrification is due chiefly to the higher voltages induced at times of short circuits on the railroad, it is possible that the repeated electrical stresses of lower voltage, due to normal railroad operation tend to decrease the dielectric strength of the insulation and thereby facilitate breakdown.

Magnetization of Loading Coils. Loading coils in very large numbers are now employed in both open wire and cable telephone circuits. These coils are liable to be permanently magnetized by any induced currents which are materially in excess of telegraph currents. While they are magnetized there is a considerable loss of transmission efficiency and it is ordinarily impossible to demagnetize them without removing them from the circuits. Moreover large currents through the loading coils may permanently reduce the permeability of the iron cores and make them unsuitable for use on long toll cable circuits.

Electric Shock. At times of short circuits on the railroad and sometimes during switching operations, electrical surges may be set up in the telephone circuits, which are of sufficient intensity to produce electrical shocks to persons at the telephone or working on the circuits at the time. While it is improbable that such shocks will be the cause of serious personal injuries, even minor shocks are objectionable and constitute a basis of complaint as the public expects telephone instruments to be perfectly safe at all times.

Acoustic Shocks. Inductive surges such as are capable of producing electric shocks to persons are also liable to cause loud noises in the telephone receivers which may result in acoustic shocks to persons using the telephone at such times. Even the relatively slight clicks which sometimes occur due to battery interruptions may be very annoying to telephone users and acoustic shocks sometimes caused by induced voltages may be much more severe.

Investigations and Experiments. Since 1905, when first notified of the intention to install single-phase electrification on a section of the New York, New Haven and Hartford Railroad, the American Telephone and Telegraph Company has done a large amount of work on plans, tests, experiments and studies of various kinds, most of it in conjunction with representatives of railroads and the electrical manufacturing companies, all with the general object of finding means for protecting the telephone and telegraph lines and service against interference from electrification installations. A considerable amount of work has been done in connection with various electrification projects which have not been installed, some of these projects having been abandoned, at least in so far as the specific plans under consideration are concerned, while in other cases the matter is being held in abeyance, awaiting more favorable conditions for undertaking construction.

Means for Preventing Interference from Alternating-Current Railway Electrifications

There are various means which have been proposed, some applicable to the railway system and some applicable to the affected communication systems, for preventing or reducing inductive interference. Some of these means have not been found successful or advantageous in practise while others have proved beneficial in varying degrees. It will be of interest to consider briefly some of these proposals.

Separation. The most effective means is to avoid the parallel, wherever practicable, by keeping the communication circuits and electrified railroads sufficiently separated. With the extension of electric traction, and the constantly increasing importance and efficiency of communication circuits, the avoidance of parallels will be increasingly important. However, this first rule for preventing interference, is unfortunately, not one which can be generally adopted in practise. Railroads and communication circuits must serve the same communities and it is necessary that the connecting routes of each be reasonably straight and direct. The field of influence of an alternating-current railroad which uses the running rails as a part of its circuit extends out to a great distance on both sides of the railroad. This makes effective separation from such electrified railroads much more difficult than separation from most other kinds of power transmission circuits.

Neutralizing Transformers. Where communication circuits are subject, under normal operating conditions of the railroad, to induced voltages sufficient to interfere with telegraph service, neutralizing transformers can be resorted to and if properly designed and connected into the disturbed circuits, such transformers will effect neutralization of a large part of the induced voltage. The transformers are provided with a plurality of windings, some of which, called primaries, are inserted in certain of the affected conductors which are grounded at or beyond the limits of the parallel, while the remaining, or secondary windings, are connected serially into other of the conductors in which the induced voltages are to be neutralized. Under favorable conditions the remaining, non-neutralized voltage, is only 5 to 10 per cent of the total induced voltage.

For a more complete description and discussion of neutralizing transformers reference is made to an article by Thomas Shaw in the *Electric Journal*, November, 1914.

Neutralizing transformers, however, have serious disadvantages from the telephone company's standpoint. The primary circuits, of which there are ordinarily from one-third to one-half the number of secondary circuits, are practically lost for telegraph purposes although they can be used for telephoning, at somewhat reduced efficiency. The secondary circuits are also reduced somewhat in telephonic transmission efficiency, but not so much so as the primaries.

Neutralizing transformers have served a useful purpose in the early stages of alternating-current railroad electrification where means of restricting the railroad's field of inductive influence were not employed. They continue to have a limited field of usefulness, particularly in making endurable moderate amounts of induced voltages which remain after preventive methods have been applied at the source of disturbance, but they leave the general problem of interference unsolved. They are not applicable to subscribers' lines nor have they been found effective in neutralizing the higher harmonics which cause noise.

Drainage Coils. Drainage coils, bridged across a telephone line, with their mid-points connected to ground, provide a low impedance path for currents induced between wires and ground and thus tend to reduce the voltage. Such coils must be exceedingly well balanced or they will themselves constitute a source of unbalance and thus augment noise. Moreover, they increase the susceptibility to noise resulting from irregularities in series resistance or impedance of the telephone circuits. Also they impair telephonic transmission efficiency.

If telegraph service or direct-current signaling is employed on circuits equipped with drainage coils it is necessary to place condensers in series with the coils. The effect of this apparatus on telegraph service is distinctly detrimental.

Drainage coils have not proved to be adapted for general use on commercial systems but are helpful on private telephone circuits of power transmission companies for reducing high electrostatic charges when such private circuits are carried close to high-voltage wires.

Sectionalization of Telephone Circuit. An affected telephone circuit may be sectionalized by cutting in repeating coils at one or more points. This may be advantageous in certain cases of exposed rural lines where by placing a repeating coil at each end of a parallel it is possible to change to a metallic circuit through the parallel. It is also sometimes useful on private telephone

circuits of power transmission companies as it makes possible the insulation of the telephone sets from the exposed telephone wires. On commercial telephone systems the usefulness of this method is very limited as it introduces large transmission losses, precludes the use of telegraph and brings in difficulties in connection with line signaling.

Shielding Conductor. A copper conductor used as a shield may be strung near the disturbed communication wires and grounded at the ends of the parallel. With a conductor of suitable impedance, the current carried by the conductor will have a neutralizing effect on the induced voltages in the near-by communication wires. The action is similar to that of neutralizing transformers but less effective. In case of an aerial cable, the cable sheath itself can be so used instead of a separate copper conductor. The benefits derivable from this method, however, are very limited.

With a view to increasing the neutralizing action of such a conductor, it has been suggested that a part of the railway current could be diverted into it. The quantitative relations involved are such, however, that great difficulties stand in the way of successful application of this scheme on a commercial scale.

Resonant Circuits. Combinations of coils and condensers, adjusted to be resonant at the disturbing frequency and connected so as to reduce the disturbing current in the receiving instruments, have been employed to some extent. These afford considerable benefit for low-speed telegraph service such as ordinary hand sending. For higher-speed operation, the benefit obtainable in this way becomes rapidly less. Many modern telegraph systems operate at speeds approaching 25 dots per second which makes it impossible to differentiate in this way between the signaling and disturbing currents.

Similar methods have been suggested for reducing noise but are not usually applicable because the harmonics which cause noise are within the range of frequencies required to give good telephonic quality.

Balance and Insulation of Telephone Circuits. It is advantageous to construct and maintain telephone circuits exposed to induction with a high degree of balance and insulation. This includes an adequate transposition system. In all cases of inductive disturbance care should be exercised that these features of the affected lines are properly attended to.

Use of Relay Sets. On direct telephone lines the bell is bridged between the two metallic conductors. On two-party selective lines one bell is connected between each side of the circuit and ground. On four-party semi-selective lines two bells are connected between each side of the circuit and ground. On four-party lines, with full selective ringing, the bells are not connected to ground except at times when an operator is ringing on the line and at such times the connection of the bell to ground is established by means of relays. On all these classes of lines both sides of the circuit are grounded at the central office.

It will thus be seen that an induced voltage between the circuit conductors and ground might ring all grounded bells, but under normal circumstances would not ring bells on a direct line or at stations equipped with relay sets. However, if the induced voltage is high enough to operate the telephone protectors, a path to ground is established through the protector, and bells on direct lines or bells at relay set stations may be falsely rung.

Biasing Bells. In regions where the induced voltages are not too high, false bell ringing can be obviated by biasing the bells, that is, by stiffening the control springs so that increased voltage is required to ring the bells. Obviously, there are very positive limitations to what can be accomplished in this manner.

Measures Applicable to Railroads. The foregoing measures for obviating inductive interference are of a palliative nature and assume a condition of the electrification which produces large inductive effects. Another class of measures for avoiding interference looks to the source of the disturbances, the electrical system of the railroad, and seeks to avoid the conditions which produce large induction. This latter class has, in general, the advantage of benefiting, not one affected circuit only, but all communication circuits within the area affected.

Double Trolley. One radical and probably effective method of preventing inductive interference from single-phase railroads would be the use of a double-trolley circuit completely insulated from ground, thus avoiding the use of the running rails as a part of the railway circuit. This method, however, is distinctly unpopular with railway men, mainly for operating reasons, on account of the complexity of the overhead construction particularly in yards, and at sidings and crossovers. Purely from the cost standpoint this method might have advantages in certain cases, where the conditions of exposure are severe and other methods of restricting the earth currents are expensive to apply.

Frequent Power Supply Stations. One of the most important methods of interference-prevention is the provision of a sufficient number of substations to supply power to the trolley-rail circuit at frequent intervals. If the substations are near enough together, the amount of stray current and the average length of path of such current can be made small. It is particularly desirable that all sections of electrified railroad which are involved in parallels be supplied with power from both directions rather than by stub end feed.

Sectionalization of Trolley System. Considerable advantage may be gained by sectionalizing the trolley, thus decreasing the length of the earth current path as well as reducing the amount of power supplied to a short circuit. However, as each separate section of trolley requires an independent power supply sufficient for its maximum demand, the total transformer capacity required for a given length of electrified road is much increased by any considerable use of trolley sectionalization. Notwithstanding this objection a limited amount of sectionalizing may be used to advantage where the exposure is severe.

Opposing Polarities. On railroad lines having two tracks it is possible to connect the two trolleys for opposing polarities, so that the current flowing in the rails and earth is only the difference in the currents of the two trolleys. An instance of this method applied to a direct current railroad is afforded by the City and South London Railway in England which has been so operated for 20 years. As applied to alternating-current railroads of 11,000 volts, this method is considered to have serious operating disadvantages in respect to cross-overs between tracks, and for this reason this plan, which was studied in connection with the revision of the Woodlawn-Stamford electrification of the New Haven Railroad in 1912, was not adopted.

Balancing Transformers. This method, which is now employed on the main line electrification of the New Haven Railroad, is of much benefit in reducing stray currents, particularly where power is supplied from both directions. Its use, however, involves a combined transmission-distribution circuit, tied together by the balancing auto-transformers, whereas the general practise in such matters seems to tend toward a separate transmission line supplying power to the trolley-track circuit through standard transformers.

Booster Transformers. Another important method of controlling railroad currents is by the use of booster-transformers

placed at frequent intervals along the electrified section. These transformers have a substantially even ratio of transformation, the primary winding being inserted serially in the trolley, and the secondary winding inserted serially in the track circuit. In this way the track current is required to be substantially equal to the trolley current at points where the transformers are located. By placing the transformers near together the leakage of current into the earth between transformers is made small.

A modification of this plan is to install a feeder, electrically connected to the rails at intervals, and insert the secondaries of the booster-transformers in series with this feeder. In this way the current is confined to the feeder instead of the rails. The London, Brighton and South Coast Railway embodies an installation of this type. This arrangement is somewhat more effective perhaps than trolley-track transformers but it involves considerable additional expense for the feeder, which must be of high conductivity. One advantage of the feeder-booster, over the track-booster arrangement, is that successful operation of the former is not so dependent on high grade maintenance of the track bonding.

Specific Electrifications

Having now considered various means which are available, or at least, worthy of being considered, for avoiding or reducing inductive interference to communication systems by alternating-current electrified railroads, we may now direct our attention to some specific electrification installations and see what has actually been done to prevent such interference and with what degree of success. In so doing I will confine my remarks to the salient features of four single-phase installations: (1) New York, New Haven & Hartford Railroad, Woodlawn to Stamford; (2) New York, New Haven & Hartford Railroad, New Canaan Branch; (3) Norfolk and Western Railroad, Bluefield to Vivian, W. Va.; (4) Pennsylvania Railroad, Broad Street to Paoli, Pa.

Woodlawn-Stamford. The original electric installation of the New York, New Haven and Hartford Railroad Company between Woodlawn, N. Y. and Stamford, Conn., began operation, in part, in the summer of 1907. This is a section of four-track railroad, about 21 miles in length, and all power was supplied by a generating station at Cos Cob about three miles west of Stamford. To move a train at Woodlawn, the current passed for 18 miles over the trolley wires and paralleling feeders from Cos Cob to the locomotive, the remainder of the circuits from

locomotive to Cos Cob, being the running rails and earth. The telephone company's New York-Boston subway is, throughout this section, situated at varying separation averaging about 2000 feet from the railroad, a sufficient distance so that the inductive effect of the trolley current would have been largely neutralized by the inductive effect of the rail current, had these two currents been equal. However due to the long rail path, a large part of the current left the rail and spread into the earth, where its effect in neutralizing the corresponding part of the trolley current was negligible.

After full electric passenger service between Woodlawn and Stamford was inaugurated, the induced 25-cycle voltage on circuits in the New Haven subway, at normal rush hour periods, was as much as 170 volts. On the Shore Line, one of the telephone company's open wire routes between New York and Boston, the corresponding induced voltage was about 300, the higher voltage on the Shore Line being principally accounted for by about a mile and a half of exposure near Greenwich, where the average separation was only about 100 feet. The open wire Shore Line circuits were also affected by noise, which was most intense during periods of train acceleration, the pitch of the noise varying with the speed of the train. The subway circuits being in metal-sheathed, underground cable, and not having any section of very close parallelism, were not made noisy. The Midland Line, another open wire route between New York and Boston, about four miles away from the railroad at the nearest parallel section, sustained corresponding induction of about 40 volts.

Wires of the Western Union Telegraph Company, which were carried on poles located on the railroad right of way, were subjected to much higher voltages. These wires, except a few which were equipped with neutralizing transformers and continued in use by the Railroad Company, were removed to a new pole line which was built a number of miles away.

The conditions as to induction continued substantially as outlined above for a period of four or five years.

Early in 1911 the railroad company made known its intention to extend the electrification to include the Harlem River branch and the New York, Westchester and Boston Railroad. The former is a six-track line used principally for freight, extending about 12 miles from its junction with the main line, near New Rochelle, to the Harlem River. The New York, Westchester

and Boston Railroad, which is partly four-track and partly two-track, was constructed principally for suburban service and extends from West Farms at 176th Street, where it forms a junction with the Harlem River branch, to White Plains, a distance of 16 miles. A branch six miles north of West Farms taps the main line of the New Haven Railroad just east of New Rochelle. These two new lines involved the direct connection, to the Western end of the previously electrified section, of additional electrified line to the extent of about 200 miles of single track railroad. Moreover, it was planned that, after the Harlem River branch was electrified, freight trains, as well as passenger trains, on the entire system west of Stamford should be operated electrically.

This proposed large extension of the electrification, with its resulting increase in load, caused considerable apprehension to us of the telephone company. Our estimates of induced voltages under the new conditions, based on the Railroad Company's estimates as to future train loads, indicated over 1500 volts on the Shore Line and nearly 1000 volts on the subway. These values corresponded to maximum normal railroad loads and the induction would have been still greater at times of abnormal conditions. Voltages of this magnitude are far beyond the endurable limit for the telephone company and the matter was taken up by the companies with a view to determining what could be done to ameliorate the situation.

In January 1912, a joint committee of engineers, comprising a representative each of the New Haven Railroad Company, the Western Union Telegraph Company, and the Telephone Company, was formed to study this question. Several different plans for modifying the railroad distribution system were laid before this committee. After six meetings during the ensuing three months, a plan was decided upon and a sub-committee of engineers was designated to work out the details.

This new distribution system involved quite a comprehensive change in the original installation. It was cut over on January 15, 1914 and has been found to bring about a great improvement with respect to induction.

Besides an additional power supply station at West Farms, which in itself effects a considerable improvement, the new distribution system includes the use of 17 balancing auto-transformers of 2000 kv-a. capacity each. These are distributed along the line, in such locations as are most advantageous for

supplying power to the trolley circuit, so as to minimize the length of path of current through the rails.

As full accounts of this distribution system have been published I will not undertake an extended description of it here.

The same system has since been used by the Railroad Company in extending the electrification from Stamford to New Haven.

At present, the induced voltages on the through circuits in the subway seldom exceed 30 volts under normal conditions of railroad operation.

The principal interference now incurred (apart from that due to the New Canaan Branch which is discussed elsewhere) is in connection with railroad short circuits in the vicinity of Stamford. Within the section from 2 miles West of Stamford to 5 miles East of Stamford, local subscribers' lines and trunk lines have been affected by false bell ringing, false flashing of line signals, and grounding of protectors, on about twenty different occasions in the past three years, an average of twenty lines being affected on each occasion. It is of interest to note that these troubles are localized within this seven-mile section of the sixty-mile electrification from Woodlawn to New Haven, and also that it is at approximately the middle of this seven-mile section that the New Canaan Branch, referred to immediately below, joins the Main Line.

New Canaan Branch. In 1907 the New Canaan Branch of the New Haven Railroad, which previously had been operated at 500 volts direct-current, was reconstructed and its trolley connected directly to the 11,000-volt trolley on the main line near Stamford.

This branch is about six miles long, single track, and the traffic is light so that under normal conditions of operation no interference with telephone or telegraph lines was produced. However, the telephone circuits have been subjected to a great deal of interference from this branch, due to short circuits. Owing to the conditions of power supply, the short-circuit current at New Canaan is about 2500 amperes or ten times the maximum load current. At points nearer the main line the short-circuit current is even greater.

The inductive effects of the New Canaan branch were augmented by the large proportion of earth current, due to the relatively high impedance of the single track. Momentarily voltages as high apparently as 1000 were imposed on trunks between New Canaan and other places and voltages up to 500 on many telephone circuits in the New Canaan exchange. These

induced voltages operated protectors, permanently grounded and put out of service many telephone lines and subjected operators to severe acoustic shocks. Due to the recurrence of these surges, the operators in the New Canaan office became so afraid of shocks that the operating efficiency was seriously impaired.

In 1913, after several unusually severe surges from short circuits had been experienced, the matter of finding some means to overcome this interference, which had already been under consideration by the telephone and railroad companies, was taken up with renewed energy. Various plans were proposed and an extended series of experimental tests and measurements were made, as a result of which, means were finally agreed upon as follows: 1. To keep the rails well bonded. 2. To insert a current-limiting reactance in the trolley, near its junction with the main line trolley, so as to restrict the short-circuit current. 3. To install 12 series booster track-trolley transformers at intervals of about ½ mile. 4. Readjust the circuit-breaker, at the junction with the main line, for instantaneous operation.

From the tests and from experience with six booster transformers, it is believed that the above mentioned measures will be effective in preventing this interference although the full installation of transformers and the current limiting reactance has not yet been completed.

It is of interest to note that the balancing transformer plan, although generally giving good results on the main line of the New Haven Railroad, does not afford an effective means for preventing inductive interference under such conditions as exist on the New Canaan Branch.

Norfolk and Western Railroad Electrification. This electrified section is between Bluefield and Vivian, West Virginia, a distance of approximately 28 miles. The railroad is double track with numerous yards and sidings and includes some heavy grades. The power house for supplying power to the electrified section is located at Bluestone, about 10.8 miles west of Bluefield. Power is transmitted by duplicate single-phase transmission lines at 44,000 volts to five substations, the distances between which, respectively, beginning at Bluefield, are 8.2, 4.6, 6.6 and 4.8 miles. At these substations the voltage is stepped down to 11,000 for delivery to the trolley-track circuit which is electrically continuous throughout from Bluefield to Vivian.

The original plans for this electrification were taken up with the telephone company who proposed some modifications for

the better protection of the paralleling communication circuits. The plans as modified include 23 series booster trolley-track transformers, the average spacings of which are, east of the power house, about a mile and a half, and west of the power house about a mile. Each transformer is 100 kv-a. continuous rating and 400 kv-a. for 2 ½ minutes.

One telephone line paralleling this road has several exposures of about 500 feet separation from the railroad and there are also local circuits of the Bluefield Telephone Company in proximity to the railroad. No trouble has been reported from induction under normal railroad conditions. At times when the electrification wires are down noise has been experienced on some of the above mentioned circuits. Also a telephone trunk circuit which crosses the railroad underground is sometimes thrown out of service when the railroad circuit is in trouble, by reason of the copper block protectors at the crossing becoming grounded. Some trouble has been sustained from corrosion of lead cable sheaths at underground crossings of the railroad in Bluefield but it has not been established whether or not this is due to electrolysis by the alternating railroad currents.

Broad Street—Paoli. Plans for electrifying the Pennsylvania Railroad's four-track main line from Broad Street to Paoli were announced early in 1913. The question of interference with the telephone company's lines was taken up with the engineers representing the railroad company and several plans looking to the prevention of interference were considered. As a result of the best information then available it was decided in 1914 to install series booster trolley-track transformers for the purpose of confining the current to the rails.

The power for this electrification is conveyed by a 44,000-volt 25-cycle transmission line to three substations, one each at West Philadelphia, Bryn Mawr and Paoli, and from these substations supplied to the trolley-track circuit at 11,000 volts. Sixteen pairs of trolley-track transformers, each transformer equipping two tracks and having a continuous rating of 80 kv-a. and 600 kv-a. for one minute, are provided, the spacing between transformers being about one mile.

The regular operation of electric passenger trains between Broad Street and Paoli was begun during the latter part of September, 1915. Extended induction tests were made on the section east of Bryn Mawr in April, 1915 and on the entire line in the following August. Further induction tests were made in the summer of 1916.

In July, 1916, the Railroad Company commenced operating synchronous condensers at Radnor and it has been found that, with these condensers in use, the booster transformers are greatly overloaded at times of short circuit. This results in the transformer iron becoming heavily saturated and the magnetizing current, consisting largely of the third harmonic, which under these conditions reaches very high values, necessarily flows through the ground. The current wave is badly distorted as a result of this overloading and the induced voltages during short circuits may actually be higher with the booster-transformers than without them; in fact, it has been found preferable to remove the booster transformers east of Bryn Mawr, and only those from Bryn Mawr west are now regularly in service.

With all booster transformers in service the maximum induced voltages, (peak values)*, during normal operation are about 10 for subscribers' lines and 25 for trunk lines; at times of short circuit, calculations based upon experimental data show that the maximum voltages may exceed 1000 on subscribers' lines and 1200 on trunk lines. With all booster transformers cut out, the corresponding figures are, for normal operation, 50 volts for subscribers' lines and 125 volts for trunks and, at times of short circuit, 225 volts for subscribers' lines and 900 volts for trunks. These figures assume two condensers in service.

This section of railroad follows through a highly developed suburban area where disturbances on telephone lines are extremely undesirable. Unfortunately, a considerable number of cases of bell ringing due to short circuits on the railroad circuit have been experienced. Not all short circuits cause bell ringing however.

During the first three months of electric operation, namely, October, November and December, 1915, the number of short circuits causing bell ringing averaged 10 per month and the bell ringing troubles over 2000 per month. During the following seven months the average number of short circuits causing bell ringing, fell to 4.5 per month and the bell ringing troubles from 735 in January to 57 in July. Since July, 1916, the number of short circuits per month which caused bell ringing have been

*On account of the-wave shape distortion described above it was found advantageous here to measure peak voltages rather than effective voltages and all values of voltage mentioned in this discussion of the Paoli electrification are peak values.

further reduced and during the year 1917 averaged about 1.5 per month.

The improvement in the bell ringing situation after January 1916 and up to August 1916, was due partly to changes in the railway control circuits and in the operating arrangement of circuit breakers, and partly to the substitution of relay sets for standard party line bells, at subscribers' stations within the regions of heaviest inductive disturbances, and the biasing of bells within the areas of less disturbances. This work required changes in subscribers' apparatus at about 3000 stations.

The synchronous condensers installed in the latter part of July, 1916, increased the maximum voltages impressed on the telephone circuits at times of short circuit on the railroad, so that while the actual number of short circuits causing bell ringing has been reduced since that date, the average number of bell ringing troubles per month has increased. From August 1916 to December 1917 inclusive, there were 22 short circuits causing bell ringing and 1589 bell ringing troubles, an average of 72; whereas for the five months' period preceding the installation of the synchronous condensers, there were 20 short circuits causing bell ringing, and 356 bell ringing troubles, an average of 18, or one-quarter of the corresponding average number in the later period. During the year 1917, 45 per cent of the bells rung were on direct lines and not grounded.

The high induced voltages causing bell ringing have been experienced over the entire electrified portion of the Main Line except within about three miles of Broad Street. It is fortunate that the region of high induced voltages does not extend to Broad Street, otherwise, owing to the density of telephone development within that district the trouble would be exceedingly difficult to cope with.

In a small percentage of cases where bells are rung the induced voltages are sufficient to leave the telephone lines grounded through the protectors, thus interrupting service.

Methods of signaling on call circuit trunks involving use of the ground have had to be abandoned.

Noise tests made on ten trunks which parallel the electrification indicate that the induced currents from the railroad cause a small amount of noise but not enough to constitute interference with service on these comparatively short lines. However, I may mention the fact that in the construction of the new Philadelphia-Reading toll cable, in which the requirements as

to freedom from noise are more exacting than in the case of the shorter Main Line trunks, the liability of noise, together with other features of interference, was considered a sufficient reason for changing to a different route in order to avoid exposure to the electrified section of the railroad. The route adopted involves charges of $1000 a year more than the route exposed to the electrification, which otherwise would have been followed.

None of the telephone company's circuits affected by the electrification are now used for telegraphy, hence there has been no interference with this type of service. It is expected, however, that telegraph service over the paralleling toll circuits will be required later and to give this service it will be necessary, unless there is some new development, to employ neutralizing transformers with their attendant disadvantages and limitations.

The high voltages induced on the telephone lines at times of short circuits, also, in the opinion of the telephone company, constitute a considerable fire hazard, although it is fortunately true that no fires have as yet been caused. All terminating trunks at the three directly exposed offices west of Bryn Mawr have been provided with carbon block and heat coil protection. As these trunks are in underground cable, they would not require these protective devices except for the induced voltages. As an additional precaution against fire the telephone company has maintained a special force of night watchmen at several central offices throughout this area where regularly there are no inside men at night, thereby incurring an expense of about $18,000 per year. As a still further precaution, trunks to certain offices west of Malvern were for a time provided with repeating coils at Malvern, but it has recently been necessary to phantom these circuits, which has required the removal of the repeating coils.

A large number of the trunks affected by induction from this electrification are equipped with loading coils. Tests on these coils show that more than 20 per cent of them are magnetized to a greater or less extent but it has not yet been determined whether this trouble has been brought about wholly or in part by induced currents from the railroad circuits or whether it is due to other causes.

Within the area of high induced potentials, telephone subscribers and employees are exposed to the possibility of electric shocks at times of short circuit on the railroad. Fortunately, however, no troubles from such shocks have yet transpired.

Telephone operators and users are also exposed to the possibility of acoustic shocks at times of surges from short circuits, although no serious acoustic shocks, due to this electrification, have been reported.

It will be seen from the foregoing that, notwithstanding all that has been done by the railroad company and the telephone company to reduce interference, there still remain, at times of short circuits, certain hazards of fire and shocks, bell ringing trouble, and other latent interference as described. While this impending interference has not, with the exception of bell ringing, actually materialized into trouble, nevertheless the possibility of such trouble is continually present and the conditions can by no means be regarded as satisfactory.

Looking to further improvement in the situation a number of plans which involve changes in the distribution system of the railroad, have been worked out. The plan which, on the whole, seems entitled to the most favorable regard, involves the installation of additional power supply transformers at Radnor and at a point 16½ miles from Broad Street, and also includes the sectionalizing of the trolley at both these points. This plan further requires the moving of the Berwyn-Malvern aerial telephone cable to a more remote location. By the adoption of this plan it is estimated that the maximum induced voltage at times of short circuits would be brought down to 250. This would largely reduce, but not wholly avoid, the fire and shock hazards, bell ringing on direct lines, and the other evils involving the operation of protectors. The cost of carrying out this plan, including labor and material only, has been estimated at $140,000.

Another plan, involving more extensive changes in the power supply and distribution system to avoid wholly the interference and hazard, would require a much larger expenditure and perhaps would not be warranted by the existing situation.

A different plan would be to install sufficient additional booster-transformers so that they would not become overloaded at times of short circuits. This would probably require placing the boosters one-third to one-half mile apart and would cost from $85,000 to $150,000. This plan has not been worked out in detail, as the railroad company objects to the introduction of insulating joints in the trolley wires at such frequent intervals.

Conclusion

It may be said in conclusion that means are now known whereby alternating railway currents can be kept sufficiently within control, except under abnormal conditions, to prevent substantial interference to neighboring communication lines, although the application of such means to the extent necessary to produce satisfactory results may involve considerable expense.

Even under abnormal conditions the interference can be greatly reduced by the application of suitable measures, but in some cases there still remains the problem of obtaining a sufficient reduction of interference without incurring a cost which the railroad companies consider excessive.

It is important in each electrification project that the railroad company and the communication companies affected cooperate in determining what interference-preventive measures shall be adopted. Each electrification requires a special study, as the best measures to employ may be quite different in different cases.

I wish to take this opportunity to testify to the broad minded and cordial manner in which the railroad companies and electrical manufacturers concerned have cooperated with us in searching for a satisfactory solution of this problem, a work which, it is probably unnecessary to add, is still in progress.

Discussion on "Inductive Effects of Alternating Current Railroads on Communication Circuits" (Warren), Philadelphia, Pa., April 8, 1918.

Marius Latour (communicated after adjournment): It may be of interest to call attention to a very simple system proposed by the writer in 1912, based on resonance, and which allows a Morse telegraphic line to be operated in spite of strong disturbing alternating e. m. fs. This system, which was put in operation on the French Midi (Southern) railway, was new at the time of its introduction there. It comprises three principal features:

1. Connected in parallel with the receiving apparatus, preferably *ahead of the sending* key, is a resonance-circuit capable of eliminating the disturbing current, and operating in combination with a definite inpedance, connected in series with the receiving apparatus, which exaggerates the shunt effect of the resonance-circuit.

2. Connected in series with the telegraphic line is a suitable impedance capable of reducing the disturbing current

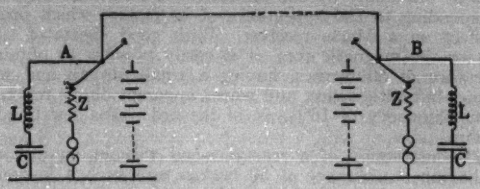

Fig. 1

while allowing the telegraphic signal currents to pass through without deformation.

3. The impedance inserted in the line is distributed all along the line so as to reduce the maximum potential difference between the telegraphic line and the ground.

I. Connection on Resonance Circuit in Parallel with Telegraphic Receiving Apparatus

The Morse receivers are connected in parallel with the two corresponding stations A, B, by resonance-circuits L, C (Fig. 1) which are tuned to the frequency of the electric railway current, in other words, to the frequency of the interfering current; and an impedance, Z, is at the same time connected in series with the receiving apparatus. The resonance-circuits, L, C, are connected ahead of the sending key. In this manner, the circuit for the interfering current over the line is never opened by the operation of the sending key at either end, because the sending key closes one branch circuit as it opens another, in

the manner shown in the diagram; and the conditions remain favorable for the continuous maintenance of resonance.

For the frequency of $16^2/_3$ cycles, which is that quite generally employed for traction by single-phase current in the case of motors of high power, it will be assumed that the inductance is 50 henries and the capacity is 2 microfarads.*

The inductance of the reactance-coil L, is constructed with an iron magnetic circuit. It was believed by some that it was not possible to obtain resonance successfully for currents of a few tens of milliamperes with an inductance comprising a magnetic circuit of iron. This opinion is even still to be found expressed in January 1914, in the German Post and Telegraph Annals. But this opinion, while it might have a theoretical basis, is not borne out by practical experience. While it may be true that the permeability of iron, in the case of low, variable, magnetic densities, is a function of the density, it is none the less true that a small air-gap is sufficient to render the total reluctance of the magnetic circuit quite uniform; the irregularities in the permeability can then manifest themselves at most only by the formation of a residual impedance corresponding to the appearance of harmonics which produce no effect on a Morse receiver. With good sheets of silicon steel for the magnetic core, it is easily possible to obtain an inductance of 50 henries, having a small air-gap and which occupies but little space, and whose apparent ohmic resistance, for the frequency of $16^2/_3$, is of the order of a few hundreds of 1 ohm.

The construction of a reactance-coil without an iron core, and having a reactance of 50 henries and such a low ohmic resistance, would be impracticable.

In reality, the frequency of the railway traction current is not absolutely constant, and it is to be expected that the resonance cannot be perfectly tuned. For example, if we suppose the frequency to vary 5 per cent above and 5 per cent below, the normal frequency, or 10 per cent altogether, the impedance L, C, can attain 500 ohms in reactance or in capacitance, according to whether the frequency is falling or rising with respect to its mean value, and it will be seen that the apparent ohmic resistance of the reactance-coil is not the only thing to be taken into consideration.

It is necessary to exaggerate the effect of the parallel resonance-circuit L, C, by placing in series with the receiving apparatus an impedance Z which increases the total impedance of the portion of the circuit containing the telegraphic receiver. This impedance, Z, may be composed either of an inductance or of an ohmic resistance. The advantage of using an ohmic

*The frequency of $16^2/_3$ is designated with this precision because one has in mind the process of transformation, by means of motor-generators, by which this frequency can be equally obtained from the higher industrial frequencies, 50 and 25 periods: $16^2/_3 = 50/3 = (2 \times 25)/3$.

resistance is that equivalent and symmetrical effects are obtained when dissonance of the shunt circuit L, C, is produced, either by an increase, or else by a decrease, of the frequency, in other words, when the shunt circuit L, C, has either capacitance or else reactance.

II. Connection of Suitable Impedance in Series with the Telegraph Line

Considering the importance of the e. m. f. which causes interference, it is desirable to reduce the strength of the interfering current diverted into the resonance-shunt. This is

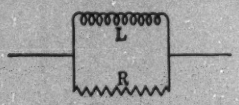

Fig. 2

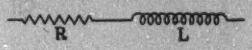

Fig. 3

accomplished by introducing, in the line, an impedance composed of an inductance, L, and a resistance, R, connected either in parallel, as in Fig. 2, or in series, as in Fig. 3. The resistance portion is, in any case indispensable, in order that the time-constant of the line may not be too high, and to prevent the Morse signals from running together. It is possible to use resonance in a new way to oppose a high impedance to the interfering current. It was for this purpose that Mr. Bethenod and the writer devised, together, an impedance composed of an inductance, L, and a capacity, C, connected in parallel (Fig. 4).

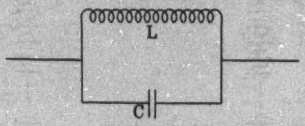

Fig. 4

If this arrangement be regulated so as to produce resonance (for example, by making $L = 50$ henries and $C = 2$ microfarads, for a frequency of $16^2/_3$) an impedance will be obtained which is equal, theoretically, to the reactance of the inductance (5000 ohms) multiplied by the ratio of this reactance to the resistance of the coil (or $500 \times \frac{5000}{500} = 5000$ ohms). Practically, a very high impedance is obtained. The arrangement shown in Fig. 4 is some times called a "stopper". The in-

ductance L is constructed in the same way as that used in the resonance shunts.

When the arrangement is not in resonance, it has a weaker impedance, which is a reactance if the frequency is too low, and a capacitance if the frequency is too high. In order to maintain the time-constant of the line at a proper ratio, it is necessary to put still more resistance, R, in series with the "stopper", as shown in Fig. 5.

In the complete arrangement (Fig. 6) comprising resonance-

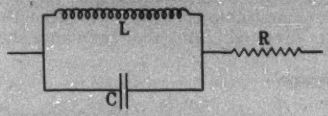

FIG. 5

shunts at the stations A and B, and a stopper, D, between the two stations, the resistance R is already connected in series with the Morse receiver, where it already fulfills the function of the impedance Z, shown in Fig. 7. In reality, it will be seen, by reference to Fig. 6 that an interfering current which can close its circuit through line leakage between station A and the point at which the stopper D is located will go through station A without being stopped by the stopper D, and that, in

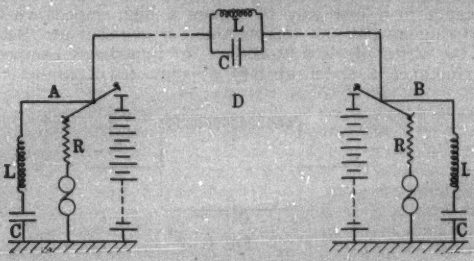

FIG. 6

like manner, the charging current of the line (due to the fact that the telegraph line and the line parallel thereto which carries the traction current form the two armatures of a condenser that is charged by the potential difference between the traction line and the ground) circulates through station A without being stopped by the stopper D. It would seem, therefore, that in order to utilize satisfactorily the properties of the stoppers, each station should directly be protected by a stopper at its point of connection with the line. However, the question

of introducing a stopper at each station brings up the question of connection stoppers in series.

Now, one of the difficulties in the use of stoppers is that they cannot be connected in series without certain precautions. Let us suppose, for instance, that two stoppers are not tuned exactly alike, as for instance, when one is tuned for the fre-

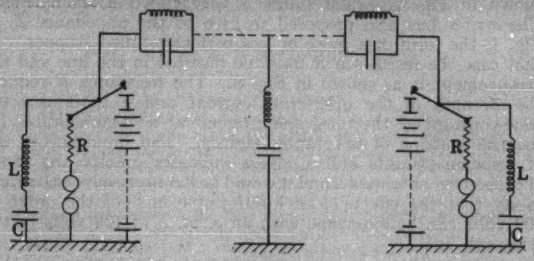

FIG. 7

quency f_1 while the other is tuned for a frequency f_2, very near to it but still different from it. For an intermediate frequency between f_1 and f_2, which will be too high for one of them and too low for the other, one of the stoppers will act as a capacity, *i. e.*, it will have capacitance, while the other will act as an inductance, *i. e.*, it will have reactance. It could thus happen

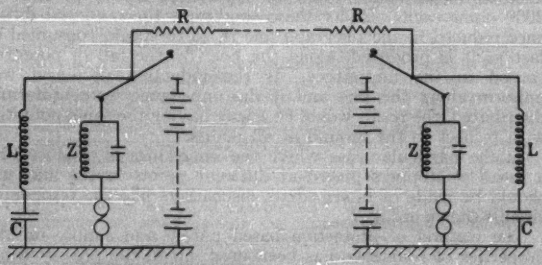

FIG. 8

that the connecting in series of two stoppers might produce actual resonance in the line and that while expecting to produce an infinitely high impedance a short-circuit might, instead, be produced.

By a similar process of reasoning it could be shown that it is impossible to connect two resonance shunts in parallel. The

connecting in parallel of two resonance-shunts which are not absolutely alike in all respects may cause the unexpected appearance of a shunt-connection of very high impedance. A stopper can be placed at the line connection at each station by placing a resonance-shunt in the middle of the line so that the two stoppers can operate independently of each other, as shown in Fig. 7, which shows a very effective arrangement. The writer has also proposed to give to the impedance Z, in Fig. 1, the form of a more or less perfectly tuned stopper. In that case the resistance R must be inserted in the line and the arrangement is as shown in Fig. 8. The resistance R reduces the strength of the interfering current and it maintains the time-constant of the telegraphic circuit at a value suitable for the transmission of telegraphic signals. The simultaneous use of resonance-shunts and stoppers increases greatly the range of protection obtainable, and it would be a conservative statement to say that the effects of an interfering e. m. f. of the order of 500 volts can be overcome with an e. m. f. of 100 volts in the telegraph line.

III. Distribution of Impedance in the Line

It is desirable to find means of reducing the potential difference between the telegraph wires and the ground in order to enable the telegraph wires to be handled without danger in making line repairs. This result is attained by distributing along the entire line the impedance which serves to limit the strength of the interfering current. For example, an impedance of 10,000 ohms would be subdivided into 5 portions of 2000 ohms each. Under these conditions the potential difference induced in the telegraph line is, so to speak, consumed as fast as it is produced, along the line, by the fall of potential caused in the impedance. If the induction produced was uniform along the line and if the impedance were uniformly distributed, the result would be a wire having the same potential with respect to the ground at all points.

In the particular case where the impedance is produced by a series of stoppers placed at different points in the line, use should be made of intermediate dischargers like the resonance-circuits shown in Fig. 7.

The method of protection based purely and simply on the properties of resonance has been used, up to the present time, only on lines working with Morse apparatus. Its efficiency would very likely be questionable for telegraph systems of the Hughes or Baudot type. However, it would not be unreasonable to expect that it could be serviceable with such systems.

Presented at the 339th meeting of the American Institute of Electrical Engineers, Pittsburgh and New York, April 9 and 12, 1918.

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NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

BY R. E. HELLMUND

ABSTRACT OF PAPER

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. Previous publications on this subject, including text books, are usually rather vague and incomplete, especially with regard to the secondary magnetizing currents and the field forms. Furthermore, numerous conflicting statements are found in previous literature, thus leaving the subject, as a whole, in a rather confusing condition. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The treatment of all cases is rather uniformly based on the following fundamental considerations. The sum of all e.m.fs. must be zero in both the primary and secondary circuits. With the impressed primary e.m.f. known, this leads to definite conditions governing the primary counter e.m.fs. The same law applied to the secondary circuits gives the condition that the induced voltages must be equal and opposite to the ohmic drops. Having thus certain laws governing the voltages to be induced in the windings, we have at once certain laws governing the fluxes for inducing these voltages. In most cases, it is then found, that only a single definite local distribution of the resultant field satisfies both the conditions for the primary and secondary windings simultaneously; having thus established the required resultant field distribution, we have at once laws for the required resultant distribution of ampere turns around the circumference to bring about such field distributions. Whenever certain portions of the circumference have conductors of either the primary winding or secondary winding alone, the resultant ampere turns found represent at once the ampere turns for the single winding located at this portion. With this fact known, a number of facts regarding the currents can be determined. Whenever both primary and secondary turns coincide at the same portion of the circumference, certain problems arise in determining the distribution of the resultant ampere turns between the two windings. A number of different considerations are used for the various cases to assist in the solution of these problems; all these considerations are, however, based on simple facts.

In order to demonstrate the method by means of the simplest mathematics, two cases of small practical application are given first merely in order to separate the fundamentals from a rather large amount of mathematics which, while necessary, in connection with the more practical cases, are of little value in connection with the understanding of the fundamental principles.

Starting out from these simple cases, the influences of various factors are taken up in the following cases, one at a time, because a simultaneous consideration of all of them make a clear understanding practically impossible.

The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.

By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.

Introduction

THE EXACT nature of the fields and magnetizing currents of single-phase induction motors, especially of the currents flowing in the rotor, have been the subject of a good deal of speculation in the past. The introduction of the phase converter as a commercial machine has given additional interest to these problems, and has furthermore introduced a number of new problems, as for instance, the determination of the output voltage. The following studies of the no-load conditions in these machines, which form the necessary basis for further studies of the load conditions should, therefore, be of interest to students and designers of such machines.

The fundamental laws and basic considerations used in the paper are very simple and should be easily understood. Also, the mathematics are relatively simple, involving, with very few exceptions, nothing but the solution of several equations with several unknown quantities; the integrals used in a few places are of the simplest kind. Nevertheless the derivations involve by necessity a rather large number of formulas, which will be of interest only to a limited number of readers. An attempt has, therefore, been made to write the paper, so that it can be read to good advantage, without following through the mathematical portions, marked by vertical rules.

The results, given graphically in a large number of figures, and the conclusions reached, should be of general interest. Some of the final conclusions and formulas will enable the designing engineer to predetermine machine performance more accurately than was possible with the previous methods, most of which are based on more or less incorrect assumptions.

Wherever a correct and complete mathematical solution has not been attempted, because it involves an impractical amount of work, the considerations have been carried far enough to indicate plainly to the student and designer how the most favorable design can be obtained and how bad combinations can be avoided.

A number of simple theoretical cases are given first, because it is believed that the demonstration of the methods employed in this paper on such simple cases will be of assistance in the understanding of the more general solutions of the problems given later on.

Case No. I

Single Primary Coil, Single-Phase Secondary

The simplest type of a phase converter is shown with its rotor in different positions in Figs. 1a to 1e. It has a single concentrated primary coil $P\,P'$, a single concentrated secondary or rotor coil $S\,S'$ and a single concentrated output or tertiary coil

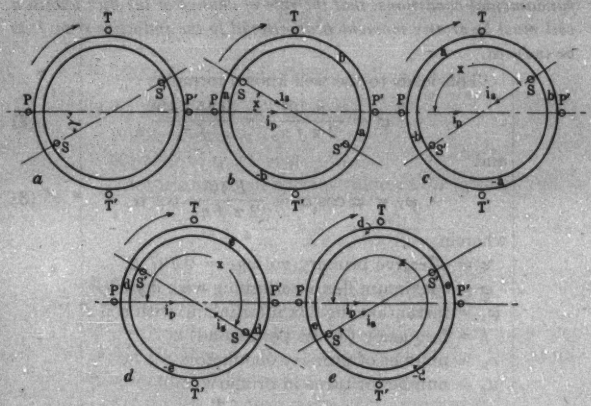

Fig. 1

$T\,T'$, located on the stator 90 electrical space degrees shifted against the stator coil $P\,P'$.

The first fundamental condition, which must be fulfilled in a machine of this kind, is that *within every closed circuit, the sum of all counter e. m. f's. must at any moment be equal and opposite to the impressed e. m. f.*

Applying this law to the primary coil $P\,P'$, we know, therefore, that if we impress a sinusoidal voltage wave with instantaneous voltage values

$$-e_p = -E \sin \alpha$$

that the counter e. m. f's. in this winding must be always opposite in direction, but of the same numerical value, namely,

$$e_p = E \sin \alpha \tag{1}$$

where E is the maximum crest value of the voltage and α the time angle.

The counter e. m. fs. consist of ohmic drops and electromagnetically induced voltages.

The ohmic drops at no-load are usually very small in a-c. machines relative to the inductive voltage and may, therefore, for the present be neglected.

The inductive voltage is always proportional to the rate of change of the total flux interlinking with the coil $P\,P'$. Since the voltage to be induced is definitely given by equation (1), the flux values within the coil are also definitely given by *the second fundamental conditions, that the rate of change of the flux within a coil must be at any moment proportional to the inductive e. m. f. to be induced.*

This leads to the well known formulas

$$\varphi = \frac{E\,10^8}{2\,\pi\,f\,n_p} = \frac{0.45\,e\,10^8}{f\,c_p} \tag{2}$$

and

$$\varphi_i = \varphi \cos \alpha = \frac{E\,10^8}{2\,\pi\,f\,n_p} \cos \alpha \tag{3}$$

wherein

e = effective primary voltage = $0.707\,E$
φ = maximum flux interlinking with coil $P\,P'$
φ_i = instantaneous flux interlinking with coil $P\,P'$
f = frequency (cycles per second)
c_p = number of primary conductors
n_p = number of turns in primary coil.

These are the only conditions governing the flux in the primary coil. It is altogether immaterial, with regard to the primary, whether these conditions are fulfilled by a stationary flux fluctuating in size, by a single rotating flux of sinusoidal space distribution, or by a number of fluxes rotating in the same or in opposite directions, or even by a combination of fluctuating and rotating fluxes. It is also immaterial with regard to the primary coil how these fluxes are locally distributed within the coil, whether these fluxes are set up by currents in the primary or the secondary coil, whether by direct currents or alternating currents, or a combination of such currents, as long as the above conditions are fulfilled.

If the secondary and tertiary coils are open circuited and inactive, so that none but the primary coil can furnish magneto-

motive forces, the entire magnetization must naturally be furnished by the primary coil $P\,P'$. It is further evident in this case that the flux distribution and magnetic densities within the coil are solely governed by the relative magnetic reluctances of the different paths.

Under the customary assumption of uniform reluctance and neglecting for the present the relatively small magnetic leakage fluxes, the above leads to the well known relations:

The instantaneous gap density is

$$B_i = \frac{p\,\varphi_i}{D\,\pi\,l} = \frac{\varphi_i}{\pi}\,K \tag{4}$$

if $$K = \frac{p}{D\,l} \text{ and}$$

if the average air gap diameter $= D$

Width of core $= l$

Number of poles $= p$

The instantaneous current value, in the primary coil, is

$$i_p = \frac{B_i}{n_p}\,\frac{l}{K_l} = \frac{\varphi_i\,K}{\pi\,K_l\,n_p} \tag{5}$$

where K_l is a constant $= \dfrac{1}{0.315 \times 2g \times G \times S}$

if g = length of air gap on one side; G = the usual gap factor allowing for the slot opening and S = the saturation factor (all dimensions in inches).

From this, the crest value of the current is

$$I_p = \frac{\varphi}{\pi}\,\frac{K}{K_l\,n_p} \tag{5 A}$$

Let us assume now that the coil $S\,S'$ is short-circuited and revolved at synchronous speed by some external means.

By applying the first fundamental condition given above to this secondary coil, we know that in so far as this coil is short-circuited and has no external voltage impressed, we must have a resultant counter e.m.f. of zero induced within the coil. If there is any current flowing within the coil, it will cause an ohmic drop voltage. Therefore, in order to get a resultant of zero we must have an inductive voltage induced internally, which is equal and opposite to this ohmic drop voltage. The resistance of the coil $S\,S'$ is usually very small so that the ohmic

drop is very small and with it also the necessary inductive voltage, as long as the secondary currents are not too large. Under the majority of conditions the required rate of change of the flux interlinking with the coil $S\,S'$ is, therefore, so small as to be negligible, as compared with that required in the primary coil. We are, thus, justified to assume at present for the sake of simplicity, that the flux within the secondary coil remains practically constant. This is, however, theoretically correct only with zero resistance in the secondary.

It is again indifferent with regard to the secondary coil how this condition of practically constant flux within the coil is maintained. We know, however, that if a flux has once been established by some means or other, within the coil, such flux will somehow be maintained practically constant. If the magnetomotive force which has established the flux disappears, or if the secondary coil is moved away from its influence, the flux within the coil will tend to diminish. Any slight decrease of flux will, however, at once induce voltages in the coil which, in turn, cause currents to flow maintaining the flux practically at its original value, if the ohmic drops are zero, or at least negligible.

Let us refer now to Fig. 1a and assume that it happens to represent the rotor position at the time angle $\alpha = 0$, with an angle γ between the coil $P\,P'$ and $S\,S'$ as shown. Assuming further synchronous speed, we know that the time $\alpha = \gamma$ will have elapsed, before the secondary coil has moved an angle γ and coincides with the primary coil. Now we know from formula (1) that the flux interlinking with the primary coil at the time $\alpha = \gamma$ must be

$$\varphi_{i\gamma} = \varphi \cos \gamma \tag{6}$$

If we neglect the leakage between the coils $P\,P'$ and $S\,S'$ this represents also the flux in the secondary coil at this time.

Since the flux in $S\,S'$ is constant, we know, therefore, that this constant flux of coil $S\,S'$, which rotates with the coil at synchronous speed must be

$$\varphi_s = \varphi \cos \gamma \tag{7}$$

Having now an equation for both the total flux φ_i in the primary and the total flux φ_s in the secondary coil, we can easily determine how the fluxes around the air gap must be distributed at any time to always satisfy both the equations (1) and (7) for φ_i and φ_s respectively.

Let us consider for instance a secondary position as shown

in Fig. 1b, with an angle x between $P\,P'$ and $S\,S'$. The time angle corresponding to this position is

$$\alpha = x + \gamma \text{ therefore } x = \alpha - \gamma \tag{8}$$

By reference to Fig. 1b, it is further at once evident that the flux between P and P', that is the primary flux φ_i must be the sum of the fluxes φ_a between P and S and the flux φ_b between S and P'; similarly, we see that the flux between S and S' that is the secondary flux φ_s must be the sum of the flux φ_b between S and P' and the flux φ_a' between P' and S'. Since φ_a' is evidently equal to $-\varphi_a$, we have only two unknown quantities which can at once be found from the two conditions just stated. With the fluxes for the different air-gap portions known and the angles for which they apply the densities can be easily found.

For the time $\alpha = \gamma$ to $\alpha = \gamma + \pi$ which it takes the conductor S to travel from P to P', we can, therefore, write the following equations

$$\varphi_i = \varphi_a + \varphi_b \tag{9}$$

$$\varphi_s = -\varphi_a + \varphi_b \tag{10}$$

It follows by addition and subtraction of (9) and (10)

$$\varphi_a = \frac{\varphi_i - \varphi_s}{2}$$

$$= (\cos\alpha - \cos\gamma)\,\frac{\varphi}{2} \tag{11}$$

$$\varphi_b = \frac{\varphi_i + \varphi_s}{2}$$

$$= (\cos\alpha + \cos\gamma)\,\frac{\varphi}{2} \tag{12}$$

The densities over the distances a and b of Fig. 1b, are, therefore,

$$B_a = \frac{\varphi_a}{x}\,K = \frac{\varphi_i - \varphi_s}{2\,x}\,K$$

$$= \frac{\cos\alpha - \cos\gamma}{x}\,\frac{K}{2}\,\varphi \tag{13}$$

$$B_b = \frac{\varphi_b}{\pi - x}\,K = \frac{\varphi_i + \varphi_s}{2\,(\pi - x)}\,K$$

$$= \frac{\cos\alpha + \cos\gamma}{\pi - x}\,\frac{K}{2}\,\varphi \tag{14}$$

Similarly, we find for the time

$$\alpha = \gamma + \pi \text{ to } \alpha = \gamma + 2\pi$$

in connection with Fig. 1D.

$$B_d = \frac{K}{2}\,\varphi\,\frac{\cos\alpha - \cos\gamma}{x - \pi} \tag{15}$$

$$B_e = \frac{K}{2}\,\varphi\,\frac{\cos\alpha + \cos\gamma}{2\pi - x} \tag{16}$$

The field densities thus obtained are shown in full lines in Fig. 2, with regard to their local distribution over the air gap

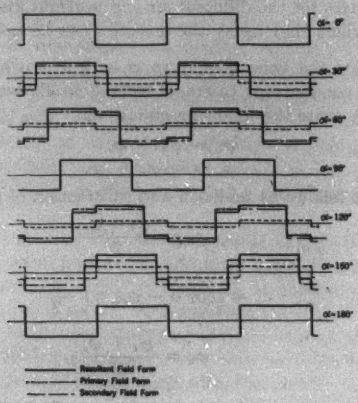

FIG. 2

circumference for a number of values of α as indicated, and under the assumption of $\gamma = 0$, which means that the secondary and primary coils are assumed to coincide at the time angle $\alpha = 0$, that is, when the primary flux has its maximum value. The areas enclosed by the full lines represent the total fluxes, as they must exist to fulfill the two fundamental conditions so far stated, for both the primary and secondary coil.

As previously pointed out, the two conditions of equilibrium could be fulfilled in a number of different ways for each of the windings individually. Since our equations, taking the two windings into account, simultaneously, give only a single solution, however, Fig. 2 represents the only field condition possible under the assumptions made.

For different values of γ somewhat different results will be obtained but usually it will be found as in Fig. 2 that the resultant field travels around with the rotor, *i. e.*, we have a rotating field; it will also be noticed that the distribution of the field varies.

As a matter of convenience in theoretical considerations, it is, of course, always possible to substitute a number of component fields for those obtained from the calculations, but the resultant of such field must always conform to our equations.

The determination of the currents flowing in the two windings must be based on *a third fundamental condition*, namely, *that the sum of all ampere turns acting upon a certain part of the gap circumference must be proportional to the magnetic density in such part*, making the usual assumption of uniform magnetic reluctance.

Referring, for instance, again to Fig. 1b, in which certain current directions have been arbitrarily assumed and indicated by arrows, it is at once evident that the density B_a between P and S, for instance, must be set proportional to the difference between the ampere turns in $P\,P'$ and those in $S\,S'$. Determining similar relation for other portions, we obtain at least two equations with the two unknown current values i_p and i_s, which follow directly from the solution of these equations.

If the secondary coil $S\,S'$ with the turns n_s carries the current i_s in a direction as marked in Fig. 1b, we find the following relations between the currents and densities.

$$B_a = K_l\,(i_p\,n_p - i_s\,n_s) \tag{17}$$

$$B_b = K_l\,(i_p\,n_p + i_s\,n_s) \tag{18}$$

By additions and subtraction of (17) and (18), we get

$$i_p = \frac{B_a + B_b}{2\,K_l\,n_p} = \frac{K\,\varphi}{4\,K_l\,n_p}\left(\frac{\cos\alpha - \cos\gamma}{x} + \frac{\cos\alpha + \cos\gamma}{\pi - x}\right) \tag{20}$$

$$i_s = \frac{B_b - B_a}{2\,K_l\,n_s} = \frac{K\,\varphi}{4\,K_l\,n_p}\left(\frac{\cos\alpha + \cos\gamma}{\pi - x} - \frac{\cos\alpha - \cos\gamma}{x}\right) \tag{21}$$

Similarly, we find for the time $\alpha = \gamma + \pi$ to $\alpha = \gamma + 2\pi$ in connection with Fig. 1d.

$$i_p = \frac{K\varphi}{4K_l n_p}\left(\frac{\cos\alpha - \cos\gamma}{x - \pi} + \frac{\cos\alpha + \cos\gamma}{2\pi - x}\right) \tag{22}$$

$$i_s = \frac{K\varphi}{4K_l n_s}\left(\frac{\cos\alpha - \cos\gamma}{x - \pi} - \frac{\cos\alpha + \cos\gamma}{2\pi - x}\right) \tag{23}$$

Fig. 3 shows in light lines the values for i_p (Curve P) and i_s (Curve S) as a function of the time angle α and as found from (**20**) to (**23**), again for the assumption $\gamma = 0$.

These light curves represent the single and only solution of our equations at any time except at the instant when the secondary and primary coils coincide. At these instances, as for example at the time $\alpha = 0$ with $\gamma = 0$, the above third fundamental condition must be fulfilled as much as at any other time. We have, however, at this instant only the flux $\varphi_a = \varphi_i$, the angle x being 0. Our condition is, therefore, given by

$$B_b = K_l\,(i_p n_p + i_s n_s)$$

that is the same as in equation (**18**), but there being no value for B_a, we have only one equation with the two unknown quantities.

The single equation known for $\alpha = 0$, etc., is satisfied with any values for i_p and i_s, as long as

$$i_p n_p + i_s n_s = \frac{B_a}{K} = \frac{\varphi}{\pi}\,\frac{K}{K_l}$$

or assuming $n_p = n_s$

$$i_p + i_s = \frac{\varphi}{n_p \pi}\,\frac{K}{K_l} = i_x \tag{24}$$

This simply means that from our third condition nothing but the sum i_x of i_p and i_s is known, while it is possible to fulfill equation (**24**) for any value of i_s between $-\infty$ and $+\infty$ by assuming i_p to correspond.

The following consideration makes, however, a determination of i_s for the time $\alpha = 0$, $\alpha = \pi$ etc., possible. Assume coil $S\,S'$ coincides with $P\,P'$ at the time $\alpha = 0$ with $\gamma = 0$, *i. e.*, when $\varphi_s = \varphi$. For any other position, the currents in $S\,S'$ tend to maintain this flux, as previously pointed out, by furnishing whatever magnetizing current is required to do so, in addition

to the primary current. In order to cause such magnetizing current to flow with even the smallest secondary resistance, an inductive voltage induced by slight changes of the flux φ_s is required. While it was permissible to neglect these small changes in our previous calculation of densities and currents, we must not overlook that a small change in the flux is necessary to make i_s flow between the times $\alpha = 0$ and $\alpha = \pi$. We further know that while $S\,S'$ travels from $x = 0$ to $x = \pi$ the primary flux has changed from φ to $-\varphi$ that is, it has reversed its direction with regard to $P\,P'$. Its direction with regard to SS' which has traveled 180 deg. in the meantime is, however, the same for both $\alpha = 0$ and $\alpha = \pi$. In other words we know that while φ_s undergoes slight variations, it must periodically resume in intervals of π the same value, which is φ in our present case. This, in turn, means that any flux decrease in coil $S\,S'$ must be followed by an increase, further, that for a full time interval π, the sum of all increases must equal the sum of all decreases. If, therefore, $d\,\varphi_s$ represents the infinitely small flux change during a time dt, we know that

$$\Sigma\, d\,\varphi_s = 0$$

Since the rate of change of the flux φ_s determines the voltage, we have $\frac{d\,\varphi_s}{dt} = e_s = i_s\, r_s$ or $d\,\varphi_s = i_s\, r_s\, dt$ where e_s = the voltage induced in $S\,S'$ and r_s the resistance. We have further

$$\Sigma\, d\,\varphi_s = r_s\, \Sigma\, i_s\, dt = 0 \qquad (30)$$

between the limits 0 and π.

The value $\Sigma\, i_s\, dt$ represents the area included by the secondary current curve over the time 0 to π, the resultant of which must, therefore, be zero, if r has a value different from 0.

From this, we can directly derive our *fourth fundamental condition*, namely, *that the voltages induced and, therefore, the currents flowing in the secondary coil must have an average value of zero, over each complete time period of* π.

In other words this means that the areas of a curve, including all positive current values must be equal to the area of a curve, including all negative current values.

An inspection of curve S, in Fig. 3, reveals that all current values of i_s between $\alpha = 0$ and $\alpha = \pi$ are positive, and it is, therefore, at once evident that the current can be negative only for infinitely short time intervals at $\alpha = 0$, $\alpha = \pi$ etc. This means that the height of the negative current area must

be $-\infty$, in order to give with an infinitely small basis an area equal to the corresponding positive current area.

The value of i_s for $\alpha = 0$, etc., having been thus determined, it follows at once from (**24**) that

$$i_p = \infty + \frac{\varphi}{n_p \pi} \frac{K}{K_l} = \infty$$

The theoretical infinite current values are indicated by heavy vertical lines in Fig. 3. The conclusion to be drawn from the

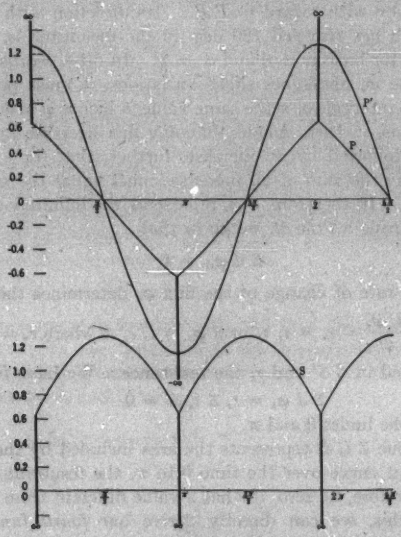

FIG. 3

completed current curves is that the primary currents are of line frequency with very marked odd higher harmonics, while the secondary currents are of double line frequency with very marked higher harmonics; the fact that the positive and negative wave shapes are materially different indicates the rather unusual existence of marked even harmonics in the secondary currents.

Fig. 3 also shows in a curve P' the value of i_p as obtained from (**5**) for the open-circuited secondary. It will be seen that

the effective i_p of curve P, without the infinitely high extension would be even smaller than that of curve P', which we know to be in contradiction to the facts. This is additional evidence for the necessity of a sudden change of the currents for $\alpha = 0$ and $\alpha = \pi$.

In line with the above, Fig. 2 shows in dotted lines the field components supplied by the stator and the rotor for all cases except for $\alpha = 0$ and $\alpha = \pi$ in which cases these values are ∞; in case of $\alpha = \frac{\pi}{2}$ the dotted lines do not appear because the field induced in the primary is zero, and that induced by the secondary coincides with the total field.

For most other values of γ, similar conditions are obtained, but the coinciding of the coils and, therefore, the infinite current values occur at different values for α.

While the current curves derived so far are correct under the assumption made, attention may already be briefly directed towards a number of facts, which have marked modifying effect in actual machines.

In practise, it is, of course, impossible to concentrate the effect of the coil conductors into a point, and, therefore, the passing of the secondary and primary coils is not actually infinitely short. Consequently, we obtain in the secondary, even though the coils are concentrated as much as practicable, certain limited but rather large current values for a rather short time, in place of the infinite negative values for an infinitely short time. Similarly, the additions to the calculated primary current wave are not infinitely large, but will appear as rather sharp corners of limited height extending over a very brief period of time.

Another equally important feature, which makes the existence of infinite current values in actual practise impossible, is the fact that any machine has magnetic leakage fluxes. These fluxes no matter how small they may be will induce very appreciable voltages affecting the working conditions to a marked degree, if the rate of change in such fluxes is large; the latter condition naturally prevails if the currents setting up the leakage fluxes suddenly tend to assume infinite values, as in our case.

While, for these reasons, the previous current curves, as well as some of those given later, do not picture the actual conditions correctly, they are nevertheless very instructive insofar as they indicate the strong tendency towards higher harmonics and

secondary phenomena caused thereby. This will be discussed more in detail later on.

The fluxes interlinking with the tertiary phase converter coil $T\,T'$ can easily be determined, since the densities at all portions of the circumference are known, if we neglect again the leakage fluxes. With the fluxes known, the induced tertiary voltages follow directly by finding the rate of change of the tertiary flux $\frac{d\,\varphi_t}{dt}$.

We have

For the time $\alpha = \gamma$ to $\alpha = \gamma + \frac{\pi}{2}$ (Fig. 1b)

$$\varphi_t = B_b \frac{\pi}{2} - B_a x - B_b \left(\frac{\pi}{2} - x\right)$$

$$= (B_b - B_a) \frac{x}{K} \tag{25}$$

For the time $\alpha = \gamma + \frac{\pi}{2}$ to $\alpha = \gamma + \pi$ (Fig. 1c)

$$\varphi_t = B_a \left(x - \frac{\pi}{2}\right) + B_b (\pi - x) - B_a \frac{\pi}{2}$$

$$= \frac{(\pi - x)}{K} (B_b - B_a) \tag{26}$$

For the time $\alpha = \gamma + \pi$ to $\alpha = \gamma + \frac{3}{2}\pi$ (Fig. 1d)

$$\varphi_t = B_e \frac{\pi}{2} - B_d (x - \pi) - B_e \left(\frac{3}{2}\pi - x\right)$$

$$= \frac{(\pi - x)}{K} (B_d - B_e) \tag{27}$$

For the time $\alpha = \gamma + \frac{3}{2}\pi$ to $\alpha = \gamma + 2\pi$ (Fig. 1e)

$$\varphi_t = B_d \left(x - \frac{3}{2}\pi\right) + B_e (2\pi - x) - B_d \frac{\pi}{2}$$

$$= \frac{(2\pi - x)}{K} (B_e - B_d) \tag{28}$$

The voltage of the tertiary coil can be found by finding the value $\frac{d\,\varphi_t}{dt} = e_t$ from these equations, after

the previously found values for B_a and B_d in terms of α have been introduced. Thus we obtain for the time

$$\alpha = \gamma \text{ to } \alpha = \gamma + \frac{\pi}{2}$$

$$e_t = \frac{\varphi}{2}\left[\frac{\pi}{(\pi - \alpha + \gamma)^2}(\cos\alpha + \cos\gamma) + \left(1 - \frac{\alpha - \gamma}{\pi - \alpha + \gamma}\right)\sin\alpha\right] \qquad (29)$$

etc.

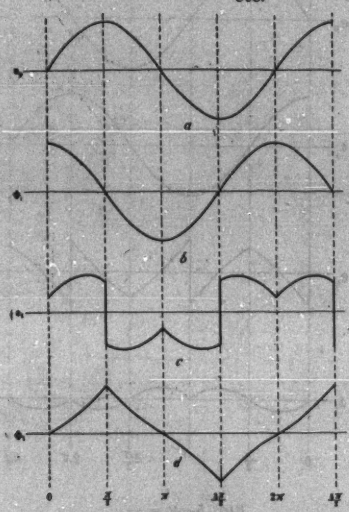

FIG. 4—$\gamma = 0$

Fig. 4 shows the time curves of the primary voltage of the coil $P\,P'$ and the flux φ_t interlinking with this coil as found from (**1**) and (**3**), further the flux φ_t interlinking with the tertiary coil $T\,T'$ and the voltage induced in this coil as found from (**25**), (**26**), (**27**), (**28**) and (**29**).

It will be noted that although the primary voltage and flux follow a sine law, the tertiary flux and voltage are far from such law. The tertiary maximum flux value is the same as that of the primary, but both the average and effective values are lower in case of the tertiary flux.

The case of Fig. 1 for $\gamma = \frac{\pi}{2}$, that is, the case in which coil $S\,S'$ and $P\,P'$ coincide at the time $\alpha = \frac{\pi}{2}$ when the primary flux is 0, is of little interest in connection with the phase-converting problems. It is interesting, however, insofar as we obtain in this case an appreciable tertiary voltage of higher frequency. The fluxes and voltages for this case are shown in

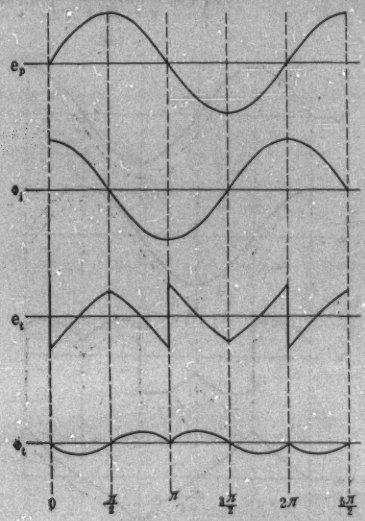

FIG. 5—$\gamma = \frac{}{2}$

Fig. 5. The results obtained are somewhat surprising, since it would appear from a casual consideration that insofar as the resultant secondary flux

$$\varphi_s = \varphi \cos \gamma = \varphi \cos \frac{\pi}{2} = 0$$

is zero, that φ_t should be zero.

Closer investigation shows, however, that while φ_s is zero, as a whole, it is at times the resultant of two opposing fluxes, which add up to zero inside the coil $S\,S'$, but give certain positive and

negative values inside of $T\,T'$ as shown in Fig. 5d. While these fluxes are relatively small, they are of higher frequency and, therefore, induce an appreciable voltage in $T\,T'$. We have, therefore, for $\gamma = \frac{\pi}{2}$ a frequency changer instead of phase converter.

While a further study of these phenomena with relation to the resistances and leakage fields would be interesting it has been omitted since it is not related to the subject matter of this paper.

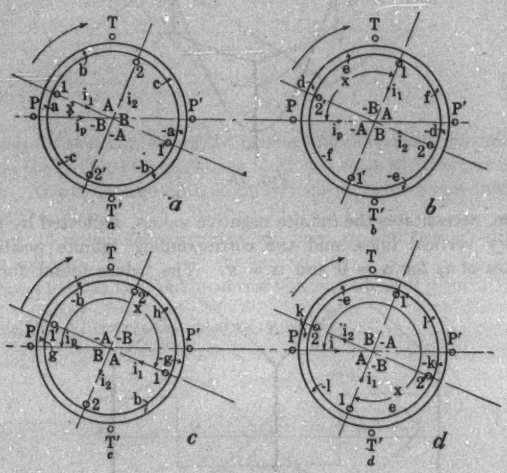

FIG. 6

CASE NO. 2.

Single Primary Coil, Two-Phase Secondary

Let us now consider a converter with a two-phase secondary with one coil per pole per phase, as shown in Fig. 6. Attached appendix gives the detail calculations carried through along the same lines as in the previous case.

The only difference in the calculations between this and the previous case is that two instead of one secondary coil has to be considered, which leads to three equations with three unknown quantities, instead of two.

Fig. 7 shows for $\gamma = 0$ the currents in the primary and in the two secondary coils. The calculation from the formulas gives again only positive currents for i_1, which for reasons previously

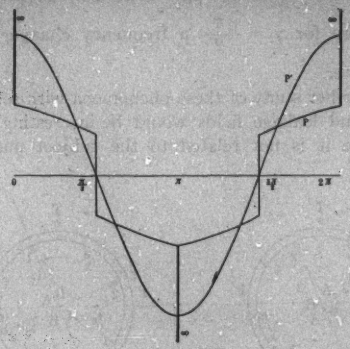

FIG. 7A

given, necessitates the infinite negative values, indicated by the heavy vertical lines, and the corresponding infinite positive values of i_2 for $\alpha = 0$ and $\alpha = \pi$. The values found for i_2

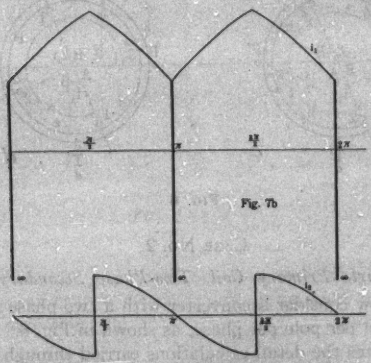

FIG. 7C

satisfy our fourth fundamental condition, without infinite values.

Fig. 8 shows the corresponding primary and tertiary fluxes and voltages.

Since we have now a polyphase rotor, the machine will be self-propelling like any single-phase induction motor. If the machine is asynchronous, the friction will cause a very small slip, so that even if the relation $\gamma = 0$ were originally established, the rotor will slip slightly behind synchronous speed, so that the value of γ changes gradually, but continuously. Therefore, other values than $\gamma = 0$ are of practical interest.

Fig. 9 has been worked out to give the currents for $\gamma = \frac{\pi}{4}$

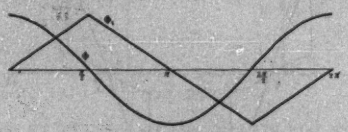

FIG. 8A

that is for the case, where the primary flux reaches a maximum, when the middle of a secondary tooth coincides with the primary coil. As will be seen both secondary currents have now infinite negative values.

Fig. 10 shows the corresponding primary and tertiary fluxes and voltages.

Fig. 11 shows the field forms for various values of α for $\gamma = 0$; while Fig. 12 covers the case of $\gamma = \frac{\pi}{4}$

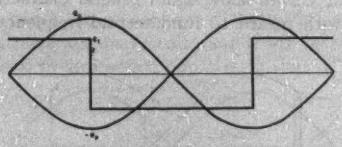

FIG. 8B

$$e_z \text{ effective} = \frac{0.636}{0.707} \text{ or } 0.9\ e_p \text{ effective}$$

As previously mentioned γ changes very gradually all the time with a light running asynchronous machine. Since the fields, currents and tertiary voltages change with γ, as will be seen by a comparison of Figs. 7 and 9, 8 and 10, and 11 and 12 respectively; this means that nearly everything in the machine undergoes a continuous change. Thus, the higher harmonics or current peaks of the primary change their location with regard to the fundamental wave. (Compare Figs. 7A and 9A); the cur-

rents in each secondary coil change materially in both size and shape, (Compare Figs. 7B and 9B, also 7C and 9C); also the tertiary voltages change their wave form materially (Compare Figs. 8B and 10B) as well as their peak values; similar conditions

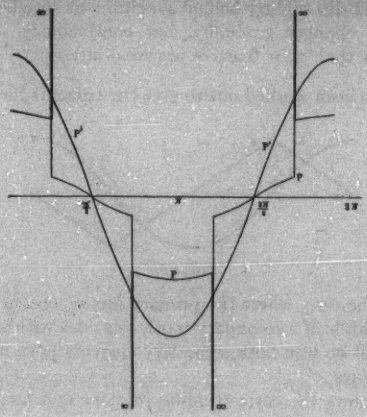

FIG. 9A

apply to the tertiary flux values (Compare Figs. 8A and 10A) interlinking with coil $T\ T'$.

The current curves show again general characteristics as in Case No. 1 with regard to fundamental frequency and higher harmonics.

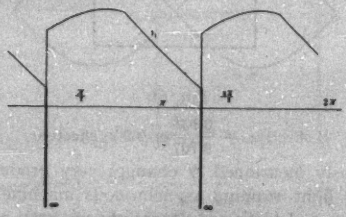

FIG. 9B

The local distribution of the resultant field changes continuously even with constant value of γ, as indicated in Fig. 11, but changes in the value of γ, cause further material changes. (Compare Figs. 11 and 12.) Among other things it will be noted

for instance that the total area for certain values of α is smaller in Fig. 12 than for similar values of α in Fig. 11. Figs. 11 and 12 show, however, in both cases that the resultant field rotates in the direction of the rotor. These figures also show in dotted

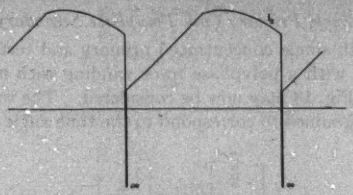

FIG. 9C

lines the fields as furnished by the primary and secondary coils alone.

The lines y y indicate the center of the resultant field and plainly show its travel from left to right; the center line of the

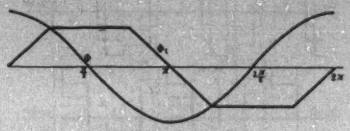

FIG. 10A

field induced by the primary currents is marked z z, and is of course, stationary; the center line of the fields induced by the secondary currents are marked x x, and it will be seen that they move in general in a direction opposite to that of the lines y y,

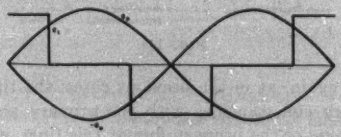

FIG. 10B

$$e_z \text{ effective} = \frac{0.636}{0.707} \text{ or } 0.9\ e_p \text{ eff.}$$

although their speed is very irregular and reverses for certain intervals of time.

In view of the irregular behavior of nearly everything in the motor, it is rather surprising that exact calculations show the effective value of the tertiary voltage to be unchanged with

changing γ namely always 90 per cent of the effective primary voltage, although as mentioned before, the wave shape of the tertiary voltage changes continuously with γ.

Case No. 3

Single Primary Coil, Polyphase Secondary

A case with single concentrated primary and tertiary coils in combination with a polyphase rotor winding with m rotor coils as shown in Fig. 13 may now be considered. The rotor position as shown is assumed to correspond to the time angle $\alpha = 0$.

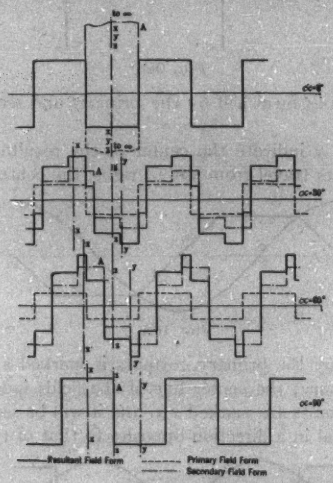

Fig. 11

By finding again, as in the previous cases, the time at which each secondary coil coincides with the primary and by determining the value of the primary flux at this time, we can easily find the constant total fluxes interlinking with each secondary coil.

The secondary coils pass coil $P\,P'$ as follows.

Coil 1 at the time γ
" 2 " " " $\gamma - \beta$
" 3 " " " $\gamma - 2\,\beta$
" n " " " $\gamma - (n - 1)\,\beta$
etc.

Consequently, the constant fluxes interlinking with these coils are

$$\begin{aligned} \varphi_1 &= \varphi \cos \gamma \\ \varphi_2 &= \varphi \cos (\gamma - \beta) \\ \varphi_n &= \varphi \cos [\gamma - (n - 1)\ \beta] \\ \varphi_r &= \varphi \cos [\gamma - (r - 1)\ \beta] \\ \varphi_{r+1} &= \varphi \cos (\gamma - r\ \beta) \\ &\text{etc.} \end{aligned} \tag{31}$$

We also have $\gamma = (r - 1)\beta$ and $\beta = \dfrac{\pi}{m}$

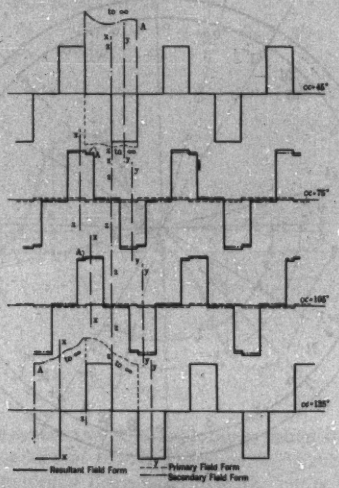

FIG. 12

The determination of the fluxes in the individual teeth and the densities, by the previous method of writing as many equations as there are tooth fluxes, would be, of course, very laborious in the present case with many secondary teeth. The calculations immediately following this prove, however, that the total flux of each secondary tooth can be found as one-half the difference of the total fluxes interlinking with the two adjacent coils, as found above in (**31**). The individual part fluxes of the rotor tooth opposite the primary coil, that is, the fluxes for the angles u and v in Fig. 13 forming part of tooth R can be determined by a

similar simple calculation. After all part fluxes are known, the densities follow, of course, directly.

Let us assume for the present a field distribution for the resultant rotor field as shown in Fig. 14, and if we assume further that the flux in tooth N of Fig. 13 is represented by the area

$$\varphi_N = n\, n_1\, p_1\, p$$

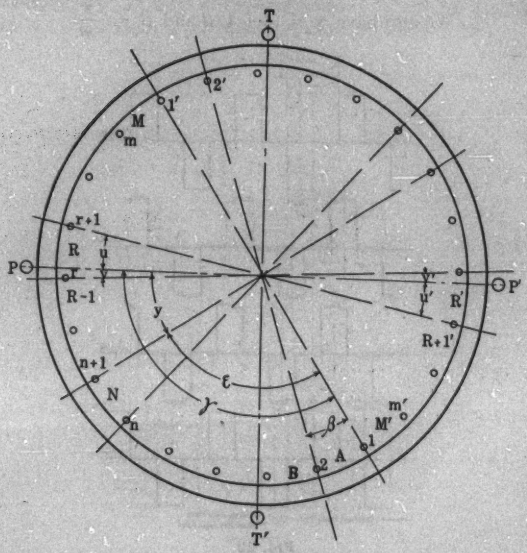

FIG. 13

and that of the opposite tooth N' by

$$\varphi_{N'} = n'\, n_1'\, p_1'\, p'$$

Let further

$$\begin{aligned} \text{area } n\, p\, P &= Y \\ \text{“ } P\, n'\, p' &= Y' \\ \text{“ } n_1\, p_1\, P &= X \\ \text{“ } P\, n_1'\, p_1' &= X' \end{aligned}$$

The resultant flux in coil n is then according to Fig. 14

$$\varphi_n = Y - Y' \tag{32}$$

and that in coil $n + 1$ is

$$\varphi_{n+1} = X - X' \tag{33}$$

It follows further from the figure that

$$Y = X + \varphi_N$$
$$Y' = X' - \varphi_{N'} \tag{34}$$

therefore, we can write (32) by introducing these values

$$\varphi_n = X + \varphi_N - X' + \varphi_{N'} \tag{35}$$

or since obviously opposite teeth carry the same amount of flux and, therefore,

$$\varphi_N = \varphi_{N'}, \text{ we get}$$
$$2\,\varphi_N = \varphi_n - (X - X') = \varphi_n - \varphi_{n+1}$$
$$\varphi_N = \tfrac{1}{2}\,[\varphi_n - \varphi_{n+1}] = \tag{36}$$
$$\frac{\varphi}{2}\quad [\cos(\gamma - (n-1)\,\beta) - \cos(\gamma - n\,\beta)]$$

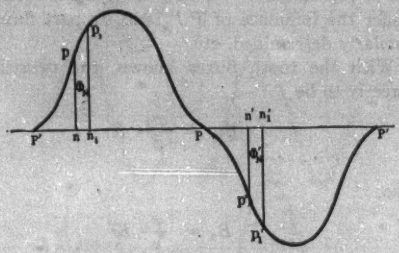

FIG. 14

It appears that the teeth fluxes found from (36) are independent of α and, therefore, constant. They are also uniformly distributed over the entire tooth width in all teeth except in the teeth R and R', which are influenced by the currents in P and P'.

It is evident from Fig. 13 that

$$\varphi_R = \varphi_v + \varphi_u \tag{37}$$

also that

$$-\varphi_i = \varphi_v + \varphi_{R-1} + \ldots\ldots\ldots \varphi_B$$
$$+ \varphi_A + \ldots\ldots\ldots \varphi_{R+1'} + \varphi_{u'} \tag{38}$$

Since obviously $\varphi_{u'} = -\varphi_u$ and since all values in (38) except φ_i, φ_v and $\varphi_{u'}$ have been found to be constant, we can write

$$-\varphi_i = \varphi_v - \varphi_u + K_2 \tag{39}$$

where $K_2 = \Sigma\,\varphi$ from φ_{R-1} to $\varphi_{R+1'}$.

By adding (37) and (39) we find

$$\varphi_v = \tfrac{1}{2}(\varphi_R - \varphi_i - K_2) \tag{40}$$

and

$$\varphi_u = \tfrac{1}{2}(\varphi_R + \varphi_i + K_2) \tag{41}$$

We know from (36) that

$$\varphi_R = \frac{\varphi}{2}[\cos(\gamma - (r-1)\beta) - \cos(\gamma - r\beta)]$$

and

$$\varphi_i = \varphi \cos \alpha$$

therefore, we can now determine φ_v and φ_u in terms of α for the time

$$\alpha = -u \text{ to } \alpha = v.$$

Since $\cos \alpha$ enters the values for φ_v and φ_u, we find that while the sum of the two is constant, each of the two values varies with the time.

For the time $\alpha = v$ to $\alpha = v + \beta$ the tooth $R - 1$ is under the influence of $P\,P'$ and its part fluxes can be similarly determined, etc.

With the tooth fluxes known, the densities follow directly to be

$$B_N = \frac{\varphi_N}{\beta} K$$

etc.

also,

$$B_v = \frac{\varphi_v}{v} K \tag{42}$$

$$B_u = \frac{\varphi_u}{u} K$$

The step lines of Fig. 15 represent the resultant field distribution for a motor, as per Fig. 13, with $m = 6$, that is, with six secondary slots per pole for various values of α and under the assumption that γ = a multiple of β, which means that one of the secondary coils coincides with P for $\alpha = 0$ and the maximum value of the flux φ_i.

It is plainly evident again from Fig. 15 that the resultant field travels with the secondary. The sine curves shown in the figure, as well as equation (36), reveal the fact that the sine curve cuts the center of each step. This fact applies to any polyphase rotor, even Case No. 2, although it was not so evident there. In other words, we see that the magnetizing currents in the secondary create as much as possible a resultant rotating field of sinusoidal

space distribution, even with the unfavorable rectangular distribution of the primary field assumed in our present case. The resultant field will approach a sinusoidal distribution the closer, the larger the number of secondary teeth and the more the secondary resistance approaches the zero value.

Every slot, primary or secondary, produces a step, unless the current in the slot happens to be zero. It will further be seen that the flux of each secondary tooth is, of course, uniformly distributed over the tooth, except in the tooth opposite to the primary slot; in this case, the ampere conductors in the primary

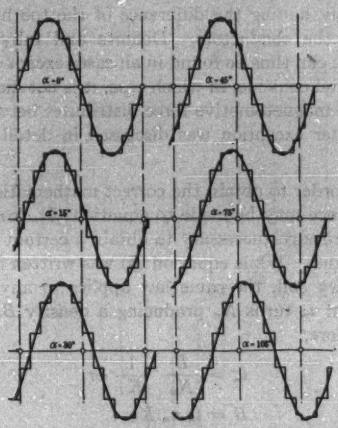

FIG. 15

slot form an additional step, without, however, changing the total flux of the particular secondary tooth.

The currents could again be determined as in the previous cases, by setting the sum of all magnetomotive forces acting upon each tooth or its parts proportional to the known densities. This would, with many coils, lead to an impractical amount of calculations. A much simpler method follows from the simple consideration that the difference between the densities adjacent to each slot is naturally caused by the ampere conductors in the slot.

If we make again the previous assumptions of uniform reluc-

tances, etc., this fact points to the natural conclusion that the difference in the densities at each side of a slot is proportional to the ampere conductors in the slot. This in turn gives, for our present problem, *a fifth fundamental condition to be met* (which now takes the place of the third condition utilized previously), namely, that with densities for all tooth portions being definitely established by other conditions, *the ampere conductors of each slot must at any time be proportional to the difference in magnetic densities at the two sides of the slot.*

Since the densities of all tooth portions are known, this condition offers a very simple way for finding the current in the various coils by simply finding the difference of the tooth densities at each side of the conductors. Definite and fully determined current values can thus be found in all cases except when a rotor and a stator coil coincide, in which case, it is not directly known how the total magnetomotive force distributes between the two coils; this latter exception was discussed in detail under Case No. 1.

In order to obtain the correct mathematical relations, reference may be made to equation (**5**), from which the ampere turns necessary to obtain a certain density may be found. While equation (**5**) was written for the single primary coil, the same law applies to any coil with a current i_n turns N_n producing a density B. We have, therefore,

$$i_n = \frac{B}{N_n} \frac{1}{K_l} \text{ or,}$$

$$B = i_n n_n K_l$$

Now it is evident that if a single coil produces a density B, we will have a density of say $+B$ at one side of the slot and $-B$ at the other side, giving a difference B_d between the densities on both sides of the slot of

$$B_d = B - (-B) = 2B = 2 i_n n_n K_l \text{ or,}$$

$$i_n = B_d \frac{1}{2 n_n K_l} \tag{43}$$

With the exception previously stated, we find, therefore,

$$i_1 = (B_{M'} - B_A) \frac{1}{2 n_1 K_l}$$

$$i_2 = (B_A - B_B)\frac{1}{2\,n_2\,K_l}$$

etc.

$$i_n = (B_{N-1} - B_N)\frac{1}{2\,n_n\,K_l} \tag{44}$$

$$i_r = (B_{R-1} - B_v)\frac{1}{2\,n_n\,K_l} \tag{45}$$

$$i_p = (B_v - B_u)\frac{1}{2\,n_p\,K_l} \tag{46}$$

$$i_{r+1} = (B_u - B_{R+1})\frac{1}{2\,n_{r+1}\,K_l} \tag{47}$$

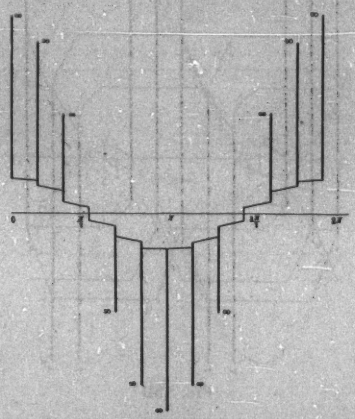

FIG. 16A

Since all values of these equations, except B_v, and B_u are constant, it follows that all current except i_r, i_p and i_{r+1} are constant for the time $\alpha = -u$ to $\alpha = v$.

For the next following time $\alpha = v$ to $\alpha = v + \beta$ all currents except i_{r-1}, i_p and i_r are constant, etc. In other words, it follows that any of the secondary currents are constant, except when either of the adjacent teeth of the coil under consideration passes P or P'.

Fig. 16 shows the primary and secondary current curves for the same case as assumed in Fig. 15. In accordance with our

fourth fundamental condition, the ∞ negative values are again indicated by the heavy vertical lines in connection with the secondary currents. Each of the $-\infty$ values reflects into the primary, so that the primary current has a number of those heavy vertical lines shown.

A close study of the secondary currents reveals that each of the currents is uniform for the largest part of the time. The uniform current value is a sine function of the space angle between the particular coil and coil 1.

When the tooth adjacent to the coil under consideration begins to coincide with P or P', the current decreases along part

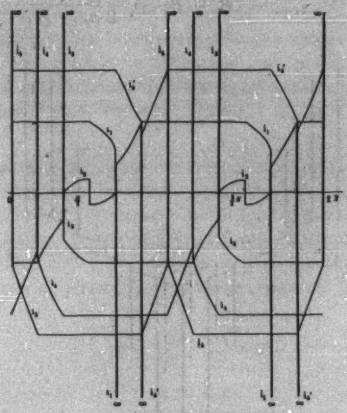

FIG. 16B

of a sine curve until the coil coincides with P or P', when an infinite negative value is reached for an instant. Subsequently, the current increases again along part of a sine curve, while the other adjacent tooth passes P, until the previous constant value is reached again and maintained until the coil approaches the next primary slot.

These general characteristics apply to all cases considered so far. The constant current value over a certain period does, however, not exist in Figs. 3, 7 and 9, of case 1 and 2, because there is at all times at least one tooth adjacent to each secondary slot coinciding with a primary slot. Therefore, we have in

these cases only those parts of the secondary current curves which are infinite or those which follow parts of sine curves. In practise, the infinite values will, of course, again be changed to definite high current peaks as pointed out in Case No. 1.

The determination of the tertiary voltages by means of the previous methods with a number of equations would again be very laborious in the present case. We have found, however, that the density over each secondary tooth is uniform and constant unless it coincides with a primary slot. Since in our case, a secondary tooth passing the tertiary slots T or T' never coincides at the same time with the primary slot P or P', as

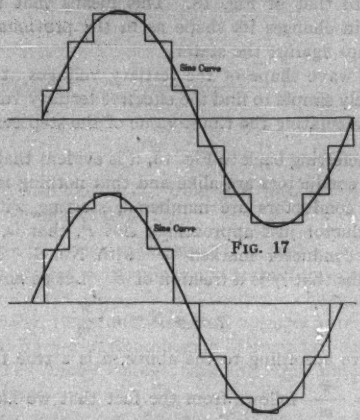

FIG. 17

FIG. 18

long as $m \geqq 2$, each tooth flux and density remains constant, while passing and cutting the tertiary conductors.

The determination of the tertiary voltage is, therefore, very simple in connection with the present case, if we apply the theory of cutting magnetic lines in place of figuring with the interlinking fluxes. The tertiary voltage is then simply to be set proportional to the density of the teeth passing the coil sides.

If we determine, therefore, from previous formulas the resultant field form, disregarding the extra step caused temporarily by the primary in each secondary tooth while it passes a primary slot, we have at the same time the tertiary voltage

wave. Figs. 17 and 18 show the resultant field and tertiary voltage wave, thus determined for six secondary slots per pole. Fig. 17 applies for $\gamma = 0$, that is, under the assumption that a secondary slot happens to coincide with the primary slot at the time $\alpha = 0$, while Fig. 18 applies for $\gamma = \frac{\beta}{2}$, that is, when the middle of a secondary tooth happens to coincide with the primary slot at the time $\alpha = 0$.

With an asynchronous machine, the small slip will gradually change the values of γ and, therefore, the conditions from that of Fig. 17 to that of Fig. 18. This means that the tertiary voltage again changes its shape as in the previous case while the rotor slips against the stator.

With the wave shape of the tertiary voltages at hand, it is comparatively simple to find the effective tertiary voltage values by simply calculating the r.m.s. value of the step curve.

Referring back to Fig. 13, it is evident that all secondary conductors are alike and that nothing is changed if the conductors are numbered, starting with 1 at the conductor just approaching slot P, that is, numbering the conductor marked "r" with No. 1. This simply means that γ is a fraction of β. Let us now set

$$\gamma = a\,\beta = a\,\frac{\pi}{m}$$

where according to the above, a is a true fraction and $\beta = \frac{\pi}{m}$ follows from the fact that we have m teeth per pole, *i.e.*, for the arc π. Introducing this in equation (**42**) for B_N, and using equation (**36**) for φ_N, we obtain

$$B_N = \frac{\varphi_N\, m}{\pi} K = \frac{\varphi\, m\, K}{2\,\pi}$$

$$\left[\cos\left(a - (n+1)\frac{\pi}{m}\right) - \cos\left((a-n)\frac{\pi}{m}\right)\right] \quad (\mathbf{48})$$

From this it can be shown that all teeth from A to M inclusive, have positive densities for values of $a = 0$ to $a = \frac{1}{2}$. The effective value of the total positive wave can, therefore, be found, by considering the teeth A to M, inclusive, while $a = 0$ to $a = \frac{1}{2}$. The effective value of the total positive wave can, therefore, be found by

considering the teeth a to m, inclusive, while $a = 0$ to $a = \frac{1}{2}$. Similarly, the teeth B to A' have to be considered while $a = \frac{1}{2}$ to $a = 1$.

The effective density value and, therefore, the effective tertiary voltage is proportional to the square root of the mean height of the area, showing the square values of all positive densities. The area of each step of the curve is

$$B_N^2 \frac{\pi}{m}, \text{ therefore, the total area is}$$

$$\Sigma\, B_N^2 \frac{\pi}{m}\Bigg|_A^M \text{ and}$$

$$e_T \text{ effective} \cong \sqrt{\frac{\Sigma\, B_N^2 \frac{\pi}{m}}{\pi}}\Bigg|_A^M$$

$$= \sqrt{\frac{\Sigma\, B_N^2}{m}}\Bigg|_A^M \tag{49}$$

By introducing B_N from (48) into (49) the tertiary voltage follows directly.

It will again be found, as in Case No. 2 that as long as $m \geq 2$, the effective tertiary voltage is constant, and independent of γ, although the wave shape is continuously changing with γ. The curve of Fig. 19 shows the tertiary voltages as obtained from (48) and (49) for various numbers m of secondary slots per pole in terms of the primary voltage, assuming equal number of turns in both windings.

The difference between the two effective voltages is caused principally by the difference in field forms, interlinking with the primary winding and that inducing voltages in the tertiary winding. Such differences are usually taken into account under the names of zig-zag and differential leakage. Therefore, we may call the difference between the values of curve 19 and one, the differential plus zig-zag leakage coefficient; its value is indicated by the right-hand scale in Fig. 19. It will be seen that even with the small number of stator slots assumed so far, a relatively small number of rotor slots is sufficient to reduce this leakage coefficient to a very low value. The leakage coefficient thus found and shown in Fig. 19 seems to conform to the following simple equation

$$\sigma = \frac{0.4}{m^2}$$

With our present assumptions, this represents all the so-called zig-zag leakage; the leakage value usually going under the name of differential leakage is zero in our case.

Case No. 4

Single Primary Coil, Infinite Number of Secondary Phases

The erratic shape of the primary current curve with infinite theoretical values has made a quantitative calculation of the effective primary current value difficult for practical use in all previous cases. The theoretical case of an infinite number of secondary coils and a single primary coil, in which higher harmonic primary currents cannot exist, lends itself better for this particular purpose and may, therefore, be considered next.

The relations derived for the previous case apply, of course,

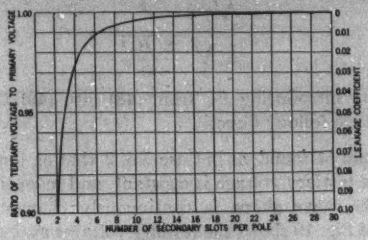

Fig. 19

here if the proper substitutions are made by simply introducing an infinite number of infinitely small secondary teeth into the equations.

Take $$\beta = d\,\epsilon$$

and $$n\,\beta = n\,d\,\epsilon = \epsilon$$

It follows from (**31**) by introducing these values

$$\varphi_{n-1} = \varphi \cos(\gamma - \epsilon + 2\,d\,\epsilon)$$
$$\varphi_{n} = \varphi \cos(\gamma - \epsilon + d\,\epsilon)$$
$$\varphi_{n+1} = \varphi \cos(\gamma - \epsilon) \qquad (\mathbf{50})$$

Similarly, it follows from (**36**)

$$B_N = \frac{\varphi_N}{d\,\epsilon} K$$

$$= \frac{\cos(\gamma - \epsilon + d\,\epsilon) - \cos(\gamma - \epsilon)}{d\,\epsilon} \frac{\varphi}{2} K$$

and
$$B_{N-1} = \frac{\varphi_{N-1}}{d\,\epsilon} K$$
$$= \frac{\cos(\gamma - \epsilon + 2\,d\,\epsilon) - \cos(\gamma - \epsilon + d\,\epsilon)}{d\,\epsilon} \frac{\varphi}{2} K$$
or
$$B_N = \frac{\varphi}{2} K \frac{d\,[\cos(\gamma - \epsilon)]}{d\,\epsilon}$$
$$= \frac{\varphi}{2} K \sin(\gamma - \epsilon) \tag{52}$$
$$B_{N-1} = \frac{\varphi}{2} K \frac{d\,[\cos(\gamma - \epsilon + d\,\epsilon)]}{d\,\epsilon}$$
$$= \frac{\varphi}{2} K \sin(\gamma - \epsilon + d\,\epsilon)$$

It follows directly from (**52**) that the local distribution of the actual resultant rotor flux follows a sine law around the rotor, so that a constant rotor field with sinusoidal distribution rotates with the rotor. This simply means that the steps of the field form found in the previous case are infinitely small, therefore, giving a smooth sine curve.

The density of the field is zero for $\gamma - \epsilon = 0$, that is, at the conductor 1. With these facts known, we can, therefore, plot the resultant rotor field φ_s as shown in Fig. 20 for various values of α under the assumption that $\gamma = 0$.

In determining now the nature of the secondary currents, we proceed again as in the previous case, except that we consider the ampere conductors over an infinitely small angle $d\,\epsilon$ of the secondary in place of the current per secondary coil.

By substitution of the values from (**52**) and (**44**), we find the secondary ampere turns $i_n\,n_n$ over any space angle $d\,\epsilon$ for any time, except the moment when the particular angle $d\,\epsilon$ passes the coil P, to be as follows:

Considering the ampere turns of an angle $d\,\epsilon$ extending an angle of $\frac{d\,\epsilon}{2}$ to either side of the coil point n we have,

$$i_n\,n_n = \frac{1}{2\,K_1}(B_{N-1} - B_N)$$

$$= \frac{\varphi K}{4 K_l} [\sin (\gamma - \epsilon + d\epsilon) - \sin (\gamma - \epsilon)]$$

$$= \frac{\varphi K}{4 K_l} d [\sin (\gamma - \epsilon)] \tag{53}$$

$$= - \frac{\varphi K d\epsilon}{4 K_l} \cos (\gamma - \epsilon)$$

It follows from (**53**) that the local ampere-turns distribution around the rotor circumference follows a cosine law, except at the point of the primary coil.

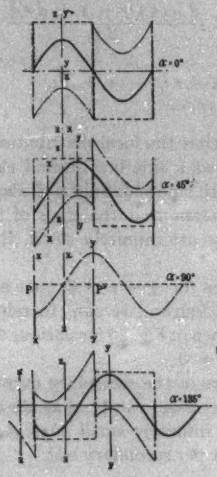

FIG. 20

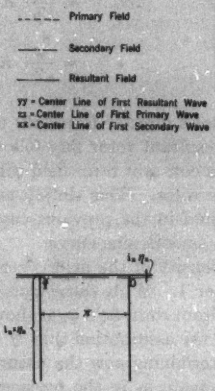

FIG. 21

This is again in line with the previous case, except that the secondary ampere conductors for each infinitely small angle are infinitely small during the periods of constant positive current.

Since the area of the positive wave of the secondary currents has in consequence of this an infinitely small value, we cannot any longer conclude that the negative current values must be infinite, when the secondary coil point passes the primary. On the contrary, we can determine certain definite negative current values.

We know from the previous case that the current in each secondary coil reverses when the particular coil passes the primary coil. The size of the reversed current can be determined from the following consideration. We found that if the flux within a rotor coil decreases after the coil has passed the primary coil, it must increase again to its original value, when it coincides again with the primary coil. In other words, if the flux φ_n decreases according to a function $f(\alpha)$ for a certain time, it must increase again according to some function $f_1(\alpha)$ before the time π has gone by. Now we know that if the current in the coil during the time of decreasing flux is a function $F(\alpha)$ and another function $F_1(\alpha)$ during the remaining time, the voltage which must be induced in the coil follow functions $r\,F(\alpha)$ and $r\,F_1(\alpha)$.

Therefore, we must have

$$\frac{d\,f(\alpha)}{d\,\alpha} = r\,F(\alpha)$$

and

$$\frac{d\,f_1(\alpha)}{d\,\alpha} = r\,F_1(\alpha) \tag{54}$$

and since we know that the sum of all $d\,f(\alpha)$ must be equal to the sum of all $d\,f_1(\alpha)$ we can write

$$r\int F(\alpha)\,d\,\alpha = r\int F_1(\alpha)\,d\,\alpha \tag{55}$$

where the first integral is to be taken for the limits between which the current is positive and the second integral for the limits between which the current is negative. In our case, we know the current to be i_n, that is, a constant and of uniform direction during the time $(\pi - d\,\alpha)$ and of opposite direction during $d\,\alpha$, we have therefore,

$$\int_0^{\pi-d\alpha} F(\alpha)\,d\,\alpha = \int_0^{\pi-d\,\alpha} i_n\,d\,\alpha = i_n\,(\pi - d\,\alpha)$$

and if the current i_{nx} in opposite direction is assumed to be constant during the infinitely short time $d\,\alpha$, we have

$$\int_{\pi-d\,\alpha}^{\pi} F_1(\alpha)\,d\,\alpha = \int_{\pi-d\alpha}^{\pi} i_{nx}\,d\,\alpha, \text{ therefore,}$$

$$i_n\,(\pi - d\,\alpha) = -\,i_{nx}\,d\,\alpha$$

or,

$$i_{nz} = - i_n \frac{\pi - d\alpha}{d\alpha} \tag{56}$$

$$i_{nz} = \frac{\pi - d\alpha}{d\alpha} \frac{\varphi}{4} \frac{K}{K_l} d\epsilon \cos(\gamma - \epsilon) \tag{57}$$

Since both $d\alpha$ and $d\epsilon$ are indefinitely small, it follows that i_{nz} must not any longer be infinite as in the previous cases.

It will be seen from (**57**) that the values for the negative secondary ampere conductors $i_{nz} n_n$ are proportional to cos $(\gamma - \epsilon)$. The time curve of the current conductors in any of the secondary coils appears, therefore, as shown in Fig. 21, if we imagine the value $i_n n_n$ infinitely small.

The determination of the primary current values follows again from the condition that the known resultant field must be proportional to the sum of the magnetizing effect of the primary ampere turns and the just determined secondary ampere turns.

With uniformly distributed secondary conductors, it is again immaterial which coil point is numbered 1, and we may, therefore, make $\gamma = 0$, which merely means that coil point 1 happens to coincide with P when $\alpha = 0$. It further means that any coil n passes P at the time $\alpha = - \epsilon$. This follows directly from Fig. 13.

A coil point with $\epsilon = - \frac{\pi}{2}$ therefore, passes P at the time $\frac{\pi}{2}$. It follows also from (**53**) and (**57**) that both the positive and negative currents in this conductor are zero. Reference to Fig. 20 for $\alpha = 90$ deg. $= \frac{\pi}{2}$ further reveals that at that time the resultant flux in the axis $Z\,Z$ of the primary coil is zero, and that the sine-shape flux curve has its maximum at the primary coil point $P\,P'$. This, in turn, means that there is no change in flux density at these points (the differential coefficient of a cosine curve being zero at its maximum value), and, therefore, no resultant ampere conductors acting. It follows directly that under this condition and with the secondary coil points at P and P' carrying zero current, the primary current for $\alpha = \frac{\pi}{2}$

must also be zero. We know further that all other secondary coil points carry at this moment positive currents as per equation (**53**), and evidently the resultant field at the time $\frac{\pi}{2} = \alpha$ must be induced by these currents alone. Since the field is constant, we know, therefore, that the positive secondary currents alone are just sufficient to induce the necessary resultant field. From this, we can at once derive our *sixth fundamental condition* applying only to the present case, namely, that *the sum of the primary ampere conductors and the negative ampere conductors of the secondary coil points coinciding with the primary coil point must be zero at all times.*

This leads to the following calculation, resulting in a formula for the primary magnetizing current.

With $\gamma = 0$, each secondary coil point n passes the primary coil at the time $\alpha = -\epsilon$ and it follows therefore from (**57**) and the condition just stated, that the primary ampere turns at the time α must be

$$i_p\, n_p = i_{nz}\, n_n = \frac{\pi - d\,\alpha}{d\,\alpha}\,\frac{\varphi}{4}\,\frac{K}{K_l}\, d\,\epsilon \cos(-\epsilon) \tag{58}$$

where $-\epsilon = \alpha$

In this equation, we may neglect the infinitely small $d\,\alpha$ as compared to π and let $\pi - d\,\alpha = \pi$. We may further let $d\,\epsilon = d\,\alpha$, both being infinitely small and obtain

$$i_p\, n_p = \frac{\pi}{4}\,\varphi\,\frac{K}{K_l}\cos\alpha \tag{59}$$

We find the crest value of the current

$$I_p = \frac{\pi}{4}\,\frac{\varphi}{n_p}\,\frac{K}{K_l} \tag{60}$$

It follows from (**59**) that the primary current follows a cosine law, as is to be expected with an impressed sinusoidal voltage and a rotating field with sinusoidal distribution.

We know from (**5a**) the value with open secondary, and, therefore, find the ratio of the open to the closed secondary magnetizing current in the primary to be

$$\frac{\frac{\pi}{4}}{\frac{1}{\pi}} = \frac{\pi^2}{4} = 2.45 \text{ for the present case.}$$

Knowing the flux set up by the primary at each instant, namely,

$$\varphi_p = \left(\frac{\pi}{2}\right)^2 \varphi \cos \alpha$$

and the resultant flux φ_s, the flux set up by the secondary currents, being the difference of the two, can now be easily plotted for the various values of α as in Fig. 20. It will be seen that the flux furnished by the secondary has a rather peculiar space distribution, being the difference between a sinusoidal and rectangular distribution field form. The traveling of the resultant field and the secondary magnetomotive forces is again the same as in previous cases.

The tertiary voltage is in this case evidently of the same value and curve shape as the impressed voltage, the zig-zag and belt leakage being obviously zero.

Case No. 5

Primary Sinusoidal Distribution, Infinite Number of Secondary Phases With Negligible Resistance and Leakage

One of the reasons why the previous cases with a concentrated primary winding were taken up so much in detail, is because it can be definitely proved in these cases, that the resultant rotating field approaches a sinusoidal distribution very closely in spite of the very unfavorable rectangular field form set up by the primary winding. With this fact established, it is obvious that with a distributed primary winding giving in itself a sinusoidal field distribution, a similar resultant field will exist. If we assume further an infinite number of secondary coils, together with the previous assumptions of an impressed primary sinusoidal voltage, a true sinusoidal resultant field will undoubtedly obtain and can simply be used as the basis for all calculations. Similar considerations, together with results obtained in Case No. 4, justify, under these conditions, the assumption of a sinusoidal primary current wave, shifted 90 deg. against the impressed voltage wave. Such an ideal case may now be considered to good advantage because the knowledge of this and the previous opposite extremes is helpful in the understanding of the more practical cases.

With the self-evident assumptions just made, the resultant ampere conductors, that is, the sum of the primary and secondary at each point of the circumference, can again be found from our fifth fundamental condition, that the ampere conductors of each

point must be the difference of the magnetic densities at either side of the point under consideration.

Having thus a means for determining the resultant ampere conductors, and knowing laws for the time curve of the primary current, as well as for the distribution of the primary and secondary conductors, we can easily write down a number of equations expressing these laws, and which are necessary for determining the numerical values of the currents.

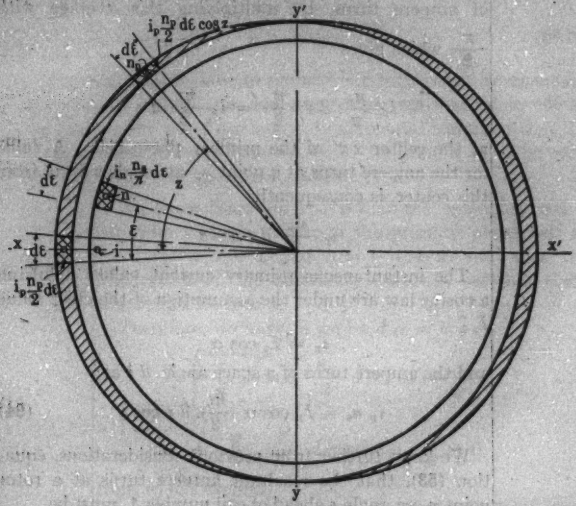

FIG. 22

It is well known that the flux needed to induce the proper counter e. m. f. with a sinusoidal winding distribution, is as a matter of course, larger than with a concentrated coil, which can be taken care of by the usual voltage winding coefficient. The latter is in the present case $c = \frac{\pi}{4}$ so that

$$\varphi = \frac{4}{\pi} \varphi_c. \tag{63}$$

if φ_c is the flux as found from formula (2) for a concentrated primary winding.

If the total number of primary turns is again n_p and if they are first assumed to be uniformly distributed over the pole arc π we would have $i_p \frac{n_p \, d\,\epsilon}{\pi}$ ampere turns for an infinitely small angle $d\,\epsilon$. With sinusoidal distribution, as shown in Fig. 22, we find a maximum value of ampere turns, by multiplying this average with $\frac{\pi}{2}$ which gives

$$i_p \frac{n_p}{\pi} \, d\,\epsilon \, \frac{\pi}{2} = i_p \frac{n_p}{2} \, d\,\epsilon$$

at the center $x\,x'$ of the primary phase belt. A value for the ampere turns at a point n_p an angle z away from this center, is consequently

$$i_p \frac{n_p}{2} \, d\,\epsilon \cos z$$

The instantaneous primary current values following a cosine law are under the assumption of the crest value I_p.

$$i_p = I_p \cos \alpha$$

and the ampere turns of a space angle $d\,\epsilon$ are

$$i_p \, n_z = I_p \cos \alpha \, \frac{n_p}{2} \, d\,\epsilon \cos z \tag{64}$$

We know further from previous considerations, equation (53), that the resultant ampere turns at a rotor point n, an angle ϵ ahead of coil number 1, must be

$$i_{rn} \, n_{rn} = - \frac{\varphi}{4} \, \frac{K}{K_1} \, d\,\epsilon \cos (\gamma - \epsilon) \tag{65}$$

It was also pointed out that the ampere turns furnished by the secondary must be at any point of the circumference and at any time the difference

$$i_p \, n_z - i_{rn} \, n_{rn} = i_n \, n_n \tag{66}$$

For the sake of simplicity, we may again number the secondary coil points so that point 1 coincides with the center of the primary phase belt at the time $\alpha = 0$ which simply means that $\gamma = 0$ in the formula for $i_{rn} \, n_{rn}$. This also means that the rotor coil n coincides

with a stator coil point, which is an angle ϵ from the center at the time $\alpha = 0$ and angle $\epsilon + \alpha = Z$ at the time α.

Thus we can write by setting $n_n = \frac{n_s\, d\,\epsilon}{\pi}$ and by introducing into (**66**) the values from (**64**) and (**65**)

$$I_p \cos\alpha \frac{n_p}{2}\, d\,\epsilon \cos(\epsilon + \alpha) + \frac{\varphi}{4}\ \frac{K}{K_l}\, d\,\epsilon \cos(-\epsilon)$$

$$= i_n \frac{n_s\, d\,\epsilon}{\pi} \qquad \textbf{(66 A)}$$

Since both the maximum primary current I_p and the secondary current are unknown in the single equation derived in (**66 A**) we again make use of our fourth fundamental condition that the resultant secondary current area over the time π must be zero, in order to obtain another equation.

We know from the above that the secondary coil n coincides with the center of the primary phase belt at the time $\alpha = -\epsilon$ and $\alpha = -\epsilon + \pi$ and that the integration of the secondary currents between these times must be zero.

Therefore, we have, if we let $d\,\alpha = d\,\epsilon$

$$\int_{\alpha=-\epsilon}^{\alpha=-\epsilon+\pi} \Big[I_p \cos\alpha \frac{n_p}{2}\, d\,\alpha \cos(\epsilon + \alpha)$$

$$+ \frac{\varphi}{4}\ \frac{K}{K_l}\, d\,\alpha \cos(-\epsilon)\Big] = 0$$

from which we find

$$I_p = -\frac{\varphi}{n_p}\ \frac{K}{K_l} \qquad \textbf{(67)}$$

For open-circuited secondary, we can find from a simple calculation

$$I_p = -\frac{\varphi}{2\,n_p}\ \frac{K}{K_l} \qquad \textbf{(68)}$$

With I_p known, we can find $i_n\, n_n$ from (**66A**) for any point n of the secondary,

$$i_n\, n_n = i_n \frac{n_s}{\pi}\, d\,\epsilon = \frac{K}{K_l}\, d\,\epsilon\ \frac{\varphi}{2} \Big(\frac{\cos(-\epsilon)}{2}$$

$$- \cos\alpha \cos(\alpha + \epsilon)\Big)$$

or

$$i_n\, n_s = \frac{K}{K_l}\ \frac{\pi}{4}\ \varphi \cos(2\,\alpha + \epsilon) \tag{69}$$

It will be seen from (67) and (68) that the primary magnetizing current with closed and synchronously rotating secondary is, therefore, just twice the magnetizing current with open secondary. This is in line with the assumption usually made in commercial designing.

Fig. 23 shows the secondary currents for a number of secondary points corresponding to various values of ϵ, as well as the primary current.

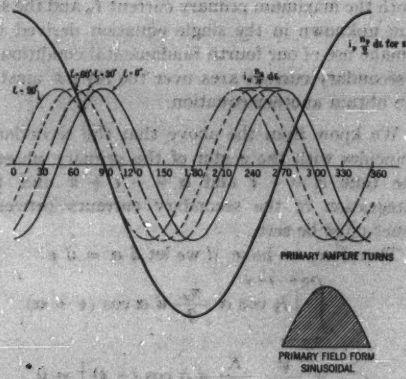

FIG. 23

It will be seen that all the secondary currents are in this case double frequency sinusoidal currents, with the same maximum values of

$$I_n = \frac{\pi}{4}\ \frac{\varphi}{n_s}\ \frac{K}{K_l} \tag{69A}$$

Fig. 23A shows the fields, or rather the magnetomotive forces as furnished by the primary and the secondary, as well as the resultant field, all of which have a sinusoidal local distribution in this particular case. This figure shows better, than those previously given, the fundamental characteristics of the fields and how they travel. The primary magnetomotive force with the axis $s\,s$ is, of course, not rotating, but stationary and al-

ternating; that is, varying in size and polarity. The resultant field with the axis $y\,y$ of practically constant size travels, relative to the stator, at uniform speed and synchronously with the secondary member to the right. The magnetomotive forces furnished by the secondary currents with the axis $x\,x$ are also practically constant in size and travel with equal speed, relative to the stator, to the left. Relative to the synchronously rotating secondary, the resultant field is, therefore, fixed, while the secondary magnetomotive force rotates with double synchronous speed in a backward direction. Such double speed field or rather

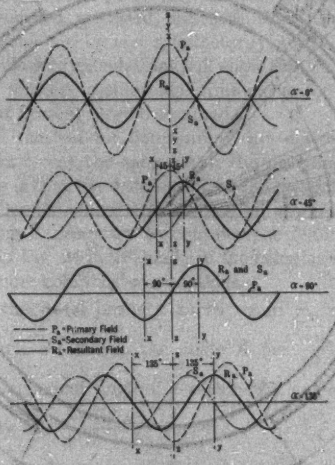

FIG. 23A

magnetomotive force in the secondary is, of course, to be expected in so far as the secondary currents are known to be double-frequency polyphase currents. While the primary field may be considered as the sum of the field set up by the secondary and of the resultant field from a purely mathematical point of view, as has been done at times, it should be kept in mind that actually the primary field is primarily induced from the line; its existence, in turn, causes double-frequency currents in a synchronously rotating rotor which, if they are imagined to exist without primary currents, would induce a field rotating opposite in direction

to the mechanical direction of rotation. As previously mentioned, the speed of this imaginary field is synchronous relative to the primary and double synchronous relative to the rotor. Stating the facts physically correct, we may say that the stationary and alternating primary magnetomotive forces of line frequency, together with the double-line-frequency secondary magnetomotive forces, combine to induce a resultant field which

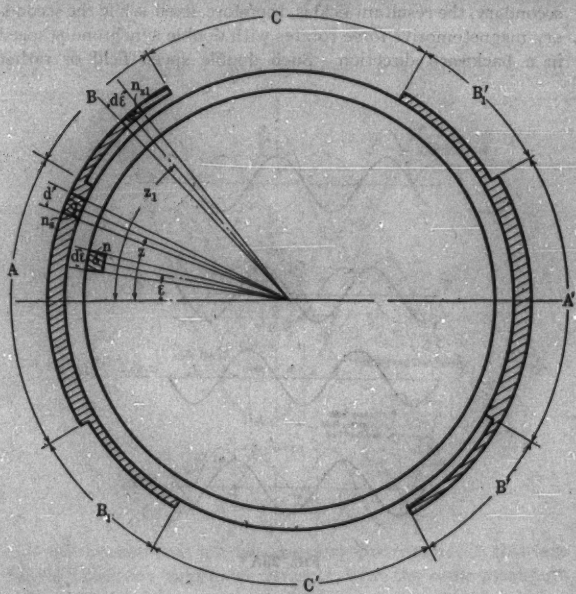

FIG. 24

rotates in the same direction as the mechanical rotation of the secondary and with the same speed relative to the primary.

CASE NO. 6

Primary Field Form Consisting of Straight Line Functions, Infinite Number of Secondary Phases

In practise, it is difficult to obtain sinusoidal distribution of the primary winding, and, therefore, winding distributions as shown in Fig. 24 are usually resorted to; such distribution can

be easily obtained by either concentric windings or chorded diamond coils.

While it is possible with such windings to induce sinusoidal counter e.m.fs. with resultant fields having other than sinusoidal distributions, it may at present be assumed that a sinusoidal distribution of the resultant field exists as in all previous cases of an infinite number of secondary phases. Such a field will always induce sinusoidal counter e.m.fs., thereby satisfying our first fundamental condition.

In view of the strong tendency for the establishment of a resultant sinusoidal field previously demonstrated, this assumption is quite justified, provided, of course, that our other fundamental conditions can be met satisfactorily under this assumption.* The following calculations will show that this is the case.

For similar reasons, a sinusoidal time curve for the primary current may at present also be assumed.

The general method for determining the desired quantities is in the present case the same as in the previous case. The only difference is that the distribution of the primary conductors does not any longer follow a single law all around the circumference, making it necessary to consider the different portions A, B, and C, in Fig. 24 individually in the calculation, this leads naturally to more complicated equations.

If the total primary turns again n_p, we find that an angle $d\,\epsilon$ within the angle A has $\dfrac{n_p\, d\,\epsilon}{(A+B)}$ turns, while an equal angle $d\,\epsilon$ within the angles B and B_1 usually has $\dfrac{n_p\, d\,\epsilon}{2\,(A+B)}$ turns.

A secondary coil point n, an angle ϵ shifted against point 1 must again have the resultant ampere turns—see equation (**53**)

$$i_{rn}\, n_{rn} = -\frac{\varphi}{4}\,\frac{K}{K_l}\, d\,\epsilon \cos(-\epsilon) \text{ if } \gamma = 0 \qquad (70)$$

*Note: It should be remembered that our considerations so far still assume uniform magnetic reluctance, negligible resistances and leakage reactances, etc. In practical machines, both single-phase and polyphase operated, it is quite possible and in practise often actually experienced, that the resultant field form has higher harmonics, caused by magnetic densities in the iron, bad distribution of resistance between bars and rings of squirrel-cage windings, etc. These are important problems in themselves and will be made the subject of a separate study, apart from the present paper.

While passing the angles C and C', which occurs during the times

$$\alpha = \frac{A}{2} + B - \epsilon \text{ to } \alpha = \frac{A}{2} + B + C - \epsilon$$

and

$$\alpha = \frac{3A}{2} + 3B + C - \epsilon \text{ to } \alpha = \frac{3A}{2} + 3B + 2C - \epsilon$$

no other ampere turns act upon this point and these ampere turns must, therefore, be furnished by the rotor, so that

$$i_n n_{nc} = i_{rn} n_{rn}$$

While the same secondary point passes any of the angles A or B, we must find the secondary ampere turns as the difference of the primary and the resultant ampere turns.

Point n passes the angles B during the times

$$\alpha = \frac{A}{2} - \epsilon \text{ to } \alpha = \frac{A}{2} + B - \epsilon$$

$$\alpha = \frac{A}{2} + B + C - \epsilon \text{ to } \alpha = \frac{A}{2} + 2B + C - \epsilon$$

etc.

During this time, we must have

$$i_n n_{nB} = - I_p \cos \alpha \frac{n_p d\epsilon}{2(A+B)}$$

$$- \frac{\varphi}{4} \frac{K}{K_1} d\epsilon \cos(-\epsilon) \qquad (71)$$

Point n passes the angles A during the times

$$\alpha = -\frac{A}{2} - \epsilon \text{ to } \alpha = \frac{A}{2} - \epsilon$$

and

$$\alpha = \frac{A}{2} + 2B + C - \epsilon \text{ to } \alpha = \frac{3A}{2} + 2B + C - \epsilon$$

During this time, we must have

$$i_n n_{nA} = - I_p \cos \alpha \frac{n_p d\epsilon}{A+B}$$

$$- \frac{\varphi}{4} \frac{K}{K_1} d\epsilon \cos(-\epsilon) \qquad (72)$$

We know again that the integral of $i_n\, n_n$ between

$$\alpha = \frac{\pi}{2} - \epsilon \text{ to } \alpha = \frac{3\,\pi}{2} - \epsilon \text{ must be zero,}$$

that is,

$$\int_{\frac{\pi}{2}-\epsilon+\frac{C}{2}+B}^{\frac{\pi}{2}-\epsilon+\frac{C}{2}+B+A} i_n\, n_{nA} + \int_{\frac{\pi}{2}-\epsilon+\frac{C}{2}}^{\frac{\pi}{2}-\epsilon+\frac{C}{2}+B} i_n\, n_{nB}$$

$$+ \int_{\frac{\pi}{2}-\epsilon+\frac{C}{2}+B+A}^{\frac{\pi}{2}-\epsilon+\frac{C}{2}+2B+A} i_n\, n_{nB} + \int_{\frac{\pi}{2}-\epsilon}^{\frac{\pi}{2}-\epsilon+\frac{C}{2}} i_n\, n_{nC}$$

$$+ \int_{\frac{\pi}{2}-\epsilon+2B+A+\frac{C}{2}}^{\frac{3\pi}{2}-\pi} i_n\, n_{nC} = 0 \qquad (73)$$

This equation applies only as long as $\epsilon < \frac{\pi}{2}$ but similar equations follows in other cases. By substitution, we find from this

$$I_p = -\frac{\pi\,(A+B)}{4\left(\sin\frac{A}{2} + \cos\frac{C}{2}\right)}\,\frac{\varphi}{n_p}\,\frac{K}{K_i} \qquad (74)$$

The flux is in this case again

$$\varphi = \frac{1}{c}\,\varphi_e \qquad (75)$$

if φ_e the flux required with a concentrated primary coil. The coefficient c is in this case

$$c = \frac{2\sin\frac{A+B}{2}}{A+B}\cos\frac{B}{2} \qquad (75\text{ A})$$

With diamond shaped chorded coils

$$\frac{2\sin\frac{A+B}{2}}{A+B}$$

corresponds to the usual distribution factor and

$$\cos \frac{B}{2} = \sin \frac{\pi - B}{2}$$

corresponds to the usual chord factor.

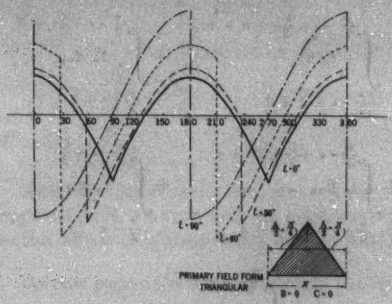

FIG. 25

It will be seen in (**74**) that ϵ has cancelled out which means that a primary current following a cosine law, together with a sinusoidal resultant field fulfills in each secondary coil located

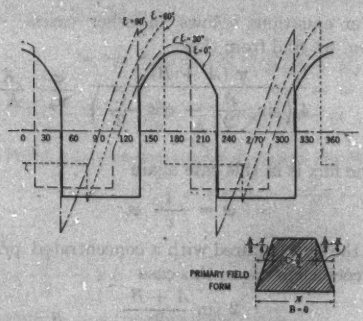

FIG. 26

at any angle ϵ away from coil 1 the condition that the average current in the coil is zero. This being the case, we have shown that the assumptions of a sinusoidal space distribution of the resultant field and a sinusoidal time curve for the primary cur-

rent made in the beginning of this case were correct under our present conditions.

Figs. 25 to 29, inclusive, show the curves for the secondary currents for a number of different field forms. Fig. 25 covers the case of a triangular field form for which the currents corresponding to $\epsilon = 0$ are rather small, while those corresponding to $\epsilon = 90$ deg. are rather large. Fig. 28 gives the case of a machine in which the winding is uniformly distributed over 120 deg. as would be the case in a full-pitch three-phase machine run single-phase. As compared with the previous case, the currents corresponding to $\epsilon = 0$ have increased, and those corresponding to $\epsilon = 90$ deg. have decreased. Fig. 27 gives the case of a machine in which the primary winding is uniformly distributed over 90

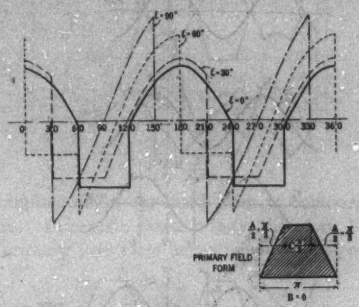

FIG. 27

deg., as would be the case in a full-pitch two-phase machine, run single phase. As compared with the previous case, the currents corresponding to $\epsilon = 0$ have again increased, and those corresponding to $\epsilon = 90$ deg. have decreased. Fig. 28 shows the case of a primary winding which is practically concentrated. It will be realized that even with a primary winding wound into one coil, the field form will not be truly rectangular, but somewhat as shown in Fig. 28. Accordingly, the secondary currents will not assume infinite values as theoretically found in connection with Case No. 4, but it will be somewhat similar to those shown in Fig. 28. Fig. 29 corresponds to a winding in which A is 90 deg., B 35 deg., and C 20 deg. As will be seen from the figure, the primary field form approaches in this case very closely a sinusoidal curve. While the primary current from (**74**) is, in

this case, only one per cent different from that obtained with a sinusoidal distribution, it will be seen that the secondary currents are still very much different from a sinusoidal double-frequency curve. This shows that even the slightest departure from the sinusoidal curve necessary for practical reasons leads to very irregular secondary current curves, although the departure from a sinusoidal curve is the smaller the closer the primary distribution approaches such a curve.

Fig. 27A shows the magnetomotive forces furnished by the primary and the secondary, as well as the resultant field for the

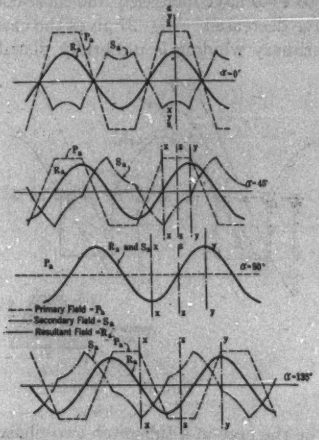

FIG. 27A

case covered in Fig. 27. It can be noted again that the secondary magnetomotive force travels in a direction opposite to the resultant field; its distribution assumes, however, in contradistinction to Fig. 23A, rather irregular shapes, being always the difference between the primary trapezoid and the resultant sine curve.

CASE NO. 7

Primary Distribution According to a Function $f(z)$, Infinite Number of Secondary Phases

It is evident that the methods used in the previous cases can be applied to any distribution of primary ampere turns. If the

primary field form is made up from portions of straight lines, each of them being a function $f(z)$, the method of Case No. 6 can be applied, no matter what the number of different straight lines is. Naturally, the equations will become rather

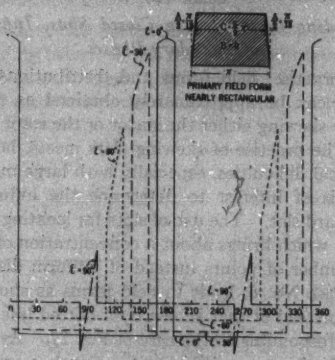

FIG. 28

long and complicated, if the field form is composed of many different lines. In certain cases, it may, therefore, be simpler to resolve the primary field form into a single equation of a funda-

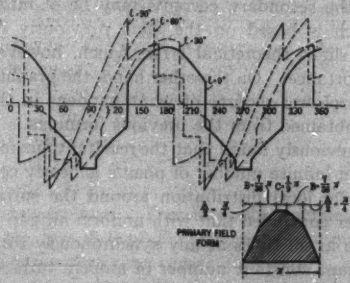

FIG. 29

mental sine wave and its higher harmonics, and then proceed as in Case No. 5. In this case, a single function $f(z)$ applies for the whole cycle. In such cases, the function may, however, be such as to lead to complications in connection with the inte-

gration necessary for finding I_p. Thus the one or the other method may be found more convenient, depending upon the case.

Case No. 8

Primary Winding Distributed in Closed Slots, Infinite Number of Secondary Phases

In actual practise, field forms and distributions of primary ampere turns can be approximately obtained as shown under Case No. 6 by skewing either the stator or the rotor one primary slot pitch. The practise of skewing slots meets, however, with certain practical difficulties, especially with large machines and, therefore, it is of interest to determine the influence of the straight standard slot. The use of slots for locating the primary ampere turns, which brings about a concentration of the ampere turns in a number of points instead of uniform distribution as assumed in Case No. 6, leads to field forms as shown in a full line Fig. 30a instead of that shown by a dotted line in the same figure and corresponding to the case of Fig. 27. As pointed out under Case No. 7, it is possible to treat such a case along the same lines as described under Case No. 6, which requires, however, a considerable amount of calculation, especially with a large number of slots.

For these reasons, the description of a graphical method for determining the secondary currents may be of interest. This method, which is based on the facts previously determined, introduces a slight theoretical error, which, however, is of no practical importance. On the other hand, the graphical method gives a much clearer picture of what happens in the machine than can be obtained from the previous formulas.

We have previously found that the resultant flux rotating with the rotor of an infinite number of phases is nearly constant and has a sinusoidal local distribution around the surface. From this we further concluded that with uniform air-gap reluctance, we must have at all times at any synchronously rotating rotor point a resultant constant number of ampere turns, the size of which follows a cosine law around the rotor. Line $A\ B$ in Fig. 30c may represent this constant resultant ampere-turn value for a certain point. Now we know that such resultant value is at any time the sum of the rotor ampere turns at this point and of the stator ampere turns which happen to be located opposite to this rotor point during the time element under consideration.

Let us assume, for instance, a case as assumed in Fig. 27 and a rotor point corresponding to $\epsilon = 0$; this means the rotor point passing the center of the primary phase belt at $\alpha = 0$, that is, at the time when the primary current reaches its maximum value.

Fig. 30b shows the dotted areas $a\ a'\ b'\ b$ representing the primary winding belts. While our rotor point travels from b_1 to a and from b to a_2, it is not acted upon by any primary ampere turns, which means that the secondary ampere turns are iden-

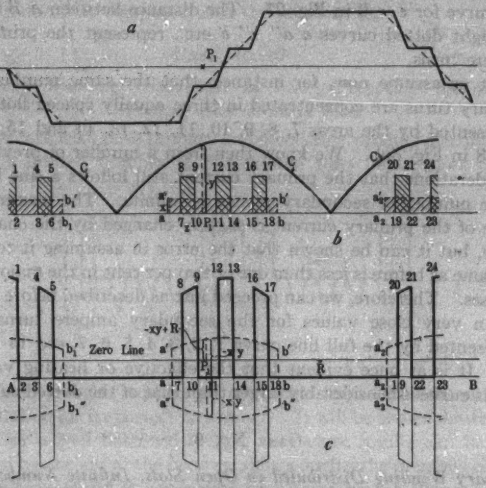

FIG. 30

tical with the resultant. Therefore, the portions b_1 to a and b to a_2 of the line $A\ B$ of Fig. 30c represent portions of the secondary ampere-turn time curve.

The sine curve C represents the time curves of the primary current as found from formula (**74**) for this case plotted so against the circumference, that y represents the time value of the primary current, when our point has reached a point P_1 on the stator circumference. If we multiply, therefore, y with the turns located at this point, we get the number of stator ampere

turns acting upon our point, when it has reached point P_1 on the stator.

These ampere turns $x\,y$ act in the same direction as the resultant ampere turns R represented by line $A\,B$ in Fig. 30c. If we add $x\,y$ to $A\,B$ as shown, we obtain the sum of the primary and resultant ampere turns, and it is also evident that the rotor ampere turns must be the difference between $-\,x\,y$ and R as shown. By proceeding similarly for all other points, we obtain the heavy dotted line $b_1'\,b_1\,a\,a'\,b'\,b\,a_2\,a_2'$, representing the time curve of the secondary current, which corresponds exactly to the curve for $\epsilon = 0$ in Fig. 27. The distance between $A\,B$ and the light dotted curves $a\,a''\,b''\,b$ etc., represent the primary ampere turns.

Let us assume now, for instance, that the same number of primary turns are concentrated in three equally spaced slots as represented by the areas 7, 8, 9, 10, 11, 12, 13, 14 and 15, 16, 17, 18 in Fig. 30b. We know then from a number of previous considerations that the primary current still follows a sine law, if the number of secondary phases is infinite. The maximum value of the primary current is slightly changed by the change made, but it can be shown that the error in assuming it to be the same as before is less than one or two per cent in the majority of cases. Therefore, we can proceed just as described before and obtain very close values for the secondary ampere turns as represented by the full line curve 1, 2, 3, 4, 5, 6, 7, etc. in Fig. 30c. It is at once evident that the effective or heating value of this curve is considerably larger than that of the dotted curve.

Case No. 9

Primary Winding Distributed in Open Slots, Infinite Number of Secondary Phases

In all previous cases, the resultant ampere turns of a given point were assumed to be proportional to the difference in density of the adjacent surface portions. This assumption is practically correct in most cases of closed and partially closed slots. With wide open slots, it will, however, be necessary to take the variations of the air gap reluctance into account. This can be done very easily with the method shown under Case No. 8. It is merely necessary to replace the straight line $A\,B$ of Fig. 30c by a curve $A\,C\,B$, as shown in Fig. 31, such curve representing the varying reluctance of the air gap caused by the slot

openings and with it, the varying resultant ampere turns required to bring about the practically constant flux condition necessary for equilibrium. The secondary ampere turns can then be found just the same as in Fig. 30, except that the distances $x\ y$ are entered, starting from the curve $A\ C\ B$ in Fig. 31 instead of from the straight line $A\ B$ in Fig. 30c.

In this manner, we obtain the heavy line curve 1, 2, 3, 4, 5, etc., giving the secondary current and ampere turns. It will be seen that higher harmonics are introduced all over the secondary current cycle, at the same time, the maximum peaks have been reduced as compared with the previous case.

Case No. 10

Primary Winding Distributed in Slots, Polyphase Secondary Winding Distributed in Slots

The case with both primary and secondary winding distributed over a limited number of non-skewed slots found most com-

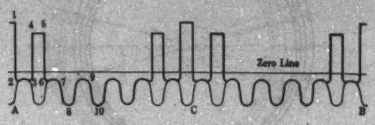

Fig. 31

monly in practise requires an altogether impractical amount of calculation for its correct treatment. It will be shown, however, that for a well balanced design, very close results can be obtained by the aid of the facts established with the previous cases.

We have found in connection with cases Nos. 1, 2 and 3 (see in particular Figs. 13 and 16) that each secondary slot reflects into the concentrated primary winding, while passing the same, a very high current peak. Such peak is infinite if the secondary coil is concentrated in a mathematical point and under the assumption of no leakage reactance. In practise, neither of the latter assumptions apply. The fact that the secondary slots, even if practically closed, distribute the effect of the secondary ampere turns over a certain distance on account of the flux dispersion, changes the infinite values of infinitely short duration to definite values still rather large and of relatively short duration. The difference between the theoretical primary current curve

and the actual caused by the fact that the effect of the secondary coil is not concentrated in a point is the same as the difference between the secondary currents of Fig. 16 and those of Fig. 28. The leakage reactance present in every machine serves further to reduce the height of the higher harmonic current peaks, especially since they are of rather high frequency. Since the secondary flux dispersion around the slots, as well as the leakage reactance, vary over wide ranges in different designs, it is difficult to give a general rule regarding the exact height of the higher harmonic current peaks actually obtained in case of Fig. 16. It is evident, however, that in this case, peaks of such magnitude may be obtained to cause an appreciable difference between the actual primary current curve and a sinusoidal curve. Since the nature of this actual curve cannot easily be determined, the

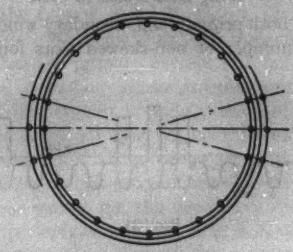

FIG. 32

method of Cases No. 8 and 9 for the determination of the secondary currents, or any of the other methods cannot be readily employed.

If the primary winding is distributed over a number of slots of the same pitch as the secondary slot, as shown in Fig. 32, conditions are practically the same as before. Each of the secondary currents has, in the case assumed, three reversed current peaks, as shown in Fig. 30, instead of one, as shown in Fig. 28, each of the three peaks being about one-third of the single peak. Since, however, three secondary coils coincide simultaneously with the three primary coils, three secondary current peaks of one-third size are reflected at the same time into the primary, giving again rather bad peaks in the primary current wave. Experience with noise, dead points, etc. has long

ago taught designers to avoid equal tooth pitch in both members, and relative tooth pitches, as shown in Fig. 33 are commonly used. In this case, only one secondary slot coincides at the time with a primary coil so that the secondary reflects for each high peak in Fig. 16, several smaller peaks into the primary. Both the higher frequency and the smaller amplitude of these peaks, together with the effect of the leakage in most practical machines, iron the higher harmonics out to such an extent that the departure of the actual primary current wave from the sinusoidal can be neglected in most well designed machines. Therefore, it is permissible to assume primary currents as calculated from (**57**) for the investigation of most practical cases, although small errors may be introduced even in well designed machines and appreciable errors in some extreme cases.

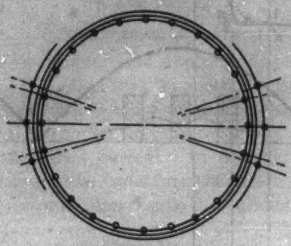

FIG. 33

With a primary current wave established, the method of Case No. 8 may be applied to a limited number of secondary coils with certain modifications.

We must now consider the fact that no matter what the distribution of the primary ampere turns is, the fluxes will always distribute fairly uniform over a secondary tooth. Assume, for instance, again the same primary winding and field as in case of Fig. 27 and Fig. 30. If we have then a secondary slot n located at a point x as shown in Fig. 34a, it is evident that the flux between the points $n - 1$ and n will distribute uniformly over the secondary tooth N, as indicated by a dotted line 1, 2. Similarly, the flux between n and $n + 1$ will distribute uniformly over the tooth $N + 1$, as indicated by the dotted line 3, 4. The result is, there-

fore, the same as if a number of primary turns proportional to the vertical line 2, 3 were concentrated in the point x. By determining the length of 2,3 in this manner for a number of locations of the point x, we obtain a curve a, b, c, d, shown in Fig. 34b which represents the equivalent primary distribution of turns. The corresponding current values are again represented by the sine curve which, as previously pointed out, is only an approximation. By multiplying the values of the two curves, we obtain,

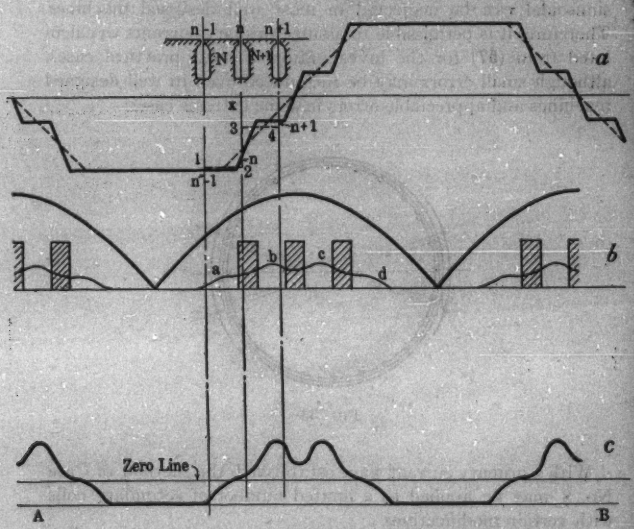

FIG. 34

as before, in Fig. 34c the time curve for the secondary ampere turns for $\epsilon = 0$. By comparison with Fig. 30c, it will be seen that the high peaks, and with them the losses, have been materially reduced by going from an infinite or a very large number of secondary slots to a number of slots only slightly different from the number of primary slots. Similar improvements will be obtained in case of open slots, if the secondary slot pitch approaches the primary slot pitch.

Case No. 11

Squirrel-Cage Rotors

In all previous cases, a polyphase rotor with coils was assumed, and accordingly, the fact that the flux within the secondary coils is approximately constant was used as a basis for the calculation. In practise, short-circuit or damping windings of the squirrel-cage type are more commonly employed. With such windings, each tooth is surrounded by a short-circuited turn, which means that the tendency is to keep the flux within each tooth constant. It has been shown, however, under Case No. 3 that the tooth fluxes must also be constant with a coil winding and it appears, therefore, that the fundamental conditions are the same in both cases and that the results obtained for coils are equally applicable to the bars of squirrel-cage windings.

Case No. 12

Synchronous Machines

The previous considerations apply principally to induction machines, but it is evident that the methods given can be readily used in connection with synchronous machines. The only thing changed is that certain or all current points in the secondary carry a constant direct current. In considering these points, the constant d-c. ampere turns furnished from an outside source must be taken into consideration in the various equations for the ampere turns.

Case No. 13

Primary Sinusoidal Distribution, Infinite Number of Secondary Phases, Resistance and Leakage Reactance Taken Into Account

So far, the resistances were assumed to be negligible in all cases; this assumption holds approximately true at no-load with large machines, as for instance, phase converters for locomotives. Single-phase induction motors are, however, principally built for very small sizes with some of which the influence of the resistances may be noticable even at no-load.

The influence of the secondary resistance in case of sinusoidal primary field distribution with an infinite number of secondary slots may now be investigated.

For zero resistances, we found the instantaneous current values of the primary and secondary, respectively, to be

$$i_p = I_p \cos \alpha$$

$$i_n = I_n \cos (2\,\alpha + \epsilon) \text{ (see (69))}$$

Resistance in the secondary may change both the phase relations and the crest values I_p and I_n of the currents. If we designate by θ the change in phase angle of the primary current and by ρ the corresponding change in the secondary current, we have, therefore, the new relations

$$i_p = I_p \cos(\alpha + \theta) \tag{76}$$

and

$$i_n = I_n \cos(2\alpha + \epsilon + \rho) \tag{77}$$

wherein at present I_p, I_n, θ and ρ are unknown.

In order to send the currents i_n through the secondary coils with a resistance r_n, we must induce voltages $e_n = i_n r_n$ in the coils. Knowing this, as well as the law of change for i_n from (77), we can easily find the law for the flux changes necessary in each coil to induce the voltages e_n. In other words, we find the conditions which the flux has to fulfill to obtain equilibrium in the secondary circuits. By determining further, as in all previous cases, the conditions which the flux has to fulfill for giving equilibrium in the primary circuit, we can definitely determine the actual fluxes satisfying both primary and secondary. The following calculation carried out along these lines gives all desirable information about the fluxes.

Assume again as in the previous cases that each secondary coil n an angle ϵ ahead of coil 1 passes the center of the primary belt at the time $\alpha = -\epsilon$.

The voltage to be induced in a coil n is according to the above considerations

$$e_n = i_n r_n = I_n r_n \cos(2\alpha + \epsilon + \rho) \tag{78}$$

Therefore, the rate of change of the flux within the coil must be

$$K_s \frac{d\varphi_{ni}}{d\alpha} = e_n = I_n r_n \cos(2\alpha + \epsilon + \rho) \tag{79}$$

where

$$K_s = 2\pi f 10^{-8}$$

From which we find

$$K_s \varphi_{ni} = \frac{1}{2} I_n r_n \sin(2\alpha + \epsilon + \rho) + C \tag{80}$$

wherein C is the integration constant.

The latter is found by considering the required flux relations with regard to the primary. Let us assume at present that the flux within each primary coil follows

a sine law and that the flux interlinking with the center of the primary coil belt has its maximum value φ at the time $\alpha = 0$, as in all previous cases. Then the instantaneous flux value interlinking with this center is

$$\varphi_i = \varphi \cos \alpha$$

Since each secondary coil n coincides with the primary belt center for $\alpha = -\epsilon$, the flux within the secondary coil n at this time must be

$$\varphi_{ni} = \varphi_i = \varphi \cos \alpha = \varphi \cos (-\epsilon) \tag{81}$$

or if we combine (**80**) and (**81**) for $\alpha = -\epsilon$

$$K_e \varphi \cos (-\epsilon) = \frac{1}{2} I_n r_n \sin (-\epsilon + \rho) + C$$

or

$$C = K_e \varphi \cos (-\epsilon) - \frac{1}{2} I_n r_n \sin (-\epsilon + \rho) \tag{82}$$

and introducing this into (**80**), we get

$$K_e \varphi_{ni} = K_e \varphi \cos (-\epsilon) + \frac{1}{2} I_n r_n [\sin (2\alpha + \epsilon + \rho) - \sin (-\epsilon + \rho)] \tag{83}$$

According to previous considerations, we know that the flux φ_N of an infinitely small secondary tooth N is one-half the difference between the fluxes of the two adjacent coils n and $n + 1$, an angle $d\epsilon$ apart; therefore, we have

$$\varphi_N = \frac{\varphi_{(n+1)i} - \varphi_{ni}}{2}$$

$$= \frac{1}{2K_e}\Big[K_e \varphi \cos (-(\epsilon + d\epsilon)) - K_e \varphi \cos (-\epsilon) + \frac{1}{2} I_n r_n \Big(\sin (2\alpha + \epsilon + d\epsilon + \rho) - \sin (2\alpha + \epsilon + \rho) - \sin (-(\epsilon + d\epsilon) + \rho) + \sin (-\epsilon + \rho)\Big)\Big] \tag{84}$$

From this we find the density

$$B_N = \frac{\varphi_N}{d\epsilon} K = \frac{K}{2K_e}\Big[-K_e \varphi \sin (-\epsilon) + \frac{1}{2} I_n r_n \Big(\cos (2\alpha + \epsilon + \rho) - \cos (-\epsilon + \rho)\Big)\Big] \tag{85}$$

In order to check our previous assumptions regarding the flux interlinking with the center of the primary phase belt, we determine now the voltage induced in the primary. Equation (85) represents the flux distribution around the secondary in terms of ϵ. The distribution around the primary in terms of Z follows directly from the fact that we always have

$$z = \epsilon + \alpha \text{ or } \epsilon = z - \alpha$$

Introducing this into (85) we get,

$$B_Z = \frac{\varphi_Z}{d_Z} = \frac{K}{2K_e}\Big[-K_e\varphi\sin(\alpha - z)$$

$$+ \frac{1}{2} I_n r_n \Big(\cos(\alpha + z + \rho) - \cos(\alpha - z + \rho)\Big)\Big] \quad (86)$$

The total flux interlinking with a coil z extending from z to $z + \pi$ is

$$\varphi_z = \int_{z=z}^{z=z+\pi} \varphi_Z = \int_{z=z}^{z=z+\pi} \frac{B_Z\, d\,z}{K}$$

$$= \frac{1}{2K_e}\int_{z=z}^{z=z+\pi}\Big[-K_e\varphi\sin(\alpha - z)\, d\,z$$

$$+ \frac{1}{2} I_n r_n \cos(\alpha + z + \rho)\, d\,z$$

$$- \cos(\alpha - z + \rho)\, d\,z)\Big] = -2\varphi\cos(\alpha - z)$$

$$+ \frac{1}{2K_e} I_n r_n [2\sin(\alpha + \rho + z) - 2\sin(\alpha + \rho - z)] \quad (87)$$

The voltage e_z induced in coil z with turns

$$n_z = \frac{n_p}{2}\cos z\, d\,z \quad \text{(See Case No. 5)}$$

is found by determining $K_e \frac{d\varphi_z}{d\alpha}\, n_z$ from (87).

We obtain

$$e_z = \frac{n_p}{4}\cos z\, d\,z\,[2\varphi K_e \sin(\alpha - z)$$

$$+ \frac{1}{2} I_n r_n [2\cos(\alpha + \rho + z) - 2\cos(\alpha + \rho - z)] \quad (88)$$

The total primary voltage follows from this by integrating between the limits $z = -\frac{\pi}{2}$ to $z = \frac{\pi}{2}$, which gives

$$e_p = K_e \frac{\pi}{4} \varphi n_p \sin \alpha \tag{89}$$

which is in line with (**63**) under Case No. 5 and, therefore, shows that equations (**85**) and (**86**) for the flux densities, satisfy the conditions of equilibrium in the primary as well as in the secondary windings.

While the expressions for the densities found in (**85**) and (**86**) are correct, they do not offer the complete solution in so far as they contain the secondary current value I_n, which is still unknown. It is, therefore, necessary to determine this current before a study of the fields is possible. In doing this, we utilize again our condition, that the resultant ampere turns at any point of the circumference must be proportional to the difference in density at either side of the point. This leads to the following formulas resulting in the determination of the currents I_p, I_n and their phase angles.

The resultant ampere turns at a coil n follow from (**85**)

$$i_r n_r = \frac{K}{K_l} \frac{d\epsilon}{4 K_e} \Big[- K_e \varphi \cos(-\epsilon) - \frac{1}{2} I_n r_n (\sin(2\alpha + \epsilon - \rho) - \sin(-\epsilon + \rho)) \Big] \tag{90}$$

From (**76**) and (**77**) we know

$$i_p n_z = i_p \frac{n_p}{2} \cos z \, d\epsilon = I_p \frac{n_p}{2} \cos z \cos(\alpha + \theta) \, d\epsilon \tag{91}$$

and

$$i_n n_n = i_n \frac{n_s}{\pi} d\epsilon = I_n \frac{n_s}{\pi} \cos(2\alpha + \epsilon + \rho) \, d\epsilon \tag{92}$$

and since

$$i_n n_n + i_p n_z = i_r n_r$$

we get the general equation

$$I_n \frac{n_s}{\pi} \cos(2\alpha + \epsilon + \rho) + I_p \frac{n_p}{2} \cos z \cos(\alpha + \theta) = -\frac{K}{4 K_l K_e} \Big[K_e \varphi \cos(-\epsilon) + \frac{1}{2} I_n r_n (\sin(2\alpha + \epsilon - \rho) - \sin(-\epsilon + \rho)) \Big] \tag{93}$$

While this equation has four unknown quantities, it applies to any value of ϵ, and we can therefore, obtain from it any number of equations by assuming a corresponding number of definite values for ϵ. As a matter of course, we assume these values so as to get simple expressions.

Some rather simple conditions are obtained by considering coils in line with the line $Y\,Y'$ in Fig. 22, because the primary winding has zero conductors in Y and Y', which eliminates all expressions with I_p in (**93**).

Let us consider for instance a secondary coil, which is an angle $\epsilon = \frac{\pi}{2}$ ahead of coil 1 and passes the points $Y\,Y'$ at the time $\alpha = 0$. Introducing these values into (**93**) we get

$$I_n \frac{n_s}{\pi} \cos\left(\frac{\pi}{2} + \rho\right) =$$

$$-\frac{K}{8\,K_l K_e} I_n r_n \left(\sin\left(\frac{\pi}{2} + \rho\right) - \sin\left(-\frac{\pi}{2} + \rho\right)\right)$$

or

$$\frac{n_s}{\pi} \sin\rho = \frac{K}{4\,K_l K_e} r_n \cos\rho$$

or

$$\frac{\sin\rho}{\cos\rho} = \tan\rho = \frac{K}{K_l K_e} \frac{r_n}{n_s} \frac{\pi}{4} \qquad (94)$$

This permits the determination of ρ and any of its functions.

Let us now consider secondary coil 1 with $\epsilon = 0$ passing points Y and Y' at the time $\alpha = \frac{\pi}{2}$. Introducing these values into (**93**) we get

$$I_n \frac{n_s}{\pi} \cos(\pi + \rho) =$$

$$-\frac{K}{4\,K_l K_e}\left[K_e \varphi + \frac{1}{2} I_n r_n (\sin(\pi - \rho) - \sin\rho)\right]$$

or

$$I_n \left(\frac{n_s}{\pi} \cos\rho + \frac{K}{4\,K_l K_e} r_n \sin\rho\right) = \frac{K}{4\,K_l} \varphi$$

or

$$I_n = \frac{K}{K_l}\,\frac{\varphi}{4}\,\frac{\pi}{n_s}\,\frac{I}{\cos\rho + \frac{K}{K_l K_e}\,\frac{\pi}{4}\,\frac{r_n}{n_s}\sin\rho}$$

$$= \frac{K}{K_l}\,\frac{\varphi}{4}\,\frac{\pi}{n_s}\,\frac{\cos\rho}{\cos^2\rho + \tan\rho\sin\rho\cos\rho}$$

(See (**94**) for $\tan\rho$)

$$I_n = \frac{K}{K_l}\,\frac{\varphi}{4}\,\frac{\pi}{n_s}\cos\rho = \frac{\varphi}{r_n}\tan\rho\cos\rho \qquad (\mathbf{95})$$

$$I_n = \frac{\varphi}{r_n}\sin\rho = \frac{K}{K_l}\,\frac{\varphi}{4}\,\frac{\pi}{n_s}\,\frac{1}{\sqrt{1 + \left(\frac{K}{K_l}\,\frac{r_n}{n_s}\,\frac{\pi}{4}\right)}} \qquad (\mathbf{95a})$$

This permits the determination of I_n.

Let us now consider the center of the primary phase belt, *i.e.*, point x and x' with $z = 0$, and assume further $\alpha = 0$, at which time the secondary coil 1 with $\epsilon = 0$ coincides with x and x'.

Introducing these values into (**93**), we get

$$I_n\,\frac{n_s}{\pi}\cos\rho + I_p\,\frac{n_p}{2}\cos\theta =$$

$$-\frac{K}{4K_l}\left[\varphi + \frac{1}{2}I_n r_n\,\left(\sin(-\rho) - \sin\rho\right)\right]$$

or

$$\cos\theta = -\frac{2}{I_p n_p}\left(\frac{K}{K_l}\,\frac{\varphi}{4} + I_n\,\frac{n_s}{\pi}\cos\rho\right)$$

$$\cos\theta = -\frac{\varphi}{2\,I_p\,n_p}\,\frac{K}{K_l}\,(1 + \cos^2\rho) \qquad (\mathbf{96})$$

Let us further consider the center of the primary phase belt for $\alpha = \frac{\pi}{2}$ at which time a secondary coil with $\epsilon = -\frac{\pi}{2}$ coincides with x and x'.

Introducing these values into (**93**), we get

$$I_n\,\frac{n_s}{\pi}\cos\left(\frac{\pi}{2} + \rho\right) + I_p\,\frac{n_p}{2}\cos\left(\frac{\pi}{2} + \theta\right) = 0$$

or

$$I_p = - I_n \frac{n_s}{n_p} \frac{2}{\pi} \frac{\cos\left(\frac{\pi}{2} + \rho\right)}{\cos\left(\frac{\pi}{2} + \theta\right)}$$

or by introducing I_n as found in (**95**) we get

$$I_p = - \frac{K}{K_l} \frac{\varphi}{2} \frac{1}{n_p} \frac{\sin \rho \cos \rho}{\sin \theta} \qquad (97)$$

or

$$I_p = - \frac{K}{K_l} \frac{\varphi}{2} \frac{1}{n_p} \frac{\sin 2 \rho}{\sin \theta} \qquad (97\text{-A})$$

By introducing this into (**96**) we get

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\sin \rho \cos \rho}{1 + \cos^2 \rho} = \frac{\sin 2 \rho}{1 + \cos^2 \rho} \qquad (98)$$

We are now in a position to calculate all currents and their phase relations correctly for any rotor resistance. We are further in a position now to calculate with I_n known from (**85**) and (**86**) the flux values for both the primary and secondary member. This has been done in Figs. 35 and 36 for a case with

$$\rho = 30 \text{ deg.}$$

It is evident from Fig. 35, which is plotted for a case of relatively high rotor resistance that the rotor resistance not only causes a variation in the size of the rotating field but also a variation in its speed relative to the stator. Fig. 36 showing the same case with the field plotted against the secondary surface shows how the field runs at times ahead of the secondary and subsequently falls back, completing such a cycle in the time $\alpha = \pi$. The combination of the variation in size and the running ahead and falling back of the field with regard to the secondary induces the voltage in the rotor, which is required to drive the rotor currents over the rotor resistance. The dotted line for $\alpha = 75$ deg. in Fig. 35 shows the maximum field interlinking with the center coil of the tertiary winding, and indicates that the tertiary voltage is shifted less than 90 deg. against the primary counter e. m. f. and different in size therefrom. The irregular field speed with regard to the primary can be seen by comparing the values of v giving the angle of space travel with the corresponding time angle α of the same curve; it will be seen, for instance, that the

field travels 39 space degrees during the first 30 time degrees, etc. The variation in the field size is indicated by the line connecting the crest values of the field, and it will be seen that this curve has double line frequency. In normal machines, the

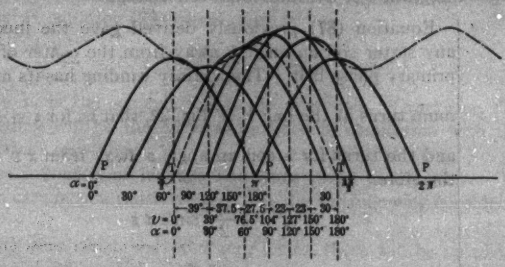

FIG. 35

influence of the resistance is very much smaller than indicated in these figures and especially, in case of the sizes used for phase converters it is negligible at no-load, with regard to its influence upon the field sizes and currents, although the phase shifting effect upon the tertiary voltage is appreciable in most cases. In

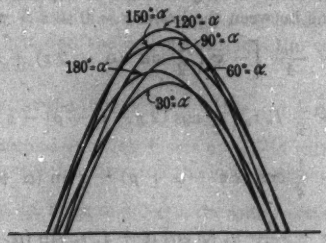

FIG. 36

small single-phase induction motors with high rotor resistance it may, however, be necessary to consider the resistance if exact results for the currents are desirable.

The tertiary voltage can now also be easily determined. Since we know the laws for the stator flux distributions, we merely

need to find the total flux for each tertiary coil by adding all fluxes within the coil. By finding from this $\frac{d\varphi_t}{d\alpha}$ n_s we get the voltage for each coil and subsequently the total voltage by a simple integration of the individual coil voltages.

Equation (**87**) previously derived gave the flux for any stator coil an angle z away from the center of the primary phase belt. The tertiary winding has its maximum turns at the line $Y\ Y'$ Fig. 22, that is, for $z = \frac{\pi}{2}$ and the turns for a coil an angle z away from $x\ x'$ are, therefore,

$$n_s' = \frac{n_p}{2} \sin z\, d\, z$$

By combining this with $\frac{d\varphi_{s'}}{d\alpha}$ as found from (**87**) we obtain therefore,

$$\frac{d\varphi_{s'}}{d\alpha} = e_s' = \frac{n_p}{4} \sin z\, d\, z \Big[2\,\varphi \sin(\alpha - Z)$$

$$+ \frac{1}{2} I_n r_n \left(2\cos(\alpha + \rho + z) - 2\cos(\alpha + \rho - z) \right) \Big] \quad (\mathbf{99})$$

The total tertiary voltage follows from this by integrating between the limits $z = 0$ to $z = \pi$ which gives

$$e_t = \frac{n_p}{4} \Big[-\varphi \left(\frac{1}{2} \sin(\alpha - 2\,z) + z \cos\alpha \right)$$

$$+ \frac{1}{2} I_n r_n \Big(\frac{1}{2} \cos(2\,z - \alpha - \rho) - z \sin(\alpha + \rho)$$

$$- \frac{1}{2} \cos(2\,z + \alpha + \rho) - z \sin(\alpha + \rho) \Big) \Big]$$

$$e_t = -\frac{\pi}{4} n_p \left[\varphi \cos\alpha + I_n r_n \sin(\alpha + \rho) \right] \quad (\mathbf{100})$$

It will be seen that the tertiary voltage consists of a vector equal and at a right angle to the primary voltage plus an out-of-phase vector proportional to the secondary ohmic drop. In the case assumed in Figs. 35 and 36, the tertiary voltage is larger than the primary and shifted against it.

The influence of resistance in the primary winding is taken

into account, the same as in any other a-c. apparatus, and, therefore, does not need to be discussed here.

The influence of the leakage reactances can be taken care of in the same way as the resistances with the only difference that the out-of-phase relation of the leakage reactance drop is taken care of in equation (**78**) which gives an equation

$$\frac{d\,\varphi_{ni}}{d\,\alpha} = I_n\, x_n \cos\left(2\,\alpha + \epsilon + \rho - \frac{\pi}{2}\right)$$

where x_n is the secondary leakage reactance. Otherwise, the procedure will be the same as outlined in connection with the secondary resistance.

The influence of leakage reactance alone can, however, be taken care of in a much simpler way, by the following consideration.

With zero resistances and zero losses, there will be no phase displacements in either the primary or secondary currents, as found under Case No. 5, and, therefore, only the size of these currents will be changed by the presence of leakage reactance. Let us assume a primary leakage coefficient of σ_p and a secondary leakage coefficient of σ_s. This means that, if a certain magnetomotive force in the secondary, for instance, induces a total flux φ_s, a part $\sigma_s\,\varphi_s$ of the total flux follow the leakage path, while the remaining part $\varphi_s\,(1 - \sigma_s)$ goes across the gap to the stator; similar conditions apply to the primary.

Assume in Fig. 37 the line $x\,x$ to go through the center of the primary phase belt and the line $y\,y$ through the center of the tertiary phase belt.

We found that at the time $\alpha = \frac{\pi}{2}$, the primary m. m. f. is zero, while the secondary m. m. f. sets up a field with the center along $x\,x$. If vector I_s represents the local position and size of the secondary m. m. f. for $\alpha = \frac{\pi}{2}$, the same vector φ_t may be assumed to represent the flux sent across the gap by this m. m. f. This flux cuts the tertiary winding on the stator, and, therefore, represents the tertiary flux φ_t in size.

Since according to our previous definition the flux $\varphi_t = \varphi_s\,(1 - \sigma_s)$ we find the total secondary flux vector

$$\varphi_s = \frac{\varphi_t}{1 - \sigma_s} \qquad (101)$$

With zero secondary resistance we know this flux to be constant and rotating with synchronous speed. It may, therefore, be represented by the vector $\varphi_s' = \varphi_s$ at the time $\alpha = 0$. The secondary m. m. f. represented by $I_s' = I_s$ acts at that time in a demagnetizing direction, as shown by vector I_s'. The secondary leakage fluxes being induced by the secondary currents only are

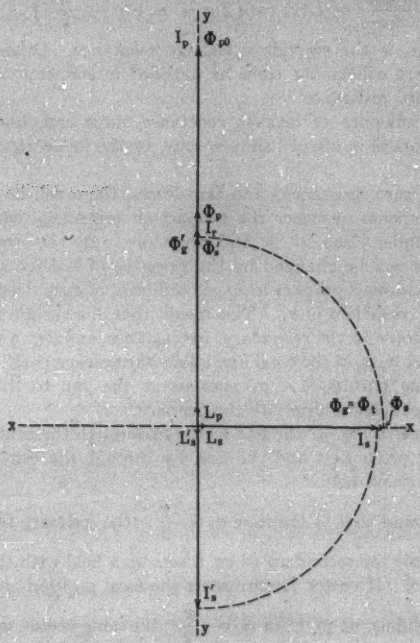

FIG. 37

of course in the direction of these currents and may be represented by the vector

$$L'_s = \varphi_s' \times \sigma_s \tag{102}$$

If L_s is the secondary leakage flux acting in negative direction and φ_s' the resultant secondary flux, we must have a flux

$$\varphi_s' = \varphi_s' + L_s \tag{103}$$

cross the air gap into the secondary core. In order to send such flux across the gap, we need a resultant m. m. f. of I_r represented by the same vector. The primary m. m. f. follows therefore as the difference of I_r and I_s' and is represented by I_p.

The primary leakage flux produced by this m. m. f. is found from the imaginary primary flux φ_{po} to be

$$L_p = \sigma_p \times \varphi_{po} \tag{104}$$

Adding this to the gap flux φ_g' we obtain the total primary flux φ_p.

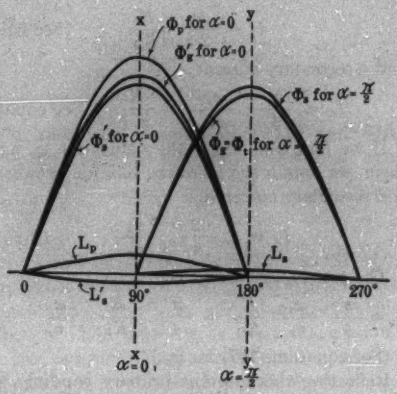

FIG. 38

By making the proper substitutions, we get

$$\varphi_p = \varphi_t \left[\frac{1+\sigma_s}{1-\sigma_s} + \sigma_p\left(1 + \frac{1+\sigma_s}{1-\sigma_s}\right)\right] = \varphi_t K_2 \tag{105}$$

Since in all but cases of exceptionally high leakage $\frac{1+\sigma_s}{1-\sigma_s}$ can be set approximately equal to $1 + 2\,\sigma_s$, we can write

$$\varphi_p = \varphi_t\,(1 + 2\,\sigma_p + 2\,\sigma_s + 2\,\sigma_s\,\sigma_p)$$

or if

$\sigma_p = \sigma_s = \sigma$, where $\sigma =$ average leakage coefficient for one member.

$$\varphi_t = \frac{\varphi_p}{1 + 4\,\sigma + 2\,\sigma^2} \tag{106}$$

The value $2\sigma^2$ is usually negligible in practise and therefore

$$\varphi_i = \frac{\varphi_p}{1 + 4\sigma} = \frac{\varphi_p}{K_2} \tag{107}$$

Fig. 38 shows the fluxes given by vectors in Fig. 37 as they are actually distributed in the machine.

The equations for the current follow from the same considerations and the following simple calculation.

In order to produce an air gap flux φ_g, we need a primary current

$$I_p = \frac{\varphi_g}{2\,n_p}\,\frac{K}{K_l} \qquad \text{(See equation } \mathbf{68}\text{)}$$

and a secondary current

$$I_n = \frac{\pi}{4}\,\frac{\varphi_g}{n_s}\,\frac{K}{K_l} \qquad \text{(See equation } \mathbf{69A}\text{)}$$

If we find from these the values for φ_g and equate them, we obtain the following relation between primary and secondary currents:

$$I_p = I_n\,\frac{2}{\pi}\,\frac{n_s}{n_p} \tag{108}$$

The secondary current is

$$I_n = \frac{\pi}{4}\,\frac{\varphi_g}{n_s}\,\frac{K}{K_l} = \frac{\pi}{4}\,\frac{K}{K_l}\,\frac{\varphi_p}{n_s}\,\frac{1}{K_2} \tag{109}$$

(See equation (**107**) for φ_g.)

Reflecting this into the primary winding, we get by utilizing (**108**) an equivalent current of

$$I_{pe} = \frac{1}{2}\,\frac{K}{K_l}\,\frac{\varphi_p}{n_p}\,\frac{1}{K_2} \tag{110}$$

In order to send the flux φ_g' across the gap, we need a resultant current

$$I_r = \frac{\varphi_g'}{2\,n_p}\,\frac{K}{K_l}$$

where φ_g' follows from (**103**)
to

$$\varphi_g' = \varphi_p - \sigma_p\,I_p\,\frac{K_l}{K}\,2\,n_p$$

Now we know that I_p must be the sum of I_{pe} and I_r, therefore,

$$I_p = \frac{1}{2}\,\frac{K}{K_l}\,\frac{\varphi_p}{n_p}\,\frac{1}{K_2} + \frac{\varphi_p}{2\,n_p}\,\frac{K}{K_l} - \sigma_p\,I_p$$

or

$$I_p = \frac{\varphi_p}{2\,n_p}\;\frac{K}{K_l}\;\frac{1+\dfrac{1}{K_2}}{1+\sigma_p} = \frac{\varphi_p'}{n_p}\;\frac{K}{K_l}\,K_3 \quad \textbf{(111)}$$

We may assume approximately

$$1 + \frac{1}{K_2} = 2 - 4\,\sigma = 2\,(1 - 2\,\sigma)$$

if we further set

$$\frac{1 - 2\,\sigma}{1 + \sigma_p} = 1 - 3\,\sigma$$

we get

$$I_p = \frac{\varphi_p}{n_p}\;\frac{K}{K_l}\,(1 - 3\,\sigma) = \frac{\varphi_p}{n_p}\;\frac{K}{K_l}\,K_3 \quad \textbf{(111 A)}$$

It will be seen from comparison of (**110**) and (**69A**), that the secondary currents are the same with and without leakage except for the factor $\frac{1}{K_2}$, which, in normal machines is somewhat smaller than 1.

Similarly, a comparison between (**111**) and (**67**) shows that the primary currents are the same with and without leakage, except for the factor K_3, which, in normal machines, is somewhat smaller than 1. The influence of the leakage upon the fields, currents, etc., is usually much larger than that of the resistances and is especially in small machines quite appreciable.

We have so far considered leakage and resistance separately. An exact consideration of the two combined gives rather complicated expressions without giving an appreciably greater degree of correctness, than is obtained by first considering the resistance only and subsequently modifying the currents with the factors $\frac{1}{K_2}$ in case of the secondary, and K_3 in case of the primary. A lengthy exact derivation of the combined effect may therefore, be omitted here.

All through the previous considerations, it has been assumed that the rotor runs synchronous. In practise, a very small slip is caused by the torque required to overcome bearing and windage friction. This causes small currents of the slip frequency in the rotor in addition to those found from our calculations. These low-frequency currents are, however, at no-load so small that they are of no practical importance whatsoever.

Case No. 14

Primary Winding Distributed in Slots, Short-Circuited Secondary Winding in Slots, (Squirrel-Cage or Individual Coils Short-Circuited) Resistance and Leakage Reactance Taken into Account.

It is evident from the foregoing that an exact consideration of the influence of resistance and leakage reactance in case of any but the simple sinusoidal distribution of the primary winding leads to altogether impracticable amounts of calculation. It has already been pointed out, however, that with windings distributed into slots the presence of resistance and leakage reactance is beneficial with regard to the elimination of the higher harmonics in the currents and that it is, therefore, permissible in most practical cases to neglect the influence of the slots upon the primary current wave form in all but extreme and undesirable designs. While the presence of resistance and leakage reactance is beneficial with regard to the currents, it must be realized, however, that the smoothing out of the current waves is obtained at the expense of the field form for the resultant field. If we have, therefore, for instance cases with few slots per pole which would call for marked current peaks without leakage reactance and resistance, the reduction of such current peaks caused by the presence of leakage reactance will result in higher harmonics for the resultant field form and may with the loaded motor lead to certain counter torques at certain low speeds resulting in dents in the speed-torque curve frequently observed in practise.*

Whatever higher harmonics in the current waves are not fully eliminated by the leakage reactance will usually result in increased effective current values and ohmic losses which, in themselves, may or may not be sufficient to show up in practical tests of commercial machines. Furthermore, the harmonics may on account of their high frequency cause very appreciable eddy losses in heavy copper conductors often used in larger machines. Large no-load losses otherwise unaccounted for can often be traced to this cause.

Any departures from the primary sinusoidal winding distribution made necessary in practise by the fact that the windings have to be distributed in winding belts, as shown in Fig. 24, with definite steps instead of being graduated according to a sine

*Note: This is not the only cause for dents in speed-torque curve but only one of many causes. A previous footnote has already mentioned that variation in the permeability of iron, bad distribution of resistance in the rotor, etc., may cause undesirable conditions along this line.

law may have effects similar to those just discussed in connection with the influence of the slots. The harmonics caused thereby in the currents are usually, however, of a much lower order than those caused by the teeth, and, therefore, the smoothing out effect of the resistance and leakage reactance will be much smaller and in most practical machines almost negligible. A practical method for determining the no-load currents rather closely consists, therefore, in the determination of the currents by the methods and formulas given under Case No. 6 and by subsequently modifying the currents thus obtained, for the influence of the resistance and leakage reactance through the introduction of the correcting factors as found in connection with the sinusoidal distribution in Case No. 13. A similar procedure will give good results with regard to the tertiary voltage of the phase converter.

The presence of secondary resistance, especially in squirrel-cage windings, as well as the presence of leakage reactance, both of which interfere to a certain extent with the formation of ideal field forms may, on account of this latter fact, introduce a certain amount of differential leakage which is otherwise impossible. It is, therefore, advisable to allow in practical machines for a small amount of differential leakage in the calculation.

Case No. 15

Primary Winding Distributed in Slots, Secondary Winding of the Slip-Ring Type Distributed in Slots, (i. e., Secondary Windings with a Small Number of Phases with Distributed Coils of Each Phase Connected in Series)

Space does not permit to be given here the complete treatment of this case which is rather involved, but a few essential points may be mentioned without giving a full proof for the statements made. With the star-connected three-phase winding frequently used in secondaries, an additional fundamental condition has to be met besides those previously considered; namely, the sum of all currents flowing at any instant towards the star point must be zero. As long as the primary winding has approximately a sinusoidal distribution thereby calling for sinusoidal current waves in the secondary, this condition is naturally fulfilled as in any true three-phase system with sinusoidal waves. In such a case, we may, therefore utilize the results obtained in connection with Cases No. 5 and 13 with regard to the primary currents and the tertiary voltages. The secondary currents follow directly from the following consideration: We have found for zero

resistance and leakage reactance in connection with Case No. 5, that the entire resultant field is at times set up by the secondary alone. We have further found that the magnetomotive force of the secondary is constant. We can, therefore, conclude from the combination of these two facts that the magnetomotive force of the secondary alone must be at all times sufficient to set up a field which is equal in size to the resultant field. Since the secondary is a three-phase winding, its currents must, therefore, be the same as the magnetizing currents in the primary of a three-phase motor with the same field and the same winding arrangement. In other words, the secondary current of a three-phase winding of a single-phase motor with zero resistance and leakage can be determined rather closely from the formulas used for the primary magnetizing current of a polyphase induction motor; still assuming, of course, that the primary distribution is sinusoidal. The effect of resistance and leakage reactance can subsequently be again taken into account by introducing the modifying factors found under Case No. 13. For delta connected and other polyphase secondary windings, the fundamental condition is somewhat different, but the conclusions as so far reached apply as well.

With primary distributions not being sinusoidal, a large multitude of different conditions may apply. In a great many cases the tendency for higher harmonic currents in the secondary is materially reduced as compared with the short-circuited (squirrel-cage) winding. With the latter, it was possible for each coil or conductor to individually adjust its current along the lines discussed. With a number of distributed coils connected in series, this is no longer possible and with proper slot relation between primary and secondary, there is a tendency to replace each high current peak of the individual coils by a larger number of smaller peaks in the phase belt. This is the same action as previously described in connection with the primary. For this reason, we find in a great many commercial machines that oscillograms of the secondary currents closely approach a sine curve even though the windings are located in slots and depart a certain amount from a sinusoidal distribution. On the other hand, we may find cases in which the special condition applying to our present case cause, in combination with the other conditions, rather erratic current waves.

The previously mentioned tendency for eliminating harmonics in slip-ring secondary windings is again obtained at the expense

of the resultant field form. For this reason, we may again have superimposed rotating fields and corresponding dents in the speed-torque curve; furthermore, the differential leakage may be appreciable in case of slip-ring secondaries.

CONCLUSIONS

Fields and Magnetomotive Forces

The actually existing main field in a single-phase induction machine running light is the resultant of magnetomotive forces furnished by the primary and secondary windings. By assuming the magnetomotive forces of only one of these members existing while those of the other members are assumed to be zero, we obtain imaginary fields which may be called primary field if only the primary magnetomotive force is assumed to exist, and the secondary field if only the secondary magnetomotive force is assumed to exist. All machines have, regardless of the primary winding distribution and the primary field form, a strong tendency to form a resultant field of sinusoidal space distribution rotating at no-load with practically the same speed as the rotor, *i. e.*, with synchronous speed relative to the primary. With an infinite number of secondary slots and with zero resistance and leakage reactance, the space distribution of the resultant field is exactly sinusoidal and the speed of the field is zero relative to the rotor and synchronous relative to the stator no matter what the primary field form may be. With a secondary winding having individually short-circuited coils or conductors embedded in a limited number of slots (usually squirrel-cage windings), the values of the average densities of the individual secondary teeth are also constant and follow a sine law, giving thereby a local distribution for the resultant field of a step form with the middle of each step located on a sine wave as shown in Fig. 15; this is still on the assumption of zero resistance and leakage reactance. These ideal field conditions are in practise slightly disturbed by the usual amounts of resistances and leakage reactances of the windings. The true sinusoidal field distribution is preserved only in case of a sinusoidal distribution of the primary winding and with infinite number of secondary slots. The presence of leakage reactance causes, however, even under these assumptions, slight fluctuations of double frequency in the size of the air-gap field. The presence of rotor resistance also causes such fluctuation in size as well as certain variations in the speed of the resultant field so that it rotates at times somewhat slower and at times

somewhat faster than the rotor. The influence of leakage reactance upon the air gap field φ_g is shown in Fig. 38; the influence of rotor resistance upon the resultant field is shown in Figs. 35 and 36, the former showing the position of the fields relative to the primary, and the latter, showing it relative to the secondary. The presence of primary resistance results, as in all a-c. apparatus, in a slight phase displacement and a small change in the size of the flux.

With non-sinusoidal distribution of the primary winding the influence of resistance and leakage reactance is similar to that described in connection with the sinusoidal primary curve. Moreover, it may, however, cause the departure of the resultant field from the sinusoidal space distribution. Such departures are slight in most commercial machines but may be appreciable in case of exceptionally large secondary resistance or large leakage reactances.

With machines of the slip ring type, which usually have a limited number of secondary phases with a number of series-connected phase coils distributed over phase belts, the resultant field may depart more from the sinusoidal distribution than in all other cases.

The size of the resultant flux in single-phase induction machines is, as in most a-c. apparatus, principally determined by the impressed voltage and frequency and the number of primary conductors and their distribution; the influence of the resistance and leakage reactance upon the size of the flux is usually rather small.

The local distribution of the imaginary primary field is naturally governed by the local distribution of the primary winding and the magnetic reluctances of the machine. Except for minor influences in the variation of the reluctances, the primary field is of constant form and stationary relative to the primary winding, but varying in size and direction.

The imaginary secondary field is, in its form and size, at any time governed by the fact that the primary and secondary densities must, together give the densities called for by the previous rules for the resultant field. This means that the secondary field has often very peculiar shapes and varies in form and size with the primary winding distribution and the reluctance of the machine.

The primary and secondary field forms as obtained when neglecting the influence of resistance and leakage reactance are shown:

In Fig. 2 for a concentrated primary winding with a concentrated single-phase secondary winding. (See Fig. 1.)

In Figs. 11 and 12 for a concentrated primary winding with concentrated two-phase secondary windings. (See Fig. 6.)

In Fig. 20 for a concentrated primary winding with short-circuited secondary having an infinite number of slots.

In Fig. 23a for a primary winding with sinusoidal distribution and a secondary winding with an infinite number of slots. (See Fig. 22.)

In Fig. 27a for a primary winding evenly distributed over 90 deg. and a secondary winding with an infinite number of slots.

The secondary field travels with double synchronous average speed relative to the secondary in a direction opposite to the mechanical rotation of the secondary. It, therefore, travels with synchronous speed relative to the primary winding opposite to the relative mechanical rotation of the secondary.

Primary Magnetizing Currents

The primary magnetizing current has a sinusoidal wave with a sinusoidal voltage impressed upon the machine and an infinite number of secondary slots and phases, regardless of the local distribution of the primary winding. The maximum and effective values of these primary sinusoidal currents are, however, governed to a large extent by the distribution of the primary winding. This is brought out by formulas given under Cases No. 5 and 6. These formulas while applying exactly only under the assumptions made, are correct within one or two per cent for most standard machines if the turns of each primary slot are assumed to be distributed between the centers of the two adjacent teeth.

With a non-skewed slot construction, a slotted primary and a short-circuited secondary with a limited number of secondary slots (squirrel-cage rotor) the curve of the primary magnetizing current may materially depart from a sine curve. Every time a secondary slot coincides with a primary slot a current peak is caused in the primary magnetizing current. These peaks will increase the effective value of the currents above those found in the previous formulas and are furthermore liable to cause appreciable eddy losses in heavy conductors. Some extreme cases of such current peaks are shown in Figs. 3, 7A, 9A and 16A corresponding to windings shown in Figs. 1, 6 and 13. These current peaks will be the more marked, the larger the number of primary and secondary slots which coincide locally at any one time. By keeping such number of coinciding slots small, the departure of the primary current from a sine curve can be made

negligible so that the formulas can be applied with a sufficient degree of accuracy.

The leakage reactance of the machine is of material assistance in eliminating current peaks and it may, therefore, be found that, under otherwise equal conditions, the magnetizing current is slightly smaller in machines with large leakage.

The fundamental frequency of the primary currents is of course, the same as the line frequency; whatever higher harmonics exist are usually odd harmonics. If the rotor speed is slightly less than synchronous, the location of the higher harmonics on the fundamental wave changes periodically with the slip.

Outside of the influences previously mentioned, resistances and leakage reactances of the motor tend to decrease the primary magnetizing currents as can be seen by comparing the formulas found under Cases No. 5 and 13.

In motors with phase-wound slip-ring secondaries, most of the previous remarks apply, except that the tendency for higher harmonics in the primary current is usually somewhat smaller than in the case of squirrel-cage rotors.

Secondary Magnetizing Currents

With synchronously rotating secondary, the fundamental wave of the secondary currents has double line frequency. With sinusoidal primary winding distribution no higher harmonics exist as indicated in Fig. 23 which shows a number of secondary currents.

Any departure from a sinusoidal distribution of the primary winding will cause the secondary currents to depart materially from a sinusoidal curve and furthermore, cause the currents in different secondary conductors to be different from each other. If the secondary speed is slightly different from synchronous speed, the current wave in each secondary coil changes its shape and size periodically with the slip. The two waves of a secondary current cycle are usually different from each other indicating the presence of even harmonics which are quite large in a great many cases.

Figs. 3, 7B, 7C, 9B, 9C, 16B and 21 show secondary current waves for primary windings concentrated into a mathematical point and having no leakage corresponding to arrangements shown in Figs. 1, 6 and 13. These cases are, of course, extreme, and in practise, impossible. Figs. 25, 26, 27, 28 and 29 show secondary current waves of primary field forms shown in con-

nection with these figures. The heavy line 1, 2, 3, 4, etc., of Fig. 30c shows the influence of the fact that the primary winding is distributed over a number of slots upon the secondary currents, in case of a secondary having a much larger number of slots than the primary. Fig. 34c shows the secondary current for the same case except that the number of secondary slots is assumed to be about the same as the number of primary slots. Fig. 31 shows again a similar case, except that wide open slots are assumed in the primary with the number of secondary slots much larger than the number of primary slots.

It is evident from these figures that the secondary currents are subject to a large number of variations. While, therefore, the simple formulas derived for a sinusoidal primary distribution may give a rough approximation for the secondary currents in single-phase machines, it should be realized that the marked higher harmonics are liable to increase not only the effective current value and the ohmic losses, but may also cause very appreciable eddy losses in the heavy conductors often employed in squirrel-cage windings, as well as in the heavy section end rings.

As in the case of the primary currents, the leakage reactance will again be beneficial in reducing the higher harmonics. Both resistance and leakage reactance also tend to decrease the fundamental wave, as can be seen by comparison of Cases No. 5 and 13.

With slip ring secondaries having phase belts, the tendency of higher harmonics in the secondary is usually much smaller than in short-circuited secondaries, although erratic wave shapes may be obtained in extreme cases.

Tertiary Voltages in Phase Converters

The tertiary voltage of a phase converter is sinusoidal if the primary voltage is sinusoidal and the secondary has an infinite number of phases. In case of a short-circuited secondary (squirrel cage) with a limited number of secondary slots the tertiary voltage has a step shape similar to that shown in Figs. 17 and 18 with the center of the steps located on a sine wave. With a distributed tertiary winding of proper choice of tooth pitch and winding distribution, the size of the steps can be reduced so that the tertiary voltage approaches more closely a sine curve. Neglecting the resistance and leakage reactance, the tertiary voltage equals the primary voltage with equal number of turns

if the secondary has an infinite number of slots. With a limited number of slots, the tertiary voltage decreases slightly with the number of secondary slots, as indicated in curve No. 19; this influence is the same as usually considered in connection with the zig-zag leakage, in reality it is here caused by the departure of the resultant flux from the sinusoidal wave shape. With the speed slightly different from synchronism, the resultant wave shape of the field varies periodically with the slip which also causes periodic variations in the higher harmonics of the tertiary voltage, without, however, affecting the effective value of the tertiary voltage.

The presence of slot and end connection leakage reactance causes a further difference between the primary and tertiary voltages, in the same direction as that caused by the zig-zag leakage or the difference in wave shape. The presence of resistance in the windings not only affects the amplitude of the tertiary voltage as compared with that of the primary, but also disturbs the 90 deg. phase relation in two-phase converters and the corresponding relation in converters wound for other numbers of phases.

General

It is realized that the rather abstract methods used in this paper are not always best adapted to explain the various phenomena in the most simple manner. It is obvious, however, that after the facts have been definitely established by such methods, it will be possible to work out other treatments of the same phenomena; these can be made both simpler to understand, and at the same time correct, by keeping continuously the results derived from this and similar papers in mind. The various methods of assuming fields of oppositely rotating direction appear to be especially advantageous in this connection.

APPENDIX I

Fig. 6 represents a phase converter with a single concentrated primary coil $P\,P'$ and a single concentrated tertiary coil $T\,T'$. The rotor is provided with a two-phase winding consisting of two concentrated coils 1 and 2 displaced 90 deg. against each other. The current and fluxes applying to these coils may be designated by i_1, i_2, φ_1 and φ_2, respectively. The four quadrants of the rotor are marked A, B, $-A$ and $-B$, and the fluxes carried by these quadrants are φ_A, φ_B, $-\varphi_A$, $-\varphi_B$. The densities B are

indicated by similar subscripts. Other assumptions are similar to those in case of Fig. 1.

Assume again, that coil 1 coincides with P at the time $\alpha = \gamma$ we know that, the flux of coil 1 has the constant value

$$\varphi_1 = \varphi \cos \gamma \tag{1}$$

The coil 2 coincides with P at a time $\frac{\pi}{2}$ earlier than coil 1, at the time $\alpha = \gamma - \frac{\pi}{2}$, so that it has the constant flux value

$$\varphi_2 = \varphi \cos \left(\gamma - \frac{\pi}{2}\right) \tag{2}$$

For any value $x = 0$ to $x = \frac{\pi}{2}$ corresponding to the time $\alpha = \gamma$ to α to $\gamma + \frac{\pi}{2}$ we have the following equations,

$$\varphi_i = \varphi_a + \varphi_b + \varphi_c \tag{3}$$

$$\varphi_1 = -\varphi_a + \varphi_b + \varphi_c \tag{4}$$

$$\varphi_2 = -\varphi_a - \varphi_b + \varphi_c \tag{5}$$

By adding (5) and (3), subtracting (4) from (3), and (5) from (4), and by introducing the values from (1) and (2), we find

$$\varphi_a = \frac{\varphi}{2} (\cos \alpha - \cos \gamma) \tag{6}$$

$$\varphi_b = \frac{\varphi}{2}\left(\cos \gamma - \cos \left(\gamma - \frac{\pi}{2}\right)\right) \tag{7}$$

$$\varphi_c = \frac{\varphi}{2}\left(\cos \alpha + \cos \left(\gamma - \frac{\pi}{2}\right)\right) \tag{8}$$

The densities are, therefore,

$$B_a = \frac{K}{2} \varphi \frac{\cos \alpha - \cos \gamma}{x} \tag{9}$$

$$B_b = \frac{K}{2} \varphi \frac{\cos \gamma - \cos \left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} \tag{10}$$

$$B_c = \frac{K}{2} \varphi \frac{\cos \alpha + \cos \left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2} - x} \tag{11}$$

Under consideration of the current directions shown in Fig. 6, we have the following relations for the currents, assuming that n_1 and n_2 are the numbers of turns in coil 1 and 2.

$$B_a = K_1 (i_p n_p - i_1 n_1 + i_2 n_2) \tag{12}$$
$$B_b = K_1 (i_p n_p + i_1 n_1 + i_2 n_2) \tag{13}$$
$$B_c = K_1 (i_p n_p + i_1 n_1 - i_2 n_2) \tag{14}$$

By adding and subtracting these and introducing the values from (9), (10) and (11) with $x = \alpha - \gamma$ we obtain

$$i_p = \frac{K\varphi}{4 K_l n_p}\left[\frac{\cos\alpha - \cos\gamma}{\alpha - \gamma} + \frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2} - \alpha + \gamma}\right]$$

$$i_1 = \frac{K\varphi}{4 K_l n_1}\left[\frac{\cos\gamma - \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} - \frac{\cos\alpha - \cos\gamma}{\alpha - \gamma}\right]$$

$$i_2 = \frac{K\varphi}{4 K_l n_2}\left[\frac{\cos\gamma - \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} - \frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2} - \alpha + \gamma}\right]$$

The tertiary flux is now

$$K\varphi_t = B_b x + B_c\left(\frac{\pi}{2} - x\right) - B_a x - B_b\left(\frac{\pi}{2} - x\right)$$
$$= - B_a x + B_b\left(2x - \frac{\pi}{2}\right) + B_c\left(\frac{\pi}{2} - x\right)$$

or by introducing the values for the densities

$$\varphi_t = \frac{\varphi}{2}\left[2\cos\left(\gamma - \frac{\pi}{2}\right) + \frac{4x}{\pi}\left(\cos\gamma - \cos\left(\gamma - \frac{\pi}{2}\right)\right)\right]$$

By determining $\frac{d\varphi_i}{d\alpha}$ we get

$$e_i = \frac{2\varphi}{\pi}\left(\cos\gamma - \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

Similarly, we find for value $x = \frac{\pi}{2}$ to $x = \pi$ corresponding

to the time $\alpha = \gamma + \frac{\pi}{2}$ to $\alpha = \gamma + \pi$

$$\begin{aligned}
\varphi_i &= \varphi_d + \varphi_e + \varphi_f \\
\varphi_1 &= -\varphi_d - \varphi_e + \varphi_f \\
\varphi_2 &= \varphi_d - \varphi_e - \varphi_f
\end{aligned}$$

$$\varphi_d = \frac{\varphi}{2}\left(\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

$$\varphi_e = -\frac{\varphi}{2}\left(\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

$$\varphi_f = \frac{\varphi}{2}\left(\cos\alpha + \cos\gamma\right)$$

$$B_d = \frac{K}{2}\,\varphi\,\frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{x - \frac{\pi}{2}}$$

$$B_e = -\frac{K}{2}\,\varphi\,\frac{\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}}$$

$$B_f = \frac{K}{2}\,\varphi\,\frac{\cos\alpha + \cos\gamma}{\pi - x}$$

$$\begin{aligned}
B_d &= K_1\,(i_p\,n_p - i_1\,n_1 - i_2\,n_2) \\
B_e &= K_1\,(i_p\,n_p - i_1\,n_1 + i_2\,n_2) \\
B_f &= K_1\,(i_p\,n_p + i_1\,n_1 + i_2\,n_2)
\end{aligned}$$

$$i_p = \frac{K\varphi}{4\,K_1\,n_p}\left[\frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\alpha - \gamma - \frac{\pi}{2}} + \frac{\cos\alpha + \cos\gamma}{\pi - \alpha + \gamma}\right]$$

$$i_1 = \frac{K\varphi}{4K_l n}\left[\frac{\cos\alpha + \cos\gamma}{\pi - \alpha + \gamma} + \frac{\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}}\right]$$

$$i_2 = \frac{K\varphi}{4K_l n_2}\left[\frac{-\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} - \frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\alpha - \gamma - \frac{\pi}{2}}\right]$$

$$K\varphi_l = B_e\left(x - \frac{\pi}{2}\right) + B_f(\pi - x) - B_d\left(x - \frac{\pi}{2}\right) - B_s(\pi - x)$$

$$\varphi_l = \frac{\varphi}{2}\left[+4\cos\gamma + 2\cos\left(\gamma - \frac{\pi}{2}\right) - \frac{4x}{\pi}\left(\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right)\right]$$

and

$$\frac{d\varphi_l}{d\alpha} = -\frac{2\varphi}{\pi}\left(\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right) = e_l$$

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A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR*

BY B. G. LAMME

ABSTRACT OF PAPER

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be produced by direct-current excitation. It follows that there is a resultant rotating primary field just as in the polyphase motor, while in the secondary there are two currents, one of low frequency, corresponding to the polyphase motor, and the other of nearly double the primary frequency. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown. This illustrates, in a simple manner, why the excitation on single-phase must be practically the same as on polyphase at full speed and falls to one-half value at standstill.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. Various discrepancies are pointed out between the resultant action of the two-motor unit and the single-phase. Modifying conditions are then taken into account which remove the discrepancies. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

THE underlying principles and the operating characteristics of the polyphase induction motor are so well understood that it is found desirable to consider the single-phase induction motor, simply as a special case of the polyphase. On this basis the single-phase motor must be considered primarily as a rotating-flux machine.

Starting with the old assumption that a single-phase alternating magnetic field may be considered as being made up of two

*It should be understood distinctly that this paper is not to be considered as a presentation of new material, for practically all of the underlying principles are old and relatively well known. It is simply an attempt to describe the operation of the single-phase motor in a way which will be easily understood by those not versed in the mathematics of the subject.

constant fields, each of half the peak value of the single-phase field and rotating at uniform speed in opposite directions, then if the single-phase flux distribution is of sine shape and varies sinusoidally in value, it may be replaced, or represented, by two sine-shaped fields of constant value rotating in opposite directions. This is the simplest case and allows a relatively easy explanation of many single-phase problems. However, when the flux distribution, or field form, due to the single-phase winding, is other than of sine shape, then the oppositely rotating components cannot be considered as of sine shape, but will assume certain varying forms as they rotate, the resultant of each instantaneous pair always giving the single-phase field corresponding to that instant.

As other than sine-shape fields tend toward complications in the physical conception of the single-phase induction motor actions, and lead more or less into the mathematical conception, the following analysis will be limited essentially to sine-shape distributions.

As a starting point and to show reasons for certain later analysis, let us assume a single-phase induction motor operating at no-load, full speed, with its polyphase secondary winding short-circuited. The single-phase primary field, of assumed sine shape, is considered as made up of the two sine-shape equal components of constant value, and of half the peak value of the single-phase field, and rotating synchronously in opposite directions. One of these fields is traveling in the same direction as, and slightly faster than, the rotating secondary. The slip of the secondary with respect to this field is of the same nature as in the ordinary polyphase motor. As the machine is carrying no load the secondary current corresponding to this rotating field is very small, being just large enough to overcome the rotational losses in the motor itself, and its frequency is equal to the slip frequency due to the forward field component.

As there is an assumed backward flux or field component of equal value, the rotating secondary winding cuts this at almost double the frequency of the line. Stated exactly, the sum of the backward and the forward frequencies, in the secondary winding, is equal to exactly double the frequency of the primary supply system. The secondary winding cutting the backward field at this high frequency tends to generate a very considerable e. m. f. and, with the winding closed on itself, short-circuit currents will flow, which tend to damp out or suppress the flux which causes them. This secondary current will rise until its magnetizing

effect is practically equal and opposite to the magnetomotive force which produces the backward field, which thus becomes almost zero in value. Consequently there are two distinct sets of secondary currents flowing, one of very small value and of a frequency corresponding to that of the forward rotation, and the other of very much larger value and of almost double the line frequency. Actual tests of the secondary circuit of a single-phase induction motor at small load, taken with an oscillograph, Fig. 1, show both of these currents as above described.

Magnetomotive Forces and Magnetic Fluxes

It is seen from the preceding that, right at the beginning of our analysis, a new condition is encountered, namely, the introduction of a secondary opposing magnetomotive force which reacts on one of the primary field components and practically neutralizes it. Also, there is a mixture of magnetomotive forces and magnetic fields, which is liable to lead to confusion. Obviously the introduction of the opposing secondary magnetomotive force rotating synchronously with the backward component of the primary introduces some entirely new features. Therefore, before going any further with the above method, it is desirable to set aside for awhile the viewpoint of two equal oppositely rotating fields and begin with a preliminary study of the magnetomotive forces and the magnetic fields resulting from them.

It may be mentioned that while the assumption of two oppositely rotating component fields, in place of a single-phase field, is well known and has been used quite frequently, the corresponding analysis, from the viewpoint of magnetomotive forces, apparently has been but little used. When magnetomotive forces, instead of magnetic fluxes, are considered, then the single-phase primary magnetomotive force, fixed in position, can be replaced by two equal components of constant value, such as would be developed by direct current, each of half the peak value of the single-phase, and rotating at synchronous speeds in opposite directions.

Returning again to our analysis, let us consider two fundamental magnetomotive forces, namely, a primary single-phase one, fixed in position and varying sinusoidally and a secondary one of constant value, of half the peak value of the primary which rotates synchronously in one direction and which is in opposition to the primary in the position where the two coincide.

Let us assume that the primary single-phase magnetomotive force is split into its two equal oppositely rotating components,

then the results may be illustrated as in Figs. 2, 3, 4, and 5. In Fig. 2, C and D represent the two components forming the single-phase magnetomotive force A. At the position chosen, C and D are of equal value and coincide in position and polarity, B, which represents the secondary magnetomotive force, is also of half the peak value of A, but is of opposite polarity. It, therefore, neutralizes one of the components C or D, thus leaving a resultant of half the peak value of A.

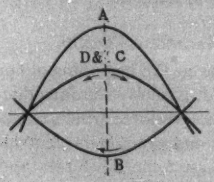

FIG. 2

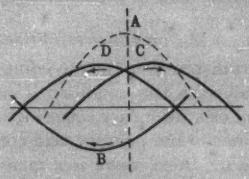

FIG. 3

In Fig. 3, the component D has shifted thirty degrees to the left, while C has shifted an equal distance to the right. The secondary magnetomotive force B is shifted thirty degrees to the left, thus neutralizing D and leaving only the component C.

In Fig. 4, D and B are shifted sixty degrees to the left, while C is shifted sixty degrees to the right. In the same way, in Fig. 5, B and D have shifted ninety degrees to the left and C has shifted a corresponding amount to the right.

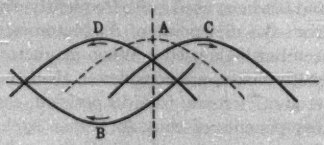

FIG. 4

Thus from the above it is seen that a single-phase magnetomotive force, fixed in position and varying sinusoidally, and a constant magnetomotive force of half the peak value of the single-phase, which is in opposition at the point of coincidence of position, and which rotates synchronously in either direction, will give a resultant constant magnetomotive force, rotating in the opposite direction, but which is of the same polarity as the single-phase magnetomotive force at the position of coincidence.

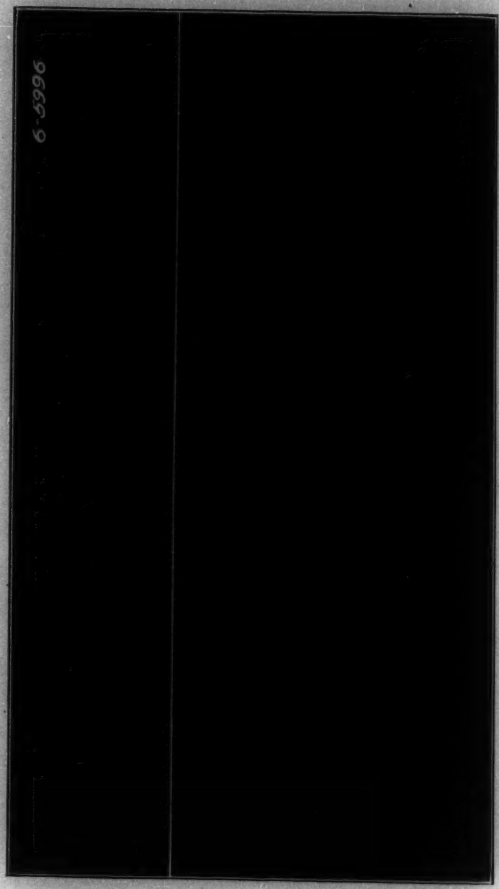

FIG. 1

[LAMME]

In other words, a single-phase magnetomotive force, fixed in position, and an opposing constant one of half the peak value rotating in either direction, will give a resultant rotating magnetomotive force equivalent to that of a polyphase induction motor.

As a continuation of the above, the resultant magnetomotive force C could be replaced by a magnetic field or flux, resulting from such magnetomotive force. If this magnetic field is plotted to the same scale as the magnetomotive force which produces it, then C, in Figs. 2 to 5, can represent a magnetic field. This field will be constant in value and of half the peak value of the field which the single-phase magnetomotive force alone would set up.

Thus according to Figs. 2, 3, 4 and 5, by the introduction of an "opposing" magnetomotive force, equal in value to one of the component magnetomotive forces of the single-phase and rotating synchronously with it, one of the two components of the magnetic field can be suppressed and only the other component left,

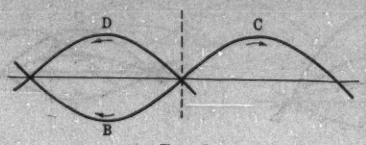

FIG. 5

the resultant is thus a rotating magnetic field, just as in the polyphase induction motor.

However, a further modification of this should be considered. Assuming again, that the single-phase primary magnetomotive force is replaced by its two equal rotating components, as in Figs. 2 to 5, then by the addition of an opposing magnetomotive force, similar to B in the same figures, but of *less value than the component* D, then the resultant of this opposing magnetomotive force and the component D is a reduced magnetomotive force of the same polarity as D. There will then remain two magnetomotive forces, each of constant value, one of half the peak value of A and the other of some smaller value, depending upon the opposing force B. These two rotating magnetomotive forces can, therefore, set up two oppositely rotating fields of *unequal* value. These are illustrated in Figs. 6, 7 and 8.

In Fig. 6, B is assumed at some less value than the component D. The resultant of D and B is shown as E. Therefore, at this

position C and E represent the two resultant magnetomotive forces and the two component fields. In Fig. 7, the conditions are shown for thirty degrees shift and here again E and C represent the two fields. In Fig. 8 the shift is for sixty degrees.

Thus by the introduction of a constant "opposing" magnetomotive force of less than either of the components of the single phase, two oppositely rotating fields of unequal value may be set up. As extreme cases of this, if the constant opposing magnetomotive force is made zero in value, the magnetic field corresponding to its position and rotation will rise to the full value of the oppositely rotating field; and, on the other hand, if the constant opposing magnetomotive force is made half the peak value of the single phase, the correspondingly rotating field becomes zero. Both of these cases are in accordance with the earlier assumptions.

The above conditions of the single-phase primary magneto-

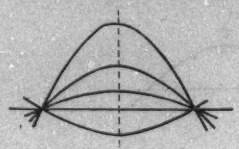

FIG. 6

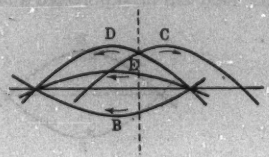

FIG. 7

motive force and a constant secondary one, in opposition, which may be of half the peak value, or some less value down to zero, and which rotates synchronously in one direction, resulting in two magnetic fields which may be of equal or unequal value, and which rotate synchronously in opposite directions, all form essential parts in the physical conception or visualization of the actions of the single-phase motor which will be given below.

It should be observed that in the above method of considering the production of a rotating field in the single-phase induction motor, the two primary components of the single-phase magnetomotive force and the secondary damping magnetomotive force *all rotate synchronously*, and such rotation is independent of the speed of the secondary core. In some methods of considering the single-phase induction motor problem, the single-phase primary winding is assumed to generate a magnetomotive force in the secondary which, by rotation of the core, is carried around until it generates a second magnetic field or flux at right angles to the

original primary flux, thus giving the equivalent of a polyphase magnetic field. However, the above method does not involve such method of treatment.

It should also be recognized that the foregoing analysis only covers no-load conditions and that with the addition of load new conditions are brought into the problem. These, however, will be brought out later, for the no-load conditions require further consideration, especially as regards the generation of the primary counter e. m. f. by the above described rotating fields. As already shown, there may be a single magnetic field rotating synchronously, or there may be two component fields of equal value rotating in opposite directions, or there may be intermediate conditions of oppositely rotating fields of unequal value, depending upon the value of the damping or opposing secondary magnetomotive force.

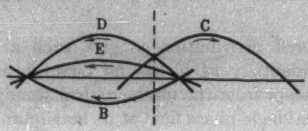

FIG. 8

COUNTER E. M. F. GENERATION AND EXCITATION

Considering next the counter e. m. f. generated in the primary, we should first look into the e. m. f. conditions produced by two oppositely rotating fields of equal values. If the secondary circuits are open, the two component fields are both present and are concerned in the generation of the counter e. m. f. This is true whether the secondary is stationary or is rotated at full speed. If, however, the secondary is closed upon itself, then when running at speed, one of the component fields is practically damped out and the other must generate the entire primary counter e. m. f. Thus, two entirely different conditions are encountered, depending upon whether the secondary is open or closed. To explain this properly, some further analysis is required, as follows:

In the first place, it may be stated that the e. m. f., produced in the primary winding by *cutting* its two component fields, is the same as that generated by the single-phase sine-shape field, varying sinusoidally and acting on the primary winding as in a

transformer. Herein lies a simple illustration of the equivalence of the transformer and the flux cutting methods for calculating e. m. fs. In Figs. 9, 10 and 11, are shown several positions of the two oppositely rotating fields and their relation to the primary winding.

In Fig. 9 is shown the magnetic flux, or field, A, which is set up by a primary winding a. This winding, of course, would

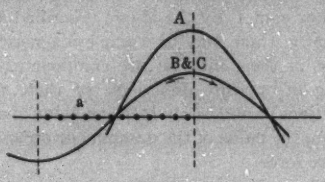

FIG. 9

require a tapered distribution to give such a field. This is mentioned incidentally as it has no direct bearing upon the explanation, except from the mathematical standpoint.

Assuming the single-phase field at its maximum or peak value, then, at this instant, the two component fields, B and C, each of half the peak value, will coincide both in position and polarity. From the transformer method of calculation, the e. m. f. gener-

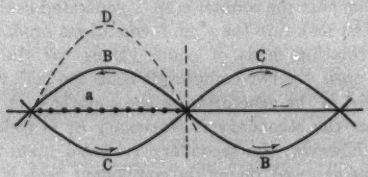

FIG. 10

ated at this instant, in the winding, will be zero, as the rate of change of the flux is zero. Also from the flux cutting method, the e. m. f. in the primary winding will be zero, for, as is evident from the figure, each belt or group of the primary winding is cutting fields which have equal positive and negative areas or values.

Considering next the conditions in Fig. 10, in which the two rotating components have traveled ninety degrees. The fields

are shown as B and C. It is evident that the resultant of these two fields is zero in value, that is, the single-phase field is passing through its zero value, and, accordingly, is generating the maximum e. m. f. by the transformer method. Also, considering component B of the rotating fields, obviously, by the cutting method it is generating maximum e. m. f. in the winding a. Also, component C is generating maximum e. m. f. in winding a. However, as one of these fields is positive in this position and is traveling in one direction, while the other field is negative and is rotating in the opposite direction, the two e. m. fs. will be in the same direction, and thus will be added. Thus, from the figure, this position will give the maximum e. m. f. in the winding by the cutting method. It can be shown by calculation that this maximum value is the same with either the cutting or the transformer methods of considering e. m. f. generation.

This shows that both of the component fluxes must be taken

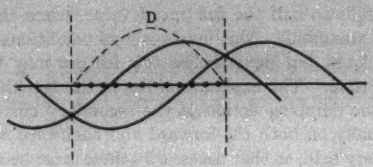

FIG. 11

into account in generating the total primary e. m. f., and if either component is decreased in value or suppressed, the total e. m. f. generated in the winding will be decreased correspondingly, unless the other component is increased a corresponding amount.

Fig. 11 is simply a continuation of the conditions of Figs. 9 and 10, in showing an intermediate position of the component field. The result is the same as if the two fields were momentarily replaced by the field D.

According to the above analysis, to produce a given counter e. m. f. in the primary, with one of the component fields damped out, the other component must be doubled in value. It was shown before that in the single-phase induction motor, running at full speed with no load, the backward field is practically damped out by the secondary current. Thus with only the forward component field remaining, either the counter e. m. f. will be halved or the forward flux component must be doubled, the latter being the case. This means, in turn, that the primary

magnetomotive force must be doubled in value. In other words, suppressing one of the two rotating field components results in doubling the no-load excitation of the motor. Furthermore, doubling the magnetomotive force of the primary and thus doubling the forward component of the field also doubles the backward component, which, in turn, is suppressed by doubled secondary current. The above conditions of doubled excitation is on the basis of sine flux distribution. With other distributions the same result holds approximately, but not exactly, due to conditions involving the shape of the field.

It is evident from the above that, with the secondary circuits open, the excitation required is of constant value regardless of the speed of the rotor core and windings; also when running at speed, the primary excitation is doubled as soon as the secondary circuit is closed. However, it is not obvious, on first consideration, that even with the secondary circuits closed the primary excitation falls to half the full speed value, when the motor is brought to standstill. This involves load conditions which will be treated later, but nevertheless this feature may be brought out at this time. The explanation lies in the fact that at rotor standstill the damping action of the secondary current will be exerted equally on both the forward and backward components of the primary field, so that necessarily these must be maintained at equal value, and, by the above analysis, this requires but half the excitation, compared with the no-load full-speed condition where the backward field is practically completely suppressed.

Load Conditions

When the single-phase induction motor is loaded, the total input current can be considered as made up of two components, namely, the no-load (practically all magnetizing) and the load current. This latter is simply the increased current in the primary due to the load and does not entirely represent energy. This load current, being single-phase, may be represented by two equal oppositely rotating magnetomotive forces in the primary of the motor, just as in the case of the no-load current. The fields which these two magnetomotive forces tend to set up are both practically suppressed by two equivalent *secondary* magnetomotive forces rotating in opposite directions. The forward secondary component corresponds to the secondary load magnetomotive force in the polyphase motor and the interaction between this magnetomotive force and the forward primary field

develops torque just as in the polyphase motor. The backward component, at first thought, would appear to develop an opposing torque, corresponding in value to that of the polyphase motor at approximately 200 per cent slip. This, however, is not the case, for at this slip the ordinary polyphase motor takes an excessive primary current tending to develop a large magnetic field, which is suppressed by a correspondingly large secondary magnetomotive force. In the single-phase induction motor, however, the primary backward rotating magnetomotive force component, due to the load current, can be only of the same value as the forward. This fact must be borne in mind as it is a very important factor in the later analysis.

To illustrate the characteristics of the single-phase induction

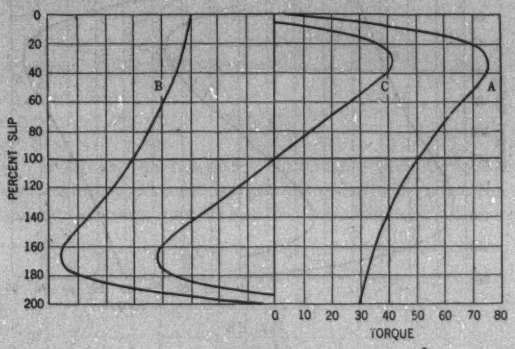

FIG. 12

motor, it may be compared with the action of two polyphase induction motors rigidly coupled together, and connected to the line to give opposite rotations. Such a set or unit has certain characteristics which are so similar to those of the usual single-phase induction motor that on first consideration one would assume them to be identical. However, a more careful study of the individual operating conditions shows that the similarity is only a general one, and a number of decided discrepancies are found.

The characteristics of the above two-motor unit and the single-phase motor may be compared as follows:

(1) The speed torque characteristics of the two motors of the polyphase unit may be represented by A and B in Fig. 12 and

their resultant by curve *C*. According to this latter curve, the resultant torque is zero at standstill, and a slight change in speed in either direction will give an effective torque tending to speed up the unit in whichever way it is started. This, therefore, corresponds to the well known starting characteristics of the single-phase motor.

(2) It may also be seen that the maximum torque the unit develops is materially less than that of either of the two component motors. This fact is also consistent with single-phase motor operation compared with the same machine on polyphase.

(3) At full speed, according to this resultant curve, the slip

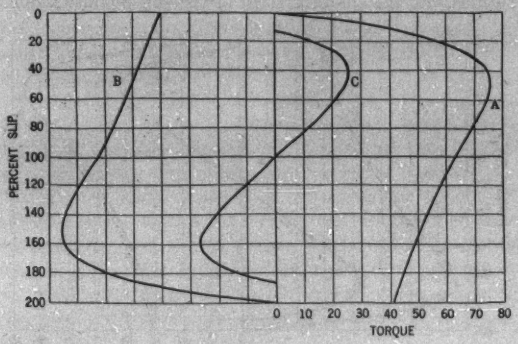

FIG. 13

for a given torque is very much larger than that of the corresponding polyphase motor. This is not true of the single-phase motor and herein lies one of the discrepancies in this method of illustrating the operation.

(4) It is well known that in the polyphase motor the maximum torque it can develop, with constant voltage applied, is independent of the secondary resistance; while, in the single-phase motor, in general, an increase in the secondary resistance will decrease the maximum torque and a decrease will have the opposite effect. This may be illustrated by repeating the curves of Fig. 12 with modified secondary resistance in the two component motors. In Fig. 13 the secondary resistance is increased and in Fig. 14 is decreased relatively to that of Fig. 12. The

resultant speed-torque curves for the three figures show that the maximum torques are materially affected by the secondary resistance. The same holds true for the single-phase induction motor.

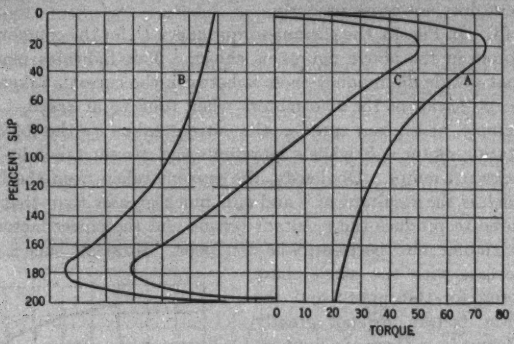

FIG. 14

(5) However, this method of illustrating the characteristics of the single-phase motor torque fails when the condition of secondary resistance is such that the maximum polyphase torque

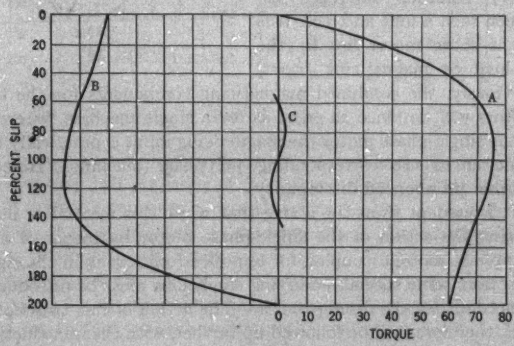

FIG. 15

is developed at about 100 per cent slip. Fig. 15 illustrates this. From this speed-torque curve it appears that the unit has a very low resultant torque, but this is not the case in the single-phase

induction motor, for with a polyphase motor developing its maximum torque at 100 per cent slip, the same machine on single-phase will give a very considerable maximum torque. Here again is a discrepancy which the assumed equivalent arrangement does not cover properly.

(6) In Fig. 16, the current-torque curve D, for the component motors in the above figures, is shown. This indicates plainly what a wide discrepancy there is between the currents taken by the primaries of the two motors when running at speed. For example, at a given speed a, the current taken by the forward rotating motor is b, while c represents the current taken by the backward motor. Obviously, the current taken from the line, which is the resultant of b and c, is much greater than that required to produce the resultant torque and the power factor of such a unit must necessarily be very poor. However, such is not the case with the single-phase motor, for the inputs and the power factors are not greatly different from those of polyphase motors of the same capacity. Herein lies a radical difference between the single-phase motor and the above assumed unit.

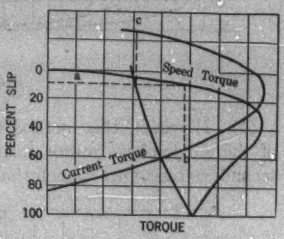

FIG. 16

(7) Another difference between such a unit and the true single-phase motor lies in the no-load or magnetizing input. Obviously, the combined magnetizing components for the two motors will be twice as great as for a single machine, whereas, in the single-phase motor the magnetizing input is practically the same as in the corresponding polyphase machine. Here is another pronounced discrepancy.

It is evident from the above that while this method of illustrating the action of the single-phase motor by means of two polyphase motors, coupled for opposite rotation, is in the right direction, some special modifying conditions must be introduced to account for the discrepancies. The action of this two-motor unit, therefore, will be followed up further, with the introduction of certain modifications derived primarily from consideration of certain characteristics of the single-phase induction motor itself.

In the first place, curves A, B and C of Fig. 12 were based upon equal and constant e. m. fs. applied to the terminals of both

motors. That this is not a correct assumption can be determined from the operating conditions in the single-phase motor. From the analysis of the component rotating fields it was shown that at full speed the backward component was practically damped out by a secondary magnetomotive force, thus leaving only the forward component, which then rose to practically double value in order to generate the required e. m. f. However, at standstill, the secondary winding holds the same rotational relation with respect to both component fields and, therefore, neither field can be damped out more than the other. Consequently, at standstill, both component fields are equal in value and the counter e. m. f. of the primary is generated by the two oppositely rotating fields, instead of a single one of double value

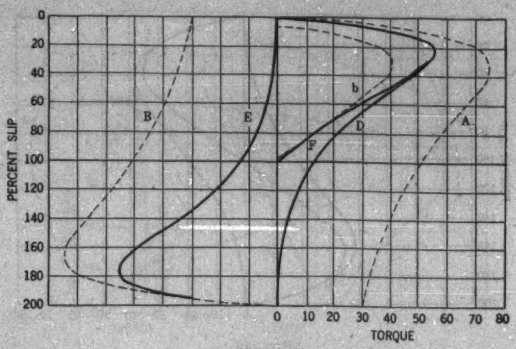

FIG. 17

as is the case at full speed. Therefore, at standstill, the forward field is of only half the value of the full speed field. This corresponds to the operation of the polyphase motor at half field strength, that is, *with half the primary voltage applied*, thus requiring one-quarter the magnetizing input. The same voltage condition applies also for the backward component at zero speed.

It would appear, therefore, that in the unit composed of two polyphase motors coupled together, the voltage applied to the terminals of the forward motor should be at practically full value at synchronous speed and should fall to half value at standstill or 100 per cent slip, and should have practically zero value at 200 per cent slip. Then assuming, as a first approximation, that the decrease in voltage from full speed to 200 per cent slip

is a straight line law, new speed-torque curves, corresponding to Fig. 13, but with the torques decreasing as the square of the voltage, can be illustrated as in Fig. 17. Here curves *A* and *B* correspond to Fig. 12, while *D* and *E* correspond to the above proportionate reductions in voltage. The resultant *F* of these latter curves is also shown.

This new resultant *F* is similar in general shape to *C* of Fig. 12, but indicates some quite different characteristics. For instance, at the higher speed values it coincides quite closely with the polyphase speed-torque curve, which is actually the case in the single-phase motor. In the second place, with high secondary resistance, as shown in Fig. 15, the speed-torque curves are modified as in Fig. 18, which shows both the former characteristic

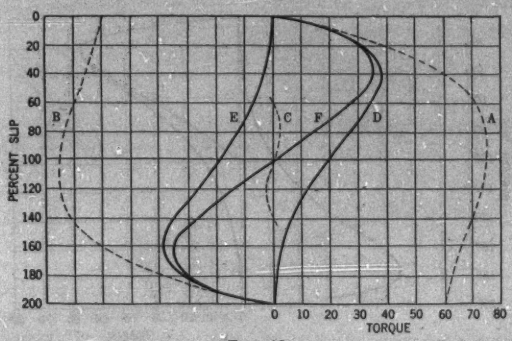

FIG. 18

and the new one. Here the resultant torque, under the new assumption is materially higher and more nearly conforms with the condition in the single-phase motor.

Under the earlier assumption of constant voltage on both motors, it was shown that the magnetizing current would be twice as great as in the single-phase motor. On this new assumption, however, at full speed, with practically full voltage on one motor and zero voltage on the other, the total magnetizing current will be only half as great, and will approximate that of one motor alone, and, therefore, that of the single-phase motor.

Furthermore, under the new assumption, the current taken by the primary of the backwardly rotating motor is quite small at high speed and, therefore, the resultant current taken from the

line is not excessive and is more nearly consistent with actual single-phase motor conditions.

Thus, with this new condition of reduced terminal voltage with reduction in speed, practically all the conditions of the single-phase motor are met, except possibly from the quantitative viewpoint. The two-motor combination thus serves as a very good illustration. There is, however, one further condition *which must be rigidly met* if the new curves are to be reasonably exact, namely, *the primary currents taken by the two motors must be equal*, for, as shown in the early part of this analysis, the forward and backward rotating components of the primary current in the single-phase induction motor are equal at all times. Consequently to duplicate this condition, the primary e. m. fs. im-

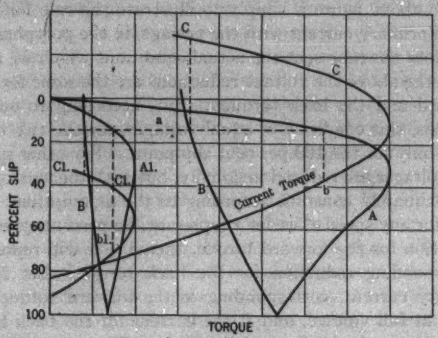

FIG. 19

pressed upon the terminals of the two polyphase motors should be varied in such a way that the primary currents will always be equal. In addition, it is assumed that the sum of the two impressed voltages is constant. This, however, is only an approximation.

The next step is to determine what is the actual law of voltage variation which will satisfy the above conditions of current and voltage. A ready means for obtaining this lies in the speed-torque and current-torque curves of the polyphase motor at constant voltage. From the current-torque curve at constant voltage corresponding curves for any other voltage can readily be plotted by varying the abscissas as the square of the voltage and the ordinates directly as the voltage. This is illustrated in Fig. 19. Here *A* is the polyphase motor speed-torque curve at

constant voltage. B represents the part below the 100 per cent slip line, but turned above the zero speed line for convenience. B can also be considered as the back torque at full voltage, but thrown to the right of the zero torque line for convenience. Curve C represents the primary current for full voltage conditions. Then at a speed a, for example, the primary currents corresponding to the forward and back torque will be b and c respectively.

Assume next that the voltage is halved for both rotations, then the new speed-torque curves will be A_1 and B_1 in which the torques are reduced as the square of the voltage. The new current curve will be C_1. The currents for speed a will now be b_1 and c_1, or half of b and c, as they are varied as the voltage.

The above figure is simply to illustrate the rule for variation of the primary current with the voltage, in the polyphase motor, and does not represent the actual conditions which we are after; for in the above the voltage reductions are the same for both the forward and the back torques. But, according to our former analysis, this condition of equal voltages, for the two rotations, holds only for the 100 per cent slip point. For other speeds the two voltages are reduced unequally, but with the sum of the two approximately constant according to the assumptions.

If, for any speed a, we let x represent the percentage of voltage reduction for the forward torque, then $1 - x$ will represent the corresponding reduction for the back torque. Let I_f be the primary current, corresponding to the forward torque for this speed at full voltage, and I_b the current for the back torque at the same speed and also for full voltage. Then $I_f x$ will represent the primary current at the reduced voltage for the forward rotation and $I_b (1 - x)$ will be the primary current for the back rotation. One of the conditions of our two-motor unit, to make it correspond with the single-phase motor, is that these two primary currents must be equal. Therefore, $I_f x = I_b (1 - x)$, and

$$x = \frac{I_b}{I_f + I_b} \text{ and } (1 - x) = \frac{I_f}{I_f + I_b}.$$

The above allows the determination of the percentage x of full voltage which must apply for each speed between zero and synchronism, when the values of the current I_f and I_b for full voltage are known.

A second method of determining the percentages of voltage

for the two rotations is available when the speed-torque curve of the motor on single-phase has been determined, by test or otherwise. By our former assumption this single-phase torque is the difference between the speed-torque curves for the forward and backward rotations with the respective voltages reduced the proper percentages. These torques for any given speed vary as the square of the terminal voltage. For example, calling T_f the forward torque, at full voltage and speed a, and T_b the back torque, and T_1 the single-phase torque for the same voltage and speed, then $T_f x^2 - T_b (1 - x)^2 = T_1$, from which x may be determined, with T_f, T_b and T_1 known.

It would appear from the preceding that, if the assumptions made are anyways close to the actual conditions, this method of analysis shows an approximate means for deriving the single-phase speed-torque curve from the polyphase curves of the same machine. Methods of calculating the primary current and speed-torque characteristics of the polyphase motor have been developed quite completely, so that it is not necessary at this place to give any details of such methods. The accuracy of the methods for calculating the polyphase curves depends almost entirely upon the correct determination of the reactance and saturation constants. All methods for the direct determination of the single-phase speed-torque characteristics also involve the use of corresponding reactance and saturation constants. Therefore, the above method brings in no new and more difficult conditions. The primary object of this paper, however, is not to develop a new method of calculation, but simply to give a better conception of the close relation of the single-phase and polyphase characteristics.

After development of the above method, an attempt was made to check it by applying certain existing test data, but without positive results, although the indications were quite satisfactory. It was discovered that in all the existing test data at the writer's command, where the polyphase speed-torque and current-torque curves have been obtained by actual test, constancy of temperature had been more or less disregarded. The effect of change in the secondary resistance on the polyphase speed-torque curve is to change the slips but not the maximum torque. The difficulty, however, in the polyphase tests available was that apparently the resistance had varied very considerably during the tests, especially at the points of high slip, where the secondary losses were very large. As a result the speed-torque curves

corresponded to those of motors in which the resistance increased as the load and slip increased. As a consequence, the torques below the zero speed line were considerably too large, which meant that in applying these curves to the above method, the back torques were presumably entirely too great, thus apparently introducing errors in the derivation of the resultant single-phase curve.

The effect of these discrepancies are shown in Fig. 20. Here, *A* shows the speed-torque curve as it should be at constant temperature, whereas, *B* shows the curve with the resistance of the secondary increasing with increased slip. The corresponding current-torque curves are also shown. A consideration of these curves would seem to indicate that the resultant single-phase curves derived from *A* and *B* should differ somewhat.

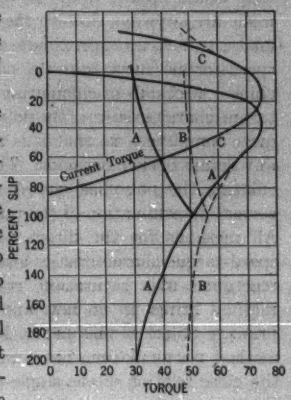

FIG. 20

It was then decided to make a more accurate set of tests on a 10-h.p., 60-cycle four-pole, three-phase motor of the wound-secondary type, so that the secondary resistance could be varied if so desired. It was also decided to obtain a test with two similar motors rigidly coupled together, with their individual primary windings in series, but with their secondaries independent. As already explained, the theory of the foregoing method calls for *equal currents* in the two oppositely rotating fields. This condition is automatically obtained by coupling two primaries in series with each other.* With this arrangement, if the power factors of the two motors were always equal, then it should be equivalent to the method already described. However, these are practically never equal except at the standstill position, although an analysis of the problem shows that the two primary voltages, with this series arrangement, are not greatly out of phase with each other over a very large part of the working range. The

*In reviewing an early draft of this paper, this suggestion, with a number of other most excellent ones, was made by Mr. R. E. Hellmund. However, it developed later that this same suggestion appeared about twenty years ago in Mr. B. A' Behrend's book, "The Induction Motor."

writer has not yet sufficiently analyzed the series arrangement to be sure that it exactly represents all the conditions of the two rotating fields in the single-phase motor, but is inclined to think that such is the case. However, the approximate method developed in this paper lends itself so readily to calculation, that it was considered worth while to check it up carefully by test to see what degree of accuracy could be obtained.

The following series of tests was planned:

(1) Three-phase speed-torque and primary current curves at 220 volts with one motor alone, with its secondary short-circuited on itself.

(2) Single-phase speed-torque and primary current curves on the same motor as (1) at 220 volts and with the secondary short-circuited on itself.

(3) Three-phase speed-torque and primary current curves on the same motor and at same voltage, but with external resistance in the secondary circuits.

(4) Single-phase speed-torque curves under same conditions as (3).

(5) Speed-torque and primary current curves with two similar motors with their primary windings coupled in series, and with the secondaries independently short-circuited on themselves, one of these motors to be that used in tests (1) and (4).

(6) Similar tests to (5), but with resistance in the secondaries as in (3).

In carrying out these tests, the torque was measured by a special dynamometer brake, the power absorbing element of which consists of a special separately-excited direct-current machine. Below zero speed, power was supplied to the direct-current machine in order to obtain negative rotation.

Difficulties in obtaining consistent tests, especially at negative speeds, soon developed, due to variations in temperature. With the very heavy currents at low and at negative speeds, the motor would heat so rapidly that all kinds of speed-torque readings could be obtained. Test after test was made and while these would agree very well for the higher speed points where the heating was small, they showed all kinds of inconsistencies for the negative speeds, in particular. The currents for these speeds also showed very wide discrepancies. Eventually it was found that those tests taken with extreme rapidity, and which covered only a comparatively small number of points, would plot in quite reasonable curves above zero speed, so that the writer was enabled thus to obtain quite consistent curves

for both torque and current between 1800 rev. per min. and standstill. Not only were the curves, consistent in themselves but those taken with different secondary resistances were fairly consistent with each other. It then remained to obtain reasonable readings for the negative speeds. Obviously it was wrong to take a large number of test points and then draw an average curve through them, for it is evident that the errors, due to heating, tend to throw the torques and currents to one side of the proper curves. Consequently the correct curves should really be boundary lines rather than averages. It was noted, in particular, that heating did not appear to affect the speed to the same extent as the torque at very large slips, and, consequently, by plotting the current in terms of speed rather than torque, less erratic curves were obtainable, and it was possible to plot speed-current curves which were quite consistent for the different conditions of secondary resistance. Furthermore, from the speed-torque and speed-current curves above the zero line, which appeared to be reasonably correct, as they were consistent with each other, it was possible to derive the constants for the general equations for speed-torque. It was found that such derived equations fitted these curves quite accurately and, moreover, they held the proper relation of constants for both high- and low-resistance secondaries. The various agreements between the calculations and the tests for the higher speeds were such that one could assume that the derived equations were practically correct and that from them the curves for the negative speeds could be plotted with fair accuracy. In this way the curves for the negative speeds were first obtained and it then remained to check them by actual test. Finally a method of testing was tried which appeared to give quite good results. This consisted in setting the apparatus at about the desired speed and torque conditions; then cooling the motor down to the required temperature preparatory to obtaining the desired test, the power was then thrown on and readings obtained in the shortest possible time, five seconds, for instance. Allowing the motor to run, additional readings were obtained at five-second intervals. A series of consecutive readings, at definite intervals apart, was thus obtained and plotted in a curve. By extending this curve back to the instant of starting, results were obtained which were undoubtedly quite close to those corresponding to the starting temperatures, and were not only quite consistent with each other, but also plotted very close to the negative exten-

sions of the calculated curves. As a result of a series of tests extending over several weeks, data was obtained which plotted in curves which agreed fairly well with each other throughout.

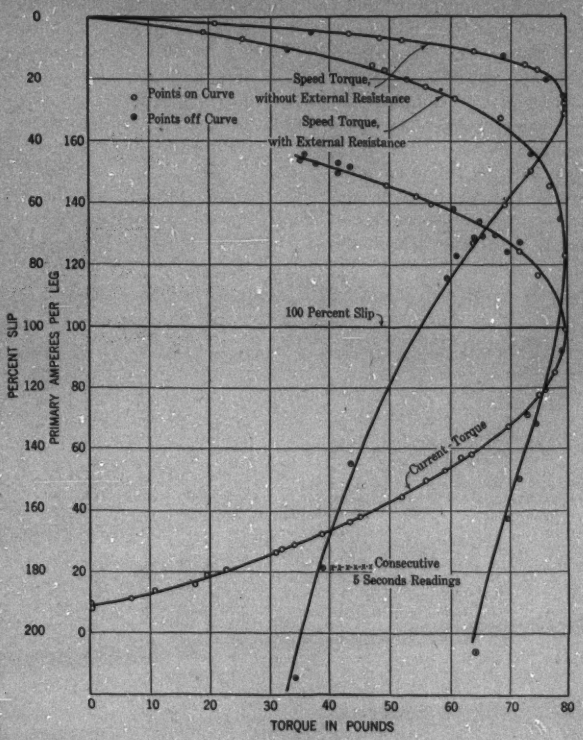

FIG. 21

RESULTS OF TESTS

Polyphase Speed-Torque, Speed-Current and Current-Torque Curves

In Fig. 21 are shown the polyphase speed-torque and primary current both with the secondary short-circuited, and with resistance added. In the speed-torque curves the circled points

represent actual test readings, while the solid line covers the points calculated from the derived equations.

In Table I, covering data on the short-circuited rotor tests, are shown the forward and back torques and the corresponding forward and back currents for the various speeds between zero and 200 per cent slip, as derived from Fig. 21; also the calculated values of the ratio of voltages, x and $(1 - x)$, by which the forward and back torques should be reduced in order to get the equivalent single-phase speed-torque curve. The corresponding reduced values for the forward and back torques are also given as calculated from the values x and $(1 - x)$. The last column shows the difference between the reduced forward and back torques, which should represent the single-phase torque, according to the foregoing analysis.

TABLE I

Slip		Torque at full voltage		Primary amperes per leg at full voltage		$X =$		Reduced torque		Resultant torque
For positive speeds	For negative speeds	T_f	T_b	I_f	I_b	$\frac{I_b}{I_f+I_b}$	$1-x$	Forward	Back	
0.02	1.98	20.	35.8	19.0	154.8	0.89	0.11	15.8	0.4	15.4
0.05	1.95	41.	36.3	34.0	154.5	0.819	0.181	27.5	1.1	26.4
0.10	1.90	61.7	36.9	55.5	154.0	0.735	0.265	33.3	2.6	30.7
0.15	1.85	71.6	37.7	71.0	153.5	0.684	0.316	33.5	3.8	29.7
0.20	1.80	77.3	38.3	85.0	153.0	0.643	0.357	31.9	4.9	27.0
0.25	1.75	79.4	39.2	96.0	152.5	0.614	0.386	29.9	5.9	24.0
0.30	1.70	79.6	39.9	104.0	152.0	0.594	0.406	28.1	6.6	21.5
0.35	1.65	78.8	40.8	110.0	151.5	0.580	0.420	26.5	7.2	19.3
0.40	1.60	77.6	41.6	113.0	151.0	0.572	0.428	25.4	7.6	17.8
0.50	1.50	74.0	43.6	121.0	150.0	0.554	0.446	22.9	8.7	14.2
0.60	1.40	70.0	45.5	128.0	149.0	0.538	0.462	20.3	9.7	10.4
0.70	1.30	65.9	47.8	133.0	147.3	0.526	0.474	18.2	10.8	7.4
0.80	1.20	62.1	50.0	136.5	145.5	0.516	0.484	16.6	11.7	4.9
0.90	1.10	58.8	52.7	139.5	143.5	0.507	0.493	15.1	12.8	2.3
1.00	1.00	55.5	55.5	141.5	141.5	0.50	0.500	13.9	13.9	0

In Fig. 22 are shown the single-phase speed-torque and current-torque curves with short-circuited secondary, as plotted from Table I, and checked by actual test. The circled dots represent actual test points, while the crosses represent points plotted from the last column in Table I. The agreement of test and calculated values are as close as can really be expected considering the difficulties in obtaining the data, and the possible errors.

Unfortunately, due to the very short time available, it was not possible to make any extended tests on single phase with a view to correcting for temperature. In consequence, the calculated single-phase speed-torque curve, which is on the basis of constant temperature, is compared with tested curves in which no temperature correction has been made. It, therefore, is not known in this case how much of the discrepancy is due to temperature.

In Table II is shown data similar to that of Table I, but for the tests with resistance in the secondary. It will be noted that the resultant of the forward and back torques is considerably lower than in Table I, which is consistent with the fact that in-

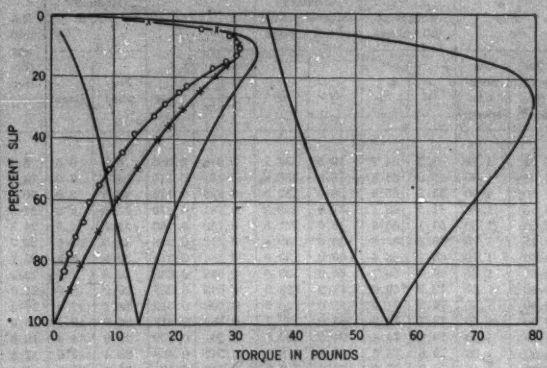

FIG. 22

creased secondary resistance reduces the maximum torque of the single-phase motor.

In Fig. 23 is shown the calculated single-phase speed-torque and the tested torques of the motor with resistance in secondary. Here the circled dots represent the actual test readings and the crosses represent the points obtained from the last column of Table II. The discrepancies are somewhat smaller than in the motor with short-circuited secondary. This should be the case, if heating is responsible for any considerable part of the discrepancy, for the currents are relatively smaller.

In order to get a crude idea as to how much of the difference may be due to this feature of temperature, a polyphase speed-

torque test was selected in which no correction had been made for temperature and where the conditions were quite closely comparable with those of the single-phase tests. From the speed-torque and current data of this polyphase test, the resultant single-phase speed-torque curve was calculated, making no attempt at corrections of any sort. This speed-torque curve is represented by the small squares in Fig. 23. This lies much closer to the tested single-phase curve, thus indicating that temperature is possibly an explanation of a considerable part of the discrepancy between the calculations and the tests. This would

TABLE II.

Slip		Torque at full voltage		Primary amperes per leg at full voltage		$X =$		Reduced torque		Resultant torque
For positive speeds	For negative speeds	T_f	T_b	I_f	I_b	$\frac{I_b}{I_f+I_b}$	$1-x$	Forward	Back	
0.02	1.98	8.2	64.3	12.5	134.2	0.915	0.085	6.9	0.5	6.4
0.05	1.95	18.9	64.8	18.0	133.8	0.881	0.119	14.7	0.9	13.8
0.10	1.90	33.3	65.4	28.0	133.0	0.826	0.174	22.5	2.0	20.5
0.15	1.85	41.5	66.1	37.0	132.3	0.781	0.219	27.5	3.2	24.3
0.20	1.80	53.1	66.9	46.0	131.5	0.740	0.260	29.3	4.5	24.8
0.25	1.75	59.8	67.6	53.0	130.8	0.712	0.288	30.3	5.6	24.7
0.30	1.70	65.1	68.4	60.0	130.0	0.684	0.316	30.5	6.8	23.7
0.35	1.65	69.2	69.1	66.0	129.0	0.662	0.338	30.4	7.9	22.5
0.40	1.60	72.2	69.9	71.2	128.0	0.643	0.357	29.9	8.9	21.0
0.50	1.50	76.5	71.4	81.0	125.5	0.608	0.392	28.2	11.0	17.2
0.60	1.40	78.7	72.9	88.0	122.7	0.582	0.418	26.6	12.7	15.9
0.70	1.30	79.6	74.4	94.0	120.0	0.561	0.439	25.0	14.4	10.6
0.80	1.20	79.6	75.9	99.5	118.1	0.542	0.458	23.4	15.9	7.5
0.90	1.10	79.2	77.8	104.0	112.2	0.519	0.481	21.4	18.0	3.4
1.00	1.00	78.9	78.9	108.2	108.2	0.50	0 50	19.7	19.7	0

also indicate that heat effects as referred to in connection with Fig. 20 are not as objectionable as anticipated. However, the writer does not believe that all the discrepancy is due to heating, but considers that this approximate method of dealing with the problem makes the back torque too small. In the arrangement with two motors in series, as mentioned before, the voltages of the two motors will not usually add up directly to give the line voltage, and the motor which represents the back torque, will have a relatively larger percentage of the total voltage than is the case with the above method of considering the problem. This will be considered further under the two-motor tests.

Two Motors in Series

In Table III is shown the test data and the calculations derived therefrom, for two motors with their primaries in series and with their secondaries short-circuited independently. In this test no external resistance was used in the secondaries. Considerable difficulty was encountered in making this test, due partly to bad alignment of the machines, as they were rigidly coupled together. Furthermore, in several of the earlier tests, the effects of temperature were disregarded and all indications were that the secondaries were quite hot during the tests. There was so much discrepancy between the various results that the

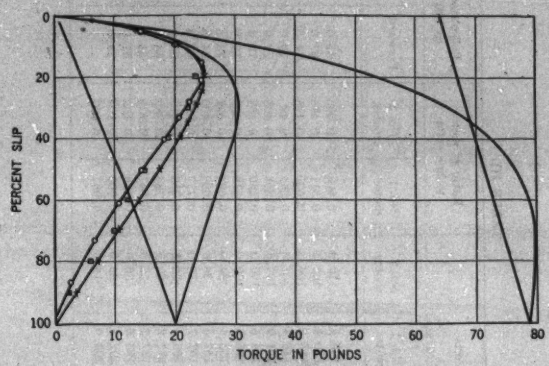

FIG. 23

writer cannot feel sure of the data shown in this table, although it was obtained under quite careful conditions of test.

In the above table the percentage of line voltage applied to each motor is shown. It is of interest to compare these percentages with those shown in Table I. This is illustrated in Fig. 24. This shows that the percentage of voltage on the forward rotating motor is higher at the higher speeds, than in Table I, but is lower at the low speeds. On the other hand, the voltage on the backward-rotating motor is higher at all speeds than in Table I. Thus, the back torque has always a higher value than in Table I. Consequently, with the reduced forward torque at the lower speeds and the higher back torque, the resultant torque

TABLE III.

Speed		Torque corrected to 220 volts	Volts			Ratio of motor to line volts		Polyphase torque at 220 volts		Polyphase torque at reduced voltage		Resultant torque
r. p. m.	Slip		Line	No. 1 motor	No. 2 motor	No. 1 motor	No. 2 motor	Forward	Back	Forward	Back	
300	0.833	1.5	218.5	109.2	110.5	0.506	0.505	61.0	51.4	15.6	13.1	2.5
600	0.667	2.5	217.0	110.9	107.5	0.511	0.495	67.0	47.0	17.5	11.5	6.0
800	0.556	4.9	218.5	115.8	106.8	0.530	0.488	71.5	44.5	20.9	10.6	10.3
1000	0.444	8.3	219.0	118.4	105.0	0.541	0.48	76.0	42.4	22.2	9.8	12.4
1100	0.389	9.7	219.0	121.2	102.5	0.553	0.468	77.9	41.4	23.9	9.1	14.8
1200	0.333	12.3	219.0	124.5	100.0	0.569	0.457	79.2	40.3	25.6	8.4	17.2
1300	0.278	15.5	219.0	126.5	100.0	0.578	0.457	79.6	39.3	26.5	8.2	18.3
1400	0.222	19.6	220.0	133.4	94.0	0.605	0.427	78.8	38.5	28.9	7.1	21.8
1500	0.167	24.4	220.0	152.5	88.0	0.693	0.400	74.5	37.6	35.8	6.0	29.8
1580	0.122	27.6	220.0	157.7	78.0	0.717	0.355	67.0	37.0	34.4	4.7	29.7
1620	0.100	27.9	221.0	166.4	70.0	0.753	0.317	62.0	36.7	35.1	3.7	31.4
1660	0.078	29.2	221.0	173.2	62.5	0.784	0.283	54.5	36.3	33.5	2.9	30.6
1700	0.056	26.1	221.0	187.2	52.5	0.847	0.238	44.0	36.0	31.6	2.0	29.6
1740	0.033	19.9	222.5	195.5	36.0	0.879	0.162	31.0	35.7	23.9	0.9	23.0
1780	0.011	10.	223.0	202.8	25.0	0.909	0.112	14.0	35.4	10.3	0.4	9.9
1800	0.0	....	224.0	208.0	14.0	0.961	0.062	....	35.3	....	....	....

derived from the polyphase curve will naturally be lower than in Table I, which appears to be the case in all the tests made.

The data in Table III indicate that the two motors have their

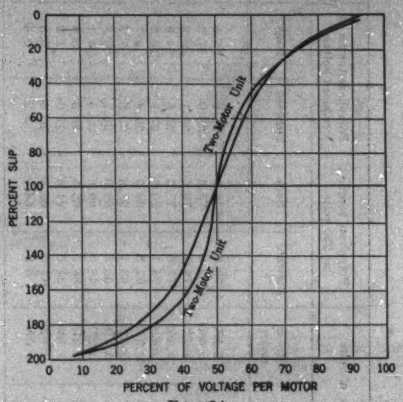

FIG. 24

primary voltages very nearly in phase at all times. The sum of the two motor voltages is never much greater than that of the line.

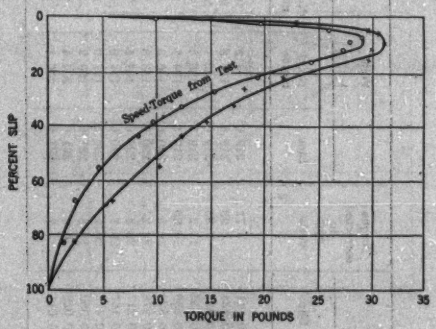

FIG. 25

In Fig. 25 is shown the calculated and test speed-torque results corresponding to Table III. The test result shows lower torques at the low speeds than can be derived from the voltage percent-

TABLE IV.

Speed		Torque corrected to 220 volts	Volts			Ratio of volts		Polyphase torque at 220 volts		Polyphase torque at reduced voltage		Resultant torque
r. p. m.	Slip		Line	No. 1 motor	No. 2 motor	No. 1 motor	No. 2 motor	Forward	Back	Forward	Back	
340	0.811	1.71	219.0	114.3	108.0	0.521	0.493	79.6	76.1	21.6	18.5	3.1
480	0.734	4.93	219.0	117.8	105.0	0.538	0.480	79.7	75.0	23.0	17.3	5.7
640	0.645	7.85	219.0	122.1	102.0	0.552	0.470	79.1	73.5	24.1	16.2	7.9
810	0.555	11.2	219.5	126.4	98.0	0.576	0.446	77.8	72.0	25.8	14.3	11.5
1010	0.439	15.25	219.5	134.2	93.0	0.612	0.424	74.1	70.1	27.8	12.6	15.2
1220	0.322	20.5	220.0	146.3	83.0	0.664	0.377	67.5	68.5	29.8	9.7	20.1
1300	0.278	22.2	220.0	153.3	79.0	0.697	0.359	63.1	67.8	30.8	8.7	22.1
1410	0.217	23.8	220.0	164.5	70.0	0.748	0.318	55.5	66.9	31.1	6.8	24.3
1500	0.167	23.1	220.0	174.0	61.0	0.791	0.277	48.0	66.3	30.5	5.1	25.4
1600	0.111	20.1	220.5	187.0	47.0	0.848	0.213	36.5	65.5	26.3	3.0	23.3
1700	0.050	12.6	221.0	199.2	30.0	0.90	0.136	31.0	64.6	17.0	1.2	15.8
1750	0.028	6.0	221.0	204.4	24.0	0.925	0.109	12.0	64.3	10.3	0.8	9.5
1780	0.011	3.2	221.0	206.1	17.0	0.933	0.076	6.0	64.0	5.2	0.4	4.8

ages applied to the polyphase torques. Part of this difference may be due to temperature conditions.

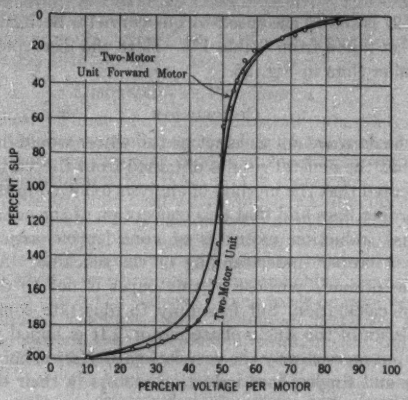

FIG. 26

In Table IV is shown the corresponding data for two motors with resistance in the secondary. Under this condition the various tests made were more consistent with each other and

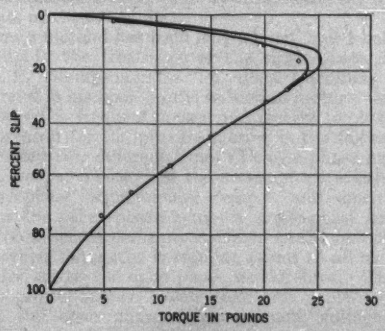

FIG. 27

the writer has more confidence in the data than in the case of Table III.

In Fig. 26 is shown the percentages of line voltage on each of

the two motors, compared with those in Table II. These show the same differences as in Fig. 24, where there was no external resistance.

In Fig. 27 is shown the speed-torque curve for both calculation and test, as taken from Table IV. Here the discrepancies are much smaller than in Fig. 25.

Conclusion

While the data are not as exact as the writer would desire, yet he feels that the general results obtained from the various tests have indicated that the method of analysis followed in this paper is along proper lines and that this conception of the action of the single-phase induction motor is of considerable assistance in obtaining a proper understanding of the machine. As stated before, the primary purpose of this paper is not to develop a method of calculation, but is simply to illustrate some of the characteristics of the single-phase motor. It is hoped that this will bring out more clearly the very intimate relation between the polyphase and single-phase induction motors in their operating characteristics.

DISCUSSION ON "NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS" (HELLMUND) AND "A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR" (LAMME), PITTSBURGH, PA., APRIL 10, 1918 AND NEW YORK, N. Y., APRIL 12, 1918.

DISCUSSION AT PITTSBURGH

J. Slepian: These two papers illustrate what a great difference in the clearness of understanding of a complex system of circuits is made by different choices of the simpler systems out of which it is attempted to build up the more complex systems.

Both papers are concerned with symmetrical induction motors having coils missing. Mr. Lamme takes as his simple system the complete symmetrical machine operating under balanced conditions. He then determines two different balanced conditions such that when super-imposed in one machine their resultant will give zero current in the coils which must be removed to get the unbalanced machine. Mr. Hellmund, however, takes as his simple system the individual circuits of the unbalanced machine. Mr. Hellmund's choice of a simpler system appears perhaps as more fundamental than Mr. Lamme's, inasmuch as the basic laws of electromagnetism were first developed for simple closed circuits, but in the actual study of an unbalanced machine most will agree that Mr. Lamme's choice works better. The papers show quite forcibly that clear conceptions and correct results can be obtained much more easily by trying to resolve the problem of an unbalanced machine into the sum of different balanced conditions, than by considering the individual closed circuits of the machine.

Mr. Lamme's method has been long known, and I believe was used correctly in the treatment of the single-phase induction motor by S. P. Thompson in his "Polyphase Currents," and also by Mr. Behrend in his book on the induction motor. Nevertheless, the method is frequently used incorrectly to this day. I believe one reason for the prevalent error is the failure to distinguish between two different types of single-phase motor, the single-phase *current* motor, and the single-phase *voltage* motor. Take a two-phase squirrel-cage motor and opening one phase, throw the other phase across a single-phase line. The primary current then is certainly confined to the one phase but the voltage when the motor is running is not at all so confined as a voltmeter across the open phase would show. This is the single-phase *current* motor. Now take this same motor and short-circuit the open phase. The primary voltage is now certainly confined to the one phase but the currents are not at all so confined as an ammeter in the short-circuited phase would show. This is the single-phase *voltage* motor.

The error commonly made is to develop a theory of this last type, the single-phase *voltage* motor, and then to apply the

results obtained to the generally used motor of the first type, the single-phase *current* motor. Of course, there is lack of agreement between this theory and the observed operation.

The working out of this theory is very simple. The condition of a given voltage on one phase and zero voltage on the other is resolved into the sum of two balanced two-phase voltages, of one-half the magnitude of the given voltage, agreeing in sign in one phase and opposite in sign in the other phase. The resultant currents, torques, etc., of the single-phase *voltage* motor are obtained by adding together the currents, torques, etc. produced by the two two-phase voltages independently. Since the speed-torque curves of an induction motor operated on balanced polyphase constant voltage are well known, the speed-torque curve of the single-phase *voltage* motor may be obtained as described by Mr. Lamme and illustrated in Fig. 12 of his paper. Of course, instead of letting the two two-phase voltages act simultaneously on the same motor, we may let them act on different motors and then add the torques by mechanically connecting the rotors. Furthermore, since the voltages are the same, we may connect the stators in parallel with one phase on one stator reversed. This combination would have the same speed-torque curve as the single-phase *voltage* motor.

The condition obtaining in the single-phase *current* motor, that the current in one phase shall be zero, is satisfied, as is easily seen, by the sum of two balanced two-phase states in which the stator currents of each state are of the same magnitude, agreeing in sign in one phase and opposite in sign in the other. If now, the polyphase and single-phase systems in use were of the constant-current type, the theory of this motor would be exactly similar to that of the single-phase *voltage* motor described above. We would take for the magnitude of the two two-phase currents one-half that of the constant current in the single-phase line. Knowing the torque-speed curves for a balanced motor operated on a balanced polyphase constant-current system, we could obtain the speed-torque curve of the single-phase *current* motor by combining two speed-torque characteristics in the manner of Fig. 12. Similarly, instead of having the two two-phase currents flow in the same machine, we may cause them to flow in two different machines and combine the torques by mechanical coupling of the rotors. Furthermore, since the currents in the two machines are to be the same, the stators may be connected in series with one phase of one stator reversed as Mr. Lamme has done. Of course, in actual practise the voltage of the single-phase line is constant and not the current, but this condition is easily met. Instead of the two two-phase currents being directly given, they must be chosen of such a magnitude that the voltages necessary to produce them must add to the line voltage in the phase receiving current. Thus the currents will have to be changed with each change of slip. This, however, is easily taken care of by anyone familiar with the theory of induction motors under balanced conditions.

The method used by Mr. Lamme is clearly one of superposition, that is, two different possible systems of currents, voltages, torques, etc. are added together and their sum is said to be also a possible system. While no objection can be raised as regards the currents and voltages, there may be some doubt as to the validity of combining the double frequency quantities as torque, Joulian heat, etc. in this way. In fact, such combination in general is not permissible. For example, the ohmic loss produced in a conductor by the sum of two currents of the same frequency is not equal generally to the sum of the ohmic losses which each current would produce in that conductor independently. It is a remarkable fact that for balanced circuits, two systems of currents and voltages, each balanced polyphase of the same frequency but of opposite phase relation may be combined and the mean resultant torque, heating, etc. will be equal to the sum of the mean torques, heatings, etc. which each system would produce separately. This justifies the above procedure.

Of course, the instantaneous torque, heating, etc. cannot be obtained in this way. This points to one difference between the mechanically connected series machines of Mr. Lamme and the single-phase *current* motor. In the former, the torque in each machine is constant in time, so that the same is true of their sum. In the latter the torque is pulsating, vanishing generally four times per cycle. The mean value, however, is the same in the two cases.

Mr. Hellmund, admittedly discusses idealized motors, the idealization consisting in assuming resistance and leakage reactance zero. Discussion of such ideal cases is profitable if this idealization makes the problem easier and if actually used motors with small resistance and small leakage reactance approximate in their operation these resistanceless and reactanceless cases. However, I don't think, that Mr. Hellmund will claim that most of his ideal cases are easy to work out or understand. Also, those cases where the rotor windings are of few phases and concentrated, do not correspond to motors in general use on unbalanced operation as in phase converters or balancers and they bring in complications of infinite currents and higher harmonics. The reason for the detailed consideration of such cases is given at the beginning of Case No. 5. It is because these cases show that as the number of phases in the rotor and the distribution of the rotor windings are increased, the main flux of the motor approaches a synchronously rotating sine distribution, irrespective of the field form which the primary would produce by itself. But could not this result be obtained more directly and more easily. I would like to show here that it can be, following the line of ideas used in Mr. Lamme's paper.

Consider the case of a motor with rotor of an infinite number of phases and having zero resistance and zero leakage reactance. By zero leakage reactance is meant that an arbitrary distribution

of m.m.f.'s coming from the stator can be entirely annulled by some properly chosen current distribution on the rotor. Suppose that the stator winding is single-phase and concentrated and gives a square topped m.m.f. wave. Suppose, for simplicity, that the machine is two-pole. The rotor is supposed to be at synchronous speed.

As is well known, the square-topped m.m.f. wave can be resolved by Fourier's theorem into harmonics. The fundamental will be a two-pole, sine-shaped field. The third harmonic will be a six-pole sine field; the fifth harmonic will be a ten-pole sine field, and so on.

Consider first the case of a sine-wave current in the stator. The height of the square topped m.m.f. wave will then change in time according to the sine law. It is clear that each of the harmonics into which the square topped wave is resolved will also have amplitudes varying sine fashion in time. Thus the alternating square topped m.m.f. field is resolved into a two-pole sine-shaped field alternating sine fashion in time; a six-pole sine field alternating sine fashion in time with the same frequency, and so on.

What now will be the reactions of the rotor upon these component fields. Consider first the fundamental m.m.f. field. As Mr. Lamme has so clearly explained, this can be further resolved into a forwardly rotating and a backwardly rotating sine-shaped field. The forwardly rotating component is synchronous with the rotor and therefore the flux it produces induces no currents in the rotor. Therefore, this forwardly rotating component produces its full flux. The backwardly rotating component has, however, a 200 per cent slip relative to the rotor. Rotor currents, therefore, will be induced, which will completely counterbalance this backwardly rotating m.m.f. component. Thus, no backwardly rotating two-pole flux will be produced.

Consider now the third harmonic of the m.m.f. wave, forming a six-pole alternating field. As before, this can be resolved into a forwardly rotating and a backwardly rotating component, but now having six poles the speed of rotation will be only one-third synchronous. Thus the rotor will have a 200 per cent slip relative to the forwardly rotating component and a 400 per cent slip relative to the backwardly rotating component. Hence any fluxes which these components would produce are damped out by currents induced in the rotor. In a similar way, the fifth and higher harmonics of the stator m.m.f. field produce no resultant flux. All that is left is the forwardly and synchronously rotating sine-shaped flux. Such a rotating sine flux would, of course, induce a sine-shaped counter e.m.f. in the stator so that if the impressed voltage is sine shaped, so also will be the stator current. The case where the stator current is not sine shaped offers some interesting features which I shall not go into here.

The real simplicity of the ideal case, just considered is now apparent. As far as concerns the counter e. m. f. generated by

the main flux, we see that it remains balanced polyphase under all conditions, if only the rotor is running near synchronism. This is brought about by the selective action of the rotor, which damps out any fluxes other than the synchronously rotating sine field.

Where the leakage reactance and resistance are not zero, but the rotor winding is distributed and has infinitely many phases, it is clear that this selective damping will not be perfect, and rotating components of the field other than the synchronously rotating two-pole field will persist. It is clear that the voltages induced in the stator by each rotating field component will be of fundamental frequency, but now the voltages will not be balanced polyphase. This is the approximate condition in the single-phase motor, phase converter and phase balancer.

When the winding of the rotor is concentrated and of few phases, not only does the damping out of the non-synchronous component of the field flux by the rotor become incomplete, but the rotor currents themselves set up new component rotating fluxes. These give higher harmonics in the stator voltages, as Mr. Hellmund's first cases show. It is easy to show that with the rotor running synchronously, the harmonics of stator voltage are always odd. Thus we see, that the complications in most of the cases considered by Mr. Hellmund arise from his supposition of concentration of the rotor windings. Since this case is quite different from machines actually used under unbalanced conditions, and since the facts concerning the machines of infinitely many phased rotors can be arrived at without considering these cases, its detailed study becomes of less practical importance.

C. Fortescue (by letter): The problems of design have occupied the attention of electrical designers so much in the past that there has been comparatively little activity in connection with purely theoretical considerations of machine operation. This state of affairs tends toward a narrow specialization on the part of machine designers which is not favorable to progress. Thus we find a tendency to consider each type of machine as an entity; we have literature on "Theory of Transformers", "Theory of Polyphase Induction Motors", "Theory of Single-phase Induction Motors", "Theory of Synchronous Motors", and so forth, without any inquiry as to the relation between the various types of electromagnetic apparatus. As a result of this there appears to be a distinct loss of generality in modes of thought among electrical designers, so much so that, because a machine has parts that rotate about an axis, it is often considered as being distinct in theory from an ordinary stationary network or even from a network having relative linear motion between its parts as for instance an electromagnet.

Those who have devoted some time to the theoretical investigation of electrical machinery are beginning to see that the same general analytic treatment applies to all the various types of machines. For example the induction motor differs from a

stationary network only in this respect, that the coefficients of mutual induction between different branches in the former become time functions and we can no longer express the e.m.f. induced in a given circuit due to a current flowing in another by $M_{12} \frac{d\, i_2}{d\, t}$ as in a simple network but must use instead $\frac{d}{d\, t} (M_{12}\, i_2)$ or $M_{12} \frac{d\, i_2}{d\, t} + i_2 \frac{d\, M_{12}}{d\, t}$. Carrying out the comparison a step further we see that even if i_2 is constant an e.m.f. is induced, $i_2 \frac{d\, M_{12}}{d\, t}$, an action familiar to designers of rotating machinery under the name of "rate of cutting of lines of force." Again we find in a-c. generators that if we supply reactive kilovolt amperes to a load a porportionate amount of d-c. ampere-turns must be supplied to the field in order to maintain the e.m.f. Conversely if reactive kilovolt amperes are supplied to the generator by means of condensers the d-c. ampere-turns must be reduced; in some cases even without any d-c. magnetization it is impossible to keep the terminal e.m.f. down to a reasonable value. These actions are strictly analogous to the similar actions in a transformer.

I regard Mr. Hellmund's paper as a bridge between the old methods of considering rotating machinery and the more recent ideas on the subject. His treatment is essentially anlaytic although he uses quantities and terms familiar to designers. Mr. Lamme's paper I regard as a practical demonstration of the truth of the comparatively new physical conception of the operation of a single-phase induction motor as being a special case of that of polyphase machines. This conception might have been deduced from the analytic solution of the single-phase motor which is old, but its full significance was not appreciated until quite recently, when a new analytic method of considering rotating machinery gave the solution in a form which points more directly to this conception.

In regard to Mr. Hellmund's paper, I wish to point out one very common error into which he falls, namely that of associating simplicity of parts with simplicity of theory. In physics it is not generally true, for in considering aggregates or groups it is often possible to simplify the equations without affecting the general result. Thus we are familiar with such expressions as "rigid body" in dynamics, "perfect fluid", etc., which are tacit admissions that the equations used are not absolutely general or fundamental. These expressions and the simple form of the usual electromagnetic equations of electrical circuits are the result of the application of the mathematical device known as the "ignoration of co-ordinates" that is to say the omission of such considerations as have no material weight in the final results. The method of analysis required for

practical problems is one which will permit of this and yet permit of further elaboration if desired.

Mr. Hellmund's method of attack lacks this desirable characteristic because it is affected by the number of physical parts that have to be considered simultaneously and, therefore, it is found advantageous in his method to begin with the two simple concentrated circuits in relative motion. The usual analytic treatment of this case with the condition of constraint assumed by Mr. Hellmund is also comparatively easy and leads to similar results. The first few cases which he takes up, though necessary in his train of reasoning, are of academic interest only and are not necessary in the analytic solution of the general problem of unbalanced motors. I think that the method of treatment leads to an over emphasis of non-essential factors, which are largely eliminated in actual designs, and is weak where essential factors such as secondary resistance and magnetic leakage are introduced.

In connection with the wave forms of induction motors; it is not generally appreciated to what a great extent the harmonics are eliminated. I have attached to this discussion a mathematical proof that a uniformly distributed polyphase winding on a cylinder of an infinite number of phases will give a sine wave space distribution of induction. It should be added that a winding of finite number of phases can theoretically be so distributed that it will give a sine wave space distribution of induction. These are interesting facts but others of more importance are that it is not the wave form of the induction which determines the wave form of currents in primary and secondary but the flux linkage distribution or $M_{12}\, i_1$ and $M_{12}\, i_2$. These still more nearly approach sine wave form with properly distributed windings. Thus the secondary flux linkage wave form of one phase of a uniformly distributed three-phase winding due to a balanced primary currents will take the form,

Flux leakage

$$= K\, i_1 \left(\cos\theta + \frac{1}{625} \cos 5\,\theta + \frac{1}{2401} \cos 7\,\theta + \ldots \frac{1}{n^4} \cos n\,\theta \right)$$

where n is always odd. The harmonics are so small that they may be ignored practically. The above does not include tooth harmonics. These may, however, be reduced to a negligible factor in the design.

In a paper presented before the Philadelphia Section of the A. I. E. E. Mr. Gilman and I showed that any unbalanced system of three-phase e.m.f. or current vectors whose vector sum is zero may be resolved into two symmetrical three-phase component systems, one of which is of positive phase sequence and the other of negative phase sequence. The same is true of any two-phase system of e.m.fs. or current vectors. It may be shown analytically that a symmetrically wound induction motor running at a given speed will have a definite impedance to each of these

components which will be designated Z_1 and Z_2. The first impedance is made up of the true resistance and a composite reactance, and a virtual resistance which represents the resistance due to mechanical work. The second impedance is made up of a true resistance, a leakage reactance and a negative virtual resistance which represents the resistance due to mechanical work, required to help circulate the negative phase sequence currents.

The case that Mr. Lamme has considered in his paper is a special case of the symmetrical polyphase induction motor

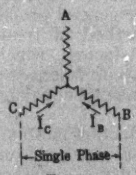

FIG. 1

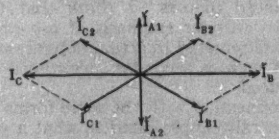

FIG. 2

operating under unbalanced terminal e.m.fs. Only one of these terminal e.m.fs. is given directly, namely the single-phase impressed e.m.f., but in this case the currents are perfectly defined in relation to each other, namely (Fig. 1) $\check{I}_B = -\check{I}_c$.

(Fig. 2) shows graphically how the current vectors $\check{I}_B$ and $\check{I}_c$ may be resolved into two symmetrical current systems $S^1(\check{I}_{a1})$ and $S^2(\check{I}_{a2})$. Fig. 3 shows the same graphical decomposition for a single-phase current into two two-phase currents, $\check{I}_s$ in this case being the single-phase current.

The terminal e.m.f. can likewise be resolved into two symmetrical e.m.fs., the star values being represented by $S^1(\check{E}_{a1})$, and $(S^2\check{E}_{a2})$, and we shall have the relation derived from the regular relations of star and delta e.m.fs.

$$\check{E}_{bc} = j\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \qquad (1)$$

Likewise the currents are subject to the constraint,

$$\check{I}_a = 0,\ \check{I}_b = -\check{I}_c \qquad (2)$$

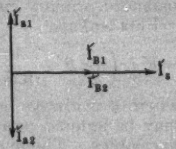

FIG. 3

and therefore by the (Fig. 2)

$$\check{I}_{a1} = -\check{I}_{a2} \qquad (3)$$

or by theory $\dfrac{\check{E}_{a1}}{Z_1} = -\dfrac{\check{E}_{a2}}{Z_2}$

and therefore $\check{E}_{a2} = -\dfrac{Z_2}{Z_1}\check{E}_{a1}$ (4)

Substituting in (1) we obtain

$$\check{E}_{a1} = -\frac{j\,\check{E}_{bc}}{\sqrt{3}}\,\frac{Z_1}{Z_1 + Z_2} \tag{5}$$

$$\check{E}_{a2} = -\frac{j\,\check{E}_{bc}}{\sqrt{3}}\cdot\frac{Z_2}{Z_1 + Z_2} \tag{6}$$

and therefore

$$\check{I}_{a1} = -j\,\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1 + Z_2} \tag{7}$$

$$\check{I}_{a2} = +j\,\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1 + Z_2} \tag{8}$$

and

$$\check{I}_b = -\check{I}_c = -\frac{\check{E}_{bc}}{Z_1 + Z_2} \tag{9}$$

equations (7) and (8) show that the single-phase performance is equivalent to that of two machines coupled together mechanically with windings connected in series, one machine being connected in opposite phase sequence to the other. If K_1 and K_2 are the ratios of transformation of primary and secondary currents for each symmetrical component, P_0 the power output is given by

$$P_0 = I_b^2\left(\frac{\omega_0 - \omega_1}{\omega_1}\,K_1^2\,R_s - \frac{\omega_0 - \omega_1}{2\,\omega_0 - \omega_1}\,K_2^2\,R_s\right) \tag{10}$$

Where ω_0 is synchronous and ω_1 slip frequency and R_s secondary resistance.

The input in volt-amperes is $P_1 + j\,Q_1$ where

$$P_1 = j\,Q_1 = I_b^2\,(Z_1 + Z_2) + P_v \tag{11}$$

P_v being the iron losses. The power factor is

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \tag{12}$$

If instead of impressing the single-phase e.m.f. across $B\,C$ (Fig. 1) it is impressed between A and $B\,C$ the terminals $B\,C$ being connected together we would have an entirely different story. There the e.m.fs. are determined and therefore, the components are $\check{E}_{a1} = \check{E}_{a2}$ and are given and the two component currents.

Systems S^1 ($\check{I}_{a1}$) and S^2 ($\check{I}_{a2}$) are inversely proportional to the impedances Z_1 and Z_2 the operation is, therefore, equivalent to that of two symmetrical motors coupled together, connected to a symmetrical three-phase system one being connected in opposite phase sequence to the other.

Finally if we wish to consider both single-phase primary and

single-phase secondary we impose the constraint (3) on both primary and secondary currents with the result that for each harmonic of current in the two windings, there will be two symmetrical components having the same relation as for the simple case except that Z_1 and Z_2 will have different values for each harmonic.

The above sketch of the principle on which the single-phase motor theory as given by Mr. Lamme depends is presented in order to show how a complex problem may be solved by the superimposition of a train of separate solutions each of which is connected by some known relation to the preceding one. It should be understood that in the case given in the preceding paragraph the effect of slip greatly complicates the problem.

Supplement to Discussion

If $F(\theta)$ be any function of position on a cylindrical surface, which is symmetrical about a diametral plane, it may be expressed by the Fourier series

$$F(\theta) = \Sigma\,(A_n \cos n\,\theta) \tag{1}$$

where n comprises all odd numbers. The coefficients A_n are obtained from the integral

$$A_n = \frac{2}{\pi} \int_{-\pi/2}^{+\pi/2} F(\theta) \cos n\,\theta\, d\,\theta \tag{2}$$

If we have a flux distribution $I\,d\,\theta'\,F(\theta)$ round a cylinder due to a filament of current $I\,d\,\theta'$ at $\theta = O$, and if we consider the filaments to be multiplied so as to form a current sheet on the cylinder, of which the current density at any point may be expressed by $I\,d\,\theta' \cos\theta'$. The field density at the point θ due to the filament at θ' will be

$$d\,B = I \cos\theta'\,F(\theta - \theta')\,d\,\theta' \tag{3}$$

The field density at the point θ due to the current sheet will therefore be

$$B = \int_0^{\pi} I \cos\theta'\,F(\theta - \theta')\,d\,\theta' \tag{4}$$

Since by (2)

$$F(\theta - \theta') = \Sigma\,A_n \cos n\,(\theta - \theta') \tag{5}$$

the integral (4) becomes

$$B = \frac{\pi}{2}\,I\,A_1 \cos\theta \tag{6}$$

B is therefore a simple sine wave field provided that $F(\theta)$ have a finite number of finite discontinuities.

C. A. M. Weber: Several years ago, in the course of my regular work, I had occasion to make what I then considered quite complete tests on a standard polyphase motor operating on both three-phase and single-phase over a range of speed from zero to 200 per cent slip.* Upon reviewing these tests with Mr. Lamme, he intimated that he believed it was possible to derive the single-phase speed-torque curve from the polyphase curves on the basis of two oppositely rotating magnetic fields as represented by two motors coupled together. A short study was made of this at that time but on account of lack of knowledge of the voltage relations involved in the two motors no definite results were obtained. However, Mr. Lamme stated that he was going to take the matter up again when he had sufficient time and expected to obtain the real solution along the above lines.

Several months ago he notified me that he had again taken up the problem and believed he could furnish a relatively simple explanation of the actions of the motor. He then explained to me the general method which he used which was practically the same as covered by his paper, but he stated that my former tests were obviously full of errors as he discovered when he applied them as a check to his method. I then proposed to repeat these tests with another set of motors, taking into account the discrepancies and errors involved in the former tests. Mr. Lamme then stated that he expected to carry out similar tests on very much larger motors. It may be of interest to know that my tests were carried out entirely independently of his and that I did not see the results of his tests until after his paper had been sent to press. You may imagine, therefore, my pleasure in noting the very close similarity of my results with his, in view of the fact that my tests were on two-phase motors of quite small capacity while his were on much larger three-phase motors.

For my tests I selected two 1-h. p., 220-volt, 60-cycle 2-phase, 4-pole wound-rotor motors with their shafts rigidly connected together. The torques of the various tests made were registered by an electric dynamometer of just sufficient capacity to handle the motors under test. Considerable pains were taken to obtain practically perfect alignment between the two unit combinations and since special arrangements were provided on the dynamometer for obtaining alignment with apparatus under test I feel that the errors in alignment, while of course not entirely eliminated, have been reduced to a practical minimum. The tests taken were as follows:

(1) Two-phase speed-torque and primary current curves on one of the motors at 220 volts with secondary short-circuited on itself.

(2) Single-phase speed-torque on one winding of one of the two-phase motors at 220 volts with secondary short-circuited on itself.

*The curves derived from these tests appeared in the *Electric Journal* of September, 1914.

(3) Single-phase speed-torque on the two-unit combination at 220 volts with the secondaries short-circuited on themselves.

The voltages impressed on each motor were also recorded.

Fig. 4 shows the forward and backward torques of one of the two-phase motors at full voltage plotted from test data, the corresponding primary current-torque curve and the single-phase speed-torque curve, shown solid, which was derived from the two-phase speed-torque and primary current curves as shown in Table I.* For comparison, the tested single-phase speed-torque curve on one winding of one of the two-phase motors is also shown. This curve is the one shown dotted.

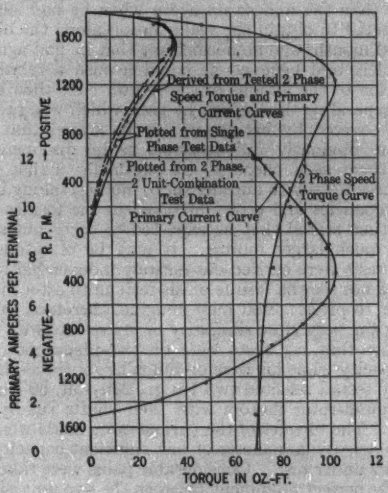

FIG. 4

For further comparison the tested speed-torque curve of the two-unit combination with primary windings in series and shafts rigidly connected together is also shown. This curve was plotted from data in second column of Table II and is the curve in dot and dash.

Considering the nature of the tests, the uncontrollable factors such as heating, saturation, leakage, etc., and the fact that comparisons are being made between several tests which could

*In order to avoid confusion the tables in this discussion were purposely arranged the same as in Mr. Lamme's paper.

not be made under identical conditions, it is indeed surprising that the various single-phase speed-torque curves check so closely. I wish to call particular attention to one discrepancy and that is the shape of the upper portion of the primary current curve which should bend over, whereas you will note it is practically straight. This result is undoubtedly due to heating as you will note the speed-torque curve at 200 per cent slip is very nearly perpendicular which of course could only occur through heating in this case. I will refer later on to the shape of these two curves in connection with the calculated curves.

Carrying the analysis somewhat further as did Mr. Lamme I measured the two-unit speed-torque recording at each torque

TABLE I.

Speeds		Torque at full voltage oz. ft.		Primary ampere per terminal full voltage		Per cent voltages		Reduced Torque oz. ft.		Resultant torque.
Positive	Negative	*Tf*	*Tb*	*If*	*Ib*	*X*	1−*X*	Forward	Back	oz. ft.
1750	1750	30	69	2.12	12.2	85.2	14.8	21.8	1.5	20.3
1700	1700	48	69.1	2.82	12.18	81.2	18.8	31.6	2.5	29.1
1600	1600	76	69.3	4.27	12.15	74.0	26.0	41.6	4.7	36.9
1500	1500	89	69.8	5.2	12.1	70.0	30.0	43.6	6.3	37.3
1400	1400	96.5	70.0	5.95	12.07	67.0	33.0	43.3	7.6	35.7
1300	1300	102	70.3	6.8	12.04	63.9	36.1	41.6	9.2	32.4
1200	1200	103	70.5	7.3	12.03	62.2	37.8	39.9	10.1	29.8
1000	1000	99	71.3	8.53	11.92	58.3	41.7	33.6	12.4	21.2
800	800	94	72.4	9.21	11.81	56.2	43.8	29.6	13.9	15.7
600	600	89.6	73.5	9.84	11.7	54.2	45.8	26.3	15.4	10.9
400	400	85	75	10.4	11.54	52.5	47.5	23.4	17.0	6.4
200	200	81.5	76.7	10.8	11.35	51.2	48.8	21.4	18.3	3.1
0	0	79	79	11.1	11.1	50.0	50.0	19.75	19.75	0

reading the voltages impressed on each motor. Table II shows the test data thus obtained together with the calculated resultant torques using the tested voltages on each motor and the two-phase speed-torque curve Fig. 4.

The last column in Table II is to be compared with column No. 2 and it will be noted again that the check is as good as could be expected under the circumstances. The individual resultant torque value in the last column marked with an asterisk is an error probably due to incorrect reading of impressed voltage, etc., and should be thrown out.

In Fig. 5 are plotted the calculated line voltages for forward and backward torques from Table I together with the tested per cent line voltages on each motor of the two-unit combination from Table II.

It will be noted that approximately the same difference between calculated and tested values occur in this case as in the cases described by Mr. Lamme with the one exception, that the tested curve does not cross the calculated curve at the higher voltage values, which may be accounted for by motor size, since the motors used by me were 1-h. p. two-phase whereas those used by Mr. Lamme were 10-h. p. three-phase. The variation of the tested voltage curve from the calculated voltage curve may be accounted for in various ways such as leakage and saturation, and the fact that although both motors of the two-unit combination were manufactured according to the same specifications it could not be expected that they would be identical in every

TABLE II.

Speed	Torque at 220 volts oz. ft.	Volts per motor		Per cent line volts		Polyphase torque at 220 volts oz. ft.		Polyphase torque at reduced voltage oz. ft.		Resultant torque oz. ft.
r. p. m.		Motor No. 1	Motor No. 2	Motor No. 1	Motor No. 2	Forward	Backward	Forward	Backward	
1800	0	192	28	87.3	12.7	0	69	0	1.1	−1.1
1700	31.5	173	52	78.6	23.6	48	69.1	29.6	3.9	25.7
1600	36.0	163.5	66	74.4	30.0	76	69.3	42.0	6.2	35.8
1500	34.5	148.0	76	67.3	34.5	89	69.8	40.2	8.3	31.9*
1400	30.75	139	79	63.2	35.9	96.5	70.0	43.6	9.0	34.6
1300	26.5	134	86	61.0	39.1	102	70.3	38.0	10.8	27.2
1200	24.0	129	95	58.6	43.1	103	70.5	35.4	12.1	22.3
1000	16.5	124	100	56.4	45.5	99	71.3	31.5	14.8	16.7
800	12.0	117	103.5	53.1	47	94	72.4	26.5	16.0	10.5
600	7.5	116	105	52.7	47.7	89.6	73.5	24.9	16.7	8.2
400	4.5	112	108	51.0	49	85.0	75.0	21.1	18.0	3.1
200	2.0	111	109.5	50.5	49.7	81.5	76.7	20.7	18.9	1.8
0	0	110	110	50	50	79.0	79.0	19.75	19.75	0

respect as is assumed in the calculations. In Fig. 6 I have plotted the tested torque values of the two-unit combination from the second column Table II, the calculated resultant torque values last column Table II and the tested single-phase torques of one winding of one of the two-phase motors, each against speed.

In considering these curves it should be borne in mind that only ordinary precautions were taken to obtain the same temperature conditions as it was discovered at the outset that identical temperature conditions were impossible of attainment.

It will be noted that the derived curve shown solid in Fig. 6 has lower torque values below the break down point than the derived curve shown solid in Fig. 4, which is attributable to

the fact that tested voltages were used in deriving the solid single-phase curve, Fig. 6, which takes into account saturation and leakage, whereas when calculated voltages were used as in solid single-phase curve, Fig. 4, saturation and leakage were not taken into account.

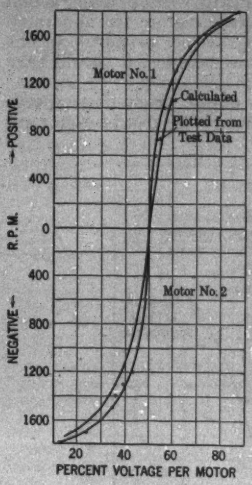

FIG. 5

After making the above tests and obtaining such remarkable checks between the various single-phase curves I decided to calculate the single-phase and polyphase speed-torque and primary current curves from the physical constants of the motor and derive from the polyphase calculated curves the single-phase speed-torque curve to determine how it checks with the calculated single-phase speed-torque curve.

Fig. 7 shows the speed-torque and primary current curves of one of the two-phase motors referred to, calculated from the physical dimensions of the motor. My calculations were based on fixed values of reactance and resistance, whereas under test these values vary. This is quite apparent when a comparison is made between the two-phase tests of Fig. 4 and the calculations, Fig. 7. The torque values from no load to break down calculated higher than the tests due to primary resistance being higher

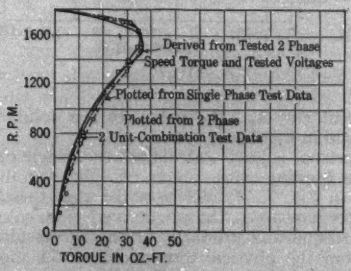

FIG. 6

under test than I assumed in the calculations. The break down torque calculated about 10 per cent higher than the tested value due also to primary resistance being assumed lower than

test. From break down torque to 200 per cent slip the calculated torques are less than tested values for similar reasons. In the calculations I assumed a normal increase in resistance, constant from no load to 200 per cent slip, whereas under actual test the increase in resistance due to temperature was abnormal. This abnormal heating caused the tested primary current curve Fig. 4 to straighten out, whereas normally it should have been as shown in Fig. 7.

Table III shows the calculations of the single-phase speed-torque curve from the calculated two-phase speed-torque and

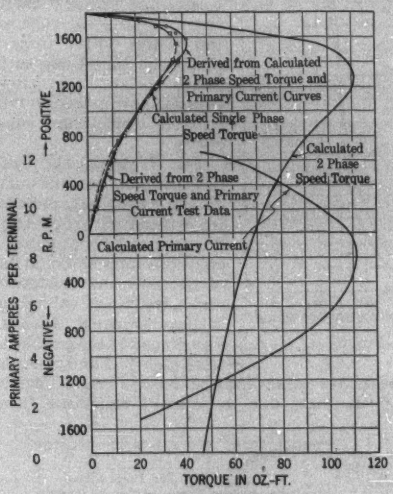

FIG. 7

primary current curves. The values in last column Table III are plotted solid in Fig. 7.

For comparison I have calculated the single-phase speed-torque of one winding of the above two-phase motor and show same dotted in Fig. 7. It will be noted that these two single-phase speed-torque curves, the one derived from calculated two-phase speed-torque and primary current curves and the other calculated from the physical dimensions of the motor, practically coincide. I feel that they would have coincided exactly had I drawn my circle diagrams large enough to reduce the effect of errors in measurement to a minimum.

For further comparison the single-phase speed-torque derived

from the two-phase speed-torque and primary current test data are also plotted in Fig. 7. The break down torque of the single-phase curve derived from test data is about 10 per cent less than the calculated value due to temperature. It will be remembered that there was about 10 per cent difference between calculated and tested two-phase break down points.

Reviewing all of the tests, calculations from test data and calculations from the physical dimensions of the motor and the various curves it would appear that the analogy of two oppositely rotating fields by means of two separate polyphase units rigidly connected together with their primary windings connected in series is a correct physical conception of the single-phase induction motor.

TABLE III.

Speeds		Torque at full voltage oz. ft.		Prim. amperes per term. full voltage		Per cent voltages		Reduced torque oz. ft.		Resultant torque oz. ft.
Positive	Negative	*Tf*	*Tb*	*If*	*Ib*	*X*	1−*X*	Forward	Back	
1750	1750	30	47	1.8	12.33	87.3	12.7	22.8	.8	22
1700	1700	52	47.2	2.86	12.32	81.2	18.8	34.2	1.7	32.5
1600	1600	85.5	48.3	4.62	12.31	72.7	27.3	45.4	3.6	41.8
1500	1500	102.7	49.4	6.0	12.29	67.2	32.8	46.4	5.3	41.3
1400	1400	108.8	50.4	7.0	12.27	63.6	36.4	44.1	6.7	37.4
1300	1300	111.4	51.5	8.02	12.23	60.4	39.6	40.6	8.1	32.5
1200	1200	111.0	52.6	8.75	12.21	58.3	41.7	37.7	9.2	28.5
1000	1000	103.0	55.0	9.57	12.14	56	44.	32.2	10.2	22.0
800	800	92	57.2	10.32	12.07	53.9	46.1	26.7	12.2	14.5
600	600	84	60	10.81	12	52.6	47.4	23.2	13.5	9.7
400	400	78.3	62.6	11.13	11.89	51.6	48.4	20.9	14.6	6.3
200	200	73.5	66	11.4	11.75	50.7	49.3	18.9	17.1	1.8
0	0	69.5	69.5	11.58	11.58	50	50	17.4	17.4	0

G. H. Garcelon: There is one point in Mr. Lamme's paper which, it seems to me, may account for the discrepancy between the tests of a single motor running single phase and a two-motor unit supplied with opposing polyphase current.

The single motor is always excited at full voltage whereas each motor of the two-motor unit is subjected to an excitation varying from one-half normal to practically normal.

This difference would naturally cause variation in the saturation and the reactance so that the coincidence of the two curves seems to me remarkably close considering the conditions.

The two-motor unit has twice as much material, so that some discrepancies are bound to appear.

R. E. Hellmund: Believing that those readers of Mr Lamme's paper who are accustomed to work with the circle diagram would be interested to see how the two diagrams of the series connected polyphase motors or the oppositely rotating

fields in the single-phase motor join together, I have worked out the diagram for a number of typical load cases. In discussing these diagrams, let us keep the two polyphase motors with oppositely rotating fields and connected in series, in mind. Considering each of these motors by itself, it is evident that the circle diagram applies as usual to the individual motor. If the voltage of the first motor has a direction $O E_1$, (Fig. 8), its resultant field may be represented by a vector $O A_1$ which gives at the same time a measure for the size of the voltage. If the leakage coefficient is represented by the ratio $\frac{O B_1}{O A_1}$, we have at once the circle B_1, D_1, A_1, around the center C_1 applying to the first motor. If we assume now the load

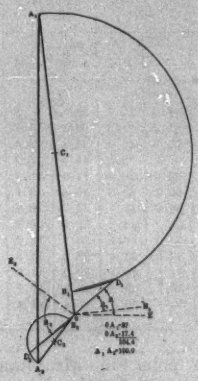

FIG. 8

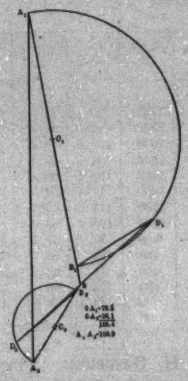

FIG. 9

point D_1 corresponding to about full load with a primary current $O D_1$ we know that on account of the series connection of the motors this current must apply to both of the motors. If, therefore, $O D_2$ equal and opposite to $O D_1$ represents the current of the second motor, we know that the circle applying to this motor must go through the point D_2. If we assume further the voltage of the second motor to be equal to $O A_2$, the direction of $O A_2$ representing the field of the second motor and the basis for the circle is definitely fixed by the knowledge of the point E_2 and the ratio $\frac{O B_2}{O A_2}$ which

must be again the leakage coefficient. We find, therefore, the circle B_2, D_2, A_2, around the center C_2 corresponding to the second motor. The power factor of the first motor for the load point under consideration corresponds to the angle a_1; that of the second motor to the angle a_2; and that of the combination to the angle a. The secondary current of the first motor is proportional to $B_1 D_1$, and that of the second motor proportional to $D_2 B_2$. Knowing that $O B_1$ and $O B_2$ represent the magnetizing currents of the two motors for their individual voltages, which are also known, we can find all the required scales, and formulas can be derived for the combination under the condition that the sum of the slip of the two motors must be 200 per cent. A complete theory for the series connected polyphase motor and the equivalent single-

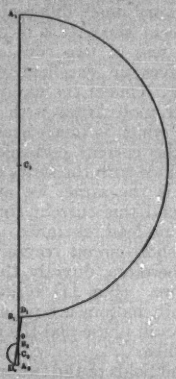

FIG. 10

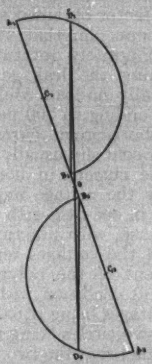

FIG. 11

phase motor could thus be derived. This would be of small practical value, however, since there is small promise that any calculating method along that line would be simpler than those now available for single-phase induction motors. The principal value of the two polyphase motors equivalent being the easy physical conception, the diagram so far derived is sufficient. It will be noticed from the diagram that both the voltages and the fields $O A_1$ and $O A_2$ are somewhat out of phase and that their arithmetical sum is, therefore, slightly larger than the total impressed voltage $A_1 A_2$, which is the geometrical sum. At the same time it will be seen that the difference between the arithmetical and geometrical sums is only 4.4 per cent, so that the small error introduced by figuring with the arithmetical sum as has been done in Mr. Lamme's paper is almost negligible.

(Fig. 9) is a similar diagram applying to the same motor for a larger load. It will be noted that the field voltage, and, therefore, the circle for the motor has decreased, while that of the second motor has increased. The difference between the arithmetical and geometrical sums of the two voltages is 5.4 per cent in this case. In practise the difference hardly ever exceeds 10 per cent and is usually much smaller. The two special cases of no-load and standstill are of interest and shown for the same motor in (Figs. 10 and 11;) (Fig. 10) representing the no-load case shows a very small circle $B_2 D_2 A_2$, for the motor which runs at 200 per cent slip. For this condition, the two fields $O A_2$ and $O A_1$ are practically in phase with each other so that the arithmetical and geometrical sums of the two are identical. This is true in nearly all motors, except those with very high resistances, because the no-load power factor of an induction motor is usually not very much different from the power factor obtained at 200 per cent slip. It is interesting in this connection that the no-load test of two coupled motors with oppositely rotating fields gives an excellent means for measuring the leakage coefficient. One of the motors runs at practically no-load while the other motor runs with a current corresponding to 200 per cent slip which is much nearer to the ideal short-circuit current than the current corresponding to 100 per cent slip usually tested. Since with the series connected test the current in both motors is the same, we are testing directly the voltage required to send this current through the motors in one case with practically no-load reactance, and in the other case, with practically ideal short-circuit reactance. The ratio of the voltages represents, therefore, directly the leakage coefficient of the polyphase motors. (Fig. 11) represents the standstill condition with which both motors are on a par, the two voltages being equal and always in phase and the diameters of the two circles being naturally alike.

The fact that the two voltages and fields of the two motors are not always in phase does not indicate by any means that there is a fundamental discrepancy between the single-phase induction motor and the equivalent of two series-connected polyphase motors; this slight out-of-phase condition of the oppositely rotating fields can exist in the single-phase motor as well. As a matter of fact, it seems that the two cases are exactly equivalent in all respects except for slight differences caused by differences in iron saturation, heating, eddy currents, bearing friction, etc., that is, such secondary phenomena which have little to do with the fundamental theory.

The close agreement between the two equivalents may further be emphasized by a number of examples of load cases which are easily understood. At no load, we know that one of two series connected two-phase motors takes practically the full line voltage and, therefore, magnetizing current corresponding to this voltage. This current i flows in both phases of each motor

giving a primary copper loss of $4\,r\,i^2$, if r is the resistance per phase in each motor. The single-phase motor takes with the same line voltage a no-load current $2\,i$, which, with the same resistance r in the single-phase, gives the primary losses equal to $(2\,i)^2\,r = 4\,r\,i^2$ that is the same as in the polyphase motor combination. At standstill, each of the polyphase motors has one-half line voltage, giving a certain locked current of $\frac{1}{2}\,I$ and primary copper losses of $\left(\frac{1}{2}\,I\right)^2 \times 4\,r = I^2\,r$. The single-phase motor has full line voltage and, therefore, a locked current of I and primary copper losses of $I^2\,r$, that is again the same as the polyphase motor combination. The same conditions apply to the secondary copper losses. In the single-phase motor secondary, we have two currents of different frequencies, superimposed upon each other. In the two-motor combination, the same currents flow separately in the two secondaries. It can be shown by a simple calculation that currents of different frequencies flowing in the same winding give the same copper losses as if they were flowing in two separate windings, each of the same resistance as is the case in the double motor combination.

The core losses at no load are obviously the same in both cases. With the single-phase motor, we have one rotating field corresponding to full line voltage; with the two polyphase motor combination, one of the motors has a rotating field corresponding practically to line voltage, while the field and core losses of the other motor are practically zero. At standstill, the resultant field of the single-phase motor is alternating in one axis and corresponds in size to full line voltage. In the two motor combination, we have fields of half the strength, which, if they were only alternating in one axis, would give less than one-half the losses per motor, since core losses vary with something like the 1.8th or 2nd power of the field strength. This is, however, practically made up for by the fact that the fields are rotating, giving the maximum density in a larger portion of the iron than would be the case with fields alternating in a single axis. So we have again a very close agreement between the two cases.

The primary ohmic voltage drop at no-load is, in the polyphase motor combination, $i \times 2\,r$ with two motor resistances in series, while in the single-phase motor we have a resistance of only r but a current $2\,i$, therefore, again the same drop. Similarly, we have at standstill an ohmic drop of $\frac{1}{2}\,I \times 2\,r = I\,r$ for the polyphase combination and the same drop $I\,r$ in the single-phase motor.

The same conditions apply to the leakage reactance drop, if the reluctance of the leakage path remains constant as is usually assumed in all induction motor theories. Since, however, the primary currents of the single-phase motor are about double the

current in the polyphase motor, they cause larger local leakage fluxes around the slots in which the single-phase winding is located. If there is, therefore, a tendency for saturation in the tooth tips *a*, shown in (Fig. 12) such saturation may cause increased reluctance in the leakage path of a single-phase motor as compared with the polyphase equivalent.

The decreasing influence of the increased reluctance upon the leakage field of the single-phase motor tends to give it slightly larger torques than the polyphase equivalent, especially near the pull-out point and speed points below.

FIG. 12

A comparison between the tested curves of Fig. 22 and Fig. 25 of Mr. Lamme's paper, which correspond to the single-phase motor and the series connected polyphase motors, respectively, shows a difference in this direction. The same fact is also borne out by a comparison of the test curves, Figs. 23 and 27.

There is nothing new, however, in having an otherwise perfectly good theory spoiled by the varying reluctance of the iron. We know, for instance, that the same change in the reluctance of the leakage path introduces marked discrepancies in most theories of the polyphase induction motor and spoils in many cases the true circle of the circle diagram.

Similar discrepancies are caused by the fact that the varying frequency of the induction motor secondary winding bring about varying eddy losses in the copper; this results in a variation of the effective secondary resistance, while most theories are based on constant resistance. The resistance variations caused by heating are even more annoying in any attempts to check theory by tests. The various unavoidable discrepancies in Mr. Lamme's curves can, therefore, not be considered as a reflection upon the correctness of the fundamental ideas advanced, but they are simply discrepancies which confront the designer in nearly all of his work.

A. M. Dudley and C. A. M. Weber: After reviewing Mr. Hellmund's paper we decided to wind a phase converter which would meet as nearly as possible some of Mr. Hellmund's fundamental conditions thereby producing by actual tests some of the wave shapes obtained by Mr. Hellmund's analysis. We could not of course obtain zero resistance nor zero leakage and therefore, we would not obtain the infinite current values shown in the paper. However, where infinite values are shown in the paper for zero resistance and leakage our oscillograms show an abrupt rise to a high value during a finite time instead of to an infinite value for an infinitesimally short time.

The converter, which was built and tested after Mr. Hellmund's paper was sent to press, consisted of a 48-slot stator with one primary coil wound pitch in diametrically opposite slots and with one tertiary coil displaced 90 deg. from the primary coil also wound pitch in diametrically opposite slots. This produced

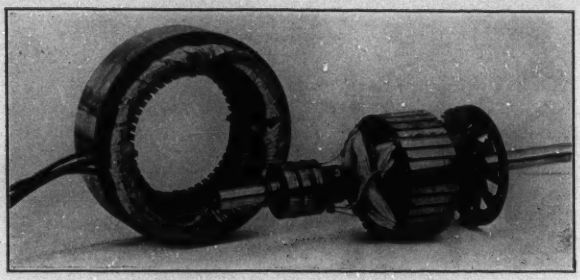

FIG. 13

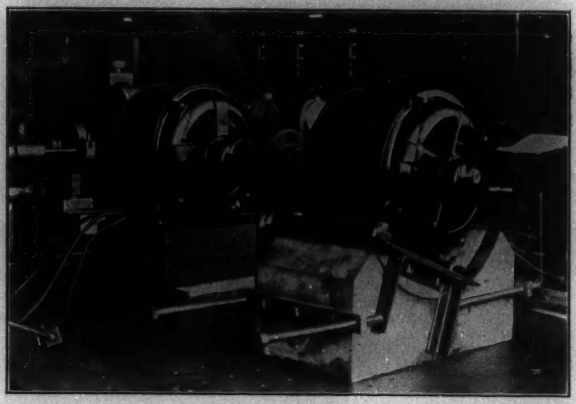

FIG. 14 [DUDLEY & WEBER]

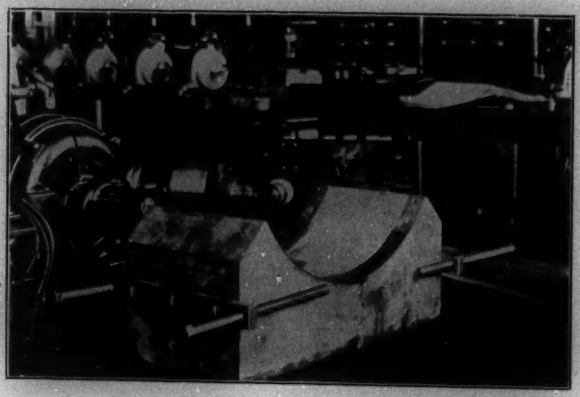

FIG. 15

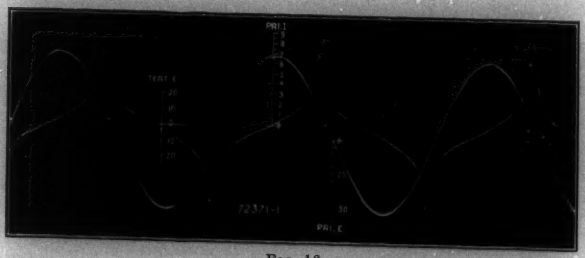

FIG. 16

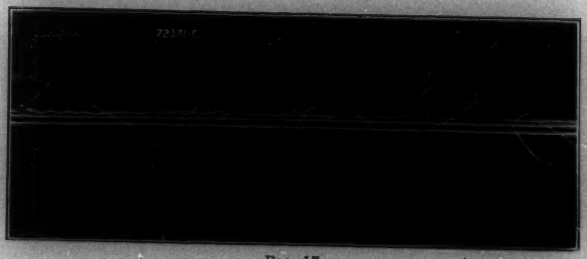

FIG. 17 [DUDLEY & WEBER]

FIG. 18

FIG. 19

FIG. 20

[DUDLEY & WEBER]

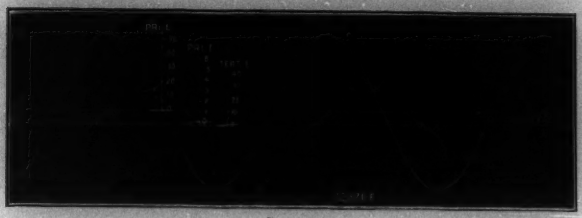

FIG. 21

FIG. 22

FIG. 23 [DUDLEY & WERER]

a stator with concentrated primary and tertiary windings and is shown in Fig. 13.

The rotor had 34 slots and was wound similarly to the stator except you will note that the phases are not in exact quadrature which of course does not affect the results obtained with single-phase rotor and very little, as you will see, when the rotor was used two-phase. This rotor is also shown in Fig. 13.

This converter was driven by a two-pole synchronous motor mounted in a cradle. The cradle was necessary in order to obtain and maintain the correct angular position of the primary and secondary coils with respect to the primary flux. Fig. 14 shows the synchronous induction motor mounted in the cradle and coupled to the phase converter. Fig. 15 shows the lower part of the cradle which is simply a block of wood with brass straps for slides.

The synchronous induction motor and primary coil of the phase converter were excited from the same source so as to maintain the proper relationship between primary coil, secondary coil and primary flux. Precautions were also taken to reduce external reactances to a minimum. The only external reactances between the generator and the apparatus under test were two sets of large transformers.

Fig. 16 was taken under the conditions of single-phase secondary coil coinciding with single-phase primary coil at the instant when the primary flux was maximum.

The primary magnetizing current should be compared with Fig. 3A in Mr. Hellmund's paper. The similarity of the primary current wave shape here shown with that in the paper will be noted except that infinite values are modified by leakage, resistance and the fact that the coils are not concentrated in a mathematical point as is assumed in the paper.

The tertiary voltage wave compares with Fig. 4C of the paper. The rotor teeth cause a varying reluctance which in turn has produced a saw tooth effect on the tertiary voltage. It will be noted that there are 34 points in a complete cycle and we used a 34-slot rotor. The effect of the rotor teeth does not show in the primary voltage because this voltage is impressed and the counter e.m.f. adjusts itself exactly. The tertiary voltage crosses the zero line on account of the leakage, resistance, etc.

Fig. 17 was made under the same conditions as Fig. 16. The primary current, the primary voltage and the secondary current are shown.

The primary magnetizing current is the same as in Fig. 16. The primary voltage is also the same as in Fig. 16 except the scale has been reduced.

The secondary current compares with Fig. 3B of the paper except that infinite values do not appear for the same reasons as mentioned in connection with the primary current Fig. 16. Note the strong even harmonics which are indicated by the difference in the positive and negative wave shapes.

Fig. 18 was taken with single-phase primary and single-phase secondary coils coinciding at the instant the primary flux was zero. It will be noted that the presence of the rotor teeth is reflected in the tertiary voltage and primary current.

The tertiary voltage shows the third harmonic with the fundamental almost missing. If the primary and secondary coils had been more nearly in coincidence when the primary flux was zero the fundamental would have practically disappeared.

Fig. 19 was taken with single-phase primary and two-phase secondary. One secondary coil coinciding with primary coil at the instant of maximum primary flux. The primary current and tertiary voltage wave shapes compare with Figs. 7A and 8B in the paper. The effect of the rotor teeth is again reflected in the primary current and tertiary voltage.

The primary current wave shape is similar to Fig. 7A in the paper except infinite values are modified for the same reasons as previously brought out, but otherwise the shape is almost identical with the one in the paper.

The tertiary voltage shape compares with Fig. 8B in the paper. It is rectangular except reflecting the effect of rotor teeth. The jog in the middle of the tertiary wave is probably caused by the element load of the oscillograph. The little jog in the primary current may possibly be due to the same cause or more probably to varying reactances caused by saturation.

Fig. 20 was taken under the same conditions as Fig. 19 and shows the secondary current in each phase. This oscillogram compares with 7B and 7C in the paper except infinite values are modified for reasons previously stated. It will be noted that the secondary currents in the different phases not only vary in value but also in form and in this particular case are quite different from each other.

Fig. 21 was taken with single-phase primary and two-phase secondary. One secondary coil coinciding with the primary coil at about 70 per cent maximum flux, that is, 45 deg. from the position in which Fig. 20 was taken. The primary current compares with Fig. 9A in the paper. The two primary current peaks correspond to the two infinite values shown in the paper. The infinite values are damped out by leakage, resistance, etc.

The tertiary voltage compares with Fig. 10B. It is rectangular in shape and follows the zero line for a certain distance. This wave, compared with the previous tertiary voltage waves, illustrates the fact that the wave form of the tertiary voltage varies with the rotor position, although the effective value (voltmeter value) remains unchanged. The effect of rotor teeth is again reflected in the tertiary voltage.

Fig. 22 was made under the same conditions as Fig. 21 and shows both of the secondary currents. This oscillogram compares with Fig. 9B and 9C in the paper. It will be noted that the shapes of the two current waves are similar and that one is the image of the other, *i. e.* the round part follows the

straight part in one wave whereas in the other the straight part follows the round part. It will be noted that the negative areas are equal to the positive areas which is one of the fundamental facts on which calculations in the paper are based.

Fig. 23 was made with single-phase secondary coil coinciding with single-phase primary coil when primary flux was approximately 70 per cent of its maximum value, about half way between Figs. 16 and 17 and shows the primary voltage and current and tertiary voltage. This oscillogram has no equivalent in the paper but is interesting in that it shows the primary current wave shape can be unsymmetrical depending upon the position of the rotor coils. All the previous oscillograms of the primary current showed a symmetrical wave shape. The tertiary voltage shows a strong third harmonic.

The object of these oscillograms was to illustrate that the methods used in the paper were correct and not to illustrate actual conditions in machines that are manufactured today. Having illustrated the correctness of the fundamental principles used in the paper for the simplest form of machine it is reasonable to assume that these same principles when applied to modern machinery will give the correct results. These oscillograms therefore are a practical demonstration of the operations of the fundamental principles used by Mr. Hellmund in his paper and are not to be used in determining the wave shapes in practical machinery.

R. E. Hellmund: Various suggestions regarding other methods for investigating the subject have been made. As already emphasized by the discussion, the choice of method is a question of taste and will depend largely upon the mental equipment and upon the mathematical tools which are at the disposal of the engineers studying the problems. There is no doubt whatsoever but that a student who is familiar with the working principles of a polyphase motor and desires to obtain simply a clear physical conception of the single-phase induction motor and its working principles can get what he wants by carefully studying Mr. Lamme's paper. The designer of single-phase machines and phase converters, however, needs something more for his work than a physical conception. It is necessary for him to have formulas by which he can predetermine machine performance with a fair degree of accuracy; further he should have a general idea of what are the best ratios between the numbers of primary and secondary slots; what is the best practical winding distribution for the primary, and so on; in other words, he should know enough to choose the most favorable combinations and keep away from poor combinations which cause unnecessary copper losses, eddy losses, etc. It has been the purpose of my paper to furnish information of this kind. The same or similar results can undoubtedly be obtained by the more mathematical methods intimated by Mr. Fortescue and possibly by methods mentioned by Mr. Slepian. In writing my

paper I made it a special point, however, to start in with a few very simple cases. The study of these cases requires nothing but the knowledge of two or three of the simplest and most fundamental laws which everybody who is an electrical engineer must know, as for instance, one of the Kirchhoff laws, the law of magnetic induction, etc. The only other thing necessary for these cases is the ability to solve two or three equations of the simplest kind with two or three unknown quantities. By giving these simple cases first, I had hoped to enable any interested party to understand the fundamental features of the methods used in the paper. After going that far, the reader can either go on or content himself with studying merely the conclusions reached in the paper for the practical cases and using the formulas given, having, however the satisfaction of knowing in a general way how they have been derived. I adopted this procedure realizing that unfortunately the average designing engineer is as a rule rather rusty in mathematics and other subjects.

On the other hand, there is no doubt but that the methods indicated by Mr. Fortescue by which he proposes to go directly after a broad general solution taking everything into account to start in with is a more elegant way of attacking the problem and undoubtedly the one which will appeal to the man who has the required mental aptitude and the mathematical tools at his disposal. I had hardly expected that my method of starting in with a very simple case and building up slowly, considering the various factors, one by one, would appeal to men who are well equipped to handle more difficult problems. I am confident, however, that the average designing engineer will prefer to start the problem slowly with simple means and then progress to the more complicated case as far as required for his purposes. The fact that the problems discussed in my paper do not come any too easily to a great number of engineers is evidenced by the fact that so far as I know, nobody has previously even attempted to study the case much in detail. It is fortunate that in the near future we will have a paper by Mr. Fortescue on a similar subject but using other methods, thereby enabling everybody to take their choice between the two different methods.

B. G. Lamme: I think I stated pretty definitely in my paper that it is not intended as a method of calculation, but is merely an attempt to give, in as simple form as possible, a physical conception of what is going on in the motor. Also the general treatment is not claimed as novel or original. There is a very large class of people which is interested in the single-phase induction motor and would like to have some idea of its action, but either cannot or will not follow any mathematical conception or analysis. Twenty-one years ago, I wrote a paper on the characteristics of the polyphase motor. At the time that motor was not very well understood except by a few engineers. I spent a great many months of hard work trying to explain the properties of the polyphase motor and, in doing so, I used con-

siderable mathematics. However, the criticism brought against that paper during its preparation, by a number of people to whom I referred it, was that they did not like the mathematics in it and I was asked to leave out all formulas and equations. This I succeeded in doing after much effort and the general public gave this paper a kindly reception, largely on account of the absence of mathematics in its treatment. I felt, therefore, that a similar treatment of the subject of the single-phase induction motor might be of some value to that portion of the electrical public which does not care to consider papers of a mathematical nature.

Discussion at New York

M. I. Pupin: We have to look upon the induction motor from two different points of view; one, from the point of view of the designer, and the other from the point of view of the operator. The operator wants to have a motor which is capable of a certain performance under certain conditions, and the designer wants to know how to design a motor that will give the performance under those conditions.

To design a motor you have to proceed, of course, from certain elements of the motor. Now, what are the elements of an induction motor or any piece of dynamo-electric machinery. Why, its electric and its magnetic circuits. That is all we have to guide us, the constants of the electric and of the magnetic circuits; they are the resistance, inductance, and capacity of each circuit which is employed in the structure. The question arises, how are we to define these constants of the various circuits. The answer to that question is simple. You have to define them as Maxwell defined the constants of a circuit. The attempts of ordinary mortals to do better than Maxwell did, must be discouraged. Let us follow Maxwell as long as we can, then when somebody is born who is more profound than Maxwell, we will bow to him, but for the present, I think Maxwell is the leader.

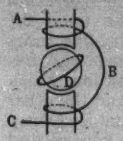

Fig. 24

Maxwell defines the constants of a circuit—the resistance of the circuit, the capacity of the circuit, and the inductance for the circuit—in such way that the action of neighboring circuits is not taken into account. Let Fig. 24 indicate an induction motor, with the rotor circuit D and stator circuit $A\ B\ C$. What is the inductance of the primary winding $A\ B\ C$? It is the inductance which it would have when the secondary winding D is open. That is the only satisfactory way of defining it; not the effective, but the value which is found when the secondary is open, and that you can calculate, of course, from the number of turns and from the constants of the magnetic circuit. In the same way, the primary resistance is the resistance which is obtained with that electromotive force which we intend to apply, but the secondary circuit is supposed to be open. The same thing is true for the other circuits.

The elements, then, of the circuit which we must have in order to construct a theory which the designer can use, are the inductance, the resistance, and the capacity of each electrical circuit, when the other circuits are open. Now, when we do that, and introduce the further condition that the primary and secondary currents produce a magnetic field, which has a sinusoidal distribution, then the mathematical theory of the single-phase induction motor becomes very simple; indeed, remarkably simple, so that anybody who knows the elements of the theory of the electrical circuit can sit down and work it out. An undergraduate can work it out—when once he is shown how to do it.

Now, what does that simple theory show? It shows that we have two magnetic fields produced by the *rotor* currents. The presence of these two magnetic fields, rotating in opposite directions, is not a theory; it is a fact. We do not have to make an assumption of that kind at all. It is a fact, from which you cannot escape. You have in each secondary winding two alternating currents. Take an oscillograph and photograph the e. m. f. in the secondary winding, and you will find that it consists of two components of different frequencies; one, which I call the *additive* frequency and another which I call the *differential* frequency. If you impress an e. m. f. of 60-cycles upon the primary or stator circuit and the motor rotates fifty-five times per second, you will have in the secondary windings two frequencies, one of frequency 60 plus the speed of rotation, which is 55, and that makes the additive frequency 115, and the other 60, minus 55, which is 5, that is to say the differential frequency is 5 periods per second. By photographing suitably, you will find that these two frequencies are distributed in such a way that the additive frequency of 115 periods per second produces a rotary magnetic field rotating contrary to the motion of the rotor, and the differential frequency produces a rotation in the direction of the rotor. It is assumed, of course, that the rotor has a symmetrical winding.

The differential frequency is the working frequency—it is the busy bee, the bee that makes the honey, and the additive frequency is the drone which produces the drag and tends to stop the rotor. The additive frequency produces a rotation in opposite direction to the motion of the rotor, trying to stop it; it is a brake. The differential frequency produces a rotation in the direction of the rotor motion; it pulls the rotor and thus produces mechanical work. So that the presence of two rotary magnetic fields is not a theory resting upon a lucky guess; it is an actual physical fact. It is not necessary to assume two rotary magnetic fields produced by the stator current at all; in fact, they have no physical existence; but the presence of two rotary magnetic fields produced by the rotor currents is a fact. That is what the mathematical theory gives you, and that is what experiments show to be actually the case. So much, then, for the mathematical theory.

Now, the mathematical theory, following the methods of Maxwell, which he first pointed out in a very simple and elegant way in 1865, suggests that, just as Maxwell defined the performance of a transformer by means of its effective inductance and effective resistance—I shall define these presently—so you can in the case of an induction motor describe the performance of a motor by finding out the effective inductance and the effective resistance of the primary winding, which is the winding where the impressed force is located.

If that were possible, it would be a very convenient thing to do, namely, to consider first the ideal inductance, the resistance and capacity of the various circuits of the motor, and from these to calculate what is the effective resistance and the effective inductance of the motor under various conditions of load. A comparison between this calculation and experiment must of course follow. The calculation is the same which Maxwell first applied in the case of a transformer.

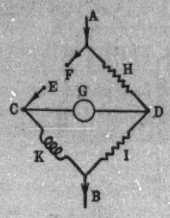

Fig. 25

The question arises then, how are we going to perform the experiment? Very simply,—when you know how to do it. We use the Wheatstone bridge. We actually take the motor and put it in a Wheatstone bridge. Several years ago, about ten years ago, I was giving a short talk to the Radio Engineers, at their request and I told them that one of the simplest ways to determine the performance of an aerial is to determine its effective resistance and its effective inductance under various conditions by means of a Wheatstone Bridge. They are doing it now in this manner, at any rate some of them, and thousands of Wheatstone bridges for aerial work have been made and I hope that some day we will make thousands of Wheatstone bridges for the study of induction motors, both single-phase and polyphase.

Now, if you will allow me I shall amplify a little because the subject of the single-phase induction motor has grown to be much bigger than some of us think. Take the well known arrangement of the Wheatstone bridge, indicated in Fig. 25 K is an inductance, H and I are resistances, and between E and F we may introduce anything we wish to measure by the indicated Wheatstone bridge arrangement. The two points $E\,F$ may be points through which power is sent anywhere, to a whole district beyond $E\,F$. If we wish to know how much power is consumed in the district and at what efficiency all you have to do is to connect them to the Wheatstone bridge and measure the effective resistance and effective inductance of that whole district and from the two you can calculate everything else. In the same manner if you put the primary of an induction motor between E and F you can measure its effective

inductance and effective resistance at various speeds, and you get curves something like those in Fig. 26. In this figure the abscissas measure the ratio of rotor speed to frequency speed of the impressed e. m. f., the ordinates give the values of the effective resistance (curve R) and of the effective inductance (curve L). The effective resistance starts at a certain value, goes up, and is at a maximum a little before synchronous speed is reached; this is also the approximate location of the maximum torque. The effective inductance is represented by the curve L. From these two you can calculate everything else. I have done it, and it works like a charm. You can do that for a single-phase induction motor, and you can do it for a polyphase induction motor, with this distinction—that the bridge for a polyphase motor is not quite as simple as that for a single-phase motor, but it can be done, and it can be done in very simple fashion; the method is very accurate.

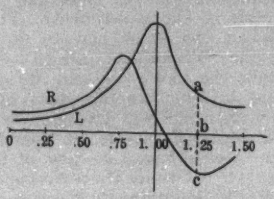

FIG. 26

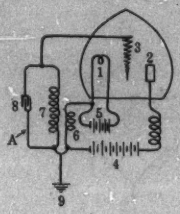

FIG. 27

You will find that the mathematical theory following strictly the simple method of Maxwell gives you results, which agree with the theory very closely, so close that the experimental points obtained on the curve agree with the theory remarkably well, particularly when feeble magnetizations are employed such as I used in my work. Of course, as I have already said, all is based on the fact that we are dealing with a sinusoidal distribution of the magnetic flux, and that is not a serious limitation, because most induction motors have a distribution of flux which approaches the sinusoidal very closely.

That being so, then, it makes the theory very acceptable. And why is the theory so desirable? It is desirable, because, so far as I know, it is the simplest method that can be put in the hands of a designer of single-phase or polyphase induction motors. But that is only one reason. The other reason is that it really presents the whole theory in a much simpler way than anything I have heard of before; simpler even than the simple presentation which Mr. Lamme gave this evening, although I must say, that when I come to read his paper a little more closely, it may perhaps not be quite as simple as he made it.

You know the action of a single-phase induction motor is extremely similar to the action of a vacuum tube amplifier, particularly those vacuum tubes which are used today as oscillators. One of the most beautiful recent inventions, made by a student of mine, Mr. E. Armstrong, is the pliotron oscillator; that is really a wonder. I am digressing, I know, but the subject is connected with the action of the single-phase induction motor, and therefore, the digression is permissible.

Fig. 27 is a diagram of a vacuum tube invented by Mr. De Forest about twelve years ago. It is a vacuum tube, with a third electrode, which is called the grid. 2 is the cold positive and 1 is the hot negative electrode. This electrode is a tungsten filament heated by battery 5, and battery 4 maintains the electron current between 1 and 2. 7 and 8 are the inductance and the capacity of a separate circuit which is coupled by mutual induction to the circuit 2, 4, 6, 1, through which the electron current circulates. Circuit 7, 8 has a period of its own depending upon its capacity 8 and its inductance 7 in the well known way. When the oscillations are started they will not die out if the inductive relation between 7 and 6 are suitably adjusted but will continue until the power of battery 4 is used up. They are so persistent and so steady that the period of oscillation will not change more than one-tenth of one per cent for a whole day. Lately we have been operating a circuit like that, and one of the mechanicians watched it a whole afternoon to see that nothing happened to the apparatus; he did not touch it during the whole afternoon and the period was not changed one-tenth of one per cent; it gave 90,000 periods per second. It is a wonderful piece of apparatus, and the invention consists in introducing this energizing circuit in inductive relation with this oscillating circuit, and that is what Mr. Armstrong did.

It was not accidentally that Armstrong made this invention. He was working in my laboratory where the theory of the induction motor was developed by experimental tests, and he heard my lectures on the subject and applied it, perhaps unconsciously, to the action of pliotrons in which he was interested. I will make now a comparison between the two actions. When the rotor passes beyond synchronous speed, then as you see from Fig. 26 you get negative effective resistance. Negative resistance sounds very strange but it is simple enough when you stop to think about it. Of course, you cannot measure directly in a Wheatstone bridge a negative resistance. It means that we have to introduce a sufficient amount of positive resistance to prevent the occurrence of trouble in the bridge, which I will describe presently.

Consider now the primary effective inductance *a b*, an effective resistance *b c* in Fig. 26. Insert a capacity in the primary, in multiple or in series with the inductance so as to overcome this inductance and introduce a resistance in series so that you will have in the primary winding zero inductance and zero resistance, what will happen? You may say that this is an

absurdity. It is not. It is true that you never can have a current equal to infinity; but the current starts toward infinity, but pretty soon the pole pieces are drawn toward the armature, and the belt thrown off and the machine stops. Theory and experiment actually agree in that case.

Another thing which happens is this; when you introduce this capacity to overcome the effective inductance the current grows and the magnetization gets so strong that the inductance changes on account of the change of the permeability; the effective resistance increases also, so that the critical relation which demands an infinite current is upset. Now, the point which I wish to bring out is this: Under the conditions specified above, the primary circuit has a period of its own; it is that due to inductance *a b* in Fig. 26 and the capacity inserted, oscillations being started in this circuit will be maintained if the effective resistance is negative, that is if the power put into the circuit by the rotation of the rotor is equal to or greater than the power consumed by the circuit. The conditions then under which these oscillations are produced by putting power into the circuit are determined by the two curves in Fig. 26.

In the same way, if at *A* you break circuit 7, 8 Fig. 27 and connect the two terminals to *E F* of the Wheatstone bridge, Fig. 25 then apply an alternating electromotive force, varying gradually its frequency and determine the effective resistance and the effective inductance of circuit 7, 8, you will get curves which are almost exactly like the two curves in Fig. 26 and you can tell that, as soon as the effective resistance begins to be negative for a given frequency, the circuit will act as an oscillator in consequence of the power supplied by battery 4.

This battery performs the same function as the mechanical power does which turns the rotor. The period is determined in this case in the same way as in the other, and therefore the two phenomena are the same with this difference, however, that whereas you can produce with the vacuum tube any frequency, you cannot produce it with the single-phase induction motor because you are limited by the speed of rotation.

It is a difficult undertaking to design a dynamo-electric machine that will give you 10,000 periods per second, to say nothing of 50,000 periods per second or even more, whereas here in the case of the vacuum tube we are not limited, and we can get any frequencies depending on the inductance 7 and the capacity 8 and the coupling of circuit 7, 8 to the rest of the system. Pliotrons are made with 1000 volts applied between 1 and 2 which will easily give you 10,000 volts in 7, 8, and a frequency anywhere between 50 and 1,000,000, and power up to 150 watts, from one single pliotron. That is a great deal of power at 1,000,000 cycles, and it will give you a perfectly satisfactory alternating electromotive force so steady that, as I said, the frequency would not change more than one-tenth of one per cent for hours.

The fact that the theory of the induction motor which I

described to you as developed along the lines first drawn by Maxwell gives correct results, which not only agree with the actual performance of the induction motor, but enable you to predict what will happen under certain conditions you never thought of, shows that the theory is correct, because after all, the supreme test of a theory is whether it can prophesy or not, and this theory can prophesy results which otherwise would not have been thought of.

The fundamental physical conception in the theory is the presence of the two rotary magnetic fields rotating in opposite directions, as Mr. Lamme pointed out. These contrary magnetic fields, as I said, are not a theory, they are an actual, physical fact, and they exist there, and you must accept them, because the oscillograph shows their presence.

E. F. W. Alexanderson: The single-phase motor has been the subject of perhaps more theoretical speculation than any other dynamo-electric machine, and the reason for this is, undoubtedly that it is in its functioning, the most complicated of all dynamo-electric machines, although in its structure it is the simplest of them all. If we are to have a thorough understanding of any dynamo-electric machine I believe it is necessary to form a mental picture of the real physical relations between every volt and ampere and line of force that exists in the machine. that is particularly difficult in the case of the induction motor, and the approximations to a mental picture that we are able to draw depend upon personal preference to a great extent.

Mr. Lamme has given us a theory which is very clearly stated and very easy to follow, and no doubt will prove of great value for students of the technique. However, as I said, for academic reasons it may be worth while pointing out that although the theory Mr. Lamme gives is for all practical purposes accurate enough for induction motors, it is, after all, an approximate theory, and I intend to show to what degree it is approximate and how it differs from an exact theory.

The theory of the single-phase induction motor has been treated on the basis of two rotating magnetic fields, and I was going to say that those fields are a mathematical abstraction. Prof. Pupin told us they should not be regarded so. Now, with the qualifications that he made, we may accept that, because he claims that they are in existence only in the rotor, but not as we ordinarily assume, in the stator. There is, however, one statement in connection with the theory of the single-phase induction motor that represents the actual physical facts, and that is the discovery of Mr. Behrend that the single-phase motor can be replaced by two polyphase motors in series.

Starting with this discovery as a premise, we can draw a few logical conclusions and arrive at practically the same result as Mr. Lamme has, although these conclusions will show just what discrepancy there is.

I have prepared a diagram showing in one composite picture,

Fig. 28, what these relations are. This diagram has exactly the same form as the well-known Behrend-Heyland circle diagram, although it is to be interpreted a little differently. The actual physical substitution that Mr. Behrend has made is that we assume that we put on a motor two windings on one phase and two windings on the other phase. We may assume that we feed the windings on the first phase with one ampere from one single-phase alternator, and feed the windings of the other phase with one ampere from another single-phase alternator in quarter-phase relation. The two second phase windings are in opposition, and therefore the second alternator has absolutely no

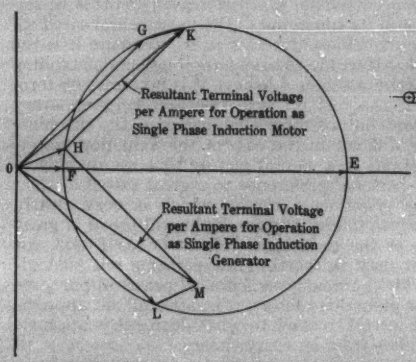

FIG. 28—INVERTED CIRCLE DIAGRAM FOR SINGLE-PHASE INDUCTION MOTOR AND GENERATOR SHOWING VOLTAGES DEVELOPED WITH CONSTANT CURRENT APPLIED

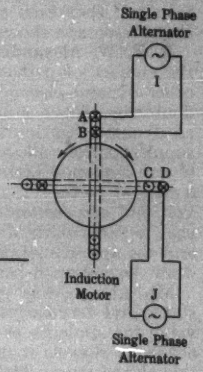

FIG. 29

inductive effect upon the motor. The primary alternator delivers the power to the motor exactly as it does in the single-phase motor. However, the combination Fig. 29 of winding *A* with its current, and winding *D* with its current is a real polyphase winding with the rotation in one direction. The combination of winding *B* with its current, and winding *C* with its current is the real polyphase winding with the rotation in the opposite direction. If we apply the current separately to windings, *A* and *D*, first we get certain voltage reactions; if we apply the current to windings *B* and *C*, afterwards we get certain other voltage reactions upon the total winding.

This circuit follows the same law as the other alternating current circuit, that is, the application of voltage reactions from two sources, will combine vectorially, and therefore, the combined voltage reaction must necessarily be the vectorial sum of the two individual voltage reactions. The voltage reactions of each of the polyphase motors can be calculated by well-known methods, and the calculation is most directly expressed in the Behrend-Heyland circle diagram. That diagram shows what currents flow with a constant voltage applied. I have inverted the diagram showing what voltages are produced with a constant current applied. We must do this in order to get a composite picture, because the windings are in series, and we cannot combine the currents, but must combine the voltages.

The proof why the inverted or voltage diagram has the same shape as the current diagram is a simple proposition of geometry which I will give as a supplement to the written discussion.

On the basis of the known relations in a polyphase motor, referring to the inverted diagram, $O\,E$ is the magnetizing voltage, $O\,F$ is the short-circuit voltage, and $O\,G$ is the normal load voltage for forward rotation. $O\,H$ is the voltage for backward rotation. Now, we let one ampere flow from alternator I, and one ampere flow from alternator J, in windings A, B, C and D. The voltage in the forward winding is $O\,G$. The voltage in the backward winding is $O\,H$. Therefore, the combined voltage is the vectorial sum of voltage $O\,G$ and voltage $O\,H$ and that means $O\,K$.

Mr. Lamme's theory has assumed that the voltages combine arithmetically, as shown in that diagram, so we are not very far from the truth to assume that it is the arithmetic sum, but after all that is not theoretically correct. If, however, we drive the machine over synchronous speed, then one of the motors consumes energy and the other begins to generate. That means the power factors of the two motors are opposed—Prof. Pupin would say one has positive resistance and the other has negative resistance. Then the diagram becomes entirely different, and it is not correct to assume we can take the arithmetic sum. As seen here, $O\,L$ is the voltage of the motor with the negative resistance, and $O\,H$ is the voltage of the motor with the positive resistance. $O\,M$ is the vectorial combination. The two vectors are nearly at right angles, and the vectorial combination represents the accurate solution, whereas the arithmetical sum is in this case not even an approximation.

I had the opportunity to direct the designing of the first phase balancer installation, the successful completion of which was reported to the Institute in 1915, and in the processes of this undertaking I succeeded in making clear to a number of associates not only the function of the phase converter, but how they can be designed and how automatic control apparatus could be worked out for automatic balancing of the voltages, and I found that in those processes it was not necessary to go into very elaborate theoretical explanations.

B. A. Behrend: From Prof. Pupin's remarks one might easily be led to the impression that the theory of alternating-current apparatus did not now follow the conceptions laid down by Maxwell. It need hardly be said that this is not the case and that Maxwell's work underlies all our alternating-current theory. When Prof. Pupin tells us how he uses the Wheatstone bridge to measure the quantities which we measure in practise with ammeters and voltmeters, it is just a little professional mystery injected into a subject which, on account of its inherent complexity, can readily dispense with it.

Mr. Alexanderson shows how to obtain the operating characteristics of the single-phase induction motor for constant current and using the method of inversion or of images, first used by Lord Kelvin, and since frequently applied by Dr. Bedell, Dr. Steinmetz, and the speaker. As it was proved in the case of the polyphase induction motor that the locus of the primary current at constant voltage is a circle, so I proved twenty-two years ago that the same condition prevails in the single-phase motor. I obtained this result by assuming that the single-phase induction motor could be represented by two polyphase motors with their stator windings connected in series acting upon the same rotor winding. From this it follows that, since the slip of the dragging motor is above 100 per cent, it is approximately true to assume the effect of the second motor to be an induced e.m.f. in quadrature with the primary current for the entire range of operation. With this approximation, I worked out a circle diagram for the single-phase induction motor.

Since that time rigorous solutions have been worked out by Messrs. Sumec and Thomaelen demonstrating that the locus of the primary current in the single-phase motor is also a circle. Mr. Alexanderson has now given us another and very interesting method of demonstration of this characteristic of the single-phase induction motor which is to be welcomed by all who see in the vivid and accurate presentation of theory the foundation of all new, great, and enduring engineering work.

L. W. Chubb: I agree with Dr. Pupin's contention that there are two opposite rotating fields.

Prof. Pupin: In the rotor only.

Mr. Chubb: I think that what is in the rotor is in the gap and what is in the gap is in the stator. The best way to get a conception of, and a working system for, a magnetic field is to get it into its components.

In Mr. Alexanderson's discussion, I understood him to say that the two machine analogy is not exactly correct. I wonder if this is not because we have two machines on the same shaft and add together the heat in the two machines. A slight discrepancy may possibly be due to the summation, rather than partial cancellation, of the two resistance losses.

I would like to call attention to another little point. If we have a polyphase winding and apply a single-phase line to two

terminals of it, we then have a single-phase current and a polyphase voltage. If now we connect two terminals together, and apply the single-phase voltage, we get polyphase current and single-phase voltage. There are other combinations of connecting the machines in parallel and connecting the machines in series, any of which can readily be treated by the method of oppositely rotating fields.

In going over these papers, it struck me that the differences between the two methods of analysis are very clearly brought out, but not as well brought out as they would have been had the single-phase motor been another special case of the unsymmetrical polyphase treatment. The single-phase motor is one of the special limiting cases of unbalanced polyphase operation and is therefore relatively more difficult, because by Mr. Hellmunds method it is one of the simple but long solutions.

In Mr. Hellmund's analysis he starts by what is, by that method, the simple case, the concentrated winding, and works up to his case Number 5 which is the first practical case. With the double rotation system, if we wish to consider Mr. Hellmund's case Number 1 it would be rather laborious, because with the concentrated winding the distribution of the field is such that we have many harmonics to deal with. Any of the later practical cases with distributed windings, can, however, be more readily treated by Mr. Lamme's type of analysis.

Again the double rotational method is by far the better to prove that the field distribution is sinusoidal with distributed secondary independent of the primary distribution. It can be seen from simple mental conception only and without any long graphical or mathematical analysis. The concentrated primary produces a square topped m.m.f. which can be represented by the equation

$$H = K\,(\sin\theta + 1/3 \sin 3\,\theta + 1/5 \sin 5\,\theta + 1/7 \sin 7\,\theta + \ldots)$$

The single-phase alternating primary current will vary the amplitude of the square topped wave with fundamental frequency. Each component will vary proportionally and each component therefore can be represented by two oppositely rotating fields of n pairs of poles, rotating at $1/n$ synchronous speed. All of these component fields, except the forwardly rotating fundamental component, will have a relatively high slip with regard to the synchronously running rotor. All components will therefore be damped out except the fundamental component rotating with the secondary and the field distribution will therefore be sinusoidal. I wish to call your attention to the fact that the distribution will not be sinusoidal except when the rotor is running at synchronous speed. If the rotor runs at one-third speed, there will be a field distribution having three times the number of poles, traveling at the speed of the rotor, if at one-fifth speed, there will be five times the normal number of poles, traveling with the rotor.

This type of treatment applies generally to all special cases of

rotating machines, symmetrical or unsymmetrical, balanced or unbalanced, single or polyphase, and such treatment as given in Mr. Lamme's paper should be encouraged as the logical and economical method.

***J. Slepian** (by letter): For text of this discussion, see page 659.

Alex M. Gray: Mr. Lamme wrote his paper principally because he had found that college students while they understood the polyphase induction motor had no physical conception of the operation of the single-phase induction motor.

The theory of a-c. machinery is developed in the classroom as follows: After the polyphase alternator and polyphase armature reaction have been discussed, single-phase armature reaction is then considered as the result of two polyphase fields rotating in opposite directions. The inverse field is wiped out by the rotor structure. Unfortunately, the same idea has been

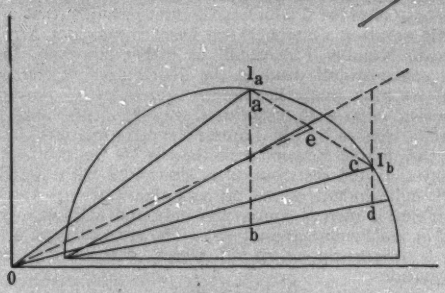

FIG. 30

applied in discussing the subject of single-phase induction motors, and many teachers do not make it clear that the two phenomena are different, in that the alternator fields are determined only by the current, whereas in the induction motor the applied voltage is fixed, so that any decrease in one of the fields is accompanied by a corresponding increase in the other, in order that the flux threading the stator winding may remain approximately constant.

We can consider the two component fields as produced by two separate polyphase induction motors connected in series, the fields rotating in opposite direction, but the rotors coupled together. The reader can convince himself that this representation is exact by referring to the literature of the subject. We will assume first that the total applied voltage is divided equally

*Discussion presented both at Pittsburgh April 10 and New York April 12, 1918.

between the two machines, and that the circle diagram representing operation of these polyphase machines is as shown in (Fig. 30). Let us consider the operation at half synchronous speed. In the case of motor A the current would be I_a and the torque represented by $a\,b$, while in machine B, which is operating as a generator, the current would be I_b and the torque represented by $c\,d$. The two motors, however, are in series so that I_a and I_b must be equal and in phase, and therefore the assumption that each motor absorbs half of the applied voltage is wrong. The voltages are in the inverse ratio of the currents I_a and I_b and their resultant is the applied voltage. In (Fig. 30), therefore, the motor torque $a\,b$ has to be multiplied by the ratio $\left(\frac{o\,c}{o\,e}\right)^2$ and the generator torque $c\,d$ by the ratio $\left(\frac{o\,a}{o\,e}\right)^2$.

Another method of treatment is that developed largely in this country by Dr. McAllister and will be found discussed in his book on a-c. motors, and also in recent texts such as Lawrence's "Alternating-Current Machinery". In these texts it is shown that there are two fields 90 deg. out of phase with one another both in time and space, one field being proportional to the speed and the other to the frequency, so that the resultant field is elliptical and is a uniform rotating field at synchronous speed. This latter method is the one now taught to students and it is not improbable that it does not give the student a clear conception of what is happening, since there are four rotor voltages and two rotor currents, and since also phase relations in time and space are liable to be mixed up.

Some teachers, without further introduction, will draw a circuit on the blackboard and state that it is exactly equivalent to an induction motor. The analytical treatment thereafter is easy, but the student does not know the machine when he sees it, far less does he know how the machine works. To prove that the circuit is equivalent to the motor and then to solve the circuit is a roundabout way of going after the matter. The equivalent circuit should never be mentioned until the machine has been thoroughly discussed along the lines indicated above.

There is one other method of treatment which is suitable only for mathematicians who happen at the same time to be electrical engineers, and that is the treatment by complex quantities. Complex quantities are very useful if you know how to humor them and if you are using them frequently, but they do have a habit of slipping out of one's mind. An instructor who revels in this method of treatment can be cured if he is put in charge of the laboratory, because it is very difficult to point out to the student just which is the imaginary part of an actual machine. I am convinced that the use of complex quantities in undergraduate instruction should be restricted to the solution of cir-

cuits and networks and should never be used in machinery. After graduation the student may study Steinmetz's texts with advantage.

***R. E. Hellmund** (Read by A. M. Dudley): For text of this discussion, see page 675.

***A. M. Dudley and C. M. Weber:** For text of this discussion, see page 680.

***C. A. Weber** (by letter): For text of this discussion see page 669.

Chas. F. Scott: The author uses a type of physical analysis in this paper which is a characteristic of his method of handling engineering and design problems. A notable former instance was in connection with the polyphase motor. The form of motor which first came into general use employed secondary resistance for starting.

Then came one of the notable results of Mr. Lamme's analysis, of this motor, for he so proportioned it that the squirrel-cage rotor could be employed with acceptable starting conditions. It was this kind of physical analysis by Mr. Lamme which led to the proportions in his design and gave us the squirrel-cage or "Type C" motor as it was then called. It was this type which made a reputation for the a-c. motor, and established it as a simple, useful and dependable machine.

Mr. Lamme presented a paper before the National Electric Light Association, at Niagara Falls in 1897, in which he described in a physical way the operation and characteristics of induction motors. It was a sort of practical paper, appealing to those who like to understand fundamental relations, but who are not mathematicians, which has become a classic. Now, the same general method which has done so much to make the operations of the polyphase induction motor known and understood, are extended by Mr. Lamme to a consideration of the single-phase motor.

Michael I. Pupin: One of the brightest remarks made this evening was made by a professor, when he said, that when they get an induction motor before the student he understands its true theory when he reaches the laboratory. I agree with him perfectly that the introduction of the complex variable into alternating-current electrical engineering has done a tremendous amount of mischief among the students. Not among men of my age, because I cannot be fooled so easily, but the student has used a complex quantity where the complex quantity ought not to be used. There is no doubt that the introduction of the complex quantity into alternating-current calculations introduces a simplicity which is remarkable in certain types of problems, but certain other types of problems should not be treated by the complex quantity, and this is one of the types. The man who uses complex quantities in the treatment of the theory

*Discussions presented both at Pittsburgh, April 10 and New York, April 12, 1918.

of the induction motor commits an unpardonable crime, he ought to be interned as a dangerous individual—he should stick to the old fashioned method of using the real quantities in connection with real problems, whenever he can.

It is not easy to teach a student to use a typewriter before he learns how to write. It would be a dangerous thing to introduce it into the public schools, before the boys and girls knew how to write, and the same thing is true in the electrical engineering class room. To introduce complex quantities before the students know how to handle real quantities is dangerous and misleading, and the students themselves do not like it. I never use a complex quantity, until I come to the power transmission lines, and the problem of the distribution of alternating-currents in a network of conductors.

In this problem you cannot use it. When it comes to a problem where there is a single-phase winding in the primary and a single-phase winding in the secondary, God help you, it is a most complex problem, and I warn you not to be misled by your curves, because if you experiment you will find that you do not get periodic curves at all. The theory of the alternator is not the converse of the theory of the single-phase induction motor. In the alternator you have a single-phase winding in the rotor, and you have a single-phase winding in the stator, but in the induction motor you have a tremendous number of circuits in the rotor. In the alternator, the field is saturated so that the magnetic reaction in the primary produced by the armature current does not amount to much. But if you should try an alternator with a laminated field, and have a few turns only in the field winding, you will get electromotive force and current curves which seem to baffle all understanding. When the primary is fed by an alternating current you do not get periodic curves at all, because, although in the primary you impress a simple harmonic alternating electromotive force, you get an infinite series of additive and differential frequencies. I observed them in some of the results shown tonight, when you have a single-phase winding in the secondary of the induction motor.

This is a case which has been handled for us by Goldsmith in his famous alternator which is used in wireless telegraphy, where he impresses 10,000 periods per sec. on the primary and gets 40,000 or 50,000 periods per sec. in the secondary by reflections as he calls it. It is a very interesting case, but it has no practical value except to make high-frequency alternators without resorting to excessive number of poles and excessive speed.

I warn the gentleman who presented the second paper, Mr. Hellmund, to proceed rather slowly. He has given a most complicated case, and I am sure that he will get into a mess, into a labyrinth of all kinds of complications, if he does not stick to the single-phase induction motor with sinusoidal distribution.

Selby Haar: When I began to study precision of measurements I was told that there was a great deal of difficult mathemathics in the theory, but that after I had learned the theory I could forget the mathematics, and the actual process of using the theory would be very simple, and would require nothing more than a knowledge of arithmetic. The circle diagram may be very much simpler than the method of complex quantities for purposes of instruction, but when I was a commercial designer of induction motors, we had to spend quite a large amount of time in educating inspectors and others who determined motor performance curves by the circle diagram, and naturally obtained different results than we did. The theory of complex quantities may be difficult, but the calculation of induction motor curves by this method is as simple as the application of the rules of precision of measurements, and, at the same time, is accurate. With a slide rule a whole set of curves covering the usual range from one-half load to one and one-quarter load may be completed well within an hour. In the circle diagram the most important points are near the end of the horizontal diameter, where the errors of measurement are greatest.

B. G. Lamme: It has been brought out in the various discussions that the method given in my paper is only an approximate one. That is correct and it is so stated in the paper itself. However, certain of the numerical tables and curves in the paper show how small the error is. If, in order to avoid minor approximations, I had introduced the complications shown in some of the discussions, it would have spoiled the paper for the purpose for which it was written. The intention was to give a simple physical conception of the motor and to do so it was necessary to make certain approximations as stated before. It may be said that none of the discussions this evening have simplified the subject in the sense of making it easier to comprehend.

As to the different methods of looking at the problem of the induction motor, I have tried this experiment,—I have taken three different men and put up to them certain new problems of induction motors and then asked them what was the first thing they thought about. One man said that the first form in which the problem presented itself to him was in the nature of an equation; the second man said that the first thing he thought about was a circle diagram; the third man said that his first thought was about the distribution of flux in the machine and the currents in the windings. If these three men had equal mathematical ability, I believe the one with the physical conception of magnetic flux and the currents could tackle the problem better than the others. In general, I have found that a man with good physical conception and fair mathematical ability can tackle new problems most successfully. Consequently, I have made a practise, with the young men under my charge, to try to teach them the physical conception first of all

and then take up the mathematical aspect. I find, as a rule, that they do much better after they get the physical conception.

My purpose in preparing this paper was not to assist that small group of people who have a very good grasp of the fundamentals of the single-phase motor, but to meet the needs of the very much larger class of people who are interested in such apparatus, but who cannot read mathematics very well and do not know what a circle diagram means, or what a vector diagram is. With many such pepole, if an equation is encountered when reading the paper, they will not study it out and, furthermore, if there are one or two equations on a page they will not even read that page. This applies to a very large percentage of the readers of the TRANSACTIONS, and I find that if a subject can be so treated as to leave out all formulas, etc., a great many people will read it who would not otherwise do so.

I wish to add that apparently there has been some misunderstanding as regards the intent of this paper. It was not the intention of the writer to claim anything new or novel in the general treatment. Many of the various methods used in this paper have been brought out long ago, some of them so many years ago that I do not recall where they originated. Mr. Behrend would be a much better man to give this than myself, as he brings out a number of these points in his book on the induction motor, published some eighteen years ago, I believe. The only credit I can claim in regard to this paper, is for the attempt at simplification, to bring it within the range of non-mathematical readers.

Mr. Hellmund: It has been brought out quite clearly in my paper that it was not intended as material for the classroom and for giving to beginners a physical conception of the working principles of a single-phase motor. It was rather intended to answer certain questions to the designing engineer and advanced student, who have raised such questions from time to time regarding secondary magnetizing currents, losses, etc. Unfortunately, it was found simply impossible to get all the results I was after without quite a good deal of mathematics.

Mr. Slepian has supplemented my paper by a very interesting discussion, showing how it is possible for an engineer familiar with higher harmonics and some other characteristics of the motor, to arrive without mathematics at one of the conclusions arrived at in the paper, namely, that a sinusoidal field distribution prevails under certain assumptions irrespective of the primary winding distribution. While this fact was known to me for a considerable number of years from certain investigation of the polyphase motor,* I knew little about the influences of the winding distributions upon the currents, and the like, until I figured them out by the use of mathematics. Only after thus definitely establishing certain facts, I was able to devise the

*See my paper A. I. E. E. 1908, p. 1373, in particular p. 1382. Also Articles in *Electrical World*, 1906, p. 329.

graphical method described in connection with Fig. 34 of my paper for determining the secondary magnetizing currents of commercial machines.

While the treatment of practical cases is given much in detail in my paper, it is evident from the discussion that this has been overlooked by some of the speakers, who have apparently paid too much attention to a few rather theoretical cases, which I have given for reasons previously stated. Referring, for instance, to Prof. Pupin's brief discussion of my paper, it should be pointed out that the case of sinusoidal primary distribution has been treated in full both under consideration of leakages and resistances. This part of the paper alone would, however, not give any answer to all questions arising in connection with commercial machines. The mere fact that the treatment of the latter is more complicated does not make the knowledge of what actually happens in practise less desirable. It was just on account of the peculiar phenomena caused by the non-sinusoidal distribution of windings in slots that designers have been puzzled from time to time and unable to explain, for instance, why certain losses were much higher than expected. I fully agree, however, that the treatment of the more complicated cases requires great care to avoid confusion and wrong conclusions. I am, therefore, very grateful to Messrs. Dudley and Weber for the trouble they went to in checking my methods by actual tests and thus demonstrated their correctness. They deserve much credit for removing thereby any doubts, which are only too natural with some of the peculiar results arrived at in my paper.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

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A REVIEW OF ELECTRICAL ENGINEERING PROGRESS

President's Address

BY E. W. RICE, JR.

ELECTRICAL engineering now covers such a wide field of scientific and technical activities that your President is presented with an embarrassment of riches in the attempt to select a subject for his address. He has, therefore, decided not to confine himself to any one feature of our Society's work, but to pick out here and there a few of many items to talk about which seemed of present interest and importance.

It is a pleasure to call your attention to the fact that we have added 1235 members of all classes during the year, our total membership being now 9370. This is a most encouraging result in these times of stress and change.

Our Institute is a national asset of increasing value to its members and to the nation. Every person who has the necessary qualifications should identify himself with the Institute, not only for the great benefits which he personally will receive, but in order that the usefulness and power of the great army of electrical engineers may be increased and rendered more available for the highest and most efficient service to our country and to the world.

The engineer is the hope of the nation, not only now, when we are at war, but even more so in the future in the days of reconstruction following the great peace. The engineer may perform valuable service when working alone, but his usefulness and power is manifestly greatly increased when acting in co-operation with thousands of his brothers.

The Institute is not only a democracy but is a democracy of educated men and such men have a heavy responsibility to society at present and will have in the future. They should be leaders and exemplars for those who have not been so fortunate as to have enjoyed their opportunities. The Institute needs its members and the members need the Institute, that the electrical engineer may fulfill his high destiny.

That the work of the Institute is of the highest quality is evidenced by the character of its meetings, its papers and discussions and the splendid work of its various committees. Its value and usefulness has increased every year of its existence and it should continue to gain in strength and usefulness because its methods are in accord with the spirit of the times.

But in order to accomplish this desirable result, it must continue, as at present, to be representative of the electrical engineering profession of the country, and therefore, must continue to expand its membership. This growth will bring with it problems inherent in all great institutions, democratic or autocratic, but I have confidence that all difficulties will be met successfully for the reason that the members of our profession are trained in the scientific view-point and methods of solving problems.

In the early days the progress of the electric science and arts was so rapid that it was relatively easy to find each year plenty of material for a review. Progress has continued and will continue, but naturally a decided tendency to saturation is shown in many directions. In some instances, this saturation can be demonstrated to be due to the fact that limits of perfection have been so closely approached that little remains of possible accomplishment. In other instances the slowing up is due to lack of knowledge, or, especially at the present time, to lack of workers, such workers having been diverted to the work imperatively needed to secure us against the attack of our enemy on the foundations of our existence.

There has been no material improvement for several years in the matter of efficiency in electrical units, such as dynamos, motors, transformers, etc. The efficiencies stated in Past-President Lincoln's address, in 1915, still remain almost exactly of the same values, and for the reasons which he so clearly pointed out.

The efficiency of conversion of mechanical into electrical energy, or the reverse, of electrical into mechanical energy, is still about 90 per cent in the average case, under practical conditions of operation; the efficiency reaching as high as 97 per cent or 98 per cent in the most favorable cases, with the large units, and falling below 90 per cent in unfavorable cases, or in the small units. The efficiency of conversion of electricity from high to low potential, as in transformers, also remains substantially the same, reaching as high as 98 plus per cent in the largest units. It is obvious, as Lincoln pointed out that no material change can be expected where such practical perfection has been reached.

The conversion of mechanical power of falling water into electrical energy by our water-wheels and electric generators has increased from about 87 per cent to 90 per cent in the largest units of 40,000 h. p. This represents about the limit which may be expected.

In the field of thermodynamic engines, represented largely by the steam turbo-generator unit, some improvement has been obtained. Lincoln stated that 75 per cent of Rankine efficiency had been obtained in some large modern steam turbo units in 1915. This has now been increased to about 80 per cent in the largest units of 35,000 to 40,000 kw. and 75 per cent is quite common practise even in such moderate sized units as 10,000 kw. This improvement, while not large, is doubly important because of the great increase in the cost of fuel. It has been realized mainly by bringing the practical design more nearly in accord with the theoretical, by increasing the number of stages or processes of steam extraction, reducing various losses, and by improving many details which, when properly looked after make in the aggregate, gains of practical importance.

Increase in the initial pressure of steam and lowering of terminal pressure, by better condenser arrangements, have also contributed to improvement, as it enables an increase in the range of temperature to be utilized. This makes possible better thermal efficiencies, even with the same per cent of Rankine efficiencies.

The following information illustrates the improvement in efficiency of turbo-electric units beginning with the first 5000 kw. installed in this country, in 1903, and continuing up to the close of 1917:

Year	Size, kw.	Steam Conditions			Lbs. per kw-hr.	Per Cent of rankine efficiency
		Steam pressure	Superheat fahrenheit	Back pressure		
1903	5,000	175 lb.	0	2 in.	24.00	37.8
1908	14,000	200 "	125°	1½ "	13.50	66.1
1911	20,000	235 "	100°	1½ "	13.20	67.0
1913	20,000	200 "	200°	1 "	10.74	75.9
1916	20,000	250 "	250°	1 "	10.00	76.5
1917	35,000	230 "	200°	1 "	10.14	78.7

It is gratifying to note that a percentage of Rankine efficiency of approximately 80 has been reached. This progress reflects

great credit upon the designers of turbo-electric machines and is a record of achievement found only in electrical development.

Concurrently with this improvement in the turbo-electric machines, great advances have been made in the design and operation of steam producing devices—the boilers, and in auxiliaries and other features of the modern power station. As a result the thermal efficiency has been rapidly improved. The thermal efficiency to which I refer may be stated as the ratio of the total energy produced at the terminals of the generator, to the total energy in the fuel burned — expressed as a percentage. It takes account of all losses from the coal under the boiler to the electricity at the dynamo terminals. It is the ratio of the heat units equivalent to one kw-hr., divided by the similar heat units in the fuel consumed to produce one kw-hr. at the generator terminals.

This thermal efficiency is after all, to the electrical engineer, the most important measure of progress. It measures the advance in station fuel economy, and as stated, many factors in addition to the improvement in turbo-generators have contributed to the result. Thermal efficiency may obviously be used to express the results of a single unit, consisting of turbo-generator, with its bank of boilers and other accessories, or it may be used to designate the combined result of all the units in a given power station.

The progress in the case of a combination unit, *i.e.* turbo-generator, with its boilers, auxiliaries, etc. has been as follows:

Year	Size of unit kw.	Thermal efficiency per cent
1903	5,000	10.15
1908	14,000	15
1913	20,000	18
1917–18	35,000	21.6

For comparison, I may state that large gas engines in steel mill practise, under best test conditions, show 25 per cent thermal efficiency, but in actual operation, an efficiency higher than 18 to 20 per cent is rare.

High compression oil engines of the Diesel type, driving electric generators, realize 25 to 26 per cent thermal efficiency when new, but are difficult to maintain at such efficiency.

The figures given must not be confused with the much higher thermal efficiencies often quoted for gas and oil engines, which refer to indicated horse power and not to electrical output.

The steam turbo-electric unit has not reached its limit of thermal efficiency. Calculations show that, with pressures of the order of 500 lb. gage, a thermal efficiency of 26 per cent should be easily realized. For any further substantial improvement, we must look to new methods, such as the use of two fluids, for example mercury and steam, as planned by Mr. W. L. R. Emmet. This method is still under development but its progress has been hampered by the pressure of war work.

As a matter of interest to electrical engineers, I may say, parenthetically, that the steam turbine in this country owes its existence and development almost entirely to the electrical engineer, and this is not surprising as the electrical engineer was familiar with the advantages of rotary machines, and perhaps it is not too much to say, prejudiced in their favor.

While, as stated, the efficiency of electrical units reached about its limit some years ago, those familiar with electrical engineering development are aware that progress has been made and is still possible in the generation, transmission and utilization of electrical energy. The struggle for improvement in efficiency has been transferred from the unit to the aggregate, called the system. We cannot have a system of maximum efficiency without units of maximum efficiency, but individual units of highest efficiency do not, of themselves, insure that the system upon which they are used will be of the highest efficiency, so progress has been made in the direction of improving the system economy or system efficiency.

To obtain the highest efficiency in practical operation, the element of time enters as a powerful factor. Our conception of efficiency should not be limited to a consideration of the relation between the instantaneous value of available heat units in coal and the electrical units produced at the point or points of consumption, but should consider the relation between the total number of heat units in fuel consumed in a given time, say 24 hours, to the total number of electrical units produced and used in the same time. The attempt to improve the efficiency of the system has shown the necessity for utilizing the generator units and transmission and distributing systems, for the maximum possible time.

This has led to the study of such questions as load factors of

generators, of stations, and of the system as a whole, to the study of the diversity factor, to the reduction of idle currents in alternating-current systems by the use of synchronous condensers, and to means for the reduction of the constant and no-load losses in all machinery, in transformers, etc.

The resulting improvement has been effected, not only by changes in designs of the units themselves, but also by their method of use, based upon the recognition of the fact that the elimination or reduction of the losses at light load will greatly improve the total efficiency, especially when the time of use of the apparatus under load is a small part of the total time.

Automatic substations for transformers and synchronous converters have come into existence; different power houses of the same system have been tied together electrically; transmission lines of different systems have been interconnected, so that the units may be usefully employed for the maximum period, or lie idle or unloaded for the minimum time.

This general development has led to marked improvement in total energy efficiency, represented by the amount of fuel burned per electrical unit sold or utilized, and has also reduced cost of operation and charges for investment. There is still room for continued improvement in this direction and the progress will be rapid due to the pressure for maximum efficiency in the use of coal and of existing investment at the present time.

Many interesting examples of the methods and devices adopted to improve station and system economy and efficiency may be found throughout the country. In California, large electrical systems have been arranged to be tied together electrically, for exchange of power. In Washington and Idaho, power systems under different management have made similar arrangements. In the South, all important hydroelectric systems have been tied together for exchange of power. The advantage, as I have stated, of such arrangements is better utilization of variable stream flow, improvement in load factor, increased reliability of service, and the net result is to improve the efficiency of the system, not only financially, but in a purely technical sense. One most important advantage is the obvious reduction of the necessary investment in reserve machinery of every description.

In Montana, eight hydroelectric plants successively use the same stream flow, the total effective head amounting to 600 feet, and not only is the natural flow of the stream thus successively utilized, but all the storage water is effectively used by each

plant in series. In this same system, the yearly load factor is stated to reach 75 per cent and the mean monthly load factor to reach 80 per cent.

The interconnection of hydroelectric plants brings about another extremely important saving, based upon the variation of rainfall in amount and time on the different watersheds which are thereby brought to serve a common system. It frequently happens that there will be plenty of precipitation on one watershed, while another watershed may suffer from long continued drought. This condition varies not only in the same year but in different years. Interconnection serves to eliminate these variations by a process of averaging, and where the inter-connected system covers a sufficiently wide area, a remarkable increase in total useful power is made available.

It has frequently happened that thousands of horse power have been wasted over the dams of one system, the watersheds of whose plants happened to have a wet year, and at the same time, a nearby hydroelectric plant, supplied by another watershed, was without water power. The result has been that one system wasted power, while the other was suffering from a power shortage which would frequently be made up by burning a large amount of high grade coal, in the operation of an auxiliary steam plant. This condition has to a large extent been remedied by the interconnections to which I refer.

It has been estimated, and it seems a conservative estimate, that through the saving in reserve equipment, improvement in load factor, and the diversity of different loads, the useful output of groups of large systems may through inter-connection be increased about 25 per cent.

Electric regeneration of power, that is, the utilization of the weight of trains running on a down grade due to the force of gravity to generate electricity which is fed back into the electric system to help other trains up grade, is an illustration of the same important improvement in the system efficiency.

I have thought it desirable to call your attention to the improvements obtained in system economy or efficiency because of the important savings in investment, in coal, in transportation, in labor and material, which in the aggregate, have already been realized. It illustrates the wonderful flexibility, value and economy of a general system transmitting energy by electricity, compared with any other possible method.

These advances have been more rapid during the last year,

due to the imperative demands for economy saving and increased efficiency imposed by the war. It is a great satisfaction that the foundation had all been well prepared during the times of peace.

The development of our industry has been so rapid that the need of intelligent and constructive standardization was realized some years ago. The Standards Committee of the Institute, formed in 1898, has been of inestimable value to the profession and to the industry. The standards adopted have been flexible enough to ensure progress and yet to discourage variations which were valueless. The standards promulgated by our committee have so appealed to the profession and to the industry that they have been cheerfully followed, and I am convinced that, as a result, the cost of electrical apparatus to the consumer has been greatly reduced over a number of years and the quality has not, been sacrificed, but has been improved. I consider that the money value of the work so done could be conservatively placed at many millions of dollars.

Sixty-cycle systems have shown, during the past few years, a more rapid growth than 25-cycle, and it is now estimated that 60-cycle systems represent about 70 per cent of the total power supplied in the country. This is undoubtedly due to the lowered cost of transformers, generators, induction motors, and similar apparatus. The relative growth of 60-cycle as compared with 25-cycle systems is reflected in steam turbine installations. In 1910 about 60 per cent of the steam turbine electric energy of the country was supplied from 60-cycle units; in 1917, this had risen to approximately 75 per cent.

This is an instance where standardization is desirable and economical. It will hasten the time so often predicted, when a network of transmission lines, carrying electrical energy, will cover the country. These will be fed by super-power stations, suitably located with respect to cheap reliable supplies of coal for fuel, and water for condensing purposes, and into the same network will also be fed energy from the various hydroelectric installations.

Marked advances have been made during the past year in the application of electricity to the electric furnace. It is estimated that the number of electric furnaces in the United States has been increased about 40 per cent in the past year and that there are now in operation over five times the number that existed five years ago. The world's output of steel from electric furnaces has now grown to approximately four million tons per annum.

Experience has demonstrated that the electric furnaces can utilize the cheapest and most inferior raw material to produce steel of the most uniform and highest quality, with the greatest regularity. The cost of steel so produced, while reasonable, considering its quality, was higher, until recently, than that produced by the open-hearth method. It is now possible to produce electric steel at substantially the cost of that produced by the open-hearth method. This result has been brought about partly by the increased cost of the open-hearth method, due to a variety of well known causes, but largely by a reduction in the cost of electric furnace operation. The marked change which has taken place in the reduction of the cost of operating electric furnaces is based upon greatly increasing the rate at which energy is delivered to the metal, both during the melting and the refining period. This has reduced the time required for an individual heat and also the kilowatt hours required per ton of metal melted, with a net result of increasing the daily output of the furnace.

As a concrete example, I mention the history of a five-ton furnace. It was originally supplied with 800 kv-a. at 80 volts. This was increased to 2000 kv-a. at 150 volts for the melting period and about 1400 kv-a. at 100 volts for the refining period. The time for the heat was reduced from six to three hours, power consumption was reduced from 877 kw-hr. to 588 kw-hr. per ton, and the number of heats per 24 hours was increased from three to five, increasing the net output from 15 to 25 tons.

Electric resistance furnaces of large sizes, for special heat treatment requiring unusual exactness, are being extensively used, producing results greatly superior to oil or gas fire furnaces.

Electric welding, both by the arc and incandescent method, is being rapidly extended and is destined to greater development in ship-building and similar operations.

Electric engineers have been devoting much time to the solution of many war problems. It is not desirable or possible to review such work at present, but when the veil is lifted, we will all be gratified with the result. We must content ourselves with the mere statement that this work has covered means for the detection of the pirate submarine; wireless signalling and telephoning for army and navy, and aircraft devices; searchlights of novel design and great power; improved methods in manufacture of ammunition and ordnance; electro-chemical work of

every description; electric welding; X-ray sets of greater simplicity and accuracy; and many other lines too numerous even to mention.

The great industrial research laboratories, the educational and governmental research departments have all co-operated enthusiastically and effectively, and the members of their staffs have labored day and night, without regard to pecuniary reward or public applause, sustained entirely by the high purpose of giving their best to the service of the country. I hope the time may come when the story may be told, so that the world may realize the debt which it owes to scientific men and engineers, without whose arduous, unselfish and almost inspired work, our cause, righteous as it is, would have no chance of a victorious conclusion.

In my address at the opening of the mid-winter convention of the Institute, in February, 1918, I called attention to the advantages which it seemed to me would follow a more general electrification of the steam railroads of the country. I merely repeat at the present time that electric locomotives have been so improved and simplified that they are competent to haul the heaviest train that can be held together with the present train construction; to operate at the highest speed permissible by the alignment of the road and independent of its grades; and that the electric locomotives can meet in the most efficient and adequate manner the transportation problems confronting the country, and offer better results than are now obtained or seem possible with steam locomotives.

There can be no question that railroad electrification is not only economical but imperatively needed to improve the present standards of steam operation. Our mountain districts are congested almost entirely by the limitations of the steam railroad systems, and the addition of more tracks, under such conditions, is not the best solution of the problem. The electrified divisions of the steam roads have been free from troubles during the past severe winter and I repeat that the coal famine which the country suffered last winter could have been largely avoided if the steam railroads had been electrified. Moreover, it should not be forgotten that steam locomotives burn about 25 per cent of the entire coal mined in the United States and that 12 per cent of the entire ton mileage movement of freight and passengers carried over our railroad tracks is represented in cars and tenders required to haul coal to supply steam for the locomotives.

It is a truism, which has been frequently stated, that war requires the mobilization of the nation's industries and their devotion to essential work. This is especially true in this country, as it has been necessary in addition to create substantially new industries on an enormous scale, such as the production of ships, ordnance, ammunition, airplanes, chemicals, etc. To operate these industries, it has been necessary to mobilize to the fullest extent our available material and labor, but material and labor can only be converted into war work by the application of power. This power, in view of its great economy and flexibility, must be electrical.

While this country was fortunate in having available a magnificent system of power stations, so great was the magnitude of the demand for increased power, created by the war industries, that it is estimated that there will be a shortage of at least 500,000 kw. of electric power in the Eastern district.

It takes from one to two years to build and equip the large units which are essential for the production of such power. This illustrates the importance of all of the methods which I have mentioned to conserve, utilize and increase the efficiency of existing equipment and investment, as such methods can produce results in a much shorter time.

It is, however, vitally important that the great electrical power producing companies of this country should be helped in every way to meet the heavy demand which is placed upon them. It has been demonstrated that the quickest, most efficient, and altogether best way to meet the demand for power is through the expansion of such existing organizations and installations.

Fortunately, there is general appreciation of the fact and comprehensive schemes are under consideration which will provide for the erection of large steam electric power stations in the mining regions. Favorable locations exist which are within reach by transmission lines of electric power stations now serving large industrial areas. By interconnection, present investment and machinery will be better utilized and a large amount of additional electric power made available, without making any increased demand upon our congested railroad facilities.

It is evident, therefore, that we need to consider and put into effect, every practical method for conserving our existing developments, and also, we should take a courageous view of the future; we should provide, for the future growth at least as liberally as has been the custom of the managers of the great public service

systems in the past. It has been their custom to build from two to three years in advance of existing requirements, in anticipation of the future. I have yet to learn of a single important instance where such foresight has not been amply justified.

I would say in conclusion that the saving in fuel, by such improvements as I have mentioned in various parts of my address, amounts to many millions of tons every year; the saving in material and investment represents millions of dollars, which manifestly represent service of the highest value to the industry and to the country. Such work is just as much the province of the electrical engineer as improvements in the design and efficiency of the electrical units, and requires the same scientific ability, vision and industry.

While I admit to considerable prejudice in favor of things electrical, I think that in no other field of engineering has there been such a remarkable improvement and a condition which so nearly approaches, in the matter of efficiency, to 100 per cent, as has been shown in the field of electricity. This phenomenal record is not the result of accident. It has been due to the enthusiastic devotion of the scientist and engineer and executives to their work. They have not been satisfied with things as they are, or with mediocrity. They have wanted the best; have not been contented with a 75 per cent to 80 per cent efficiency when something better was obtainable. The causes of inefficiency have been scientifically attacked; the losses have been studied and their causes discovered and removed.

Concurrently with the improvements in the efficiency of conversion, the engineer has studied ways and means to reduce the amount of material and the amount of labor required to produce a given effect, and has been equally successful in increasing the effective use of material and labor, and as a result, until interrupted by the war, the cost of electrical machinery and devices of every description has shown a progressive reduction, not only without sacrifice of quality, but with great improvement in quality. This truly marvelous work, we can safely affirm, is the foundation of the phenomenal growth, prosperity and present commanding position of the electrical industry, which is a monument to the broad vision, intellectual honesty, faithful work and the correct economic viewpoint of the electrical engineer and his co-workers.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

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ANNUAL REPORT OF TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors,

The past year has been one of unprecedented difficulties for all railway transportation companies. With labor conditions at their worst, the maximum tonnage ever offered, rolling stock and motive power sadly deteriorated due to a long period of lean income—all added to a winter of unprecedented severity, it is little wonder that the trunk lines practically collapsed under the strain. The collapse of the steam railways, however, did not extend to electrified lines, which once more demonstrated the superiority of electricity as a motive power, particularly in cold weather. This was especially true on the electrified steam railways, such as the Norfolk & Western, and the Chicago, Milwaukee & St. Paul Railways, the latest railways to be electrified; and the Pennsylvania, New York Central, New Haven Railways, and others. These roads, one and all, operated successfully through the hardest weather. The record is one of which all electrical engineers may be proud.

The advantages of electrification are now almost universally recognized. The main features are:

1st, Increased capacity.

2nd, Economy in fuel consumption.

3rd, More reliable service.

4th, Greater safety in operation on mountain grades and through tunnels.

5th, A higher class of service.

The first two advantages were especially emphasized by President Rice in his masterly plea before the Midwinter Convention for the electrification of railways as a war measure, and the Committee urges the most careful consideration of this matter by the Government and the railways.

There are many places where electrification will show an actual economy in operation, especially on lines such as the Norfolk & Western and in the Broad Street Terminal of the Pennsylvania Railroad. In both of these cases, the capacity of the railroads had been reached with steam, and it would have

been utterly impossible to handle the traffic which they have had in the past year with steam locomotives. It is impossible to estimate or to appraise all of the advantages of electrification. Electrification will eventually be adopted because the character of service will be so much better that the railways cannot afford to do otherwise. During the war, however, the amount of electrification work done will probably be limited to places where it is absolutely essential, or especially advantageous, in order to facilitate handling the traffic necessary for the proper prosecution of the war.

In this connection, it is interesting to note that the C. M. & St. P. Ry., in continuing its electrification work with 3000 volts d-c., is electrifying the two engine divisions from Seattle east, which will give them a total of approximately 650 miles of electrified main line. Work is progressing on this new line very rapidly, and it is expected that operation will begin early in 1919. It is reported that this line will be the official route from Chicago to Seattle.

The Pennsylvania Railroad has recently put in electric operation the Chestnut Hill division of the line running into Broad Street Station, practically duplicating the single-phase equipments on the Paoli division.

In street railway circles, the past year has been one of great hardships. The long continued cold weather, combined with the heavy falls of snow, pyramided the troubles and left the street railways in many cases almost as bad off as the steam railways.

A number of things combined to produce this result. First and foremost, is the general labor and material situation which makes it difficult, if not impossible, to secure good maintenance. In addition, the street railways have been working for several years under increasing financial difficulties, so that in many cases the equipments were in no condition to stand the extra strain. This led to a very unusual number of break-downs when the severe weather was encountered, and the long continued cold gave no opportunity to recuperate.

Another feature which contributed in a greater or less degree was the light weight campaign which has taken such a strong hold of all of the railways of the country. Ten years ago there was great need for this campaign. The standard street railway equipment consisted of nothing less than four 40-h.p. motors, which, being of the non-ventilated type, were comparatively heavy. Very few people had paid any particular attention

to the weight of railway equipments, either of motors, car bodies or trucks. Consequently, the weight per passenger was very high on practically all street railways. The idea became prevalent that it costs a railway 5 cents per lb. per annum to haul their equipments around, so that it has been the end and aim of practically every man having anything to do with the design of cars and equipments, to cut the weight. This has been done in a more or less scientific manner, but like all campaigns of this kind, the pendulum has swung past the limit in some respects. probably due to the fact that it is impossible to recognize the limit until it has been passed and trouble encountered. Fortunately, this has not been sufficiently widespread to do more than point to the limit.

Possibilities for trouble with light weight motors have been approached from four angles: First, open ventilation leading to the introduction of snow into the working parts of the motor, with resulting danger to insulation; second, higher armature speeds, leading to more rapid deterioration of armatures and wear of bearings; third, reduction of weight by increasing the stresses in material; fourth, the danger from overloading due to lack of sufficient thermal capacity to withstand abnormal loads.

The ventilated motor has come to stay. Its advantages are too many to give up because of the small amount of damage resulting from snow. The logical thing to do when there is danger from this, is to put tight covers over the motor openings in the winter time. The additional margin due to the lower ambient temperature in the winter will, in practically all cases, be sufficient to keep the motor temperature within safe limits.

Higher armature speeds should be approached with conservatism, taking care that the armature is sufficiently substantial to withstand the additional strains, and that bearings are of liberal dimensions, with adequate lubrication.

The use of high-grade steel is recommended, but is preferred as an additional factor of safety, rather than to get minimum weight. It must be remembered that it is not always possible to secure special grades of material for making repairs.

Trouble from overloading with the light weight motors has come as a very disagreeable surprise to operators. It is subject to careful analysis and the reasons are easily understood. The chief reason is that the limitations of the ventilated motor are not generally understood.

The trade has been educated to believe that the continuous

rating of the railway motor is the one that determines its service capacity; and that if its continuous rating is equal to the integrated loads in service, it will be ample to perform the work. This method of selecting motors was quite satisfactory with the old non-ventilated type of motors, where the motor had sufficient thermal capacity to absorb the losses generated in the short applications of heavy loads, without reaching abnormal temperatures. The modern highly ventilated motor, however, has relatively a very high continuous rating, as compared with the one-hour rating. The latter, as is well known, is really the gage of the thermal capacity of the motor and of its capacity for handling heavy intermittent loads. It will be seen at once that when the motor that is selected for its continuous rating is required to develop four times this load for a few minutes under some abnormal condition, a non-ventilated motor would be loaded to only about 60 per cent above its one-hour rating, while the highly ventilated motor would be loaded to two and one-half or three times its one-hour rating, resulting in a much greater rise in temperature. Where this abnormal load is applied at low speed, as is apt to be the case, the trouble is further accentuated, due to the decreased ventilation secured with fan-cooled motors.

The logic of this situation is simply that motors of the ventilated type for a given service will require a higher continuous rating than one of the non-ventilated type. Due regard must be taken to the capacity for short-time over loads in order to avoid reaching a dangerous temperature under abnormal conditions.

There seems to be a distinct tendency towards more conservative selection of equipments for street car service, since it has been definitely established that the cost of maintenance of motors which are worked beyond their capacity, added to the cost of unreliable service, is so high as to off-set any possible saving resulting from the lighter weight. This is also leading to a return to four-motor equipments, simply because of their greater reliability under abnormal conditions. It is hoped, however, that any return along these lines will be taken with the greatest care and with the maximum utilization of the experience that has been secured up to date. Having been at both extremes, it should now be possible to adopt a mean position which will give the very best results.

Great savings in energy can be effected without cutting the

weight of the equipment, by attention to improved methods of operation and, especially, by the further use of field control. It is hoped also that eventually it will be possible to make use of regenerative control for elevated and subway equipments, at least. This has worked out so satisfactorily on mountain grades that its extension to car equipment seems very desirable.

The activity of the Railway Committee in the past year has been at a low ebb, largely due to the fact that everyone in any way connected with transportation business, has been too busy to take up much committee work. It is hoped that another year may see an improvement in this respect, so that a constructive programme may be carried out.

N. W. STORER, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY

To the Board of Directors,

The Committee on Electrical Machinery submits the following report for the year of 1917-18.

At the first meeting of this new committee called in October to discuss the policy to be adopted, it was found that industrial conditions were such as to make full and regular meetings an impossibility, and that the activities of the Committee would therefore have to be restricted. There was, moreover, a feeling among those present that it would be difficult this year to secure papers from the men in the industry, and it was decided that the Committee limit its activities to the presentation of the subject of "Polyphase Commutator Motors," and to be prepared if necessary to present a critical review of "The Development of Turbo-Alternators". Contrary to our expectations, however, an unusual number of papers were presented for consideration and the Committee was kept busy sifting out the desirable material.

It was clear from the discussion of the three papers on "A-C. Commutator Motors" that little is known about the possibilities of the polyphase commutator machine, and that the available literature in English is so meagre and the methods of attack of the various writers so radically different that it would seem to be the duty of the Institute to find someone who knew the subject and who also knew how to present it to work up the existing material into a comprehensive treatise for publication in the TRANSACTIONS.

We believe that one of the principal functions of the technical committees is to keep the literature of their subject in good shape. The manufacturers will see to it that new developments are brought to the attention of the members. In line with this point of view the Committee has had a critical bibliography prepared on the subject of "Unbalanced Pull in Electrical Machinery" to be attached to a paper to be published in the PROCEEDINGS this summer.

In the April PROCEEDINGS there was published a paper by Mr. Lamme which created considerable discussion. Mr. Lamme had found that the graduates of our universities have not a clear

conception of the operation of single-phase induction motors, and he wrote a paper giving a method of treatment which he had found satisfactory for the graduate students of the Westinghouse Co. Such papers, although they do not present new material, are of special interest to the younger members of the Society and might well be presented under the auspices of the Educational Committee. From the discussion of Mr. Lamme's paper it was obvious that there were decided differences of opinion among teachers and practising engineers as to the relative advantages of vector diagrams, equivalent circuits and complex quantities from an educational point of view.

While the chief function of the technical committees is to secure papers in their respective fields, The Committee on Electrical Machinery considers that it might well be used as a clearing house for suggested changes to the Standardization Rules, a place where suggestions dealing with machinery might be thoroughly thrashed out before being submitted to the Standards Committee.

ALEXANDER GRAY, *Chairman.*

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors,

I beg to submit on behalf of the Lighting and Illumination Committee the following report for the year 1917-18.

It is with regret that the Committee finds it necessary to report that again this year it did not seem feasible to hold a session of the Institute for the consideration of papers on illumination. The Committee held a meeting early in the year and proposed the following subjects as suitable ones for presentation before the Institute:

(1) Intensive and Ornamental Street Lighting, as projected for a number of cities in the South and Southwest.

(2) A general discussion of Industrial Lighting Codes;

(3) A discussion of Standardized Methods of Lighting Cantonments, Aviation Fields, etc., provided the report of the I.E.S. Committee on this subject will be available for public presentation.

The Chairman was authorized to make an inquiry regarding the possiblitiy of securing papers on one or more of these subjects and report the results subsequently to the Committee, but following a canvass of the situation it was found impossible to arrange such a program. Consequently, any thought of requesting one of the sessions of the Institute to discuss papers on illumination had to be abandoned. When conditions once again become normal, it will be possible to provide interesting programs on this aspect of electrical engineering, but for the time being it would seem that the Committee can do no more than remain intact and wait.

A brief summary of progress in electric illumination during the past year is appended.

Progress in Electric Illumination

The general trend of practise for direct lighting is very decidedly toward units of low brightness. The extended use of the high-powered incandescent lamps has stimulated the appreciation of good diffusing devices which will give satisfactory light distribution but by their low brightness minimize glare. The enormous increase in commercial activities, particularly in those

lines which are connected with supplies for the Government, has made night work the rule and brought a realization of the importance of proper illumination from the standpoint both of the maintenance of quality and quantity in production and of the health and comfort of the worker. Progress toward this end is evidenced in the revision of industrial lighting codes in several states and by the appointment of a National Committee on Lighting to act as a sub-committee of the Advisory Commission-Council for National Defense for the preparation of suggested regulations to govern industrial lighting, which have subsequently been published in the form of a Code of Lighting by the Committee on Labor with a suggestion that the Code be put into effect in every state in the country.

War conditions have also brought about a more careful consideration of protective lighting and the best way to utilize it. Thus it has been found that in many cases inexpensive reflectors of the ordinary type may be used for lighting open spaces in and around a plant leaving the special flood lighting units for those locations requiring particular treatment. In many cases the use of a large number of properly shaded low-intensity units will avoid dangerous shadows better than high powered sources, even though the light flux from the latter is greater.

A sphere formerly considered impregnably held by the arc lamp has been finally invaded by the incandescent lamp. Motion picture projection work required light flux of extremely great intensity and the small area and high intrinsic brilliancy of the source of light in the arc has enabled it to meet the requirements in a way hard to duplicate. By using a mirror back of the filament and for a condensing lens one of the Fresnel type, it has been found possible to make an incandescent lamp which will give satisfactory results within a certain limited field of motion picture work.

The motion picture theatre has in itself become an arena in which unique lighting effects are being experimented with continuously. Thus in several cases, by the use of several circuits in each fixture, lamps of different colors may be lighted and thereby give a color tone to the whole illumination.

The action of the Government in attempting to save fuel by restricting its use for lighting purposes has shown in many localities the important part played by display lighting in maintaining the illumination of streets and sidewalks.

EDWARD P. HYDE, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors,

The Committee on Transmission and Distribution submits the following report for the year 1917-1918:

Experience during the preceding year indicated that the Committee was entirely too large. At the writer's suggestion the membership was reduced during the present year from 24 to 14 members. We now suggest that the Committees be further reduced to not exceeding 10 members. It would be an act of courtesy on the part of a member to decline the appointment when it is offered him if he can take no part in the work of the Committee.

It has been very difficult to make progress in the problems before us on account of every one, almost without exception, being employed on war work or very urgent duties contributing to the war. Some of the members who were most helpful in the past have gone into the government service and have not been able to continue the work which they started last year. Mr. W. D. Peaslee of the Oregon Agricultural College, who has been investigating the insulator problem from a chemical and microscopic standpoint, is now captain in the United States Army. Professor Harris J. Ryan, who is our strong right arm in insulator matters, is giving practically all of his time to government work. However, some progress has been made.

High-Tension Insulators

Last year we had papers by Messrs. Austin, Peaslee and Ryan pointing out clearly that progress in the design of high-tension insulators must provide for (1) reduction of porosity to the lowest possible limit, (2) joints designed to avoid cracking from expansion and contraction and (3) ample mechanical strength. Mr. G. I. Gilchrest has now made some very careful studies of insulators, both from the laboratory point of view where distribution of electric stresses was considered, and from the practical point of view where troubles and failures by operating companies under wide varieties of conditions were examined. We hope that insulator manufacturers will give careful con-

sideration to the design of insulators that Mr. Gilchrest has evolved. This paper shows very clearly where and why many of the old insulator designs failed. While it is quite probable that a perfect insulator will never be obtained, one has but to compare the insulator of today with that of five years ago to see that great progress has been made.

At least one large transmission company is very hopeful of the wood stick insulator for voltages up to 100,000. This insulator has been in successful service in the West for some years on 60,000-volt service. We had a paper listed for this meeting on the wood stick insulator by Mr. H. H. Cochrane, and hoped to get full details of it and record of the service it has given, but at the last moment Mr. Cochrane asked to have the matter go over until a later date, as some difficulties of impregnation had been encountered.

Lead Sheath Cables

Last year at the annual meeting a whole session of the Institute was devoted to the discussion of dielectric losses in cables. It was shown that cables insulated with mineral-base compounds had greatly reduced dielectric losses over those insulated with vegetable-base compounds. It was further shown that cable ratings under some conditions were more than doubled and on the average could be increased 20 or 30 per cent when the mineral-base insulating compounds are used. A start has been made in the matter of preparing specifications covering dielectric losses in cables. Engineers of some of the principal cable manufacturers have agreed to cooperate. Before such specifications can be formulated, at least two fundamental points must be considered and agreed upon:

First, a standard method of making tests must be established. Very few, even of the cable manufacturers, are equipped for measuring dielectric losses and probably no users of cables have facilities for properly making these measurements. Some very much simpler and more easily workable apparatus than is now available must be developed before commercial routine tests of this kind can be applied to the output of the cable factories. If a portable testing set for measuring dielectric loss were devised it could be used to test newly installed cable as well as to secure experimental data on old feeder cables under various conditions of age, temperature, charred insulation, etc.

Mr. S. M. Farmer recently presented a paper describing a method of determining power loss in three-conductor cables, the

loss being measured directly under three-phase conditions. It is hoped that the method he followed may be developed for factory tests.

In the second place, data must be collected showing the limits of the losses. To secure the data on losses is a difficult matter at this time when men as well as laboratories are occupied to full capacity on war work.

Additional information is being obtained on the characteristics of insulating compounds. For instance, it has been found that cable insulated with mineral-base compound will not withstand a high insulation test when cold, as the insulation resistance is much reduced under such conditions. All this complicates the problem of preparing specifications. Evidently much research work must yet be done, but as before stated, all work of this sort is much hampered by the war.

Mr. E. B. Meyer in a very practical paper gives the experience of one large distributing company in supplying high-voltage cable service to customers in cities where overhead wires are permissible and where the expense of conduit is not justified. The method while adopted as a war expedient has doubtless a wide field as a permanent method of installation. At least it can be used until the demand for energy in the particular locality is large enough to justify the expense of underground conduit.

Mr. W. H. Cole this year presents a paper as result of the experience of Edison Electric Illuminating Company of Boston with split-conductor cables and balanced-relay protection against interruptions caused by short circuit or grounds. It is very gratifying to note the success which has been attained notwithstanding the complexity of the system and the great care which must be exercised in installation of the equipment. All users of underground cables will appreciate Mr. Cole's work in this field.

Suggestions for the Future

For the future work of this Committee we would like to recommend that investigation of the insulator problem be continued. While at present porcelain seems to be the most available material, yet it is not beyond possibility that some other material such as fused quartz or even glass may be used.

Professor Ryan is now supervising some extensive tests at Leland Stanford, Jr. University of aging effect on insulators carried through a large number of temperature cycles corres-

ponding to the daily and seasonal variations experienced in practise. The result of these tests will be of great value.

Further investigation of the fused quartz insulator along the lines suggested last year by Mr. Peaslee will be well worth while as soon as some one can find time to do it.

As indicated above, the matter of dielectric loss in cables should receive most careful attention, both by cable manufacturers and users. Manufacturers must not be hampered by impractical and half-baked specifications. On the other hand, conservatism of manufacturers must not block the road to progress, and as soon as the laws governing dielectric loss can be determined and a practical method of measuring these losses developed, the most efficient cable will be called for and must be produced.

L. E. IMLAY, *Chairman*

ANNUAL REPORT OF ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors,

The Committee on Electrochemistry and Electrometallurgy submits the following report for the year 1917-1918:

It did not appear to be possible during the past year to secure any papers of interest on electrochemical and electrometallurgical subjects. One paper was submitted to the chairman of this committee for consideration which seemed more suitable for presentation to the American Electrochemical Society than to the Institute.

One of the most vitally important matters to Electrochemistry and Electrometallurgy is the development of water powers and consequently it appeared to be a subject which would properly come under the consideration of this Committee more particularly as regards the matter of legislation since this has probably done more than anything else to hamper their development. Correspondence with the Chairman of the Public Policy Committee brought out the fact that this subject has been referred to the standing Committee on Public Affairs of the Engineering Council.

Both as regards the question of securing papers for presentation to the Institute and as regards all matters pertaining to electrochemistry and electrometallurgy much more valuable results could probably be reached were some scheme worked out by which there could be cooperation of this committee and the American Electrochemical Society. With this end in view a suggestion was made to the Board of Directors of the American Electrochemical Society that it consider the question of approaching the A.I.E.E. with the object of forming a joint committee which would take care of subjects that are of common interest to the A.E.S. and the A.I.E.E.

Such a Committee would be in a good position to take care of various papers which may be of importance to both societies, to bring to the attention of both societies matters in which they have a common interest and finally to make arrangements when possible for joint meetings of the Societies.

At a meeting of the Board of the American Electrochemical Society held April 28, 1918, the Directors acting on this suggestion appointed a Committee "to co-operate with" this Committee. The Committee appointed by the American Electrochemical Society is as follows:—

Dr. Colin G. Fink, Chile Exploration Co., 202nd Street and Tenth Ave., New York.

Howard C. Parmelee, Metallurgical & Chemical Engineering, McGraw-Hill Co., Inc., 36th St., and Tenth Ave., New York.

C. G. Schluederberg, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

A letter was received from Professor Karapetoff of Cornell University describing the Research Section of the *Electrical World* which he is conducting and asking for suggestions from this committee. The members of the committee were notified of Professor Karapetoff's request.

Suggestion for Future Activities

In view of the action of the Board of the American Electrochemical Society it is suggested that this Committee should communicate with that appointed by the A.E.S. with the object of discussing the feasibility of cooperation for the benefit of the Institute and the Society. This matter, however, should be left to the Committee for 1918-1919 as the term of office of the present committee is so near an end.

As regards hydroelectric power there is no more important matter for consideration by this Committee and if some scheme for cooperation with the American Electrochemical Society is developed it should be specially studied by this Committee.

Electrochemistry and Electrometallurgy 1917-1918

The entrance of the United States into the Great War had the effect of stimulating enormously electrochemical and electrometallurgical processes.

As regards electrolytic work probably one of the most highly stimulated manufactures is that for the production of chlorine; but the most noticeable stimulation is found in electrometallurgical work.

For years there has been much experimental work carried out in electric furnaces for the melting of non-ferrous metals like brass, bronzes, etc., but no really satisfactory furnace was developed. Then, as one of the consequences of the Great War

graphite crucibles became scarce and expensive with the result that renewed attempts were made on the electric furnace with the result that during the past year various furnaces have been designed of which some show distinct promise of meeting with ultimate success.

There has also during the past year been an enormous increase in the production of electric furnace ferro alloys of all sorts particularly ferrosilicon and ferromanganese. The latter had been produced in electric furnaces before the war but could not compete with the blast furnace.

There has also been a considerable increase in the use of electric steel furnaces and this probably would have been much greater were it not for the great difficulty of getting a sufficient supply of carbon electrodes.

The most interesting electrochemical development of the year, however, is the construction of the cyanamid nitrogen-fixation plant at Muscle Shoals, Ala. Two unique features in this plant are the use of steam for the generation of the electric energy and the use of 60-cycle current for the calcium carbide furnaces. These features will give interesting subjects for study; on the one hand the energy item in the cost of making ammonium nitrate by the cyanamid process and on the other the power factor of carbide furnaces with high-frequency current.

FRANCIS A. J. FITZGERALD, *Chairman*

ANNUAL REPORT OF INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

In presenting our annual report, your Committee on Industrial and Domestic Power desires first to call attention to two of our members who are now in Military Service in the war of 1917, viz: our chairman, Capt. E. H. Martindale now in active service in France, and Lieut. (j. g.) A. M. MacCutcheon, U. S. Navy, also in active service, U. S. S. Louisiana.

We honor their action, and have missed their advice in our work this year, particularly that of our Chairman.

Others of our Committee from whom we have not heard may be similarly employed, and in that event their names should be joined with the above.

During the two years beginning in August, 1914, under the leadership of Mr. David B. Rushmore, work of the committee began to be centered on a study of our activity by industries. This thought was carried out in several meetings of the Institute.

In the Fall of 1916 under Capt. Martindale, it was decided that the major portion of our activities should be devoted to this study. To show the value thereof, it was decided to make an investigation of the machinery, motors, controllers and accessories involved in each process in three industries or parts thereof, and reach conclusions from this as to next procedure. On this basis, the whole committee was divided into three subcommittees headed by Lieut. A. M. MacCutcheon, to investigate a portion of the Metal Working Industry; Mr. R. B. Williamson, to investigate the Cement Industry; Mr. R. M. Goodwillie, to investigate the Passenger Elevator Industry.

The committees have worked hard, collecting much data, but without having their work completed by the end of the last term.

In the Fall of 1917, by reason of his military service, Capt. Martindale was obliged to relinquish active handling of his work as Chairman, the position of Vice Chairman was created, and the writer appointed to the position. The executive duties have been carried out in the way it was believed Capt. Martindale desired.

On account of the absence of our Chairman, and also because

of the many immediate and pressing duties of service of all individually, in connection with the war, it was deemed advisable to maintain our plan just as outlined one year previously, center our thought on its progress and application, and not attempt the preparation or presentation of papers to the Meetings and Papers Committee, and this course has been followed.

The work of the sub-committees in collecting data is well in hand. In the examination of the cement industry for example, Mr. Williamson's committee has first tabulated all the processes involved from the point when the raw product is brought to the cement plant up to the point when the finished product is delivered, barrelled and ready for shipment. Each process may be considered a movement of some kind. Where electric power is applied to this movement, the preferred type of motor, control, and accessories and alternates are indicated. In a similar way, Mr. MacCutcheon's Committee has examined a portion of the metal working industry. Thus mainly the principal data have been collected and compiled. The work has given our whole committee opportunity to thoroughly consider the good features and objections, and in this connection two immediate difficulties; viz: to know when the work is done, and how to present it to the Institute, are not solved. These two points with others brought out by this investigation have been carefully canvassed by the whole committee, and the discussion and recommendations which immediately follow are based upon a majority vote also of the whole committee, with no dissensions.

The work has been carried far enough so that each feels sure that if the data could be gotten together for all processes, kept correct and made available, these would be of immense benefit to the industry as a whole, and particularly at this time. The collecting and preparing of the data presents very serious difficulties as follows:

1. *Tabulation of what industries are within the scope of our committee's activities.* The scope of this committee overlaps that of the committees on mines, steel mills, electrochemical work, marine work, and perhaps others. It is recommended that its scope be plainly defined.

2. *What constitutes an industry.* A tabulation of industries can be prepared by our committee. Such tabulation is bound to be at variance in thought from any tabulation prepared by another committee. We believe that a tabulation should be prepared for the electrical industry, as a whole, and that this can-

not be done short of a careful study of the subject, and the interrelated processes. As Mr. Goodwillie of our committee states, the real essential element is the load characteristics of the machine itself from the starting, accelerating, running, slowdown and stopping conditions which in turn compel considerations of starting friction, inertia, running frictions and load characteristics. It is probably a matter of consideration by some special technical committee, and not by a technical committee devoted to some special work like our own unless the work as a whole for some reason be specially assigned to us. Much study has been put already on this subject by allied associations, and by private enterprises. Much of the benefit of this study can be secured very likely. The list of industries and processes involved will be subject to constant modification as the work proceeds. A plan for caring for these changes is essential.

3. *Magnitude of the work involved in the study.* It is clearly beyond the power of our committee to perform this work alone in any measurable period of time. External assistance is vital and some plan to secure the assistance necessary. Even if it were completed, it is doubtful if the committee alone as it stands could keep it completed.

4. *Preparation and filing for permanent record.* It is apparent that the proper place for recording this data is not in the PROCEEDINGS of the Institute. The expense of doing it in this way would be prohibitive. It would lack availability for use and change as required. A plan for permanently caring for the data should be perfected as will be separately discussed.

5. *Authority for work.* In the execution of this work, time and expense beyond the limits of the committee would be bound to accrue. Definite authorization from the Board of Directors to proceed is necessary, and with it the statement that the work would be continued from year to year, for it is obvious that to start work and continue for a year or so only to change or abandon would involve needless loss of time, and discouragement.

6. The ethics of the study from the standpoint of the professional engineer, from that of patent rights and from the unconscious publicity given to private *enterprise is comparing correct process attainment.* This is considered one of the most difficult phases to cover correctly. As Mr. Dudley states, it is believed that with the exercise of tact with each case, that difficulty should not be encountered. Thus far in its work the sub-committees have been able to steer clear of this as a diffi-

culty. The work must be undertaken strictly from a technical standpoint.

7. *Conflict with other technical committees in other associations.* The whole reason for this work is to avoid duplication of effort. The American Institute of Electrical Engineers is the recognized association devoted to our art from an unbiased technical and professional standpoint. If this work be undertaken by us with carefully prepared plan and be definitely endorsed, the need for similar work by other bodies such as the N.E.L.A., A.I. & S.E.E., etc., should disappear. The committee is confident that help from them will be fully available and given. There should be no conflict with other committees in the Institute. For this, the management must care.

8. *Miscellaneous Difficulties.* Undoubtedly these exist but it is believed that what has been previously stated will cover the major difficulties, and that the minor ones will settle themselves as they arise.

The study which our committee has thus far put on the work has convinced us that it is broader in scope than that of our committee. If it is right to investigate the large group of industrial applications with which our committee is concerned, why should not the whole field be covered. Consideration of this thought has been inevitable, and is crystallized into a number of suggestions or recommendations which are presented as the opinion of a majority of your committee.

We believe that there should be instituted in the office of the Secretary of the Institute in New York City, and under his complete charge, a file of industries using electrical power, the processes involved in each industry, the movements involved in each process, and the electrical apparatus, which is recommended for each movement. We believe that this recommendation should be strictly from the technical standpoint, the apparatus to be described so that it will be technically understood. We believe that the file should be kept up by the machinery of the Secretary's office, and in such form as will make it most available to all who should have access to it. It is assumed that this file should be in card form, and in this connection, Messrs. Weichsel and Dudley of our committee suggest the compilation of pamphlets or a hand book from these files in the belief that they would have ready sale.

It is further suggested by the committee that as Institute Sections are able to take care of them properly, copies of the

files be placed on record at Section Headquarters all under control of the Secretary of the Institute.

Manifestly the data for these files can be secured only through the cooperation of a large number of members, and one perhaps the hardest study would be to enlist this cooperation. Fundamentally, the plan must have the absolute endorsement of the Board of Directors, and then it should be planned to secure this large support. Our committee feels that either the duties of the Industrial and Domestic Power Committee should be broadened to assume the work, or (and this is favored by a majority of the committee) that a new technical committee should be formed to take care of the job. No technical committee alone can get the data unless it have broad powers, and a membership large enough to reach into the activity of the Nation industrially and geographically, in a very comprehensive manner indeed. As a committee, we feel very sure that in some way the individual member must be reached; must be encouraged to furnish data, with proper recognition. We feel that if this action on the part of the individual member in being of service to the Institute and to the industry causes him to recognize his true relation to the Institute of giving to it rather than of receiving from it, it should be of the greatest value.

We further respectfully suggest that in the event of establishing a committee to secure this data that it should work in very close conjunction with the Secretary of the Institute, and with the other technical committees.

Briefly we suggest, that the new committee plan and secure the data, the existing committees censor the data and advise with the new committee, and that the Secretary's office compile and carry the data.

Such a file should be the truest permanent record possible of the application of electrical power and its progress. It should tend to harmonize policies of application of electrical power; not to standardize details. It should prevent an enormous amount of duplication of effort, not only in association work, but in private enterprise as time goes on; should tend to pool, for the good of all, data which are today largely open to all and yet not collected so as to be of mutual benefit.

Your committee sincerely hopes that these views will have your consideration.

We desire to comment on another matter. Our committee's scope today includes domestic as well as industrial power. It

has seemed to some that the domestic end of our activity should be taken from us and given to some other committee whose work is more nearly allied thereto. On a poll of opinion seven were in favor of this action, five did not reply and two were in opposition thereto.

The past year has been crowded with work which your committee feels covers very largely application of earlier progress than it does the invention or perfection of new work. Constant effort to keep existing apparatus operating at the maximum rather than to perfect or develop more efficient operation has marked this period when our Nation for the first time in twenty years was plunged in war. This holds particularly for the production of textiles and clothing generally, steel, coal, and in matters affecting transportation.

In the Chicago territory there has been installed during the past year a 7000-h.p. induction motor having a maximum torque equivalent to 30,000 h.p., which so far as normal rating and maximum torque is concerned is the largest industrial motor which we believe has ever been built.

In the textile industry, the matter of individual drive of spinning frames is progressing, many hundred frames having been equipped during the past year.

The first electrically propelled battleship has been launched during the past year. The application is new.

The year marks the first application of small electric generators, air driven, to air-planes. This extends the use of industrial power to the air along with gasoline engines.

In summary, your committee would value quite specific instructions as to its duty, and scope, and shall be much interested to note any action that may be taken on its recommendations.

A. G. Pierce, *Vice Chairman*

ANNUAL REPORT OF COMMITTEE ON ELECTROPHYSICS

To the Board of Directors,

The Committee on Electrophysics submits the following report for the year 1917-18:

During the year two sessions of the Institute have been devoted to electrophysics under the auspices of this committee as follows:

On the evening of Nov. 9, 1917, Mr. Chester W. Rice presented a paper entitled "An Experimental Method of Obtaining the Solution of Electrostatic Problems with Notes on High-Voltage Bushing Design" which forms a very valuable contribution to our knowledge of the electrostatic field. The paper, published in the November PROCEEDINGS, has a direct bearing on the problem of insulator design and the mathematical appendix gives the solution of two electrostatic problems, the usefulness of which extends beyond the immediate problem of design.

On the evening of Feb. 15, 1918, at the Midwinter Convention, the Institute listened to a most interesting lecture by Dr. A. C. Crehore on "Some Applications of the Electromagnetic Theory of Matter"; the lecture is published in the April, 1918, PROCEEDINGS. Great progress is being made in the theory of atomic structure, to which Dr. Crehore's own work has formed no small contribution, and the placing of these most recent advances by Dr. Crehore before the Institute in non-mathematical form was indeed most opportune. It will be recalled that a similar lecture on "Modern Physics" was given by Prof. R. A. Millikan at the Midwinter Convention in 1917.

The committee has felt that the cooperation and mutual understanding between physicists and engineers, as pointed out in the report of the committee for last year, are of the utmost importance. The close relation has been maintained between this committee and the Technical Physics Committee of the Physical Society. It is believed that joint meetings of the Institute and the Physical Society from time to time should be continued as in the past and such a meeting is planned for the Philadelphia session next October.

The maintenance of a supply of trained physicists and en-

gineers, at all times important, the committee has considered to be particularly important in time of war in order to meet the needs of governmental and industrial service. This matter was discussed on April 11th at a meeting of the committee held in New York for the purpose, and the conclusion was reached that to insure the continuous output of technically trained men from the universities and technical schools of the United States it was most important that steps be taken to provide for the maintenance of adequate teaching staffs, which are in danger of being depleted through the application of the draft and through voluntary enlistment. This matter was brought to the attention of your Board on April 12 and it was the unanimous opinion of the members of the board that immediate action was necessary.

FREDERICK BEDELL, *Chairman*

ANNUAL REPORT OF PROTECTIVE DEVICES COMMITTEE

To the Board of Directors,

The Committee on Protective Devices submits the following report for the year 1917-18.

On account of the decision of a number of companies to cancel all committee work during the period of the war the work of this Committee has been carried on largely by correspondence. One of the members of the Committee entered the military service of the country and a number of others have had increased duties on account of their assistants entering the Army or Navy. These changes have served to considerably retard the work of the Committee.

During the year the Committee has issued the questionnaire on relays, referred to in the previous report, to about fifty of the leading operating companies in the country. Not enough replies have been received from the questionnaire up to the present writing to permit of any summary or resume from the inquiries. It is recommended that the work on the questionnaire be pushed to a conclusion during the coming year.

Several members of the Committee have called attention to the proposed interconnection of transmission and distribution systems throughout the country as a measure of economy and fuel saving, and it is recommended that the Committee investigate this subject in particular to determine what protective features are necessary in such tie lines for the purpose of ensuring continuity of service and stability of operation.

Other subjects which might be taken up by the Committee are the following:

Relays on generators, transformers, synchronous converters, etc.

Lightning arresters for transmission lines.

Preparation of definite recommendaions regarding the standardization of relay nomenclature and rating of circuit breakers.

Detrimental effect of power reactors.

D. W. Roper, *Chairman*

ANNUAL REPORT OF COMMITTEE ON APPLICATION OF ELECTRICITY TO MINES

To the Board of Directors,

I give below a brief report of the activities of the Committee on the Application of Electricity to Mines.

Owing to the resignation of Mr. H. H. Clark, the writer was appointed as chairman of the Committee on the Application of Electricity to Mines. Pressure of other work has prevented devoting as much time to Committee work as I would like to have given. No meetings of the Committee as a whole have been held, but the chairman has arranged with the Meetings and Papers Committee to take charge of the October meeting which will be held in Philadelphia. We propose at this meeting to present three papers bearing on the subject of the Application of Electric Power to Coal Mining. We also hope at this time to have an informal talk by a member of the Fuel Administration Bureau at the dinner preceding the evening session.

The American Institute of Mining Engineers has a committee on the Application of Electricity to Mining and I do not believe that the best results can be accomplished by the independent action of these committees in two entirely independent societies; and I would suggest as a future activity for the Committee on the Application of Electricity to Mines in the A.I.E.E. that of making some working arrangements with the corresponding committee of the American Institute of Mining Engineers whereby both national societies may get the benefit of the work of the Committees of each. Possibly this can be accomplished by making the annual meeting of these two committees joint meetings, the A.I.E.E. joining with the mining engineers at one meeting and vice versa for the next, the papers being published by both societies.

K. A. Pauly, *Chairman*

ANNUAL REPORT OF INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors,

This committee was appointed during the year of 1917 with the general purpose of promoting interest through the presentation of papers and discussion along the lines covered by the name of the committee. The field for the consideration of instruments and for measurements had not heretofore been made a matter of separate committee work and discussion heretofore has been coupled with the consideration given papers in which both instruments and measurements were incidental to another subject.

Meetings of the committee were held and arrangements were made for the Meetings and Papers Committee to assign one session of the mid-winter convention in February for the presentation of papers on measurements. One afternoon session was assigned and four papers were presented. Attendance at the session and the discussion indicated sufficient interest to justify the continuance of the Committee's activities along these lines.

Two impressions were obtained from the papers and discussion, which while obvious, seem of sufficient interest to report. Two of the papers presented dealt with the investigation and development of a substitute for the standard cell for certain classes of work. This substitute was a thermocouple, eliminating entirely the use of the chemicals required by the standard cell.

One of the papers presented dealt with the measurement of dielectric losses in cables, and while it was a single paper only, it was in reality the latest of a series of several papers on the same subject presented before the Institute in the last few months. The discussion indicated a desire or a necessity for the standardization of methods of measuring the very small energy losses in the dielectric and the specifications covering the purchase and acceptance tests of cables.

No matters of nomenclature or standardization arose requiring the attention of the committee on standards. No matters of policy or coordination with other committees arose requiring the attention or action on the part of the Board of Directors.

Commenting briefly on the progress of the industry falling within the scope of the committee, it can be stated that as might be expected no new development work along purely commercial lines is being undertaken by the manufacturers of apparatus at this time. Developments have undoubtedly taken place, however, in apparatus along the lines of military and naval activity, such as radio and other signal apparatus which will without doubt furnish valuable and interesting material when available at some time in the future.

S. G. RHODES, *Chairman*

ANNUAL REPORT OF EDUCATIONAL COMMITTEE

To the Board of Directors,

Lack of time on the part of the members of the Educational Committee is responsible for the regrettable fact that only one meeting could be arranged during the year.

At this meeting it was decided that two papers, if possible, be prepared; one dealing with electrical engineering education given in colleges at the present time, and the other giving a summary of the educational facilities offered by the large manufacturing and power companies.

The second paper was not completed, the first is given in the synopsis prepared by the Chairman and presented herewith.

The Committee recommends that the particular scope suggested above be considered as an important part of the duties of future Committees so that the members may be able to keep in touch with educational methods and ideas, and assist the leaders of education in shaping their policies.

Synopsis of Electrical Engineering Education given in American Colleges, 1917

The importance of engineering and engineering education, always recognized in this country, has never been more fully realized than today. The engineer is called upon not only to supervise engineering work and to design machines but is more and more involved in administrative work so that—large as his task has been—it will be greater in the future.

It seemed, therefore, opportune to the Educational Committee that a report be presented to the Institute giving a brief summary of the present status of electrical engineering education in this country, as shown by the latest catalogues. A questionnaire was mailed to a large number of colleges giving four-year courses in electrical engineering and the returns are tabulated below.

The studies were divided in ten groups as shown and the figures given represent the percentage of time out of the entire four-year curriculum devoted to each group.

Group 1, includes strictly electrical engineering subjects.

Group 2, includes English, English literature, rhetoric, etc.

Group 3, Foreign languages.

Group 4, Mathematics, excluding descriptive geometry.

Group 5, Physics, including elementary mechanics.

Group 6, Chemistry.

Group 7, General engineering subjects. This heading includes such phases of the engineering curriculum as are usually given to all branches of engineering students. It includes, for instance, drawing, surveying, descriptive geometry, advanced mechanics, applied mechanics, thermodynamics, etc.

Group 8, General subjects. These include prescribed courses in history, law, economics, etc.

Group 9, Electives, technical and general.

Group 10, Physical training, physiology, hygiene, military work, etc.

Table I gives in alphabetical order the colleges which responded. At the bottom of the tabulations are given the total averages.* This should, therefore, indicate what weight is given to the various groups in the average college at the present time.

It is interesting to note that general engineering subjects, that is, subjects which are essentially common to all classes of engineering students, cover 31 per cent of the entire time, and the purely electrical engineering subjects are given 21.6 per cent. Thus the engineering topics occupy approximately one-half of the entire time of the students.

English, including literature and rhetoric, is given only 5.5 per cent, foreign languages 3.2 per cent, general subjects such as economics, history, law, etc., are given 3.4 per cent. Science subjects, mathematics, physics, and chemistry are given 27 per cent of the total time.

Tables II, III, IV, V, VI, emphasize particular studies. So for instance, Table II gives approximately one-half of the entire list in accordance with the prominence of purely electrical studies. Norwich University leads, it devoted 33.5 per cent, or roughly, 50 per cent more than the average time to that subject.

Table III emphasizes English studies. The Agricultural and Mechanical College of Texas leads with 12.5 per cent or more than twice as much as the average. Some leading colleges give no instruction in English. These may, however, to some extent, take care of this feature in a more rigid entrance examination.

*Since deducing the average a couple of colleges have been added which may have very slightly modified the actual value.

Table IV emphasizes foreign languages. It is of interest to note that a large percentage of the colleges give no foreign language course at all.

The average time given to electives is 3.9 per cent which seems rather low. There are, however, a number of institutions which permit of a wide choice. Tufts leads with a percentage of 21.

The preparation of these tables is intended to facilitate a discussion on the very vital subject of electrical engineering education as given in colleges. It is not intended that a "standard" course should be evolved—that would indeed be unfortunate—but it does look as if almost all colleges could to advantage increase some of the non-technical courses, such as English, economics and foreign languages. Some colleges should perhaps adopt a course along scientific lines, neglecting somewhat intruction strictly in engineering and concentrating on mathematics, physics and chemistry. Others may also lay less stress on their engineering subjects and devote more time to English, foreign languages and to problems of economics. The third class, perhaps the largest, should do essentially what the average college is doing today.

ERNEST J. BERG, *Chairman*

TABLE I

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Agricultural and Mechanical College of Texas	28.8	13.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Armour Institute of Technology	28.6	2.4	00.0	7.2	8.8	6.4	35.0*	6.8	17.9†	4.8
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Carnegie Institute	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Case School	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Clemson Agricultural College	13.4	9.7	0.0	9.9	8.0	4.2	34.6	6.9	0.0	13.3
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
Cornell University	20.6	0.0	0.0	7.8	9.0	7.0	46.5	3.9	2.6	2.6
Drexel Institute	20.5	7.0	0.0	8.0	10.0	8.0	31.0	4.5	0.0	3.0
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.2	23.7	2.7	7.5	3.6
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7.0	26.6	4.7	2.3	7.8
Lafayette College	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Massachusetts Institute of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
Mississippi Agricultural & Mechanical College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Montana State College of Agriculture and Mechanic Arts	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
New Hampshire College	21.0	4.1	0.0	12.2	10.1	6.1	35.7	0.0*	5.4	5.4
New Mexico College of Agriculture & Mechanics Arts	17.8	2.8	3.8	9.1	8.2	5.8	42.7	2.8	1.0	5.8
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Norwich University	33.5	3.9	7.9	0.9	9.9	3.9	13.2	5.9	1.3	10.5
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
Oklahoma Agriculture and Mechanical College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Oregon Agricultural College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
Pennsylvania State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Polytechnic Institute of Brooklyn	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
Rensselaer Polytechnic Institute	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
State Agricultural College of Colorado	25.5	5.0	0.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	20.7	4.7	3.1	3.7
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
State University of Kentucky	12.7	6.2	0.0	10.6	7.8	5.3	53.2	0.0	0.0	4.2

* Armour—Includes shop work

†New Hampshire College—General subjects included in electives

TABLE I—Continued

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Stevens Institute of Technology	8.1	3.9	4.2	5.6	6.3	7.5	53.9	2.1	0.0	8.4
Syracuse	25.0	4.0	0.0	16.3	10.2	7.5	35.0	2.0	0.0	0.0
Throop College of Technology	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Tulane University of Louisiana	12.3	6.4	0.0	12.8	7.9	8.9	48.4*	0.0	2.0	1.3
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
University of Alabama	18.4	3.2	0.0	13.2	10.5	5.3	34.7	4.7	5.8	4.2
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of California	15.0	0.0	0.0	11.0	12.0	7.0	40.0	0.0	8.0	7.0
University of Cincinnati	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
University of Detroit	16.7	3.8	4.2	13.8	18.6	10.5	30.5	2.9	0.0	0.0
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9†	0.0	0.0	2.7
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
University of Kansas	17.0	7.0	7.0	12.0	11.0	9.0	28.0	5.0	2.0	2.0
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
University of Michigan	20.7	4.3	11.4‡	12.9	10.0	5.7	25.0	0.0	10.0	0.0
University of Minnesota	20.6	4.1	0.0	13.7	8.2	4.1	33.3	4.1	8.9	2.8
University of Missouri (Rolla)	18.6	6.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
University of Missouri (Columbia)	16.16	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
University of New Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....#
University of North Dakota	21.0	7.0	5.5	11.0	9.5	8.0	33.0	2.0	1.5	1.5
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
University of Oklahoma	25.2	6.3	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
University of Pennsylvania	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
University of Tennessee	10.4	6.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
University of Utah	14.6	4.6	0.0	12.0	7.7	9.2	41.5	4.5	0.0	1.5
University of Vermont	27.6	4.3	6.5	8.6	5.4	5.4	27.0	2.2	2.2	10.8
University of Virginia	23.4	0.0	0.0	11.8	9.0	5.9	44.0	0.0	0.0	5.9
University of Washington	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.3	6.3
University of Wisconsin	21.0	4.0	0.0	11.9	7.9	4.0	31.5	4.0	11.8	4.0
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6.3	38.1	0.0	8.7	3.2
Washington University	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
University of West Virginia	21.0	6.0	0.0	11.0	7.0	7.0	44.0	0.0	0.0	4.0
Worcester Polytechnic Institute	20.9	4.8	7.7	11.4	8.8	6.8	32.1	5.3	0.0	2.2
AVERAGE	21.6	5.5	3.2	11.8	9.1	6.3	31.1	3.4	3.9	4.1

*Tulane—Includes shop work
†Florida—Includes shop work
‡Michigan—Includes cultural electives
New Mexico—Freshman gymnasium not included in percentages.

TABLE II.
ELECTRICAL ENGINEERING

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Norwich University	33.5	3.9	7.9	9.9	9.9	3.9	13.2	5.9	1.3	10.5
Montana State College of Agriculture and Mechanic Art	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
Agric. & Mech. College of Texas	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Armour Institute	28.6	2.4	00.0	7.2	8.8	6.4	35.0	6.8	17.9	4.8
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
Carnegie Institute	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Polytechnic Institute of Brooklyn	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
University of Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....
Lafayette College	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
University of Vermont	27.6	4.3	6.5	8.6	5.4	5.4	27.0	2.2	2.2	10.8
State College of Washington	27.2	6.2	0.0	12.7	6.2	6.8	29.7	4.7	3.1	3.7
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
University of Oklahoma	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
Univ. of Pennsylvania	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
State University of Colorado	25.5	5.0	0.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
Case School	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Oklahoma Agric. and Mechanical College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Oregon Agric. College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	2.0
Massachusetts Inst. of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of Virginia	23.4	0.0	0.0	11.8	9.0	5.9	44.0	0.0	0.0	5.9
Rensselaer Polytechnic Institute	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
Univ. of Cincinnati	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
Univ. of Washington	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.2	6.2
Washington University	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
Pennsylvania State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7

(33 Colleges in all)

TABLE III.

ENGLISH

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Agricultural and Mechanical College of Texas	28.8	12.5	00.0	13.3	4.5	4.5	26.7	3.3	1.7	6.7
Mississippi Agric. and Mechanical College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Clemson College	13.4	9.7	0.0	9.7	8.0	4.2	34.6	6.9	0.0	13.3
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	4.2
Throop College	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6.0	29.4	5.3	0.0	6.6
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
Penn. State College	22.0	7.5	7.5	12.5	7.5	5.5	20.0	7.5	0.0	0.0
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0.0
Drexel Institute	20.5	7.0	0.0	8.0	10.0	8.0	31.0	4.5	0.0	3.0
University of Kansas	17.0	7.0	7.0	12.0	11.0	9.0	28.0	5.0	2.0	2.0
Univ. of Tennessee	10.4	6.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
University of Oklahoma	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
Polyt. of Brooklyn	28.0	6.6	3.7	10.7	9.4	7.0	28.0	2.8	1.9	1.9
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Tulane University	12.3	6.4	0.0	12.8	7.9	8.9	48.4	0.0	2.0	1.3
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Missouri	18.6	6.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
State University of Kentucky	12.7	6.2	0.0	10.6	7.8	5.3	53.2	0.0	0.0	4.2
State College of Washington	27.2	6.2	0.0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
Bucknell University	23.0	6.0	5.5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
Univ. of West Virginia	21.0	6.0	0.0	11.0	7.0	7.0	44.0	0.0	0.0	4.0
Oklahoma Agric. and Mech. College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Rensselaer Polytechnic Institute	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1.3
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7.0	26.6	4.7	2.3	7.8
University of Cincinnati	22.4	5.3	5.0	8.0	10.8	10.4	34.2	2.3	0.0	1.6
Massachusetts Institute of Technology	23.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9	0.0	0.0	2.7

TABLE IV.

FOREIGN LANGUAGES

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
University of Michigan	20.7	4.3	11.4	12.9	10.0	5.7	25.0	0.0	10.0	0.0
Union college	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2.8
University of Florida	13.4	5.2	7.9	11.8	10.5	6.6	41.9	0.0	0.0	2.7
Norwich University	33.5	3.9	7.9	9.9	9.9	3.9	13.2	5.9	1.3	10.5
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2.3
Lehigh University	28.6	6.5	7.8	11.0	13.0	6.5	16.2	1.9	1.9	6.5
Worcester Polytechnic	20.9	4.8	7.7	11.4	8.8	6.8	32.1	5.3	0.0	2.2
Pennsylvania State College	22.0	7.5	7.5	12.5	7.5	5.5	30.0	7.5	0.0	0.0
Throop College	19.5	8.4	7.0	8.2	10.0	6.0	30.0	6.7	0.0	4.2
University of Kansas	17.0	7.0	7.0	12.2	11.0	9.0	28.0	5.0	2.0	2.0
University of Tennessee	10.4	5.9	6.9	11.6	4.6	5.2	39.4	0.0	3.4	11.6
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
University of Vermont	27.6	4.3	6.5	8 6	5.4	5.4	27.0	2.2	2.2	10.8
Case School	25.2	6.4	6.4	10.4	9.1	5.5	25.9	1.3	1.3	8.5
Washington University	22.3	5.0	6.3	11.3	8.8	5.4	32.7	4.4	1.3	2.5
Rensselaer Polyt. Inst.	23.3	5.9	6.2	13.0	9.2	6.2	33.8	1.1	0.0	1 3
Georgia School of Technology	21.1	8.5	6.1	11.0	10.8	6.0	25.3	2.5	0.0	8.7
University of Notre Dame	29.6	4.7	6.0	11.8	10.2	0.0	22.0	10.7	1.2	3.6
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
Maryland State College of Agriculture	25.7	10.8	5.6	10.5	4.5	4.5	21.6	9.3	0.0	7.5
Bucknell University	23.0	6.0	5 5	9.0	12.6	9.4	23.0	8.0	3.0	0.5
University of North Dakota	21 0	7.0	5.5	11.0	9.5	8.0	33 0	2 0	1.5	1.5
North Carolina State College	19.4	7.7	5.3	11 9	8.5	6.0	29.4	5.3	0.0	6.6
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
University of Cincinnati	22.4	5.3	8.0	8 0	10.8	10.4	34.2	2 3	0.0	1.6
Carnegie Institute	28.3	2.9	4.6	8.0	9.6	4.6	30.0	3.3	0.0	8.7
Univ. of Pennsylvania	26.0	3.5	4.6	8.0	7.6	6.1	33.5	1.5	0.0	9.2
George Washington University	25.7	4.3	4.3	8.6	7.1	8.6	33.6	2.8	5.0	0.0
Stevens Institute	8.1	3.9	4.2	5.6	6.3	7.5	53.9	2.1	0.0	8.4
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of Detroit	16.7	3.8	4.2	13 8	18.6	10.5	39.5	2.9	0 0	0.0
Tufts College	24.0	4.0	4.0	9 0	6.0	6.0	22.0	2.0	21.0	2.0
Lafayette College	27.7	3.9	3.9	11.0	6.4	6.4	24.5	11.0	3.9	1.3
Pennsylvania College	15.0	9.2	3.9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
New Mexico College of Agric. and Mech Art.	17.8	2.8	3 8	9.1	8.2	5.8	42.7	2.8	1.0	5.8

TABLE V.

MATHEMATICS EXC. DESCRIPTIVE

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Colorado College	26.5	4.7	7.8	20.3	13.3	4.7	15.7	4.7	0.0	2 3
Mississippi Agric. and Mech. College	15.3	10.9	0.0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Union College	22.8	8.6	8.6	17.2	5.6	4.5	20.0	5.5	4.4	2 8
Syracuse University	25.0	4.0	0.0	16.3	10.2	7.5	35.0	2.0	0.0	0.0
University of Missouri (Columbia)	16.1	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Idaho	21.5	7.8	0.0	15.0	7.8	5.2	36.2	1.3	0.0	5.2
State University of Iowa	26.9	6.6	0.0	14.7	8.8	5.9	28.3	6.6	0.0	2.2
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
University of Texas	20.7	9.5	0.0	14.3	15.8	8.0	27.0	4.7	0.0	0.0
University of Colorado	30.0	7.1	0.0	14.3	7.1	7.1	31.4	3.0	0.0	0 0
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6.3	38.1	0.0	8.7	3.2
University of Illinois	27.7	4.3	5.7	14.0	9.2	5.7	25.6	8.5	0.0	4.3
Univ. of New Mexico	28.0	4.2	0.0	14.0	9.1	5.6	25.9	2.1	11.2	....*
University of Detroit	16.7	3.8	4.2	13.8	18.6	10.5	39.5	2.9	0.0	0 0
Univ. of Minnesota	20.6	4.1	0.0	13.7	8.2	4 1	33.3	4.1	8.9	2.8
University of Oklahoma	26.2	6.8	0.0	13.7	14.3	5.3	29.0	3.4	0.0	1.3
University of Maine	20.2	6.5	6.5	13.4	8.7	6.0	26.5	4.6	4.0	3.6
Agric. and Mech. College of Texas	28.8	12.5	00.0	13.3	4.5	4 5	26.7	3.3	1.7	6.7
University of Alabama	18.4	3.2	0.0	13.2	10.5	5 3	34.7	4.7	5.8	4.2
Montana State College of Agric. and Mech. Arts	33.0	3.0	0.0	13.0	16.0	5 2	15.2	7.0	5.2	2.4
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
Oregon Agric. and Mech. College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
Rensselaer Polyt	23.3	5.9	6.2	13.0	9.2	6.3	33.8	1.1	0.0	1.3
University of Michigan	20.7	4.3	11.4	12.9	10.0	5 7	25.0	0.0	10.0	0 0
Tulane University	12.3	6.4	0.0	12.8	7.9	8 9	48.4	0.0	2.0	1.3
Oikahoma Agric. and Mech. College	25.0	5.9	0.0	12.5	11.0	5.7	30.9	1.5	3.0	4.5
Penn. State College	22.0	7.5	7.5	12.5	7.5	5 5	30.0	7.5	0.0	0.0
Pensylvania College	15.0	9.2	3 9	12.4	14.4	9.2	23.5	11.8	0.0	0.7
State College of Washington	27.2	6.2	0 0	12.4	6.2	6.8	29.7	4.7	3.1	3.7
New Hampshire College	21.0	4.1	0.0	12.2	10.1	6 1	35.7	0.0	5.4	5.4
University of Kansas	17.0	7.0	7 0	12.0	11.0	9 0	28.0	5.0	2.0	2.0
University of Utah	14.6	4.6	0.0	12.0	7.7	9 2	42.5	4.5	0.0	1.5
North Carolina State College	19.4	7.7	5.3	11.9	8.5	6 0	29.4	5.3	0.0	6.6
Univ. of Wisconsin	21.0	4.0	0 0	11.9	7.9	4 0	31.5	4.0	11.8	4.0

TABLE VI.

ELECTIVES

INSTITUTION	Electrical engineering	English	Foreign languages	Mathematics excluding descriptive	Physics including el. mechanics	Chemistry	General engineering	General subjects	Electives	Physical Training
Tufts College	24.0	4.0	4.0	9.0	6.0	6.0	22.0	2.0	21.0	3.0
Armour Inst. of Technology	28.6	2.4	00.0	7.2	8.8	6.4	35.0	6.8	17.9	4.8
University of Washington	22.4	1.5	0.0	9.4	9.4	7.9	26.0	0.0	17.2	6.2
State Agric. College of Colorado	25.5	5.0	00.0	10.0	8.5	7.0	21.3	2.5	16.2	4.0
University of Missouri (Rolla)	18 6	5.4	3.5	10.5	7.6	8.5	26.7	1.2	14.8	2.3
University of Missouri (Columbia)	16.1	4.8	0.0	16.1	8.1	4.0	29.8	4.0	13.7	3.2
University of Wisconsin	21.0	4.0	0.0	11.9	7.9	4.0	31.5	4.0	11.8	4.0
Univ. of New Mexico	28.0	4.2	0.0	14.0	9.1	5.5	25.9	2.1	11.2	....
University of Arkansas	28.5	4.2	4.2	14.6	5.5	5.5	22.9	0.0	10.4	4.2
University of Nebraska	17.8	3.2	0.0	16.0	12.0	4.8	32.8	0.0	10.4	3.2
University of Minnesota	20.6	4.1	0.0	13.7	8.2	4.1	33.3	4.1	8.9	2.8
University of Wyoming	16.7	4.8	0.0	14.3	7.9	6 3	38.1	0.0	8.7	3.2
Massachusetts Institute of Technology	33.8	5.2	3.2	8.8	8.2	8.3	25.5	4.2	8.3	4.5
University of California	15.0	0.0	0.0	11.0	12.0	7.0	40.0	0.0	8.0	7.0
Purdue University	20.4	7.9	6.6	14.4	9.6	3.5	21.5	5.9	7.6	2.6
Iowa State College	21.7	7.5	0.0	15.0	11.5	6.8	23.7	2.7	7.5	3.6
University of Arizona	17.3	6.4	5.1	10.9	5.1	5.1	32.7	0.0	7.1	10.3
Massachusetts Agric. and Mech. College	15.3	10.9	0 0	18.8	7.0	4.5	26.1	5.9	6.3	5.2
Ohio State University	20.1	2.6	5.2	13.0	13.0	5.2	31.2	0.0	5.8	3.9
University of Alabama	18.4	3.2	0.0	13.2	10.5	5.3	34.7	4.7	5.8	4.2
Oregon Agric. College	25.0	4.5	0.0	13.0	9.5	4.5	30.0	0.0	5.7	7.5
New Hampshire College	21.0	4.1	0.0	12.0	10.1	6.1	35.7	0.0	5.4	5.4
Montana State Agric. and Mech. College	33.0	3.0	0.0	13.0	16.0	5.2	15.2	7.0	5.2	2.4
George Washington University	25.7	4.3	4.3	8.6	7.1	8 6	33.6	2.8	5.0	0.0
Union College	22.8	8.6	8.6	17.2	5.6	4 5	20.0	5.5	4.4	2.8
University of Maine	20.3	6.5	6.5	13.4	8.7	6 0	26.5	4.6	4.0	3.6
Lafayette College	27.7	3.9	3.9	11.0	6.4	6 4	24.5	11.0	3.9	1.3
University of Tennessee	10.4	6.9	6.9	11.6	4.6	5 2	39.4	0.0	3.4	11.6
State College of Washington	27.2	6.2	0.0	12.4	6.2	6 8	29.7	4.7	3.1	3 7
Bucknell University	23.0	6.0	5.5	9.0	12.6	9 4	23.0	8.0	3.0	0 5
Oklahoma Agric and. Mech. College	25.0	5.9	0.0	12.5	11.0	5 7	30.9	1.5	3.0	4.5
Cornell University	20.6	0.0	0.0	7.8	9.0	7 0	46.5	8.9	2.6	2.6
Johns Hopkins University	20.3	5.5	3.2	9.0	13.6	7 0	26.6	4.7	2.3	7.8
University of Vermont	27.6	4 3	6.5	8.6	5.4	5 4	27.0	2.2	2.2	10.
Tulane University	12.3	6 4	0.0	12.8	7.9	8.9	48.4	0.0	2.0	1

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

The Marine Committee submits the following report for the year 1917-18.

Two merchant vessels equipped with alternating-current lighting and motor service including engine room auxiliaries as well as deck auxiliaries have been completed and put in service. As mentioned in last year's report these equipments followed land practise adopting 250-volt, 60-cycle, three-phase alternating current. The total power provided was 200 kw. divided in two equal units.

These equipments were thoroughly tested and found satisfactory by extensive trial trips of the vessels but the vessels themselves have not been in service a sufficient time to warrant conclusions to be drawn as to service conditions. The vessels are both oil carriers and the heaviest auxiliary loads were those connected with the cargo oil pumps. It is to be expected that the owners will maintain records to show whether the vessels show marked improvement in the loading and discharging of cargo. It was upon this basis and the danger coincident to the use of d-c. motors that the application was made. The two electrically propelled merchant vessels have not yet been completed. Their equipments, however, are now under construction.

Much work has recently been projected on the basis of using oil engine-driven generators and electric motors for ship propulsion. For reasons connected with the low speed of the oil engine and increased efficiency, these plants have been designed for 25 cycles. It is understood that two such vessels may be so equipped.

Present Activities

Your committee has not been able to make further progress this year regarding the full revision of the electrical rules of Lloyd's Register of British and Foreign Shipping. The war conditions have prevented the necessary conferences, but minor matters of installation have been referred to the Lloyd's Register from time to time and approval given in accord with American practise. It was the consensus of opinion of this committee at its last meeting that the present time would not permit of

the preparation of technical papers either for presentation at meetings or for publication in the TRANSACTIONS. There has been, however, a favorable tendency towards the writing of popular articles on marine subjects in order to aid the general public in its conception of the extent of the uses of electricity in the marine field.

Suggestions for the Future

It is the purpose of your committee to continue to make suggested changes in the rules of the various classification Societies, and as experimental equipments emerge into established practise this committee will make the proper recommendations. As was inferred above, the intensive work of the individual members of the committee now prevents the writing of technical papers. It is believed that the time is approaching when it will not only be expedient but necessary to have such papers prepared and published in the TRANSACTIONS. The tendency in the field of ship propulsion seems to be approaching nearer to the use of electric drive due probably to increased interest on the part of ship owners and marine engineers in the efficiency of such systems, and the possibility of obtaining electrical apparatus with less difficulty than other types of propulsive machinery.

The committee desires to call your attention to the desirability of closer coordination of its work with the other technical committees of the Institute. The growth of the shipbuilding industry in this country and allied problems makes this most desirable.

H. A. HORNOR, *Chairman*

ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors,

The Committee held two meetings during the year, which, however, were poorly attended. It was early recognized undesirable to request from any engineer any labor for committee work except that which would be absolutely necessary or would become of vital importance to the operation of plants during the war.

At the last meeting, held December 14, 1917, two members in addition to the Chairman undertook to investigate and collect all available information on the broad questions of savings in production and utilization of power. Mr. Gorsuch has collected a good deal of information on what has been accomplished in utilizing waste gases in industries and tying together the electric power distributing lines with such by-product power plants; also studies of fundamental factors affecting economies in operation of power plants and favorable conditions under which they may be secured.

Mr. Putnam and the Chairman undertook to review the present-day relative economic values of new water power developments vs. steam power developments and their dependency and co-ordination, having in view the advisability or not of investing new capital in new water power developments during the time of the war, in contra-distinction of what the economic factors would have been previous to the war and what may be after the war.

The Chairman in collecting these subjects had in view the possibility of eventually securing proper papers for presentation at one of the meetings of the Institute if it were found desirable to cover at such meeting the subjects from the standpoint of the broad national policy during the war period.

In pursuing the study, the Chairman soon found out that the subject of power is a complex one and has many ramifications, so that a comprehensive solution could not be attained without securing the co-operation of representatives of different organizations interested in the application of water powers, steam powers, best methods of securing highest fuel economy by con-

centration of power generation, inter-connection of systems, possibilities of economies in use of wastes and gases from by-product coke ovens, powdered fuel, etc.

The Chairman, on the occasion of the Mid-Winter Convention, took the opportunity of suggesting to the President that it might be advisable to initiate the organization of a National Engineering Commission for considering and discussing plans and ways of advancing the recommendations made in his address. Such a Commission would naturally broaden out to study and report on policies affecting economics of power generation for general power application, steam electrification and special industries requiring continuous use of power. It was believed that such a study and recommendation would be of immense value to the industries, Government and State in shaping their policies in the generation and utilization of power.

A conference to discuss the subject thoroughly and outline plans could not, on account of the pressure of other matters, be arranged to include all who, in the opinion of the Chairman, should be present.

Some individual work was, however, done and considerable material is now available for use if the new administration should decide to carry out the plan.

Respectfully submitted,

PHILIP TORCHIO, *Chairman*

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.

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SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

BY W. H. COLE

ABSTRACT OF PAPER

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given in order that the extent of the work described may be visualized, supplemented by a description of the results obtained in actual operation.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

IT IS not the intention to offer this paper as a complete treatise on the general subject of balanced protection for lines and apparatus. It seems advisable to deal as briefly as possible with some of the experiences of the Edison Electric Illuminating Company of Boston in its pioneer work. Only such detailed description and discussion as appears necessary to explain the scope of the work, will be attempted. Such a recital may be of general interest, and of some value to transmission engineers. It is submitted with the hope that a general discussion will follow, disclosing experiences of other engineers along related lines.

Some years ago our engineers became convinced that line protection devices, which function with respect to time, to value of current, or to direction of power flow, were, in an extensive

and rapidly growing transmission network, exceedingly difficult, and frequently impossible to adjust and maintain in such a relation one to another, as to provide for the automatic disconnection of any faulty element, without simultaneously permitting the disconnection or shutdown of elements not themselves involved in the fault.

Following this recognition of the inadequacy of such protective gear, careful investigation was made of fault discriminating systems then in use abroad. As a result our company determined to make use of one or more methods based on the current balance principle. Up to that time the input-output method commonly known abroad as the Merz-Price system, seemed to be the most popular, although the older idea of balancing conductors in pairs also had seen some application.

Our company at first gave serious consideration to the input-output method, as it appeared to have enjoyed a considerable degree of success. Before we were prepared to install this scheme in connection with transmission cables, however, attention was directed to a reassembly of the older proposition of balanced pairs, which consisted of an arrangement of paired conductors in the form of a special cable. Since the pairs were to operate parallel connected at each end, and therefore with substantially no potential between the members of a pair, the amount of insulation between them could be comparatively light, and, consequently, one belt of primary insulation only, was required for each two paired conductors. This resulted in a cheaper and more compact transmission unit, for a given capacity, than two separate or independent primary insulated conductors, of equivalent capacity. It also appeared to be a less costly proposition than the "input-output" scheme, especially when the cost of duct space for the necessary pilot cables was considered.

Weighing the advantages of the two methods, led to a decision to make one or more installations of the special form of paired-conductor cable. This type of cable has now become quite well known as the "split-conductor" type. This name may not be the most expressive, since the arrangement is obviously not so much the splitting of conductors, as it is the assembly in an economical manner, of two separate conductors to be operated as paired conductors in a balanced protection scheme. It may be, however, desirable to perpetuate it when referring to any arrangement of paired conductors assembled

and operated with a common primary dielectric. The nomenclature used herein is, therefore, based on an assumed division or splitting of conductors into an equivalent arrangement.

While paired conductors have been arranged or proposed in forms other than the "concentric twin," Fig. 1 and Fig. 2, such as the so-called "D twin," Fig. 3, and the "sector twin" Fig. 4, no form other than the "concentric twin" will be speci-

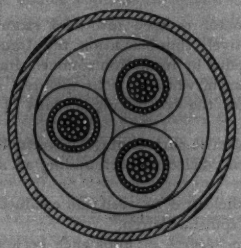

FIG. 1

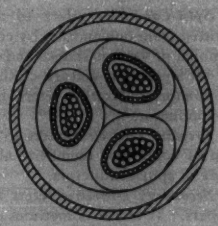

FIG. 2

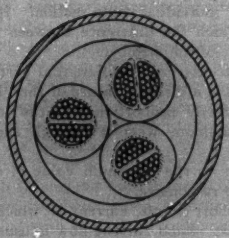

FIG. 3

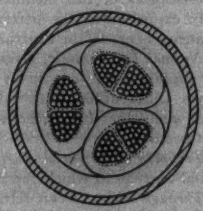

FIG. 4

fically considered herein, since this form, at least up to the present moment, seems to embody more desirable features than any other.

As a matter of historical interest, it may with propriety be stated here, that the generic type of balanced protection hereinafter discussed, had its birth about 17 years ago, when it was suggested by Mr. E. M. Hewlett of Schenectady, N. Y. A form of balanced protection for paired conductors also was proposed

by Mr. Leonard Andrews of Hastings, England, in 1902, embodying the first use of the differential reactor hereinafter described. Balanced protection schemes are, therefore, relatively old suggestions.

In order that the nomenclature used in this paper may be clear in meaning, it seems advisable to define some of the terms particularly applicable to "split-conductor" cable, and to balanced protection, as follows:

1. *Split-Conductor* refers to a conductor divided into two strands separated from one another by comparatively thin insulation, the strands assembled in various shapes and surrounded by insulation commensurate with the operating voltage of the system in which it is to operate.

2. *Conductor Member* refers to one of the conducting strands of a split-conductor.

3. *Primary Dielectric* refers to the dielectric surrounding an assembled group of conductor members, also to the dielectric surrounding an assembly of split-conductors.

4. *Secondary Dielectric* refers to the dielectr c between the conductor members. It is also sometimes called "split-insulation."

5. *Impedance Differential* refers to the degree of unba ance between the impedances of the current paths of a plit-conductor line. It is usually expressed as the percentage, which the difference between the impedances, bears to the impedance of one of the paths.

6. *Resistance Differential* refers to the degree of unbalance between the resistances of the members of an individual cable section. It is usually expressed as the percentage, which the difference between the resistances, bears to the resistance of one of the members.

7. *Normal Differential Current* refers to the vector differential current in an unfaulted split-conductor line. It is a direct measure of the "Impedance Differential." Its value is directly proportional to the total current and, therefore, reaches its maximum when a through short-circuit occurs.

8. *Tripping Differential Current* refers to the minimum value of differential current to which the relays are responsive.

9. *Through Short Circuit* refers to a short circuit, current to which is fed through any line under consideration. Such current flow is usually termed "through short-circuit current."

10. *Balancing or Differential Transformer* refers to the transformer devices used for comparing the currents in the conductor members, whereby a current proportional to the vector differential current is derived and circulated through the relay circuit.

11. *End-Impedance or End-Reactor* refers to an impedance device used to insure the creation of tripping differential current at either end of a faulted line, when the fault is at or near the opposite end of the line.

12. *Split-Con act Switch* refers to a special form of switch or breaker having three fixed contacts per pole. These contacts are bridged by a three-point yoke or blade. Two of the contacts are the term ni of the two members of a split-conductor. The normal closing movement of

the yoke or blade is such that the two conductor-member contacts are connected together, prior to their connection through to the bus. The switching sequence is exactly the reverse when disconnecting the line from the bus. The special function of the two-conductor member contacts with the associated portion of the yoke or blade, is to introduce impedance into one path to an end fault, in order to insure a tripping current differential at the remote end of the line.

13. *End Fault* refers to a primary fault which occurs at or so near the end of a line as to require external devices, such as end-impedances or split-contact switches, to assist in creating tripping differential currents at the opposite end of the line, to insure disconnection at that end.

14. *Middle Fault* refers to a fault which occurs toward or at the middle of the length of the line, and which does not require end-impedance devices to insure tripping differential current at both ends of the line.

15. *Secondary Fault* refers to a fault in the secondary dielectric permitting current flow from one conductor member to the other.

16. *Arithmetical Balancing* refers to any method of balanced protection, wherein the effect of arithmetical difference only, between the compared currents, is operative to cause operation of the disconnecting gear.

17. *Vectorial Balancing* refers to any method of balanced protection, wherein the effect of the vectorial difference between the compared currents, is operative to cause operation of the disconnecting gear.

18. *Primary Capacitance* refers to the ordinary capacitance of a cable, such as is usually measured between one conductor and the other two bunched with the sheath.

19. *Secondary Capacitance* refers to the capacitance of the secondary dielectric, as measured between one conductor member and its mate.

20. *Transposition Joint* refers to any joint in which the members of one or more of the conductors are transposed.

21. *Straight Joint* refers to any joint in which the conductor members are held in the same relative position through the joint.

22. *Self Healing* refers to automatic healing of a punctured dielectric of the saturated type. With balanced protection, it often happens that a line is so rapidly disconnected, the fault current does not attain high values, nor does it carbonize much material. The result is a fairly clean puncture which immediately becomes more or less completely sealed with hot oil or compound. If the potential is restored, it may take considerable time for the carbon particles to align sufficiently to cause repetition of the breakdown.

Split-conductor Cable Design

In split-conductor cable design, the engineering problems do not differ materially from those arising in ordinary cable design, except with respect to the division of the conductors. The chief additional considerations are, first, the dimensions and type of the secondary dielectric and, second, the permissible resistance and impedance differentials.

In order intelligently to design the secondary dielectric, it is

necessary to predict the probable value and duration of voltage stresses in the secondary dielectric, and when they may be expected to occur.

Cables are subjected to two classes of potential stresses, *i.e.*, those occurring during preliminary high potential tests, and those occurring in service. In general, only those occurring in service need be regarded as factors affecting cable design.

If, while in service, one member of a split-conductor becomes involved in a primary fault, say to earth, it is clear that some portion of the other member will for a finite period of time be at star potential above the faulty one. The rapidity of the change from this difference of pressure to a lesser one will depend upon the position of the fault with respect to the end of the cable; the frequency at which the greater one occurs, upon the type of system, *i.e.*, whether grounded or isolated neutral. The current flowing to the fault causes a differential IR drop from the terminals to the fault, and under some conditions this drop may result in establishing the star potential across the secondary dielectric. The linear amount of secondary dielectric affected by the higher stresses is dependent upon the position of the fault with reference to the length of the cable, and also upon the type of protective gear. The cumulative duration of stresses in the secondary dielectric for a given fault, is a function of the time required by the switch gear to disconnect the line from all sources of supply.

Since balanced protection schemes provide for rapid disconnection of faulted lines, it is a reasonable assumption that the higher secondary stresses will be more or less transient, for which a conservative allowance may be made when determining the thickness and grade of the secondary dielectric.

The possible effect of high potential testing of the primary dielectric, upon the secondary dielectric, requires further consideration. As stated above, the probable maximum secondary pressure due to operation, will not exceed the star potential of the system. Primary testing at double operating pressure may, if a primary failure occurs, result in stressing a part of the secondary dielectric with nearly four times the star potential. Occasional secondary failures, consequent upon primary failures due to such high-tension testing may, therefore, be expected.

Since high potential testing after installation, if accomplished without failure, causes no abnormal stress in the secondary

dielectric, it is good commercial judgment to ignore the possibility of secondary failures resulting from primary high-tension tests, and design for operating star pressure maxima only.

The local heat set up by the arc, upon the occurrence of a primary fault, and its possible effect upon the secondary dielectric, must be recognized. As the outer member usually is very thin, we may expect it to be destroyed rapidly at the point of fault, accompanied by extremely local and intense heating or burning of the adjacent secondary dielectric. On the same basis of reasoning, the secondary dielectric often may be completely destroyed at this point, even though insulating material substantially thicker than required to withstand star pressure is provided. This risk coupled with the fact that the maximum pressure across this dielectric is coincident with the point of fault, makes it extremely probable that if any breakdown of the secondary dielectric does occur, it will occur at this point. It is assumed, of course, that this dielectric is uniform in value throughout its length.

At the beginning, we were forced to consider a choice beween a secondary dielectric of sufficient thickness to withstand successfully at least the star voltage of the system impressed across a dielectric being rapidly weakened by the arc at a fault, and a dielectric of minimum safe thickness from a mechanical standpoint, but of more than ample value for the normal operating voltage between conductor members. A choice of the first meant more expensive cable, while the second involved the danger of failure of the protective apparatus then available, to operate, if a secondary fault should occur. Our reasoning was, that the major portion of the secondary dielectric being substantially an idle investment under all normal conditions, any reduction in its thickness with consequent cheapening of the cable was justified, provided a form of protective gear could be devised whereby a failure of the secondary dielectric under any circumstance would not prevent prompt disconnection of the line so affected.

The required form of gear subsequently became available, resulting in our standardizing, tentatively at least, a secondary dielectric thickness of 3/64 in. (1.19 mm.) paper for all round type, paper-insulated, concentric split-conductors for1 5,000-volt working pressure; 1/16 in. (1.58 mm.) paper for the same class of conductors for 25,000 working pressure; and 5/64 in. (3.96 mm.) paper for both 15,000 and 25,000-volt sector type,

paper-insulated, concentric split-conductors. The additional thickness in the sector split type is due to what appeared to be mechanical necessity, *i.e.*, relatively sharp corners of the inner member, but it will undoubtedly be reduced as the art of manufacture improves. So far as our experience indicates, these thicknesses, if properly applied and thoroughly impregnated, with the conductor members free from mechanical defects, are ample to meet the conditions imposed by manufacture, installation, and service, with our standard form of protective gear.

For economic reasons it seemed better to assume some risk of a possible secondary failure, if the cable was also simultaneously nvolved in a primary fault and, therefore, bound to be disconnected, than to specify a secondary dielectric of a thickness calculated to withstand any possible stress to which it might be subjected, knowing that this maximum stress cannot occur except under primary fault conditions in the same line.

The impedance differential in a completed line must be considered jointly with the characteristics of the protective gear at the ends of the line, and its magnitude limited in accordance therewith, since if the maximum normal differential current is permitted to be high, thereby requiring a high setting of the relays, the means for creating the required value of tripping differential current under and fault conditions must be of corresponding magnitude. This limitation is of particular importance, if the end-impedance devices are the reactive type.

In order that the impedance differential of a completed cable line shall be as small as desired, it is first necessary to care properly for the resistance differential in the individual cable sections, by limiting its value, or preferably its variation from a predetermined percentage of the resistance of one of the conductor members. In other words, it is not so important that the conductors agree with each other to a small percentage, as it is that the ratio of their resistances, one to the other, shall not differ by more than a small percentage. Second, it is necessary that the conductor members shall be so transposed in the jointing of the cable sections as to insure that the resistance differential of the completed line is within the desired limits, which also provides for relative changes in temperature and resistance of the conductor members. Third, the reactance differential of the completed line must be kept down to the proper value, and it is ordinarily accomplished by the transposing required for resistance balancing. As it is a much larger differential in the

individual cable sections than the resistance differential, running as high as 40 per cent, it may be the larger component of the impedance differential. It is very necessary that this differential be carefully equalized in a completed line. Its value is fixed by the design of the conductor and, therefore, is out of the control of the manufacturer.

A reasonable specification for resistance differential of a cable section is that it shall not exceed two per cent when the nominal resistance ratio of the conductor members is required to be 100: 100. If it is desired to specify a nominal resistance ratio of, say 100: 102, then the requirement should be that the ratio of resistance of the conductor members shall be not greater than 100:101, nor less than 100:103.

Our experience, without feeder reactors, has shown that no material difficulty need be expected in getting completed lines to balance closely enough to permit the maximum through short-circuit current to flow, without setting up normal differential currents of tripping value. Short lines originating at a power plant are, of course, subject to the heaviest through short circuits and, therefore, require the most care in balancing. While long lines do not require as close balancing, they are easier to balance than short lines, since the law of averages has a more effective application.

During the war, the copper wire market has been such, that considerable difficulty has been experienced by manufacturers, in securing copper drawn with the accuracy desirable for the production of well balanced split-conductors. This has forced the acceptance of cable not quite up to the pre-war standard, and has required more attention in the matter of transposing conductors, in order to secure the desired balance in the completed lines. It is the aim to secure impedance balance in completed lines to within one tenth of one per cent. With uniformly well balanced cable sections, this result is ordinarily secured with from one to five transpositions. The present practise is to use no less than three, dividing the line into four sections.

Relays for Current-balance Protection

Relays should be of the instantaneous overload type, operative on small amounts of energy, hand resetting, and the moving parts should have small inertia. Secondary devices are necessary to close the contacts in the tripping circuit. An auxiliary break in the tripping circuit should be provided, to be actuated by the

movement of the main switch mechanism. At least two manufacturers in this country have produced a satisfactory form.

The hand-reset feature is very important, since it constitutes reliable means of diagnosing the class of fault causing the operation of the protective gear.

Split-Conductor Protective Gear

Single-line diagrams, Figures 5, 6, 7, and 8, show diagrammatically some of the forms of protective gear we have used or considered, the form shown in Fig. 5 now being fairly well standardized.

At the risk of some repetition it may be advisable to explain

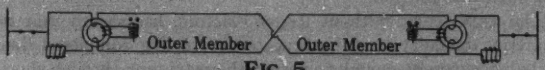

Fig. 5

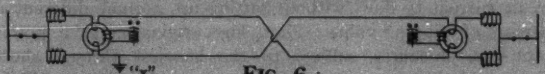

Fig. 6

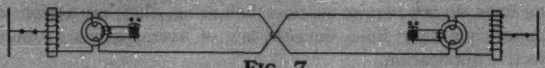

Fig. 7

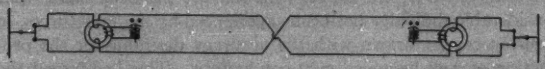

Fig. 8

the functions of the several devices shown, and discuss their different characteristics.

Common to all current-balancing schemes, an apparatus for weighing or comparing the currents in the paired conductors is required. The simple differential transformer is undoubtedly the best device for the purpose so far suggested. It consists of a standard type of current transformer core upon which are wound two primary coils, and one secondary coil. The primary coils are each respectively connected into one of the paired conductors in such a manner that under normal conditions of current flow, the coils will have equal and opposite effects upon the transformer core. If the primary coils are symmetrically disposed with reference to each other and to the core, no core flux will be set up, and consequently no current will flow in the secondary wind-

ing or relay circuit connected thereto. This is the condition that should obtain in a perfectly balanced split-conductor under normal conditions. Should a fault occur, however, resulting in a disturbance of the normal current flow, then the balanced-current condition no longer obtains, and the difference of current values in the balancing transformer primaries results in establishing a core flux, and consequently a current flow in the secondary coil and relay circuit. As soon as this secondary current reaches the tripping value, the line is automatically disconnected at one or both ends, depending upon the position of the fault.

If all faults in split-conductor lines were confined to the middle of the length of the line, and to one conductor member only, tripping differential current would be assured at both ends of the line, even though the balancing transformer windings have very little impedance. Balanced-current protection for paired conductors would, with such premises, be an ideally simple proposition.

Since it is absolutely necessary that the scope of the protection shall include the entire line, and extremely desirable that line disconnection be effected while the fault current is moderate in value, it is necessary to provide for what are known as end faults.

End fault protection is secured by the use of so-called end-impedance devices. Up to the present, two types have been developed, the reactive type and the non-reactive type. The reactive type has been proposed in several forms, three of which are shown in Figures, 5, 6 and 7. The non-reactive type as embodied in the three point per pole, or split-contact switch, Fig. 8, is the best known form of that type.

The theory of design and application of end-impedances is discussed quantitatively in the appendix to this paper, kindly prepared by Prof. C. A. Adams, but possibly a non-mathematical statement of the action of end-impedances may be helpful.

Assume that a failure occurs on a split-conductor, say at X, Fig. 6, affecting one member only, and for the moment that no impedance devices are in circuit, current will flow to or from the fault in two directions. At the fault end of the conductor, the flow through one primary coil of the balancing transformer will be reversed in direction with respect to the flow through the other primary coil. The current flowing in the first mentioned primary coil will be supplemented by current from the bus at the fault end of the conductor, if this bus is supplied by conductors

other than the faulty one, or possibly by current from synchronous apparatus connected to this bus. As the effect of the current flow in the transformer primaries becomes cumulative under conditions of relative reversal of current, it will be seen that ample secondary or tripping current is assured, with consequent disconnection of the conductor at this end.

From the above, it is clear that end-impedance devices *are not* required in order to insure operation of the disconnecting gear at the end of a line adjacent to an end fault. Since, however, an end fault is so located with respect to the distant end of the line, that the impedance in one current path from the bus at the distant end is substantially equal to the impedance in the other current path, the two components of the total current flow from the distant end will also be substantially equal, or at least they will not differ by more than the normal differential current, due to the inherent impedance differential of the conductor. As the relays must not operate on normal differential currents, it is necessary, in order to produce disconnection of the distant end, that the impedance of one of the current paths be altered with respect to the other, to the extent required to produce a tripping differential current in the balancing transformer at the distant end of the line.

Referring again to single-line diagrams Figs. 5, 6, 7 and 8, and assuming an end fault on each, in a position equivalent to that shown in Fig. 6, and that all lines have been disconnected from the bus at the fault end in the manner previously described, the tripping differential current to insure disconnection at the distant end is assured as follows: In Fig. 5, the impedance in one current path to the fault becomes increased with respect to the other, due to the inclusion in the first path of both reactors. In Fig. 6 the impedance of one path is increased, due to the inclusion therein of three of the reactors as against one reactor only in the other path. In Fig. 7 the impedance of one path is increased, as the coils of the reactor at the fault end become cumulative in effect, due to relative reversal of current in one of them. In Fig. 8 the impedance of one path is increased (to an infinite value) by the actual opening of one path to the fault at the so called split-contacts.

Discussion of End-impedance Devices

While any of the types or forms shown, may be designed to provide adequate end-fault protection, they differ enough in their

characteristics to warrant considerable study before a choice is made. Choice should be governed by such considerations as, strength of design, space required for their installation, effect on regulation, effect on switchgear and switchboard design, rapidity of action, cost, etc.

The form shown in Fig. 5 is a special arrangement of simple reactors, rather than a distinct form. It was developed as a result of a desire to reduce, for a given duty, the reactor capacity necessary in the forms shown in Figures 6 and 7, and as well to insure that tripping current differentials would be set up, should both the conductor members be simultaneously involved in a fault.

The form shown in Fig. 6 has had but little application and is shown and described merely for purposes of comparison. It is obviously the least desirable of all forms shown, since the capacity required for a given duty, the space required for installation, and the effect upon regulation, are all of an order twice that inherent to the scheme shown in Fig. 5. The arrangement compares favorably, however, with that shown in Fig. 7, except as to iron losses and effect upon regulation.

The form shown in Fig. 7 is best described as the differential type. It was suggested by Mr. Leonard Andrews of Hastings, England, about 16 years ago, and has had some application abroad, not only in the form illustrated, but also when provided with a super-imposed secondary winding of proper impedance, thereby combining in one device, the function of an end-impedance with that of a balancing transformer. Its chief claim for consideration, is its non-inductive characteristic, and absence of core loss, under conditions of normal current flow. At first glance it might be thought that this form of reactor could be designed with a minimum amount of iron in its core, since under normal conditions there is no flux therein, and thus it could be smaller and cheaper than other reactive forms of end-impedance. This type of reactor is, however, not only differential itself, but cooperates differentially with its mate at the opposite end of the line, so that it is the vectorial difference between their respective effects that produces the required differential current for tripping purposes, at the end remote from an end fault. Any attempt, therefore, to reduce the size and cost of this form of reactor, by designing it for high saturation in the core when functioning for end-faults, is ineffective, since the volt-ampere capacity must be so increased, as to offset any hoped for gains due to reduction of iron in the core.

The form shown in Fig. 8, is merely the combination of a main and auxiliary switch, the auxiliary contacts when opened separating the conductor members from each other. This isolation of the conductor members from each other is, in effect, equivalent to the introduction of infinite impedance into one path to an end-fault, when the opening occurs as a result of such a fault. If the opening is due to regular switching, the members are merely simultaneously disconnected from the bus at that end.

As before stated, the selection of end-impedance devices should be made with several considerations in mind.

From the standpoint of strength of design, the balance appears to be in favor of the reactive type, since it has no moving or wearing parts requiring adjustment or renewal.

The space requirement usually is a local consideration. American practise in line cell construction, however, seems to favor the reactive type, as the reactors in most cases may be installed in what would otherwise be unused space. The additional space required for split-contact switches usually is more difficult to obtain.

The reactive type, in some forms, admittedly has some effect upon voltage regulation. In any form the effect is not serious, particularly when we consider the growing use of current limiting reactors. The form shown in Fig. 5, in the capacities used by the Boston company, has about the same effect upon regulation as would a one per cent feeder reactor. The reactive form shown in Fig. 7, as well as the split-contact type shown in Fig. 8, has no effect upon regulation.

The relative effect of the two types of end impedances upon switch gear design, is of great importance. It is obvious that standard types of switches are adequate with reactive schemes of end protection; no departure from standard practise or design is necessary. The non-reactive type however, as embodied in the split-contact switch, requires a special design of switch. For small capacities this may not be a serious matter. For the large capacities, such as are common in American practise, a serious factor is introduced, *i. e.*, the necessity of designing the three-point or split-contact switches, so that all three breaks per pole have the same breaking capacity. This means that if a switch is of a type requiring two pots per pole for standard work, it will require an additional pot of equal capacity in order to convert it into a split-contact switch.

It may be thought that since the so-called split-contacts each

carry only half the full line current, each might be designed for half duty, when compared with the third or so-called non-split contact. This assumption may be correct if the breaking of through current only is considered. If we assume a case when this type of switch is called upon to perform a dual duty, such as clearing an end fault, it can be shown that one of the split-contact breaks may have a duty exceeding that of the main or non-split-contact break.

Three-point or split-contact switches must be so designed that when being closed both conductor members shall be connected together before either one of them is connected to the bus. This must be assured, for if by chance one of them should be closed on a bus before the other, the instantaneous current flow through one conductor member only, will result in the simultaneous opening of the switches at both ends. In order to prevent such an occurrence, the relative position of the contacts and yoke or blades is made such, that the split-contacts will be connected together just prior to their joint connection to the bus or non-split-contact. The time interval between the two closures must be extremely short, in order to make the switch as effective as possible when rupturing current, since in opening, it is of great importance that all breaks per pole occur as nearly simultaneously as possible, in order to divide the breaking duty equally.

When a three-point switch operates on an end fault, the first break takes place at the main or non-split contact, closely followed by a break at the split-contacts. One of the split-contact breaks handles the component of the fault current flowing over the corresponding conductor member. The other split-contact break must handle the same component, plus any current flowing to the fault from the bus at the fault end of the line via the main break. The third or non-split contact break handles only the current flowing to the fault from the last mentioned bus. Thus it will be seen that one of the split-contact breaks may have a far greater duty to perform than any other.

Close analysis of the relative effect of the two types of end-impedances upon switch gear and switchboard design seems to indicate the superiority of the reactive form of end-impedances.

It has been claimed that the split-contact switch provides for more sensitive control, by virtue of its function of inserting infinite impedance into one path to an end-fault, thereby concentrating the fault current in one path only, at the end remote from the fault, thus creating at that end a greater tripping

differential current than would be obtained if reactive impedance devices of permissible size were used. This advantage is real only in a limited degree, and only if the fault current is so small, that when it is divided between the two paths to the fault, as must be the case if reactive end-impedances are used, the differential current is less than that required to operate the relay. Such small fault currents are encountered only when the fault is one to earth, in a system having small capacitance to earth. In such systems the use of the split-contact switch may be justified, particularly as such a system will ordinarily be of small magnitude. In systems of large magnitude, the cable systems also are usually extensive, and fault currents to earth are ample to insure tripping differential current with reactive end-impedance devices of reasonable size, even if the system is operated with an isolated neutral. Whenever the system is of such size or type as to justify the use of reactive end-impedance devices, disconnection of lines under end-fault conditions is effected more rapidly with such devices than would be the case with split-contact switches, since tripping differential currents are set up simultaneously at each end. With split-contact switches, a tripping current differential exists at the fault end, only, until the operation of the switch at the fault end, effects the introduction of impedance and the setting up of a tripping current differential at the opposite end of the line.

On the whole, it would therefore appear, that considering the rapidity of action, at least on large systems, the reactive form of end-impedance device has a distinct advantage over the non-reactive type.

The matter of relative cost of the two types is at present unsettled, as the development in this country has proceeded along reactive lines. It is believed, however, that taking all features into consideration, the cost differential will be in favor of the reactive type.

In order to check the soundness of our conclusions as to end-impedance devices, it has been thought advisable to secure some actual operating experience with the more promising forms. As a result, we have placed in operation during the past two years two installations of the split-contact type of end-impedance, Fig. 8, also we have on order for installation during the current year, two installations of the differential type of end-impedance, Fig. 7.

Balanced Protection for Ordinary or Independent Conductors

While the experience of the Boston company in the balanced protection of paired ordinary conductors has not been extensive, the work to date has been of great importance in establishing a basis for future practise. It is believed, therefore, that a few references to this phase of the subject are permissible at this time.

As it is obvious that there is no material difference in operating principle between the older and well known arrangement of paired independent conductors, and the so-called split-conductor scheme, any choice between the two arrangements tends to be an economic one. It is incorrect, as is sometimes done, to refer to the earlier method as a combination of ordinary conductors on the "split-conductor plan", since such arrangements were proposed many years previous to the split-conductor suggestion.

In general the protective apparatus already described is applicable to any combination of paired conductors, but when the paired conductors are situated each in different cables or lines, they are each insulated for the primary voltage of the system, and are, therefore, independently suitable for operation if for any reason the companion conductor is out of service. It follows, therefore, with any arrangement of paired conductors, other than actual split-conductors, that each of the paired conductors ought to and usually does have its own switches independent of those for its parallel associate. The use of individual switches for each conductor, moreover, results in a combination equivalent in most respects to the split-conductor arrangement, employing split-contact switches in place of end-impedances or reactances, so that end-faults are adequately provided for.

With a paired independent conductor line, it is not possible to secure and maintain the degree of balance inherent to a split-conductor line, for even were it possible to start out with a fair balance, conditions local to one conductor or the other would soon affect the balance to a considerable degree; for example, should one line of a pair be out for repair and the other remain in service, it is obvious that when put in parallel again, the difference in conductor temperature, which might then exist, would be the cause of a material impedance differential. If a through short circuit should occur under such circumstances, the normal differential current would attain a very high value.

It is frequently desirable to consider paralleling, with balanced protection, conductors which differ materially in length, cross

section, or reactance. In such cases it is impossible to operate without correspondingly large normal differential currents. Since the end fault condition in such lines will be cared for by the independent switches for each conductor, the remaining necessity is that of compensating for the large normal differential currents set up by through short circuits, and with the same devices provide for selective disconnection, should faults occur that result in differential currents materially smaller than those due to through currents of maximum values.

If the ordinary type of overload relay is used in this class of balanced protection, the settings might need to be so high as to preclude operation on section faults, or in any event to prevent obtaining the maximum benefit of balanced protection.

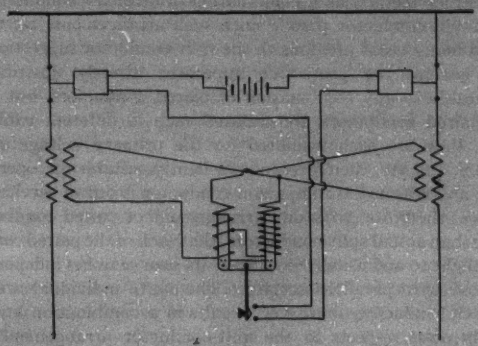

FIG. 9

In order to obviate the necessity of the high relay settings referred to, and at the same time to insure non-operation on through short circuits, relays have been developed both in this country and abroad, variously called, "biased relays", "percentage-balance relays", and "ratio-balance relays".

We have experimented for nearly four years with ratio-balance relays in the forms shown in Figs. 9 and 10. One pair of cables has been equipped with the form shown in Fig. 9, and has been in operation for over three years. In all cases of actual trouble, as well as under artificial fault conditions, the relays have functioned as predicted. While in the case of a single pair of lines, both are disconnected upon the occurrence of a fault in one, the other may be immediately put to work with straight instantane-

ous overload protection, since the relays may be arranged to so function for either line, when the companion line is out of service.

The form shown in Fig. 10 is particularly well adapted for use with groups of three or more parallel lines, in which case the relays may be electrically interlocked, so that the faulty line only is disconnected. When so applied, all the discriminative features of balanced current protection may be secured without the use of pilot wires, special cables, reactors, or split-contact switches.

The relays shown in Figs. 9 and 10 are so constructed, that the restraining force is proportional to the vector sum of two compared currents, and the operating force is proportional to the

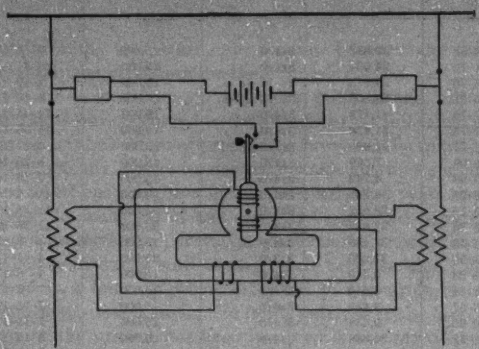

FIG. 10

vector difference between the compared currents. With proper winding ratios, the restraining effect will predominate until the normal relation between the compared currents is altered, due to a fault involving one of the conductors, to such an extent as to cause the ratio between the vector sum and vector difference of the compared currents, to exceed a predetermined limit. After this limit is reached, the operating force becomes predominative and the relay operates to disconnect the faulty conductor.

In order to choose the proper winding ratio for ratio-balance relays, it is necessary to know what the normal differential current will be under extreme conditions, and the percentage it bears to the simultaneous vector sum of the compared currents. Once adjusted for this normal relation, they cannot operate on

any value of through current, since the restraining force will then always be in excess of the operating force.

It is expected that upon the return of more normal conditions, further progress will be made in the application of ratio-balance protection to many of our standard transmission lines, by means of the devices described, or that may hereafter become available.

Schedule of Split-conductor Cable Installations

The following table shows the installations of split-conductor cable now included in the Boston Edison Company system:

Line No.	Length in feet	Size	Voltage	Date placed in service
34–82	32,032	3x4/0	15,000	9– 4–13
31–76	24,054	3x1/0	25,000	8–31–14
26–76	29,273	3x. 10 sq. in.	25,000	1–10–15
26–77	29,305	3x. 10 sq. in.	25,000	5–16–15
47–76	41,478	3x4/0	15,000	11–14–15
47–75	41,478	3x4/0	15,000	11–30–15
31–77	24,069	3x. 10 sq. in.	25,000	5–2 –16
16–99	7,599	3x4/0	15,000	8–25–16
* 49–98	4,742	3x4/0	15,000	9–10–16
* 49–95	4,742	3x4/0	15,000	11– 4–16
† 23–75	23,652	3x3/0	25,000	11–26–16
† 23–76	23,652	3x3/0	25,000	11–26–16
54–99	33,488	3x4/0	15,000	4– 7–17
* 43–59	32,205	3x3/0	25,000	4– 1–17
‡ 8–53	5,510	3x4/0	15,000	7– 8–17
‡ 8–54	6,510	3x4/0	15,000	7– 8–17
‡ 30–87	9,439	3x4/0	15,000	7–15–17
‡ 30–96	9,436	3x4/0	15,000	7–15–17
49–51	24,889	3x4/0	15,000	8–12–17
39–51	23,480	3x4/0	15,000	8–12–17
51–69	8,789	3x4/0	15,000	10–21–17
* 57–69	9,189	3x4/0	15,000	10–21–17
* 9–69	6,926	3x4/0	15,000	10–21–17
52–77	3,758	3x4/0	15,000	10–21–17
52–78	3,772	3x4/0	15,000	10–21–17
‡ 8–87	7,384	3x4/0	15,000	11–18–17
* 58–80	6,742	3x4/0	15,000	11–18–17
§ 53–53	7,754	3x4/0	15,000	
§ 53–54	7,754	3x4/0	15,000	
§ 53–70	3,516	3x4/0	15,000	
§ 21–72	10,202	3x4/0	15,000	
§ 21–73	10,202	3x4/0	15,000	
101–102	50,044	3x4/0	15,000	5–1–18
101–103	49,272	3x4/0	15,000	5–1–18
§101–104	48,733	3x4/0	15,000	
§101–105	48,691	3x4/0	15,000	
§103–59	11,552	3x3/0	25,000	
§ 20–59	21,841	3x3/0	25,000	

*Line not yet operating as split-conductor line.

†Lines at present operating in series with standard overhead lines.

‡Lines at present operating in series with standard conductor cable.

§Lines under construction to go in service 1918.

The total length of the above cables is 747,590 ft., or approximately 142 miles.

Lines 26-77 and 31-77 are connected in series at present and operate as one line, which is protected with the English form of split-contact switch gear, Fig. 8. Line 26-76 is equipped with a modification of the form of gear shown in Fig. 6. Lines 26-77 and 31-77 are to be separated during the current year and the new terminals are to be protected by the form of gear shown in Fig. 7, the opposite terminals remaining as before, protected with split-contact switch gear. The remaining lines are protected, or will be upon their completion as split-conductor lines, with the form of gear shown in Fig. 5.

Results of Operation

Not including the preliminary testing of lines under artificial fault conditions, to determine their characteristics, and to check the theory of operation, a sufficient number and variety of actual faults have occurred to demonstrate that full reliance may be placed on the system of protection, provided the fundamental requirements are met in its design, installation, and operation.

Line 34-82 has now been in service nearly five years and with the possible exception of one case, to be referred to later, the balanced protection apparatus in connection therewith has functioned in accordance with theory, each time the line has been involved in a fault, or when subjected to the stress of feeding through short-circuit current to faults external to the line. This line up to the present has been involved in one secondary fault (due to defective stranding of the outer member of one conductor), one end fault, and at least six through faults, three of which were bus short circuits at one terminal of this line.

Line 47-75 has been in service about two and one half years and has been involved in one fault, a primary fault in a joint, one conductor to earth. This fault created so little disturbance that the station operator was inclined to feel doubtful as to the need of the switches opening. Tests were made to locate the fault, but after a while the operators concluded that none existed, and the line was placed back in service. Outside of some static discharges of small volume at the line terminal, which soon ceased, the line appeared as good as ever. In order to clear up the action of the relays, it was deemed advisable to inspect the line. Men were sent over the line, opening manholes and examining the cable therein, with the result that a joint was found

with a hole blown through the lead sleeve, and fresh compound expelled in considerable quantity. The line was then promptly taken out of service, the faulty joint opened, whereupon it was found that one conductor had broken down to earth (corresponding to the relays that operated), but the fault had become healed due to hot compound running back into the wound after the line was disconnected. From all outward appearance, this line probably would have operated for years in its then condition. This case is described at some length, as such an occurrence may have been the cause of relays operating on our first line, No. 34-82, on one occasion without the cause ever having been determined. Self healing, therefore, may be expected. This experience, confirmed by results obtained in high potential testing, teaches that any case of apparently unnecessary operation should be carefully investigated before conclusions are drawn.

Line No. 31-77 has been in service about two years and has been involved in two faults. The first fault was due to a connector in a split-conductor pothead (English Type) becoming loosened, thereby increasing the impedance of one of the current paths and setting up a tripping differential. This caused several operations of the protective gear at each end before a diagnosis of the fault was made by our operators. The second fault was due to mechanical injury; a laborer driving a pick into the cable, grounding one conductor. The protective gear, (split-contact switch type) operating in the expected manner, isolating the line at both terminals. This line was again made alive, but was automatically disconnected again, since the fault was well established. The second operation resulted in breaking down both inner and outer members at the point of fault. The cable at the point of injury was laid on the solid system, *i.e.* armored, laid directly in the ground and covered with a two-in. (5.08-cm.) spruce plank. Repairs were effected by making a new joint in this location.

Line No. 31-76 has been in operation for nearly four years, but on account of defective joints and some dry spots in the secondary dielectric which were eliminated by high-tension tests, it was not finally accepted until about three years after installation. In the course of making the many primary high potential (50,000-volt) tests, the secondary dielectric failed, due to the stresses set up therein as a result of primary failures, at four or five of the dry spots, also in two cases where the outer member was defective, once as to stranding (crossed strands) and once where

a strand had been brazed. This line could not be kept continuously out of service for testing, however, and on one occasion it was put back in service with the positive knowledge that an incipient secondary fault due to the testing above referred to was present in one conductor. No cable was available at the time to replace the faulted section, and as the fault was of very high resistance, it was decided to assume the risk of operating the line with balanced protection. This proved to be poor judgment on our part, since this fault developed in service to such an extent as to cause operation of the protective gear, the operation, of course, being perfectly normal for this type of fault. Unfortunately it occurred simultaneously with one of the outages of line No. 31-77 described above with which it was ring connected, and thereby contributed to an interruption of service. It was also unfortunate in this case that the first mentioned fault in line No. 31-77 had not been diagnosed and remedied at an earlier date. Such experiences, however, are of great importance as affecting future operating practise.

Line No. 54-99 was placed in service April 7, 1917 and within a few weeks afterward broke down between main conductors, resulting in its disconnection by the balancing gear. This fault occurred about one and one half miles from the power house. No disturbance was noticed when it was disconnected. Relays operated on two of the conductors, normally indicating a short between them. The operators put the line back again, however, and it ran about three minutes when it came out again, the same relays operating as before. The line was again made alive from a separate bus and generator at somewhat higher pressure than the system pressure, and remained in for four minutes, coming out for the third time upon the operation of the same two relays. This appeared to satisfy the operator that the line was faulty. The fault was located and removed, and was found to be due to a breakdown between the two phase wires corresponding to the two relays that operated in each of the three cases of disconnection. Since a short circuit of similar type and location, on a feeder protected with overload relays with as short a setting as one half second, usually makes itself known in a more or less violent manner, the performance of the protective devices on this line appears to be a substantial tribute to the efficacy of the balanced method of protection. In this case, it appears that each disconnection of the line was so rapid that the fault current built up to a value not materially greater than sufficient to create a tripping differen-

tial, and as soon as disconnection occurred the puncture healed up, due to the hot cable compound flowing into the wound. Undoubtedly enough carbon was suspended in the puncture path to gradually line up and bridge between phases, thus causing the second and third disconnection. It is a question how long this process could have been continued, but it would seem that two disconnections with the same relay operation in each case would have been conclusive evidence that a fault did in fact exist. Operators will undoubtedly require considerable experience in order to become convinced that quiet disconnection of transmission cables equipped with balanced protection is sufficient evidence of line failure.

As before stated, some trouble has been experienced by the manufacturers in getting perfect saturation of the secondary dielectric. In the case of the above mentioned primary fault in line No. 54-99, the condition of the secondary dielectric was not checked prior to putting the line back in service after repair, as is now our practise. It was soon found that one of the conductors involved in the primary fault had a secondary fault which resulted in the protective gear again operating. This secondary fault was located and removed, and was found to be due to defective stranding of the outer conductor member. The continued attempts to operate under the main fault condition stressed this defective point severely, and to an amount in excess of the stresses caused by the factory or installed test. It is significant that this secondary fault, as well as the two others (three in all) occurring in service to date, were located in defective construction, which permits of the reasonable assumption that none would have occurred if the construction had been perfect, and also that the values of secondary dielectric chosen for our service, are ample if properly incorporated in the construction.

It is perhaps unfortunate, but nevertheless unavoidable, that so much time is required to obtain extensive operating experience. It is obvious, however, that new cables are not expected to fail soon after having been carefully installed and tested.

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APPENDIX

BY COMFORT A. ADAMS

The purpose of this appendix is to show the relative amounts of reactance required to produce a given current differential in the three arrangements shown in Figs. 5, 6 and 7, under end-fault conditions, since it is only under these conditions that reactance is required.

In order to simplify the final general analysis assume that the added reactance has in each case a constant value. The effect of saturation will be considered later.

Let x equal the value of a single reactance in the arrangements of Figs. 5 and 6, and of the series or round-the-end reactance of one of the differential reactors of Fig. 7.

Assume that the switch at the fault end has opened—

Let $\dot{I}_1$ be the current from the far end to the fault, by the longer path, $\dot{I}_2$ that by the shorter path, both counted positive in the same direction from supply end to fault end, and $\dot{I} = \dot{I}_1 + \dot{I}_2$ the total fault current.

Let r_1 = the resistance of a single conductor member.

Let x_1 = the combined reactance of the two conductor members considered as a single conductor, in which the current is the vector sum of the two currents in the two members. This is sufficiently accurate for all practical purposes.

Arrangement of Fig. 5. Equating the potential drops from supply to faults by the two paths gives

$$\dot{I}_1 r_1 + 2 j x \dot{I}_1 + j x_1 (\dot{I}_1 + \dot{I}_2) = \dot{I}_2 r_1 + j x_1 (\dot{I}_1 + \dot{I}_2)$$

$$\dot{I}_1 (r_1 + 2 j x) = \dot{I}_2 r_1$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 2 j x}{r_1}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j x}{r_1 + j x}$$ where $\dot{I}_d$ is the differential current.

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + x^2}}$$ where q_d is the per cent differential current.

Let $\frac{x}{r_1} = q_x$ then

$$q_d = \frac{q_x}{\sqrt{1 + q_x^2}}$$ or for a given q_d the necessary q_x is

$$q_x = \frac{q_d}{\sqrt{1 - q_d^2}} \tag{1}$$

Arrangement of Fig. 6. Equating potential drops to fault as before —

$$\dot{I}_1 r_1 + 3 j x \dot{I}_1 + j x_1 \dot{I} = \dot{I}_2 r_1 + j x \dot{I}_2 + j x_1 \dot{I}$$

$$\dot{I}_1 (r_1 + 3 j x) = \dot{I}_2 (r_1 \times j x)$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 3 j x}{r_1 + j x}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j x}{r_1 + 2 j x}$$

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + 4 x^2}} = \frac{q_x}{\sqrt{1 + 4 q_x^2}}$$

$$q_x = \frac{q_d}{\sqrt{1 - 4 q_d^2}}$$

For purposes of comparison with Fig. 5 this result must be multiplied by two since there are twice as many reactors in this case. The relative magnitude of the reactance required will then be —

$$2 q_x = \frac{2 q_d}{\sqrt{1 - 4 q_d^2}} \tag{2}$$

Arrangement of Fig. 7. Equating the potential drops to fault as before

$$\dot{I}_1 r_1 + j \frac{x}{4} (\dot{I}_1 - \dot{I}_2) + j x_1 \dot{I} + j x \dot{I}_1$$

$$= \dot{I}_2 r_1 + j \frac{x}{4} (\dot{I}_2 - \dot{I}_1) + j x_1 \dot{I}$$

$$\dot{I}_1 \left(r_1 + \frac{3}{2} j x\right) = \dot{I}_2 \left(r_1 + \frac{1}{2} j x\right)$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j\,x}{2\,(r_1 + j\,x)}$$

Numerically

$$\frac{I_d}{I} = q_d = \frac{x}{2\sqrt{r_1^2 + x^2}} = \frac{q_x}{2\sqrt{1 + q_x^2}}$$

or, for a given per cent differential current.

$$q_x = \frac{2\,q_d}{\sqrt{1 - 4\,q_d^2}} \tag{3}$$

Conclusion. Referring to equations (1), (2) and (3) it appears that, for the same per cent differential current, the arrangements shown in Figs. 6 and 7 require the same total reactance, and in each case slightly more than double that required by the arrangement of Fig. 5.

As between Figs. 6 and 7, the latter has the advantage, since although the total reactance is the same, it is in two units for Fig. 7 against four units for Fig. 6, which will cost less and take up less space.

Since q_x and q_d are so nearly equal within normal range of values, these formulas supply a simple and convenient method of computing the approximate per cent reactance necessary for any particular differential current. For example, if $q_d = 0.20 = q_x$ (approx.) (Fig. 5), and the normal ohmic drop on the line inquestion is 0.05, the full-load reactive drop due to the reactors will be $0.20 \times 0.05 = 0.01$ or 1 per cent of the line voltage. This is an outside figure.

For the case of Fig. 6, this will be twice as great or two per cent.

In the case of Fig. 7, the reactors offer no appreciable reactance under through conditions.

Saturation of Reactor Cores. The effect of the core saturation is to reduce the reactance of each reactor and also to distort the voltage drop across each reactor.

The reduction of reactance works differently in the three cases. In Fig. 5 it reduces the differential current in almost the same proportion; but in Figs. 6 and 7 the differential current is still further reduced by the fact that the reactance in the short-path to ground, which is undesirable as it reduces the current differential, is reduced less than that in the long path to ground, since its core is less saturated. This difference is however not considerable although, as far as it goes, it increases the advantages of the arrangement shown in Fig. 5.

Since the reactance of the reactors absorbs such a small part of the voltage of the system, the saturation of the reactor cores does not influence the current wave shape appreciably but does seriously influence the wave shape of the voltage drop across each reactor. The flux wave is flat topped and the voltage wave very peaked under conditions of core-saturation.

Under fault conditions this distortion results in very high-frequency harmonic voltages between splits or conductor members, which in turn produce high-frequency charging currents between conductor members, which in part reduces the voltage distortion by supplying harmonic exciting currents for the reactors.

This voltage distortion and consequent danger of breakdown of the secondary insulation will be approximately twice as great for Figs. 6 and 7 as for Fig. 5.

Under normal conditions this phenomenon appears to some extent in Fig. 5, but the currents are not large enough to approach the danger point.

These distortion effects can be considerably reduced by the employment of a gap in the magnetic circuit of the reactors.

DISCUSSION ON "SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION" (COLE), ATLANTIC CITY, N. J., JUNE 26, 1918.

John B. Taylor: What is the practise of this company as regards grounding the neutral, which would determine the extent of the unbalancing on the usual cable fault? That is, how much earth resistance, how much limited current, may there be on the system?

W. H. Cole: Replying to Mr. Taylor's question as to the grounding of the neutral, our 15,000-volt system neutral is grounded through an 8-ohm resistance, which limits the short-circuit current in the case of fault to earth to a maximum value approximating 1000 amperes.

The setting of the relays used with split-conductor cables depends upon what value of current we are willing to allow to develop before the relays can operate. Our split-conductor relays are standardized on the basis of operation at values of fault current not to exceed 100 per cent full line current when the fault is located near one end of a line, or in the most unfavorable position to create a tripping differential current. Under such conditions, the tripping differential current at the opposite end of the line will be equal to about 10 per cent of the fault current, or 20 amperes with a fault current of 200 amperes. The extent of current unbalance in a split-conductor line, due to a fault therein, is affected more by the position of the fault with reference to the length of the line, than by the value of the fault current, the unbalancing being greater the nearer the fault is to the centre of the line. It is also true that a certain minimum value of fault current must flow in any case to produce the required tripping differential current at both ends of a split-conductor line.

W. A. Del Mar: I would like to know whether the extra layer of insulation which is between the two conductors has any appreciable effect on the carrying capacity of the cables?

W. H. Cole: We really know little about it. We believe, however, that we need not rate a split-conductor cable any lower than a standard cable of equivalent conductor cross-section, particularly as such a split-conductor cable will ordinarily have a larger outside diameter. I believe Mr. Meyer of the Public Service Electric Company has had experience which has enabled him to draw some conclusions with respect to the relative carrying capacity of the two types of cable under discussion.

Paul M. Lincoln: We have all been faced of late years with the importance of operating systems in parallel in order to take advantage of diversity factor. As soon as we begin to connect systems in parallel, it becomes all the more essential that faults of cables and other parts of the system shall be eliminated as soon as they occur, and this method outlined here of eliminating these faults as soon as they occur is, I think, one of the most promising that has been suggested. It is a method which is

bound to come into use more and more, as its advantages are understood.

I would like to ask how much more it costs to install cables on the split-conductor plan than if the cables were installed on the straight single-conductor plan? I do not believe that the cost is a great deal more, but if Mr. Cole can give us any specific figures on the difference in cost, I think it would be of interest to all of us.

W. H. Cole: In reply to Mr. Lincoln's question as to costs, I dislike to discuss the matter of relative costs of standard conductor and split-conductor cables, for the very obvious reason that until the demand for a new article of manufacture is well established, it is more or less special and is subject to comparatively heavy development charges.

A prominent manufacturer's cable engineer has informed me that in his opinion a 3-conductor 4/0, 15,000-volt, paper-insulated cable such as is extensively used by our company ought not to cost much above 7 per cent in excess of the cost of a standard conductor cable of equivalent capacity. This figure was quoted when cable of the capacity referred to was selling for approximately $1.00 per foot.

Against the excess cost of a split-conductor cable, we compared the cost of pilot cables required with the input-output method of balanced protection, such pilot cables costing approximately 10 cents per foot exclusive of the cost of duct space required by them. Including the cost of duct space for the pilot cables which will run from ten to fifty or more cents per foot, depending upon the number of such cables installed in one duct, we felt that the split-conductor cable had a better basis for consideration in any scheme of balanced protection for new lines.

In the manufacture of split-conductor cable there is one more stranding and one more insulating operation. Some difficulty has been experienced in saturating the secondary insulation in paper-insulated cables. I am quite sure that manufacturers will soon be able to supply the demand for split-conductor cables and at such a price that no engineer will feel that he is paying an unreasonable price for the sake of having balanced protection.

The excess cost of installation over that of ordinary conductor cables is principally due to the splicing or jointing, as jointing of cables is of such a nature that it ordinarily must be a continuous process to prevent the entrance of moisture. It has not been found expedient for a splicer to make more than one of our standard split-conductor joints in a working day of eight hours as compared with two joints on an equivalent ordinary conductor cable.

Including the extra cost of installation, our experience indicates that split-conductor cable costs at the present date from 10 per cent to 15 per cent more than the ordinary conductor type.

E. B. Meyer: In reference to the question of carrying capacity, I think that Mr. Cole is slightly mistaken in the state-

ment he made. We had quite some difficulty last summer in keeping a number of our tie feeders in operation. Some of these were of the split-conductor type, but were not operating at the time as split-conductor cables, and the failures due to overheating were caused primarily by the peculiar position in which the cables were placed in the duct, and the way it was necessary to operate them. We were carrying the load at that time at a high load factor, and so I am not prepared to say whether the carrying capacity of the split-conductor cable will be reduced. My opinion is it will be about the same as the solid-conductor cable.

R. W. Atkinson: I think I can throw some light on the question of relative carrying capacity of split-conductor and ordinary cables. The difference is so very slight that a theoretical analysis is more direct and conclusive than would be the result of tests unless these were of extended nature; therefore, I will consider the question from that standpoint. The split-conductor cable is larger than the ordinary, both in conductor surface and outside surface; therefore the same number of watts per foot dissipated will produce less temperature rise of sheath above surroundings and less temperature rise of copper above sheath, than in the case of the ordinary cable.

This takes account only of the main insulation. The insulation between members, and the inner member itself will be somewhat warmer than the outer member. The difference is small in actual amount and is unimportant because the inner insulation may safely be subjected to somewhat higher temperature than the outer. When these different considerations are carefully weighed, it is found that the difference in carrying capacity of the two types of cables is entirely negligible, fully justifying the practise of considering them equal.

J. R. Craighead: Mr. Cole says it is practicable to arrange two split-conductor cables, after final adjustment, so that their impedance differential is 0.1 of one per cent. He made the statement a moment ago that the fault current was usually not over 200 amperes, and that this, corresponding to a tripping current of 20 amperes, means that the tripping current is 10 per cent of the total fault current. If that is the case, what is the point of attempting to reduce the differential to as low a point as 0.1 of one per cent. Applied to the 200 amperes, a differential of 0.1 of one per cent would mean 0.2 of an ampere or one per cent of the tripping current of the relay.

This also affects the design of the balancing transformer. It is possible to make current transformers for balancing purposes to practically any degree of perfection of balance that is desirable, but when the requirement is beyond reasonable figures, or such as can be obtained by the use of separate coils insulated from one another by simple means, the expense and the difficulty of design of the transformer are considerably increased. I recently experimented with some transformers, in which a part of the

requested design was that the secondary current due to passing normal primary current through both primary windings in opposite directions should be limited, to about 0.04 of one per cent of the normal secondary current. That is closer than 0.1 of one per cent.

If these figures are necessary we should know why, and how far they are necessary, and if not, it would be well to know about what limit can be practically used in order to keep design down to the cheapest and smallest practicable sizes.

Oliver C. Traver: We have talked about relays with brains and intelligence, etc., and I think that this is the time to feel that we have a relay which has something more than brains, this is a device which has entered into the superhuman class.

I was told a short while ago by one of the engineers of a very large operating company, that if they could operate their system in parallel they would be able to save a million dollars' worth of cable, rather, they could remove from the system a million dollars' worth of cable. This is on the order of being able to pay one thousand dollars a pole for relays in order to accomplish this. That particular company is actually working toward the accomplishment of this vast saving. As operating companies begin to realize the importance of the matter, they stop expecting a twenty-dollar device to hold a thousand-dollar position.

The shorter the run of cable, and the fewer cables in parallel, the greater will the split-conductor method appear to advantage. This is true from the standpoints of both first cost and operation. The longer the split-conductor cable, the greater will be the extra price paid for the protection of the line. This extra price must then be balanced against the cost of the relays of high accuracy and reliability, which are now available, for conditions today are different from "some time ago" when as Mr. Cole suggests, "line protection devices, which function with respect to time, to value of current, or to direction of power flow, were,—exceedingly difficult, and frequently impossible to adjust and maintain" in proper relation one to another.

Longer lines result in higher impedances. Higher impedance puts a limit on fault currents such that a relay can more easily distinguish the fault on which it should function from the one which it should allow to pass.

For multiple lines, balanced groups of relays can be used with greater flexibility, which will give rapid relief from a fault. The more lines in multiple, the easier the problem becomes.

Although two standard cables are more expensive than one split-conductor cable for the same total ampere capacity, still it is also legitimate to take into consideration the advantage of having one workable line in case of trouble in its companion. On the other hand, two standard cables are less costly than two split-conductor cables of the same capacity. So where two or more parallel lines are involved, cost favors the use of balanced protection for these lines, rather than split-conductor protection.

When two standard cables are used, standard types of current transformers which also carry some instrument load can be used in place of the differentially wound current transformers illustrated in Fig. 8 of the paper.

A similar scheme is shown in Fig. 9 and I would like to mention that a "biased" relay is available to select the switch in the line, which carries the greater current. The use of this discriminating device to me, seems to be one of the most promising of the various balanced-relay schemes for parallel line protection.

I have herein suggested some preferable alternatives to the use of split-conductor cables for certain cases. This is not intended to, nor should it, detract from the importance of the work done by Mr. Cole, for I believe the split-conductor cable to be one of the most important developments in selective protection in some time.

If one after, careful investigation, concludes that a system of protection is needed for any line, which will be independent in its selective action of any other part of the system, he should consider split-conductor protection, for it apparently comes nearest to giving complete protection of any scheme so far proposed.

W. H. Cole: In closing the discussion, I will first reply to Mr. Craighead's inquiry regarding impedance differential. In the paper I made the statement that we aim to secure impedance differentials in completed lines not to exceed 0.1 per cent. The purpose of limiting the impedance differential to low values is to insure correspondingly low values of normal differential current under through short-circuit conditions, thereby permitting low settings for the relays. If the relays are set to operate on a differential current equal to 10 per cent full line current, it is obvious that the impedance differential must be small enough to insure that this value of differential current shall not be reached upon the occurrence of any possible value of through short-circuit current, due to faults or conditions beyond the cable under consideration. The reference in my reply to Mr. Taylor, of a minimum tripping differential of 20 amperes with a fault current of say 200 amperes, referred of course to conditions involving a fault in the cable under consideration.

Regarding balancing transformers, their design is a matter to which attention must be given by manufacturers in connection with the production of suitable relays for split-conductor protection.

In general it may be said that the performance of balancing transformers should be such that their "normal differential current" shall not be greater than that permissible in the line itself under through short-circuit conditions. We have made use of standard double-wound current transformers with the primaries connected differentially, which under test with high current values indicate that no extreme or expensive designs are required for the duty under discussion.

Mr. Traver has called attention to the desirability of pairing or grouping ordinary conductors under some conditions, as being more desirable than the indiscriminate use of split-conductors. With this suggestion, I am in full agreement with Mr. Traver, particularly when the system is large and consequently involves a relatively large number of lines between distribution and generating centers. Split-conductor cables, however, are self-contained units and have many advantages not inherent to cables previously considered as standard. There is room for both of the general schemes discussed in the paper. Any choice between them "tends to be an economic one" in practically every case where they are under consideration.

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AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

BY E. B. MEYER

ABSTRACT OF PAPER

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

CENTRAL station companies have had to meet a number of difficult problems during the past three years but the most important has been that of supplying enormous power demands imposed upon them, particularly since the United States has become one of the allies in the European war.

On account of the rapidity with which most of the materials covered by war contracts must be delivered, industrial companies found that the building of isolated plants was out of the question, not only because of the time necessary for erection, but because of the low rates and excellent service furnished by utility companies.

At the present time the central station engineer in dealing with the customer has to provide for thousands of kilowatts rather than hundreds, which were the usual demands previous to the war.

These large demands have made it necessary to solve numerous operating problems in connection with the transmission system and to devise special methods of construction in order to serve the industries upon which the Government is depending to help win the war.

The Public Service Electric Company, which operates in 200 municipalities throughout the State of New Jersey, supplies light and power to approximately 170 manufacturing plants engaged

directly or indirectly on Government contracts. The material furnished under such contracts consists of high explosives, chemicals, shells, textiles, rubber goods, motor cars, castings, wire and cable, wireless apparatus, ships of various types, and many other accessories. In addition to this, the Public Service Electric Company has contracts with the United States Government for furnishing light and power to Camp Dix, Camp Merritt, Raritan Ordnance Depot, Colonia Base Hospital, and the Quartermaster's Department located at the Port Newark Terminal. The supplying of these industries has made enormous demands on the generating system of the company so that at the present time about 80 per cent of the company's output in commercial power is for war work.

One of the special methods adopted by the Public Service Electric Company in meeting war time demands, was that of furnishing the customer with primary service by the use of aerial cable run on poles and supported by messenger wire, a type of construction similar to that used in telephone work.

This type of construction was first used by the company about seven years ago when it was found necessary to connect two large generating stations through tie feeders.

The matter of running overhead wire was considered but found impracticable because the line in several places would have to cross freight yards, trestles, and bridges, and the owners of these structures objected to open-wire high-tension lines.

Most of the section between these two stations was soil of a marshy character, through which it would have been impossible to run a duct line without the use of foundation piling.

It was therefore concluded that the use of aerial cable furnished the most satisfactory solution of the problem. In this installation ordinary lead-covered cable of the same type as that used for underground work was run on a pole line with 50-ft. (15.2-m.) pole spacing. To protect the sheath from mechanical injury there was applied a covering consisting of several layers of jute and marlin with an outer armor of soft steel tape.

The use of lead-covered cable for aerial work was found undesirable, however, because of the excessive weight of the cable and the fact that it could not be installed on standard pole line construction, and a special form of cable was developed to overcome these objections.

In Fig. 1 is shown the modified form of cable for 13,200-volt operation, which is made up with 7/32-in. (5.5-mm.) paper conductor insulation, a 3/32-in. (2.3-mm.) paper jacket and a

4/32-in. (3.17-mm.) reinforced rubber covering over the paper jacket. The reinforced rubber covering is similar in construction to that of the ordinary garden hose, being made up of several plies of fabric and rubber. The entire cable is saturated with rubber compound and covered with tape and a weatherproof braid, thoroughly impregnated with a waterproofing compound. For mechanical protection, the whole core is encased in an armor made up of galvanized steel tape. The use of this form of construction reduces the weight of the cable approximately 50 per cent and permits the use of lighter pole line construction.

Reinforced rubber cable was developed by Mr. Philip Torchio, Chief Electrical Engineer of the New York Edison Co. who has furnished the following information regarding the process of manufacture.

"The process of manufacture of the reinforced rubber covering

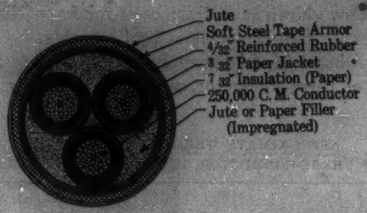


FIG. 1—REINFORCED RUBBER COVERED AERIAL CABLE

consists in calendering both sides of the cotton fabric, previously dried and waterproofed, with a 30 per cent Para rubber compound, so as to obtain a thorough filling of rubber, which, under the process of calendering, becomes partially vulcanized. The prepared fabric is then cut into tapes. These are applied to the electrical conductor in the usual manner, all contact surfaces and interstices being filled with a rubber cement. The insulated conductor is then dried under moderate heat. According to whether the reinforced rubber covering is applied over an insulating layer of rubber compound, or a layer of cambric or paper, the finished cable may or may not be subjected to vulcanization. In the latter case, the partial vulcanization of the rubber in the reinforced rubber is further advanced during the drying process and during leading in the case of leaded cables; otherwise further vulcanization takes place with aging and under service.

The finished material is perfectly homogeneous. Its specific insulating and dielectric constants are lower than those of rubber, paper and varnished cambric insulation, and for that reason, among others, it is preferable to combine a thickness of reinforced rubber with one of the other materials. By placing the reinforced rubber outside a thickness of a higher dielectric compound near the copper wire, the potential gradient is reduced so that the lower dielectric strength of the reinforced rubber does not materially decrease the total dielectric strength of the cable."

Many engineers have been of the opinion that paper insulated cable with the reinforced rubber jacket would not give satisfactory service when subjected to the heat of the summer sun, but in spite of the fact that the cable is exposed to the elements throughout the year the Public Service Electric Company has never experienced a service interruption through the failure of any of the aerial cable in use in the transmission system.

In Fig. 2 are shown several types of reinforced rubber multiconductor cable with and without lead covering.

The following table gives approximate weights and outside diameters of three-conductor cables, insulated for 13,200-volt operation:

APPROXIMATE WEIGHT AND DIAMETER
OF THREE-CONDUCTOR, 13,200-VOLT-AERIAL CABLE

Size	Weight per foot-lbs.	Dia. inches
No. 4	3.50	2.25
No. 2	4.05	2.41
No. 1	4.45	2.50
1/0	4.80	2.57
2/0	5.70	2.66
4/0	6.70	2.91
250,000 cm.	7.05	3.00
350,000 cm.	8.50	3.22

The principal advantage of aerial cable for tie feeder installations is that it makes little difference how many working lines are carried on a single pole line. Additional cable may be run, existing construction changed, transferred or repaired without taking out of service any line except the one on which the actual work is being done. Lightning discharges seem to have little effect because the messenger wire which carries the aerial cable is permanently grounded.

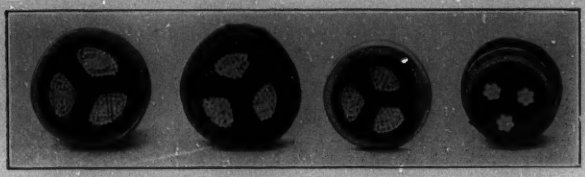
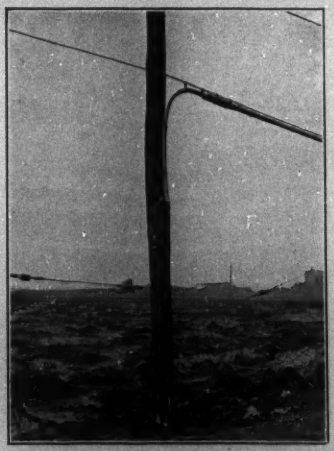


FIG. 2—REINFORCED RUBBER MULTI-CONDUCTOR CABLE

FIG. 3—AERIAL CABLE LINE WITH FIVE 13,200-VOLT CIRCUITS



[MEYER]

FIG. 7—CONNECTION BETWEEN AERIAL AND UNDERGROUND CABLE.

Fig. 3 shows a pole line carrying five 13,200-volt feeders. This line has been in operation for a period of over seven years without the occurrence of a single failure.

The usual aerial cable installation requires the use of Class B chestnut poles, with a normal spacing of from 90 to 100 ft. (27 to 30 m.). Where conditions make it necessary, sections as long as 150 ft. (45 m.) are permissible, but in such cases the adjacent sections should not exceed 130 ft. (39 m.). Sections longer than 150 ft. (45 m.) should receive special attention, and Class A poles should be used on long sections and at points of special strain. The location and frequency of guys is largely dependent on local conditions and can, in most cases, be decided upon by a competent line superintendent.

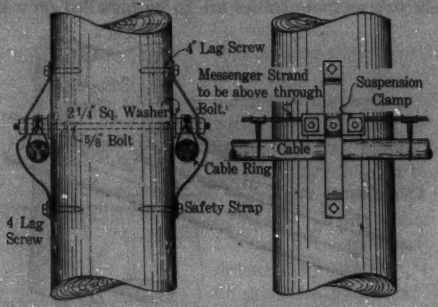


FIG. 4—METHOD OF SUSPENDING MESSENGER CABLE

Attention is called, however, to the fact that the stress at dead ends and corners is very great, frequently being as much as 25,000 lb. (11,339 kg.). These points of special stress need to be well guyed. Both the anchors and the guys should be designed with a factor of safety so high that the messenger will fail before the pole will pull over. In all cases it will be necessary for guy stubs to be reinforced by an anchor guy.

For the suspension of the messenger a double ended 5/8-in. (15.8-mm.) through bolt is recommended, as illustrated in Fig. 4. The use of a safety clamp is also desirable. This clamp serves the double purpose of reinforcing the through bolt and preventing the cable from falling to the ground in case the rings fail. Careful tests made on the method of suspension show that it will withstand the maximum loads to which it will be subjected.

The type of clamp used is similar to that used by the American Telephone and Telegraph Company, the size depending on the diameter of the messenger strand adopted. The clamp is designed expressly for construction of this character and is not built like a guy clamp which is designed to grip two strands instead of one. It affords a greater lever arm for the bolts to work upon in grasping the messenger and supports the messenger strand closer to the bolt, decreasing the bending moment on the bolt due to the weight of the cable.

The messenger strand should always be placed above the bolt in order that the weight of the cable will not be supported by the clamp. Various forms of cable rings may be used in supporting the cable on the messenger wire.

Where two or more cables are to be installed on a pole line,

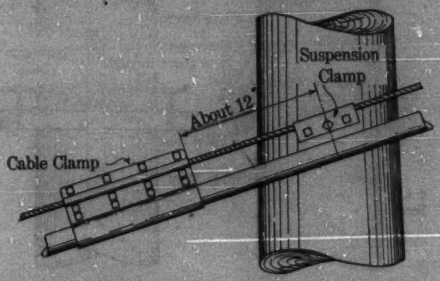


FIG. 5—CLAMPING CABLE TO MESSENGER ON STEEP GRADES

they are usually suspended in pairs, two from each through bolt. The messenger wire is extra strength 5/8-in. (15.8-mm.), seven-strand, galvanized, steel wire. The wire composing the strand should be free from scale, inequalities, splints or other imperfections, not consistent with the best workmanship. It is usual in purchasing galvanized steel wire of this character to have it conform to a specification covering the galvanizing. This is necessary as otherwise inferior grade wire might be obtained.

It is very important that the messenger wire be drawn as tight as possible, in order to prevent sagging when subjected to the weight of the transmission cable. If this is not done, an unsightly installation will result. After the messenger wire has been given its final pull and properly dead-ended, the placing of the aerial cable is the next step. In pulling the cable up to the

messenger wire it is very important that precautions be exercised to prevent mechanical damage or excessive strains which would tend to weaken or damage the insulation.

It is customary in aerial installations to ground the messenger strand. Where the soil is dry or soil conditions unfavorable for grounding, a ground connection should be installed at every second pole. Where the earth is damp and soil conditions are favorable, a ground should be installed at every fourth pole. In marshy ground and in places where conditions are particularly favorable, a ground at every eighth pole will be sufficient. Where possible, this ground connection should be well bonded to some metallic subsurface structure. If this is not possible, the standard artificial pipe ground should be installed.

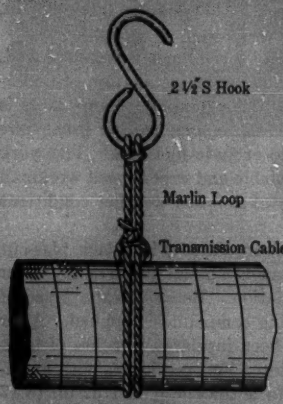


FIG. 6—METHOD OF FASTENING S HOOKS TO AERIAL CABLE

It is also desirable that the steel tape on the cable be bonded to the messenger strand with bonding wire at every cable joint, as proper bonding is necessary in order to furnish the required protection against lightning. Where cable is run through trees and likely to be damaged by abrasion it should be protected by several layers of galvanized tape similar to that later described for use in protecting the joint.

In Fig. 5 is illustrated a method of clamping the cable to the messenger wire. On steep grades where the angle between the cable and the horizontal is greater than 30 deg. the use of such a cable clamp is recommended. This clamp can be made up as required and should be used on every fourth pole. It is designed to take the greater portion of the down hill pull on the cable, which otherwise would be carried by the cable rings.

In erecting the cable, the first reel is set up in the usual manner and the cable run off to the first pole, at which is placed a sheave of approximately 12 in. (30 cm.) diameter, the top of the sheave being located about 5 in. (12.7 cm.) below the messenger wire. On the four or five succeeding poles similar sheaves or cable

rollers are placed, and in feeding out the cable 2.5-in. (6.35 cm.) "*S*" hooks, spaced 18 in. (45.6 cm.) apart, are fastened to it. These hooks are fastened to the cable with a small piece of marlin, made up in a loop knot, as illustrated in Fig. 6.

A lineman is stationed at each pole to change the "*S*" hooks from one side of the pole to the other, which process is repeated until the entire length of cable has been installed in place.

The "*S*" hooks, which were used as a temporary support, are now removed and permanent rings put in place. This is done by a lineman supported on a boatswain's chair, which is moved along the section supported by the messenger wire.

In running the transmission cable, either a motor truck, horses, hand or power winch may be used.

In the splicing of aerial cable, no special means are employed, but the usual precautions observed in the installation of underground cable must be followed. The jointing of any cable is more or less a matter of individual experience and great care must be exercised in all cases to exclude moisture. The work should be carefully done by a reliable and experienced workman and no splicing should be undertaken when weather conditions are unfavorable.

Each conductor of the cable is insulated with black bias-cut varnished cambric tape of a thickness of about 30 per cent greater than the machine applied insulation. Between each layer of tape, varnished cambric insulating compound is applied. After the individual conductors have been insulated a jacket of bias-cut black cambric tape, well painted between layers with an insulating compound, is applied to a thickness of 4/32-in. (3.17 mm.). Over the jacket of cambric tape several layers of the best grade rubber tape, 5/32-in. (3.9 mm.) in thickness, are applied and painted between layers with a high grade rubber compound. The completed joint is then covered with three or four layers of friction tape well painted with rubber compound. The joint is then ready for the application of a soft steel galvanized tape over which is finally applied an outer covering consisting of three or four layers of friction tape painted between the layers with a good grade of waterproof compound.

Where it is necessary to make connection from an aerial cable to an underground system a standard form of lead-covered cable is used and installed in a lateral pipe as shown in Fig. 7. The joint between the underground and aerial cable is made up in the manner just described, and there is slipped over the joint a

FIG. 8—AERIAL CABLE ENTRANCE TO SUBSTATION

FIG. 9—26,000-VOLT AERIAL CABLE INSTALLATION [MEYER]

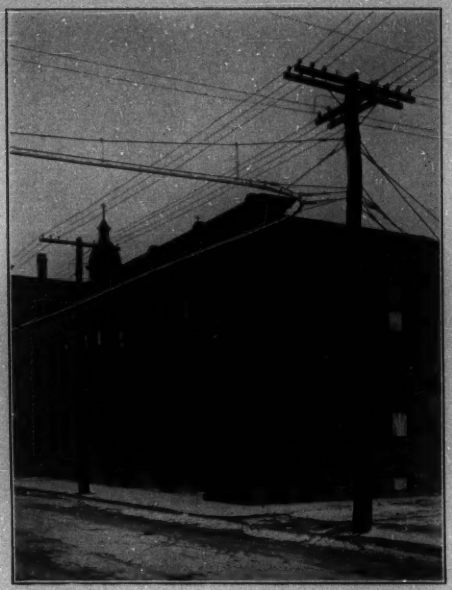


FIG. 10—CATENARY METHOD OF SUSPENDING AERIAL CABLE

[MEYER]

FIG. 11—26,000-VOLT CABLE CROSSING UNDER RAILROAD BRIDGE AND HIGH TENSION POWER CIRCUITS

lead sleeve, one end of which is wiped to the lead-covered cable. The other end is well taped to prevent moisture from penetrating the cable.

Two aerial cables entering a power substation are shown in Fig. 8. In this installation iron pipe laterals are run up the pole and underground lead-covered cable is spliced on to the aerial cable as just described.

While most of the existing circuits are operated at voltages under 15,000, the excellent results obtained with aerial cable has led the company to use this type of construction on all special work for operation at 26,000 volts.

In Fig. 9 is illustrated a completed 26,000-volt aerial cable installation. It should be noted that this method of construction permitted the erection of a high-voltage line without reconstructing the existing low-voltage open-wire distribution circuits.

To keep the cable in good condition it is necessary to paint it every four or five years with some form of insulating paint. This serves to keep the outside jacket from disintegrating and protects it from the action of the elements.

There is in service on the various transmission lines of the Company approximately 65,000 ft. (19,812 m.) of aerial cable operating at 13,200 volts and about 16,000 ft. (4,876 m.) either operating or in course of construction for 26,400-volt service.

Fig. 10 illustrates a catenary method of installing aerial cable. The line so erected was built to furnish 3,000 kw. to a customer who required service within a few weeks after the signing of the contract.

It was impossible within this short length of time to obtain the standard aerial cable with reinforced rubber insulation, and it was found necessary to take ordinary lead-covered cable out of stock.

The erection of lead-covered cable by the methods commonly used in installing aerial cable, on account of the weight and long pole spacing, would have resulted in throwing too great a stress on the messenger wire and lead cable. It was, therefore, decided to use the catenary form of construction so as to reduce the strain with the result that the transmission cable hangs perfectly level and without sag.

In Fig. 11 is shown an illustration of aerial cable crossing under a railroad bridge and electric railway power circuits. Open air potheads or line terminals are installed between the cable and the open wire transmission conductors. No protection in the

form of lightning arresters is provided as after a number of years of operation without failure from any source, the cost of the installation of arresters seemed to be unwarranted.

Aerial cable construction is somewhat more expensive than ordinary open wire construction but its cost is less than that of an underground conduit system. As the costs of the various types are so largely dependent on local conditions, no comparative estimates will be given here. In general, the cost of an aerial cable line is about midway between underground and open wire construction.

While this paper deals primarily with the use of reinforced rubber cables, there are numerous installations throughout the country where other forms of insulation have been used with satisfactory results.

It is not the intention of this paper to recommend exclusively the use of reinforced rubber insulation, but primarily to bring out the fact that aerial cable construction may be used advantageously for power transmission, where open-wire construction would be undesirable and the time and cost of underground construction would be prohibitive.

Where line extensions have to be made over marshy ground which would require the use of foundation piling for a conduit line, over private property such as railroad freight yards, trestles or bridges where very special overhead construction would be necessary, or on important streets on which open wires are not permitted, and subway construction would be expensive or impossible, aerial cable furnishes an ideal form of construction.

DISCUSSION ON "AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION" (MEYER), ATLANTIC CITY, N. J., JUNE 26, 1918.

W. F. Dawson: The author pointed to the possibility of having trouble with the insulation of this aerial cable from blistering in the sun. I want to ask if they have any particular protection for the cable, because it makes great difference in the amount of heat attracted, whether it is coated with a light colored paint or an ordinary tar product. Glass thermometers, exposed in the sun will show 15 to 20 deg. fahr. lower temperature than if they are exposed inside of black cases and I would expect a corresponding difference if black paint were used instead of white or gray.

E. B. Meyer: No particular attention has been paid to that. I realized that there would be considerable difference where the cable had been painted. Some of the cable has recently been covered with a light coating, but we have not had much experience with it so far.

H. L. Wallau: I want to ask the carrying capacity of the aerial installation. Judging from the pictures submitted, in practically all cases these cables have been in series with cables installed in subways, in which case the limiting current will be determined by the underground insulation. I ask if Mr. Meyer has had any of this cable installed exclusively overhead, and can he operate those cables at a higher rating than the subway installation because of that reason?

Another point I notice is that the weight per foot for the 0000 is given at 6.70. Our practise is to use 0000 3-conductor cable either 6 by 6 or 8 by 2 installation, with 1/8 in. lead, if I remember aright, and our cable weighs about 9 lb. per foot, and the weight of the cable listed in the paper is almost 7 lb. per foot, and so I was wondering whether the figure 50 per cent was correct or not.

In reference to lightning protection, has any trouble ever been experienced by the connection of a stretch of cable to two stretches of overhead line?

E. B. Meyer: In reference to current carrying capacity, I do not recall a single installation entirely made up of aerial cable. All of the circuits have more or less underground cable in series with the overhead cable, and, in addition, quite a little of the open wire construction. Our principal difficulty has been the fact that the aerial cable, which, in the past, we have made the same size as the open wire, would limit us to the capacity of the underground and overhead cable sections.

It is to be our practise in future work to increase the size of the aerial cable, and also the underground cable, where in series with the open wire, so as to get the full capacity of the circuit.

There are a number of installations where aerial cable is used in the center of connection to open wire lines. To my knowledge, no difficulty has been experienced, even when subjected to very severe lightning storms.

As to the weight, it is my impression from figures that were prepared, that the aerial cable was about 50 per cent less in weight than the underground cable. There may be some slight discrepancy I have not checked up.

W. A. Del Mar: I am somewhat surprised to hear the last two speakers, especially Mr. Meyer, express an opinion that the limiting feature in carrying capacity is the underground cable. I should have thought it would have been the overhead cable in sunlight. Cable can easily attain a temperature in direct sunlight of 120 or 130 deg. fahr. There is not very much margin between that and the maximum permissible operating temperature of the insulation. It, therefore, seems to me, that in obtaining a result showing the contrary, the underground cables may have been operated at less than their normal carrying capacity.

At pressures higher than 20,000 or 30,000 volts, there would be practically no carrying capacity left after the cables attained the maximum temperature which they could attain in sunlight.

E. B. Meyer: Most of our aerial cable installations are out in the open where they are subjected to the rays of the summer sun, but apparently there is a sufficient circulation of air, and in that case the opportunity of dissipating heat seems to be somewhat better than a cable enclosed in a conduit line, so we have experienced no difficulty in the way of a diminished carrying capacity.

On the 350,000-cir. mil 3-conductor cable, particularly on the tie feeders, we have limited the current to 300 amperes at 13,000 volts, and a number of the 00 cables have carried in excess of 200 amperes without failure. The principal difficulty has been in carrying the current for a long period of time or at a high load factor, particularly where there are a number of cables in the duct.

John B. Taylor: The possibility of working cables to the utmost is very important, when we hear about saving a million dollars by a device that will release some of our cables .

I note that the New York Central Railroad use in the summer time a sort of cheese cloth tent over their cables, so I assume their experience is that cables out in the open have a harder time keeping cool than those under ground. Cables should be painted black or white, it depends on whether the greater amount of heat to be disposed of comes from the sun on the cable or from the copper outwardly. In one case the cable should be painted white for the best results, and in the other case it should be painted black for the best results. I do not know why the New York Central use the contrivance I have referred to, but if there is some New York Central man here he would be able, possibly, to give us some of the reasons which cause them to go to the trouble of stretching several miles of saturated fabric over the lines of cables out doors.

Philip Torchio: I think Mr. Taylor is making a comparison for two different conditions. Mr. Meyer has an open overhead

cable, while the New York Central has a cable installed in a pipe. In the former case there is free circulation of air and, as Mr. Meyer explained, it is pretty hard even with load to raise the temperature of the body as a whole considerably above the temperature of, say, 125 deg. fahr. due to the direct sun's rays, but if the body is enclosed in a pipe and the circulation of air limited, the temperature may go up higher. I think that may explain the difference in results.

J. Allen Johnson: In a certain installation with which I am somewhat familiar, of a 2200-volt lead cable, strung on a messenger wire, considerable difficulty was experienced with the joints. The unequal expansion on the lead cable and messenger cable resulted in the breaking of the sheath at the joint away from the sheath of the cable.

I would like to ask Mr. Meyer if they have experienced any difficulty of that character, and if so, what course has been taken to overcome it?

E. B. Meyer: Answering Mr. Johnson's inquiry, I have heard of a number of cases where lead cable has given some difficulty, particularly at the joints, due to difference in expansion, perhaps, of the wire and the cable too. I will say that in the Public Service Electric Company's installation, particularly the earlier installation, made with the standard form of underground cable, and the splice made up in exactly the same manner as it is made in the manhole, to my knowledge we have never lost a joint, and have had no difficulty in keeping the cable in alignment. The cable is supported with the messenger ring, which clamps under the messenger wire, and I am unable to account for the trouble experienced by other companies. I can only speak of the satisfactory results we have obtained.

H. R. Summerhayes: I was very much interested in this cable installation, Mr. Meyer mentions, as they use all lead-covered cables, and also the cable covered with rubber fabric, and he speaks of other cables, but is it not true that there is nothing else that will stand the out of doors conditions, nothing else but rubber or lead which will keep the insulation dry and will stand the electrical and weather conditions to which the cable is subjected?

E. B. Meyer: Answering the last question, I will say that we are experimenting with other types of cable. There are some insulated with a varnished-cambric jacket with the ordinary rubber jackets on the outside. That particular type has not been in service long enough for us to obtain any definite results with it. I do understand that in Pittsburgh they have used the varnished-cambric cable, and that it has operated in a satisfactory manner, but they also have had some trouble in making up the joints.

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APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

BY G. I. GILCHREST

Abstract of Paper

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is thereafter briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Thereafter, the necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type, as regards the resistance of each to the conditions that cause failure of the insulator in service.

In conclusion, a summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

Introduction

USUALLY any design problem of engineering may be quite easily separated into two rather distinct phases. The one phase is termed "theoretical" and infers that the service experience, processes of manufacture, cost of materials, cost of manufacture, etc., are placed secondary in importance in the search of an ideal design. The other phase is termed "practical" and infers that the design has been evolved mostly from a consideration of service experience. It is generally conceded that a design evolved by either method may have certain advantages. The object of the following investigations has been to link together these two phases in a specific application, namely; the design of pin-type transmission-line insulators.

In order to save repetition, the present article will deal, for

the most part, with laboratory tests of various new designs and the comparison of these designs with those now in commercial use. The theoretical elements of the problem which laid the foundation for the following developments are clearly given in two papers published in the 1913 A. I. E. E. TRANSACTIONS. One paper, by C. Fortescue, is subjected *The Application of Theorem of Electrostatics to Insulation Problems*. The other, by C. Fortescue and S. Farnsworth, is subjected *Air as an Insulator*.

Since an attempt will be made to deal with the practical and theoretical phases of the problem, two logical questions at once arise:

First, are the insulator designs installed in service at the present time satisfactory?

Second, can one type of design be developed that will be satisfactory in all localities?

The first question is answered by a resume of current engineering literature which offers convincing evidence that there is a field for improvement. A comparison of the flashover voltage versus overall dimensions and weight of the present insulator designs would seem to warrant an attempt toward uniformity. Furthermore, the divergence of certain characteristics of some designs from the average curves indicates that some of the designs must be far from efficient.

The causes of such chaos in the present day insulator designs are quite obvious and have been frequently stated. First of all, the progress in transmission engineering has been rapid. The expanding transmission companies demanded insulator designs which would offer a good factor of safety. There was no previous operating experience to use as a basis in new developments and consequently it was often necessary for the transmission engineer to propose his own design. Moreover, the majority of our present insulator types were designed when the electrical and mechanical characteristics of porcelain were less understood than at the present day. As a result, the type of design has fluctuated and various features were, at one time or another, accentuated as most important. That is, at one period a long leakage path was required, regardless of voltage distribution per shell from capacity current or leakage current; then again a high puncture voltage, then a high mechanical strength, and so on. Naturally, many mistakes were made and a large proportion of the older insulator designs have failed in service application.

Causes of Insulator Failures

The knowledge that certain insulator types have failed in service is of little value in the redesign of insulators unless actual conditions of service and cause of failure are known. Also, the cause of failure of a particular type in one locality should be compared to the cause of failure in other localities. Hence, before attempting to develop a new type of design, a study was made of available data on insulator deterioration and the opinions of operating engineers in various parts of the country were obtained.

From discussions with these engineers, from published data on insulator deterioration, and from observations of insulators that had failed in operation, it would seem that the following items are the main causes of pin-type insulator failures:

1. Improper distribution of dielectric field.
2. Improper distribution of surface leakage.
3. Porosity.
4. Mechanical breakage.
   a. From handling.
   b. Mischievous shooting and stone throwing.
   c. Insufficient strength as a support.
   d. Brittle material.
5. Lightning.
6. Birds and animals short-circuiting line.
7. Unequal expansion of metal, cement and porcelain.
8. Internal stresses in material.
9. Defective batches.

Items 3, 4d and 9 are the problems in which the ceramic engineer is vitally concerned and will not be considered further in this paper. These items have doubtless been of great importance in the past but more scientific and painstaking factory control must minimize them in the future. ("Electrical Porcelain", *Electric Journal*, February and March, 1918, by T. A. Klinefelter and G. I. Gilchrest).

The manner in which these items have caused ultimate failure of certain designs are very briefly enumerated below:

1. *Improper Distribution of the Dielectric Field.* Failure to consider the electrostatic field distribution as regards every part of the unit has resulted in designs which have an unequal voltage distribution per shell even when the unit is dry and clean. When the rain sheds are closely spaced the air between them is ionized with the result that preliminary discharges take place before flashover. Parts of the unit in the dielectric field are,

thereby partially short-circuited and other parts overstressed. Flashover voltage will, therefore, be low in relation to overall dimensions. These conditions are usually augmented as the insulator becomes dirty and wet.

2. *Improper Distribution of Surface Leakage.* The most serious trouble from surface leakage is probably experienced on sections of line along the California coast, sections near Great Salt Lake, Utah, etc. Moreover, the climatic conditions of certain localities, especially where a "dry season" is followed by a "rainy season", augments the difficulty. For example, along the California coast the insulator surface gradually becomes coated with dirt during the "dry season" of the year. At night a strong breeze drives a fog containing more or less salt spray over the transmission lines. The dirt on the insulator surface then becomes moist and conducting and a rather high leakage current is often the result. Where wooden construction is used, charring of the wood at points of highest ohmic resistance may take place, resulting in the burning of pins and cross arms. The leakage resistance per shell of many of the older designs are such as to give a very uneven voltage distribution per shell under service conditions. Moreover, the voltage drop over the surface between closely spaced sheds often becomes sufficient to cause static discharges between them. The effective leakage surface of the insulator is thereby decreased and the arcing imposes an electrical impact on the insulator at the same time. Of course, this same trouble occurs on sections of lines near factories, steam railroads, cement mills, smelter plants, etc., to a more or less degree.

3. *Porosity.* The deterioration of porcelain insulators in service was given little consideration during the early days of transmission engineering. The majority of transmission engineers preferred an insulator having a porcelain body which offered a high resistance to mechanical breakage. As a consequence, the porosity of the material, which varies inversely to the mechanical strength as regards resistance to mechanical impact, was considered of secondary importance. The results that the condition has caused in service have been clearly presented before the Institute by Professor H. J. Ryan.[1]

4. *Mechanical Breakage.* Mechanical breakage has been a frequent source of annoyance, and has worked havoc in a number of ways.

1. "*Ceramics in Relation to the Durability of Porcelain Suspension Insulators.*" A. I. E. E. TRANSACTIONS, Vol. XXXV, 1916.

(a) The deep thin sectioned sheds are easily broken in handling. This results in a loss of insulator units. What is of more importance, the danger of installing defective units is considerable, since many fine cracks may pass the usual construction crews' inspection.

(b) Some of the operating engineers, especially those located in the middle West, or near mining camps, claim that 80 to 90 per cent of the defective insulators removed from the line were first injured by rifle shooting or stone throwing.

(c) Many designs are not sufficiently strong as a support due to the thin sections of porcelain and small area under mechanical stress, or to the fact that the deep center shed necessitates a high pin. Such designs fail in service when unusual stresses occur, such as are caused by sleet storms, by poles giving way during freezing and thawing of the ground, or heavy rains, etc.

5. *Lightning.* It is generally conceded that a direct stroke of lightning will destroy any insulator that comes within its path. However, some of the older designs, especially those having deep inner shells and heads of large diameter, were very vulnerable to any sudden impact voltage. In the first place, the impulse ratio (flashover voltage at high frequency divided by flashover voltage at normal frequency) of such insulators is rather high and in the second place the ratio between flashover voltage in air and puncture voltage under oil is comparatively low.

6. *Birds and Animals Short-Circuiting Line.* Some transmission companies have found it necessary to place shields on the poles in certain sections in order to prevent squirrels climbing the poles and short-circuiting the lines. In other localities it has been necessary to increase the height of insulator pins or the wire spacing in order to prevent cranes, eagles, etc., from short-circuiting or grounding the line.

7. *Unequal Expansion of Metal, Cement and Porcelain.* In many cases solid metal pins or heavy cast thimbles have been cemented into the insulator. Apparently the unequal expansion of the metal, cement and porcelain has caused the cracking of the porcelain and ultimate failure of the insulator.

8. *Internal Stresses in the Material.* Corners of small radii and non-uniform sections of the porcelain shells have possibly produced internal stresses in the material during the manufacturing processes and these have developed cracks later on in service. Also, the relation between the shape of the shells in

the cemented area and the shape of the cemented area itself has been such as to allow the full effect of unequal expansion of the porcelain and cement which is caused by temperature changes or absorption of moisture.

Investigation of Dielectric Field

In the papers of Fortescue and Farnsworth, several insulator forms were evolved mathematically, and the dielectric field explored by means of an electrolytic bath. It was believed that the data from which these papers were written in conjunction with the available data of other investigators, of both analytical and experimental nature, afforded sufficient basis from which to formulate preliminary designs.[2]

After a careful summation of the data at hand, it seemed that the logical method of attacking the problem would be to have several theoretical insulator designs produced out of a usual commercial porcelain body. The dielectric field of these should then be investigated under a voltage of approximately the same value that would be impressed in service. Thereafter practical considerations, such as deterioration of the various commercial units in service, manufacturing limitations, etc., should be taken into account with the intent of arriving at a compromise between the theoretical and practical features.

Method of Determining Form of Dielectric Field

The dielectric field was determined by the following procedure: The insulator was fastened rigidly in a position such that the plane of the field to be determined extended horizontally. A piece of fullerboard was fitted over a half section of the insulator in this plane. In all cases the apparatus was so arranged that the cross-arm supporting the insulator was grounded as in service where steel construction is used. Finely divided asbestos was then sifted evenly onto the sheet of fullerboard, voltage at 60 cycles of the desired value applied, and the sheet was gently tapped until the particles had adjusted themselves. Permanent records were obtained by placing a sheet of photographic printing paper over the fullerboard, obtaining the field as above, and exposing the paper after the particles had become arranged.

2. "Distribution of Potential about High-Voltage Line Insulators," by C. T. Allcutt and W. K. Skolfield. *Journal of Electricity, Power and Gas*, June 17, 1916. *An Experimental Method of Obtaining the Solution of Electrostatic Problems with Notes on High-Voltage Bushing Design*, by C. W. Rice. A. I. E. E. Transactions, Vol. XXXVI, 1917, p. 905.

That the stronger portion of the field around an insulator was not disturbed materially by the presence of the fullerboard or the asbestos particles was proven by suspending a piece of finely drawn glass in parts of the field by means of a silk fibre supported by small insulated rods. As nearly as could be checked, the glass indicated the same direction of the field as the asbestos particles.

Theoretical Insulator Designs

The dielectric fields of five theoretical designs were determined. Wherever a customary transmission cross-arm and line-wire are used, there are two principal planes of the dielectric field which show the greatest difference, *i.e.*, the plane of the cross-arm and the plane of the line wire. These two planes are 90 degrees apart and in passing from one to the other the transition is gradual. During the investigation, records were taken of the dielectric field of these two principal planes and of a plane midway between the two. In this paper the diagram taken in one plane of the unit is usually shown. Diagrams of three planes of two designs are shown in order to illustrate the variations that occur. The plane in which the particular field was taken is indicated by the reduced top projection at the upper left portion of each figure.

Dielectric Field Forms and Illustrations of Theoretical Designs

Fig. 1 shows the field form of a bushing having dimensions of ring and rod chosen such as to give maximum breakdown voltage over the surface for the mean diameter of torus ring.

Fig. 2 shows a 60-cycle flashover on bushing of Fig. 1.

Fig. 3 gives the field form of a design using a confocal system of ellipsoids and hyperboloids of revolution.

Fig. 7A, 60-cycle flashover on shape shown in Fig. 3.

Fig. 4, field form between special metal cap and pin as might be used as terminals of a line insulator.

Fig. 7B, 60-cycle flashover between cap and pin as shown in Fig. 4.

Fig. 5, field form of line insulator without rain sheds.

Fig. 7C, 60-cycle flashover on shape shown in Fig. 5.

Fig. 6, field form of pin-type insulator, the porcelain of which has a curvature similar to that of Fig. 5. However, the porcelain body is separated into three sections and metal rainsheds added to give wet arcing distance.

Fig. 7D, 60-cycle flashover on unit given in Fig. 6.

In Table I are given the length of path over the insulator surface between electrodes and the 60-cycle flashover voltage.

TABLE I

Shape in figure	Length of surface		Effective kilovolts flashover voltage		
	Inches	Centimeters	Total	Per inch	Per centimeter
1	4.25	10.8	87	20.4	8.1
7a	6.5	16.5	148	22.8	9.0
7c	8	20.3	115	14.4	5.7

From a consideration of Table I it is evident that a flashover value of between 20 and 23 kilovolts per inch (8 and 9 kilovolts per centimeter) of surface may reasonably be expected if the unit is designed with contours of the surfaces approximating the flow lines of the dielectric field. Of course, the flashover on the unit without rain sheds is somewhat lower, being 14.4 kilovolts per inch. The lower flashover on this unit is due to two conditions, *i.e.*, the porcelain surface does not follow the dielectric field in all planes and the small tie wire produces corona and subsequent static discharges at a relatively low voltage. Placing a static shield on the top of this unit increased the flashover voltage 18 per cent.

With the field form between cap and pin as given in Fig. 4, and the voltage values given above in Table I, theoretical insulator designs could be determined for such electrodes. Such designs should follow surfaces indicated on Fig. 4, as (*a*) and (*b*). The highest flashover voltage for a given surface distance between electrodes would thereby be obtained. Moreover, the flashover voltage of such a unit could be closely approximated if the electrodes have sufficient radius of curvature at points of contact with the insulating material and a good seal is made between the metal and the insulating material.

Modifications of Theoretical Design to Meet Operating and Manufacturing Conditions

Insulators based on such theoretical data would be excellent from the electrical and mechanical standpoints if they were to operate in clean, dry air. However, the commercial insulator must maintain the transmission system during the heaviest of

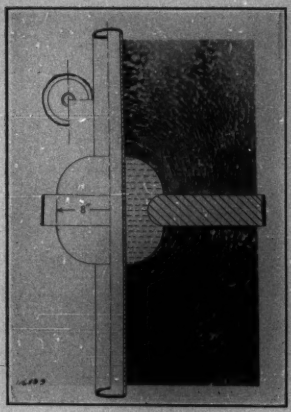


FIG. 1—DIELECTRIC FIELD ABOUT THEORETICAL BUSHING

FIG. 2—60-CYCLE FLASHOVER BUSHING OF FIG. 1

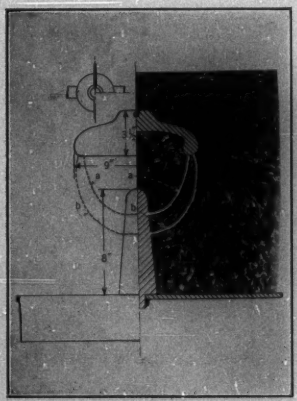
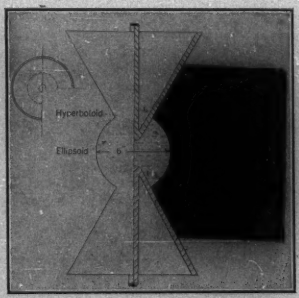


FIG. 4—DIELECTRIC FIELD BETWEEN METAL CAP AND PIN



[GILCHREST]

FIG. 3—DIELECTRIC FIELD ABOUT THEORETICAL DESIGN USING CONFOCAL SURFACES OF REVOLUTION

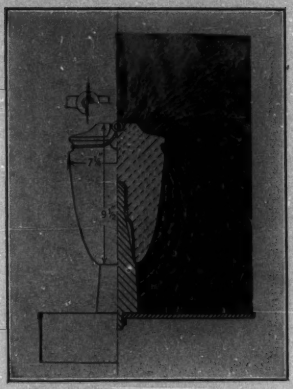


FIG. 5—DIELECTRIC FIELD ABOUT LINE INSULATOR WITHOUT RAIN SHEDS

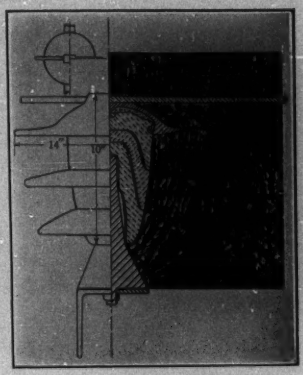


FIG. 6—DIELECTRIC FIELD ABOUT INSULATOR WITH METAL RAIN SHEDS

[GILCHREST]

FIG. 7—60-CYCLE FLASHOVERS ON THEORETICAL PORCELAIN SHAPES

snow and rain storms. Moreover, it must have sufficient leakage distance to prevent flashover or even high power loss from surface leakage when the surface becomes dirty and wet.

The production of one-piece insulators for high-voltage service, although possible, would be costly. Also, the puncturing voltage of a one-piece unit would be low for a given thickness, since the stress in an insulating material between metal electrodes of different potential varies as a logarithmic function. The separation of the unit into parts that are cemented together, more uniformly distributes the stress of the dielectric if the unit is properly designed. It also decreases the probability of complete failure of the insulator and facilitates factory production, lessening the cost of the commercial unit.

The use of a special cap would be desirable from a dielectric standpoint. However, the voltage characteristics under rain are the same whether the usual line and tie wire or a special cap are used. Moreover, the cost and ease of replacement, cost of construction, etc., favor the line and tie wire construction.

Proposed Commercial Insulator Design

With the above limitations of the theoretical designs and the causes of insulator failures in mind, the type of unit indicated in Fig. 8 was evolved.

Summed up briefly this type of design embodies the following features:

1. Surfaces *a* conform to the flow lines of the electrostatic field.
2. Surfaces *b* of the rain sheds conform to the equipotential surfaces.
3. Lines of mechanical stress are parallel to the electrostatic flow lines.
4. The leakage resistance per shell is about equal, being increased gradually from the head to the center shell.
5. Approximately equal capacity per shell.

Comparison With Older Designs

It is not possible to much more than indicate in the following discussion the methods employed to compare the proposed type of design given in Figs. 8A and 8B with the older commercial insulators. Samples of various commercial designs were produced and were subjected to rather thorough laboratory tests at the same time tests were made on insulators of the proposed design.

It should be noted that the insulators of the new type used in the comparative tests do not exactly correspond to the proportions of Fig. 8. In order to lessen the cost of investigation, insulator sheds of several diameters were obtained from one set of molds by trimming the individual shells before burning. This also accounts for the unfinished appearance of the edges of sheds, etc., in some of the experimental designs.

In the following comparison it is not assumed that the evolved design should be final in each detail. The main goal toward which work is being directed is uniformity of all the elements entering into the designs with the idea in view of arriving at a

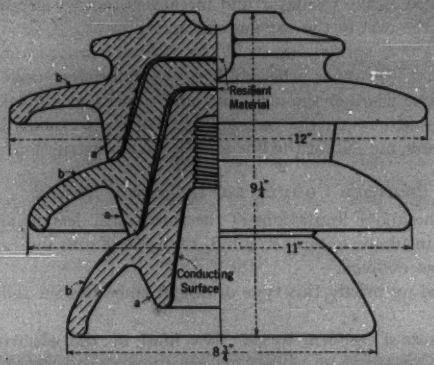


FIG. 8A—THREE-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN—INSULATOR *A*

type of design which will be equally successful in resisting failure in service whatever the requirements are in that particular section. In the following comparisons the items causing failure in service are discussed in the order given at the beginning of the paper.

1. *Dielectric Field Distribution.* The shortest air path under electrostatic stress should be at least long enough to prevent overstressing of the air at any point. In the theoretical discussions referred to in the introduction it was proved that wherever porcelain and air are in series in a dielectric field the voltage gradient per unit distance through the porcelain will be ¼ to 1/5 the voltage gradient through the air. It is obvious that

any thin section of air between porcelain sheds of a customary line insulator will be over-stressed even at the normal line voltage of the insulator.

In order to make a comparison of the dielectric fields of various insulators, their field forms were determined as in the investigation of the theoretical designs. It is believed that the following field forms and illustrations sufficiently indicate that many present types have not been designed with a full appreciation of the advantages of shapes that conform to the electrostatic flow lines in obtaining the most efficient distribution of the stresses in the dielectric field.

Fig. 9 (insulator *C*) gives the dielectric field of a unit of the type used in the early developments of high-voltage trans-

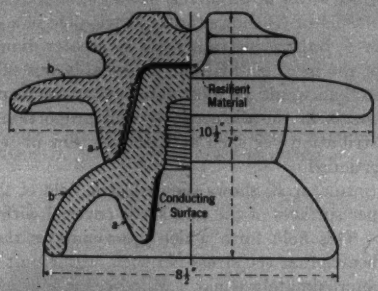


FIG. 8B—TWO-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN A—INSULATOR *B*

mission. The air between sheds just below the cement section is highly stressed. Because of the height of the pin in proportion to other dimensions of the unit the stress toward the base of the pin and the supporting cross arm is very low. Moreover, the third shell of the insulator is spaced so close to the insulator pin that it does not take its proportion of voltage stress when either dry or wet flashover occurs.

Fig. 10 (insulator *D*) shows the dielectric field of a three-piece insulator of a somewhat more recent design. The center shed is better spaced than in insulator *C*. However, the air just below the cement sections is highly stressed and the short rain shed of the second shell gives an unequal voltage distribution at flashover, dry or wet.

Figs. 11, 12 and 13 (insulator *E*) show the dielectric field of a four-piece unit of comparatively recent design. The sheds of this design are more uniformly spaced, but the air between sheds just below the cement sections is highly stressed. The stress throughout the dielectric field of this unit is an improvement over the types *C* and *D*. However, the short second shed and protected fourth shed give unequal voltage distribution at flashover dry or wet.

Figs. 14, 15 and 16 (insulator *F*) show the dielectric field of a unit of the proposed design. The shortest air path between shells is sufficient so that the air is not overstressed at working voltage of the insulator or until flashover occurs. Moreover, the rain sheds are so spaced that each section of the unit takes its share of the stress at flashover, dry or wet.

Fig. 17 (insulator *G*) shows the dielectric field of a unit similar to insulator *F*, but having rain sheds of greater diameter. The diameter of the head of this unit is probably out of proportion and greater than would be most satisfactory for service. However, the stress in the dielectric is well proportioned and the voltage distribution per shell at flashover, dry or wet, is fairly well proportioned.

Fig. 18 (insulator *F*) shows the dielectric field of the insulator having upper surfaces of the rain sheds covered with a conducting paint. This field form which approximates the rain conditions indicates that the stress per shell on the unit during rain would be approximately equal. Moreover it indicates that the stress in the dielectric field is more uniform during rain.

Fig. 19 (insulator *F*) shows the dielectric field of the insulator when equipped with Nicholson Arcing Rings, and indicates that the most highly stressed portion of the field about the insulator is not changed. However, the most highly stressed portion of the field between the line wire and cross arm is now between arcing rings and flashovers would, therefore, occur between rings.

Fig. 20 (insulator *F*) shows the dielectric field when static shields are placed at the top and base of the insulator. This combination would give a very fine distribution of stresses in the dielectric but would be rather expensive commercially.

60-Cycle Flashover Tests

Flashover on most of the older insulator types is caused by the corona formation at the line and tie wires and the edges of the

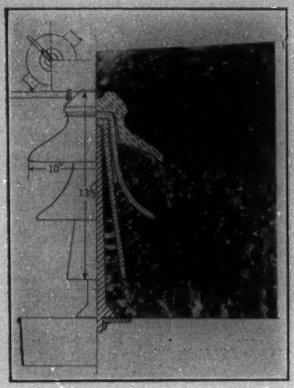


FIG. 9—INSULATOR *C*—DIELECTRIC FIELD ABOUT LINE INSULATOR OF EARLY DESIGN

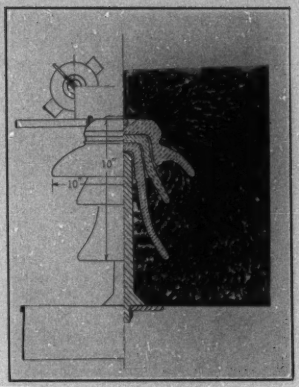


FIG. 10—INSULATOR *D*—DIELECTRIC FIELD OF INSULATOR OF FAIRLY RECENT DESIGN

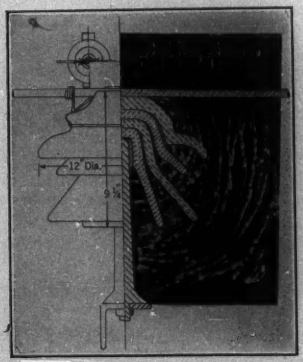
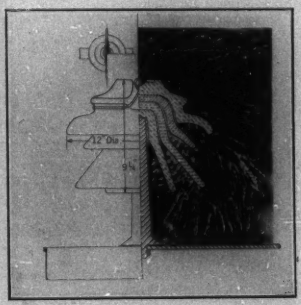


FIG. 11—INSULATOR *E*—DIELECTRIC FIELD ABOUT INSULATOR OF RECENT DESIGN



[GILCHREST]

FIG. 12—(SEE FIG. 11)

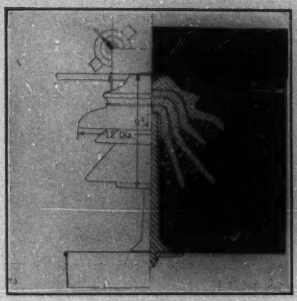


FIG. 13—(SEE FIG. 11)

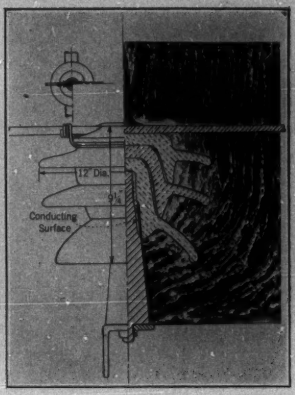


FIG. 14—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

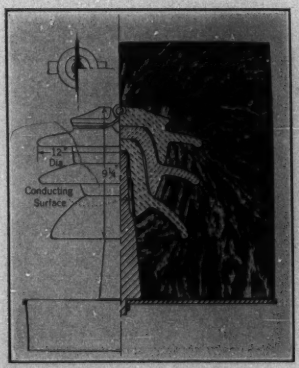
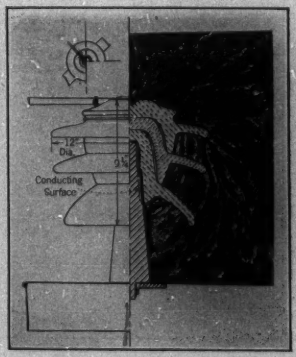


FIG. 15—(SEE FIG. 14)



[GILCHREST]

FIG. 16—(SEE FIG. 14)

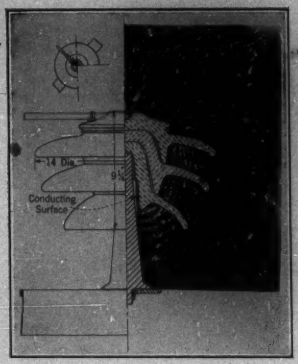


FIG. 17—INSULATOR *G*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

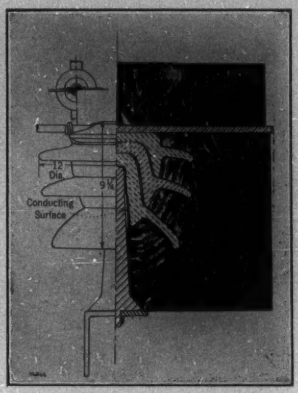


FIG. 18—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN UNDER CONDITIONS APPROXIMATING RAIN

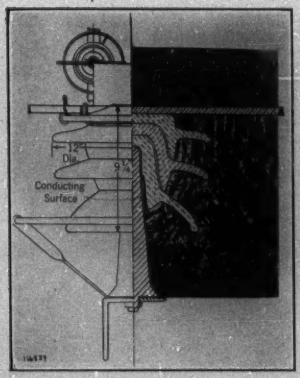
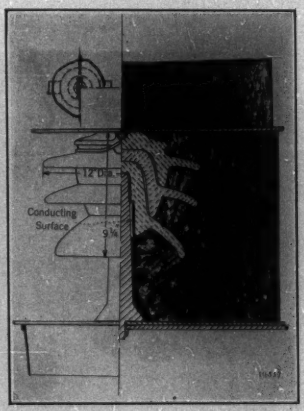


FIG. 19—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN INSTALLED WITH NICHOLSON ARCING RINGS



[GILCHREST]

FIG. 20—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR HAVING STATIC SHIELDS AT TOP AND BASE

FIG. 22—60-CYCLE DRY FLASHOVER

FIG. 23—60-CYCLE WET FLASHOVER [GILCHREST]

cement joints between shells. As the voltage applied to the insulator is increased, the area of the corona formation increases and static streamers gradually spread over the surface of the insulator sheds. The static streamers increase in length until the air insulation between them finally fails and flashover follows. Obviously, the path of the flashover will start along the path of these streamers and thus trouble may be caused by the intense heat of the power arc and rain sheds may be stripped from the insulator.

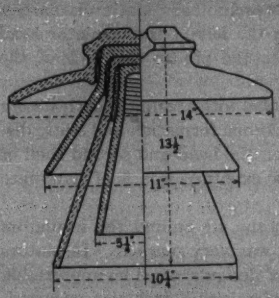


FIG. 21A—INSULATOR *I*

In the proposed type of design there are no static streamers from the edges of the cement section between shells up to flash-over voltage. The corona formation at the tie and line wires therefore, builds up until flashover occurs by breaking down an air path between the line and pin or cross arm. The proof of these statements may be seen in the following illustrations. The axes of the two units in each of the following figures giving comparative flashovers

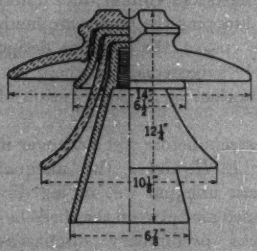


FIG. 21B—INSULATOR *K*

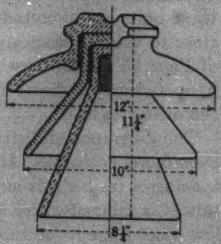


FIG. 21C—INSULATOR *H*

were at the same distance from the camera lens and hence the dimensions are directly comparable.

Figs. 22 and 23 show illustrations of dry and wet flashovers respectively, on insulators *G* and *H*. The design of insulator *H* is given in Fig. 21c.

Fig. 24 shows an illustration of wet flashover on insulators *J*

and *I*. The design of insulator *I* is given in Fig. 21A. Insulator *J* is of the proposed type similar to the unit given in Fig. 8.

Fig. 25 illustrates of one of the early types of high-voltage insulators and insulator *J* in parallel. The finer lines over the surface of the old type unit are preliminary static discharges. The final power arc passes from the left of the insulator head around in front of and finally to the pin at the back of the insulator.

Fig. 26 shows insulator *F* in parallel with the unit of early design shown in Fig. 25, and shows the corona formation and static discharge over the head and between the head and second shell of the old type unit. The camera exposure was $\frac{3}{4}$ of a minute at *F*-8.

The difference in the stress in the air around the insulators just below flashover voltage dry was very marked. Insulators *F*, *G* and *J* of the proposed type of design showed no appreciable corona except at line and tie wires until flashover occurred. Flashover occurred from tie wire or line wire to pin or cross arm, there being no tendency for the arc to start between the rain sheds. Considerable corona formation and static streamers could be detected on insulators *E*, *H* and *I*. Static streamers began to spread out over the surfaces between shells of insulators *H* and *I* at 80 per cent of flashover voltage, and unless these units are mounted on rather low pins the power arc holds close to the insulator surfaces. Of course, the old type design of Figs. 25 and 26 has been entirely superseded but these two figures clearly indicate the entire neglect of a consideration of the dielectric field.

The difference of distribution of stress before wet flashover is even more noticable. In insulator *E*, *H* and *I* the unequal spacing of rain sheds and consequent unequal wet arcing distances, combined with a highly stressed air between the sheds below the cement sections produces preliminary discharges (marked *p*) between rain sheds. These preliminary discharges throw electrical impacts onto parts of the insulator and short-circuit portions of the porcelain between the line and pin. Consequently, when a line surge occurs during a rain storm or when the unit is wet and dirty the factor of safety of these insulators in resisting puncture or flashover is actually no more, and sometimes is less, than it would be minus one of the shells.

Insulators *F*, *G* and *J* of the proposed design show no prelimi-

nary discharges except static from tie or line wires to pin or cross arm. Static discharges (marked *s*) are shown on each of these units. All of the leakage surface and thickness of porcelain between line and pin are, therefore, effective up to failure by flashover.

2—*Surface Leakage.* As previously stated, the leakage surface of many of the older designs gives a very uneven voltage distribution per shell. Table II gives the resistance per shell of various insulators tested during this investigation. The values were obtained by an integration of the surface, *i.e.*, surface resistance equals $S \frac{d\,s}{2\,\pi\,y}$ where $d\,s$ is an element of surface and y the radius of that element from the axis of the insulator.

It is obvious from this table that certain of the older designs, especially those having a short second shell, long inner shells, etc., have a very unequal surface resistance per shell. If the insulator surface becomes dirty and wet so as to pass a leakage current of even a thousandth of an ampere the voltage distribution would depend upon this current and the capacity current could be neglected. The voltage gradient over the insulator surface thus often becomes sufficient to cause discharge between sheds and pin or cross arm or over the short sheds. An electrical impact is thereby applied to parts of the insulator and portions of the porcelain body between line and pin are short-circuited. It is believed that the continued overstressing of parts has been the cause of many insulator failures in the past.

TABLE II

SURFACE RESISTANCE PER SHED IN PER CENT OF TOTAL RESISTANCE

Insulator	Number of shed			
	First	Second	Third	Fourth
A	28	30	42	..
B	45	55	..	..
E	14	13	32	41
F	26	29	45	..
G	26	31	43	..
H	18	29	48	..
I	12	16	32	40
K	15	11	30	44

The surface resistance of the proposed designs as typified by insulators *A* and *B* in Table II is gradually increased from the top

to center shells, the increase being considered as an advantage since the center sheds will usually become dirtiest.

A novel feature of the proposed design is illustrated in Fig. 27 showing insulators *D*, *E*, *F* and *H*. These units were set on a cross-arm line, and tie wire attached as in service, voltage applied and plaster of paris dust blown around them. The surfaces along the lines *a* of the proposed design (Fig. 8) are practically free of dust.

The reason for this is quite apparent. All the force acting in the dielectric field along this surface *a* is tangential and would tend to force the particles to the sheds above or below. The same action was noted when the units were subjected to atomized salt water. This feature would doubtless have some value in dust laden sections since the dust would tend to settle mostly on the lower shed and rain and wind would clean this to some extent.

It is necessary to clean the insulators in long portions of line in certain sections of country as the coast districts of California. It is very apparent that the proposed type of design may be cleaned much more readily and thoroughly than any of the older types.

3. *Porosity.* As denoted previously, the porosity of porcelain is a specific problem of the ceramic engineer rather than the designer. As clearly pointed out in a recent paper[3] by Prof. Ryan we apparently have no method of detecting the very slightly porous material which may cause trouble. Since porosity is a function of the body composition, manufacturing process and burning, even with the most careful production and testing, a small amount of this slightly porous material is not detected. The thicker portions of the porcelain in the cemented area of the proposed type of design should minimize the number of the pieces that will give trouble later in service.

4. *Mechanical Breakage.*

(a) From Handling: The increase of thickness of the rain sheds and addition of a drip edge will materially decrease the percentage loss from this cause.

(b) Mischievous Stone Throwing and Rifle Shooting: (1) The following photographs give comparative flashover voltages dry and wet, on units having various rain sheds broken by throwing a small weight at the insulator. Figs. 22 and 23 show the dry and wet flashovers on insulators *G* and *H* respectively. Table III gives the flashover voltages of broken units in per

[3] loc. cit.

FIG. 24—60-CYCLE WET FLASHOVER

FIG. 25—60-CYCLE DRY FLASHOVER [GILCHREST]

FIG. 26—60-CYCLE TEST—COMPARATIVE CORONA FORMATION

[GILCHREST]

FIG. 27—PLASTER PARIS DUST DEPOSITED WITH INSULATORS UNDER 60-CYCLE VOLTAGE

FIG. 28—TOP SHED BROKEN—60-CYCLE WET FLASHOVER

[GILCHREST]

FIG. 29—SECOND SHED BROKEN—60-CYCLE DRY FLASHOVER

FIG. 30—SECOND SHED BROKEN—60-CYCLE WET FLASHOVER

[GILCHREST]

FIG. 31—SECOND AND THIRD SHEDS BROKEN—60-CYCLE DRY FLASHOVER

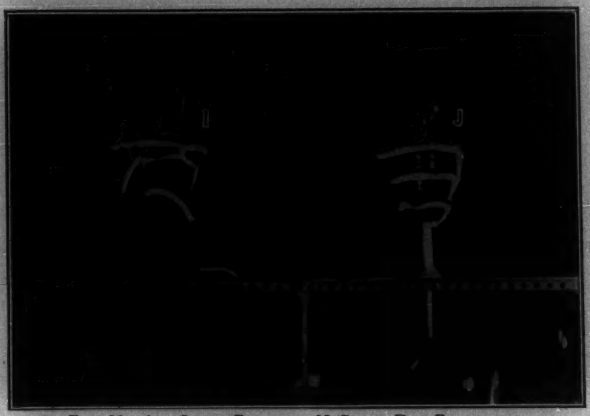


FIG. 32—ALL SHEDS BROKEN—60-CYCLE DRY FLASHOVER

FIG. 33—AFTER 15 SHOTS WERE FIRED AT EACH

[GILCHREST]

FIG. 34—AFTER NUMBER OF SHOTS AS INDICATED

FIG. 35—60-CYCLE DRY FLASHOVERS ON INSULATORS *H*, *I* AND *J* AFTER BREAKAGE FROM RIFLE SHOTS

FIG. 36—RESISTANCE TO SIDE-PULL [GILCHREST]

cent of flashover of the unit when unbroken. Reference to illustrations is made in the Table. Fig. 24 shows the wet flashover on units *J* and *I*. Table IV gives comparative flashovers of broken units, as in Table III.

TABLE III

Sheds broken....	Top		Second		Second and third	
Illustrated in Fig.	28		29	30	31	
Dry or wet.....	dry	wet	dry	wet	dry	wet
Insulator *G*.....	85	80	100	100	68	74
Insulator *H*.....	79	77	82	97	54	70

TABLE IV

Sheds Broken.....	Top	Top and third	All sheds	
Photographs in Fig.....	32			
Dry or wet.....	dry	dry	dry	wet
Insulator *J*.....	87	78	59	30
Insulator *I*.....	85	70	34	15

As would be expected from a study of the dielectric field diagrams, the breaking of the second shed of the proposed type of design has practically no effect upon the flashover values of the insulator. In fact, as shown in the illustrations, the paths of the dry and wet flashovers did not follow over the broken shed. When sheds are broken, the corona formation and static streamers build out over the surface of the older type of design at a lower voltage than when the units are intact. The paths of flashover over these older types, therefore, follow the surface of the insulator. In the proposed type of design the absence of streamers from the porcelain surface causes the arc to keep clear of the insulator. A power arc will, therefore, be less liable to cause complete failure of a broken unit of the proposed design.

One of the most important features of the proposed design is that when the units are hit by stones, etc., the rain sheds will not crack or break beyond line *a* Fig. 8, due to the shape of the individual parts. The rain sheds of the older types of designs when hit are very likely to crack or break up into the cemented section. The first voltage surge or even normal line voltage will, therefore, often puncture the remaining shells. In fact, in the two series of tests photographed, both the older type of units

punctured during the dry arcover after sheds were broken. Static streamers shot over surface of insulator *H* in Fig. 31 and then puncture occurred. Insulator *I*, Fig. 32, flashed over and then punctured before the circuit breaker of the testing transformer operated.

(2) One each of units *H*, *I* and *K* and two of *J* were subjected to rifle shots. Twenty-two caliber long bullets were shot at the insulators from about 30 yards distance and in a line at 45 deg. to their axes. The following photographs show the comparative breakage and the ability of the broken units to thereafter withstand electrical test. The shooting was done by men disinterested in the design of the insulators and they were requested to do as much damage as possible.

Fig. 33 shows insulators *I* and *J* after 15 shots were fired at each. The top, second and third shells of *I* were broken, the second shell being cracked into the cemented section. The second shell of *J* was chipped in two places, the rest of the insulator being intact. Fig. 34 shows insulators *H*, *K* and *I* after 14 shots were fired at *H*, 12 at *K* and 28 at *I*. The second and center shells of *H* were cracked and the center of *K*. The sheds of *J* were chipped off in a few places but the shells were not cracked. These five units were then set with their axes at right angles to the line of fire. Not more than 5 or 10 shots were necessary to strip the main part of the remaining sheds from Insulators *H*, *K* and *I* while one unit of type *J* still retained a considerable portion of its sheds after approximately 100 shots had been fired at it. The sheds remaining on the two units *J* were then knocked off by a hammer, to illustrate to those present that the surface of the insulator that follows flow line *a* would not be cracked thereby.

Fig. 35 shows the first dry flashover test made on these units after the shooting. Units *H*, *K* and *I* punctured at voltage of 33, 43 and 56 kilovolts, respectively. Unit *J* flashed over at 105 kilovolts, the remaining porcelain body bounded by line *a* still being intact.

(*C*) *Insufficient Strength as a Support*: Two samples as per Fig. 36, were tested to determine the resistance to side pull. In each case load was applied at the wire groove which was one foot from the base of pin. The parts from which insulator *L* was formed were obtained by trimming off the rain sheds of individual shells of unit *J* before burning and the mechanical test should, therefore, be about the same as of unit *J*. The pin of

unit L was cemented directly into the insulator. A separable pressed steel thimble was cemented into insulator F. The one-inch bolt of the pin cemented into insulator L failed at 4400 ft-lb., and 3100 ft-lb. bent the pin of insulator F as shown, the position of insulator being such that additional load could not be applied. Both units were electrically intact after these tests.

(d) *Brittle Material.* All units used in these comparative tests were made of the same porcelain body and hence the question of brittleness, which is a ceramic problem, does not enter.

5. *Lightning.* The impulse ratio of the proposed design is lower than that of most of the older types of design. The actual value has not been determined. This statement is based on tests of the proposed design in parallel with older types. The old types of design were set on a pin of such height above the cross arm as to cause the old type of unit to flash over when set up alone at a voltage slightly greater than that of the proposed type of design alone. When tested in parallel the static discharges over the porcelain surface of the older type would often cause the proposed type to flashover first.

Furthermore, the body of the porcelain bounded by the flow lines a should have an impulse ratio close to one. A very high impulse voltage might, therefore, puncture through the rain sheds of the insulator leaving this body of the unit intact. The thicker section of porcelain between line and pin will also materially increase the factor of safety of the unit.

(6) *Unequal Expanion of Metal, Cement and Porcelain.* The introduction of a resilient material between tops of shells should eliminate the tendency of certain older designs to split off. Greater radii of curvature at the tops of the insulator shells and a cement section sloped from the axis should tend to eliminate the trouble from any difference of coefficient of expansion of the porcelain and cement.

(7) *Internal Stresses in the Material.* Internal stresses set up in the insulator parts during manufacture should be very much decreased by the elimination of small radii in corners and sudden changes of cross section of the material.

Conclusions

Briefly stated, it is believed that the advantages of the proposed type over the older commercial types in resisting failure in service would be as follows:

1. When the insulator is dry, the corona and static forma-

tions are practically limited to the tie wire and line wire up to flashover voltage.

2. When the insulator is wet, no corona or static formation occurs up to flashover voltage. The flashover voltages for given overall dimensions are thereby increased.

3. The leakage resistance per shell is increased gradually from the head to the center shell. This takes into account the probability of the lower sheds becoming dirtier than the tops. The voltage distribution per shell is, therefore, equal when the insulator becomes dirty and wet and a heavy leakage current passes over the insulator.

4. Since the capacity per shell is about equal, the voltage distribution per shell will be equal when the insulator is clean and in dry air.

5. Since the distribution of voltage per shell depends upon the capacity current and leakage current, the distribution of voltage per shell in these designs should be approximately equal under all operating conditions.

6. The resistance of the insulator to side pull for a given weight and given electrical strength is relatively high. This is due to the feature of the design whereby the flow line *a* of the electrostatic field and the mechanical stress lines coincide.

7. The design of the individual shells is such that when they are tested before assembly the surface conforms to the electrostatic flow lines *a*. This allows testing of the individual parts to a higher percentage of service voltage than was possible in case of the individual shells of older designs.

8. Due to the shape of individual parts and of the assembled unit, the insulator sheds when hit by stones, riffle, balls, etc., do not break beyond surface *a*. The unit, therefore, offers a considerable percentage of its original resistance to flashover after the sheds are broken. The same feature tends to protect the insulator from complete failure during flashover in service.

9. Each characteristic of the insulator which would vitally affect durability in service has been treated uniformly throughout the line.

DISCUSSION ON "APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS" (GILCHREST), ATLANTIC CITY, N. J., JUNE 26, 1918.

Charles Fortescue: The primary facts to be dealt with in the application of electricity, the two main things, are conduction and insulation. These two facts involve other elements which should be termed the electric circuit, the dielectric circuit and the magnetic circuit. The former and the latter have been given a great deal of attention, but not much attention has been given to the dielectric circuit.

There are two reasons for this. One reason is that in the early days the methods of insulation for low voltages were so obvious that there was not much necessity for giving the dielectric circuit much consideration. Latterly, however, as the conduction problem became more difficult it was necessary to increase the voltages used, and, therefore, for that reason the dielectric circuit became of increasingly greater importance. The problems involved in the dielectric circuit are in three dimensions. They are not easy of solution. They involve some of the most difficult mathematics that are known. Therefore, it is essential to find some practical method of dealing with these problems, and some of these methods have been outlined in various reports from time to time. Mr. Farnsworth and myself brought out a method of plotting out the equipotential surfaces by using an electrolyte. Mr. Rice in a paper a short while ago also mentioned some methods, in fact, he took our method and improved upon it very materially.

Mr. Gilchrest has made use of some of these methods and other methods evolved by other people to produce a practical insulator.

In a paper presented to the Institute some years ago I brought forward an apparent paradox, namely, that by the use of conductors the effectiveness of an insulator can be increased. There is real truth in this. To insulate a body or system of bodies, it is necessary not only to consider the insulating body, but we must also consider the shapes of the bodies to be insulated. We must so form these bodies that they may be insulated with the maximum effectiveness.

In the large porcelain insulator problem, we have to insulate a line wire, we will say, from a cross arm, and we have to evolve an efficient form of insulator, and consider what form of conducting surface will be best suited under the various conditions; that is to say, when it is raining, or when the surfaces are dirty, etc., and this brings up another point, and that is the question of what is meant by theory and practise.

The term "theory" is being used rather loosely to cover an approximate solution of a problem, taking into account what are considered the most important factors. Strictly speaking, theory should account for all the facts, as we know them, and as we observe other facts they are to be introduced into the

theory. There are many ways of doing this. One way is to make as close an approximation as possible with known theory and then find out how much it is necessary to deviate from this approximation in order to take account of, or care for, other considerations that come from actual practical work.

Now, in this insulator of Mr. Gilchrest's, we have to consider, as I said before, the best form for the insulating body and conductors and we must take into account the fact that certain surfaces attract dirt, or dirt under certain conditions tends to accumulate upon them. The problem is to produce something in which the distribution of stress will be affected as little as possible by changes in the external conditions. The working surfaces, properly speaking, should conform to the flow lines, or the Faraday tubes. There is a very good reason for this. One is, it cuts down the actual intensity at the surfaces of the insulator. You do not have as much ionization. Another reason is, since the intensity is parallel to the surface, it is preventive of the accumulation of dirt on the actual working surfaces. On the other hand, if, instead of conforming to the flow lines, the boundary surfaces are at right angles to the flow lines, even if they get dirty, they will have the effect of equipotential surfaces, just the same as if we extended the internal equipotential surfaces out into the air. As we know, if a conducting surface coinciding with an equipotential surface is not very thick, it has almost no influence on the shape of the Faraday tubes.

There are other considerations, such as the proper proportioning of the capacity of each part of the insulator, etc., the theory of which has been presented in papers before the Institute. These have all been taken into account in the design of this insulator and have been dealt with in this paper.

There is a common belief that there are two ways of looking at these matters, the theoretical way and the practical way. I wish to emphasize the fact that there should not be—, that there should be only one way to consider them, and that without casting any reflections upon practical experience, might be termed the theoretical way. In other words, the theoretical way should account for all that practical experience shows.

There is a school which takes the stand that in designing insulators practical experience alone counts; in other words, we are expected to arrive at an efficient design of an insulator by a process of evolution or the survival of the fittest. I think that such a process of arriving at a design is rather hard on the people who have to bear the brunt of being the innocent victims of the experimentors. In a few words, I think a proper consideration of all the facts involved ought to enable one to arrive at a good design, which shall be theoretically correct, and this means, also, that the insulators should stand up in practise, and I believe that the work which Mr. Gilchrest has done is a big step in this direction.

Charles F. Scott: When a new device is described, we do not get the full significance of it unless we get it in historial

perspective. An interesting chapter in insulator history was furnished by the International Electrical Congress, held in 1904 in connection with the Exposition at St. Louis. The Transmission Section brought together the transmission engineers from the Pacific Coast and from the Rocky Mountains, as well as from the East. They were the actual, practical operating men, who came together to interchange experiences and to learn what they could.

The foremost problem in several days of papers and discussions was the insulator. Voltages had risen to 50,000 volts, 60,000 volts, and I think 66,000 volts. They had been of that order for a number of years and no advances had been made. Transformers of double ratio had been installed for 88,000 volts and a lower voltage, but the high voltage was not used because no suitable insulator had appeared.

These papers—some of them specifically on insulators and insulator design, and many of them with regard to operating conditions—tell of insulator troubles, and in discussing the future use of higher voltages, the insulator was the one thing which seemed to set the limit. The pin form of insulator was the only one considered. These apprehensions have been justified by subsequent history, as no plants are operating in this country at any substantially higher voltages with pin insulators than there were at that time. There was no suggestion offered, as I recall now, except some modification of a pin-type insulator, which it was proposed to be make larger and larger, until one engineer suggested that it would resemble a Chinese pagoda in size. There was a limit. The solution came, and the method of operating at higher voltages was worked out by leaving that type of insulator and going to something else, the suspension insulator.

Mr. Gerry, of the Missouri River Power Company, read a paper at this Congress, in which he discussed the insulator, and described a number of elementary experiments to show that the electrostatic field has a good deal to do with the character of the discharge on insulator surfaces, and said that these elements must be considered in insulator design. Dr. Perrine, one of the leading engineers of that time, both in practical experience in one of the largest transmission plants in the West, and also because of his well known theoretical knowledge, said that Mr. Gerry had put his finger on the point to which attention must be given in the future, as we had neglected the electrostatic element in insulator design. The Institute TRANSACTIONS of the past fourteen years since the Electrical Congress contain, almost annually, papers descibing the difficulties and troubles with insulators in one way or another, which have been epitomized very well at the beginning of the paper presented today. One must have all these difficulties in mind to appreciate this paper, which runs along so beautifully, showing how theory has been applied to practise and the

two together have produced something that looks so simple and obvious, that it almost makes us feel that there is nothing striking about it. A new step has been taken in the evolution of the insulator. There have been scores of different types, many of which follow some one idea, but have carried it to an extreme and violated some other principle.

The paper shows very nicely, as Mr. Fortescue pointed out, that as long as we deal with the electric circuit alone we get along very well until we get up into the region of high voltage, and then the electrostatic field comes in; at first its importance was not recognized, but now it has been recognized, and this new solution results.

This paper indicates how admirably the subject has been handled from the standpoint of research and experimental development.

The great value of the paper, then, comes in this notable step in design which leads to a more simple and practicable and desirable form, in a field in which insulators by the thousand, yes, by the million, are in use, and it becomes, therefore, of the highest importance in adding to the safety and reliability of high-tension transmission.

Another point that has not been brought up, which is one of interest and consequence, is this,—if this insulator is so much better for the field which has been occupied by pin-type insulators on the order of 66,000 volts, may not this new design enable transmission lines of still higher voltage to use the pin type of insulator advantageously?

V. Karapetoff: One point that is not brought out in Mr. Gilchrest's paper is the question of refraction of lines of force when they pass from the dielectric of one permittivity κ_1 into a dielectric having a different value of permittivity, κ_2. The lines of electric force experience a refraction similar to the refraction of the magnetic lines of force when they pass at an angle from a ferrous material into the air. This refraction is different from that of rays of light, in that the law which holds here is that of tangents and not of sines.

A college girl in her examination paper, which had the question, "State briefly the law of refraction of light." wrote "The sins of the angels are constant." Well, here it is the ratio of the tangents that is constant.

If the angle of incidence is θ_1 and the angle of refraction θ_2, then the law of refraction is

$$\tan \theta_1/\tan \theta_2 = \kappa_1/\kappa_2$$

For a proof of this law see for example, V. Karapetoff's "Electric Circuit," p. 163 (second edition.) I mention this fact, because it is not mentioned in the paper, yet the field is being studied there outside the insulator, and not within the insulator, while we are primarily concerned with the dielectric stresses within the insulator.

It should not be assumed that by mentally extending the lines

of force as shown by asbestos dust on the outside we can judge about the stresses in the porcelain. This law of refraction should be taken into account. For instance, assume the ratio κ_1/κ_2 to be equal to 5. Then if θ_2 in the air is equal to 45 deg., the angle θ_1 in porcelain comes out equal to 78 deg., so that refraction is of very considerable magnitude. In determining what takes place in the insulator from the field outside the insulator, this law of refraction must by all means be considered, and, therefore, it would be very desirable to evolve a method similar to that outlined in the paper, but in which the medium in which we are experimenting is the body of the insulator itself and not the surrounding air.

I wish to cite in this connection an old experiment in which an electrostatic field is investigated in liquid paraffine, which later is solidified. A very light dust, lycopodium powder which is obtainable in drug stores, is mixed with the paraffine while it is in a molten state. A vessel is filled with liquid paraffine, electrodes of a desired shape are dipped into the paraffine, and an electrostatic field is produced between the electrodes. The individual particles of powder assume the directions of the lines of force. Then the paraffine is allowed to solidify, and it can be cut in any desired plane, and the actual field investigated. This method had the advantage over the method described in the paper in that it is a 3-dimensional method, whereas the method described by Mr. Gilchrest gives results in only one plane at a time.

L. W. Chubb: In Mr. Gilchrest's work on the new insulator, some of the practical men made light of the idea of trying to develop an insulator by scientific and theoretical considerations. Such things as rain, dust, dirt, salt spray, and other things, they said, made the practical considerations paramount.

Mr. Gilchrest has taken the theoretical considerations into account in connection with the practical, and so placed the cemented points and formed the surfaces that the usual disadvantages of the theoretical design are not present.

Prof. Karapetoff's discussion regarding the refraction of the lines of force is very interesting. The refraction was, of course, considered in the design of the insulator. If he takes the limiting values of 90 deg. to zero for ϕ, it can be shown that the refraction does not play any part.

It was also mentioned that we are interested in the field within the porcelain. The shape of the field does not necessarily change appreciably if the insulator is properly designed.

S. Barfoed (communicated after adjournment): In 1914 I attempted to design an insulator making use of the same sources of theory as Mr. Gilchrest mentions, but devoted my attention to the suspension type. It also occurred to me that if the electrical stress in the insulating material was placed at right angles to the stress as usually found in commercial insulators somewhat less attention need be paid to the porosity of the insulating materials. Further, that if cracks were developed running

more or less parallel with the lines of electrostatic force, such cracks would be of approximately the same length as the leakage surface path over the insulator and thus not constitute a puncture in the sense it now is taken. Should this reasoning result in the use of more cheaply manufactured insulating materials, the various processes involved might be conducted with greater certainty as to the results. This thought led to some designs of which some samples are herewith given.

Fig. 1 shows a type of insulator where use is made of fibrous impregnated material of such strength that large mechanical forces can be transmitted. It may be hollow or solid, and its

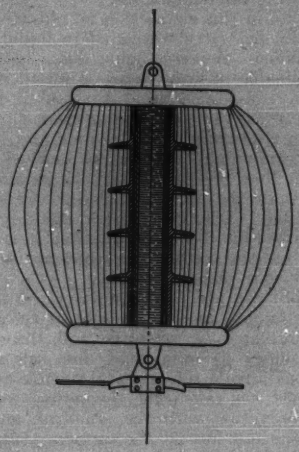


FIG. 1

surface may be covered in any suitable manner. At either end of this structure are placed metallic shields for uniformly distributing an electrostatic field over the surface of the insulating body. If hollow, the interior space may be filled in with an insulating compound or oil to prevent the accumulation of moisture. It is seen that the electrostatic stress is lengthwise of the insulating body instead of transversely thereto, giving the highest possible puncture value.

If this insulator was made sufficiently long no rain sheds need be employed whatever. The outer water-proof covering would merely be a glass or porcelain tube which need not be of the same high class material as now demanded from the insulator manufacturers.

In Fig. 2 the insulating material has been put in compression

instead of in tension as in the former case, but here likewise a control of the electrostatic lines of force over the surface of

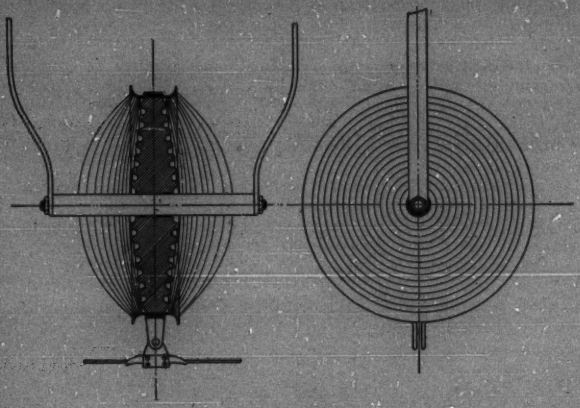


FIG. 2

the insulator is had by means of a circular band surrounding the disk shaped insulator body. The electrostatic stress is lengthwise, instead of transverse to the body.

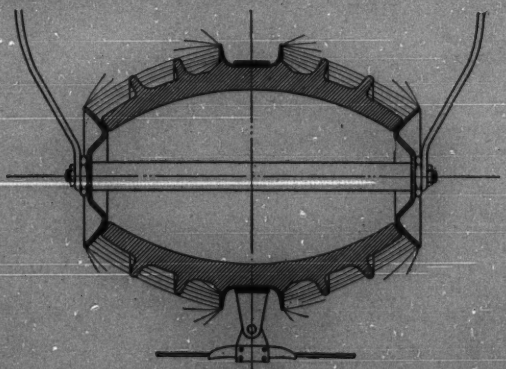


FIG. 3.

In Fig. 3 the same idea is merely carried out in a different manner.

The paper by Mr. Gilchrest is of very great value in showing the lines which must be followed in the future if we are to remove one of the most serious causes preventing commercial progress in high-voltage engineering as applied to the transmission of power.

G. I. Gilchrest: In reply to Mr. Scott's inquiry as to the range of voltages, I might say that we have had units made up which we believe are entirely satisfactory for 66,000-volt operation—that is not a catalogue rating, but actual operating voltage, and we have contemplated going to higher voltages possibly 110,000.

I am inclined to take issue with Prof. Karapetoff in regard to the field inside of the insulator being of vital importance. In the operating field the flashover of the unit is the limiting feature of the design, and that is determined by the distribution of the stress over the surface of contact between the air and the insulator. In this design, if the insulating material is made of sufficiently good dielectric, there is no trouble in the inside of the unit.

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LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

BY CHESTER T. ALLCUTT

ABSTRACT OF PAPER

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap" because of its particular effectiveness in protecting against line disturbances of steep wave front.

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows. Particular reference is made to some of the conditions that may exist when a high-frequency impulse is superimposed on a low-frequency wave. In this connection is shown the desirability of a gap having a selective action, making it sensitive to steep wave fronts, and a number of forms of gap having this selective property are described.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

The results of the tests are highly favorable and indicate that the new gap may have a wide application in the field of lightning protection.

IN GENERAL, a high-voltage lightning arrester consists of two distinct parts: First, a spark gap for discharging abnormal voltages; and second, means for preventing the normal line voltage from maintaining a power arc across the gap. In past years the satisfactory interruption of the power arc has been the most serious part of the lightning arrester problem, and, in consequence, the attention of manufacturers has been concentrated on the development of this feature. It is only comparatively recently that the gap itself has been receiving its due share of attention.

It is quite a simple matter to provide an electrical "safety valve" in the form of a spark gap to discharge abnormal voltages of low frequency, but when we have to deal with steep

wave fronts, or impulse voltages, as they have been termed, there are certain complications due to dielectric spark lag. The existence of the phenomenon of dielectric spark lag was first brought to the attention of the Institute by E. E. F. Creighton[1] more than ten years ago. A later paper by Steinmetz and Hayden[2] gave the results of certain tests on the transient breakdown voltage of different gaps as a function of the energy available. Other papers[3] on impulse tests on porcelain shed some further light on the subject of dielectric spark lag but still left our knowledge of the subject in a rather incomplete state. It remained for F. W. Peek, Jr.[4] to present to the Institute the first quantitative data on the breakdown strength of different forms of gap under the action of transient voltages of carefully predetermined characteristics.

Mr. Peek's extensive investigations have definitely established the fact that some forms of spark gap require a very much higher voltage to discharge a high-frequency impulse than is required to discharge a continuously applied e.m.f. The name "impulse ratio" has been given to the ratio of the impulse breakdown voltage of a gap to the continuously applied breakdown voltage. The impulse ratio was found to vary with the shape of the gap electrodes, the length of the gap and the shape of the impulse wave applied. In case of the needle gap, for example, the impulse ratio was found to be considerably greater than two under some conditions. The sphere gap, on the other hand, was found to have an impulse ratio of substantially unity through a wide range of gap setting. The general conclusion reached by Peek was that a gap in which the discharge is preceded by corona will, in general, have an impulse ratio greater than unity, the impulse ratio increasing with the non-uniformity of the field and with the steepness of the wave front applied.

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1. E. E. F. Creighton. Methods of Testing Electrical Porcelain. A. I. E. E. TRANSACTIONS, Vol. 25, p. 365, May 1906.

2. J. L. R. Hayden and C. P. Steinmetz. Disruptive Strength with Transient Voltages. A. I. E. E. TRANSACTIONS, Vol. 29, p. 1125, June, 1910.

3. A. Chernyschoff and C. A. Butman. Different Methods of Testing Electrical Porcelain. *Electric Journal*, Vol. 12, p. 282, 1915.

L. E. Imlay and P. H. Thomas. High-Frequency Tests of Line Insulators. A. I. E. E. TRANSACTIONS, Vol. 31, p. 2333, December, 1912.

4. F. W. Peek, Jr. The Effect of Transient Voltages on Dielectrics. A. I. E. E. TRANSACTIONS, Vol. 34, p. 1695, September, 1915.

The relation of his experimental work on transient voltages to the subject of lightning protection was discussed by Peek in a later paper[5]. In this paper he indicates the desirability of a low impulse ratio in a lightning arrester gap and shows the superiority of the sphere gap as a protective device, over the simple horn gap usually employed in connection with high-voltage arresters.

Protection Against Impulse Voltages

The papers referred to above have resulted in a great advance towards the solution of the problems involved in securing adequate protection against line disturbances of steep wave front. By the simple expedient of using a lightning arrester gap having spherical electrodes, the operating man may be perfectly sure that the voltage to ground at the point at which the arrester is installed will never appreciably exceed the 60-cycle discharge voltage of the gap. This is a great step in advance, but the question at once arises: Does it give the best possible protection against steep wave fronts? In this connection it must be pointed out that the danger to apparatus from an impulse voltage may be all out of proportion to the actual magnitude of the impulse. Consider, for example, a transformer winding designed for 50,000 volts. If 100,000 volts, at normal frequency, be applied to the winding the stresses on the insulation will only be doubled. But, on the other hand, if a high-frequency impulse of 100,000 volts be applied, the stresses on certain parts of the insulation may be many times normal because of the "piling up" of the steep wave front on the end turns of the winding. Consequently, if we are to protect the winding by means of a spark gap shunting it, the gap should be selective in its action; *i.e.*, it should discharge a high-frequency impulse at a *lower* voltage than an abnormal e.m.f. of line frequency.

The need for the selective action referred to above is emphasized by a consideration of the possible combinations of a high-frequency impulse with the normal line-voltage wave. Let us consider the three combinations shown in Fig. 1, and their effect on a winding protected by a gap.

Case A.—Impulse Occurring at Zero Point of Line E.M.F. Wave. In this case the presence of line voltage does not affect the action of the impulse. The insulation stresses in a winding

5. F. W. Peek, Jr. Lightning, *General Electric Review*, Vol. 19, p. 586, July, 1916.

and the discharge characteristics of a gap protecting the winding will be the same as if the impulse alone were applied.

Case B.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Adding Thereto. In this case the total e.m.f. impressed on the gap and tending to discharge it is equal to the sum of the line voltage and the impulse voltage. The insulation stresses between turns of the winding, on the other hand, are largely dependent on the value of the impulse alone, since line frequency does not cause a high voltage between turns. It would appear that, in this case, the gap affords a better protection

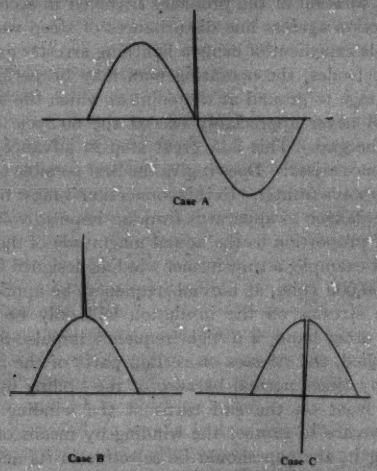


FIG. 1—IMPULSE VOLTAGES SUPERIMPOSED ON 60-CYCLE WAVES

than in Case A. For example, suppose the gap to be set at double line-voltage. In Case A, an impulse of double line-voltage may be impressed on the winding without the gap functioning, while in Case B the maximum impulse that can be applied to the winding without causing the gap to discharge is just equal to line voltage.

Case C.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Subtracting Therefrom. In this case it is obvious that the gap furnishes much less effective protection than in the preceding two cases. A very considerable impulse voltage may

be impressed on the winding without causing any rise in voltage across the gap. The presence of the line-frequency wave prevents the impulse from causing a rise in voltage across the gap but does not prevent the full destructive effect of the impulse from being felt by the winding. Assuming, once more, that the gap is set at double line voltage, it will be seen that an impulse of double line voltage may be applied without causing any rise whatever in the voltage across the gap, while it will require an impulse of *three times* line voltage to cause the gap to discharge.

The fact that such a condition is possible has not been generally appreciated, although it is clear that it may be a source of grave danger. Adequate protection against such a combination of high-frequency and line-frequency demands the use of a gap which is highly sensitive to steep wave fronts. It is true that, if the impulse is oscillatory, the second half-cycle may cause an ordinary gap to discharge, but such a discharge is too late to protect the winding against the destructive effects of the first half-cycle of the impulse.

The Impulse Protective Gap

A rather extensive experimental investigation of impulse phenomena has lead to the development of a new type of gap for lightning arrester service, which possesses in a marked degree the selective properties referred to in the preceding discussion. This gap has been termed the "impulse protective gap" because of its particular effectiveness in discharging line disturbances of steep wave front. The general principle underlying the action of the impulse gap is comparatively simple. In its most elementary form it consists of two gaps in series, each gap being shunted by a relatively high impedance. At line-frequency these impedances are proportional to the respective discharge voltages of the gaps which they shunt, but they are designed to change at different rates with changes of frequency so that, under the action of a high-frequency impulse, one of the impedances becomes much greater than the other and causes most of the high-frequency voltage to be impressed on one of the gaps. The breakdown of this gap will result in the total voltage being impressed on the remaining gap, which will break down in turn.

Fig. 2 is a diagrammatic representation of a gap constructed according to the above principle. One of the two equal gaps, g and g', is shunted by a condenser C' and the other by an equal

condenser C and an inductance L. At line-frequency, the inductive reactance of L is inappreciable compared with the condensive reactances and, therefore, causes no unbalance in the voltage across the two gaps. To a high-frequency impulse, however, the condensers offer very little impedance and most of the high-frequency voltage appears across the inductance L and thus across the gap g. Discharge of the gap g is, in general, immediately followed by a discharge across g'.

It has been found by experiment that better results are obtained by using a single gap, having an intermediate electrode, rather than two distinct gaps. Fig. 3 shows a number of different gaps constructed in this manner. A shows a horn gap having an auxiliary electrode mounted midway between the horns. The auxiliary electrode is connected to one of the horns through a condenser C' and to the other through a condenser C and an inductance L. The action is similar to that of the gap shown in Fig. 2.

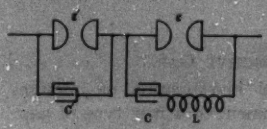


FIG. 2

In place of the reactance L, a resistance may be employed, as shown in Fig. 3B. This is the preferred form of gap, from a commercial standpoint, since the inductance used in A must necessarily be quite bulky, unless condensers of unreasonably high capacity be employed. With the structure shown in B, it has been found possible to secure excellent results with capacities as low as 10^{-11} farads.

Fig. 3C shows another form in which a reactance coil is used as an auto-transformer to produce a larger voltage between the auxiliary electrode and the main horn than is produced by the simple reactance shown in A.

Fig. 3D shows another modification, in which two intermediate electrodes are employed. R_1 is of greater resistance than R_2 so that a high-frequency impulse causes the right-hand gap to break down first, followed successively by the two others. For extremely long gaps it is probable that this form would give better results than B, but for commercial voltages it has been found possible to secure a gap of ample protective power by using but one auxiliary electrode.

Preliminary tests were made on a number of different gaps similar to those shown in Fig. 1. The early experiments included tests to determine the effect of different shapes of electrodes,

both for the main horns and for the auxiliary electrode. The results of these preliminary experiments indicated that the scheme shown in Fig. 3B was the most promising for commercial development. A wire of small diameter (practically a pointed electrode) was found to be most satisfactory for the auxiliary electrode. The pointed auxiliary electrode gave results so much superior to a spherical or cylindrical electrode that further experiment with these latter forms was dropped.

As a result of these preliminary experiments it was decided to make complete tests on the two forms shown in the illustrations, Fig. 4 and Fig. 5. These two gaps differed only in the shape of the horn electrodes. The gap shown in Fig. 4 had

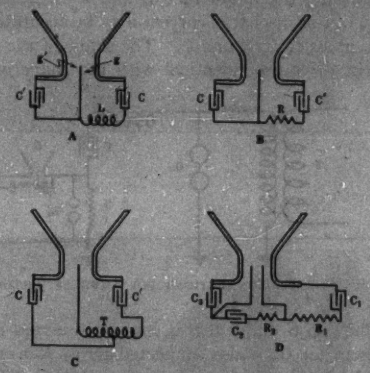


FIG. 3—IMPULSE PROTECTIVE GAPS

plain horn electrodes made of 3/8-in. (0.95-cm.) diameter brass rod, while the other gap had brass hemispheres 6.25 cm. in diameter clamped to the horns, this presenting spherical discharge surfaces. In both gaps, the auxiliary electrode consisted of a blunt point of copper wire 0.025 in. (0.064 cm.) in diameter. During the tests, each horn was supported on a pillar insulator composed of three porcelain insulator units. These pillars rested on a wooden table which brought the gap to a height of about 5 ft. (1.5 m.) above ground. The porcelain units composing the pillars were of special design, having an unusually high electrostatic capacity, approximately 2×10^{-10} farads. Two of these units in series were used as the condensers between

the auxiliary electrode and each of the horns (corresponding to C and C' in Fig. 3B). A water tube was used for the resistance. In most of the tests its value was approximately 100,000 ohms.

Test Methods

The method employed in obtaining impulse voltages of predetermined characteristics was practically identical with that described by Peek in his 1915 paper.[6] Fig. 6 shows a diagram of connections. When the voltage of the transformer T is gradually raised, the sphere gap G_r will eventually discharge. At this instant, the condenser C is charged up to the breakdown voltage of the gap. When G_r breaks down, the condenser discharges through the inductance L and the resistance R. The wave form of this discharge current may be accurately calculated from the values of L, R and C. The shape of the voltage wave across R is, of course, identical with the shape of the current

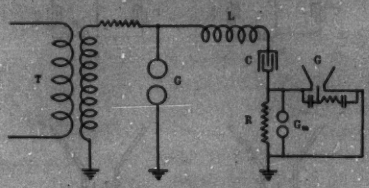


Fig. 6—Connections for Impulse Tests

wave. The gap G under test is connected across the resistance R. A sphere gap G_m is also connected across R in order to obtain a direct measurement of the impulse voltage. The voltage of the impulse applied to the gap under test may be varied by changing the setting of the gap G_r. For a more complete discussion of this method of obtaining impulses the reader is referred to Peek's paper.

The wave adopted as a standard for these tests was a critically damped impulse having a wave front corresponding to a 500,000-cycle sine wave. This wave is shown in Fig. 7. The constants of the oscillating circuit used in producing this impulse were approximately, $C = 10^{-9}$ farads, $L = 0.25$ millihenrys and $R = 1000$ ohms.

The procedure adopted in making the tests was as follows: First, the gap under test was adjusted to the desired setting.

6. Loc. cit. 4.

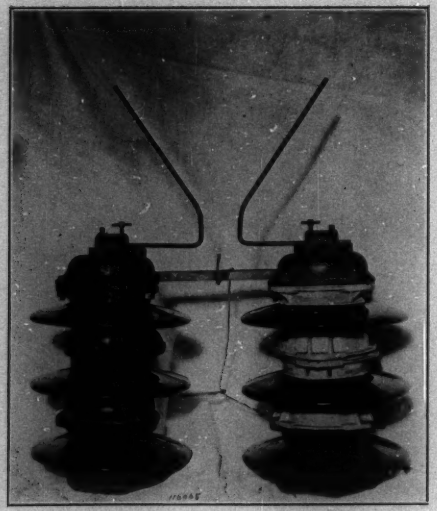


FIG 4

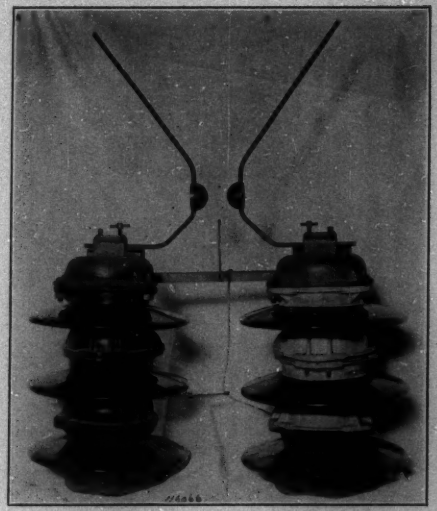


FIG. 5

[ALLCUTT]

Then successive impulses were applied at intervals of from 15 to 30 seconds and the gap G_r adjusted until a setting was found at which about 50 per cent of the impulses would discharge across the gap being tested. Then the measuring gap G_m was adjusted until the impulse discharges were shared equally by it and the gap under test. The setting of G_m that fulfilled this condition was taken as a measure of the impulse discharge voltage of G. Thus it will be seen that the impulse discharge voltage of the gap under test was in every case determined by direct comparison with a sphere gap.

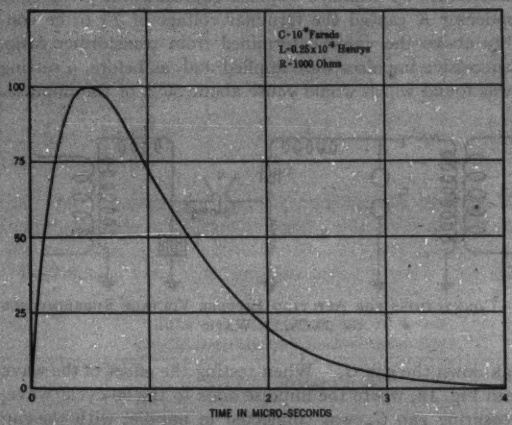


FIG. 7—500,000-CYCLE CRITICALLY DAMPED IMPULSE WAVE

The circuit used in testing the effect of an impulse voltage superimposed on a 60-cycle wave is shown in Fig. 8. Two high-tension transformers T_1 and T_2 are employed. These transformers are excited from the same 60-cycle supply circuit. T_1 applies a 60-cycle voltage to the gap while T_2 excites the impulse circuit which comprises a condenser C, an inductance L and a resistance R. As in Fig. 6, G_r is a spark gap used to regulate the impulse voltage. A condenser K of relatively large capacity is connected across the transformer T_1 external to the protective reactance L_p. This condenser presents a negligible impedance in comparison with the impedances shunting the gap G, so the

full value of an impulse voltage across the resistance R may be regarded as being impressed on the gap under test. If the voltage of the transformer T_2 be slowly raised, the impulse resulting from the discharge of G_r will occur at the peak of the 60-cycle wave. By proper selection of the relative polarities of T_1 and T_2 this impulse may be made to add to, or subtract from, the 60-cycle wave produced by T_1, thus giving a resultant voltage wave, impressed on the gap G, similar to B or C in Fig. 1.

In making tests with the outfit described above, the voltage of T_1 was first set at the desired value. This voltage was carefully determined by means of a sphere gap, since the presence of the condenser K caused the terminal voltage of T_1 to be considerably above the value determined from transformer ratio. Then successive impulses were applied and, as before, a setting of G_r was found which would cause about 50% of the impulses

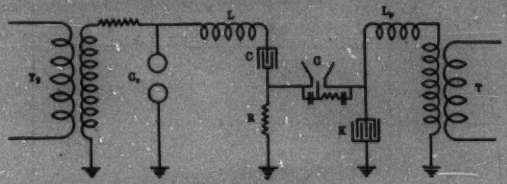


FIG. 8—CONNECTIONS FOR APPLYING IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE

to break down the gap G. When testing the effect of the wave shown in Fig. 1B, where the impulse adds to the 60-cycle wave, the measuring gap G_m was connected in parallel with the gap under test and a direct comparison obtained between the two. In testing case G_r where the impulse subtracts from the 60-cycle wave, the value of the impulse voltage was computed from the setting of G_r and checked by an occasional measurement with a sphere gap connected across the resistance R.

The 60-cycle discharge voltage of the gaps tested was determined in the usual way by comparison with a standard 25-cm. sphere gap.

RESULTS OF IMPULSE TESTS

The first series of experiments undertaken was for the purpose of determining the effect of the presence of the auxiliary electrode on the 60-cycle discharge voltage of the two forms of gap tested. Careful tests were made at gap settings ranging

from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests indicated that the presence of the auxiliary electrode does not appreciably lower the 60-cycle discharge voltage. With the auxiliary electrode in place, breaks were somewhat less consistent

TABLE 1. IMPULSE PROTECTIVE GAP
3/8-in. Diameter Horn Electrodes.

Gap setting, inches	60-Cycle discharge, kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	37	16	16	0.45
1.0	60	28	28	0.47
1.5	73	40	40	0.55
2.0	81	53	55	0.68
2.5	88	67	73	0.83
3.0	93.5	80	84	0.90

than with the plain gap, but the irregularities were not of sufficient magnitude to indicate an appreciable lowering of the breakdown voltage.

Preliminary tests with impulse voltages were then made in

TABLE II. IMPULSE PROTECTIVE GAP
6.25-Cm. Spherical Electrodes Mounted on Horns.

Gap setting, inches	60-Cycle discharge, kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	39	12.5	12.5	0.31
1.0	71	30	30	0.42
1.5	99	47	49	0.50
2.0	121	60	65	0.54
2.5	137	74	82	0.60
3.0	147	85	93.5	0.64

order to determine the best values for the capacitances and resistances used in the circuits connecting the auxiliary electrode with each horn. The results of these experiments showed that the capacitances and resistances could be varied through wide

limits without affecting the impulse discharge voltage. The values of the capacitances employed in the succeeding experiments were, therefore, largely determined by convenience. In practically all the experiments discussed in the following pages, the condenser connected between the auxiliary electrode and each horn had a capacity of approximately 10^{-10} farads. As previously stated, each of these condensers consisted of two pillar-type porcelain insulator units in series (see Figs. 4 and 5). Using capacitances of the value given above, tests made with

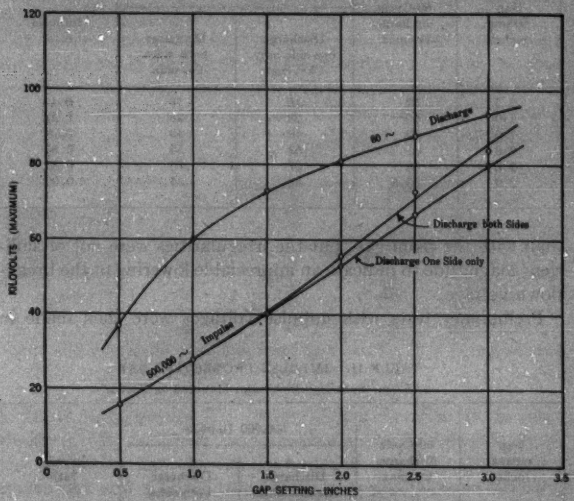


FIG. 9—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP ¾-INCH DIAMETER HORN ELECTRODES

resistances varying from 50,000 ohms to several megohms showed no variation in the impulse discharge voltage of the gap. Accordingly, 100,000 ohms was selected as a convenient value of resistance to use in the remainder of the tests. This resistance is high enough to cause a very great unbalance in the voltage across the two sides of an impulse gap under the action of high frequency, and, on the other hand, is low enough to present a negligible impedance to 60-cycles in comparison with the capacitances employed.

Having decided on the constants given above, the two forms of impulse protective gap shown in Figs. 4 and 5 were tested for both 60-cycle breakdown and 500,000-cycle impulse breakdown, with various gap settings ranging from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests are given in Tables I and II.

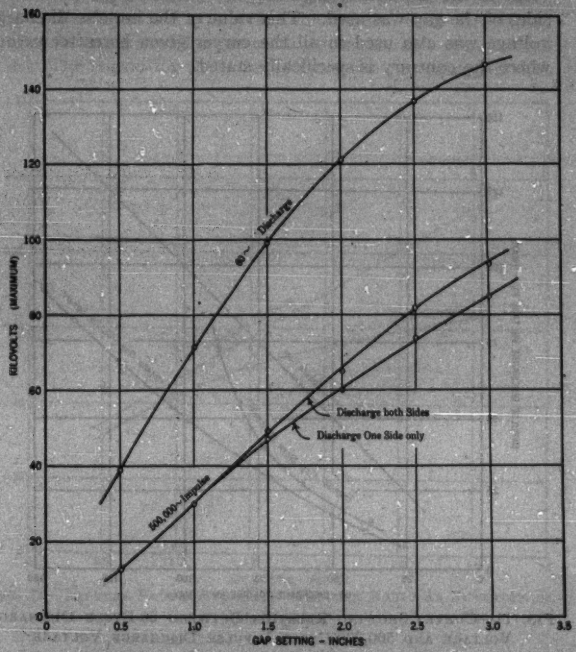


FIG. 10—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP 6.25-CM. SPHERICAL ELECTRODES MOUNTED ON HORNS

It will be noted that, under the heading "500,000 cycle impulse discharge voltage", there are two columns, one giving the impulse voltage which would discharge across one side of the gap only, and the other giving the impulse voltage required to cause a discharge across the whole gap. For small gap settings the difference between these two voltages was found to be too small

to measure; that is to say, successive applications of an impulse of just sufficient voltage to cause an occasional discharge, resulted in part of the discharges being across the whole gap and part of them being between the auxiliary electrode and one horn only. In computing the impulse ratio, given in column 5, the value of the impulse voltage required to discharge across both sides of the gap was used. This value of the impulse discharge voltage was also used in all the curves given hereafter except where the contrary is specifically stated.

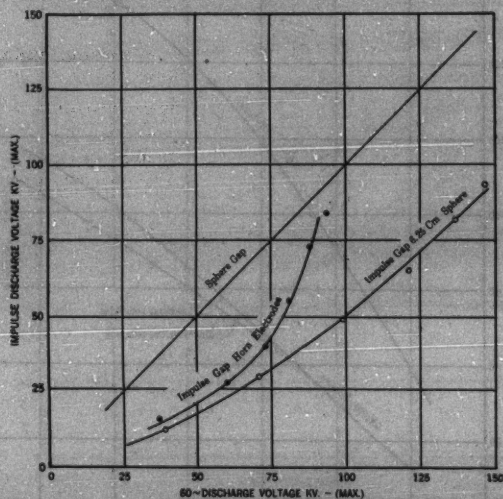


FIG. 11—CURVES SHOWING RELATION BETWEEN 60-CYCLE DISCHARGE VOLTAGE AND 500,000-CYCLE IMPULSE DISCHARGE VOLTAGE

The data given in Tables I and II are presented graphically in Figs. 9 to 12 inclusive. Figs. 9 and 10 give the 60-cycle and impulse discharge voltages as a function of gap setting for the two forms of gap shown in Figs. 4 and 5 respectively. It will be seen that in every case the impulse discharge voltage is *less* than the 60-cycle breakdown, indicating an impulse ratio of less than unity. In Fig. 11, the impulse discharge voltages are plotted against the 60-cycle discharge for the two forms of impulse gap

and for the plain sphere gap. Fig. 12 shows the impulse ratio of the two gaps as a function of gap setting. It will be noted that with the form of gap having the plain horn electrodes, the impulse ratio increases rapidly with the larger gap settings and will probably become higher than unity for gap settings much in excess of 3 in. (7.6 cm.). Where spherical electrodes are used, it is evident from the curves that the impulse ratio will remain well under unity even for gap settings considerably greater than 3 in. The reason for the superiority of the gap equipped with

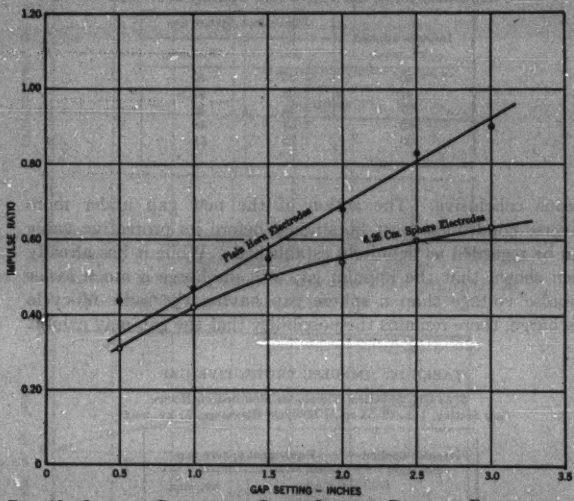


FIG. 12—IMPULSE PROTECTIVE GAPS—IMPULSE RATIO AS FUNCTION OF GAP SETTING

spherical electrodes may be made clear by reference to Figs. 9 and 10. By comparison of these curves it appears that, for a given gap setting, the impulse discharge voltages of the two gaps do not differ greatly. That is to say, a very great change in the shape of the main electrodes does not affect the impulse discharge voltage to a corresponding degree. We know, however, that the shape of the main electrodes will very materially modify the 60-cycle discharge. In view of these facts it is obvious that the lowest impulse ratio will be obtained by so shaping the main

electrodes as to give the highest possible 60-cycle breakdown voltage. This condition is approximated with spherical electrodes.

A consideration of the foregoing results seems to lead to the conclusion that the impulse gap has great possibilities as a protective device. In themselves, however, these results are by no

TABLE III. IMPULSE PROTECTIVE GAP

3/8 in. Diameter Horn Electrodes
Gap Setting, 1½ in. (3.8 cm.); 60-cycle Discharge 73 kv. max.

Impulse applied kv. max.	Equivalent Sphere gap	
	cm.	kv. max.
40	1.30	40
47	1.45	43.5
57	1.55	46
82	1.63	48
100	1.75	51

means conclusive. The action of the new gap under more adverse conditions must be studied before its protective value can be regarded as definitely established. While it has already been shown that the impulse gap will discharge a much lower impulse voltage than a sphere gap having the same 60-cycle discharge, there remains the possibility that the gap may exhibit

TABLE IV. IMPULSE PROTECTIVE GAP

6.25 Cm. Spherical Electrodes Mounted on Horns.
Gap Setting, 1 in. (2.54 cm.); 60-cycle Discharge, 71 kv. max

Impulse applied kv. max.	Equivalent sphere gap	
	cm.	kv. max.
30	0.95	30
45	1.05	32.5
71	1.15	35
86	1.20	37
100	1.20	37

large time lag when an impulse considerably in excess of its discharge voltage is applied, thus permitting the voltage to rise to a dangerous value before discharge takes place. This condition was tested by determining the "equivalent sphere gap" of an impulse gap subjected to a transient voltage greater than its discharge voltage. The results of two series of tests made are

given in Tables III and IV and in the curves in Figs. 13 and 14. In obtaining the above results a sphere gap was connected in parallel with the impulse gap under test. Then a setting of the

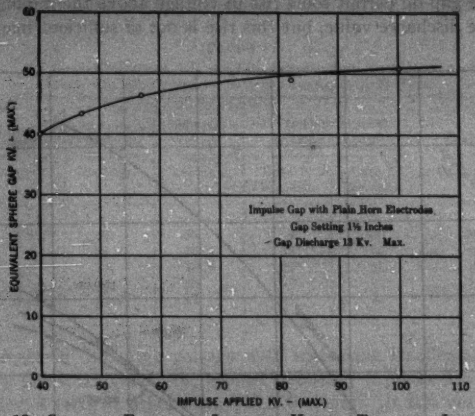


FIG. 13—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

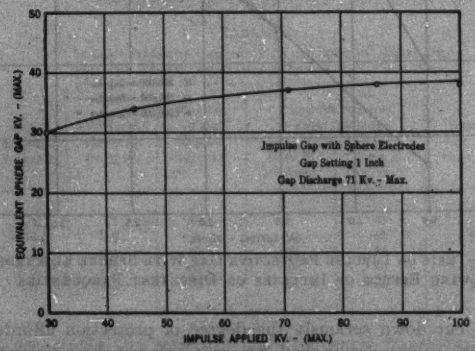


FIG. 14—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

sphere gap was found that would permit the two gaps to share the discharges equally when a given impulse voltage was applied. If we assume the time lag of the sphere gap to be negligible, this

"equivalent sphere gap" setting is a measure of the actual voltage to which the impulse gap permits an impulse of super-discharge voltage to rise. Figs. 13 and 14 show that the two forms of impulse gap do permit some rise in voltage above the minimum impulse discharge value, but this rise is not of sufficient magni-

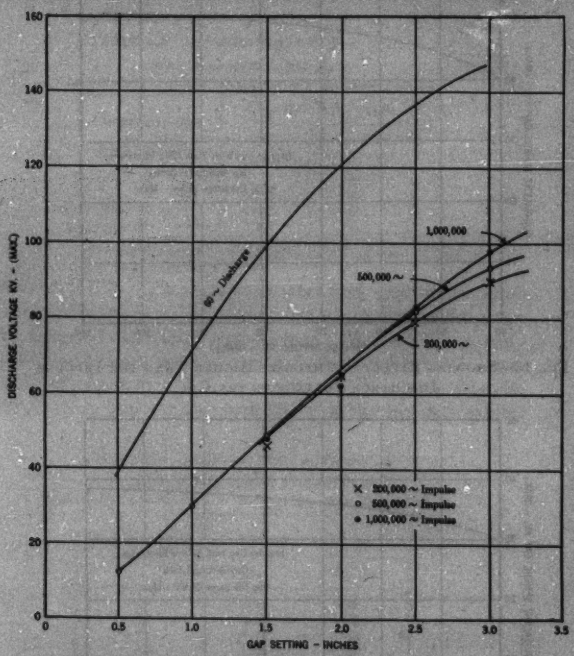


FIG. 15—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERE ELECTRODES SHOWING EFFECT OF IMPULSES OF DIFFERENT FREQUENCIES

tude to cut down seriously the degree of protection afforded. Even with an impulse applied, having a value of three times the minimum discharge voltage, breakdown occurs at very much less than the 60-cycle discharge voltage.

In addition to the above tests, it was thought desirable to undertake further experiments to determine the discharge char-

acteristics of the impulse gap with frequencies of other than 500,000 cycles. Fig. 15 shows the results of some of these tests. It will be seen that the impulse discharge voltage increases slightly with increasing steepness of wave front applied. This

TABLE V.

Impulse Protective Gap with ½-in. diameter Horn Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-Cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	60	30	9	30	36.5
1.5	73	36.5	16	45	44.5
2.0	81	40.5	20	52	49.5
2.5	88	44	40	71	54
3.0	93.5	47	41	90	57

increase in discharge voltage with increasing frequency is not very great, so it was not deemed necessary to make a complete series of tests at other than 500,000 cycles. It is a reasonable assumption that the tests made at this latter frequency will represent a fair average value of the discharge characteristics of

TABLE VI.

Impulse Protective Gap with 6.25 cm. Spherical Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	71	36	9	32	44
1.5	99	49.5	18	47	60.5
2.0	121	60	22	68	73.5
2.5	137	68	32	84	83
3.0	147	73	45	96	89

the gap under the action of high-frequency transients liable to occur in practise.

Results of Tests

Impulse Voltages Superimposed on 60-Cycle Wave

In making tests with an impulse voltage superimposed on a 60-cycle wave, the 60-cycle voltage applied was equal to one-half

the discharge voltage of the gap. With this 60-cycle voltage applied, the value of the impulse voltage required to cause discharge was determined both for an impulse adding to the 60-cycle wave and for an impulse subtracting therefrom. This test may be regarded as closely simulating conditions that may actually occur in practise. Having the gap set for a discharge voltage equal to double the 60-cycle voltage applied, was taken as a fair approximation of the usual practise with regard to the setting of lightning arrester gaps.

The results of the tests made are given in Tables V and VI.

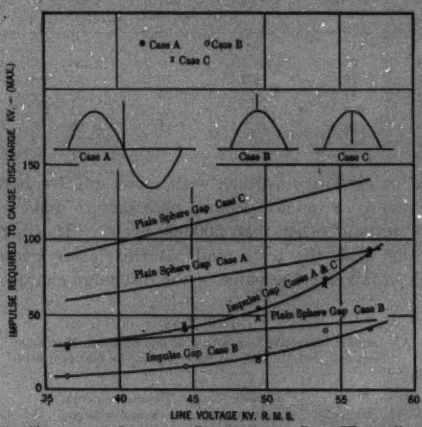


FIG. 16—TESTS ON IMPULSE PROTECTIVE GAP WITH PLAIN HORN ELECTRODES SHOWING EFFECT OF IMPULSES SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

In column 3 are given the real peak values of the 60-cycle voltages applied at the different gap settings. This is, of course, equivalent to the peak value of voltage from one line to ground, which is applied to a lightning arrester gap in service. In order to form a more convenient basis for correlating the test results with service conditions, the root-mean-square value of the three-phase line voltage, equivalent to this peak voltage to ground, is given in the last column. Figs. 16 and 17 show, graphically, the impulse voltage required to cause discharge, as a function of this equivalent line voltage. The curves given for Case A are taken

from the results, given in Tables I and II, for the impulse voltage alone. It is interesting to note that, for both forms of impulse gap, the results for Cases A and C are practically identical. In addition to the curves for the impulse gaps, the characteristics of a plain sphere gap are shown for purpose of comparison. The curves for the sphere gap are computed, assuming an impulse ratio of one. It might be stated here that actual tests on a

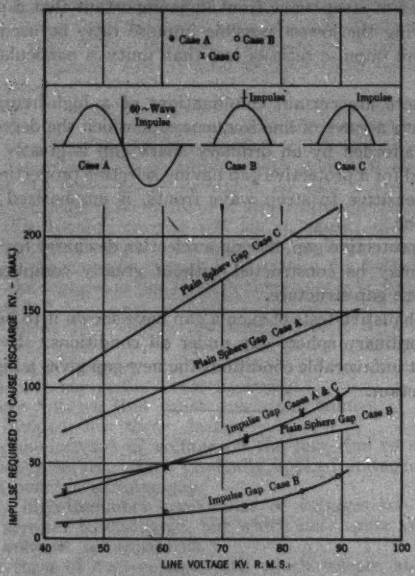


FIG. 17—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERICAL ELECTRODES SHOWING EFFECT OF IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

sphere gap subjected to an impulse subtracting from a 60-cycle wave (Case C) closely checked the computed curves given.

Figs. 16 and 17 are particularly effective in showing the very superior protection afforded by the impulse protective gap when subjected to an impulse subtracting from the 60-cycle wave (Case C). For example, on a 66,000-volt line an impulse protective gap would discharge such an impulse having a value of

but 56 kv. max. (See Fig. 16), while a plain sphere gap would require an impulse of 161 kv. max. to cause a discharge under the same conditions.

Conclusion

The conclusions arrived at from the foregoing experiments may be summarized as follows:

1. In order to secure adequate protection against line disturbances of steep wave front, it is important that a protective gap having the lowest possible impulse ratio be used. A gap having an impulse ratio of less than unity is particularly desirable.

2. There are certain combinations of a high-frequency impulse with a wave of line frequency for which the degree of protection afforded by an ordinary spark gap is greatly lessened. The need for a protective gap having selective properties, rendering it sensitive to steep wave fronts, is emphasized by these conditions.

3. A protective gap, having a selective discharge for steep impulses, may be constructed without greatly complicating the usual horn gap structure.

4. Exhaustive tests of such a gap have shown it to be superior to the ordinary sphere gap under all conditions. Even under the most unfavorable conditions the new gap gives a high degree of protection.

DISCUSSION ON "LIGHTNING ARRESTER SPARK GAPS—THEIR RELATION TO THE PROBLEM OF PROTECTING AGAINST IMPULSE VOLTAGES" (ALLCUTT), ATLANTIC CITY, N. J., JUNE 27, 1918.

Paul M. Lincoln: In the design of lightning arresters, we must recognize two distinct functions. First is the necessity of discharging the surges which come in from outside sources. They must be taken care of, and discharged through a path where they can do no harm instead of being allowed to go through apparatus where their discharge may result in disaster to the apparatus; that is one function. The second function is to let go of the dynamo current after it has started through the lightning arrester.

This piece of analysis apparatus proposed by Mr. Alcutt I want to point out, is of advantage on both of these scores. First, it allows a setting of the arrester gap which will permit the surges of lower value to go through and second, it allows the spacing of the horns to be increased, so that if the dynamo current of the order of 60 cycles or so starts through the lightning arrester, the dynamo current will let go easier; that is, the spacing of the horn gaps is permitted to be larger for the frequencies of the dynamo current. Therefore for both of these functions the suggestion put forth in this paper is of very material assistance.

F. W. Peek, Jr.: Before discussing directly Mr. Allcutt's results, I will briefly review some of my own work which has a bearing on this paper; this review will, I hope, make clearer what I have to say in the direct discussion.

The over-voltages that cause insulation failures in practise may be divided into three classes:

1. Gradual increase of voltage on the line, due to static or low-frequency surges. Such gradually applied voltages readily discharge over an arrester gap.

2. Very high-frequency oscillations of voltages too low for any gap arrester to discharge, but which may cause very high internal voltages in apparatus.

3. The form of over-voltage with which we are at present concerned,—lightning impulses of very steep wave front, when the voltage across the apparatus increases from normal to a very high value in perhaps a millionth of a second.

In 1913 I became interested in determining means of producing impulses of known wave shape and voltage, and in measuring impulse voltages. This work was stimulated by lightning troubles that I was asked to investigate on the lines of one of the large operating companies. In this particular case certain switch bushings with a 60-cycle arc-over voltage of about 100 kv. were protected by aluminum arresters with horn gaps set at about 50 kv., 60 cycles. Lightning always arced over the bushings, and never arced over the arrester gaps. The 60-cycle arc-over voltage of a gap or insulator was

evidently no criterion of the lightning arc-over voltage. It was obvious that a lightning arrester gap should be designed, if possible, for a low lightning arc-over voltage, and insulators and apparatus for a high lightning arc-over voltage. This difference between the lightning and 60-cycle arc-over voltage, illustrated by the above practical case was caused by the "lag" of the gap.

It had long been known that there was some sort of a time lag between the application of voltage and the discharge of a gap, and an interesting paper by Hayden and Steinmetz showed that energy was necessary. The importance of the shape of the electrode on the arc-over voltage was however not suspected. Lightning arresters were, in fact, at that time given the E. N. G. or *equivalent needle gap* test. That the needle gap was just the wrong one to use will appear later. There were no definite

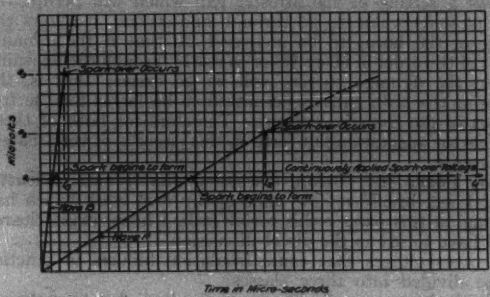


FIG. 1—DIAGRAMMATIC ILLUSTRATION OF WHY THE "LIGHTNING" OR IMPULSE SPARK-OVER VOLTAGE IS HIGHER THAN THE CONTINUOUSLY APPLIED

data in regard to time, the reason being that there was no generator for producing known impulses or a means of measuring them.

From theoretical considerations it seemed to me that the lag or time to break down a gap should depend upon the dielectric field and, therefore, upon the shape and spacing of the electrodes. A gap of a shape and spacing to cause a uniform field should be "fast"; a gap producing a non-uniform field should be slow, for reasons that will be given later. A sphere gap should, therefore, be fast, and a needle gap slow. Time lag manifests itself by an increase in the arc-over voltage as the rate of application of or steepness of the wave front is increased. The mechanism of breakdown is as follows:

When a 60-cycle voltage is slowly applied to a gap, and gradually increased, spark-over will occur at some definite

voltage. This is the minimum voltage that will cause sufficient ionization for the gap to discharge, and it requires a relatively long time.

Lightning voltages, or voltages of steep wave front, start at zero, or line voltage, and increase at the very rapid rate of millions or billions of volts per second. When such a voltage is applied across a gap or insulator, spark-over does not occur at the instant the minimum or 60-cycle spark-over voltage is reached as a considerable time is required at this voltage. When this voltage is reached, the spark begins to form but is only completed after the rapidly rising voltage has reached some higher value. The slower the gap the higher the voltage will rise. (See Fig. 1.). In a uniform field break-down takes place over a relatively short path everywhere at the same time. In the case of a non-uniform field represented, for instance, by the needle gap, corona forms around the electrodes before arc-over. A vast amount of air must be ionized. The condition is equivalent to putting the corona or arc resistance in series with an ever increasing capacity represented by the

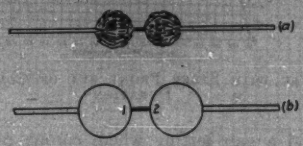


FIG. 2—DIAGRAM ILLUSTRATING SPARK FORMATION ON SPHERES AND ON POINTS

unbroken dielectric. Time is thus required to bring all of the space between the electrodes up to the breakdown gradient. (See Fig. 2).

In order to illustrate this theory experimentally, the sphere horn was devised. The original sphere horn shown in Fig. 13 was built in March 1914. The first tests were made with an oscillator, this oscillator did not give any definite or known wave shape, but illustrated the principle.

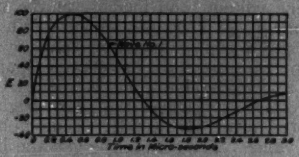
It was realized that in order to make any progress a systematic experimental investigation, to supplement the theoretical one, was necessary. It was only possible to get definite results by means of an impulse generator giving waves of known voltage, wave front, shape, duration, etc. Such a generator was designed and built, and made definite data possible. The results of this investigation were published by the A. I. E. E. in 1915.* A few of the laws resulting are as follows:

1. Two gaps, or insulators, with equal 60-cycle spark-over

*F. W. Peek, Jr. "The Effect of Transient Voltages on Dielectrics." TRANS. A. I. E. E. Aug. 1915. Vol. XXXIV., Part II, p. 1857.
F. W. Peek, Jr., "Lightning," *G. E. Review*, July 1916.

voltages, may have entirely different lightning or impulse spark-over voltages because of the time lag.

2. The time lag is greatest in a non-uniform field or for



APPROXIMATE SINGLE HALF SINE WAVE IMPULSE (ONLY FIRST HALF OF THE WAVE NEED BE CONSIDERED AS THE VOLTAGE IS COMPARATIVELY LOW AFTER THE FIRST CREST)

FIG. 3—IMPULSE WITH STEEP FRONT BUT OF COMPARATIVELY LONG DURATION

electrodes and a gap setting where corona precedes arc-over; it is minimum for a uniform field.

3. The time lag for any given electrodes and spacing is not constant, but depends upon the steepness of the wave or the

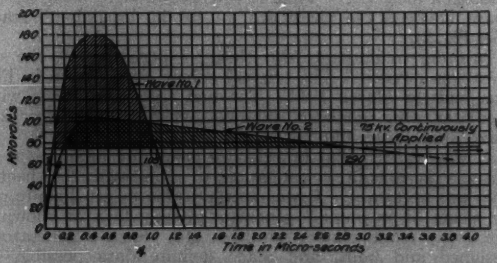


FIG. 4—CONTINUOUSLY APPLIED AND IMPULSE VOLTAGES TO JUST CAUSE SPARK-OVER OF A 10-CM. GAP BETWEEN NEEDLES

rate at which the voltage is applied. The arc-over voltage increases and the time lag decreases with increasing steepness of wave front. The ratio of the 60-cycle to the impulse spark-over voltage was termed the "impulse ratio."

Some of the other interesting facts noted were:

The impulse spark-over voltage, unlike the 60-cycle spark-over voltage, of an insulator or gap is not greatly affected by rain. Wet and dry insulations have practically the same impulse breakdown strength. The corona starting point is practically the same for impulse voltages as it is for operating voltages. When two fast gaps are placed in series there is appreciable lag unless the arrangement is such that both gaps spark-over simultaneously. Corona produced by voltages lasting less then a micro-second can be quite readily seen. In some tests, voltages were used of such a steep wave front that a "drop" of 25 kv. was obtained over a 2-in. copper tube a yard long. Figs. 3 to 8 give some actual curves obtained in the tests.

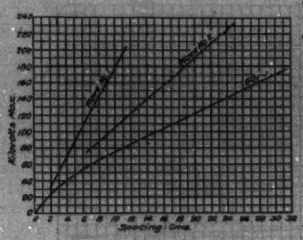


FIG. 5—NEEDLE GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2 SHOWN IN FIGS. 3 AND 4

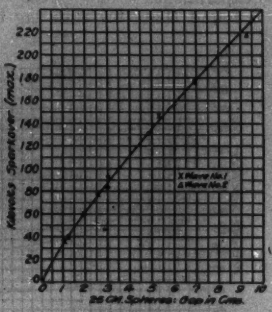


FIG. 6—SPHERE-GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2, SHOWN IN FIGS. 3 AND 4 (DRAWN CURVE 60 CYCLES—POINTS, WAVES NOS. 1 AND 2)

It will be of interest to take a specific example to illustrate time lag:

At 10 cm. spacing between needle gaps, the 60-cycle spark-over voltage is 75 kv.; the impulse spark-over voltage for a given wave is 180 kv.; the time lag is of the order of 0.5 micro-seconds. For spheres set at 75 kv., 60 cycles, the spark-over voltage for this impulse is 75 kv. and the time lag is so small it cannot be detected, it is probably from 0.001 to 0.0001 that of a needle.

Referring to the special form of sphere-horn gap which is the subject of Mr. Allcutt's paper:

During my early work I found that while the 60-cycle and impulse spark-over voltages of a sphere gap were approximately

equal, the impulse spark-over voltage of two spheres in series was much higher than the 60-cycle spark-over voltage.

In other words, there was no measurable lag for a single sphere gap, but considerable lag when two sphere gaps were placed in series. The reason for this is that the two gaps do not break down simultaneously unless the voltage divides between them in proportion to their respective strengths. One gap sparks over and puts the resistance of its arc in series with the other gap. The resistance of an arc depends upon the current flowing in the arc. The resistance remains high because the current is limited by the capacity of the sphere that has not yet broken down. Time is required to charge this capacity up to breakdown voltage through the resistance. This can be illustrated by experiment.

I found that this time lag could be eliminated if the gaps

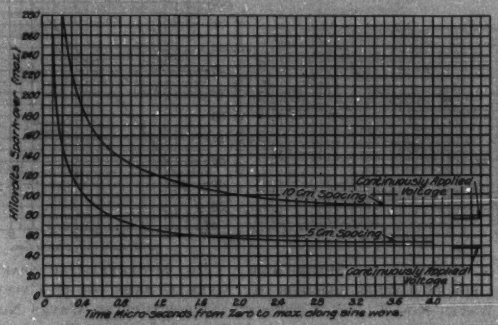


FIG. 7—TIME AND VOLTAGE TO SPARK-OVER NEEDLE GAPS (SINE WAVE IMPULSE)

were made to divide their voltages in proportion to their respective strengths so that simultaneous arcs occurred over both gaps and thus acted as a single arc, as in a uniform field.

In the gap shown by Mr. Allcutt, the division of voltage is not greatly affected under ideal conditions at 60 cycles by the auxiliary electrode. The auxiliary electrode is held at mid-potential because it is connected at the mid-point between the two equal condensers. (See Fig. 9). The capacity current is too small at 60 cycles to cause any appreciable "drop" across the resistance. If the condenser circuit were opened on one side, the gap on that side would break down at 60 cycles at about half voltage. This is exactly what happens under impulse. For steep wave fronts the resistance has the effect of opening the condenser circuit on that side. (See Fig. 10). The gap

on that side breaks down. The voltage does not immediately disappear across the arc. The gap has lag. Corona precedes spark-over. The complete spark-over voltage is higher. Whether it is above or below the 60-cycle setting depends upon the impulse. The effect is the same as that which would result from a needle gap which could be set at, for instance,

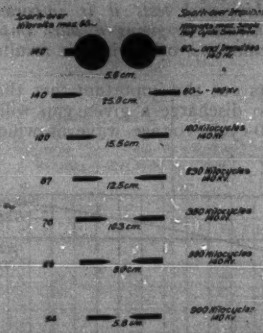


FIG. 8—SETTINGS OF NEEDLE AND 12.5 CM. SPHERE GAPS FOR EQUAL IMPULSE SPARK OVER VOLTAGES OF VARIOUS WAVE FRONTS
(IMPULSE VOLTAGE SETTING 140 KV. MAX.—WAVES SINGLE HALF SINE—100 KILOCYCLES AND 900 KILOCYCLES)

100 kv. for 60-cycle operation and instantly and automatically reduced to a 50 kv. 60-cycle setting whenever an impulse came on the line. For moderately steep wave fronts the spark-over voltage would be greater than 50 kv. and less than 100 kv., but for very steep wave fronts, the impulse ratio of the 50-kv. gap would be greater than two, or the spark-over voltage would be greater than 100 kv. This lag is shown in Mr. Allcutt's

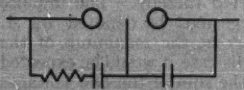


FIG. 9

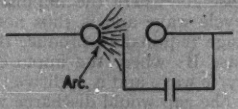


FIG. 10

data, Figs. 13 and 14. Super-voltages are equivalent to voltages of increased wave fronts and increase the spark-over voltage of the gap.

Fig. 11 gives data which I have obtained on such a gap at 3-in. and ½-in. spacings. You will note the characteristics are as discussed above. Starting with a sphere and a selective

sphere at the same 60-cycle setting, 3 in., the sphere arc-over voltage remains practically constant for any wave front. At a moderate wave front the selective-gap has about 5 to 10 per cent lower arc-over voltage than the sphere, while for the steeper wave fronts the voltage is higher. *For steep wave fronts the sphere is the superior gap.* My tests show less advantage for the selective gap than Mr. Allcutt's. (See tests, Tables I and II. *In making such tests it is important to be sure that the impulse generator is giving a pure impulse and there are no secondary oscillations,* otherwise the results will be quite different.

Mr. Allcutt points out the important fact that the impulse voltage necessary to discharge a given gap will depend upon the point on the 60-cycle voltage wave at which the impulse reaches the gap.

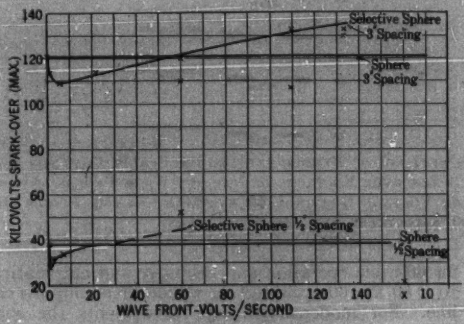


FIG. 11

There are three cases which require consideration:

Case A—where the impulse occurs at the zero point of the 60-cycle wave and is thus not affected thereby. The impulse voltage required to produce discharge may be denoted by A.

Case B—where the impulse occurs at the maximum of the 60-cycle wave and is additive. In this case it would be expected that the gap would discharge when the sum of the two voltages reached approximately the A discharge voltage of the gap. Thus, if L is the maximum of the 60-cycle line voltage, approximately,

$$B = A - L$$

Case C—where the impulse occurs at the maximum of the 60-cycle voltage wave, but in the opposite direction. In this case it would be expected that it would be necessary for the

impulse voltage to reduce the 60-cycle voltage to zero and rise to a value A in the opposite direction, or approximately,

$$C = A + L$$

For Case C, the impulse voltage would rise to a higher value than in case A and B before the arrester discharged. End

TABLE I

SELECTIVE SPHERE-HORN
(6-¼-cm. Spheres with Auxiliary Electrode)
One side grounded.

Gap setting		60-cycle spark-over kv. max.	60-cycle spark-over ½ gap kv. max.	Impulse spark-over total gap	Apparent impulse ratio	Impulse ratio
In.	Cm.					
				83 Kilocycles (Single half cycle)		
½	1.27	38	23	28	0.74	1.21
2	5.08	105	67	81	0.77	1.20
3	7.62	121	88	113	0.94	1.29
				500 Kilocycles (Single half cycle)		
½	1.27	38	23	32	0.84	1.40
2	5.08	105	67	87	0.83	1.30
3	7.62	121	88	109	0.90	1.24
				2000 Kilocycles (Single half cycle)		
¼	1.27	38	23	34	0.90	1.48
3	7.62	121	88	114	0.94	1.30
				Impulse spark-over total gap	Apparent impulse ratio	Wave front volts/sec.
3	7.62	121	88	113	0.94	0.4×10^{11}
3	7.62	121	88	109	0.90	5×10^{11}
3	7.62	121	88	114	0.94	22×10^{11}
3	7.62	121	88	110 – 120*	0.91 – 1.00	60×10^{11}
3	7.62	121	88	106 – 133*	0.88 – 110	110×10^{11}
3	7.62	121	88	130 – 132*	107 – 110	133×10^{11}

Above data for Case A—Impulse applied at zero of 60-cycle wave.
*Range of equal spark-over.

turns of transformers wou d, therefore, be subjected to the highest stresses for Case C.

Tests which I have made for both the plain sphere and the select ve sphere check approximately with the conclusions which were arrived at above from theoretical considerations.

Mr. Allcutt came to the same theoretical conclusions as

outlined above for the sphere, for *A*, *B* and *C* and these theoretical results are plotted accordingly in his Fig. 17. He found from tests on the selective gap that for Case B the impulse voltage was reduced as would be expected, but for Case C, quite contrary to expectations, the impulse required to spark-over the gap was the same as in Case A. It is difficult to reconcile his tests *B* and *C*, since if an additive wave reduces the voltage, a subtractive wave should increase the voltage. My tests show this to be the case for both gaps—in other words, that the *relative* protective values of the sphere and the selective sphere are for an impulse of a given wave front approximately the same for all three cases.

The results of my tests are given in Table III. These tests were a l made with the impulse at which the selective gap discharges at the lowest voltage as shown in Fig. 11. Attention

TABLE II

Impulse and 60-Cycle Spark-Over Voltages at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Gap setting		Impulse frequency kilocycles	Sphere 60-cy. or any impulse kv.max.	Selective sphere-horn					
				Pittsfield tests			Mr. Allcutt's test		
In.	Cm.			60-cy. kv. max.	Impulse kv.max	Apparent impulse ratio	60 cy. kv. max.	Impulse kv. max.	Apparent impulse ratio
½	1.27	500	38	*38	32	0.84	†39	12.5	0.31
2	5.08	500	105	105	87	0.83	121	65	0.54
3	7.62	500	121	121	109	0.90	147	93.5	0.64

*60-cycle values for one sphere grounded as in an arrester.

†60-cycle values for spheres not grounded.

Above data for Case A—impulse applied at zero of 60-cycle wave.

is called to the great difference between my tests and Mr. Allcutts. I found an average difference of only about 15 per cent between the sphere and selective gap for Case C *with the impulse giving the minimum break-down on the selective gap.* As the steepness of the impulse wave front is decreased this difference disappears and the discharge voltages become equal; as the steepness is increased the difference disappears and finally the break-down of the selective gap becomes greater than that of the sphere. (See Fig. 11.)

The impulse ratio figures given by Mr. Allcutt do not correspond to what I originally termed the impulse ratio, in my paper. The object of this ratio was to indicate the relative speed of different gaps. A gap without time lag has an impulse ratio of one; this ratio cannot be less than one. Mr. Allcutt gives values of less than one, although the gap is slow. The

FIG. 12A—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilovolts maximum.

Insulator	Spheres	Points
120	77	77

The spheres protect by discharging the impulse. The points have the same *60-cycle* voltage setting but do not discharge.

FIG. 12B—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilo volts maximum.

Insulator	Spheres	Points
120	137	77

Although the sphere *60-cycle* spark-over voltage setting is, in this case, higher than that of either the insulator or the points, the spheres protect because the sphere "lightning" spark-over voltages is the lowest of the three.

[PEEK]

FIG. 12C—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage settings in kilovolts maximum.

Insulator	Spheres	Points
120	———	77

The points do not protect because of their very high "lightning" spark-over voltage.

The spark-overs shown are due to a succession of steep wave point impulses applied to the line.

FIG. 13A—SPHERE AND HORN GAPS HAVING A STEEP WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv-max.)
Spheres 250 Horns 140

[PEEK]

FIG. 13B—SPHERE AND HORN GAPS HAVING A SLANTING WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv.-max.)
Spheres 250 Horns 140

impulse arc-over voltage divided by the 60-cycle voltage of half the gap wou d give this figure for the selective gap. See Table II. I mention this merely to prevent confusion in regard to my original use of the term. Mr. Allcutt's ratio is an *apparent impulse ratio*. As an instance, take the 3-in. setting for a 500-kc. wave; Mr. Allcutt finds an impulse spark-over voltage o' 93, while my tests give an impulse voltage of 109. My tests give an *apparent* impulse ratio of 0.90; Mr. Allcutt's 0.64. Part of this discrepancy is due to the fact that Mr. Allcutt takes the non-grounded value of 121 kv. for the 60-cycle voltage while the grounded value of 121 kv. should be used.

Any gain due to the selective sphere-horn is, in effect, that

TABLE III

Effect on the 60-Cycle Voltage on the Impulse Spark-Over
Comparison of Sphere and Selective Sphere-Horn at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Spacing		Selective gap impulse kv. max.			Sphere gap impulse kv. max.			*Ratio Selective/Sphere		
In.	Cm.	A	B	C	A	B	C	A	B	C
		Pittsfield Tests								
1	2.54	49	54	97	68	52	108	0.73	1.04	0.90
2	5.08	87	54	115	105	60.5	148	0.83	0.90	0.78
3	7.62	113	67	147	121	86	160	0.94	0.78	0.92
		Mr. Allcutt's Tests								
1	2.54	30	9	32	71	35	107	0.42	0.25	0.30
2	5.06	65	22	68	121	67	181	0.54	0.36	0.37
3	7.62	93.5	45	96	147	74	220	0.63	0.61	0.44

*Tests made at 500 kc., or at the wave front giving the minimum discharge on the selective gap.

Applied 60-cycle voltage = half 60-cycle discharge voltage of the gap.

Case A—Impulse applied at zero of 60-cycle wave.

Case B—Impulse applied at maximum of 60-cycle wave-additive.

Case C—Impulse applied at maximum of 60-cycle wave-subtractive.

of raising the operating voltage and not of time lag. Any apparent laboratory gain is thus lost if practise makes a large increase in spacing necessary. Greater gains of a similar nature may be made in other ways. Rain lowers the 60-cycle arc-over voltage of a gap. It is, for this reason necessary to set a gap installed outside, at about double the voltage of a gap installed inside. A fast gap protected from rain has been devised which makes a much greater practical gain for all wave fronts than that obtained by the selective gap, at its minimum discharge voltage. This is obvious since the average apparent gains due to *impulses* under the best conditions with the selec-

tive gap, Figs. 9 and 11, are only 10 to 15 per cent, while a covered gap cuts the voltage in half. A *covered fast gap is of greatest importance.*

So far the problem has only been discussed from the arrester standpoint. While it is important to design an arrester gap of low impulse arc-over voltage, it is equally important to design leads, insulators, and insulation for high impulse puncture and arc-over voltages. Great strides have been made in this direction.

Percy H. Thomas (Read by L. W. Chubb): Mr. Allcutt has devised an apparatus which will cause a discharge over the spark gap more easily as the result of a voltage impulse of steep wave front than as a result of commercial frequency current. Furthermore, the effectiveness of the arrangement does not vary numerically with the frequency but rather may be said to be effective for all frequencies above a critical intermediate range, which is the author's intention and the desired characteristic. From a certain point of view the apparatus described by Mr. Allcutt has characteristics especially adapted to the protection of commercial electrical apparatus from high-frequency discharges. From certain other points of view, however, considerations might be urged to minimize its importance. For example, experience may show that it is not desirable to cause the spark gap to discharge with very high-frequency impulse. In commercial circuits there may be very many relatively harmless impulses of sufficient abruptness to cause Mr. Allcutt's apparatus to act when no actual protection is required.

It might happen, for example, with the device adjusted in such a way that high-frequency impulses of voltages not too high to be safe did not discharge that an adjustment would result which did not discharge commercial frequency impulses easily enough. In other words, is not the ideal condition a discharge ratio of unity? The abrupt discharge to ground of a voltage impulse on the terminal of a transformer may be far more severe on the end turns than the original impulse. This results not only because the swing of voltage is greater with the discharge to ground but because it may be of even a higher frequency than that of the original impulse.

It may also be said that it is only under the most favorable and very unusual conditions that a disturbance of a frequency as high as 500,000 cycles can be met with in transmission lines. The size of conductors and all apparatus connected with a commercial system are of such high capacity and inductance that practically all impulses will be of much lower frequency. Again, these very high-frequency discharges are of very small energy content, so that they would ordinarily be absorbed in any commercial system or any commercial apparatus without producing any considerable rise in potential. Especially if their initial voltage be so high as to break down any commercial lightning arrester.

It is not safe, however, to object to such new devices as are proposed by Mr. Allcutt on purely theoretical grounds, for actual trial often reverses conclusions based on theoretical considerations.

It would seem to me that a particularly favorable field for the trial of this devise would be distribution systems carrying house lighting transformers from 2000 to 5000 volts. These transformers suffer frequent injury, and adequate arc suppressing devices are available in case of the use of very sensitive lightning arresters.

L. W. Chubb: This work of Mr. Allcutt's is very interesting and the tests in all cases made in shunt with the sphere-gap, leave no doubt regarding the quickness of the gap and the fact that it limits the voltage to a reasonable figure.

Some time ago we ran impulse tests on electrical porcelain and other dielectric materials, and the results show distinctly that voltage impulses of short duration far above the ordinary breakdown voltage of the insulator may be applied repeatedly without rupture. Each impulse, however, produces a permanent injury and eventually the insulation will give way under the repeated stress. High-frequency impulses coming in and striking the end turns of a transformer will not break it the first time, or for many times. Each time such an impulse comes it will cause a certain permanent injury, and repeated disturbances will eventually cause failure.

The selective properties of Mr. Allcutt's gap seem to me to be of great importance for it takes care of the first half cycle of a wave train which starts in opposition to the instantaneous value of line voltage. The ordinary gap, under the same conditions, set to double line voltage will require three times normal voltage to discharge on the first half cycle of the wave train, which will seriously injure the insulation of inductive end turns. The usual gap will, of course, discharge on the second alternation in this case, but this is too late as the voltage rise has not been restricted and the harm has been done.

C. T. Allcutt: I am very glad to hear from Mr. Peek on this subject and to find that he is conducting some experiments to check the results I have obtained. It would appear from the data and curves that Mr. Peek has included in his discussion, that there is a serious lack of agreement in our results. It is unfortunate that Mr. Peek has neglected to give any data concerning the construction of the gaps tested by him or the test methods employed. If such information had been included in his discussion I feel confident that the cause of the apparent discrepancies could be readily pointed out.

The particular forms of gap for which I have presented test data are but two of a large number of forms of gap, based on the same principle, that have been tested in our laboratory. The gaps described are, naturally, the preferred forms, *i.e.*, those that give the best results consistent with a commercially prac-

ticable construction. Any considerable modification of the construction described may result in a large increase in the impulse discharge voltage.

I do not believe that Mr. Peek appreciated the importance of having the auxiliary electrode of such small dimensions so as

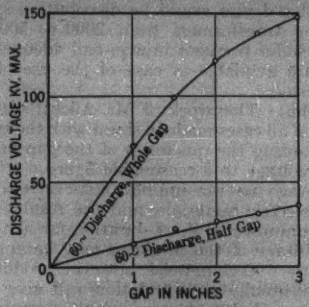


FIG. 14—IMPULSE GAP

to be, in effect, a point electrode. Perhaps the fault is mine for having insufficiently emphasized the importance of this point in my paper. For example, Mr. Peek states that the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes is about one-half the discharge voltage of the

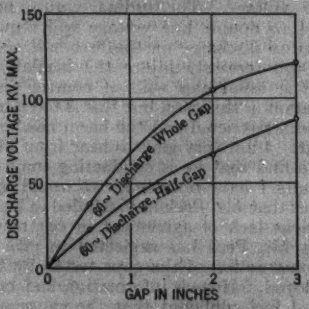


FIG. 15—PEEK'S GAP

main gap. This is clearly *not* the case where a point auxiliary electrode is used. In the sphere-horn gap tested by me, the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes was approximately *one quarter* the discharge voltage of the whole gap. (See Fig. 14). In the gap

tested by Mr. Peek, conditions were quite different, as is shown by Fig. 15, which was plotted from the data given in Table I of his discussion. It is evident that Mr. Peek used an auxiliary electrode of quite different dimensions from that described in my paper. This alone is sufficient to account for his failure to secure results comparable with my own, as the impulse discharge voltage is largely dependent on the 60-cycle discharge voltage of the half gap.

In the footnote to his Table II and again in the text of his discussion, Mr. Peek intimates that I used the standard ungrounded sphere gap calibration curve as representing the 60-cycle discharge characteristics of the gap under test. As I stated in my paper, the 60-cycle discharge voltage of the gap under test was obtained by comparison with a standard 25-cycle sphere gap. All tests were made with one electrode grounded. The presence of the horns and the height of the gap above ground both tended to make the 60-cycle discharge curve somewhat higher than the standard grounded calibration curve for 6.25-cm. spheres. Mr. Peek apparently was misled by the fact that the experimentally determined 60-cycle discharge curve for the sphere horn gap with one side grounded, happened to correspond quite closely with the standard *non-grounded* curve for 6.25-cm. spheres.

In another part of his discussion Mr. Peek objects to my use of his own very apt expression "impulse ratio." Mr. Peek introduced this term some three years ago with these words: "We have termed the ratio of the impulse breakdown voltage to the continuously applied breakdown voltage the "Impulse Ratio." In my paper I defined impulse ratio practically in Mr. Peek's own words and attempted to use the term in strict accordance with his own definition.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high-power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e.m.f." type of arrester, represented by the aluminum cell and the oxide film arrester.

It is shown that the necessity of taking care of recurrent discharges in high-power systems had led to the universal adoption of the aluminum cell arrester in such systems, in spite of its disadvantage of requiring daily attendance in charging, and of containing an electrolyte and oil.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester; the dielectric film, which punctures under the discharge, and reseals after the discharge, is formed from the solid materials between the terminal plates, compressed PbO_2, and therefore no spontaneous chemical action occurs which dissolves the film, as in the aluminum cell, in which the film forms from the aluminum electrode, gradually dissolves, and therefore requires daily charging.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

THREE periods can be distinguished in the development of lightning arresters, corresponding to the three periods of the use of electricity:

(1) Electric circuits of negligible power, telegraph and telephone.

(2) Electric power circuits of negligible electrostatic capacity;

d-c. lighting and railway, a-c. secondary and 2300-volt primary distribution.

(3) High-voltage electric power circuits, transmission lines, etc.

(1) In electric circuits of negligible power, such as telegraph and telephone circuits, a simple minute spark gap to ground afforded protection by discharging lightning to ground, and was sufficient until the recent years, when with the general introduction of electric power circuits the problem arose, in electric communication circuits to take care of crosses with power circuits.

(2) In electric power circuits, a simple spark gap to ground became insufficient for protection, since the power current, following the lightning discharge as arc, short-circuited the system and burned up the arrester. The problem then arose, to safely open the short circuit of the machine current to ground, through the lightning arrester, after the lightning discharge has passed, and to leave the arrester in operative condition to receive following lightning discharges.

Of the various devices developed heretofore, the magnetic blow-out lightning arrester still is used in direct-current railway circuits.

The first scientific investigation of this problem, is recorded in the paper[1] by A. J. Wurts, presented before the A. I. E. E. at the meeting of May 1894. Since that time all lightning arresters for alternating-current power circuits of negligible electrostatic capacity are based on the multigap principle between non-arcing metals, whatever constructive forms the arrester may assume—as the present compression chamber lightning arrester. The multigap arrester operates on the principle, that the lightning discharge over the multigaps closes the circuit to ground, but the power arc following the discharge extinguishes at the end of the half wave of the alternating current, as the non-arcing character of the gaps does not permit the reverse current of the next half wave to start. The multigap arrester thus short-circuits for a part of a half wave. It obviously is suited only for alternating currents.

For years difficulties were met with the question of resistance; without series resistance, in large systems the short-circuit power

1. TRANS. A. I. E. E., 1894, Vol. XI, p. 337, "*Discriminating Lightning Arresters and Recent Progress in Means for Protection Against Lightning*", by A. J. Wurts.

even of a part of a half wave may be sufficient to disable and destroy the arrester, while the use of a series resistance, while limiting the power current and thereby protecting the arrester, also limited the discharge capacity and thereby reduced the protection. This problem was solved by the use of multigaps shunting the series resistance, so that moderate discharges passed over the resistance, while high power lightning discharges found a path without series resistance, over the shunted gaps, and at the same time, the shunting resistance made the power arc at the shunted gaps unstable and thus assisted in the extinction of the short circuit at the end of the half wave.[2]

(3) As soon however, as circuits came into use, which had considerable electrostatic capacity, such as high-voltage transmission circuits, extended underground cable systems, or lower voltage circuits (including generator circuits) inductively connected with such circuits, the multigap arrester failed by frequently, or even usually destroying itself by the discharge, burning up.

In such circuits, oscillations between capacity and inductance may occur, started by a lightning discharge or any internal disturbance such as switching etc., resulting in recurrent high-frequency oscillations, of which the arcing ground on a transmission line is typical and probably best known. With such continual discharges, often several per half wave, the multigap arrester short-circuits at the first oscillation, for the remainder of the half wave, and while the multigap functions properly and opens the short circuit at the end of the half wave, the oscillation of the next half wave again short-circuits, and so on, so that the effect is that of a continuous short circuit, and no lightning arrester, no matter how large, can dissipate the short-circuit power of a big system for any appreciable time.

For such systems, in which recurrent high frequency oscillations, as arcing grounds, may occur, a lightning arrester is necessary, which does not short-circuit the machine current even for a fraction of a half wave, but merely discharges the over voltage, the oscillation which, however high in voltage it may be, is small in energy compared with the short-circuit power of

2. See A. I. E. E. TRANS., 1907, Vol. XXVI, p. 425, "*Protection Against Lightning, and the Multigap Lightning Arrester*" by D. B. Rushmore and D. Dubois.

the system, as it represents only the stored energy of capacity and inductance. The only arrester of this character heretofore was the electrolytic, or aluminum cell lightning arrester, developed by E. E. F. Creighton, J. L. R. Hayden, F. W. Peek and others. It acts towards an over-voltage discharge like a counter e. m. f. equal to the normal circuit voltage, and the discharge current passing through the arrester thus is the short-circuit current of the over voltage, while the normal machine voltage does not discharge, is held back and not disturbed. The aluminum cell arrester thus can discharge continual disturbances, over-voltage oscillations occurring at every half wave, for a considerable time, half an hour to several hours, before it is endangered by the temperature rise due to the accumulated energy of these discharges.

The aluminum cell arrester comprises a series of cells—usually conical and stacked into each other—of aluminum electrodes with an electrolyte, of which neither the salt nor its ions appreciably dissolve alumina. In "forming" the cell, by an alternating current passing through it, the electrodes are coated by a thin non-conducting film of alumina, which grows in thickness until it holds back the impressed voltage. Any over-voltage punctures this film, but the current passing through the puncture holes, again forms alumina and closes the holes. Thus the aluminum cell acts like a self repairing electrostatic condenser of a disruptive strength equal to the impressed voltage: about 250 to 300 volts per cell.

The practical experience of the last ten years has proven the aluminum cell arrester as the only type capable of affording protection in modern high-power circuits, and proven this so conclusively, as to lead to its universal adoption in such circuits in spite of the inconveniences incident to the need of daily attention in charging, the use of a liquid electrolyte, and the difficulty of testing the arrester without taking it apart, except by watching the appearance of the charging arc, or measuring the charging current.

These inconveniences incident to the aluminum cell arrester were well realized however, and as soon as the minor troubles met with the aluminum cell arrester in the early years had been overcome engineers went energetically to work on the problem of developing a lightning arrester of the characteristics of the aluminum cell arrester, but which does not require any attention

beyond that given to every apparatus in a well managed system, that is, an occasional inspection, at least once or twice a year.

Numerous researches were made by the engineers whose splendid work I here acknowledge: Messrs. H. D. Brown, E. E. F. Creighton, Crosby Field, V. E. Goodwin, J. L. R. Hayden, N. A. Lougee, and G. B. Phillips. Some of these researches have not yet led to results ready for communication, but I am glad to present to you here as the result of the work of these men, and more particularly of E. E. F. Creighton, Crosby Field and N. A. Lougee, in the *Oxide Film Lightning Arrester* a new type of lightning arrester, which has all the characteristics and advantages of the aluminum cell arrester, but does not require any charging and thus requires no special attention, contains no liquid electrolyte, no inflammable material, and like the aluminum cell arrester, can be located outdoors as well as indoors.

The oxide film arrester, like the aluminum arrester, acts like a counter e. m. f. equal to the normal circuit voltage, freely discharging any over voltage, but holds back the normal machine voltage. Thus the discharge is limited to the energy of the over voltage, as in the aluminum arrester, and like the latter, the oxide film arrester can continuously discharge recurrent surges, such as arcing grounds etc., without endangering itself, for a considerable time, sufficiently long to notice and eliminate the disturbance.

Compared with the almost entire absence of knowledge of lightning phenomena in electric circuits, under which Mr. Wurts had to work in developing the non-arcing metal multigap lightning arrester, our present knowledge of lightning phenomena is very great. Nevertheless, there are so many disturbances in large electric systems, which we cannot or only incompletely reproduce in our laboratories, that the final decision on the success, that is, the effectiveness and permanence of a lightning arrester, still is best given by the experience in industrial systems.

Therefore, after extensive laboratory tests had been completed and had proven the oxide film arrester as of the same characteristic as the aluminum arrester, but requiring no special attention, a number of industrial installations were made, and more added the next year and the third year. Now, however, when a considerable number of installations of these arresters, for voltages from 110 to 33,000, have been in successful operation, some for over three years, and have proven their protective value and

their permanence, I consider it desirable to bring the arrester to your attention.

Of the numerous tests made on the performance of the arrester, it may be sufficient here to give only two, by the oscillograms Figs. 1 to 4, showing the action on a recurrent oscillation, in Figs. 1 and 2, and on a single high-power impulse, Figs. 3 and 4.

The tests, oscillograms Figs. 1 and 2, were made in the usual manner: a surge or continual oscillation was produced by a large condenser, connected to an alternating-current supply and discharging over a spark gap through an inductance. The latter was chosen so as to give a frequency of 1200 cycles to the oscillation, and thereby bring it well within the range of the oscillograph. This surge was impressed upon the apparatus to be protected, a transformer energized by another alternating-current circuit. Fig. 1 shows the oscillogram without protection of the transformer, and Fig. 2 the oscillogram with an oxide film cell shunting the transformer and thereby protecting it.

In Fig. 1, the lowest curve shows the voltage of the 320-volt 47-cycle power supply circuit of the transformer, with the oscillations superimposed on it, rising to surge peaks of 2800 volts. The upper curve shows the oscillating currents passing through the transformer, rising to current peaks of 40 amperes. The middle curve is absent, as no arrester is used in this test. In Fig. 2 however, where an oxide film cell is shunted across the transformer, the middle oscillogram shows the current oscillations passing through the arrester, with peaks of 35 to 41 amperes. The lower curve in Fig. 2 then shows again the circuit voltage wave impressed upon the transformer, with the oscillations cut down by the oxide film cell. This voltage wave, the lower curve in oscillogram Fig. 2, well illustrates the characteristic action of this type of "counter-e. m. f. arrester"—to which the oxide film arrester and the aluminum cell belong. The oscillation peaks are sharply cut off at a maximum voltage of 60 per cent above circuit voltage: the value for which the spark gap was set. As the result, the oscillations are very greatly cut down from the high values which they have in the unprotected circuit Fig. 1 (2800 volts), and become unsymmetrical. The half waves of oscillation in the same direction as the circuit voltage are greatly reduced, by the limitation of the voltage to 60 per cent above-normal, while the reverse half waves—which lower the instantaneous circuit voltage—are less affected. Corresponding thereto the discharge current through the arrester, the middle oscil-

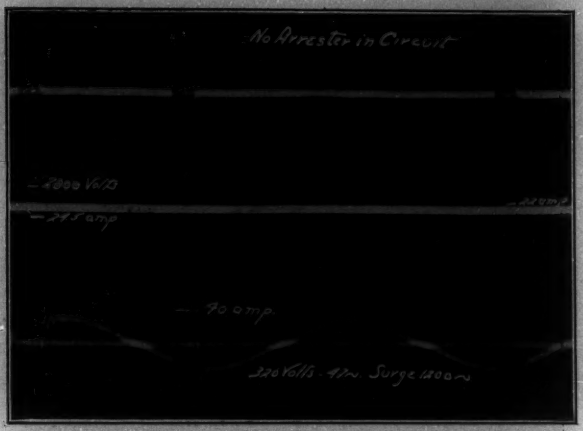
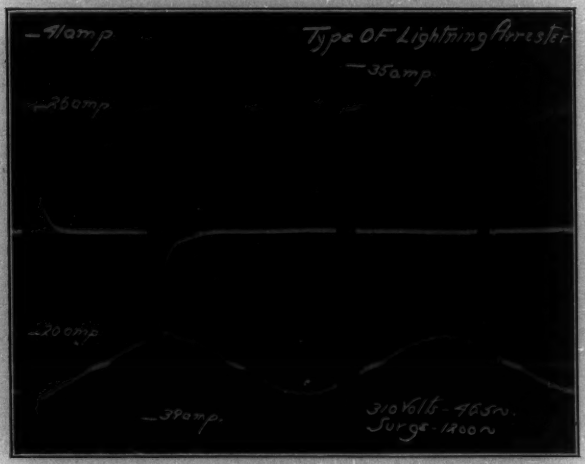


FIG. 1—SURGE DISCHARGE AS IN FIG. 2—NO ARRESTER IN CIRCUIT

Top vibrator—current through transformer.
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 2—SURGE DISCHARGE THROUGH ARRESTER CELL

Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

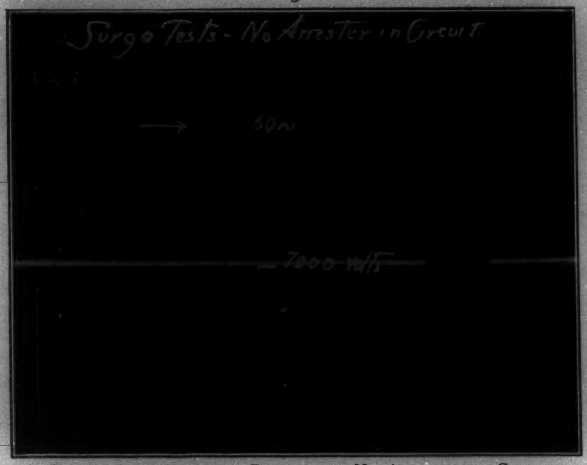
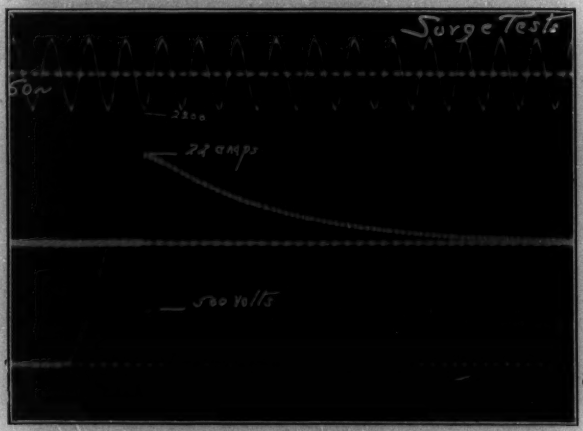


FIG. 3—SINGLE IMPULSE DISCHARGE—NO ARRESTER IN CIRCUIT

Top vibrator—60-cycle timing wave
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 4—SINGLE IMPULSE DISCHARGE AS IN FIG. 3—ARRESTER IN CIRCUIT

Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

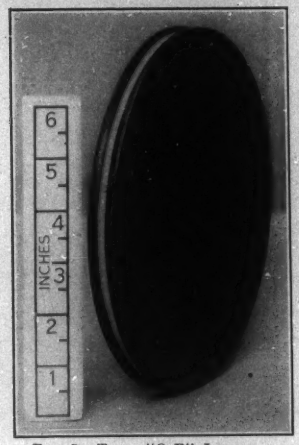
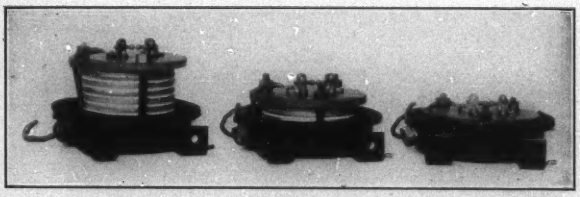


FIG. 5—TYPE "O F" LIGHTNING ARRESTER CELL



[STEINMETZ]

FIG. 6—TYPE "O F" LIGHTNING ARRESTERS WITH COVERS REMOVED RATINGS 325-650, 900-1350, 2100-2600-VOLT ALTERNATING CURRENT OR DIRECT CURRENT RESPECTIVELY

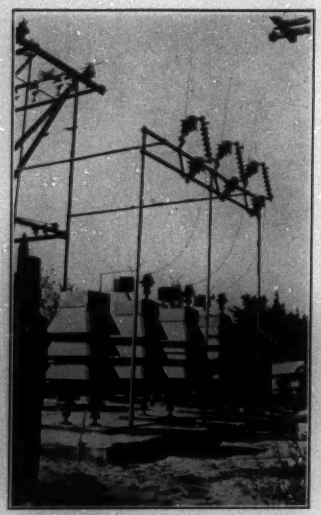
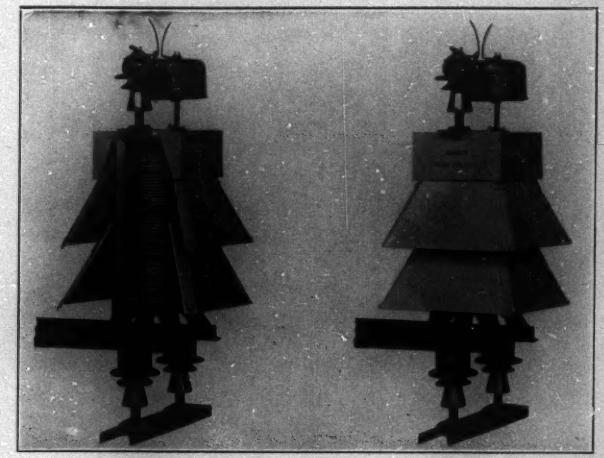


FIG. 8—TYPE "O F" LIGHTNING ARRESTER FOR OUTDOOR SERVICE INSTALLED ON A 33,000-VOLT CIRCUIT



[STEINMETZ]

FIG. 7—PHASE SECTION OF 15,000-25,000-VOLT OUTDOOR TYPE, "O F" LIGHTNING ARRESTER WITH SIDE OF HOUSING REMOVED TO SHOW INTERIOR ARRANGEMENT

logram in Fig. 2, is unsymmetrical also: in the first and second oscillation of Fig. 2, the first and third half wave of oscillating voltage is cut off, and the first and third half wave of the oscillating discharge current therefore higher than the second half wave of the oscillating discharge, which latter corresponds to a half wave of oscillating voltage in opposition to the circuit voltage, therefore not raising the circuit voltage. The third oscillation of Fig. 2 happens to start with a half wave of oscillating voltage in opposition to the circuit voltage, and the first half-wave oscillating discharge current through the arrester, in the middle curve, thus is smaller than the second half wave; the second half wave of the oscillation is cut down in the voltage and therefore gives the maximum discharge current, 35 amperes in this case.

This feature is well brought out by the oscillograms Figs. 1 and 2, due to the use of a different frequency, 60 cycles, for the power supplying the oscillator. This caused the successive oscillations to occur at different parts of the half waves of the 47-cycles circuit which was to be protected.

Fig. 3 and 4 then show the protective action of the oxide film arrester on a 550-volt direct-current circuit, against a single (non-oscillatory) impulse produced by opening a highly inductive circuit (railway motor). Fig. 3 shows on the lower curve the oscillogram of the impulse in the 550-volt circuit, rising to 7000 volts. The upper curve merely is a 60-cycle timing wave, to enable measuring the duration of the impulse. Fig. 4 shows the same circuit, with an arrester shunting it. The impulse voltage rises to the value for which the discharge gap of the arrester is set, in this case 2200 volts. Then the arrester discharges, and the voltage instantly drops back to normal, while a slowly decreasing discharge current through the arrester dissipates the magnetic energy of the impulse.

The cell of the oxide film arrester, shown in Fig. 5, consists of two circular metal plates as electrodes, which are kept apart by a porcelain ring, as shown in the figure. The space between the electrodes inside of the porcelain ring, is filled with the active material, lead peroxide PbO_2, which is put in under moderate pressure. This active material is a good conductor, but has the characteristic, that by the action of an electric discharge, it is converted in the path of the discharge, into a lower oxide, which is an insulator. Thus when an alternating current is passed through such a cell, the active material at the electrodes grad-

ually converts into a non-conductor, and forms a thin insulating film at the electrode. This grows in thickness, until it cuts off the further flow of current and holds back the voltage, about 250 to 300 volts per cell. Then only a small leakage current, of a few milliamperes, passes at normal voltage, but if an over voltage of any kind appears at the cell, the insulating film of lead oxide punctures and freely discharges through the lead peroxide, but in doing so, converts the surface of the lead peroxide in the path of the discharge into the lower non-conducting oxide, and thereby closes the puncture holes, repairs or reseals the film.

In manufacture, naturally, just as in the aluminum cell arrester, the insulating film is not produced after assembly of the cell by the slow process of passing a current through, but the film is put on before assembly, in the oxide film arrester, by dipping the plates in a suitable insulating varnish, which gives them a coating just thick enough to hold back the circuit voltage. Then after assembly, voltage is put on the cell for testing it and sealing any holes or defects which may exist in the varnish film.

In the oxide film arrester, the electrodes have nothing to do with the arrester action, and any suitable material can be used. First we used brass, but now use sherardized iron, the latter having a higher melting point and thus standing high power discharges, which would melt holes in the brass electrodes.

In this arrester, the action, which holds back the normal voltage but passes freely an over voltage, thus resides in the active material between the electrodes, and it is this material which forms and reforms the film. As this material is a solid, no chemical action occurs such as the gradual dissolution of the alumina film in the aluminum cell arrester, but the film remains intact permanently, and thus no daily "charging", that is, repairing of the film, is required.

A number of such cells, depending on the voltage of the circuit, are piled on top of each other, with a spark gap in series, and, for low and moderate voltages, incased as shown in Fig. 6.

As the cells are hermetically sealed, by the metal of the electrodes being spun over the porcelain separating ring, the cells can be installed outdoors as well as indoors, requiring in outdoor installation merely some protection by petticoats, as shown in Fig. 7, to keep the rain from short-circuiting the cells. Fig. 8 shows such an outdoor installation of a 33-kv. arrester, with three-phase stacks and the ground stack of cells, protected against the

weather by metal petticoats. Fig. 8 shows also the spark gaps on the line side of the arresters. They are protected sphere gaps, to give instantaneous discharge, with a horn attachment to allow the arc to flare up and thereby help in its extinction.

As well known, the plain horn gap has the disadvantage of requiring an appreciable —though a very short (microseconds) —time for discharge, and an extremely sudden high voltage, as a very steep wave front, thus may pass the horn gap and flash over elsewhere. Therefore in modern high-voltage lightning arresters the horn gap is shunted by a properly proportioned sphere gap, the latter being "instantaneous" in its action. In outdoor use however, rain lowers the discharge voltage of the sphere gap, and thus requires a setting which gives a higher discharge voltage in dry weather than necessary. Therefore a protected sphere gap has been designed, which overcomes this disadvantage in the open sphere gap, and is shown in Fig. 8.

The need of using this spark gap in series with the arrester, is the only still remaining undesirable feature which the oxide film arrester shares with the aluminum cell arrester, the multi-gap arrester and other types. While by the work of Mr. F. W. Peek, on the time lag of electric discharges[3], the means have been given to make the discharge gap "instantaneous", that is, faster than any other discharge path over gaps or through insulation in the system, so that the arrester takes care effectively of any over voltage above its discharge voltage, it does not discharge voltages lower than the discharge voltage of its spark gap, even if these lower voltages may involve some danger to the system by their high frequency. Such low voltages, while they cannot endanger the main insulation between circuit and ground, may, if of sufficiently high frequency, lead to local accumulations of voltage across inductive parts of the circuit, as regulators, current transformers, end turns and coils of generators and transformers, and there cause damage by puncturing insulation between turns and causing internal short circuits.

Against these high frequency disturbances of moderate voltage, the only existing protection is the addition to the arrester of a capacity discharge path permanently connected from the circuit to ground. Such capacity path should be without resistance to flatten steep wave fronts, and contain a moderate series

3. A. I. E. E. TRANS. 1915, Vol. XXXIV, Part II, p. 1857, *The Effect of Transient Voltages on Dielectrics* by F. W. Peek, Jr.

resistance, to dissipate high-frequency energy and stop cumulative oscillations in their beginning. Before I leave the field of electrical engineering, I hope still to see an arrester, of the type of the oxide film or the aluminum cell, which has no spark gap, but is permanently shunted across the circuit, and thus capable of taking care not only of over voltages, but equally well of steep wave fronts and high-frequency oscillations, even if of lower than the circuit voltage. Such an arrester then would give universal protection.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CROSBY FIELD

ABSTRACT OF PAPER

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

THIS paper will be confined to a brief statement of the scientific principles underlying a new type of lightning arrester called the "oxide film arrester."* The functioning of this arrester depends upon the fact that certain dry chemical compounds can be changed with extreme rapidity from very good conductors of electricity to almost perfect non-conductors by the application of a slight degree of heat. Lead peroxide is a good example of such a substance. It has a specific resistance of the order of one ohm per inch cube. The resistance varies with the pressure to which it has been compressed. At a temperature of about 150 deg. cent. the lead peroxide (PbO_2) will be reduced to red lead, commercially known as minium (Pb_3O_4). This has a specific resistance of about 24 million ohms per inch cube. At slightly higher temperatures this minium will be reduced through the sesquioxide (Pb_2O_3) to litharge (PbO), which last named is practically an insulator. [A megger reading of infinity is obtained on a column 3 millimeters long (0.11 in.) and 5 square millimeters area (0.2 sq. in.)].

Again the oxides of bismuth give similar characteristics. There

*U. S. Patent No. 1,238,660—Crosby Field.

are, furthermore, several other compounds and mixtures of compounds that will give these same results.

Lead peroxide is normally in the physical state of a powder. If this powder be placed between two electrodes and a current passed, the temperature due to the resistance at the contact of the peroxide and the metal will cause heat to be generated locally at the surface. When this heat is sufficient to create a temperature of about 150 deg. cent. a film of the lower oxides of lead

FIG. 1

A single cell of the oxide film lightning arrester consisting of a ring of porcelain with two circular steel disks, one spun on each side of the porcelain and insulated by the porcelain. The inside of the ring of porcelain between the plates is filled with peroxide of lead.

forms, producing a film of insulation which stops the current. This method of film formation over any large area is rather irregular, and of course the oxide is not used in such a fashion in the commercial arrester. Instead of this formation of litharge film any insulating film may be put on the electrodes initially. As insulating film spread on the metal plates there have been used thin layers of the following; glass, water glass, halowax, cloth, balsam, shellac, oil, paints, lead paints, varnishes, and lacquers of all available kinds. In all cases the results are sim-

ilar, varying only with the voltage at which puncture of the film of insulation occurs.

The foregoing statements define the principle of the commer-

FIG. 2

Shows the disassembled parts of a single cell of the oxide film lightning arrester. From left to right are a steel disk spun on a ring of porcelain, a pile of brown peroxide of lead, the other steel disk and an asbestos washer.

cial oxide film arrester. Fig. 1. It comprises two sheet metal electrodes, set about 0.5 in. (12.7 mm.) apart, one or both covered with a thin insulating film and the space between the plates

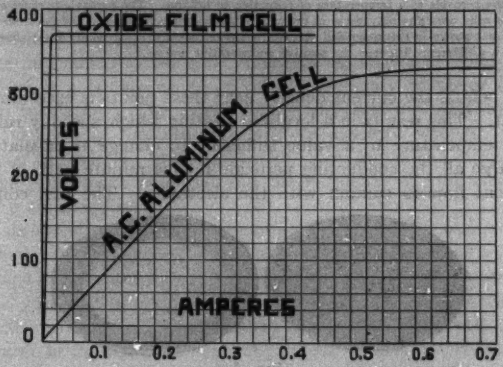

FIG. 3

The comparative volt-ampere characteristics of the oxide film cell and the a-c. aluminum arrester cell. The oxide film has only a few milliamperes of leakage current up to the critical film voltage when the film gives way more suddenly than the film in the aluminum cell. The critical voltage of the oxide film can be made approximately as low as the hydroxide film on the aluminum cell.

filled with some such substance as that described above, as, for example, lead peroxide. Fig. 2 shows the disassembled parts of a single cell. At a permissible voltage of 300 volts per cell the insulating film prevents any appreciable current flowing under

normal conditions. As soon as the voltage rises slightly above normal the film punctures in one or more microscopic points, the lightning charge meets with practically no resistance and flows to earth. Fig. 3. The dynamic current starts to follow but because of the fact that the insulation was punctured in such

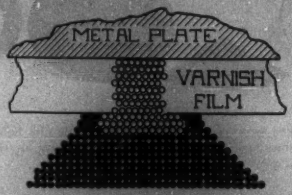

FIG. 4

A magnified, imaginary representation of one of the films on one metal plate. As shown the film is punctured by a spark and filled with a litharge plug which is represented by the open circles. The cross-section in the discharge path, a short distance away from the metal electrode, is sufficiently expanded to make the current of low enough density not to heat the peroxide to a temperature of reduction to litharge. The peroxide is represented by the solid dots and only those in the path of the discharge are shown. At the other electrode, not shown in this magnified diagram, a similar effect may be taking place although there is a difference between the positive and negative crators.

fine points, the current density near these points is exceedingly great. This results in a localized heating which speedily raises the temperature to a value sufficient to change to insulating litharge all the conducting peroxide in this minute path of the current flow in contact with the electrodes. The film conse-

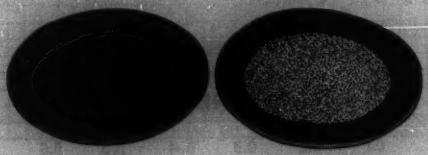

FIG. 5

New electrode and also one from a cell having had passed through it several thousand discharges. The light colored plugs of litharge are plainly visible in the many spots on the surface of the plate.

quently reseals, stopping the further flow of dynamic current. This action is so rapid that its duration cannot be measured on an oscillograph giving two thousand cycles per second, that is to say, the action of resealing occurs in less than one four thousandth part of a second after the excess of lightning voltage has ceased. Fig. 5 shows the visible spots of insulating litharge plugs on the surface of an electrode.

This film can be made of litharge itself, as well as any of the insulating materials above named. For example, metal plates may be inserted in any of the well-known lead electroplating solutions, and thus a very thin lead peroxide film (measuring a few hundred thousandths of an inch) formed. By proper heating this will be changed to litharge and this form of electrode can be used. Peroxide may also be sprinkled over any metal plate and the plate heated, which will reduce the peroxide to litharge. Again, the metal chosen for the electrode itself may be lead and if heated in the air a thin film of litharge will be formed on the surface. Again, an aluminum electrode may be put in any of the common electrolytes, and a thin aluminum film be built up. This may be used with the peroxide powder. Of these methods of forming the film the most preferable is by dipping in varnish or lacquer highly burnished surfaces of brass, steel, or copper, and is consequently used in the commercial arrester. The ohmic resistance of the arrester during discharge is quite low (less than 1 ohm per cell). Thus, when the insulating film is punctured the arrester offers very slight impedance to the flow of energy at abnormal voltages.

There is a certain range of voltage necessary to pierce any given insulation. The exact voltage depends not only upon the thickness of the insulation and its dielectric strength but also on the relation of the dielectric spark lag to the duration of the super-spark potential and the frequency of alternations of the transient surge.

If an arrester is to give protection of insulation in shunt with it, the arrester must relieve the abnormal electric pressure before damage is done to the insulation. Although tests are frequently made with the arrester and the insulation it is to protect in parallel, a more convenient method has been standardized and is known as the equivalent sphere gap test. Both the insulation and the arrester are compared by comparing each to the equivalent sphere gap.

The equivalent sphere gap of the oxide film arrester may be analyzed, as in other cases, into separate and distinct parts. First, there is the equivalent sphere gap of the main gap in series with the cells. Second, the equivalent sphere gap to initiate a discharge through the insulating film on the plate surface of the cell. Third, there is the equivalent sphere gap of the resistance drop of the current discharging through the powdered peroxide in its path. Fourth, there is the equivalent sphere gap of the inductance of the arrester.

Commenting on these factors in their relation to this arrester, the main gap is itself a sphere gap which has the fastest spark of any practical gap. The gap setting, like that of the aluminum arrester, is only slightly above that of the normal voltage of the circuit.

The equivalent sphere gap of the film is several times greater than the thickness of the film because solid material has a greater dielectric spark lag than air, but with this multiple of the thickness of the film the equivalent sphere gap is still low. Since peroxide is a good conductor, the series resistance in the path of the discharge is insufficient to give an undesirable voltage drop. As to the inductance of the arrester, it has a minimum value due to the fact that each cell is only 0.5 in. long, as shown in Fig. 1, and these cells are placed one on top of another. In other words, the total length of the arrester (which constitutes the inductance) is short as compared to the necessary length of conductor from line to earth.

One of the obstacles that had to be overcome in the making of this arrester was the increase in the resistance after a great many heavy discharges had passed through it. The predominant reason for the increase seems to be explained by the following theory. The current passing through this small puncture in the film heats up very rapidly not only the powder but also the air contained within the interstices of the powder. The particles are thereby thrown out of contact with each other, thus producing a fluffiness. The decrease in the number of contacts decreases the actual cross sectional area of conduction, hence increases the resistance. This raises the equivalent sphere gap. This action is accelerated, of course, by the giving off of the oxygen itself evolved in the reduction from lead peroxide to the lower oxide. If, however, this same arrester be violently jarred or the filling powder be compressed, or any other method utilized to restore the particles to their previous intimate contact, the equivalent needle gap will fall again. While increased fluffiness appears to be the predominant cause of change of the equivalent sphere gap, the increased thickness of the film of litharge at the point of puncture of the film is finally a factor of moment. The total area of the film must be sufficient to give a reasonable number of years of life to the arrester. There are other factors relating to the details of manufacture which give a limited degree of control over this change in equivalent sphere gap.

In all the commercial oxide film arresters used for alternating

current the power factor is nearly unity. For special purposes however, the power factor can be made anything desired from 10 per cent. to unity. This is obtained by combining with the conducting oxide other non-conducting materials. This principle has been made use of for condensers but it has not been found desirable to incorporate it in the arrester.

To summarize—an arrester operating under a new principle has been made which comprises in essence one or more metal electrodes covered with an insulating film, and separated by a conducting powder, which has the peculiar characteristic of becoming a non-conducting powder upon the application of heat. Voltage higher than that which can be withstood by the insulating film punctures it in one or more points of about 0.005 cm. diameter. Dynamic current flowing gives a high current density in the conducting powder adjacent to these punctures which in turn heats it up rapidly, reducing the powder to a non-conductor, and sealing the holes in the film. The powder being a poor heat conductor localizes this action, so that very little more powder is reduced than is actually necessary to seal up these minute punctures.

The critical spark voltage and that part of the equivalent sphere gap controlled thereby is a function of the thickness and kind of material used for the film.

Comparison of the "Oxide Film" with Well Known Arresters

The earliest form of non-electrolytic film arrester was known as the dry aluminum arrester.* It was a direct attempt to utilize the dry film which forms on the surfaces of pure aluminum immediately after it comes in contact with the oxygen of the air. The hydroxide film is easily formed in electrolyte and on drying becomes a dry film which gives sufficient action to prevent a discharge up to a given critical voltage, depending upon the thickness of the film. The film can also be formed by a spark or arc of a conductor in contact with a plate. Naturally this conductor should be of a non-metallic nature. In the earliest form tried powdered carbon was used mixed with dioxide of manganese which gives a liberal supply of oxygen at the heated point.

One of the objects of the development of this arrester was to decrease the cost of manufacture and it was found with the new principle involved in the oxide film arrester, where the powder

* U. S. Patent, E. E. F. Creighton.

furnishes the film rather than the plate, that the aluminum could be replaced by a cheaper metal, such as steel, and, as already described, the initial film known in the early stages of development as the "paint skin" type could be furnished by a layer of varnish. On first sight, knowing the extreme thinness of the hydroxide film on wet aluminum cell it might not seem that the dry cell would give the same general characteristics as the wet cell. But a comparison of the volt-ampere curves shows the same general characteristics. For a-c. voltages of 300 volts average per cell the current in the dry cell is of negligibly small value up to 40 milliamperes. The power factor is nearly unity and the current flow is due to very slight leakages through the films. In the case of the aluminum electrolytic cells there is an equivalent condition, the d-c. leakage current of the order of one milliampere being due to leaks through the hydroroxide film. In the a-c. aluminum arrester the leakage current on the plate area used is much greater due to the destructive action of the alternating current on the hydroxide film. Furthermore, the wet cell with its thinner film is a condenser of appreciable capacitance which takes a charging current of about 0.5 ampere at 60 cycles. When the voltage reaches a certain critical value which is between 300 and 400 volts for the wet aluminum cell and between 300 and 500 for the oxide film cell (or higher if the paint film is made thicker) the current is allowed to pass freely through the cells, limited only by the ohmic resistance of the cell independent of the film. Since the oxide film arrester has no dissolution of the film, as occurs in the wet aluminum cell, charging is not only unnecessary but undesirable. This extends the use of the oxide film arrester to localities where there are no attendants.

Although the wet aluminum plate becomes frosted to an appreciable thickness by the passage of current in long use, the actual thickness of the film, as represented by the critical voltage, is not changed. In the oxide film arrester, however, the film less than one 1 mil thick (0.025 mm.) initially thickens up by the addition of successive spots of litharge for each successive discharge. This represents the wear on the arrester and limits its total life. Fig. 3 shows comparative volt-ampere characteristics of the oxide film arrester and the a-c. aluminum arrester. Since both of these arresters have a leakage current which wears the plates of the cells when alternating current is supplied, it is necessary, as previously stated, to place a spark gap in series with the cells. This spark gap is set at a value

slightly above the normal potential of the circuit so that nothing but abnormal voltages will cause a discharge.

The foregoing data show that the oxide film arrester has general characteristics closely like the standard aluminum electrolytic arrester. It has the obvious advantage which comes from being dry rather than wet, it will not congeal and needs no daily charging.

In making the characteristic volt-ampere discharge curves the oxide film arrester does not lend itself as readily to the test as the wet aluminum cell. While its critical film voltage is evident, the change from no conduction to full conduction is more sudden. Therefore, the discharge rate at double potential is best shown by throwing double potential on the cell and subsequently reducing the voltage to its normal value per cell.

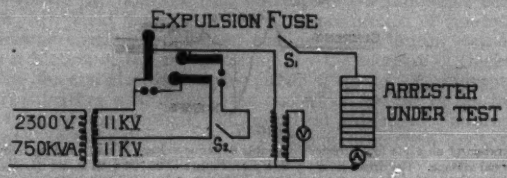

FIG. 6

Arrangement of three expulsion fuses which short-circuit gaps in the path of discharge to first throw double voltage on an arrester under test and then reduce the voltage to normal value by shifting the contacts on a transformer from full coil winding to half coil winding.

This gives a very considerable quantity of electricity through the cells and is a severe test.

In order to conveniently alter the voltage from double value to normal value the circuit is arranged as shown in Fig. 6. The object of the connections may be briefly stated: A transformer with two coils in series impresses voltage on the arrester and then the contacts of the arrester are automatically shifted to one coil giving half the voltage. A heavy pendulum closes switch *S*-1 and sets the oscillograph into operation. The full voltage on the transformer is thrown on to the oxide film arrester under test, marked *O-F*, which has a number of cells sufficient only for half the voltage of the transformer. In other words this throws double voltage on to the arrester and the heavy current passing through the cells causes fuse *F*-1 to blow. The operation of this fuse short-circuits half the transformer and throws the other half across the arrester. This is done by

means of gaps and fuses as follows: When the expulsion fuse *F*-1 blows, the conducting gases are shot into the open gap *G*-1 which closes the circuit through fuse *F*-2 to the mid-point of the transformer. This short circuit on half the transformer causes fuse *F*-2 to blow and the hot gases discharging from fuse *F*-2 close the gap *G*-2 which throws the mid-point of the transformer on to the arrester through the switch *S*-2 which is closed just previous to starting the test. These several operations occur with a rapidity depending upon the size of fuses used. It is possible

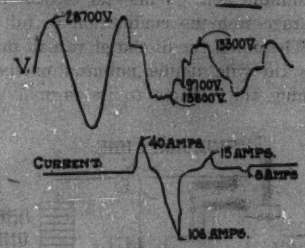

FIG. 7

Operation of an oxide film arrester on double dynamic voltage initially and its recovery on normal voltage. The upper record with a peak voltage of 28,700 volts shows the arrester connected to the circuit as the voltage wave starts to decrease on its third peak. The voltage immediately drops to the critical value of the cells—about 13,500. On the lower record the current is seen to rise to 40 amperes. On reversal of the voltage to 13,800 in the negative direction as shown by the upper record, the current as shown by the lower record rises to 106 amperes. The switching operations produce several electro-magnetic kicks as shown by the irregular voltage wave as it rises in the next cycle to 13,300 peak value. At this lower applied voltage the current in the series of cells, as shown by the lower record, is 15 amperes. In the subsequent half cycle the current rises only to 8 amperes. In the last half cycle of voltage shown, where the switching operation is complete, and the wave assumes its normal smooth form, the current in the cells is too small to be registered by the oscillograph. Its value is of the order of milliamperes. This figure is a copy of an oscillogram. The copy was made desirable by overlapping of the discharge on the two ends of the film.

by this means to throw momentarily 22,000 volts on an 11,000 volt arrester and note the character attending its discharge and recovery. Fig. 7 shows such an operation on an oxide film arrester. The initial discharge current is 40 amperes during the first half-cycle due to the point it strikes in the descending wave during the second half-cycle it is 106 amperes. After the third half-cycle the litharge film has so completely sealed up the path of discharge that the current is too small to show on the oscillogram. The leakage current with no series gap is of the order of a few milliamperes.

DISCUSSION ON "THE OXIDE-FILM LIGHTNING ARRESTER" (STEINMETZ) AND "THE OXIDE-FILM LIGHTNING ARRESTER" (FIELD), ATLANTIC CITY, N. J., JUNE 27, 1918.

C. P. Steinmetz: As Mr. Field's paper represents the state of the work when he left this field a few years ago, I wish to bring it up to date in a few words.

On the first page of Mr. Field's paper he refers to the resistivity of the filler powder as one ohm per inch cube. At present this resistivity has been reduced to 0.1 of an ohm per centimeter, or 0.4 ohms per inch cube, that is, 25 times the conductivity, and correspondingly increased discharge rate, and as a result thereof, the resistance is materially less than 0.1 ohm per cell.

At the time that Mr. Field left it was found by laboratory experiment that there is an increase in resistance, after a great many heavy discharges made on the arrester in the laboratory, resulting in a gradual, though noticeable, increase of the equivalent sphere gap, and resulting decrease of the protective action. This, Mr. Field has explained by the production of gas by the discharge causing a fluffiness, and as a contributing cause by the reduction of the actual section, by the increase of thickness of the film and also the wearing action of the leakage current.

This feature was, indeed, the one which led me in my paper to refer to the necessity of industrial experience in existing plants, as a final criterion in lightning protective apparatus, since with all our skill and ability and thought, we cannot perfectly reproduce the actual conditions, and this was one of the reasons which led us to hold back the arrester in order to get fuller information on its life, and I am glad to say that after two to three years of industrial service, and after the arrester has received many thousands of heavy discharges, cells have been taken off and retested, and we have found no increase of resistance and no increase of the equivalent spark gap.

Although we have thus not found how long a life the arrester has, we have evidence that the life of the arrester, if it is not accidentally destroyed, is so long that in two or three years' industrial service no appreciable or measurable deterioration occurs.

After finding this out by experience, we could realize, why our tests were not representative of industrial practise. We passed continuously and in rapid succession very many thousands of heavy discharges through it, and since every discharge produces a little gas, gradually gas must accumulate, and finally result in separation of the particles, increasing the resistance and the equivalent spark gap. In nature, however, we have individual discharges, or even a large number of discharges following each other, but then followed by a period of rest, when the gas produced can be dissipated, and that is the reason why the increase of equivalent spark gap did not occur, and so far we have found in two or three years no evidence of deterioration of this arrester under industrial service.

With regard to the wear produced by the leakage current. Such a wear exists in the aluminum arrester, where the leakage current is an electrolytic current, but it does not exist in the oxide film arrester, as the conducting material is of that class of conductors which I have called pyro-electric conductors, having a definite metallic conductivity at ordinary temperature, which, only at high temperature changes to electrolytic conductivity, and therefore this leakage current does not deteriorate.

As regards the discharge rate, I want to call attention to the oscillogram on the last page of Mr. Field's paper, which, as he states, shows the characteristic of the arrester in holding back the normal circuit voltage, and discharging with practically no resistance the over-voltage. In this oscillogram double voltage has been thrown on the arrester, taking the discharge, dropped the voltage, not to nothing, as a short-circuiting arrester would do, thus not short-circuiting the machine power, but to the "counter electromotive force" of the cell, discharging only the over voltage. As this was of limited power, the current discharge is only about 100 amperes, but this does not represent what we call the discharge rate of the arrester, which is the current which would pass at double voltage, because the current in this oscillogram is not the current taken at double voltage, but at very little above normal voltage.

The test to really find the discharge rate of the arrester is to impress double voltage with so much power back of it, that the voltage does not drop, and it is a difficult test, because it requires an enormous power supply back of the arrester, and therefore I cannot give you entirely definite values, but only the information that the discharge current at double voltage is materially above 1000 amperes, and apparently higher than that of the aluminum arrester and the protection apparently greater than or at least fully as high, as that given by the aluminum arrester.

In a future paper, Mr. Lougee, who continued the development after Capt. Field, expects to give more complete information on tests and performances of this arrester.

N. A. Lougee: The oxide-film arrester, although a new arrester, has been shown by tests and operating experience to be very reliable and to give excellent protection. Very many tests have been made on this arrester duplicating as far as possible in a laboratory all the operating conditions it might be subjected to, in order to be as certain as possible that its operation would be successful before it left the laboratory. Moreover, complete data have been obtained by means of discharge alarm recorders attached to the majority of the arresters which have been installed to date. We hope in a later paper to give an account of the extensive laboratory tests and operating results which we have obtained.

Table I, I believe, will be of interest in showing briefly how and to what extent the high-voltage units of oxide film arresters have been installed.

TABLE I.

			1915	1916	1917	1918 up to June 1st	Total up to June 1, 1918
Number of cells installed		Indoors	200	2932	1448	1544	6124
		Outdoors	...	...	1440	2900	4340
		Total	200	2932	2888	4444	*10,464
Cells used for life runs, Tests and Development:			3488	2436	1712	465	8101
Number of Arresters installed	11—15 Kv.	Indoors	2	5	6	10	23
		outdoors	..	..	5	6	11
		total	2	5	11	16	34
	22—25 Kv.	indoors	..	7	4	2	13
		outdoors	..	..	3	4	7
		total	..	7	7	6	20
	33—38 Kv.	indoors	..	4	..	..	4
		outdoors	..	..	1	4	5
		total	..	4	1	4	9
	Total	indoors	2	16	10	12	40
		outdoors	..	..	9	14	23
		total	2	16	19	26	63
Number of recording devices installed:			1	9	16	19	45
Number of arresters destroyed:				1	1		2
Cause				Isolated transformer stations destroyed by fire.			

*In addition hereto, 10,000 cells are in production and will be installed during the present season.

In addition to these cells, we installed in 1914 about forty 650 d-c. "O F" arresters on electric cars and car circuits. These arresters were of a preliminary type and of different design than is used at present. These first experimental units gave excellent results and many are still in operation.

As noted from the table, we installed the first high-voltage arresters in 1915 and since then we have gradually increased the number of installations and added an outdoor design. I wish also to call attention to the great number of cells we have used for development work. These 8000 are of the present cell construction—before this design of the present cell was perfected, a great number of preliminary units, of course, were used in the development. About three-quarters of the installations at present are equipped with a discharge alarm recorder so that we have definite information that the arresters are successfully taking care of all kinds of disturbances.

I think it will be of interest at this time to say a little more concerning the life of this arrester. This fact was something impossible to determine from laboratory tests alone, and consequently we were anxious to learn what effect actual operating conditions would have on the cells, before we considered it desirable to publicly announce the development of this arrester.

One cell from each of the four stacks of two different 11,000-volt "O F" arresters were returned to us for test after these arresters had been in operation about a year. Very careful tests on all these cells showed that the initial equivalent sphere gap had not increased in any case, that the 60-cycle voltage breakdown had not increased, and oscillograms showed these cells to give excellent protection against a surge voltage. The interior of the cells, which were opened, showed no deterioration of the filler, in fact the cells as a whole were exactly as good as new cells. The arresters from which these cells were taken were equipped with discharge alarm recorders and these recorders showed an advance in counter reading of about 3000 discharges in each case since the arrester was installed.

Last winter one complete phase from each of the two arresters mentioned above was returned for further tests. These arresters during the two years they had been installed showed several thousand discharges from the discharge alarm recorder, and moreover operators in each station had kept a careful record so these particular arresters were known to have had more than normal operation. Initial equivalent sphere gap measurements on a complete phase as a whole showed a value similar to what is obtained with new cells. Some of the cells were opened and no deterioration at all could be seen.

Cells from a few other installations have been returned to us and in every case to date no deteriorations in the protective value of the cells or of the interior of the cells has been found.

By having cells returned from arresters every once in awhile, we will eventually learn what the average life of an arrester

under normal service should be, and at present there is every indication that the life of the oxide-film lightning arrester is at least as long as that of any other lightning arrester.

L. W. Chubb: I would like to know whether tests have been made to find out whether one point heals up and an entirely different point breaks down, on a heavy discharge, or whether the first place that breaks partially heals up and progressively breaks down along the edge causing the point of discharge to travel along like the wandering of an arc on an electrode. The reason I ask is because, if the original puncture heals completely there must be an interruption and rise of voltage in order to break down a new point of the film. If the electrostatic charge of the film is not sufficient to heal one spot the voltage must drop and rise with each successive puncture giving an interrupter action and dangerous conditions similar to an arcing ground. The electrolytic arrester has enough capacitance and stored energy to heat a scintillation or breakdown without an appreciable drop of voltage and thus gives the true counter e.m.f. effect.

The oscillograms Figs. 3 and 4 in Dr. Steinmetz's paper, show the discharge of an electromagnetic field, without and with the arrester in shunt. In the first case the voltage rises to 7000 volts, in the second case the voltage rises to the gap voltage of 2200 volts and then drops to 560 volts, after the arc is established. The gap, of course, has an arc drop of approximately 50 volts, leaving about 510 volts creating a current of 22 amperes. I cannot understand how the resistance per cell can be as low as one ohm or 0.1 ohm per cell in view of this result. The oscillograph is too slow to follow very high-frequency oscillations and therefore gives only a smooth mean curve.

One of the advantages of the electrolytic type of arrester is the presence of high capacitance. In many cases high-frequency and steep-wave-front impulses can be discharged across the gap and absorbed without great rise of voltage or piercing of the film. I would like to know whether the new arrester has sufficient capacitance to obtain this condensive effect.

In both papers the equivalent spark gaps of different parts of the arrester are mentioned, but no figures are given. I would like to know what the equivalent spark gap of the arrester is. It seems that a varnish film which will hold line voltage so as to extinguish the gap after discharge will require several times line voltage before it pierces on an impulse, as the impulse ratio of solid dielectric films is known to be very high.

C. T. Allcutt: About the only point I want to mention in connection with Dr. Steinmetz's paper is the matter which Mr. Chubb brought out regarding the electrostatic capacity. As far as I can figure from the dimensions given, and from certain figures regarding the charging and leakage current of the arrester given by Capt. Field, the capacity of the oxide film arrester is in the neighborhood of 0.01 of the capacity of

the aluminum arrester, and as a condenser, its effect in discharging impulse voltages would not be very great. Apparently, it would be necessary, in discharging an impulse, to have that impulse actually pierce the dielectric film of the arrester, which might be expected to involve some time lag and some decrease in voltage. The difference in capacity is very great, and in that point, certainly the oxide film arrester does not possess all of the advantages of the electrolytic arrester, as it would seem.

C. P. Steinmetz: I will answer the three questions asked. Whether the puncture occurs at the film or a sealed-up puncture hole, is merely a question of probability. In resealing after a discharge the litharge spot thickens up so as to hold back the circuit voltage, and it is merely a question of probability whether the next discharge which punctures goes through an unbroken film or through a litharge spot, since just as in the aluminum cell, the resealed puncture hole is of the same strength as the rest of the film.

As to the capacity effect: The capacity of the cell is higher than it was in Capt. Field's time, due to the decreased resistance of the film, but it is less than that of the aluminum cell. The capacity at normal frequency is of less magnitude than the resistance, but it is, as you see from the construction, sufficiently large to act as a capacity path for those extremely steep wave fronts, and extremely high frequencies at which the time lag of the insulation comes into consideration, though at 60 cycles the capacity is practically negligible.

The oscillograms of the impulse voltage given in my paper as Figs. 3 and 4, are identically the same kind as would be given by an aluminum cell, that is, the impulse voltage rises to the breakdown voltage of the gap, and at that point the voltage drops to the circuit voltage, and the 560 volts is not the resistance drop across the arrester, but is what you might call the "counter electromotive force of the arrester", which holds back the normal circuit voltage, the voltage of the direct-current railway generator which was on the circuit. This would occur in identically the same way with the aluminum cell, and it is a characteristic of this type arrester not to discharge the normal machine voltage, but only over-voltage. But from this follows nothing regarding the discharge resistance. The discharge resistance is obtained by dividing the excess voltage over the normal circuit voltage, by the discharge current.

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THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

BY HAROLD S. OSBORNE

Abstract of Paper

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits. The new arrangements are the result of a systematic investigation of the degree of flexibility which could be obtained in the coordination of telephone transpositions and transpositions in outside disturbing circuits, particularly three-phase power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

The application of these and other arrangements of coordinated transpositions to specific cases must take into account the variations in separation and other changes in the power and telephone circuits. It is considered to be beyond the scope of the present paper to go into a discussion of the problems involved in these specific applications.

Introduction

IN a well-engineered telephone system the power transmitted into a long-distance telephone circuit is almost entirely absorbed in line losses. The circuit terminates in that marvelous little electric motor, the telephone receiver, which requires only a few millionths of a watt for its operation. As the power delivered to the line by the telephone transmitter may be measured in hundredths of a watt, only a fraction of one per cent of the transmitted energy need be received at the far end of the line. This is fortunate, for as the telephone currents are of relatively high frequency, the energy losses suffered by transmission over a given circuit are very much greater than would be the losses for 60-cycle current. Moreover, the telephone currents must be

carried over very great distances. It has therefore been necessary to do an enormous amount of work in developing means for increasing the efficiency of long telephone circuits in order that country-wide service may be given with a network of conductors which should not be prohibitively expensive.

The large ratio between the energies at the transmitting end and at the receiving end of a long distance telephone line imposes very severe limitations on the amount of inductive effect which can be permitted between different telephone circuits. If the transfer of energy between two circuits is more than a millionth of the energy transmitted there is danger that one can overhear on one circuit what is said on the adjacent end of the other. This must of course be avoided, and an even stricter limitation of the transferred energy is desirable.

The method which is used to prevent excessive transfer of energy between telephone circuits on the same pole line is the installation of a systematic arrangement of transpositions of the conductors of each circuit.

Present standard telephone practise for open-wire circuits provides ten wires spaced about one foot (0.3 m.) apart on a crossarm and any number of crossarms spaced two feet (0.6 m.) apart on a pole. The currents carried by these circuits are of very high frequency compared with those used in power practise and tend to make much of small mutual impedances or admittances between the circuits. It is small wonder then that when in 1886 the first long distance telephone line was built between New York and Philadelphia it was found that, listening at the distant end, an observer could not tell which circuit was used for transmission but could hear equally well on all of them. The systematic study of telephone transpositions began at that moment and has continued until the present day.

One factor which has much complicated the study of telephone transpositions is the common use of phantom circuits. These are more accurately described by the French terms *derived* or *superposed* circuits. A phantom circuit is formed by superposing a third circuit on two 2-wire telephone circuits called the side circuits of the phantom. The phantom and its side circuits thus use the same copper conductors, and it is evident that great care in balancing the inductive relations is necessary to prevent the transfer of energy between circuits so intimately related.

Long distance telephone lines have not grown up without

neighbors, and in the design of telephone transposition systems it has been necessary to take them into account. Telegraph systems and power transmission systems have been contemporaries with the telephone system in the process of development. These systems have not in general been obliged, for any reason of their own, to restrict their electric and magnetic fields as has been necessary with telephone circuits. From the first, however, the limitation of the effect of these outside fields has been a factor in the design of telephone transposition systems.

It is evident that when a telephone circuit is paralleled by an electric power circuit carrying hundreds or even thousands of kilowatts, the mutual inductive connection between the two circuits must be extremely minute if we are to avoid induction in the telephone circuit of power of the same magnitude as that of the voice currents or even much more. In fact, this difficult problem would probably be quite impractical of solution if we had to rely alone upon balance produced by transpositions. Fortunately, another very weighty factor works for the solution of the problem. The fundamental frequency of commercial power circuits is out of the easily audible range and quite out of the range of frequencies which the telephone line must transmit in order to reproduce intelligible speech. As a matter of fact the noise produced by power circuits in telephone circuits is practically not at all due to the fundamental but due to the harmonics or high-frequency components of the power circuit which lie within the range of telephone frequencies. The energy represented by these harmonics is represented by kilowatts or even by watts rather than by thousands of kilowatts. In spite of this fact the degree of balance required in cases of close parallel is very precise indeed.

Of all the means used to reduce the inductive effect between power circuits and telephone circuits the coordinate transposition of both classes of circuit is probably the most generally important. The application of this means to parallels between telephone and power circuits is in practical cases attended with difficulties because instead of two circuits running accurately parallel for long distances the separation usually varies at frequent intervals and very irregularly.

The usual case, in which the power circuit is three-phase, presents another complication which is fundamental to this type of circuit. The telephone circuit is a single-phase circuit and its conductors have only two relative arrangements on the pins.

The three-phase circuit has three possible arrangements of its conductors on the pins, and three sections of line, each with a different arrangement, are necesssary for what is called a "barrel" of the power circuit. The combinations of relative pin positions of the two circuits when both are transposed are, therefore, six in number. The difficulties introduced by this relation have been discussed in an earlier paper* before the Institute.

Although the importance of neutralizing as far as practicable the effect of outside circuits has always been a factor in the design of transposition systems, the possibilities of coordinating the telephone transpositions with the transpositions in three-phase power circuits were not at first fully developed. With the great growth in both telephone circuits and three-phase power circuits this matter has been growing rapidly more important in telephone transmission.

In connection with the very extensive investigations of the Joint Committee on Inductive Interference in California it was therefore considered advisable to undertake the development of a new system of telephone transpositions which would embody the maximum possibilities of coordination of transpositions in the two classes of circuit. There has resulted from this work a system of transpositions known as the "exposed-line" transposition system.

Outline of Methods of Design

The design of a transposition system requires the mastery of a very difficult and complicated technique. It is attempted here to give only a general outline of the methods employed and some of the difficulties encountered and overcome. Some of the mathematical work is given in the appendices.

Conductive and Inductive Connections. The mutual connection between two circuits may be of any of the following characters

Resistance
Leakage
Inductance
Capacitance

Two circuits may be said to be unbalanced with respect to each other by a mutual connection of any of the four above characters when the mutual connection enters into the circuit in such a way that current in one circuit tends to cause a current in the other circuit.

Inductive Interference as a Practical Problem. Messrs. Griswold & Mastick, Transactions, A. I. E. E., Vol. XXXV, 1916, p. 1051.

By resistance and leakage we denote conductive connections respectively series and shunt. Although two metallic circuits which have no resistance in common cannot have a mutual resistance unbalance, care is necessary to avoid a mutual resistance unbalance between a phantom and its side circuits. This is however a matter apart from transpositions. The leakage from telephone circuits is almost all directly to ground. This is normally kept very low because of the large effect which high leakage would have upon the efficiency of the circuits. For these reasons leakage is not an appreciable factor in the interference between telephone circuits, except under abnormal conditions, such as those produced by baling wire thrown over a telephone line.

Two circuits strung side by side have both mutual inductance and mutual capacity unbalances. The two terms represent the coefficients by which are measured the unbalanced action of the mutual electromagnetic and electrostatic fields respectively, and both represent inductive rather than conductive effects. It is the purpose of transpositions to correct for these as largely as may be.

Inductive Effects Between Telephone Circuits. The general equations for the mutual inductive effects between long parallel circuits are hopelessly complex. For the benefit of those interested a discussion is given in Appendix A of the mathematics by which these inductive effects may be computed in some simple special cases. In Appendix B are given equations for the most simple general case, that is, the case of inductive effects between two long symmetrical parallel wires with ground return. Examination of equations (**7**) to (**10**) in this appendix shows that the current or voltage in either conductor is represented by the sum of two complex exponential terms. Little consideration is required to convince one that similar general equations for forty parallel wires would be impracticable either to establish or to use.

However, in studying the inductive effects between telephone circuits (commonly called cross-talk), use can fortunately be made of a beautiful simplification. Although the circuits are very closely associated on the same pole line, the requirement for good operation after the circuits are transposed is that the transfer of energy from one circuit to another shall be exceedingly small. This being the case, the reaction of the induced current on the inducing circuit is evidently negligible. The prob-

lem can therefore be studied by computing the current and voltage of the inducing circuit as though no currents whatever were induced in other circuits and then computing the inductive effect of these currents and voltages on each adjacent circuit under the assumption that no current flows in any of the other circuits.

In order that this computation may be carried out, it is necessary that the coefficients of induction between the circuits be determined. The mutual inductance unbalance between all pairs of circuits on a 40-wire pole lead can be readily computed. It was estimated, however, that to compute the direct capacities between circuits would require the time of one man for at least ten years. It was therefore found much cheaper and more practical to make measurements of the capacities between the wires in a short section of line under construction. The results obtained in this way are given in Table I.

Resultant Unbalance between Long Circuits. With the data and by the use of the approximations outlined above, it is possible to compute the inductive effect between any two short lengths of telephone circuit on the pole lead. In carrying out the computation of the effects for long circuits, however, a complication arises due to the fact that the current and voltage in the disturbing circuit are different both in magnitude and phase angle at different points of the line. The change in magnitude, because of attenuation effect, varies from 0.05 per cent to over 0.5 per cent per kilometer on open wire telephone circuits of different construction, and the change in phase angle varies from a small value up to nearly 10 degrees per kilometer. Also, in the propagation of induced current along the disturbed circuit to its terminal, this current undergoes a similar change in magnitude and in phase angle. Therefore, the current induced between two circuits in one kilometer length of line cannot be perfectly balanced against that induced in another kilometer length, and no matter how often or under what arrangement the transpositions are placed, they cannot serve to perfectly balance out the induced effects. There is always a resultant effect which is known as the "type unbalance". The transposition system must be so designed that this type unbalance is kept below the desired limits for all combinations of the circuits on the lead.

A brief mathematical discussion of type unbalance is given in Appendix C.

TABLE I.

CAPACITIES IN MICROMICROFARADS PER KILOMETER IN

Direct Capacities Betwe

Wires ↓	Wires →																					
	10		9	8	7	6	5	4	3	2	1	20	19	18	17	16	15	14	13	12	11	
	1		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10	1		1785	716	360	239	142	108	92	89	103	968	603	323	161	100	55	39	37	40	58	
9	2			1579	593	333	164	114	90	81		631	639	469	249	144	68	43	35	34	47	
8	3				1509	659	255	157	111			347	469	571	432	267	116	66	45	39	50	
7	4					1792	475	256				188	262	435	541	440	214	116	68	50	56	
6	5						1030					122	155	274	464	603	384	209	110	70	75	
										20	11		1547	511	193	109	48	27	22	26	36	
										19	12			1383	429	193	68	34	22	22		
										18	13				1318	433	134	63	32			
										17	14					1621	332	141				
										16	15						854					
																				30	21	
																				29	22	
																				28	23	
																				27	24	
																				26	25	

Wires

Wires

I.

IN A 40-PT. NO. 12 N. B. S. G. 40-WIRE POLE LEAD.

etween Wires.

30	29	28	27	26	25	24	23	22	21	40	39	38	37	36	35	34	33	32	31	Earth
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	Earth
341	191	112	68	49	32	27	25	32	50	208	124	82	60	[illegible]	34	39	48	44	60	[illegible]
208	144	102	70	52	32	25	22	26	40	125	80	58	44	36	34	32	30	35	45	[illegible]
131	105	94	78	65	41	31	25	27	39	81	55	43	35	[illegible]	28	26	27	28	36	141
88	78	84	89	86	61	42	30	29	40	62	45	39	34	34	32	27	27	30	36	181
65	57	67	85	96	81	58	40	35	44	50	33	30	[illegible]	[illegible]	31	28	29	28	40	[illegible]
884	546	277	132	74	35	25	20	24	36	334	204	131	[illegible]	64	46	37	35	36	48	152
547	570	412	217	119	48	30	19	17	26	192	144	111	73	62	39	28	26	27	33	192
278	412	503	377	227	86	45	24	19	22	114	103	100	[illegible]	62	43	30	24	21	26	184
133	217	379	483	403	178	91	43	25	24	67	68	82	[illegible]	[illegible]	50	40	34	33	34	134
75	118	232	413	526	339	182	87	46	37	49	52	67	[illegible]	101	85	73	49	41	32	197
	1544	507	198	104	45	30	24	26	39	978	625	345	[illegible]	131	71	53	47	47	64	464
		1373	428	193	63	35	21	17		606	628	473	207	152	71	47	36	35	38	236
			1324	491	134	65	30			319	457	581	[illegible]	[illegible]	122	65	42	38	38	189
				1626	336	149				160	245	428	541	456	309	116	65	46	46	140
					853					101	147	272	470	603	368	215	119	66	36	177
								40	31		1767	693	343	229	131	90	80	78	88	776
								39	32			1562	538	327	157	105	80	72		528
								38	33				1491	642	242	145	88			497
							Wires	37	34					1799	465	244				37
								36	35						1099					428

(Continued)

The type unbalance does not measure the entire inductive effect between two telephone circuits, for this is contributed to also by another factor which is sometimes very important. It is impracticable to locate the telephone transpositions at exactly the theoretically correct points. In laying out a new line it is sometimes possible to set poles exactly at the theoretical transposition point, and when this cannot be done the nearest pole is generally chosen as the transposition pole. However, the transposition itself occupies two pole spans. Moreover, small changes in separation of wires cannot be avoided, even going round a curve introduces a certain amount of irregularity in the separation of wires. These and other irregular factors increase the inductive effects above those due to the type unbalances and must be taken into account in designing the transposition system.

Phantom Circuits. One factor which introduces a large complication into the design of transposition systems is the general

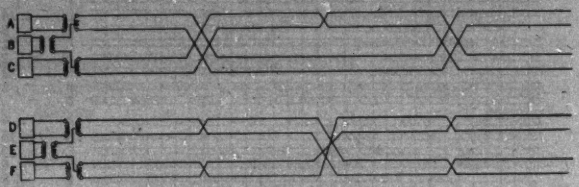


FIG. 1—PHANTOM TELEPHONE CIRCUITS

use of phantomed circuits in long-distance telephone lines. This is standard practise for long distance telephone circuits, and makes it possible to obtain three circuits from each two pairs of wires instead of two. The principle involved in phantoming is generally understood. The arrangement is indicated diagrammatically in Fig. 1.

By the transposition of a phantom circuit the pin positions of its side circuits are interchanged, and the coefficients of inductive effect between the side circuit and other circuits on the lead are changed. In order that the phantoming may be possible without interference, it is therefore necessary not only to very carefully balance the terminal transformers and other apparatus connected to the telephone lines, but also to use a transposition system that provides for a very high degree of balance of the inductive effects between the phantom and its side circuits and also between these circuits and the other circuits on the pole line.

It is evident that in designing the transpositions in the side circuits it is necessary to take into account the transpositions in the phantom. For example, in the figure, side circuit C would be considered balanced to side circuit F if the transpositions in these two circuits only were to be considered. However, the transpositions in the phantom circuits change the coefficients of induction between circuits C and F in different parts of the section shown and therefore prevent the transpositions in the side circuits from balancing this section.

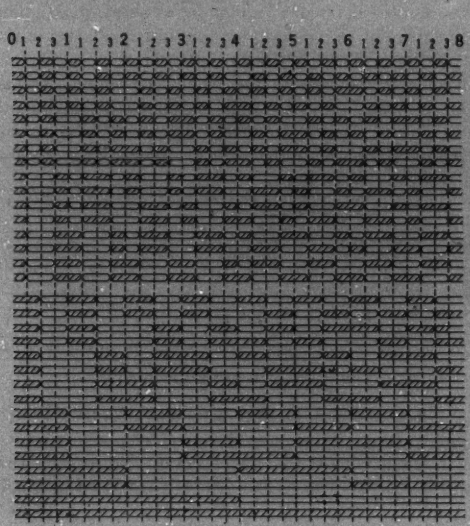


FIG. 2—TYPICAL ARRANGEMENTS OF TRANSPOSITIONS

Equations showing the method of computing the equivalent exposure and the type unbalance between circuits which are phantomed are given in Appendix D.

In order to fit in with standard telephone practises a transposition system must provide for the phantoming of any two horizontally adjacent pairs of circuits and any two vertically adjacent pole pairs. The system must also provide that any of these combinations of circuits may be phantomed and any others non-phantomed.

Typical Arrangements of Transpositions. In carrying out the design of a transposition system it is convenient and economical to arrange the transpositions for each circuit in accordance with one or another of a list of selected typical arrangements. In Fig. 2 are shown the types, 32 in number, suitable for use in a transposition section made up of 32 parts of equal length. Reading from bottom to top, the types are arranged in the drawing so that each type has one more transposition per section than the preceding type.

The type unbalance between two telephone circuits is determined by the relative transposition of the two circuits, that is, the points at which one circuit is transposed but not both. The typical arrangements of transpositions have the characteristic that the exposure between any two circuits so transposed can be represented by some other type; that is, the exposure between any two types is the same as the exposure between some other type and a circuit without transpositions. For example, exposure f to $k =$ exposure F to $K = A$, exposure J to $m = k$. For transposition sections divided into 64 equal parts, 32 more typical arrangements can be added to the 32 shown in the figure by adding a transposition at the midpoint of each elemental length of each type.

Typical Arrangements for Exposed Lines. The purpose of the exposed-line transposition system is particularly to provide increased facility in balancing the induction from outside circuits in so far as this can be done by transpositions. The effect of outside circuits in inducing voltages between the telephone wires and ground which may be productive of very serious interference cannot be remedied by transpositions in the telephone circuits, but can frequently be greatly reduced by transpositions in the outside circuits. The telephone circuit transpositions must be designed to equalize the voltages induced in the two sides of each telephone circuit with the greatest possible flexibility in the location of the transpositions in the outside circuits. This imposes great limitations on the typical arrangements of transpositions which can be used.

The outside disturbing circuits can be classified as follows:

1. Non-transposed circuits such as telegraph circuits. This applies also to the residual components of current and voltage in power circuits.

2. Two-wire transposed circuits such as single-phase power circuits.

3. Three-phase power circuits.

In order that the disturbing effect of circuits of the first class may be reduced as far as possible, no typical arrangement of telephone transpositions can be used whose type unbalance factor to non-transposed circuits exceeds a certain value. This eliminates a few of the types having the least numbers of transpositions.

Two-wire outside circuits can best be transposed in accordance with some of the typical arrangements for telephone circuit transpositions. A balance to these circuits requires then that certain types be set aside for their use and that the exposures between the transposed telephone circuits and the outside circuits transposed in accordance with these types be kept below the limiting values set for type unbalance for these cases.

The inherent complication in balancing the inductive effects

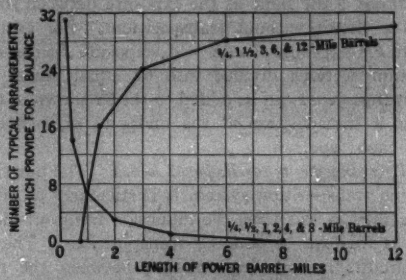


FIG. 3—NUMBER OF TYPICAL ARRANGEMENTS OF TELEPHONE TRANSPOSITIONS WHICH COORDINATE WITH THREE-PHASE POWER LINES TRANSPOSED WITH DIFFERENT LENGTHS OF BARREL

between telephone circuits and three-phase power circuits arises, as has been noted, from the fact that a three-phase circuit has three different arrangements of conductors on the pin positions and a two wire telephone circuit has but two. There are, therefore, six combinations of relative pin positions of the two circuits, all to be used for equal lengths of line.

In considering the severe restrictions placed on the selection of telephone transposition types by the necessity of balancing to transposed three-phase power circuits, the following conditions may be noted:

1. If the barrels of the power line are very long, they can be made to coordinate with almost all of the typical arrangements by having each

non-transposed section of power circuit, that is, each third of a barrel, equal in length to one-half or one-fourth of the telephone transposition section, for most of the typical arrangements balance within this length. This is shown in Fig. 3, assuming 12.8 km. (8 miles) to be the length of the telephone transposition section. Thus with a 19.2-km. (12-mile) power barrel the telephone circuits may be balanced for each 6.4-km. (4-mile) non-transposed length of the power circuit. However, as the length of the power barrel is progressively decreased to 9.6 km. (6 miles), 4.8 km. (3 miles) 2.4 km. (1 ½ miles) etc., the number of typical arrangements of telephone transposition which can be used very rapidly decreases.

2. If the barrels of the power line are very short, they may be made to coordinate with all the typical arrangements by having a complete barrel between telephone transpositions. In the case shown in Fig. 3 this requires a 0.4-km. (¼ mile) barrel. As the length of barrel is increased, however, to 0.8 km. (½ mile) 1.6 km. or 3.2 km. (1 to 2 miles) etc., the number of typical arrangements of telephone transposition which can be used, very rapidly decreases.

3. Barrels in the power line of relative lengths other than those mentioned do not in general coordinate with telephone transposition arrangements.

As a chief requirement of the exposed-line system was that it should provide a maximum possible flexibility in the location of transpositions in parallel three-phase power circuits, this was made a matter of most careful study. The results obtained are outlined in the description of the system given in the next section of this paper.

Determination of Limiting Permissible Exposures. Before the attempt can be made to select, from among the typical arrangements of transpositions which can be used, a type for each telephone circuit on the lead, a table is made up of the limiting type unbalances which can be permitted between pairs of circuits, assuming that a given limit of the induced current between circuits cannot be exceeded. The procedure is as follows:

1. A computation is made of the total amount of unbalance which could be permitted between each combination of circuits on the lead, taken two at a time. This unbalance is expressed in terms of the length of untransposed exposure between the circuits which would give the limiting value of induced current, no current being induced in the rest of the circuit.

2. The total allowable unbalance for each pair of circuits is reduced by a figure representing the unbalance which must be expected to arise from irregularities with the type of construction to be employed.

3. The remaining unbalance represents the limiting permissible effect of type unbalance in a long length of line.

4. From data regarding the attenuation of currents along the telephone circuits, and from the total limiting effect of type unbalance, a com-

putation is made of the limiting type unbalance per transposition section. This sets a direct limit on the relative exposure type between each pair of circuits.

The Design of a Transposition System. With the information regarding the requirements of the proposed transposition system prepared as indicated above, the creation of a design which shall meet the requirements of maximum economy and efficiency is a problem requiring a great deal of skillful and conscientious application on the part of a trained expert. The problem is of course one of selecting from the typical arrangements which can be used, the best type for each circuit such that the type unbalance between no two circuits will exceed the established limit.

In order that a design may be complete it must provide for any number of crossarms on the telephone lead. However, it is generally desirable to take into consideration only four crossarms at the beginning of the design. Considering all the circuits, phantom and two-wire, which may be placed on the lead, taken in combinations of two there are about 2500 such combinations on a standard 40-wire pole line.

In carrying out the design of the exposed-line transposition system it was desired to make the requirements both of efficiency and economy as exacting as could possibly be met. These requirements were set, therefore, at the beginning of the work, so high that no design could meet them, and then gradually reduced until it was possible to complete the design. As the design depends on a number of interdependent requirements, this process not only requires a very large amount of work but demands the continual exercise of trained judgment on the part of the designer to bring about the mutual adjustment of requirements so as to produce the best final result.

The process of determining whether or not a design can be established to meet a given set of requirements is very technical and will not be described here in detail. It involves a systematic consideration of all the possible arrangements. These possibilities are very great in number—about seven million in the case of the *E* section design—and are, of course, not considered individually, but in groups, and successive groups of possibilities are rejected when it has been proved that they could not meet the requirements.

The systematic grouping of the possibilities, and the means adopted for the examination and elimination of the groups depends a great deal upon the experience and skill of the designer. The general method which has been successfully employed can

be roughly described as a consideration of the design in steps. First the attempt is made to carry through a preliminary design in which pairs of circuits which cannot be exposed to each other untransposed for the entire length of the transposition section (P exposures) are provided with different typical arrangements of transpositions, but in which no other requirements are considered. If this design is possible, the other balance limitations of the types selected are added, one by one.

When a design is finally reached it is usually found that any one of a considerable number of arrangements will meet the balance requirements almost equally well. The most economical of these is chosen for use. In determining upon the relative economy of different designs, account is taken of the extent to which circuits on different pin positions will exist in the telephone plant and the extent to which it might be possible, in retransposing existing lines, to use the present transpositions as a part of the transpositions called for in accordance with the new system.

Exposed-Line Transposition System

Special Electrical Requirements. The exposed-line transposition system was designed to meet the following special electrical requirements:

1. The greatest practicable flexibility for the arrangement of transpositions in power circuits paralleling the telephone circuits, the power circuit transpositions co-ordinating with those in the telephone circuits so as to produce a balance of the inductive effects. This requirement to be met for transpositions both in three-phase circuits and in single-phase circuits.

2. A suitable balance against the inductive effects of telegraph circuits or other ground return circuits paralleling the telephone circuits.

3. A degree of freedom from interference between the telephone circuits (crosstalk) suitable for modern high grade telephone circuits used in connection with loading and with telephone repeaters.

General Requirements. In addition to these special requirements, it must meet the following requirements for any satisfactory transposition system:

1. *Neutral Points.* The system must be arranged so that neutral points, that is, points at which a maximum degree of balance is obtained between all circuits on the telephone line are

provided where there are discontinuities in line construction or in the parallel. In the case of open wire toll lines of the Bell telephone system, there must be neutral points at regular intervals of about 12.8 km. (actually 7.88 miles). In addition there must be neutral points wherever there is an important junction between telephone lines and at other points of discontinuity. It is, therefore, desirable that the system include a design intended for a maximum length of 12.8 km. and another design or designs for shorter maximum distances.

2. *Cost.* The completed section must be designed for as low a cost as practicable without sacrifice of the electrical requirements. This means that as far as other conditions of

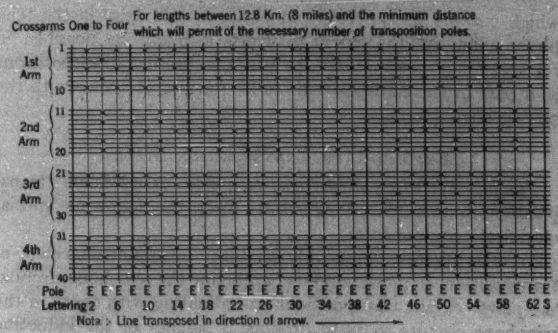


FIG. 4—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

design permit, the typical arrangements having the least numbers of transpositions should be placed on the circuits most often used. In general, it is desirable that phantom circuits have fewer transpositions than two-wire circuits as phantom circuit transpositions are the more expensive.

3. *Transposition Poles.* It is desirable that the transposition poles be as few in number as practicable not only on account of expense but because the use of large numbers of transposition poles limits the minimum length of a transposition section. The use of many poles may also tend to increase the induction between telephone circuits due to irregularities.

E Section and L Section. As a result of the study, two designs have been completed which are called respectively the *E*

section and the L section. Diagrams showing transpositions according to the E section for crossarms 1 to 8 of a pole line are shown in Figs. 4 to 9, inclusive, and according to the L section in Figs. 10 to 15, inclusive. It will be noted by examination of the figures that the designs provide for the phantoming of any two horizontally adjacent pairs or of any two vertically adjacent pole pairs on the lead. In carrying out the designs provision has been made for extension if necessary to any number of crossarms.

The E section is designed for a maximum length of approximately 12.8 km. It was found impossible to meet the electrical

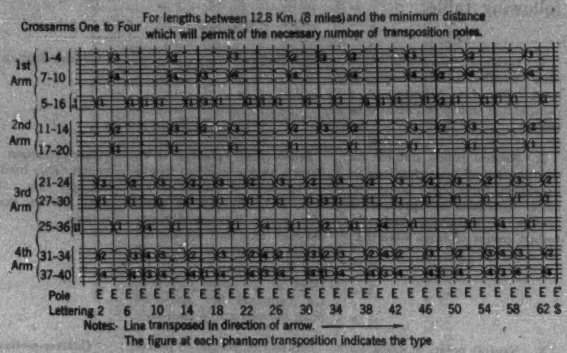


FIG. 5—E SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

requirements for this case with the use of 32 transposition poles per section. It was possible, however, to design the E section so that in the first four crossarms the non-phantom circuits and the phantom circuits which are most commonly used require only 32 transposition poles, so that in some toll lines not exceeding four crossarms, the larger number of transposition poles will not be necessary. For circuits on crossarms below the fourth and for some infrequently used phantomed circuits on the upper crossarms, 64 transposition poles are required.

The E section may be used for as short a length of line as supplies the required number of transposition poles. For distances less than 6.4 km., however, it is economical to use the

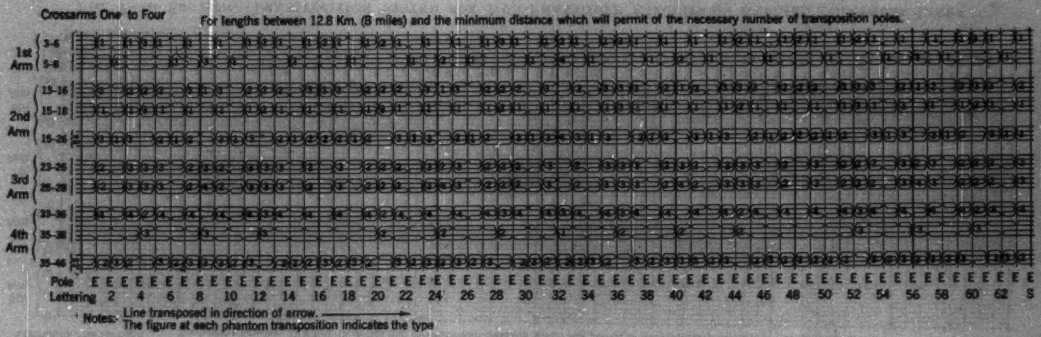
L section. It has just half the number of transposition poles for the corresponding circuits.

When two or more *L* sections are used consecutively in the same loading section, junction transpositions should be installed at the *S* poles between them in order to reduce the induction between the telephone circuits. These junction transpositions are indicated in the drawings for as many as four consecutive *L* sections.

Use of E and L Sections on Exposed Lines. The results obtained in meeting the special electrical requirements regarding balance to the inductive effects of outside circuits are indicated in the following table:

	E Section	*L Section*
1. Element of section in which each circuit is transposed at least once, *i. e.* balances to a parallel non-transposed disturbing circuit.	8th Section	Quarter-Section
2. Neutral points suitable for the location of transpositions in paralleling three-phase power circuits, separation of approximate centers of disturbing and disturbed groups of wires *more* than 6 meters.	8th Section points; *i. e.*, barrels having nominal length of 4.8 km. (3 miles) or 9.6 km. (6 miles) can be used. Five points dividing distance between any two 8th-section points into six equal parts, all five transpositions to be rotated in the same direction; *i. e.*, nominal 0.8 km. (0.5 mile) barrels can be used also.	Quarter-section points; *i.e.* nominal 4.8 km. (3 mile) barrels can be used.
3. Neutral points suitable for the location of transpositions in paralleling metallic single-phase power circuits—separation as defined above *more* than 6 meters.	8th-section points. 3 points dividing distance between any two 8th-section points into four equal parts.	Quarter-section points.
4. Neutral points suitable for the occurrence of discontinuities—separation as defined above *more* than six meters.	8th-section points	Quarter-section points
5. Neutral points suitable for the location of transpositions in disturbing power circuits or the occurrence of discontinuities in disturbing power or telegraph circuits—separation as defined above *less* than 6 meters.	Mid-section point	Mid-section point
6. Neutral points suitable for the occurrence of discontinuities in telephone circuits.	S-poles	S-poles

The results given in the table are further illustrated in Figs. 16 and 17 which show premissible relative locations of the tele-


Crossarms One to Four
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
1st Arm
3-6
5-8
2nd Arm
15-16
15-18
15-26
3rd Arm
23-26
25-29
4th Arm
33-36
35-38
35-46
Pole Lettering
E E E

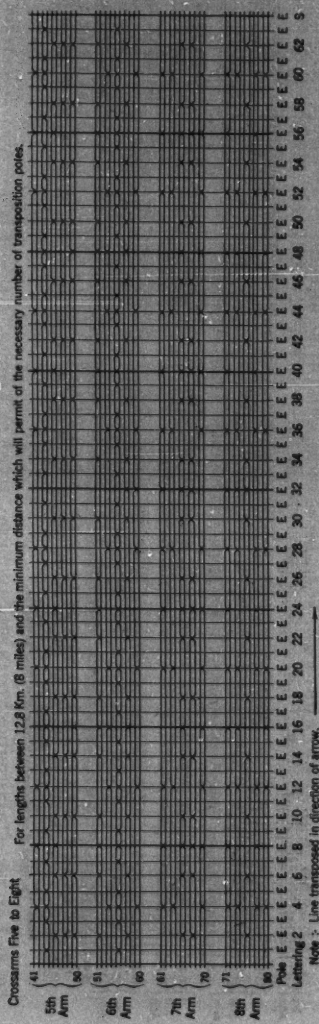


FIG. 7—E SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

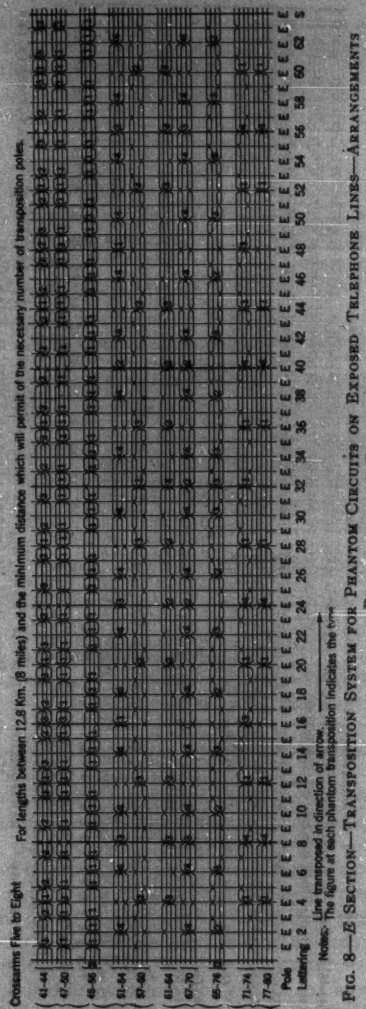
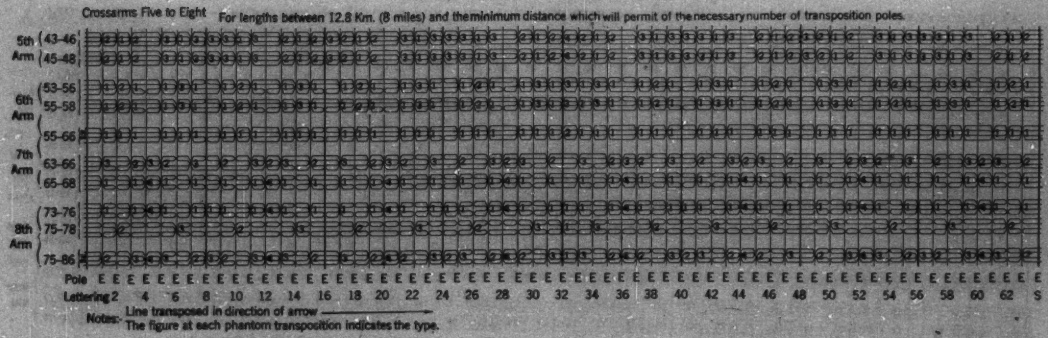


FIG. 8—E SECTION—TRANSPOSITION SYSTEM FOR PHANTOM CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS


Crossarms Five to Eight
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
5th Arm
43-46
45-48
6th Arm
53-56
55-58
55-66
7th Arm
63-66
65-68
8th Arm
73-76
75-78
75-86
Pole E E E

phone transposition poles and of transpositions and discontinuities in the outside parallel circuits or changes in separation between the two classes of circuit.

It will be noted that the E section can be used to coordinate with transpositions in three-phase power circuits which create for the maximum length of E section 0.8 km. (½ mile), 4.8 km. 3 miles) or 9.6 km. (6 miles) barrels in the power circuit. As it was considered very important to obtain the maximum practicable degree of flexibility in this respect, this matter was given very thorough study and a large amount of work was required to arrive at this result. It should be noted, moreover, that in the use of a 0.8-km. barrel, two barrels occupying a 1.6-km. section

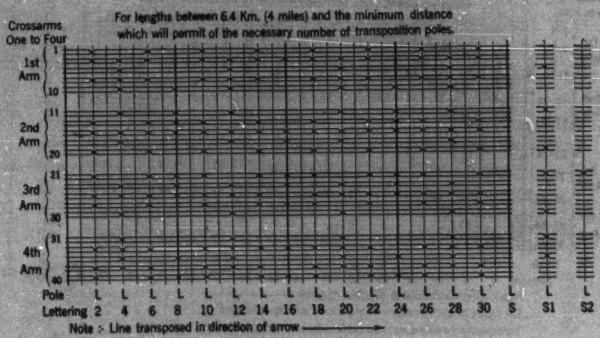


FIG. 10—L SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

of the transposition system are required to obtain a balanced condition.

In the L section the requirements for the avoidance of excessive interference between the telephone circuits are more exacting than in the E section because the use of a smaller number of transposition poles makes available a correspondingly smaller number of typical arrangements of transpositions. It was found possible, however, to design this section so that it balances to a three-phase power line transposed at the quarter section points.

The exposed-line transposition system has been designed primarily for use on toll lines and not for use on joint lines with

power distribution circuits. It is, however, somewhat better for this case than the other transposition systems in use.

When large inductive effects are experienced from circuits less than 20 ft. (6 m.) away from a side circuit, the variation in separation between the telephone circuit and the exposing circuits as the side circuits shifts from one pair of pin positions to another must be taken into consideration. The *E* section is not designed to give a high degree of balance under this condition for all circuits in each eighth-section, but does give balance to uniform outside induction effects for most circuits in each eighth-section and with one exception, for all circuits in each

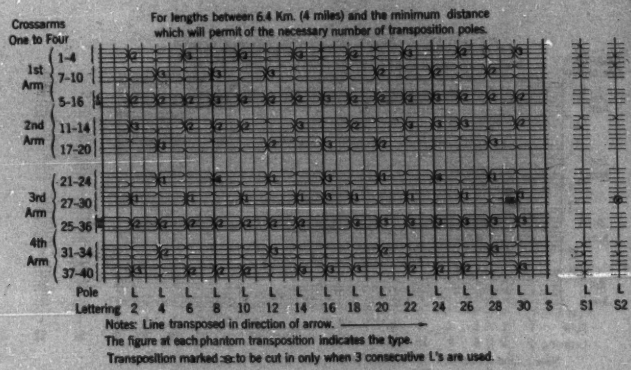


FIG. 11—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUIT ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

half-section. Under similar conditions the *L* section gives a balance for most circuits in each quarter section and for all circuits in each half section.

It should be clearly pointed out that in the use of the telephone transposition sections to coordinate with transpositions in power circuits, it is important that the transpositions in the two lines be coordinated both as regards relative length and as regards relative location of telephone transposition section and transpositions in the power circuits. The locations of the transpositions in either line must, therefore, be made with reference to the requirements of the other. It is frequently the case that

the locations of the transpositions in the power circuit are to a large degree determined by the changes in separation and other characteristics of the parallel.

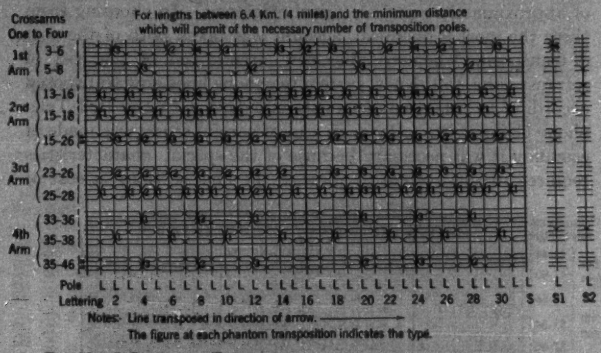


FIG. 12—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

The *E* and the *L* section are both designed to give a high degree of balance of the inductive effects between the telephone

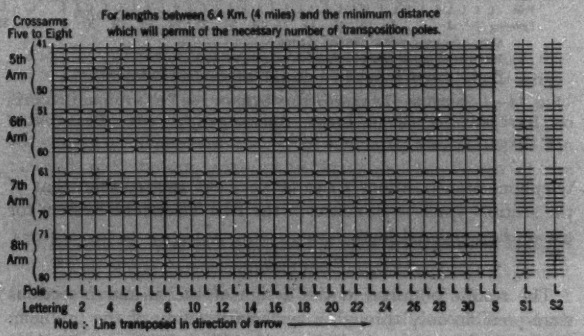


FIG. 13—*L* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

circuits. In using them, therefore, it is important that the transposition poles be spaced as accurately as practicable in

order to avoid diminishing the effectiveness of the system in this respect because of irregularities.

The design work led to a large number of possible arrangements of approximately equal electrical efficiency. In choosing from these the most economical, account was taken both of the cost of transposing when stringing new circuits and of the cost of retransposing to the new system telephone lines now transposed according to other arrangements standard with the Bell Telephone Companies. Although a most careful study was made of economy it was necessary because of the special characteristics required in this system to allow in the final sections a cost of

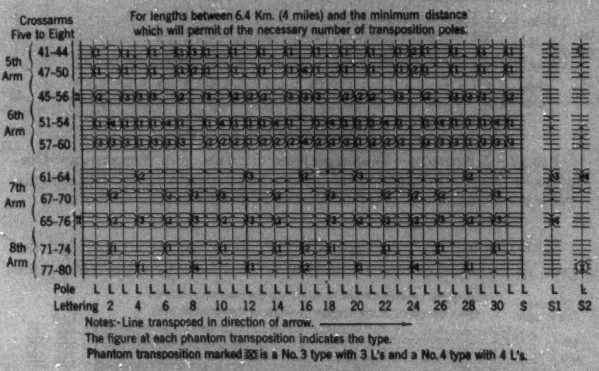


FIG. 14—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

telephone transpositions 25 to 50 per cent greater than that of the section which is standard in the Bell Telephone System for use on non-exposed lines.

CONCLUSION

The results obtained in the design of the *E* and *L* sections were possible only through a systematic study of the problem. The work directed particularly to the design of these two sections occupied about two man-years' time of the specially trained experts who carried out the work. The investigation has been so thorough that the conclusion seems warranted that the *E* section and the *L* section are substantially the best arrangement

of transpositions which could be devised to meet the requirements established for them.

The work of many members of the engineering department of the American Telephone and Telegraph Company has contributed to the results. The theory of induction between telephone circuits and the special analytical methods of design have been developed in the department during a considerable number of years. Special mention should be made of the work of Mr. G. A. Campbell who, years ago, laid the foundations for the theory of induction between telephone circuits, and of

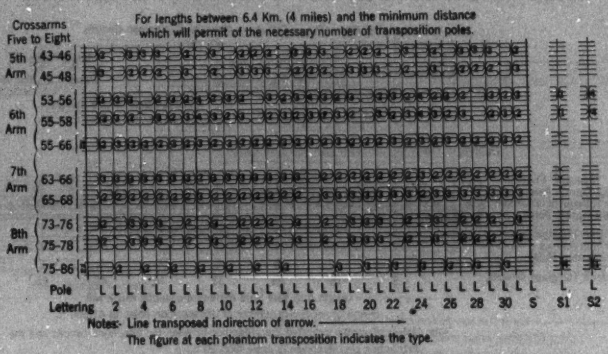


FIG. 15—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

Messrs. A. G. Chapman and R. E. Leonard, who perfected the methods of systematic search and carried out the design of the *E* and *L* sections.

APPENDIX A. INDUCTION BETWEEN PARALLEL WIRES

1. *Two Wires with Ground Return—Short Length of Exposure.* If the current and voltage in a short element of circuit No. 1 (Fig. 18) are represented by I and E, the inductive effect which this element of circuit No. 1 exercises on parallel circuit No. 2 can be represented as follows:

The voltage induced in circuit No. 2 because of current I is

$$V_2 = -j\,\omega\,I\,M\,l \tag{1}$$

where ω equals 2π times the frequency, l the length of element of circuit considered and $M\omega$ the mutual impedance between the circuits per unit length. In the case of a perfectly conducting earth surface, M is made up entirely of the mutual inductance caused by the linkage of flux from circuit No. 1 with circuit No. 2. With a perfectly conducting earth surface this mutual inductance can be readily computed by the method of images. Under actual conditions the mutual inductance is affected by the resistance of the earth, which increases the linkages between the two grounded circuits because it causes the return current from circuit No. 1 to distribute more or less widely throughout the

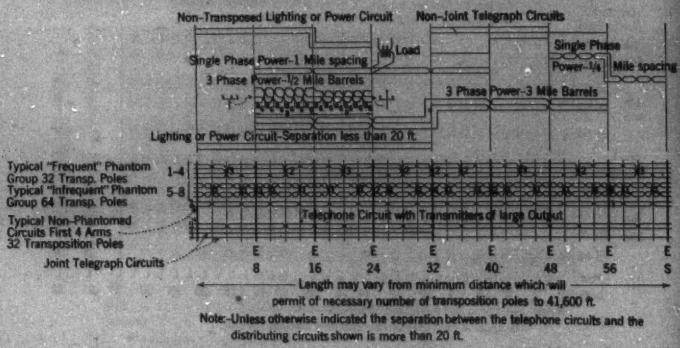


FIG. 16—CHARACTERISTICS OF EXPOSED LINE E SECTION —DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSMISSION POLES AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

earth rather than to flow in a current sheet on the earth surface. In many cases, also, the fact that the two circuits both use the earth return having a finite resistance introduces an appreciable resistance term into the mutual impedance. Sometimes this term is very important.

The voltage induced in a short section of wire 2 because of the voltage between wire 1 and ground is

$$e_2 = E\frac{C_M}{C_2 + C_M}, \qquad (2)$$

In this equation C_M and C_2 are the direct capacities from wire 2 to wire 1 and to the ground, respectively; that is, they are the

capacities measured by the charges on wire 1 and on the earth when wire 1 is connected to the earth and unit potential is applied between wire 2 and the combination of wire 1 and earth.

If the short section of wire 2 is connected to the earth, the charging current flowing to ground is

$$i_2 = j\,\omega\,l\,E\,C_M \tag{3}$$

That is the induced voltage acts like a generator having a voltage e_2 connected between wire 2 and ground through a condenser whose capacity is equal to the total grounded capacity of the exposed section of wire 2. This is indicated in Fig. 19.

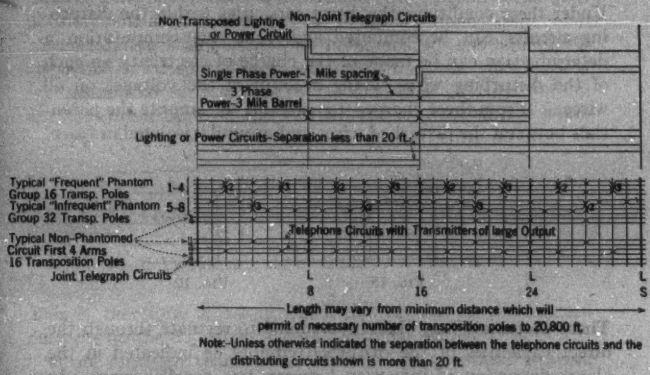


FIG. 17—CHARACTERISTICS OF EXPOSED LINE *L* SECTION—DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSPOSITION POLES, AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

2. *Small Induced Effects.* In cases where the induced currents and voltages are small compared with the inducing currents and voltages so that their reaction on the inducing circuit can be neglected, the magnitude of the induced effects can be obtained by solving for the values of current and voltage in the disturbing circuit, neglecting the presence of the other circuit, and then applying equations similar to those given above for each element of the parallel between the two circuits.

3. *A number of Parallel Wires.* The computation of induced currents is much complicated when there are several disturbing wires or several disturbed wires or both.

The computation of the electromagnetic induction may be complicated by the reaction between the induced currents in different disturbed circuits. In cases where the induced effect is small, however, the electromagnetic induction can be computed for this case as for a single circuit.

In determining electrostatic induction with a large number of parallel circuits, it is evident that it becomes exceedingly complicated to compute the direct capacities which are used. It is possible to greatly simplify this work in cases in which the disturbed and disturbing circuits are separated so that each group has little effect on the direct capacities of the other group. Under these conditions the direct capacities within the disturbing circuits can be computed and from this computation a determination can be made of the charge of electricity on each of the disturbing wires taking account of the voltages on all wires. From these charges it is possible to compute the potentials between the disturbed wires and ground if isolated in space.

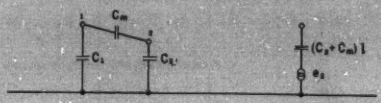


FIG. 18 FIG. 19

These potentials may then be assumed to regulate through the direct capacities of the disturbed circuits as indicated in the above discussion of induction between two parallel wires. In computing the flow of current with many disturbed wires the effect on one wire of the other wires must, of course, be considered.

This outlines briefly the method which is in use for computing the induction between three-phase power circuits and telephone circuits. The method sometimes described in the text books is incorrect in that it does not take account of the mutual reactions between the three conductors of the three-phase circuit which materially modify their resultant electrostatic field.

4. *Long Parallel Circuits.* In the case of two long circuits which closely parallel each other without means for neutralizing inductive effects, the reaction of the induced currents on the disturbing circuit cannot be neglected. If the circuits are very long there tends to be the same distribution of current and potential at the receiving ends of the disturbing and disturbed

circuits as there would be if equal voltages were applied to the two circuits at the transmitting end. The simplest case of this sort is when the two circuits are symmetrical. The equations for this case are given in Appendix B. It may be seen that even in this case the equations are so complicated that their use in practise would be burdensome. Where there are many long parallel wires the problem becomes one of utmost complexity.

5. *Parallel Telephone Circuits.* In the case of telephone circuits many conductors are run very closely parallel for long distances. It would seem, then, that the general solution of the inductive effect between telephone conductors would be so complicated as to be hopeless for practical use.

In attacking this problem, however, we fortunately are not interested in the general case. If telephone circuits are suitable for commercial service they must be so arranged that the inductive effects between any two circuits are exceedingly small.

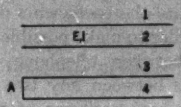


FIG. 20

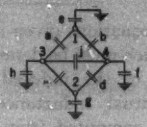


FIG. 21

This is, therefore, a case in which the reaction of the induced currents on the disturbing circuit can be neglected.

If one considers then a short section of close parallel between a disturbing circuit 1-2 carrying a voltage E and current I and a disturbed circuit 3-4 as indicated in Fig. 20 the voltage due to electromagnetic induction can be expressed as in equation (4)

$$e = j\,\omega\,I\,M\,l \tag{4}$$

In computing the current flowing at A due to electrostatic induction one may represent the direct capacities in the form of a Wheatstone bridge as in Fig. 21. In this figure the direct capacities a, b, c and d represent the Maxwell capacity coefficients. Capacities e, f, g and h represent the sum of the direct capacities between the wires, 1, 4, 2 and 3 respectively and all other wires and ground. This grouping is permissible because the induced effects all being small, the other circuits on the pole line are practically at ground potential, and have the same effect on the mutual capacity unbalance between the circuits 1, 2 and 3, 4 as they would if actually connected to earth.

The induced current is represented approximately by equation (5)

$$i = j\,\omega\,E\,\frac{a'\,d' - b'\,c'}{a' + b' + c' + d'} \tag{5}$$

$a' = a + \frac{h\,e}{s}$, $b' = b + \frac{e\,f}{s}$, etc., s being equal to $e + f + g + h$.

When the circuits are transposed so that the electrostatically induced current is small it results that in a balanced transposition section a', b', c' and d' are all very nearly equal. Under these conditions the magnitude of the electrostatically induced current in one small section of line is represented with a sufficient degree of accuracy by equation (6).

$$i = j\,\omega\,E\,\frac{a' - b' - c' + d'}{4} \tag{6}$$

The equations given above indicate the coefficients of induction which must be determined in order to make possible computations of the induced currents between telephone circuits. The mutual inductance unbalance between any two circuits can be computed from the geometrical positions of the conductors. The mutual capacity unbalance can be computed readily from the table of measured values of direct capacity for all wires on a 40-wire pole lead. (Table I.) The capacities of course vary with the number and to a small extent with the size of conductors on the lead. Forty wires is a typical condition and has been used as representative. When desirable, correction has been made for the use of wires of larger size.

The above equations therefore present a thoroughly practical method of computing the current induced between short sections of two telephone circuits which are well transposed against each other. The correctness of the results obtained in this way has been checked by experiment.

APPENDIX B—THE INDUCTION BETWEEN TWO LONG PARALLEL SYMMETRICAL CIRCUITS

Assume two long symmetrical wires, A and B, as shown in Fig. 22, operating with ground return. Assume the linear self impedances of the circuits to be z_1 and z_2 respectively, the linear mutual impedance to be m, and the linear direct admittances to

ground and linear mutual admittance to be y_1', y_2', and y_m respectively.

The differential equations for inductive effect between the two circuits are as follows:

$$\frac{-d v_1}{d x} = i_1 z_1 + i_2 m \qquad \frac{-d v_2}{d x} = i_2 z_2 + i_1 m$$

$$\frac{-d i_1}{d x} = v_1 y_1 - v_2 y_m \qquad \frac{-d i_2}{d x} = v_2 y_2 - v_1 y_m$$

In the above formulas, $y = y_1' + y_m$, $y_2 = y_2' + y_m$, and m and y_m are both written as positive quantities.

For the case of two symmetrical wires let

$$z_1 = z_2 = z$$
$$y_1 = y_2 = y$$

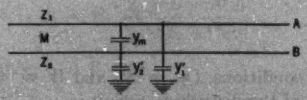


FIG. 22—INDUCTION BETWEEN TWO SYMMETRICAL GROUNDED CIRCUITS

Use is made of the following auxiliary equations:

$$A = 2\sqrt{\frac{z - m}{y + y_m}} \qquad B = \frac{1}{2}\sqrt{\frac{z + m}{y - y_m}}$$

$$\gamma_s = \sqrt{(z - m)(y + y_m)} \qquad \gamma_p = \sqrt{(z + m)(y - y_m)}$$

Equations are given below for a number of conditions which are of interest. In these equations, v_1 and i_1 represent the voltage and current on the disturbing circuit A at any distance x from the transmitting end, and v_2 and i_2 represent the voltage and current on the disturbed circuit B.

Case I. Voltage impressed on circuit A at $x = O$ is E_1. Circuit B connected to earth at $x = O$ through impedance W. Both circuits infinite in length.

$$v_1 = E_1 \frac{2 B (A + 2 W) \epsilon^{-\gamma_p x} + A (2 B + W) \epsilon^{-\gamma_s x}}{W (A + 4 B) + 4 A B} \tag{7}$$

$$i_1 = E_1 \frac{(A + 2 W) \epsilon^{-\gamma_p x} + (4 B + 2 W) \epsilon^{-\gamma_s x}}{W (A + 4 B) + 4 A B} \tag{8}$$

$$v_2 = E_1 \frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} - A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{9}$$

$$i_2 = E_1 \frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} - (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{10}$$

Case II. If the resistance, leakance, and internal reactance are negligible and the equivalent height of the wires above ground is the same for static and magnetic induction, $\gamma_s = \gamma_p$ and

$$\frac{A}{4\,B} = \frac{z - m}{z + m} = \frac{y - y_m}{y + y_m} \tag{11}$$

Then, if $W = \infty$, it follows that

$$v_2 = v_1 \frac{y_m}{y} \tag{12}$$

Under these conditions, ($\gamma_s = \gamma_p$ and $W = \infty$) the sending end impedance of the disturbing circuit is

$$z_0 = \sqrt{\frac{z\,y}{y^2 - y_m^2}} \tag{13}$$

This equation shows that under these conditions there is no reaction from circuit B at the terminal of circuit A.

Case III. If $z\,y_m + y\,m = 0$ (14)

Then $B = A/4$ and it follows that

$$v_2 = \frac{E_1}{2}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x})$$

$$i_2 = \frac{E_1}{A}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x}) \tag{16}$$

that is, at $x = 0$, the induced voltage and current are 0.

If y_m is negligible, equation (14) requires that $m = 0$. This may be approximated by the use of compensating transformers. The above relation suggests, however, that by properly adjusting the value of m an approximate neutralization of induced voltage might be obtained in some special cases in which the electrostatic induction is considerable.

Case IV. When $W = \infty$, then at $x = O$,

$$v_2 = E_1 \frac{y(D-1) + y_m(D+1)}{y(D+1) + y_m(D-1)} \tag{17}$$

where $\frac{\gamma_p}{\gamma_s} = D$.

The ratio of this voltage to that computed by equation (12) gives an idea of the error involved in the assumption which leads to equation (12).

Case V. If at $x = O$,

$$W = \frac{v_1}{i_1},$$

then

$$W = \sqrt{A\,B}$$

and

$$v_2 = E_1 \frac{2\sqrt{B} - \sqrt{A}}{2\sqrt{B} + \sqrt{A}} \tag{19}$$

This impedance, $\sqrt{A\,B}$, is

$$\sqrt{A\,B} = Z_0 = z_0 \sqrt[4]{\left[1 - \left(\frac{m}{z}\right)^2\right]\left[1 - \left(\frac{y_m}{y}\right)^2\right]} \tag{20}$$

Where $z_0 = \sqrt{\frac{z\,y}{y^2 - y_m^2}}$ = the infinite line impedance for a (13) single wire.

Case VI. At a long distance from the origin,

$$v_1 = v_2 = \frac{E_1\, e^{-\gamma_p x}}{2} \text{ if } W = O. \tag{21}$$

$$\text{Also } v_1 = v_2 = E_1 \frac{4\,B}{A + 4\,B}\, e^{-\gamma_p x} \text{ if } W = \infty. \tag{22}$$

It will be noted that the voltages on the disturbed and disturbing wires tend to become equal at a considerable distance from the origin, and that the two expressions for v_1 and v_2 with maximum and with minimum shielding are nearly the same.

APPENDIX C—TYPE UNBALANCE BETWEEN TRANSPOSED TELEPHONE CIRCUITS

The induction between long transposed telephone circuits on the same lead is the vector sum of the induction between all

short elements of the circuits, taking into account the attenuation and phase change of the inducing currents and voltages from the transmitting point, and the attenuation and phase change of the induced currents in their propagation to the end of the circuit from the point at which the inductive effect occurs.

A simple typical arrangement of transpositions between two telephone circuits in a short balanced section is indicated in Fig. 23 in which similar telephone circuits P and Q are divided into four sections of lengths A, B, C and D by the transpositions in the two circuits. The resulting induction between the two circuits in this section is the sum of two terms:

a. Unbalance Due to Irregularity in Length. As this is a short section of line, one may consider as a first approximation that the current and voltage in the inducing circuit P are the same throughout this section. Currents induced in sections B and D,

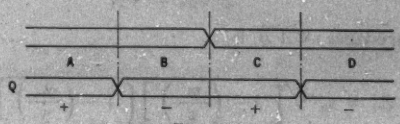


FIG. 23

therefore, tend to neutralize currents induced in sections A and C, provided the length $A + C$ is equal to the length $B + D$. When this is not the case there is a resultant induced current in circuit Q due to irregularities approximately proportional to the resultant unbalanced length $A - B + C - D$.

b. Type Unbalance. If lengths A, B, C and D are all exactly equal there will nevertheless be a resultant induced current in circuit Q because of the fact that the currents and voltages in either circuit P or Q change in magnitude and phase as they are propagated along the circuit.

In this appendix are given equations illustrating the method of computing type unbalance.

If P is a section of a long telephone circuit the current and voltage in it can be represented by equations (**23**) and (**24**).

$$I = I_0 e^{-\alpha l} \sin (\omega t - \beta l) \tag{23}$$

$$E = E_0 e^{-\alpha l} \sin (\omega t - \beta l) \tag{24}$$

In these equations α and β are the attenuation constant and the phase-change constant of the telephone circuit and of course vary with the frequency of the current.

The induced current in a short element of circuit a distance L from the end of the circuit, can be represented as equation (**25**).

$$d\,X = K\,d\,l\,e^{-\alpha l} \sin(\omega\,t - \beta\,l) \tag{25}$$

where $K\,d\,l$ is the induction between short elements at the end of the circuits and is computed with the help of equations (**4**) and (**6**) for the electromagnetic and electrostatic components. (Appendix A.) This induced current will be propagated on circuit Q back to the beginning of the circuit and under the condition for transmission without reflection, that is, terminal impedance equal to line impedance, will have when it reaches the beginning of the circuit, the value given in equation (**26**)

$$d\,X_0 = K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\,\beta\,l) \tag{26}$$

A similar equation can be written for each element of the circuit. The induction due to length A is obtained by integrating expression (**26**) for the length between zero and A as shown in equation (**27**)

$$X_A = \int_0^A K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\,\beta\,l) \tag{27}$$

The general expression for this integral is rather complicated. In practical telephone circuits, however, the effect of the attenuation in the short section of circuit between successive transpositions is very much less than the effect of the change in phase angle. Under these conditions the attenuation can be neglected for short lengths and the effective value of the expression (**27**) reduces to that given in equation (**28**)

$$X_A = K\,A\,\frac{\sin\beta\,A}{\beta\,A} \Big/ \sqrt{\beta\,A} \tag{28}$$

Similarly, the crosstalk at the end of the circuit due to section B of the exposure between the lines can if B is equal in length to A be represented as in equation (**29**).

$$X_B = -\,K\,A\,\frac{\sin\beta\,A}{\beta\,A}\,e^{-2\alpha A} \Big/ \sqrt{3\,\beta\,A} \tag{29}$$

In practical cases the ratio $\dfrac{\sin\beta\,A}{\beta\,A}$ is practically unity

and the exponential expression $e^{-2\alpha A}$ is also practically unity. Under these conditions the induced current at the end of the circuit caused by sections A and B of the parallel can be represented approximately as in equation (**30**).

$$X_{AB} = 2\,K\,A \sin \beta\,A \,\underline{/\pi/2-2\,\beta\,A} \tag{30}$$

Similarly, the resultant induction in circuit Q from sections A, B, C and D can be represented as in equation (**31**)

$$X_{AD} = 4\,K\,A \sin \beta\,A \cos 2\,\beta\,A \,\underline{/\pi/2-4\,\beta\,A} \tag{31}$$

This simple example corresponds to one of the typical arrangements of transpositions, namely, that denoted by M in Fig. 2. A similar equation can be worked out for each typical arrangement, and numerical values worked out for any given values of β and A.

For much work a sufficiently good approximation can be made in the formulas for type unbalance by assuming that

$$\sin n\,\beta\,A = n\,\beta\,A$$

$$\cos n\,\beta\,A = 1$$

The resulting simple formulas give an easy means of judging of the approximate relative effectiveness of different arrangements. They are given for different typical arrangements under the heading *Approximate Formulas* in Fig. 2. When X represents the phase change per mile these formulas represent numerically the type unbalance for a 12.8 km. (8 mile) section. The angles of the type unbalance for different types of the same length differ only by 90 deg. or multiples of 90 deg., and these differences only are indicated in the approximate formulas.

The type unbalance varies, of course, with the frequency. In the design of telephone transpositions type unbalances are computed for a frequency found to represent the average crosstalk effect.

It remains to point out the relation between resultant unbalance due to type for a long length of line and type unbalance for one transposition section. Considering current of a single frequency, the ratio between these two for N sections is

$$\frac{1 - e^{-2N\gamma L_0}}{1 - e^{-2\gamma L_0}} \tag{32}$$

where L_0 is the length of the section.

Equation (**32**) assumes that the wires have the same relative pin positions at the beginning of each section If the pin positions are the same in alternate sections and the opposite in successive sections formula (**32**) becomes

$$2 \sin \beta L_o \frac{1 - e^{2N\gamma L_o}}{1 - e^{-4\gamma L_o}} \tag{33}$$

For a large number of sections, and for the attenuations prevailing on good telephone toll lines equations (**32**) and (**33**) become approximately

$$\frac{1}{2 \gamma L_o} \tag{34}$$

and

$$\frac{\sin \beta L_o}{2 \gamma L_o} \tag{35}$$

Actually, results obtained by these simple formulas must be corrected for the fact that the telephone current is a compound of currents of different frequencies, for which the values of β and γ are different. In the case of loaded lines a correction can be made rather simply. The change in phase angle from one loading section to another is large and varies widely for different components of the telephone current. Good results are therefore obtained by assuming that the effects of successive sections add together by a root-sum-square law. The resulting ratio between long line unbalance and type unbalance for a single section is

$$\sqrt{\frac{1 - e^{-4N\alpha L_o}}{1 - e^{-4\alpha L_o}}} \tag{36}$$

Since N is large and αL_o small, this becomes approximately

$$\frac{1}{2 \sqrt{\alpha L_o}} \tag{37}$$

APPENDIX D, FORMULAS FOR TYPE UNBALANCE IN PHANTOMED CIRCUITS

Under the heading, *Phantom Circuits*, in the paper it is pointed out that the use of phantoms in telephone line construction complicates the computation of type unbalance because each transposition of a phantom circuit interchanges the pin positions

of its two side circuits and, therefore, changes the coefficients of induction between these and other circuits.

The equations for computing the resultant induction per section due to type unbalance are given below. These equations are given for the case in which a typical arrangement of transpositions is associated with each side circuit of the phantom independent of its interchange of pin positions with the other side circuit of the phantom. This is the arrangement which has been found most suitable for use in the exposed line transposition system. Transposition systems can be designed with a typical arrangement associated with the wires on a given pair of pin positions. With this arrangement, each side circuit of the phantom is transposed partly in accordance with one typical arrangement and partly in accordance with another as it changes from one pair of pin positions to the other with transpositions of the phantom circuit. Under some conditions, this arrangement has advantages in the economy of retransposing non-phantomed circuits to form phantoms. It is used in the transposition system which is standard with the Bell Telephone Companies for non-exposed lines.

The formulas for the resultant induction per section due to type unbalance are as follows:

Suppose two circuits, 1 and 2, to be combined in a phantom R and two circuits, 3 and 4, to be combined in a circuit Q.

Denote by T_1, T_2, T_R, etc., the type unbalances corresponding to the typical arrangements of transpositions on circuits 1, 2, R, etc. Further, let K_{1R}, K_{2R}, etc. represent the induction per unit untransposed length between circuits 1 and R, between circuits 2 and R, etc.

For Side Circuit to Phantom of Which it is a Part.

$$X_{1R} = S\,T_{1R} + D\,T_1 \tag{38}$$

$$X_{2R} = S\,T_{2R} - D\,T_2 \tag{39}$$

$$\text{Where } S = \frac{K_{1R} + K_{2R}}{2}$$

$$D = \frac{K_{1R} - K_{2R}}{2}$$

T_{1R} represents the type unbalance for the typical arrangement corresponding to the relative exposure between type 1 and type R.

T_{2R} is similarly defined.

For Side Circuit to a Non-phantomed Circuit or a Phantom Other Than its Own.

$$X_{1Q} = S\,T_{1Q} + D\,T_{1RQ} \tag{40}$$

$$X_{2Q} = S\,T_{2Q} - D\,T_{2RQ} \tag{41}$$

When
$$S = \frac{K_{1Q} + K_{2Q}}{2}$$

$$D = \frac{K_{1Q} - K_{2Q}}{2}$$

The nomenclature is the same as before.

T_{1RQ} represents the type unbalance corresponding to the relative exposure between the arrangement giving unbalance T_{1R} and typical arrangement Q.

For example, if circuit 1 is transposed to Type A
" R " " F
" Q " " L

$$T_{1RQ} = T_{(A\,F)\,L} = T_{KL} = T_O$$

That is, T_{1RQ} is in this case the type factor for arrangement O, which is obtained by combining successively types A, F, and L.

For the Two Side Circuits of a Phantom

$$X_{12} = K_{12}\,T_{12} \tag{42}$$

For Side Circuits in Two Different Phantoms

$$X_{13} = S\,T_{13} + D_a\,T_{13R} + D_b\,T_{13Q} + D_c\,T_{13RQ} \tag{43}$$

$$X_{14} = S\,T_{14} + D_a\,T_{14R} - D_b\,T_{14Q} - D_c\,T_{14RQ} \tag{44}$$

$$X_{24} = S\,T_{24} - D_a\,T_{24R} - D_b\,T_{24Q} + D_c\,T_{24RQ} \tag{45}$$

$$X_{23} = S\,T_{23} - D_a\,T_{23R} + D_b\,T_{23Q} - D_c\,T_{23RQ} \tag{46}$$

Where

$$S = \frac{K_{13} + K_{14} + K_{24} + K_{23}}{4}$$

$$D_a = \frac{K_{13} + K_{14} - K_{24} - K_{23}}{4}$$

$$D_b = \frac{K_{13} - K_{14} - K_{24} + K_{23}}{4}$$

$$D_c = \frac{K_{13} - K_{14} + K_{24} - K_{23}}{4}$$

In the above equations the nomenclature is as before.

T_{13RQ} denotes the type unbalance corresponding to the relative exposure between the types giving type unbalances of T_{13} and T_{RQ}.

The D terms in the above equations are generally not negligible compared with the S terms. This fact requires that in designing, for example, transpositions between side circuits in two different phantoms, it is necessary to take account not only of the type unbalance of the relative exposure between the two circuits, but also of type unbalances formed by building up combinations of the typical arrangements of transpositions on the two side circuits and on the phantoms of which they form a part.

DISCUSSION ON "DESIGN OF TRANSPOSITIONS FOR PARALLEL TELEPHONE AND POWER CIRCUITS" (OSBORNE), ATLANTIC CITY, N. J., JUNE 27, 1918.

V. Karapetoff: In the appendix, Mr. Osborne uses the expression "mutual impedance." This term is new to me, and I should like to have a definition of it. If this quantity is equal to $2\pi f$ times the mutual inductance, then it would seem proper to call it "mutual reactance," in the same way in which the usual reactance is equal to $2\pi f$ times the coefficient of self induction.

It is also pointed out that the ground may modify this coefficient of mutual impedance and this may be a justification for using the term "impedance" instead of "reactance," to indicate that it is composed in a quadratic relation of resistance and reactance. But then the difficulty arises that the mutual impedance seems to be a coefficient which characterizes the two circuits, and if the resistance is to be taken into account, it is not clear the resistance of which of the two circuits is to be used in the expression for mutual impedance. Our Standardization Rules define "mutual impedance" in application to telephone work but I am not sure that it is applicable here. I also notice that another author (Mr. Fortescue) in a paper presented at this convention uses the expression "mutual impedance." It seems to me that we ought to get a general definition of this quantity, and introduce it into the Standardization Rules so that there may be no doubt about its meaning and use.

Charles Fortescue: In the event of a power company having installed a transmission line without transpositions, what would be considered the ethical arrangement between the power company and the telephone company, in case of inductive interference? If transpositions of the power lines are determined on before the installation is made, I can easily see that the cost would be a very small item, but after the power line has been installed the cost of transpositions might be quite a large item, and I should think it would be a case for a compromise between the telephone company and the power company under such circumstances.

Another condition that one may run across is, quite frequently the power transmission line is not symmetrically designed. Of course, a perfectly symmetrical three-phase transmission line would have lines at the apexes of an equilateral triangle, but quite frequently the lines are equally spaced in the horizontal plane, and quite frequently they are unequally spaced, depending on conditions which may have existed before the line was installed. I am very much interested in this, as it happens to be part of my duty to look into these cases of trouble which arise, from such causes as this, and they are sometimes very difficult to adjust. There is always a great deal of controversy as to who is to blame, and who should be the one to correct the trouble.

I might refer to the question that Prof. Karapetoff brings up. I have myself used the expression "mutual impedance" in some cases. In alternating-current circuits where we have currents which have a common part in a part of the circuit, and are purely mutually inductive in other parts of the circuit, and which it is convenient to consider separately, the expression "mutual impedance" comes in very handily, and is a very useful quantity to use. As far as I know, it has never been defined, but I think it is a perfectly definite and well understood quantity, and it is commonly used among telephone engineers.

A. G. Chapman: Certain experimental data relating to mutual interference between telephone circuits may be of interest. Mr. Osborne shows in Fig. 2 of this paper typical arrangements of transpositions for telephone circuits and in Appendix C discusses the method of computing approximately the inductive effects between two telephone circuits transposed in accordance with any two of these arrangements. It is interesting to note that increasing the number of points at which one circuit and not the other is transposed may increase rather than decrease the inductive effects between the two circuits. For example, if the method of computation outlined in Appendix C is followed, the *N* exposure shown on Fig. 2 will be found to be much more effective against the mutual inductive effects commonly termed crosstalk than the *M* exposure, although the latter type of transposition arrangement has three transpositions while the former has but two. The reason for this is clear if it is noted that the second half of the *M* exposure starts out with the wires in the same position as in the first half and, therefore, the inductive effects into the two halves tend to add. With the *N* exposure the second half starts out with the wires reversed and, therefore, the inductive effects into the two halves tend to subtract. Extending this reasoning further, it may be shown that the *A* exposure having 15 transpositions is less effective than the *N* exposure since in the case of the former the small inductive effects in each successive eighth of the length, all tend to add.

In order to check the theory of the effect of different arrangements of transpositions roughly outlined above, tests were made of the relative effectiveness of the *M*, *N*, and *O* exposure types. Fig. 1 shows the method of making the measurements. A single-frequency source of alternating current was used in order to facilitate the comparison with computed values. In order to get the effect of a short exposure in a long telephone line, the distant ends of both circuits were terminated with 600-ohm non-inductive resistances, approximately the impedance of a long circuit of the construction used in the tests. The test end of the disturbed circuit was also terminated with 600 ohms resistance. A measured amount of single-frequency alternating current was sent out on the disturbing telephone circuit. A sensitive receiving circuit of high impedance was then alternately

bridged across the 600-ohm termination at the test end of the disturbed circuit and across a comparison circuit which impressed at the terminals of the detector a known voltage of the same frequency as that impressed on the disturbing circuit. By adjusting the voltage of the comparison circuit and listening to the sounds heard in the receiver in its two positions, it was possible to measure the voltage impressed by the disturbing circuit across the 600-ohm termination of the disturbed circuit. The current through this termination could then be readily computed.

The single-frequency tests were made on two adjacent specially transposed No. 12 N. B. S. gauge pairs in a 40-wire toll line. The inductive effects were measured in a transposition section about 7 miles (11.2 km.) long. The transpositions were carefully located so that the crosstalk would be due principally to type unbalance and not due to irregularities in wire and pole

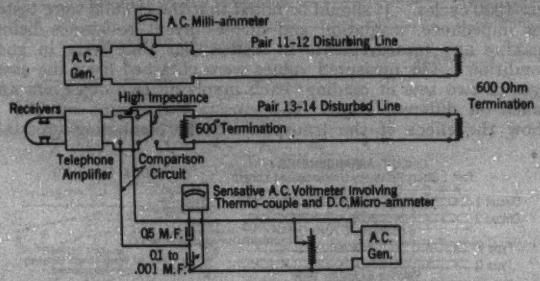


FIG. 1—METHOD OF MEASURING SINGLE-FREQUENCY CROSS-TALK

spacing except for combinations of circuits having very highly efficient transposition types.

The crosstalk in a short relatively untransposed length was first tested and then the crosstalk in a length of about 7 miles for the three exposure types was measured. By comparing the latter measurements with the former it was possible to express the exposure of any type in terms of an equivalent untransposed length. The following table shows a comparison between the equivalent untransposed lengths obtained in this manner and corresponding values computed theoretically from the equations of propagation of current on the circuits. From a comparison of the readings of different observers and of successive readings of the same observer, it was estimated that the precision of observations was ordinarily about 10 per cent. In the case of the *N* exposure, the precision was much less since the method was not suitable for accurately measuring the small amount of crosstalk involved. In addition the effect of the

irregularities would, of course, be much greater in connection with this exposure since the crosstalk due to the type unbalance is very small.

M, N AND O EXPOSURE TYPES

MEASURED VERSUS COMPUTED VALUES OF EQUIVALENT UNTRANSPOSED LENGTHS

Type exposure	Frequencies tested	No. of readings	Average ratio measured to computed	Computed length 1000 cycles
O	840, 1,000 1,100	12	0.88	0.75 mi.
M	840, 1,000 1,100	12	0.99	0.38 mi.
N*	1,000 2,000	6	0.42	0.044 mi. or 230 feet

*Measured values too small to be read accurately.

Fig. 2 shows the arrangements of transpositions tested and a comparison of computed and measured values of crosstalk for 1000 cycles. It should be noted that the circuits were tested for impedance, insulation and direct-current resistance before making any measurements to insure that they were in good condition. The measured values were obtained from the averages of two sets of readings each involving two observers and taken on different days. The crosstalk diagrams on Fig. 2 show the effect of the transpositions in combining crosstalk

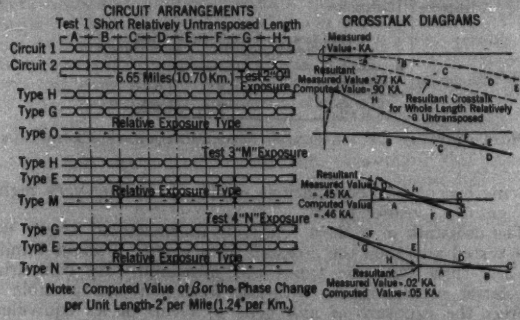


FIG. 2—SINGLE-FREQUENCY (100-CYCLE) "CROSSTALK" TESTS

at the terminal due to the various eighths of the transposition section. The average measured valve of the ratio of received to disturbing current for the test on the short untransposed length, that is K A of the diagram, was 725 millionths while the computed value was 770 millionths.

Certain talking tests made later may be of interest since they show absence of crosstalk between circuits having an N exposure which, as previously discussed, is a relatively efficient

transposition type compared with the other types here discussed. These tests were made in a manner similar to that shown in Fig. 1 for the single-frequency test, with the difference that the ratio between the received and disturbing currents was measured by shunting off a known fraction of the disturbing current and comparing it with the received current.

Fig. 3 gives data on the telephone lines employed in these talking tests. As indicated about 28 miles (45 km.) of a toll line was transposed by means of four *E* sections. The circuits tested had an *N* exposure in each transposition section and had comparatively large coefficients of induction between them, thus requiring an effective transposition arrangement. Tests made on two successive days indicated that the energy transferred from one circuit to the other was about one hundredth

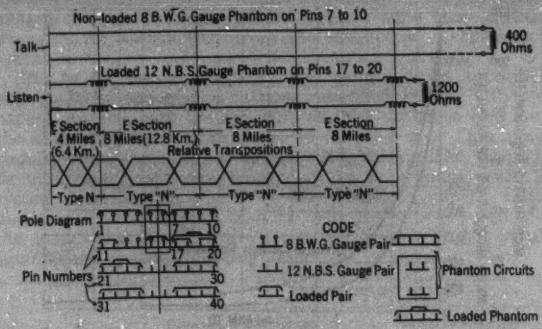


FIG. 3—"CROSSTALK" BETWEEN ADJACENT PHANTOM CIRCUITS HAVING TYPE "N" RELATIVE EXPOSURES

of a millionth of that transmitted into the disturbing circuit. About this amount of energy transfer was estimated due to the measured small irregularities in the locations of the transposition poles. The energy transfer estimated due to type unbalance was negligible in comparison.

In design of telephone transposition systems, a study must be made of the minimum separation which is permissible between circuits non-transposed to each other. It is, of course, desirable to design arrangements of transpositions for but a few crossarms and then repeat these arrangements on the lower arms. The minimum separation between similarly located relatively untransposed circuits on the first and lower arms is, therefore, of interest.

In Table I of his paper, Mr. Osborne gives a table of the capacities between various combinations of wires and between the wires and ground for a 40-wire pole lead, and in Appendix A

discusses the method of computing from these measured values, the coefficients of electric induction between the various circuit combinations. In order to study the question of the repetition of a transposition system on the lower crossarms, it was necessary to make similar capacity measurements on an 80-wire pole lead. The lower curve of Fig. 4 is obtained from the data of these tests and shows the variation with separation of the coefficients of electric induction between a phantom on the end of the first arm and similarly located phantom circuits on lower arms. The magnetic coefficients of induction were readily computed from the known conductor spacings.

H. Mouradian: It may possibly further the understanding of the practical significance of the paper if I should state, briefly,

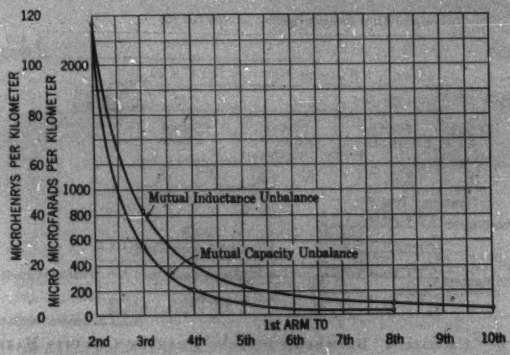


FIG. 4—ELECTRIC AND MAGNETIC COEFFICIENTS OF INDUCTION BETWEEN PHANTOM CIRCUITS ON PINS 1 TO 4 AND PHANTOMS ON CORRESPONDING PINS ON LOWER ARMS WITH STANDARD 10-PIN ARMS SPACED 2 FEET APART

some of my own experiences in the matter of the inductive interference on telephone lines in connection with two specific cases of exposure:

(1) In the first case of exposure, of approximately eight miles (12.8 km.) the power circuit was 33,000-volt, three-phase, star grounded neutral. The power line was built in 1905 or 1906. The power company installed on this eight-mile stretch two complete barrels,—a barrel every four miles. Special transpositions were also installed at the time, one special transposition on each telephone toll circuit midway between every two successive cross-talk transposition points on the telephone line throughout the length of the exposure. These two systems of transpositions, one on the power circuits and the other on the telephone circuits, were not, however, co-ordinated. The

result was that the telephone circuits, exposed as described above, were very "noisy", in fact, so noisy that the main industrial company in one of the towns served by the telephone lines actually made a complaint that, if the telephone company did not improve its service, it did not want the service. The next phase in the history of this case brought about a coordination of the telephone transpositions. The situation was further improved by the installation of two special telephone transpositions, instead of one, between every two successive cross-talk transposition points. The noise on the telephone circuits diminished and was found not to exceed 200 to 300 units of noise. Then, by further work on the special telephone transpositions, with a view to locating them as advantageously as possible with due regard to the irregularities in the exposure, the noise on the telephone circuits was finally reduced to approximately 60 to 70 units. At the present time this condition is considered satisfactory from the standpoint of commercial operation.

(2) In the second case I have in mind a direct application was made of one of the systems described in Mr. Osborne's paper. In this case the exposure was for six miles (9.6 km.), the separation fairly uniform at 50 ft. (15.2 m.). The power circuits were three-phase, 23,000-volts, delta. The telephone circuits have just been transposed in accordance with the system described on Fig. 9 of Mr. Osborne's paper. Determinations of the amount of noise were made before and after the installation of the special transpositions with the results summarized in the following table:

	Noise Readings			
	Distant end of telephone line open		Telephone station at distant end of telephone line	
	Before	After	Before	After
Side circuits.....	175	37	172	17
Phantom circuits.....	775	112	887	37

It will be noted from the above table that a material reduction has been obtained through the use of the system of transpositions above referred to. I think I might say, in closing, that the paper discloses several important practical solutions of the rather complex problem of inductive interference.

H. S. Osborne: Professor Karapetoff raises a question about the term "mutual impedance." This term is defined in Rule 916 in the Telephone Section of the Standardization Rules. The definition given there is entirely general, and as Mr. Fortescue points out, the term is used in other than telephone work. It seems that the term is a valuable one and that it should be changed in the Standardization Rules so as to be given there its more general application.

In response to Mr. Fortescue's question regarding the ethical arrangements between power and telephone companies in the case of inductive interference, I would say that my own belief is that, in general, the best practise requires that when either of the two or more schemes for reducing the interference is possible that one should be selected which gives the least total cost, counting up costs of changes in both the power circuits and the telephone circuits. Both systems are run for the benefit of the public and the interest of the public would seem to be best conserved by the application of this principle of least total cost. This principle can be applied independent of the question of the equitable distribution of expense between the two companies.

Mr. Fortescue speaks about the effect of the configuration of conductors of the power circuit. The triangular arrangement of wires is in almost every case materially better than either the horizontal or vertical arrangement in limiting the external field of the line. A great deal of information about this is given in the forthcoming book of Technical Reports of the Joint Committee on Inductive Interference in California.

Address presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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ENGINEERS AND THE WAR

BY MAJOR GENERAL WILLIAM M. BLACK
Chief of Engineers, U. S. Army

I FEEL some diffidence in addressing you this evening, partly because, although I might have qualified as an electrical engineer in 1880, I know I could not do it now, and then, I feel that a man who has the privilege of talking to a number of men such as are assembled here at this time has a tremendous responsibility, because the task set before us is one that has been unequalled in the history of the world. We represent a creed, not only in our living but in our Government, which we believe is necessary for the salvation of this world, for bringing to this earth the Kingdom of God.

We are fighting a nation that has a creed directly opposite to ours, a creed that leads them into the belief that Might makes Right, and that anything that is for the supposed good of the State is Right. This attitude has led the people of that nation to break every law that humanity has made for the amelioration of the horrors of war, laws that have gradually grown up through the ages. These laws are being ruthlessly sacrificed by these people in strict accordance with the creed which they hold. That creed and that people must simply be swept from off the face of the earth, and that is the task that we have undertaken.

It is an honor to be privileged to address you on a subject of such importance to our country, the duty of the Engineers in war. Although the part played by Engineers in this war is great and the responsibilities of the profession are correspondingly large, this war, like all wars in the past, is and must be a war carried on in accordance with the principles of the art of war, which are unchanging and which have been recognized and taught ever since organized armies were first created.

Do you realize that almost the only absolutely modern method of warfare now in use is the warfare of the air? The invention of submarines was made during the American Revolution, and

submarines were used successfully, though to a limited extent, in our own Civil War. Gas and flame fighting are of ancient origin. Trench fighting is hardly better known today than it was to the veterans of Grant and Lee, of Sherman and Johnson. The advances in human knowledge have caused corresponding improvements to be possible in the weapons of warfare. Increased knowledge of chemistry has produced more powerful explosives and improved methods in metallurgy have enabled these explosives to be utilized, by making possible heavier and more powerful guns. Improvements in the means of transportation have enabled larger bodies to be moved more quickly and more readily and to be subsisted and supplied with greater certainty. The telegraph, the telephone and the wireless have afforded a means of prompt communication and have enabled, larger bodies of men to be given co-ordinated action. With such changes, battles are fought on the same principles and won or lost from the same causes as in the time of Alexander the Great. This war has been called a people's war and so it is in the sense that due to modern facilities the entire resources of the people can be utilized to day as they could not have been utilized in the days of old. It has also been called an Engineer's war because in the quickness of movement and in the works necessitated by these modern inventions the services of Engineers become more conspicuous and perhaps more necessary than in the past. But engineering in warfare has always been essential and it is even doubtful whether the science of engineering does not owe its birth to the works of war. An Engineer myself, I would be the last to belittle the work of our profession. It is a matter of pride that the men of our profession, due to the nature of their employment in time of peace, are, of all the civil professions, most prepared to serve the country in war, but to serve the country adequately in war, the Engineer must add to his peace equipment for professional work. The profession of arms is a profession in itself and it is the profession which deals with the very greatest in magnitude of all the endeavors of men. The effective use of an army which is properly constituted exemplifies the best that men can do in organization, in discipline and in the devotion to duty which causes a man to regard his own life as a thing of small moment toward the attainment of the end sought.

There would be a great amount of effort saved if our people recognized more clearly the existence of the technicalities of

the profession of arms. The Government in Washington is simply deluged with suggestions and so-called inventions for the winning of the war. The records show that about 98 per cent of all of these are without military value and that time and labor have been thrown away by men eager to help, but entirely ignorant of the history and conditions of warfare.

An example with which some of you are familiar is the electrical gun. For years the possibility of such a weapon has been a fascinating line of study to electricians. The principle of the solenoid is the germ. If a series of solenoid coils were to be energized and de-energized in succession sufficiently and rapidly, such a series around a tube can be made to impart a movement of translation and rotation to a projectile. But practical results are today impossible. A six-inch service rifle having a length of 20 feet, fires a projectile weighing 110 lb. with a muzzle velocity of 2600 ft. per sec., or in other words, the projectile leaves the muzzle with a kinetic energy of translation of 115,500 ft-lb. This energy has been stored in the projectile during its travel through the bore of the rifle, or say in 1/65th of a second. The average power expended has therefore been at the rate of 7,607,500 ft-lb. per second or about 14,000 h. p. or 10,500 kilowatts. These figures are simply approximations and neglect entirely the power required for imparting velocity of rotation and for overcoming the friction in the bore. You can easily estimate the weight and dimension of the generating equipment which would be required for even a moderately powerful gun were all the mechanical and electrical problems of its manufacture solved, and making due allowance for the short-load periods. You can understand the impracticability of transporting the electrical plants required for any number of such guns, and the impossibility of distributing this power over shell-swept ground to guns whose position must be constantly shifted, and which must be put in action on a few seconds' notice. I think that you will agree that until new discoveries give a much improved method of storing or generating electricity, smokeless powder will continue to be the most compact and convenient form of stored energy for guns.

And yet there has been a good deal of time and money wasted in trying to perfect such a gun by men whose patriotism is undoubted, and whose ignorance, also, is undoubted. In other words, if a man has an invention or an idea of an invention, by all means let him work on it, but before he goes to Wash-

ington and takes up the time of men busy trying to devise means to beat the Boches, let him make sure that he knows the conditions of war and what he is trying to do to meet those conditions, and then if he is sure of the means, let him present his ideas and inventions to the people in Washington.

We want all we can get and want the best we can get. We want the inventive power of our country if it can be exercised to do good.

There was a proposition made seriously at Washington recently that the United States should provide a fund, of I do not know how many million dollars, and make a Home for Inventors, where any one who thought he had an invention would be able to go, and work it out at the public expense; and recently, although we had a committee of experts there to pass upon these inventions, the results were so utterly unsatisfactory to the inventors that they came in a perfect horde upon the Secretary of the Navy and the Secretary of War, so much so that they had to make a brand new committee of three men, who could be much better occupied, to go ahead and do this same thing; and I am only waiting for the next drive of inventors to show that this Committee will not suit them one particle better than the old one did.

In addition, without doubt, there are many men in our country of the highest patriotism who are sore-hearted because they are not given something to do directly toward the winning of the war. They do not understand, that some condition peculiar to themselves, possibly age, possibly physical condition, possibly mere ignorance of war and its conditions, compel it that the bit that they must do for their country at this time is to continue in their work in civil life and do their part in keeping up the normal life of the country—in itself a service of importance.

The part which Engineers are now playing in the war is a very great one. The records of the American Institute of Electrical Engineers show that out of a total membership of 9443, there are 973 in the service, or 10.3 per cent of its roster. The American Society of Civil Engineers with 8753 active members has 14⅓ per cent in the service. The American Institute of Mining Engineers 10.4 per cent, and the American Society of Mechanical Engineers 10.1 per cent. But these records are not complete. At the outbreak of the present war there were in the Engineers Corps of the Regular Army about 300 officers and approximately 3500 enlisted men. At the present time,

there are about 8000 commissioned officers and 200,000 enlisted men, made up of men formerly engaged in works of an engineering character. It is probable that this does not represent much more than one-half of the number of the profession now serving in the Army.

Let us consider the nature of the work of the Engineer, passing from front to rear of the Army.

First in importance is the work of the sappers. They go before and remove obstacles, clearing away obstructions, building bridges and roads, making the trench systems complete, mining, providing light, water, lines for supply (light railways or roads) and military mapping. In this category enter practically all of the branches of the profession. Further to the rear are found the construction and operation of railways; road and bridge construction; the construction of veritable towns for supply depots, with all their accessories, drainage, sewerage, lighting and water supply; construction of quarters and of hospitals; and furthest to the rear, the construction of the ports of debarkation with their wharves storehouses, railway lines, yards and shops, all with their sanitary systems. Separate from these activities, but necessary for their supply, are the Forestry troops who turn the growing timber into lumber of the dimensions required for the various services. Locomotive and car shop troops are performing essential services. Topographic Corps, Sound Ranging Corps and Camouflage Corps are also among the varied activities of the Engineers.

What preparation is required for the fulfillment of these varied duties? For the actual technical work of construction or installation the civil training of the Engineer should prove sufficient when the plans which embody the military features have been prepared, or when the military technique has been learned and assimilated. A fundamental of this military technique is that the time element is to be considered rather than money cost and that the work must be done with whatever materials are available. This requires clearness of conception of the results required, resourcefulness and organization—factors also required for civil work.

As stated earlier, due to the very small numbers of the personnel of the Corps of Engineers of the Regular Army, reliance had to be placed in the members of the profession in civil life. Confidence in their devotion to country and in their ability has not been misplaced. The results already accomplished prove

this fully. Could more have been done? Undoubtedly, had the profession been better prepared for the call.

Will you permit me to say a few words concerning the general training of our Engineers, based on a professional experience of more than forty years? The conviction has been forced upon me that in educational matters, as in many other affairs of life, we Americans are inclined to go too fast. The basis for any professional career where the highest is to be attained must be a sound general education. Does anyone of you regret the lessons gained in your own experience? Is not the experience of humanity as shown in properly written history of almost equal value? Would Russia now be in the sad condition existing had her people known that the experiments she is trying have always resulted disastrously? Yet is history thus considered in an ordinary technical course? Again, do you not find a knowledge of the general principles of law and of the special rules of the laws of contracts of value? Are these considered essentials? What is the handicap of an Engineer who is unable to express his ideas clearly in spoken and written English? Is this taught thoroughly in our technical courses?

It goes without saying, that the study of pure and applied mathematics is found in all technical courses. But, are these subjects well grasped before their application in special technical courses is studied? Is any faculty of an Engineer of greater value than the ability to form a mental picture of his problems and of its solution? Yet is that study which assists most in this faculty—descriptive geometry—properly apprehended? Is there any branch of the profession which in its application is not based on a knowledge of topographical work, on a knowledge of construction materials and of how these should be used? Is the study of these branches of civil engineering insisted upon sufficiently in the mechanical and electrical courses? In effect would not our professional men be better equipped for their civil work were they not in too great a hurry in their youth to enter directly into life's combat? Does not this war teach that without a long and elaborate preparation down to the last details, an attempted "drive" must fail?

These remarks apply to all Engineers, both military and civil. In the rush of war men cannot always be hand-picked for special jobs and frequently it becomes necessary for an available man to be used for the work immediately necessary, irrespective of his previous training. In this supreme test of

humanity the best man is he who is prepared to meet any emergency—perhaps not in the most finished way—but to meet it.

There are things that the engineers in this country can do. If they do know enough to give us some ideas for helping along in the killing of Boches, for God's sake let us have them. If they do not, what they can do is to help the supply of men for the winning of the war. We are now short of officers of Engineers, very short, and we are going to be very much shorter. We must have educated engineers for this work, and we must not only have the men for the line work of the army, but we must have mechanicians and artisans and laborers for the special work.

All of you men have spheres of influence—do your best in them, and if you can be used otherwise, and the problem comes up in which we need you, you may be sure you will be called upon. There is this problem now, the supply of men, in which you can help, either by your own personal sacrifice, in going out, or by influencing others.

Now as to soldier work. The movements of drill and the construction methods peculiarly military are easily learned. The knowledge of the art of war which will enable these to be applied promptly and properly is more difficult. But most difficult to acquire is the peculiar mental discipline which makes the soldier. The Army is a huge machine which must work co-ordinately in all of its parts. That competition, which in civil life causes one body to advance further and faster than another, is out of place in an Army. *All* must work together and for one common end. Each man must so subordinate his will and desire to the common good as to work willingly and earnestly in the sphere allotted to him. This does not mean that all initiative is to be suppressed. On the contrary each man must use his initiative to the utmost, but in his own allotted sphere of action. Each must learn to obey and obey from the heart. Through such obedience comes the knowledge of how to command when command becomes a duty. All of this is hard to learn. But each man who is called upon to help in this war must learn it, if he would help effectively.

By all means let us have military training in our schools, but let it be true military training and not tin soldier work.

There is another line of technical military knowledge which must also be studied hard. The machinery for the organization, training, supply and leadership of troops; the methods of ob-

taining, accounting for and issuing supplies; of keeping returns of the men; and the channels of command must be studied. To civilians in general this is wholly unknown, but if a man is to be of service in the Army, it must be learned until its use becomes automatic.

What have the Engineers done? War was declared April 6, 1917. By the middle of July, nine regiments of Railroad Engineers had been raised and organized and two had actually started for France. In each regiment were two officers of the Corps of Engineers of the Regular Army, the Colonel and the Regimental Adjutant. The remaining officers were all from the Engineer Reserve Corps, some receiving their commissions only when on the point of sailing. Of course, few of the officers had had any previous military training and the tasks of organization were most difficult. Since then, there have been organized:

Five Corps Regiments consisting of Sapper, Searchlight and Sound Ranging troops; 43 Sapper Regiments and trains; 2 Mounted Battalions and trains; 5 Ponton Trains; 4 Inland Waterway Companies; 40 Railway Regiments and Battalions including all classes of Standard Gage and Light Railway troops necessary for the construction, operation and maintenance of railways; 1 Railway Transportation Corps; 1 Highway Regiment; 1 Gas and Flame Regiment; 1 Gas Training Service; 5 Forestry and Auxiliary Forestry Regiments; 1 Surveying and Printing Battalion; 1 Military Mapping Service; 2 Supply and Shop Regiments; 1 Water Supply Regiment; 1 Quarry Regiment; 1 Mining Regiment; 1 Electrical and Mechanical Regiment; 2 Crane operating Companies; 1 Camouflage Battalion; 18 Truck and Auto Companies and 44 Depot Detachments.

The greater part of these organizations is now overseas. Some are serving with the British Army, some with the French, but the majority is with our own troops, in service both at the front and in the rear.

I wish I could go into greater detail as to the work of these Sound Ranging Corps, because it comprises some new electrical work of the highest character, and the apparatus for it has been perfected in this country. We took the best devised at the beginning of the war, and our physicists went to work and have made marked improvements. Perhaps you do not know what sound ranging is. The artillery is stationed in the

rear of the line. There is almost no direct artillery fire any longer—that is, as a rule, the gun is fired from a point where the target cannot be seen at all. The first thing to be destroyed, invariably, is the enemy's artillery, then the trenches are attacked. The obstructions of wire are torn to pieces, the trenches themselves are practically leveled, and after that is done, in the assault, there is what is termed the barrage fire. That, I suppose you know means a fixed or slowly moving curtain of shells dropped on a certain given line and through which passage is almost impracticable.

On both sides the artillery is carefully camouflaged so it cannot be seen from aeroplanes. To show what care is taken, even the tracks that are made in taking the guns to the front are wiped out, the guns themselves are covered, so that neither from an observation balloon nor an airplane from the enemy's line can the position of the gun be seen, and in order that the flashes of the gun cannot be located, there are dummy guns placed at intervals, and flashes from these guns made by electricity, so that the position of the real guns cannot be known.

In order to determine the position of the real guns, there are delicate instruments which have been devised, which are placed at intervals along the line. These instruments are for the purpose of registering the sound of the gun. There is, first of all the sound of the gun in firing. That is preceded frequently, if the range be great, by the sound of the shell passing through the air, and sometimes by the bursting of the shell itself, before the sound of the gun comes. These are all recorded, and the velocity and the direction of the sound is known. By having these instruments at different points on the line, the position of any one gun can be "spotted," and "spotted" so closely that our own artillery fire can be directed and the gun blotted out. That is one of the improvements of modern warfare rendered possible by the advance in general human knowledge, particularly in electrical knowledge, and these instruments are very exact.

This service of the rear is of great importance and magnitude. Picture to yourselves what is required to transport, house, supply and maintain a million men three thousand miles from home, producing nothing and in their work expending enormous amounts of materials.

Taking the question of storage alone, the provision of space required for an army of 1,000,000 for ninety days aggregates 20,000,000 square feet of floor space of covered storage and

double that amount of uncovered storage space, with the necessary railway tracks for receipt and shipment and for classification yards, aggregating about 650 miles. Add to this an equal mileage of highways, adequate provision for water supply, sewerage and electric lighting and power and you can realize the work involved in this one item. Add to this the constructions which have been built at the Ports of debarkation (at one of which 375,000 square feet of wharf space had to be provided), the hospitals, barracks, shops, and the lighting, water and sewerage systems required, and some conception of the actual new construction work done, can be formed.

It is estimated that the supply of the army requires the transportation to the front of 25 lb. per man per day. This makes heavy demands on the French railway systems, good as they are. These have had to be supplemented in all but the main line trackage, and a large amount of motive power and of rolling stock has had to be supplied and operated.

Among the special services, the work of the Geologists must be mentioned, and in the line of improved apparatus, it may be stated that new instruments and methods for airplane photography have been devised and introduced. Other new auxiliary aids for fighting have been worked out, some of which have already proved their value on the battle field.

Yes, the Engineers are doing their work well. Be it in constructions in the rear, or under fire, be it in the transportation of ammunition to the firing line, the construction of strong points and obstacles, the construction and destruction of bridges in the face of an enemy, or as in recent instances, under the feet of the enemy, or be it with their rifles in beating back an attack, they are doing and *dying*. All glory to our Comrades in arms in France! There is not a red-blooded American who does not envy them.

But is there not a war duty for us also, for us who are held on this side of the ocean? Yes, undoubtedly. To some is allotted a task in supply, to some a task in manufacture, to some a task in organization. But that is not all. The life of the nation must go on. Her civil machinery must function undisturbed. With so many called away from these civil duties, the onus of the work will be the heavier for those who are left. Let us each then do his bit as and where it presents itself knowing that if each does his best, with love of country and forgetfulness of self as guides, the results are sure.

Gentlemen, I have been in the front line trenches, and a more abominable place for a man to be in, apart from any enemy, you can hardly imagine. It is raining there much of the time. The mud is up to your ankles, even though the trench be drained and the best attempt made to keep the drainage in good condition. The men at the front are distant from their supports, and they are scattered, maybe a dozen at a place, directly in the face of the enemy, and they have to stay there, and have to be prepared to resist any attack that comes, with a certain knowledge that should the attack come in force, or should a real earnest effort be made to take the line, they in the front line are almost sure to perish, and they are not minding it. They are staying there in the face, not of possible death, but in the face sometimes of almost certain death. They are doing their work, and our engineers have the most dangerous part of that work to do.

You remember in that first British drive to Cambrai, the Boches got in the rear of the firing line. They struck a part of one of our Engineer Railway Regiments, building light railways to the front to bring up ammunition. These men had not their arms with them. All they could do was to scatter and get in shell holes, and as soon as the Boches went by to rally and go back, and take the arms thrown down by the wounded or dead, and then pass over, and they formed their part of the line and did their share of the fighting.

The evening of the great drive, I had the honor of dining with Sir Charles Douglas Haig. He then told me of one of the battalions of the Sixth Engineers, which had been with the British about six weeks, and in that time they had built twenty bridges with spans, ranging from 16 to 60 feet, and he spoke of the wonderful work they had been doing. Two days afterward these men took their place and held about a mile of the British line, and held it so that the Boches could not get through.

The same happened with one of the railway regiments, again caught in the same way, and they formed a part of that miscellaneous army that Gen. Carey got together of the supposedly non-combatant troops, and the line which they formed prevented the capture of Amiens. They held the Germans. There, again, the engineers played their part valiantly.

Just two days ago a cablegram came across from the other side, about one of the companies, Company C, I think, of the First Regiment, which had been ordered to do particularly

dangerous work in the neighborhood of the Marne. As they started out, two of the officers were killed at once, but the rest went on and did what they were ordered to do.

Then again, you have noted that when the Germans got down to the Marne and were crossing a bridge, some of the engineers waited until the head of the Germans had gotten completely across the bridge, which was filled with the Germans, and they blew up the bridge, and those who got there wished they had not.

Our men are doing their work on the other side wonderfully, and those are our brothers in the profession who have gone over and now wear the uniform.

(General Black then exhibited some moving pictures showing the organization of the Engineers and how they do some of their work, and finally some of the work that they have already accomplished in France, after which he added):

In landing in France, you are struck at once by the number of maimed men you see on the street and by the women in black, and then, as you get to know them, you will see that these people are from their suffering simply the more determined to carry this war through to a successful finish. The same thing is true in England. You are struck by the grim determination to win success at whatever cost. There, due to a difference of temperament, things are taken somewhat differently. I was at a tea in England, and was presented to an English woman of title. She was gowned just as any woman would be for an afternoon reception. In speaking to her I mentioned I had two boys in France. She said, "I did have four, I have only two now," and then seeing me look a little startled, she said, "You know we feel our private griefs must not be allowed to show, that we must not wear mourning, that we must go ahead with our ordinary duties and try to keep up the social life."

We have only just begun to fight. We have not begun to suffer. If you were there, and could see what these people have done, and what they are prepared to do, and then feel as you would feel, how completely they trust us to bring this war to a successful finish, you would feel, too, that it is up to us to equal them in endurance, equal them in sacrifice, and to see that freedom, individual freedom, is preserved for all time in this world. I thank you very much.

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ELECTRIC POWER FOR NITROGEN FIXATION

BY E. KILBURN SCOTT

ABSTRACT OF PAPER

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

In the direct method electric energy is the only factor, whereas by the indirect much plant of a very diverse and complicated character is required. Also there are difficulties in connection with the platinum catalyst necessary to convert ammonia into nitric acid. It is claimed that the direct method is better because of the simplicity of plant and of operation, and the possibility of working with off peak power. The suggestion is made that a number of plants for making nitrates by the direct arc process should be erected at existing power houses. Keeping the generating plant more fully employed would improve the load factor and reduce costs.

By making nitrate in a number of centers the transportation of same to the explosive factories would be reduced and the risk of interruption of supplies in case of accident or sabotage would be less than in having a few very large factories.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate from air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

ONE of the most powerful combinations in the world is that connected with the exploitation of Chile Nitrates, and to extend the uses of that material and regulate prices, etc., there is a Chile Nitrate Committee supported by the various interests concerned.

It was created for propaganda work amongst farmers and others, to facilitate the use of nitrate as a fertilizer but since the advent of air nitrates some attention has been given to discrediting the methods of fixing nitrogen from air. This has been done partly by paragraphs in the press throwing doubt on the financial and technical success of such methods, etc. German

influence, working through political clubs and the press, also assisted the Chile nitrate propaganda while at the same time German scientists were being assisted in every possible way to develop air nitrate processes in their own country.

Years before the war, some of us saw that the question of supplies of Chile nitrate for the manufacture of explosives would be an important factor, and in 1911, at the Portsmouth meeting of the British Association, and later at a meeting of the Society of Arts in London, I sounded a note of warning.

Immediately after the war started, the German government appointed an electrical engineer, head of the Allgemeine Elektricitäts Gesellschaft, to expedite the manufacture of explosives. Air nitrate plants already in operation were greatly extended, and new nitrate plants and the power houses necessary to supply them with electricity, were put in hand. One such power house built early in the war at Bitterfeld, develops 185,000 kv-a. from lignite coal.

Even after three and a half years of war, the Allies still remain practically dependent for explosives on supplies which have to be brought thousands of miles across the sea from a foreign country.

So far as Great Britain is concerned this policy of dependance on supplies from oversea, appears to have been dictated by the Ministers of munitions, two of whom have been lawyer politicians, and one a doctor of medicine. They were probably acting partly on the advice of the head of the department of explosives supplies, also a member of the legal profession.

Considering how much this is an engineer's war, and how expert are the men in charge of the departments in Germany which handle such matters, does it not seem absurd that any of the technical affairs on the Allied side should depend on persons whose particular ability in life is to make speeches that catch the popular vote.

The transportation of Chili nitrate requires much shipping that might be used for other purposes and also occupies attention on the part of the Navy, in order to keep open the sea routes. In 1917 the nitrate imported to United States from Chili amounted to 1,742,540 tons (see figures of Imported Supplies given toward the end of paper) and presumably at least as much more would go direct to Europe from Chili. The average export taken over every day in the year is at least 10,000 tons, and as a round trip is about three months, an easy calculation

shows that about 1,000,000 tons of shipping valued at $150,000,000. is tied up in this Chili nitrate carrying trade.

Huge sums have also to be paid for the nitrate, and to shippers and brokers for transportation, insurance, etc. On the other hand by being independent of Chili, the Germans have kept their money at consequent financial and economic advantage to the country.

With the right men at the head of affairs, possessing expert first hand knowledge of the engineering and chemical problems involved, the Allies could have become equally independent of Chili and much money and man power which have been absorbed in building obsolete chemical and other plants would have been saved.

To their credit, certain scientists and engineers of this country not only saw the danger, but insisted on the authorities taking action by providing money to establish plants for the manufacture of nitrates.

At the same time, in this country as well as in England, there has been time lost, owing to certain parties maneuvering to obtain the adoption of their own process to the exclusion of others. In so large a field as nitrogen fixation there must necessarily arise numerous improvements in the various processes so that it is not possible today for anyone to gage or forecast their future relative economic values.

This is particularly the case with processes in which electrical energy plays a leading part for it is a sort of ingrained habit of the electrical engineer to simplify and revolutionize existing methods that they eventually become essentially electrical. The whole history of electrical progress, and especially of electrochemistry and metallurgy establishes that fact.

Investigation boards or committees dealing with nitrate problems should be largely made up of engineers who have expert first hand knowledge of electric power conditions and of apparatus, etc. It is not right that chemists should have practically sole power to pass upon processes in a field which electrical engineering is capturing so completely as the production of nitrates.

I feel that the merits of the arc flame process for making nitric acid have not been adequately and sympathetically considered, and this paper is written with the special object of stating them. I wish also to remove the misconception that the arc flame process is dependent on water power and that it can only be in-

stalled economically on a very large scale. The matter is one of special interest to electrical engineers especially those concerned with the production and supply of electrical energy.

Methods of Making Nitric Acid

Atmospheric nitrogen can be fixed either as nitric acid or as ammonia. The first named forms a basis of all explosives, whilst ammonia is used for one explosive viz. ammonium nitrate and for that purpose it must be combined with nitric acid.

The only way to make nitric acid direct from the air is to combine the nitrogen and oxygen of the air in the electric arc, and then combine the nitrous gases so produced with water to form acid. Owing to its simplicity this is called the *direct* method.

To make ammonia from atmosphere nitrogen the most straightforward method is the synthetic which has been adopted at Nitrate Plant I at Sheffield, Ala. By this method pure nitrogen and pure hydrogen are raised to a high pressure and temperature and in the presence of a catalyst they combine to form ammonia.

Another method called the *indirect* is used at Nitrate Plant II at Sheffield and briefly this is to make carbide of calcium and treat it with nitrogen to form calcium cyanamid, the cyanamid being then acted on by steam to produce ammonia.

Having obtained the ammonia it has now to be made into nitric acid, and this is done by the catalytic process of Prof. Ostwald using platinum as the catalyst. The method adopted at Sheffield, Ala. is to take about half the ammonia produced, and convert it into nitric acid and then combine this acid with the remainder of the ammonia to form ammonium nitrate. Clearly this makes just half the quantity of ammonium nitrate that would be possible, if all the ammonia was to be combined with nitric acid made in some other way, as for example by the direct arc process.

For some considerable time there has been great shortage of ammonia, and therefore, to use any of it as raw material for another nitrogen compound—nitric acid—seems short-sighted policy.

The unit value of nitrogen in ammonia, is just as high as the unit value of nitrogen in nitric acid, so it is economically and financially unsound to at any time make nitric acid from ammonia as a raw material. For this reason it is impossible for the manufacture of nitric acid via cyanamid and ammonia to compete under normal conditions with other methods.

Those interested in the indirect method have drawn comparisons between it and the direct electric arc process, with the object of showing that the indirect is the better. A tabular statement of all the operations involved in the two processes as given below is the best way to make the comparison.

	Indirect Method Employing Calcium Cyanamid to Make Ammonia and Oxidising the Ammonia to Acid by a Catalyst	Direct Method Employing the Arc Flame Furnace Only
Factories	1. To make calcium carbide. 2. To make cyanamid. 3. To make nitric acid.	1. To make nitric acid.
Operations	1. Burning limestone.	1. Blowing air through electric arc flame to produce nitrous gases.
	2. Grinding lime.	2. Absorption of gases in towers to produce acid.
	3. Grinding coke or anthracite.	
	4. Mixing lime and carbon in correct proportions.	
	5. Making calcium carbide in electric furnaces.	
	6. Grinding carbide to fine powder in neutral atmospheres.	
	7. Making liquid air to produce pure nitrogen.	
	8. Packing calcium carbide into retorts.	
	9. Making calcium cyanamid by adding nitrogen and by heat of electric resistors.	
	10. Emptying cyanamid from retorts.	
	11. Grinding cyanamid to a fine powder.	
	12. Hydrating cyanamid to rid it of unchanged carbide.	
	13. Superheated steam.	
	14. Treatment of cyanamid with steam in autoclaves to produce ammonia.	
	15. Cooling the ammonia and elimination of water.	

	16. Oxidation of ammonia to produce weak nitrous gases by means of platinum catalyst.	
	17. Absorption of gases in towers to produce acid.	
	INDIRECT METHOD	DIRECT METHOD
Raw Materials and Renewals	1. Lime.	1. Air.
	2. Coke.	2. Water.
	3. Carbon electrodes in carbide furnaces.	3. Metal electrodes.
	4. Carbon resistors in cyanamid retorts.	
	5. Pure nitrogen.	
	6. Superheated steam.	
	7. Air.	
	8. Water.	
Electric Energy for	1. Carbide furnaces.	1. Arc flame furnaces.
	2. Grinding carbide.	
	3. Cyanamid retorts.	
	4. Grinding cyanamid.	
	5. Heating catalyst.	
	6. Motors for power, etc., including several cranes.	
Skilled Labor for	1. Carbide furnaces	1. Arc flame furnaces.
	2. Cyanamid retorts.	2. Absorption plant.
	3. Packing cyanamid.	
	4. Grinding machinery.	
	5. Making pure nitrogen.	
	6. Making ammonia.	
	7. Catalytic process.	
	8. Absorption plant.	

It is frequently stated that the amount of electric energy required for a given quantity of nitric acid produced by the indirect process, is less than that required by the direct, and this is put forward as a strong argument in favor of the indirect method. Clearly, however, the only way to compare two methods is to take into account *all* the factors which go to make up the total cost, and appraise them all at their proper values.

If two processes are to be compared as regards one factor only, then it may with equal justice be claimed that the electric energy represented by a few motors and lights required for a plant making acid from sodium nitrate, is less than the electric energy required by all other processes for making acid. Such a statement does not prove anything, and yet it is similar to the one put forward by the advocates of the *indirect* process.

Even when comparisons are made on an energy basis, they are useless, unless *all* forms of energy are included. For example, the steam for the autoclaves of the *indirect* process has to be at 150 lb. per sq. inch superheated to 350 deg. fahr. and this requires

considerable heat energy in the form of burning coal. Steam is also required for evaporation of water from the products of the end process, and for the distilled water for the acid absorption towers. The heat energy required for these various purposes should therefore be added to the electric energy taken by carbide furnaces, the cyanamid retorts, and for heating the catalysts.

On the other hand, the direct arc process raises its own steam, from heat of the hot gases coming from the furnace. This steam is sufficient for the end process and for distilled water and also under certain circumstances, may be used for generating electric power, so that the process can work regeneratively. At the Norwegian plant at Rjukan II, for example, there are three 4000 kw. steam turbo-generators, generating electricity with steam raised by the furnace gases.

The energy which can be regenerated in this way may be over 15 per cent, and this should of course, be deducted from the total electric energy measured into the furnace, when estimating the energy used for a given yield.

Obviously the cost of plant using the indirect method, will be very much greater than that in the case of the direct, for if we assume that the cost of a carbide furnace and its accessories is about the same as that of an air nitrate furnace with its accessories, then, the indirect process embraces in addition:

1. A complete plant for making cyanamid.
2. A liquid air plant for making pure nitrogen.
3. Powerful machinery for grinding the carbide and the cyanamid.
4. Steam boilers and autoclaves for making ammonia.
5. A complete catalytic plant for oxidizing the ammonia to nitric acid.

In the indirect method it is essential to have all the materials, gases, etc., absolutely pure, for example at the cyanamid works at Odda in Norway it was necessary to carry a pipe up the mountain side so as to ensure a supply of pure air to the liquid air plant.

When carbide is converted into cyanamid some of the former remains unchanged and in order to obviate danger of explosion a special treatment of the mixture is necessary to ensure a total decomposition of the remaining carbide.

To convert ammonia into nitric acid a catalyst is employed and this usually takes the form of a net of platinum wire, the surface of the wire being covered with finely divided platinum

black. The catalyst is readily poisoned by impurities in the gases etc., when the yield then falls considerably. The process is therefore a very delicate one and requires much skill to operate successfully. To "reactivate" the metal, it is necessary to subject it to an acid treatment and eventually to remelt, in which process it is impossible to avoid loss of this expensive metal.

Russia is almost the sole source of platinum, and whilst our Ally, could be depended upon. Today with Germany practically controlling that country, the position is serious. The various allied Governments have had to commandeer platinum as it is essential for several war purposes. With utmost deliberation and foresight the Germans are working to control the worlds storehouse of platinum in the Ural Mountains and any processes which depend upon this rare metal are going to be very seriously handicapped. I consider that those who have had a hand in starting new processes dependent on platinum are very blameworthy. Non-technical politicians could not be expected to know these things but those who did know should have informed them.

By the direct method the cost of air is nil, and the cost of water is practically that of pumping. On the other hand, the materials required in the indirect method are very expensive and especially difficult to obtain at the present time. Over three fourths of the cost of working the indirect process is represented in materials liable to price fluctuation. These are now much higher than before the war, and will remain at the higher level after the war.

In the direct method less than one-fifth of the total cost is represented in materials dependent on market rates, and the principal item of cost, namely electric power, will, if anything, tend to come down in price.

The direct method is very simple to operate, whilst the indirect requires much skilled and unskilled labor, and some of the operations are dangerous to health. Therefore, the more labor demands increase, the more will the indirect method be handicapped in this respect. There are many separate links involving exact operating, to make the whole run smoothly and the slightest hitch in connection with any one link necessarily holds up the whole system.

The manufacture of cyanamid has to be carried out in retorts of relatively small size involving much labor to set up, etc. This is to enable the nitrogen gas to penetrate to all parts of the con-

tained carbide. The times of the reaction and the cooling down, etc., are definitely fixed and it is quite impossible to work the process with off-peak power; also should there be an accident or failure of current for a time, the cyanamid retorts and the carbide furnaces may be injured.

All nitrate processes have a military bearing as regards preparedness, in which is involved the question of transportation. The heavy and bulky raw materials necessary for the indirect process places it at a serious disadvantage from this point of view, especially at the present time when the railways are so congested. With the direct process there is no carriage of raw materials.

The indirect process has been strongly advocated in that after the war, cyanamid will be much used as a fertilizer. On the other hand a large number of objections have been voiced against such use, as witness the following extract from a book by Dr. Brion: "Cyanamid cannot be used with a large number of soils such as very sandy or moor soils, or with such soils as tend to become acid. Further it cannot be used for growing tobacco nor for some kinds of fodder. It is useless as a top dressing and can be applied only in dry weather when it must be plowed in at once. Cyanamid attacks the eyes of men handling it."

Whilst some of these objections may have been overcome by making the cyanamid granular and probably also some of them are over-emphasized, it still remains true that cyanamid is by no means as good a fertilizer as nitrate.

The effect of the calcium in calcium cyanamid in the presence of moisture is to cause the reversion of phosphoric acid and therefore it can only be used in limited quantities in a combined fertilizer.

In a legal action in the State of Maine between the Armour Fertilizer works and Ellis Logan, in April, 1916, there was expert testimony that calcium cyanamid destroyed a crop of potatoes and that not more than 60 to 70 lb. of it should be used per ton of fertilizer.

Electric Power

As a basis load for a power house the direct arc process presents the advantage that it can be established anywhere, because the raw materials being only air and water, considerations of transportation do not enter into the situation.

It is particularly suitable for off-peak or off-season loads, for there is no fused material to solidify, and little to deteriorate in case of stoppage. Some of the furnaces can be switched on and off like an arc lamp, without detriment to brickwork or structural details, or to the process of manufacture.

As there seems to be some doubt as to the possibility of running arc furnaces intermittently on a commercial scale, I would mention that about seven years ago a nitric acid factory was built at Legnano, Italy to utilize 10,000 horse power, especially during the night (see Utilization of Atmospheric Nitrogen by T. H. Norton p. 68). Of course this plant has been considerably extended, especially since the war. I am also credibly informed that in Germany there is a very large arc process plant working with off-peak power. At any rate there is no difficulty in doing it, whereas it is impossible to work intermittently with any other method of fixing atmospheric introgen.

In some ways, it is an advantage to run a plant for 8000 or less hours per year, instead of the full number, because the spare time can be conveniently used for renewals and repairs, Less spare plant is thus required and the plant can be operated by two shifts of men.

Because the plants in Norway are very large and only use hydroelectric power, a mythology has grown up, that the arc flame process can only be worked commercially on a very large scale, and with water power. It is, however, worth while to build plants in units to utilize 10,000 kw.

As a matter of fact hydroelectric power may be a disadvantage because of its distance from industrial centers, for either the factory has to be placed in an out of the way position, or else the power has to be transmitted over a long transmission line. I am of the opinion that electrochemical factories should be placed near the power supply, and the ideal position is alongside the power house especially if off-peak power is used.

In a national emergency it is surely better to bring into immediate use all the surplus equipment that already exists, than to start building new power houses, whether hydraulic or steam, and seeing that the direct-arc flame process is suitable for working with off-peak power, I suggest that a number of nitrate plants be forthwith erected at existing power houses.

By erecting say, ten or more nitrate plants of say 10,000 kw. each at power houses near places where nitrates are required, there would be great saving in transportation and early de-

liveries of nitrate could be made. Further there would be less risk of temporary interruption of supplies in case of accident or sabotage.

As a matter of fact there are power houses which could easily spare more than 10,000 kw. for over 20 hours a day and through the week end. Also there are power houses fully equipped with steam plant now standing idle which in the present crisis they might just as well be brought into use even if the cost of generation is high.

To give an idea of plant available, the following particulars of power houses will be of interest: From the existing plants in New York City, Mr. F. Hedley has recently estimated that about 115,000 kw. of off peak power might be supplied. In Chicago the existing power houses have a capacity of 470,000 kw. with additional 42,000 kw. now being installed. The maximum peak load of December 26, 1917 was 290,000 kw. for only half an hour, and by shutting off heaters on the surface and elevated cars during this short period, the engineers estimated that at least 90,000 kw. would be available.

There is no doubt that by restricting what may be called the luxurious uses of electricity, a considerable amount of generating plant in various parts of the country can be released for urgent war purposes, and this can be done without much increasing the coal consumption. There are also, the water power plants.

I was in Birmingham, England when for some months the street car service was stopped every day for between meal times, in order to save coal. The people readily adapted themselves to the conditions and the extra walking did the business people good whilst lack of street cars in the afternoon kept the women at home who were not engaged in war work.

In the numerous coal fed power houses of this country, a very large amount of fuel is wasted by having to keep the fires banked in the boilers during the valley periods of the load. Therefore there would be considerable saving by linking such stations to an industrial process which depends essentially on the utilization of electric energy, and the furnaces of which can be started and stopped with the facility of switching on and off an arc lamp.

The direct arc process fills these conditions perfectly, and it is the only chemical or metallurgical process which can be utilized easily with off-peak power. It takes the alternating current exactly as generated, for the furnaces can be built to utilize voltages generally available. There is no fused material to

become frozen when current is shut off as in the case of carbide of calcium furnaces, steel furnaces, etc.

Some electrolytic processes may be run intermittently but they require direct current, and therefore there is great additional cost for transforming machinery etc.

Hitherto power station managers, have been content to merely supply electric energy and have taken little interest in the purposes for which the energy was used, at any rate from a financial point of view. The time seems to have arrived when this policy should be changed, and every power house, should have its own base load factory to utilize off peak power. I know of no better form of such load, than the manufacture of nitrates from the air by the direct arc process.

By filling in the valleys the load factor is much improved and this would have the immediate effect of reducing costs. There has, however, been too much consideration given to questions of cost. With U-boats on the high seas trying to stop supplies of Chile nitrate, the railways congested with traffic and electrical works engaged in making munitions, it is of little use discussing power costs. The thing to do is to make *full* use of plants already installed.

Recently much has been heard of the suitability of Muscles Shoals, Alabama as a site for the manufacture of nitrates because of the water power which is to be developed there, but it will take at least four years to complete the hydraulic works. In the meantime a large steam power house is being built in order that the cynamid process may be put in to early operation. This includes a 60,000-kw. turbo generator and should anything happen to it, the nitrate plant would be stopped as the various steps of the indirect cyanamid process are so interlocked.

Viewed from this standpoint it would seem to be better in every way to have the manufacture of indispensable materials for explosives manufactured in a number of smaller plants, in widespread centers and by other processes, than the indirect or cyanamid method.

Coke Oven and Nitrate Plants

At the present time ammonium nitrate is required in very large quantities for burster charges for shells, torpedoes, mines grenades, etc. This is made from two components, viz., nitric acid and ammonia, both of which are difficult to transport, the

first because it is a corrosive acid and the second because in every ton of aqua ammonia there are about 2½ tons of water.

An industrial plant capable of furnishing electric energy as well as a supply of ammonia would be ideal, and it so happens that this is the case with a regenerative coke oven plant. Half the total gas made is available and this can be easily turned into electric energy whilst at the same time the nitrogen contained in the coal provides about the right amount of ammonia necessary to combine with the nitric-acid made from the electric energy by the arc flame process.

In order to show how ideal such a system is for making ammonia nitrate, I have prepared following diagram.

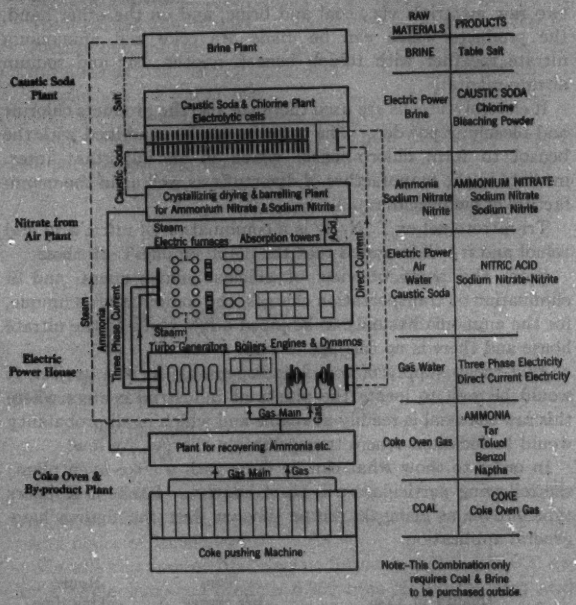

DIAGRAM LAYOUT OF NITRATE FROM AIR PLANT WITH ELECTRIC POWER HOUSE USING COKE OVEN GAS

The scheme provides for a combination of a battery of coke ovens with an ammonia recovery plant together with an electric

power house in order to utilize the surplus gas. Alongside the power house, there is an electrochemical plant for the manufacture of nitric acid from air by utilizing the three-phase high-tension current. A nitrate house is provided for the purpose of combining the ammonia from the coke ovens with the nitric acid from the elctrochemical plant.

It happens that the by-product of the acid factory is sodium nitrate-nitrite, which is made by combining the gases remaining from the acid towers with caustic soda or soda ash. Electrolytic cells may be laid down as shown in the diagram for purpose of making caustic alkali from brine.

It will thus be seen that the complete project requires only two raw materials viz., coal and brine, and on the other hand, the products which can be made are coke and ammonium nitrate together with toluol, benzol, naptha, tar and sodium nitrate-nitrite.

If electrolytic cells are used there are also the products chlorine and bleaching powder. The chlorine can be combined with the benzol to form chloro-benzol which is an important intermediate in the manufacture of dye-stuffs as well as in the manufacture of picric acid.

Tri-nitrotoluol or T. N. T. is a combination of nitric acid and toluol and it will be noted that the plant supplies both these.

From the point of view of efficient management, and of elimination of transportation charges, the combination is unique, for the ammonia has only to be piped a few yards to the nitrate house and there is no carriage of acid.

As a cheap supply of coal is indispensable for the project, it would be well to locate the plants at industrial centers where this raw material is readily available and which in all probability would be locations where transportation charges are low.

In order to show what can be done with a coke oven plant, the following particulars will be of interest. I take a Koppers type of oven as being the better known, but the figures have general application.

Quality of Coal	Tons per charge	Hours coking time
Low volatile coal	13½	18
Mixture containing 80 per cent high volatile 20 per cent low volatile	12½	16½
High volatile coal	11½	15

A battery of ovens varies in size but we may as well take a round number of 100 for which the average yields are as follows:

Number of ovens	100
Tons of coal per oven	12½
Hours coking time	16
Total yield of coke	72 per cent
Yield small coal and breeze	5 per cent
Net yield good coke	67 per cent
Ammonium sulphate per ton of coal	25 lb.
Reckoned as ammonia per ton of coal	6½ lb.
Tar per ton of coal	9 gal.
Light oil per ton of coal	3 gal.
Total gas per ton, of coal	11,000 cu. ft.
British thermal units	550 per cu. ft.
Surplus gas	55 per cent
Surplus gas per ton of coal	6,000 cu. ft.

Such a battery of ovens, each of which distils 12½ tons of coal in 16 hours, will deal with

$$\frac{100 \times 12.5 \times 24}{16} = 1{,}900 \text{ tons per day}$$

Assuming 6000 cu. ft. of surplus gas per ton of coal and 550 B.t.u. per cu. ft. the total heat value per hour will be

$$\frac{1900 \times 6000 \times 550}{24} = 260{,}000{,}000 \text{ B.t.u.}$$

If employed in gas engines using 13,000 B.t.u. per h.p-hr. the power will be

$$\frac{260{,}000{,}000}{13{,}000} = 20{,}000 \text{ h. p., or say, } 14{,}000 \text{ kw.}$$

If steam boilers and turbines are used instead of gas engines the power will be less, so to be on the safe side, we will take the round figure of 10,000 kw.

We will also assume that electric furnaces utilizing 10,000 kw. for a whole year, can produce 6,300 tons of 100 per cent acid. Nitric acid capable of furnishing theoretically 8000 tons of ammonium nitrate as indicated below:—

	NH_3	+ HNO_3	= NH_4NO_3
Molecular weights	17	63	80
In short tons	1700	6300	8000

Allowing 25 lbs. of sulphate of ammonia or 6½ lb. of ammonia per ton of coal, a total consumption of 1900 tons of coal per day should give.

$$\frac{1900 \times 365 \times 6.5}{20{,}000} = 2250 \text{ tons per annum}$$

It will thus be seen that there is plenty of ammonia to combine with the acid made by the surplus gas, even if a higher yield of acid is allowed per kw-yr. and more power is generated.

I purposely leave out of discussion, questions as to types of nitrogen fixation furnaces and of yields obtained. I may say, however, that it is not right to assume that yields are limited to those usually obtained from certain well known furnaces which must of necessity work with single-phase current.

The amount of ammonium nitrate will be less than the theoretical figure because the efficiency of the reaction is not 100 per cent, also it is usual to convert a certain amount of the gas into sodium nitrate-nitrite. A safe figure would be 6000 tons and at this rate it can be shown that with electric energy at 5 mils per kw-hr. and ammonia at 12 cents a pound, the ammonium nitrate can be made at less than half the price the Government is now paying.

In order to show how large a business the nitrogen industry has become, the following figures (compiled by Dr. Paul J. Fox) give the nitrogen balance sheet for the United States for 1917.

IMPORTED SUPPLIES	Tons of 2,000 lb.	Tons of Nitrogen
Chile saltpetre 95 per cent $NaNO_3$.	1,742,540	272,880
Ordinary saltpetre, potassium, nitrate	4,609	645
Ordinary saltpetre and gunpowder containing 75 per cent KNO_3..	1,500	210
Ammonium sulphate.....	8,135	1,725
Ammonium chloride.....	1,073	280
DOMESTIC SUPPLIES		
Coke oven ammonia—NH_3.....	113,760	93,625
Gas works ammonia—NH_3.....	12,500	10,288
Calcium cyanamid at 20 per cent nitrogen	12,800	10,534
NITROGEN EXPORTED	Tons of 2,000 lbs.	Tons of Nitrogen
Nitric acid, 15 per cent nitrogen...	486	73
Picric acid, 18 per cent nitrogen...	26,610	4,790
Dynamite, 12 per cent nitrogen	8,962	1,255
Gunpowder and smokeless powder, 13 per cent nitrogen.....	223,270	29,025
Ordinary saltpetre.....	875	123

In addition to the above, there are also about 8800 tons represented nitrogen in the following items which are the figures for 1917.

	Value
Loaded cartridges	$42,000,000
Fuses	34,000,000
Shells and projectiles	74,000,000
All other	202,000,000
Total	253,000,000

It will be noticed that ammonium nitrate is not included in these figures, but I assume it would be about 50,000 tons for 1917.

In Great Britain the consumption of ammoniun nitrate is now probably 400,000 tons a year, and the production here will have to be at least as much. To make this, the theoretical proportion of ammonia required is about 85,000 tons and of nitric acid about 315,000 tons.

It will thus be seen that the coke oven plants in the country could supply all the ammonium nitrate required if they were equipped for the purpose. I do not suggest that existing ovens should be so used, because much of the gas is already allocated for various purposes but for new installations of coke ovens it would be worth while to consider the use of the surplus gas and of the ammonia in the way suggested.

The idea of using coke oven gas for generating electric power for general supply is gaining ground and there are a good many such installations in the North of England where electric energy has been sold at lower rates than appears to be general in the United States. When a company owns the power house as well as the coke ovens it can of course sell the surplus gas to itself at a very low rate or even at zero. A battery of coke ovens is a good paying proposition independent of any money received for surplus gas.

The company operating at Fairmont, West Virginia is completing a new power house at the mouth of a coal mine which will have two 10,000-kw. units, and the company has 12,000 kw. in gas engine units which will be worked with gas from coke ovens, so that by 1919 there will be 15,000 kw. to spare.

Until recently most coke oven ammonia was converted into sulphate, but owing to the war demand for nitrate, more and more of it is being made into aqua-ammonia of about 29 per cent strength. In some cases this is being transported many

hundreds of miles prior to conversion into ammonium nitrate and since each ton of ammonia necessitates the transportation of about 2½ tons of water, the bearing of this, on the present railway congestion is at once apparent. Tank cars have to be used and they must return empty, so the freight on the actual ammonia carried is extremely high.

There are many coke ovens of the wasteful bee-hive type in operation, which do not recover by-products and the replacement of these by modern coke-ovens would be a great immediate economic gain and meet present war conditions. Coke ovens can be built quicker than large dams for water power and they have the great advantage of giving many products besides coke which are useful for war purposes.

In the present emergency coke ovens are of great value because they give coke for making steel, gas for power purposes, ammonia for nitrate manufacture, and toluol and benzol for explosives.

After the war ammonium nitrate will be in demand for fertilizer as well as for safety explosives and other purposes The high percentage of nitrogen which it contains viz., 35 per cent and the ease with which it can be converted into other compounds makes it especially useful for conveying nitrogen in the fixed form over considerable distances.

It is more profitable to make nitrate than sulphate, because, pound for pound, the nitrate contains nearly twice as much fixed nitrogen and the nitrogen commands a higher price per unit when in the form of ammonium nitrate.

DISCUSSION ON "ELECTRIC POWER FOR NITROGEN FIXATION" (SCOTT), ATLANTIC CITY, N. J., JULY 27, 1918.

C. P. Steinmetz: I decidedly agree with Mr. Scott in the conviction that the final solution of the nitrate problem is to be found in the arc flame process or direct process. I have read numerous reports during the last fifteen years aiming to prove the contrary, but I have not been convinced, and I have always had the feeling that the chemical engineers who made these reports, while endeavoring to be thoroughly impartial, were so much more familiar with chemical methods and relatively less familiar with electrical phenomena, so that the conclusions derived by them have been influenced unintentionally by their familiarity with the one class of work and unfamiliarity with the other class of work.

I agree that the direct process requires more electric power than the cyanamid process, for instance, but at the same time there are other processes, like the ammonia process, which require no electric power, and the reason why the cyanamid process requires much less power is that of the various operations only a very small part, the first quarter, is carried out electrically. We may put the statement the other way and say that in the direct process nothing but electric power is required, and all the work is done by electric power, while in the other, the indirect processes, electric power plays only a secondary part, or no part at all.

The present situation, as we all realize, is peculiar, that we have to face an emergency. It is not a question whether the one process is better or more economical than the other one, but the question is, first, which process can be brought into operation on a larger scale in the quickest manner. I am not sufficiently familiar with all the data which are available, to judge about this, but I am willing to accept the judgment of those who have gone over the field and decided that the cyanamid process offers the best possibility of getting in operation on a large scale in the quickest possible time, but that in addition thereto extensive experimentation and development should be carried out which may lead to more efficient, and therefore final, methods, but I believe a very great mistake is being made by our nation by not considering in this proposed development, or practically not considering, the arc process.

We must realize that without a powerful nitrate industry we are defenselessly at the mercy of any power which controls the ocean, and that we must have a strong nitrate industry indigenous to this country, not merely for the time of this war, but forever after, and such a nitrate industry can subsist on a large enough scale to be useful in emergencies only if an extensive use is found in peace times, and that use must be in the fertilizer industry.

That means an extremely cheap method of production, and that means as far as power is concerned, it must be capable of

operation with the cheapest power, that is, secondary power or off-peak power.

Of all the methods of nitrification the only one which is capable of efficient intermittent operation is the arc process, or direct process. The arc is equally as efficient in the second or milli-second after it is started as it is after continuous operation for days. That is not the case, however, with the cyanamid process, where even a very short time interruption means freezing of the carbide furnace and serious loss and interruption.

I saw some time ago a confidential report by a prominent engineer sent by one of our largest utility corporations to Norway to study the Norwegian process, to see whether it could be utilized in the use of off-peak power, and the report in its conclusion said that the Norwegian process,—as I may call the arc process as it was some years ago operated in the Norwegian plants,—if it is operated, not continuously, but half the time, then even if the power cost absolutely nothing, it would not be economical, because the interest on the investment would be greater than the value of the nitrate was at that time. Possibly it is different now, but I have no reason to doubt that the conclusions were correct. I do not know whether the process has been improved at the present time, but that, together with several other features, is a part of the general situation in regard to the Norwegian method.

We hear and have definite information that the Norwegian arc process nitrate plants are commercially successful and profitable, but we also hear that the conditions there are peculiar, such as cannot be found in our country, and the peculiarity is claimed to be the very low cost of power. That is not strictly true in all respects, because off-peak power here can be secured cheaper in many cases than the Norwegian power. The peculiar condition of the Norwegian plant, as it existed some years ago when these reports were made, was a limited power with very cheap capital, so that under those conditions the greatest economy required a development of the highest possible efficiency regardless of the cost of the investment. It is about the same condition which we meet in our largest public utility corporations, like Chicago and Detroit, which, as our President has pointed out, have reached the highest efficiency in electric production, exceeding 20 per cent from the chemical energy of coal to the busbar. An attempt to use apparatus of this high efficiency in an isolated station, supplying a hotel, or in a central station feeding a small city, would be hopelessly uneconomical, for the reason that, because of the size and the load factor the cost of the capital in such a small plant is much higher. We have to balance the efficiency which we can secure against the interest on the investment required to secure the efficiency. This in my opinion, is the characteristic feature of the Norwegian plant, which makes it impossible to transfer it without any change to American conditions, especially to conditions of

off-peak power, where not only the higher costs of capital under American conditions—especially under the present conditions—but furthermore, the still higher cost of capital due to its intermittent use, makes a plant aiming at the highest efficiency, at the collection of the last traces of the nitrate, makes such a plant uneconomical. This is the feature which, in my opinion, has very largely been killing the arc process in this country, because no competent electrical engineer realizing this situation was fathering it, but it has rather been done by the attempt to transplant the Norwegian method here under circumstances to which it is not adopted. It means redesign and redevelopment of this method, to a very much lower cost of investment, even at the sacrifice of some efficiency. We can better afford to waste 25 per cent of the nitrate if thereby we can get the cost of investment down to a small fraction of what the Norwegian plant costs, and thereby make it economical for off-peak power.

That is the principal point I want to bring to your attention, and furthermore, there is another matter which I can only give from hearsay, that is, that the financial control of the Norwegian nitrate plants were secured some time ago by German industries, and then afterwards were sold, and since that time we have been given to understand, intentionally or unintentionally, that the new German processes are superior, and that there is nothing in the arc process, because they have given up their control of them.

Whether that is really so, or whether they realize that the conditions in Germany are different from ours with regard to the economy of the arc process—the almost complete absence of larger water powers, the great abundance of skilled chemists and skilled chemical labor bringing about conditions which are very much more in favor of chemical processes and against the arc processes. I leave it to you to think over which may have been the reason.

But there is one feature which has seriously mitigated against the arc process, and that is the following: Some time ago a very careful investigation was made by the famous German chemist Nernst, on the temperature equilibrium of oxygen, nitrogen and nitric oxide, and he showed that at the temperature of the carbon arc the equilibrium corresponds to about 4 per cent nitric oxide. That would mean an efficiency of conversion of 13 per cent, if all this 4 per cent could be saved, but we know at these temperatures dissociation is so rapid, that we can expect to save only a part of the nitric oxide from destruction, in dropping the temperature as rapidly as possible. That means the possible efficiency must be much lower than 13 per cent.

It is true that the temperature equilibrium at the carbon arc temperature corresponds to 4 per cent of nitric oxide, and very much lower percentages at lower temperature. It has been proven however that the nitrification in the arc is not a temperature effect, but is an electrical effect, and the proof of that you can find in the TRANSACTIONS of the A. I. E. E. in the discussion

of a paper by my assistant Mr. J. L. R. Hayden, who made an extensive investigation some years back on nitrification, and has given the results in his discussion. This, however, seems to have been forgotten, since at that time the problem was of lesser importance than now. He showed that in low temperature low-voltage arc, nitric oxide can be produced far in excess of the concentration corresponding to the temperature equilibrium at the temperature of the arc, and therefore the effect cannot be a mere temperature effect. He furthermore showed that the relative efficiency of different temperature arcs does not correspond to the temperature. The carbon arc, which is the hottest, is very far from being the most efficient nitrifying arc. The iron arc and the copper arc each have about the same temperature, but not the same nitrifying efficiency, but rather they are on the two extremes of the series investigated. The copper arc is the least efficient, and the iron arc is the most efficient, showing thereby that the effect of the arc in producing the combination of nitrogen and oxygen is not a temperature effect, but a direct electrical effect.

The reason why I have confidence in the arc process is not only due to the absence of the need of any other raw material but air, which makes it specially suitable for use anywhere where power is available—as pointed out by Mr. Scott, at the coke oven for instance—it is not only the possibility of the use of intermittent power, which holds out the hope of developing a powerful nitrate industry in peace times, but it is also the vast possibility of increasing the efficiency of nitric acid production by the arc process, over that reached in the Norwegian plants. Under favorable conditions, I understand, the present efficiency is from 60 to 80 grams per kilowatt-hour. The theoretical efficiency of nitric oxide production is 2500 grams of NO_3H per kilowatt-hour, so that the present best results are an efficiency of 3 per cent.

You can realize, therefore, the vast possibilities there are in increasing the efficiency, and if at an efficiency of only 3 per cent the process has become commercial under favorable conditions, we can see that there are possibilities in this method which will bear considerable efforts being made in studying the development further, and carrying on the development, which I am sorry to say has practically been neglected since the early pioneer days of Charles Bradley, the electrical engineer, who started a plant at Niagara Falls, but failed, due to the cost of investment in his particular method being out of proportion to the returns.

I want to conclude by repeating that my study of the problem—and I have been closely interested in it, not only theoretically, but I have done considerable experimental work during the past twenty years or more,—has led me to the conclusion that while the cyanamid process may have immediate use by being the quickest available, that the process which I consider

as the most promising, in the final solution of the problem, is the direct or arc process.

Wm. A. Del Mar (communicated after adjournment): The arc process for the fixation of nitrogen is, in my opinion, the only use to which the off-peak capacity of large power stations can be put, as it is the only electrotechnical process that can thrive under the three characteristic conditions attending the use of such power, namely, absorption of large amounts of power, use of alternating current at the generated voltage and intermittent supply.

The cost of transforming apparatus bars all direct-current electrochemical processes such as electrolytic metal refining, alkali productions, etc. and many thermal processes. Other thermal processes are debarred because they cannot be operated intermittently. Any processes which are exceptional in these respects are too unimportant to absorb the power available from large stations.

Hence if off-peak capacity is ever to be utilized on a large scale, the only promising field for research is the arc-process for nitrogen fixation.

S. Barfoed (communicated after adjournment): As nitric acid is the end product desired it is evident that with the increasing demand upon the nation's man power naturally that system which can produce nitric acid with a minimum of labor is the most logical one to use, other things considered equal. The raw materials needed for the direct method being air and water lend themselves to transport in pipes requiring no attention. Also, the final product, nitric acid, is to be handled in the same manner by means of the new acid proof ferro-alloys. Consequently labor is not needed in this particular part of the cycle of operations.

In the operation of any arc furnaces used by the direct method a row of furnaces may be attended by but two men required for recording and renewing electrodes at intervals.

During the last two and one-half years the writer has been engaged upon the development and operation of a 1000-kv-a. arc furnace using metallic oxide electrodes. The results obtained in this furnace and the simplicity of its operation, also the ease with which it may be kept in repair justifies me in saying that the direct method for making nitric acid as compared with the indirect method has the very advantages which Mr. Scott mentions.

The simple way of getting the end product, nitric acid, with the direct method as the first and only operation must surely be evident to the layman even if he should be a lawyer-politician.

In discussing the proper system to use in nitrogen fixation it must be kept in mind that all the various processes have their proper place of usefulness, all depending upon the raw materials available and the end product it is desired to produce and have the market absorb. The choice of the system is practically

decided only by the geographical location of the plant. In places where coal has been obtainable at reasonable prices and where men are rather plentiful but where water power is comparatively expensive, the end product may to advantage be cyanamid. In the greater part of the West coal is not obtainable at prices that would make the indirect method of making acid or even the cyanamid process possible under any circumstances.

The scheme outlined for combining a battery of coke ovens with ammonia recovery plant with an arc furnace plant as outlined by Mr. Scott is entirely feasible and is one to which the writer has given considerable thought, but the conditions for its use during peace times must be favorable. Even in certain sections of the West in spite of the existing water powers such a scheme can be worked out successfully for producing ammonium nitrate. Where districts are equally well off as to coal and water the several methods of producing fixed nitrogen can very well exist side by side and not exclude one another, keeping in mind the suitability of each process with a view as to raw material and the end product.

I endorse the formula "Do it Electrically" when nitric acid is to be produced.

I do not believe that at the present time there are many power stations where surplus power may be had. Also, I believe that the load factor of the majority of stations is so high that the time during which off-peak power is available would be too short for making the operation of arc furnaces satisfactory. In the West there is rather a scarcity of power and new developments by existing companies are not likely to be made in time to be of any use during the war, and that is largely because the power it is thought to develop cheaply is to be of great magnitude. There are however, throughout the West numerous small water power sites of 10,000 h. p. or thereabouts which can very cheaply be developed and completed in a short time and which would be entirely suitable for a nitric acid factory. Statements are often made that such water powers cannot be developed under a certain figure. Such statements of course do not mean anything unless they are qualified as to just what is meant to be included in a development. As generally spoken of I believe what is meant is that the development should include step-up transformers, high-tension switch gear, transmission lines, lowering transformers, substation equipment and distributing lines, etc. For such systems the figures as often given may be correct, but it would be a mistake to draw any conclusions from them when it comes to making a hydroelectric development for use with arc furnaces producing nitric acid.

It is often found that the nitric acid factory can be located in close proximity to the power station and it will be at once evident the great saving that can be made by so doing in the cost of the plant. The development would include the hydraulic works and the generating equipment which generates power directly at the voltage required by the furnaces.

Looking at the matter in this light I have made several investigations where water power of around 10,000 h. p. can be developed for a figure around $60.00 per h. p. and lower. Where this can be done the power cost will be a minor item compared to the fixed charges on the acid factory.

F. W. Sperr, Jr. (communicated after adjournment): A very important phase of the plan which Mr. Kilburn Scott has developed is that it succeeds in reconciling the interests of two great industries—the nitrogen fixation industry and the by-product coke industry. It cannot be denied that, owing largely to the emphasis that has been placed upon fixation by the cyanamid process, there have hitherto been many elements of mutual antagonism in these two industries.

The by-product coke oven must be considered as an essential factor in our national security and military success. It is the greatest and most dependable source of tri-nitro toluol, the most important high explosive, and must be relied upon for our supply of benzol, creosote, naphthalene, anthracene and a host of other products of military value.

Many people still have the old-fashioned idea that beehive coke works better in a blast furnace than by-product coke. As a matter of fact records from a large number of blast furnaces show a saving of over 200 pounds of coke per ton of pig iron due to the substitution of by-product coke for the beehive product. The significance of this in connection with our fuel shortage and the importance of the steel industry is easy to recognize.

In the establishment of by-product coke ovens, we are still sadly behind both economic and military requirements and every agency should be interested in encouraging the construction of new plants as rapidly as possible.

It is to be regretted that the fear of loss through destructive competition with subsidized ammonia produced by the cyanamid process has operated in certain cases as a discouraging factor. In normal times, ammonia is the principal source of revenue, besides coke, to the by-product coke plant. Ammonia produced by the cyanamid process cannot compete with by-product coke oven ammonia except with the aid of a subsidy. Large quantities of subsidized ammonia or a cheap competing fertilizer like cyanamid, thrown upon the agricultural market might well induce hesitation in the building of by-product coke ovens.

On the other hand, Mr. Scott's proposition to combine the manufacture of nitric acid from the atmosphere with the operation of a by-product coke plant, utilizing the surplus gas from the latter to furnish the requisite power, and the ammonia to combine with the nitric acid forming ammonium nitrate, appears to be as economically sound in time of peace as it is desirable in time of war. From the standpoint of the by-product coke oven owner, it looks entirely feasible and encouraging. Under normal conditions, the nitric acid might be sold separately, or it might still be used together with the ammonia as ammonium

nitrate. In this way the expense of purchasing sulphuric acid to make ammonium sulphate would be avoided and a concentrated fertilizer more valuable than ammonium sulphate would be produced. It is to be hoped that arrangements can be made to build a plant for a large scale trial of this very promising proposition.

E. K. Scott: Members who have to deal with electric powerhouse problems, are naturally interested in improvement of load factor. I know of no load which is more suitable for filling up valleys of a load curve than the arc method of fixing atmospheric nitrogen, for the furnaces can be started and stopped at any time without detriment, as there is no fixed material to freeze when current is off.

A good situation for an air nitrate factory would be on the same plot of land as the power house and the two might well be under the same directorate.

It has often been said that the manufacture of air nitrates is dependent on a very cheap source of power and the implication is made that it must be water power and on a very large scale. I wish to combat this for I consider it feasible to successfully operate moderate sized plants with off peak power from existing power houses.

It is a mistake to unduly emphasize the question of power for there are other equally important factors, *e. g.* transportation of raw materials and finished products. Nitric acid is required in very large quantities for explosives, for dyes, for celluloid and other industries and being difficult to carry, it is best made near to the place where it is to be used.

The nitric acid made in Norway, cannot be exported owing to difficulty of carriage and even when made into nitrate of lime and nitrate of ammonia the cost of carriage to a market abroad is considerable. Take ammonia for example which for many years has been made in Norway with ammonia shipped from England. It has been estimated that the charges for carriage, insurance, transhipment, etc., of the aqua ammonia to Norway and of the finished nitrate of ammonia to England came to more per ton of product than the cost of the electric energy with which the ton was made.

Other things being equal it is often better to be near to a steady market and pay a fair price for power than to go to a considerable distance and get cheaper power. Steam power houses have often an advantage over hydroelectric plants because the latter are usually in inconvenient places.

I fully appreciate the development of hydroelectric power, but think that the building of a coke oven plant with electric power house and nitrate factory is more to the National advantage than if the same money were spent on hydraulic works, etc. The coke oven proposition has certainly wider usefulness.

It may be of some interest to mention that electric energy has been supplied for many years to certain electrochemical

factories on the North East Coast in England at prices which compare favorably with Niagara. A considerable amount of power developed in that area is from coke ovens and blast furnace gas and by linking the power houses together it has been found possible to bank some boilers in the large coal fed power houses in Newcastle over the week end and obtain supply from the coke oven and waste heat power houses of Durham and North Yorkshire which of course must be kept going at all hours.

Dr. Steinmetz referred to the investment on an air nitrate plant. A large part of this goes in the absorption towers and the building required to house same. Although chemists have always been accustomed to absorb gases in such towers I think a smaller and less costly method can be developed on engineering lines.

I purposely did not enter into questions regarding reactions, etc., that go on in nitrogen fixation furnaces, but I agree with Dr. Steinmetz that the action is not a purely thermal one. The fact that nitric acid may be formed on the windings of extra high-tension machines is an indication.

Suggestions of limitations in electrical methods are usually traceable to those who are interested in old fashioned chemical methods, as for example, that of getting nitric acid from Chili nitrate, but an audience of electrical engineers will appreciate that when electricity once gets a foothold in any industry it holds on.

Some of us remember the days when electric lighting was struggling against firmly entrenched gas interests. The electric street car was a long time before it finally ousted the horse car, especially in England, whilst the introduction of electric power in mines is still opposed by advocates of compressed air and so on.

Electric engineering is now developing rapidly in the largest fields of all, namely, electrochemistry and electrometallurgy. Very large amounts of energy are being called for which means greater power development and wider opportunities for electrical engineers and manufacturers of electrical equipment. Members will no doubt hear from time to time of other methods of making nitrates which do not require electric energy or require less than the arc process. They need have no fear that electrical methods will not outlast all others and I personally believe the arc process is best of all.

The name of Charles S. Bradley has been mentioned. He was a pioneer in this line of work before the Norwegians or Germans, and I think the fact is worthy of commemoration.

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AMERICA'S ENERGY SUPPLY

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given, and it is shown that the total potential hydraulic energy of the country is about equal to the total utilized fuel energy.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator. However, the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control to the main synchronous station. The economic advantage of the induction generator station is, that its simplicity permits elimination of most of the hydraulic development by using, instead of one large synchronous station, a number of induction generator stations and collecting their power electrically.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy, that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically. It is shown that the value of the recovered power would be an appreciable part of that of the fuel, and that organized and controlled by the central stations, this fuel power collection would improve the station load factor, give the advantages of the isolated plant without its disadvantages, and produce a saving of many millions of tons of coal.

I. The Available Sources of Energy

A. Coal

THE only two sources of energy, which are so plentiful as to come into consideration in supplying our modern industrial civilization, are coal, including oil, natural gas, etc., and water power.

While it would be difficult to estimate the coal consumption directly, it is given fairly closely by the coal production, at least during the last decades, where wood as fuel had become negligible and export and import, besides more or less balancing each other, were small compared with the production. Coal has been mined since 1822, and in Fig. 1 is recorded the coal production of the United States, from the governmental reports. The annual production is marked by circles, the decennial average marked by crosses for every five years. Table I gives the decennial averages, in millions of tons per year.

TABLE I

AVERAGE COAL PRODUCTION OF THE UNITED STATES

(decennial average)

Year	Million tons per year	Per cent increase per year
1825	0.11	....
30	0.32	22.4
35	0.83	19.7
40	1.92	17.0
45	4.00	14.5
50	7.46	10.45
55	10.8	8.35
60	16.6	8.72
65	25.9	9.22
70	40.2	8.58
75	56.8	7.42
80	82.2	7.95
85	122	6.80
90	160	5.40
95	206	5.75
1900	281	6.96
05	404	6.60
10	532	....

In Fig. 1 the logarithms of the coal production in tons are used as ordinates. With this scale, a straight line means a constant proportional increase, that is, the same percentage increase per year, and in the third column of Table I are given the average percentage increase of coal production per year.

This Fig. 1 is extremely interesting by showing the great irregularity of production from year to year, and at the same time a very great regularity over a long period of time. Since 1870 the average production may be represented by a straight line, the values lying irregularly above and below the line, which

represents an annual increase of 6.35 per cent and thus represents the average coal production[1] C by the equation

$$C = 45.3 \times 10^{0.0267\,(y-1870)} \text{ million tons}$$

or

$$\log C = 0.0267\,(y-1870) + 7.656$$

where y = year.

Before this time, from 1846 to 1884, the coal production could be represented by

$$C = 7.26 \times 10^{.0365\,(y-1850)} \text{ million tons}$$

or

$$\log C = 0.0365\,(y-1850) + 6.861$$

representing an average annual increase of 8.78 per cent.

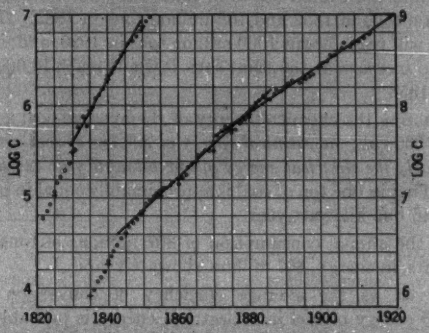

FIG. 1—COAL PRODUCTION OF THE UNITED STATES

It is startling to note how inappreciable, on the rising curve of coal production, is the effect of the most catastrophic political and industrial convulsions, such as the Civil War and the Industrial panic of the early 90's; they are indistinguishable from the constantly recurring annual fluctuations. It means, that the curve is the result of economic laws, which are laws of nature.

Extrapolating from the curve of Fig. 1, which is permissible, due to its regularity, gives 867 million tons as this year's coal consumption. As it is difficult to get a conception of such enormous amounts, I may be allowed to illustrate it. One of the great wonders of the world is the Chinese Wall, running

1. Soft coal and anthracite, and including oil reduced to coal by its fuel value.

across the country for hundreds of miles, by means of which China unsuccessfully tried to protect its northern frontier against invasion. Using the coal produced in one year as building material, we could with it build a wall like the Chinese Wall, all around the United States, following the Canadian and Mexican frontier, the Atlantic, Gulf and Pacific Coast, and with the chemical energy contained in the next year's coal production, we could lift this entire wall up into space, 200 miles high. Or, with the coal produced in one year used as building material, we could build 400 pyramids, larger than the largest pyramid of Egypt.

It is interesting to note that 100 thousand tons of coal were produced in the United States in 1825; one million tons in 1836; 10 million tons in 1852 and 100 million tons in 1882. The production will reach about 1000 million tons in 1920, and, if it continues to increase at the same rate, it would reach 10,000 million tons in 1958.

Estimating the chemical energy of the average coal as a little above 7000 cal., *the chemical energy of one ton of coal equals approximately the electrical energy of one kilowatt year* (24 *hour service*). That is, one ton of coal is approximately equal in potential energy to one kilowatt-year.

Thus the annual consumption of 867 millions of tons of coal represents, in energy, 867 million kilowatt-years.

However, as the average efficiency of conversion of the chemical energy of fuel into electrical energy is probably about 10 per cent, the coal production, converted into electrical energy, would give about 87 million kilowatts.

Assuming however, that only one half of the coal is used for power, at 10 per cent efficiency, the other half as fuel, for metallurgical work etc., at efficiencies varying from 10 per cent to 80 per cent, with an average efficiency of 40 per cent, then we get 217 million kilowatts (24 hour service) as the total utilized energy of our present annual coal production of 867 million tons.

B. The Potential Water Powers of the United States

Without considering the present limitation in the development of water powers, which permits the use of only the largest and most concentrated powers, we may try to get a conception of the total amount of hydraulic energy which exists in our country, irrespective of whether means have yet been developed

or ever will be developed for its complete utilization. We therefore proceed to estimate the energy of the total rain fall.

Superimposing the map of rain fall in the United States, upon the map of elevation, we divide the entire territory into sections by rain fall and elevation. This is done in Table II, for the part of our continent between 30 and 50 degrees northern latitude.

TABLE II

TOTAL POTENTIAL WATER POWER OF UNITED STATES

In. rain fall	Ft. elevation	Area $m^2 10^{12} \times$	Avg. elevation m.	Avg. rainfall cm.	Kg-m. per m^2; $10^8 \times$	Kg-m. total $10^{15} \times$
>10	>5000	0.54	2100	12.5	263	142
	1000–5000	0.29	900		112	32.5
10–20	>5000	1.18	2100	37.5	787	930
	1000–5000	1.96	900		338	660
20–30	1000–5000	0.32	900	62.5	563	183
	100–1000	0.97	150		94	91
30–40	1000–5000	0.35	900	87.5	788	275
	100–1000	1.40	150		131	184
40–60	1000–5000	0.27	900	125	1130	305
	100–1000	1.03	150		188	194
						$\Sigma = 2996$
						3000

As obviously only the general magnitude of the energy value is of interest, I have made only few sub-divisions: five of rain fall and four of elevation, as recorded in columns 1 and 2 of Table II[1]. The third column gives the area of each section, in millions of square kilometers, the fourth column the estimated average elevation, in meters, and the fifth column the average rain fall, in centimeters. The sixth column gives the energy, in kilogram-meters per square meter of area, and the last column the total energy of the section, in kilogram-meters, which would be represented by the rain fall, if the total hydraulic energy of every drop of rain were counted, from the elevation where it fell, down to sea level.

As seen from Table II, the total rain fall of the North American Continent between 30 deg. and 50 deg. latitude represents 3000×10^{15} kg-m. This equals 950 million kilowatt years (24 hour service). That is, the total potential water power of the United States, or the hydraulic energy of the total

1. The lowest elevation, < 100 ft., is not included, as having little potential energy.

rain fall, from the elevation where it fell, down to sea level, gives about 1000 million kilowatts.

However, this is not available, as it would leave no water for agriculture; and even if the entire country were one hydraulic development, there would be losses by seepage and evaporation.

An approximate estimate of the maximum potential power of the rain fall, after a minimum allowance for agriculture and for losses is made in Table III, allowing 12.5 cm. rain fall for wastage, and 37.5 and 25 cm. respectively for agriculture where such is feasible.

TABLE III

AVAILABLE POTENTIAL WATER POWER OF THE UNITED STATES

Avg. rainfall cm.	Avg. elevation m.	Area m^2 10^{12} ×	Wastage cm.	Agriculture, cm.	Available rainfall cm.	Kg. m. per m^2 10^3 ×	Kg-m. total 10^{15} ×
12.5	2100	0.54	12.5	....	....	....	.....
....	900	0.29	12.5	....	....	....	....
37.5	2100	0.39	12.5	25	....	....	.....
....	2100	0.79	12.5	....	25	525	415
....	900	0.98	12.5	25	....	....	....
....	900	0.98	12.5	....	25	225	220
62.5	900	0.21	12.5	37.5	12.5	112	23
....	900	0.11	12.5	....	50	450	50
....	150	0.97	12.5	37.5	12.5	19	18
87.5	900	0.35	12.5	37.5	37.5	337	118
....	150	1.40	12.5	37.5	37.5	56	78
125	900	0.27	12.5	27.5	75	674	182
....	150	1.03	12.5	37.5	75	112	116
							$\Sigma = 1220$

This gives about 1200×10^{15} kg-m. as the total available potential energy, which is equal to 380 million kilowatts (24 hour service). Assuming now an efficiency of 60 per cent from the stream to the distribution center, gives 230 million kilowatts (24 hour service) as the maximum possible hydroelectric power, which could be produced, if every river, stream, brook or little creek throughout its entire length, from the Spring to the ocean, and during all seasons, including all the waters of the freshets, were used and could be used. It would mean that there would be no more running water in the country, but stagnant pools connected by pipe lines to turbines exhausting into the next lower pool. Obviously, we could never hope to develop more than a part of this power.

C. Discussion

It is interesting to note that the maximum possible hydraulic energy of 230 million kilowatts, is little more than the total energy which we now produce from coal, and is about equal to the present total energy consumption of the country, including all forms of energy.

This was rather startling to me. It means that the hope that when coal once begins to fail we may use the water powers of the country as the source of energy, is and must remain a dream, because if all the potential water powers of the country were now developed, and every rain drop used, it would not supply our present energy demand.

Thus hydraulic energy may and should supplement that of coal, but can never entirely replace it as a source of energy. This probably is the strongest argument for efforts to increase the efficiency of our methods of using coal.

A source of energy which is practically unlimited, if it could only be used, is solar radiation. The solar radiation at the earth's surface is estimated at 1.4 cal. per cm.^{2} per min. Assuming 50 per cent cloudiness, this would give an average throughout the year (24 hours per day), of about 0.14 cal. per cm.^{2} horizontal surface per min., and on the total area considered in the preceding table, of 8.3 million square kilometers of North America between 30 and 50 latitude, a total of approximately 800,000 million kilowatts (24 hour service), or a thousand times as much as the total chemical energy of our coal consumption; 800 times as much as the potential energy of the total rainfall.

Considering that the potential energy of the rainfall from surface level to sea level, is a small part of the potential energy spent by solar radiation in raising the rain to the clouds, and that the latter is a small part of the total solar radiation, this is reasonable.

Considering only the 2.7 million square kilometers of Table III, which are assumed as unsuited for agriculture, and assuming that in some future time, and by inventions not yet made, half of the solar radiation could be collected, this would give an energy production of 130,000 million kilowatts.

Thus, even if only one-tenth of this could be realized, or 13,000 million kilowatts, it would be many times larger than all the potential energy of coal and water. Here then would be the great source of energy for the future.

II. Hydroelectric Station

A. The Modern Synchronous Generator Station

In developing the country's water powers, up to the present time only those of greatest energy concentration have been considered; that is, those where a large volume and a considerable head of water was available within a short distance.

This led to the present type of hydroelectric generating station, as best solving the problem. The equipment of such a station comprises the following apparatus:

Three-phase synchronous direct-connected generators.

Hydraulic turbines of the highest possible efficiency.

Hydraulic turbine speed governing mechanism.

An exciter plant comprising either exciters directly connected to the generators, or several separate exciter machines, connected to separate turbines.

Exciter bus bars.

Voltmeter, and ammeters in exciters and in alternator field circuits.

Field rheostats of the alternators.

Low-tension busbars, either in duplicate, or with transfer or synchronizing bus.

Circuit breakers between generators and busbars, usually non-automatic.

Circuit breakers between transformers and busbars, usually automatic, with time limit.

Voltmeters and potential transformers at the generators.

Synchronoscopes or other synchronizing devices.

Ammeters and current transformers at the generators.

Voltmeter and potential transformer at the busbars.

Ammeters and current transformers at the step-up transformers.

Totaling ammeter for the station output.

Integrating wattmeter.

Relays, interlocking devices etc., etc.

Step-up transformers.

High-tension busbars, possibly in duplicate.

High-tension circuit breakers between transformers and high-tension busbars.

High-tension circuit breakers between high-tension busbars and lines.

Lightning arresters in the transmission lines, with inductances etc.

Ground detectors, arcing-ground or short-circuit suppressors, voltage indicators etc.

Automatic recording devices (multi-recorder), rarely used though very desirable.

Due to the vast amount of energy controlled by modern stations, the auxiliary and controlling devices in these stations have become so numerous as to make the station a very complex structure, requiring high operating skill and involving high cost of installation. At the same time, not only are all these devices necessary for the safe operation of the station, but we must expect that with the further increase of capacity of our electric systems, additional devices will become necessary for safe and reliable operation. One such device I have already mentioned—automatic recording apparatus, such as the multi-recorder.

With this type of station, it is obviously impossible, in most cases, to develop water powers of small and moderate size. A generating station of a thousand horse power will rarely, and one of a hundred horse power will hardly ever be economical.

On the other hand, a hundred horse power motor installation is a good economical proposition, and the average size of all the motor installations is probably materially below one hundred horse power.

Looking over Tables II and III, especially the latter, in the preceding section, it is startling to see how large a part of the potential water power of the country is represented by comparatively small areas of high elevation, in spite of the relatively low rainfall of these areas. As most of these areas are at considerable distance from the ocean, most of the streams are small in volume. That is, it is the many thousands of small mountain streams and creeks, of relatively small volume of flow, but high gradients, affording fair heads, which apparently make up the bulk of the country's potential water power.

Only a small part of the country's hydraulic energy is found so concentrated locally as to make its development economically feasible with the present type of generating station.

Therefore, some different, and very much simpler type of generating station must be evolved, before we can attempt to develop economically these many thousands of small hydraulic powers, to collect the power of the mountain streams and creeks.

B. Simplification of Hydroelectric Station

In the following in discussing the simplification of the hydroelectric station to adapt it to the utilization of smaller powers, we limit ourselves to the case where the smaller hydraulic stations feed into a system containing some large hydraulic or steam turbine stations, to which the control of the system may be relegated.

1. We may eliminate the low-tension bus bars, with generator circuit breakers and transformer low-tension circuit breakers and connect each generator directly to its corresponding transformer making one unit of generator and transformer, and do the switching on high-tension busbars, and locating high-tension busbars and circuit breakers outdoors. While it is dangerous to transformers to switch on the high-tension side, due to the possibility of cumulative oscillations, this danger is reduced by the permanent connection of the transformer with the generator circuit, and is less with the smaller units used in small power stations, and thus permissible in this case.

However, the simplification resulting therefrom is not so great, as ammeters, voltmeters and synchronizing devices with their transformers are still retained on the low-tension circuits.

2. As it is not economical to operate at partial load, proper operation of a hydraulic station on a general system is, to operate as many units fully loaded as there is water available, and increase or reduce the number of units (of turbine, generator and transformer, permanently joined together), with the changing amount of available water, thus using all the available energy of the water power.

In this case, the turbine governors, with their more or less complex hydraulic machinery, may be omitted. If then the generators are suddenly shut down by a short circuit which opens the circuit breakers, the turbines will race and run up to their free running speed, until the gates are shut by hand. However, generators, and turbines must stand this, as even with the use of governors, the turbines may momentarily run up to their free speed in case of a sudden opening of the load, before the governors can cut off the water. Where this is not desirable some simple excess speed cut-off may be used.

3. When dropping the governing of the turbines, and running continuously at full load, the question may be raised whether generator ammeters are necessary, as the load is constant, and is all the power the water can give, and it might appear, that am-

meters with their current transformers, etc. could be omitted. However, with synchronous generators, the current depends not only on the load, but also on the power factor of the load and with excessively low power factor due to wrong excitation, the generators may be overheated by excess current, while the power load is well within their capacity. Thus ammeters are necessary with synchronous generators. As soon, however, as we drop the use of synchronous generators, and adopt induction generators, the ammeters with their current transformers may be omitted, since the current and its power factor is definitely fixed by the load. At the same time, synchronizing devices become unnecessary, together with potential transformers, generator voltmeters, etc. A station voltmeter may be retained for general information, but it not necessary either, as the voltage and frequency of the induction-generator station are fixed by the controlling synchronous main station of the system.

4. With the adoption of the induction generator, the entire exciter plant is eliminated, as the induction generator is excited by lagging currents received from synchronous machines, transmission lines and cables existing in the system. This avoids the use of exciter machines, exciter busses, ammeters, voltmeters, alternator field rheostats, etc., in short, most of the auxiliaries of the present synchronous station become unnecessary.

The solution of the problem of the economic development of smaller water powers is the adoption of the induction generator.

Stripped of all unnecessary equipment, the smaller hydroelectric station thus would comprise:

Hydraulic turbines of simplest form, continuously operating at full load, without governors.

Low-voltage induction generators direct connected to the turbines.

Step-up transformers direct connected to the induction generators.

High-tension circuit breakers connecting the step-up transformers to the transmission line. In smaller stations, even these may be dispensed with and replaced by disconnecting switches and fuses.

Lightning arresters on the transmission line where the climatic or topographical location makes such necessary.

A station voltmeter, a totalling ammeter or integrating wattmeter and a frequency indicator may be added for the informa-

tion of the station attendant, but are not necessary, as voltage, current, output and frequency are not controlled from the induction generator station, but from the main station, or determined by the available water supply.

It is interesting to compare this induction generator station lay-out with that of the modern synchronous station given above. However, it must not be forgotten that *the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control, to the main synchronous stations* of the system, and the induction generator stations thus are feasible only as adjuncts to at least one large synchronous station, hydraulic or steam turbine, in the system, but can never replace the present synchronous generator stations in their present field of application.

C. Automatic Generating Stations

With the enormous simplification resulting from the use of the induction generator, it appears entirely feasible to make smaller hydroelectric generating stations entirely automatic, operating without attendance beyond occasional—weekly or daily—inspection.

Such an automatic generating station would comprise a turbine with low-voltage induction generator, housed under a shed, and a step-up transformer, outdoors, connecting into the transmission line with time fuses and disconnecting switches.

It is true that in the big synchronous generating stations of thousands of kilowatts, the cost of the auxiliaries, as exciter plant, regulating and controlling devices, etc., is only a small part of the total station cost, and little would therefore be saved by the use of induction generators. No induction generators would, however, be used for such stations. But the cost of auxiliaries and controlling devices, and the cost of the required skilled attendance, decreases far less with decreasing station size than that of the generators—whether synchronous or induction—or in other words, with decreasing size of the station, *per kilowatt output*, the cost of auxiliaries and controlling devices and of attendance increases at a far greater rate than that of the generators, and very soon makes the synchronous station of the present type uneconomical.

It is also true that in the big modern hydraulic power systems, the cost of the generating station usually is a small part of the cost of the hydraulic development. Therefore any saving in

the cost of the generating station would be of little influence in determining, whether the hydraulic development would be economical. With decreasing size of the water power the cost of the hydraulic development *per kilowatt output* usually increases so rapidly as very soon to make the development of the water power uneconomical, no matter how simple and cheap the station is.

However, the value of the induction generator is not so much in the reduction of the cost of the generating station, as in the reduction of the cost of the hydraulic development, by making it possible to apply to the electric generator the same principle, which has made the electric motor economically so successful: *Collect the power electrically, just as we distribute it electrically.*

We do not, as in the days of the steam engine, convert the electric power into mechanical power at one place, by one big motor, and distribute the power mechanically, by belts and shafts, but we distribute the power electrically, by wires, and convert the electric power to mechanical power, wherever mechanical power is needed, by individual motors throughout mill and factory.

In the same way we must convert the hydraulic, that is, mechanical power into electrical power by individual generators located along the streams or water courses within the territory, wherever power is available, and then collect this power electrically, by medium-voltage collecting lines and high-voltage transmission lines, and so eliminate most of the cost of the hydraulic development, to solve the problem of the economical utilization of the country's water powers. If we attempt to collect the power mechanically, that is, by a hydraulic development gathering the waters of all the streams and creeks of a territory together into one big station, and there convert it into electric power, the cost of the hydraulic development makes it economically hopeless except under unusually favorable conditions, where a very large amount of power is available within a limited territory, or where nature has done the work for us in gathering considerable power at a waterfall, etc.

It is the old problem, and the old solution: If you want to *do it economically, do it electrically.*

Naturally then, we would use induction generators in these small individual stations just as we use induction motors in individual motor installations; but where large power is available, there is the field of the synchronous generator, and the induction generator is undesirable, just as the synchronous motor

is preferable where large power is required—unless the synchronous motor is excluded by conditions of starting torque, etc.

At first, and for some time to come, we would not consider going to anywhere near as small sizes of induction generators, as we do in induction motors. However, there are undoubtedly many millions of kilowatts available in water powers throughout the country, which can be collected by induction-generator stations from 50 h.p. upwards, and that at fair heads, requiring no abnormal machine design (no very low speed).

Consider an instance—a New England mill river with a descent, in its upper course, of about 1100 ft. (335 m.) within five miles (eight km.), of varying gradient. At three places, where the gradient is steepest, by a few hundred feet of cast iron pipe and a small dam of 20 to 30 ft. (6 to 9 m.) length and a few feet height—just enough to cover the pipe intake—an average head of 150 ft. (45 m.) can be secured, giving an average of 75 h.p. each, or a total of 225 h.p. or 170 kw. This would use somewhat less than half the total potential power. The development of the other half, requiring greater length of pipe line, or involving lower heads, would be left to meet future demands for additional power.

The installation of an electric system of 170 kw. would hardly be worth while, but there are numerous other creeks throughout the territory from which to collect power, and within a few miles passes a high potential transmission line, coming from a big synchronous station, into which the power collecting lines coming from the induction generator stations would be tied and from which they would be controlled.

Thus, the large modern synchronous station has its field, and is about as perfect as we know how to build for large concentrated powers; but beyond this, there is a vast field, and therefore *an economic necessity of the development of a different type of hydraulic generating station to collect the scattered water powers* of the country; and that is the *induction generator station*, to which I wish to draw the attention.

I must caution, however, not to mistake small power and low head power. There are on the lower courses of our streams some hydraulic powers, which are relatively small due to their low heads, and which can not be economically developed by the synchronous generator, due to the low head and correspondingly low speed. The designing characteristics of the induction generator, with regard to low-speed machines, are no better—if

anything rather worse—than those of the synchronous generator, and the problem of the economical utilization of the low-head water power still requires solution. It is not solved by the induction generator; the latter's characteristic is simplicity of the station, giving the possibility of numerous small automatic generating stations.

III. Induction Generator Station

A. Characteristics of Induction Generator

An induction motor at no load runs at, or rather very close to synchronism. If it is driven above synchronism by mechanical power, current and power again increase, but the electric power is outflowing, and the induction machine consumes mechanical power, and generates electrical power, as an induction generator.

The maximum electrical power, which an induction machine can generate as an induction generator is materially larger than the maximum mechanical power, which the same machine, at the same terminal voltage, can produce as an induction motor.

Resolving the current of the induction machine into an energy component and a wattless or reactive component, the energy current is inflowing, representing consumption of electric power (which is converted to mechanical power) below synchronism. It becomes zero near synchronism, and above synchronism the energy current is in the reverse direction, or outflowing, supplying electric power to the system (which is produced from the mechanical power input into the machine), and the induction machine then is a generator.

The wattless or reactive component is a minimum at synchronism, and increases with the slip from synchronism, and is in the same direction, whether the slip is below synchronism, as a motor, or above synchronism, as a generator. That is, the induction machine always consumes a lagging current (representing the exciting current and the reactance voltages), or, what amounts to the same, produces a leading current. The latter way of putting it is frequently used with induction generators, by saying that the current produced by the induction generator is leading, while the current consumed by the induction motor is lagging. Instead of saying however, that the reactive component of the current generated by the induction generator is leading, we may say, and this makes it often more intelligible, that the induction generator generates an energy current and consumes a lagging reactive current, while the induction motor consumes an energy current and consumes a reactive lagging current.

As with the increasing voltages and increasing extent of our transmission systems the leading currents taken by transmission lines and underground cables are becoming increasingly larger, the induction generator appears specially advantageous, as tending to offset the effect of line capacity. We may thus say that the induction generator (and induction motor) consumes a lagging reactive current, which is supplied by the synchronous generators, synchronous motors, converters and other synchronous apparatus in the system, and by the capacity of lines and cables. Or we may say that the lagging current consumed by the induction generator neutralizes the leading current consumed by the capacity of lines and cables. Or we may say that the leading current produced by the induction

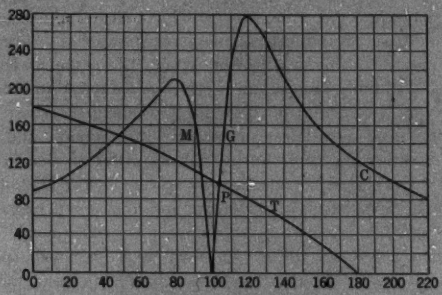

FIG. 2—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT CONSTANT TERMINAL VOLTAGE

generator supplies the capacity of lines and cables: these are merely three different ways of expressing the same facts.

In Fig. 2 are shown the torque curves, at constant terminal voltage, of a typical moderate size induction machine. M is the torque produced as an induction motor below synchronism, and G the torque consumed as an induction generator above synchronism, synchronism being chosen as 100 per cent. T is an assumed torque curve of a hydraulic turbine.

As seen, the point P where G and T intersect, is 4 per cent above synchronism, and this induction generator thus operates on full load at 4 per cent slip above synchronism or no-load. Assuming now, that the power goes off, by the circuit breakers opening. The turbine then speeds up to 80 per cent above

synchronism, where the curve T becomes zero. If at this free running turbine speed the circuit is closed and voltage put on the induction generator, the high torque consumed by the induction generator causes the turbine to slow down, and as at all speeds above 104, the torque consumed by the induction generator is very much higher than that given by the turbine, the machine slows down rapidly, to the speed where the induction generator torque has fallen to equality with the turbine torque, at speed 104, and stable condition is restored.

Inversely, if the flow of water should cease, the induction machine slows down to a little below synchronism, and there continues to revolve as induction motor.

In starting, the circuit may be closed before admitting the water, and the turbine started by the induction machine as a motor, on the torque curve M, running up to speed 100, and then, by admitting the water, the machine is speeded up 4 per cent more and thereby made to take the load as generator. Or the turbine may be started by opening the gates, running up to speed 180, and then, by closing the circuit, the induction machine in taking the power slows the speed down to normal.

With larger machines, the most satisfactory way of starting as involving the least disturbance, probably would be, first to open the gates partly while the turbine speeds up, and when it has reached a speed in the neighborhood of synchronism, say between 95 and 105, the circuit is closed and the water gates opened fully.

B. Instability Conditions of Induction Generator

In Fig. 2, the torque consumed by the induction machine, at all turbine speeds above full load P, is much higher than the torque of the turbine. However, the induction generator torque curve has a concave range, marked by C, and if the induction generator should be such as to bring the generator torque curve at C below the turbine torque curve T, the speed, when once increased beyond the range C, would not spontaneously drop back to normal. While in Fig. 2, C is much higher than T, Fig. C represents the theoretical, but not real case of constant terminal voltage at the induction machine. The voltage however is kept constant at the controlling synchronous main station, and thus must vary with the load in the induction generator station. Assuming an extreme case, of 10 per cent resistance and 20 per cent reactance in the line from the induction machine station to

the next synchronism station, we get the modified torque curve shown in Fig. 3. As seen, at full load P, there is practically no change; about 4 per cent slip above synchronism. The maximum torque of generator G and motor M, and the torque at the concave part of the induction generator curve, C, have greatly

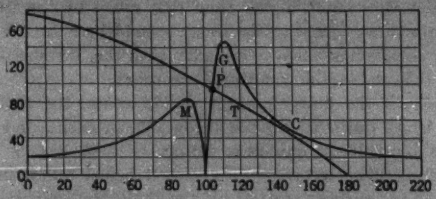

FIG. 3—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS STATION

decreased. However, C is still above T, that is, even under this extreme assumption, the induction generator would pull the turbine down from its racing speed of 180, to the normal full load speed of 104, though the margin has become narrow.

Assuming however an induction machine with much less slip,

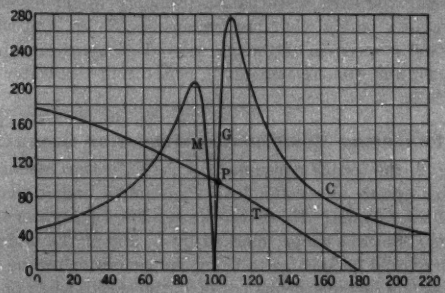

FIG. 4—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT TERMINAL VOLTAGE

with only half the rotor resistance of Figs. 2 and 3. At constant terminal voltage, this gives the curves shown in Fig. 4. The full load P is at speed 102, or 2 per cent above synchronism, and while the curve branch C is much lower, the conditions are still perfectly stable. Assuming however, with this type of low

resistance rotor, a high line impedance, 10 per cent resistance and 20 per cent reactance, as in Fig. 3. We then get the condition shown in Fig. 5. The range C drops below T, and the induction generator torque curve G intersects the turbine torque curve T at three points: P, P_1 and P_2. Of these three theoretical running speeds, $P=102$, $P_1=169$ and $P_2=113.5$, two are stable, P and P_1; while the third one, P_2, is unstable, and from P_2, the speed must either decrease, reaching stability at the normal full load point P, or the machine speed up to P_1.

If with the conditions represented by Fig. 5, the turbine should—by an opening of the circuit for instance—have speeded up to its free running speed 180, closing the circuit does not bring the speed back to normal, P, but the machines slow down only to speed P_1, where stability is reached, at very little output and very large lagging currents in the induction generator. To

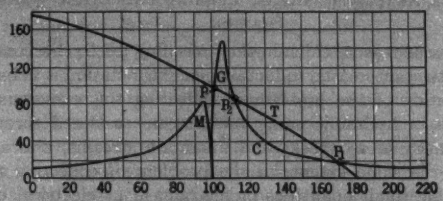

FIG. 5—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS MAIN STATION

restore normal condition then would require shutting off the water, at least sufficiently to drop the turbine torque curve T below C, and then letting the machines slow down to synchronism. They would not go below synchronism, even with the water gates entirely closed, as the induction machine as a motor, on curve M, holds the speed.

A solution in the case Fig. 5 would be the use of a simple excess speed governor, which cuts off the water at 5 to 10 per cent above synchronism.

However, the possibility of difficulty due to the "dropping out of the induction generator" as we may call it in analogy to the dropping out of the induction motor, are rather less real than it appears theoretically. In smaller stations, such as would be operated without attendance, as automatic stations, the torque curve of the induction generator, as a small machine, would be

of the character of Figs. 2 and 3, and thus not liable to this difficulty. The low resistance type of induction machines, as represented in Figs. 4 and 5, may be expected only with the larger machines, used in larger stations. In those, some attendant would be present to close the water gates in case of the circuit breakers opening, or a simple cheap excess speed cut-off would be installed at the turbines, keeping them within 10 per cent of synchronism, and within this range, no dropping out of the induction generator can occur.

It is desirable however to realize this speed range of possible instability of the induction generator, so as to avoid it in the design of induction generators and stations.

APPENDIX

Collection of Fuel Power by Steam Turbine Induction Generator

A. The Automatic Steam Turbine Induction Generator Station

The same reason which in the preceding led to the conclusion that in the (automatic) induction generator station is to be found the solution of the problem of collecting the numerous small amounts of hydraulic energy, which are scattered throughout our country along creeks and mountain streams, also applies, and to the same extent, to the problem of collecting the innumerable small quantities of mechanical or electrical energy, which are, or can be made available wherever fuel is consumed for heating purposes. Of the hundred millions tons of coal, which are annually consumed for heating purposes, most is used as steam heat. Suppose then, we generate the steam at high pressure—as is done now in many cases for reasons of heating economy—and interpose between steam boiler and heating system some simple form of high pressure steam turbine, directly connected to an induction generator, and tie the latter into the general electrical power distribution system. Whenever the heating system is in operation, electric power is generated, as we may say as "by-product" of the heating plant, and fed into the electric system.

The power would not be generated continuously, but mainly in winter, and largely during the day and especially the evening. That is, the maximum power generation by such fuel power collecting plant essentially coincides with the lighting peak of the central station, thus occurs at the time of the day, and the season when power is most valuable. The effect of such fuel

power collection on the central station should result in a material improvement of the station load factor, by cuttnig off the lighting peaks.

The only difference between such steam turbine induction generator stations, collecting the available fuel power scattered throughout the cities and towns, and the hydraulic induction generator stations collecting the powers of the streams throughout the country, is that in the steam turbine plant an excess speed cut-off must be provided, as the free running steam turbine speed is usually not limited to less than double speed, as is the case with the hydraulic turbine. Otherwise however, no speed governing is required. A further difference is, that the greater simplicity and therefore lower investment of the steam turbine plant would permit going down to smaller powers, a few kilowatts perhaps.

It is interesting to note, that even with a very inefficient steam turbine, the electric generation of such fuel power collecting plant interposed between boiler and heating system, takes place with practically 100 per cent efficiency, because whatever energy is wasted by the inefficiency of the steam turbine plant, remains as heat in the steam, and the only loss is the radiation from turbine and generator, and even this in most cases is useful in heating the place where the plant is located. The only advantage of a highly efficient turbine, is that larger amounts of electric power can be recovered from the fuel, and the question thus is that between the investment in the plant, and the value of the recovered power.

If then the total efficiency, from the chemical energy of the fuel to the electric power, were only 3 per cent, it would mean that 3 per cent more coal would have to be burned, to feed the same heat units into the heating system. At an average energy value of 30,000 kj. per kg. of coal, this would give per ton of coal, 900,000 kj. or 250 kw-hr. At a bulk value of ½ cent per kw-hr. it would represent a power recovery value of $1.25 per ton of coal. This is quite considerable, more than sufficient to pay the interest on the investment in the very simple plant required.

At first, the steam turbine induction generator plant, proposed for the collection of fuel power, would appear similar to the isolated plant which, though often proved uneconomical, still has successfully maintained its hold in our northern latitudes, where heating is necessary through a considerable part of the year. However, the difference between the steam turbine in-

duction generator plant and the isolated steam electric plants in our cities, is the same as that between the automatic hydroelectric induction generator station, and the present standard synchronous generator station: by getting rid of all the complexity and complication of the latter, the induction generator station becomes economically feasible in small sizes; but it does so only by ceasing to be an independent station, by turning over the functions of regulation and control to the central main station and so becoming an adjunct to the latter. But by this very feature, the turbo induction generator plant might afford to the central station, the public utility corporation, a very effective means of combatting the installation of isolated plants, by relieving the prospective owner of the isolated plant of all trouble, care and expense and incidental unreliability thereof, supplying central station power for lighting, but at the same time utilizing the potential power of the fuel burned for heating purposes. The simplest arrangement probably would be, that the fuel power collecting plants scattered throughout the city would, as automatic stations, be taken care of by the public utility corporation, their power paid at its proper rates, those of uncontrolled bulk power, while the power used for lighting is bought from the central station at the proper lighting rates.

As this however means a new adjustment of the relation between customer and central station, and is not merely an engineering matter like the hydroelectric power collection, I have placed it in an appendix.

B. Discussion

We realize that our present method of using our coal resources is terribly inefficient. We know that in the conversion of the chemical energy of coal into mechanical or electrical energy, we have to pass through heat energy and thereby submit to the excessively low efficiency of transformation from the low grade heat energy to the high grade electrical energy. We get at best 10 to 20 per cent of the chemical energy of the coal as electrical energy; the remaining 80 to 90 per cent we throw away as heat in the condensing water, or worse still, have to pay for getting rid of it. At the same time we burn many millions of tons of coal to produce heat energy, and by degrading the chemical energy into heat, waste the potential high grade energy which those millions of tons of coal could supply us.

It is an economic crime to burn coal for mere heating without

first taking out as much high grade energy, mechanical or electrical, as is economically feasible. It is this feature, of using the available high grade energy of the coal, before using it for heating, which makes the isolated station successful, though it has every other feature against it. To a limited extent, combined electric and central steam heating plants have been installed, but their limitation is in the attempt to distribute heat energy, after producing it in bulk, from a central station. Here again we have the same rule; to do it efficiently, do it electrically. In the efficiency of distribution or its reverse, collection, no other form of energy can compete with electric energy, and the economic solution appears to be to burn the fuel wherever heating is required, but first take out its available high grade energy, and collect it electrically.

Assume we use 200 million tons of coal per year for power, at an average total efficiency of 12 per cent, giving us 24 million kw. (referred to 24-hr. service) and use 200 million tons of coal for heating purposes, wasting its potential power.

If then we could utilize the waste heat of the coal used for power generation, even if thereby the average total efficiency were reduced to 10 per cent, we would require only 240 million tons of coal, for producing the power, and would have left a heating equivalent of 216 million tons of coal, or more than required for heating. That is, the coal consumption would be reduced from 400 million to 240 million of tons, a saving of 160 million tons of coal annually.

Or, if from the 200 million tons of coal, which we degrade by burning it for fuel, we could first abstract the available high grade power, assuming even only 5 per cent efficiency, this would give us 10 million kw. (24-hr. rate), at an additional coal consumption of 10 million tons, while the production of the 10 million kw. now requires 100 million tons of coal, more or less, thus getting a saving of 90 million tons of coal; or putting it the other way, a gain of 9 million kw.—12 million horse power—24-hr. service, or 36 million horse power for an 8-hr. working day.

It is obvious that we never could completely accomplish this; but even if we recover only one-quarter, or even only one-tenth of this waste, it would be a vast increase in our national efficiency.

Thus the solution of the coal problem, that is, the more economic use of fuel energy, is not only the increase of the thermodynamic efficiency of the heat engine, in which a radical advance

Per ton of coal: Chemical energy, 20×10^6 kj: Heat energy of steam from boiler, at 75 per cent. boiler efficiency, 22.5×10^6 kj.

	Boiler press		Dist. press		Carnot-efficiency per cent.	Output at 50 per cent. efficiency		Value of power at ¼c. per kw-hr. $...	Avg. kw., assuming 25% time of use	Tons of coal per kw.	Size of induction generator, per 100 tons coal annually
	atm:	lb.	atm:	lb.		kj. × 1000	kw-hr.				
Steam Heating	6	90	1.25	19	12.3	1390	385	1.92	0.176	5.7	25 kw.
	15	220	1.25	19	19.8	2230	620	3.10	0.283	3.5	45 kw.
Vacuum Heating	0	90	0.48	7	18.1	2030	565	2.82	0.258	3.9	40 kw.
	15	220	0.48	7	25.0	2830	810	4.05	0.37	2.7	55 kw.

is limited by formidable difficulties; but is the recovery of the potential energy of all the fuel, by electric collection.

C. Turbo Induction Generator

Assume then that wherever fuel is burned to produce steam for heating purposes, instead of a low-pressure boiler giving a few pounds over-pressure only, we generate the steam at high pressure, at six atmospheres (90 lbs.) or, in larger plants, even at 15 atmospheres (220 lb.) passing the steam through a high pressure turbine wheel directly connected to an induction generator tied into the electric supply system, and then exhaust the steam at 1.25 atmospheres (19 lb.) into the steam heating system, or at 0.48 atmospheres (7 lb.) into a vacuum heating system.

At a fuel value of the coal of 30,000 kj. per kg. we have (see table)

From this it would follow that the average magnitude of the steam turbine induction generator plant for power collection from fuel in heating plants, would be about one-quarter to one-half kw. per ton of coal burned annually, under the assumption, that the use of the heating plant is equivalent to full capacity during one quarter of the time, and the turbine induction generator plant 50 per cent larger, to take care of maximum loads.

As seen, the value of the recovered power would be a substantial percentage of the fuel cost.

With 100 million tons of coal used for heating purposes annually, assuming an average recovery of 600 kw-hr. per ton, this gives a total of 60,000 million kw-hr. per year. One-quarter of this is more electric power than is now produced at Niagara, Chicago, New York and a few other of the biggest electric systems together.

DISCUSSION ON "AMERICA'S ENERGY SUPPLY" (STEINMETZ), ATLANTIC CITY, N. J., JUNE 27, 1918.

A. M. Schoen: In connection with the discussion of the question of our fuel supply and the need of additional power, it occurs to me to mention that about the year 1911, I think it was, I had the honor of representing the Institute at a hearing before the Committee on Agriculture from Congress on the subject of forest conservation. At that time the interests of the Institute lay in the conservation of the waterpower by protecting forests. There was a paper presented at that meeting by Mr. M. O. Leighton, engineer for the U. S. Geological Survey, and in that paper he went comprehensively into a survey he had made of the water powers of the Southern Appalachians. He called attention to the fact that by throwing dams across the ravines at the heads of a number of these streams the water could be impounded, and the low water stage of certain rivers could be materially raised.

The rainfall in the Southern Appalachians is very heavy, I think next to the Puget Sound region the heaviest we have in this country. Mr. Leighton's work was done with a view to improving rivers at that time navigable and rendering navigable others suffering from extreme low water. The same data, however, would indicate that through this method of protecting against freshet and drought, the low water stages could be so improved as to furnish sufficient water to change a considerable part of the power developed at some of the hydroelectric plants from secondary to primary, thus utilizing machinery at present idle during a considerable part of the year. With the present appeal from the Government to help save the coal, now would seem to be the accepted time to make use of this additional power. Mr. Leighton's paper was published in 1908 and will doubtless be found of interest in this connection.

B. A. Behrend: Dr. Steinmetz's interesting suggestion meets with one or the other of the horns of a dilemma which has confronted engineers in regard to the distribution of heat. If Mr. Steinmetz's idea is carried out literally as he wishes it to be, it is necessary to install in each house a boiler, a steam turbine, and an induction generator, so that the latent heat of the steam can be utilized for heating purposes. If this is not done, it is necessary to group together a number of houses in which the old difficulty of distributing heat has to be faced. Mr. Steinmetz does not wish us to consider the second alternative which has been carried out in several plants, particularly in Milwaukee in a station installed there some twelve years ago by Mr. John I. Beggs, for which the speaker designed the electric generating units.

Considering, therefore, the scheme as advocated by Dr. Steinmetz, we have to familiarize ourselves with the idea that dwellings will have to be equipped as indicated above, and

whether this is feasible, or whether it is possible to operate such plants without attendants of any kind is a question the consideration of which must be left to the individual judgment of those who have heard Mr. Steinmetz's paper.

In this connection, allow me to point out the suggestion made many years ago by Lord Kelvin to reverse the Carnot cycle and to return the heat thus gained into the building to be warmed. In making this proposal, Lord Kelvin stated that it would not be resorted to until a shortage of fuel was experienced. (See Mathematical and Physical Papers by Lord Kelvin, Vol. I, p. 516.)

S. Barfoed (communicated after adjournment): In large sections of the Pacific Coast one mentioned source of energy supply, namely oil, is being used in an ever increasing amount, in spite of the fact that water power is plentiful in the undeveloped stage.

This oil, which is being burnt for its heat value only is the country's legitimate lubricating stock. It is needed in an ever increasing amount for machinery of all descriptions and should not be used as a source of energy supply merely because it will burn.

The various factors contributing to this misuse are well known to engineers and need no longer be discussed by them.

How the energy sources may be utilized and coordinated to best advantage and with a minimum of waste from an engineering point of view seems almost paltry compared to the vexatious work necessary to bring about an understanding of the problem on the part of the general public and its chosen representatives on the various legislative and executive bodies.

The trained engineer thinks generally not along the lines of least resistance, but he will have all he can do to carry out but a small fraction of Dr. Steinmetz' simple and beautiful dream while our present generation is alive.

Ralph Bennett (communicated after adjournment): In the first section of his article Mr. Steinmetz discusses in a general way the total possible energy to be derived from the use of falling water and of the sun's rays.

The conclusion drawn is that, even with a workable automatic generating station the energy of the water must be supplemented by other sources.

But if it be granted that such stations are feasible and that the electrical network extends for distances so great that local climatical changes may be taken as more or less equalized then other sources not stated become available.

The total energy of the winds is almost incalculable. Efforts to utilize them have been successful only in a small way because the need is for local power at definite times while the wind is a variable source, so far as any small areas are concerned, and cannot be controlled except to reduce the energy delivered by any single power unit.

Two methods of overcoming this difficulty have been generally suggested. Both assume local isolated plants but one stores energy in electrical storage batteries while in the other scheme the energy is stored in water lifted into reservoirs. In both cases the cost of equipment and maintenance is so great that other sources of energy are more reliable and economical.

As simple units feeding into a far extending network wind engines might have a much greater application.

The average wind velocity over the United States is in the vicinity of 10 m. (16 km.) per hr. and 50-ft. (15.2 m.) mills will produce about 5 h. p. at this velocity.

The number of mills which can be used is limited only by the reduction in current velocity which would result from over close spacing. If only one per acre were permissible on the 1,920,000,000 acres of continental United States there would be developed a continuous output averaged over the country of over 8,700,000,000 h. p.

The actual practicable production from this source may really be very considerable in the not distant future for there are known zones of highly sustained wind velocities and commercial mills are already well developed in small sizes. Only a power market capable of paying the high interest charges on the investment is needed to make such a plant possible.

Two other intermittent sources of large amounts of energy are the waves and the tides. Both have been the source of endless experimentation by inventors. But few successful motors have developed. The main difficulty appears to be the extremely rugged construction required as compared with the average energy developed. But connected to networks fed from many and scattered sources the output here might, as in the case of windmills, vary with the power available and not with the local demand.

The coasts of the United States extend for over 21,300 mi. (34,279 km.). Along most of this length there is a tidal range of 5 ft. or better and a constant wave action.

Possibly there may be available over 5000 h. p. per mile of coast from the waves alone and neglecting any energy which might be developed off shore.

This would amount to 106,500,000 h. p. for the entire shore line.

Doubtless successful motors could be built if the market would warrant the expenditure.

Tidal motors are a more difficult problem since the only method considered at all possible is the use of tidal basins and only in rare instances can these be had.

Other sources of stored energy besides coal are in extensive use. At the present time the oil production of the United States equals the equivalent of one-eighth of the coal used.

Vast oil fields are as yet untouched. No complete exploration for new fields has been made.

Oil shales and bitumen charged rocks exist in enormous beds and from them as well as the exhausted sands of old oil fields further supplies of fuel can be drawn as the demand requires.

One half of all the timber cut is wasted, yet the fuel and by-product value of this "refuse" is greater than that of the lumber commercially used.

With the extension of the networks and a policy of free acceptance of offered power the use of these and numerous other sources of smaller or larger supplies of energy will become possible and the necessary use of the coal and high-grade oil reserves can be materially restricted without hardship.

C. P. Steinmetz: In discussing fuel consumption, I have included oil and natural gas, reduced to coal by their approximate fuel value. This is the reason why the tonnage of coal given in my paper is larger than that actually mined. It includes the coal equivalent of the other fuels.

Very interesting and of some importance, is the subject regarding the use of wind power, made by Mr. R. Bennett. At present, due to the great irregularity of the wind, the windmill is of use mainly for such power demands, where great irregularity is permissible, that is, pumping for drainage, etc. Storage of the energy by storage battery is not economical—unless in very exceptional cases—due to the high investment, low efficiency and rapid depreciation of the battery. The only way of utilizing wind power to a greater extent, thus would be by electric collection, feeding whatever power the wind gives into a general system together with hydraulic power, steam power, etc. The only economical way this could be done, appears to me, by the use of an induction generator attached to the wind-wheel. Such low-voltage induction generator would thus be located at the top of the tower and there would also be located the step-up transformer raising to the voltage of the collecting lines. This will probably be about 10,000 volts, that is, low enough not to increase the cost of the transformer and sufficiently high to carry the power for some distance. Even the collecting lines would not come down from the wind-mill tower but jump directly to the next tower, that is, the wind-wheel tower would be used as transmission line tower.

Such a system of collecting wind-power then would comprise a high-voltage trunk line—100,000 volts—main collecting lines issuing at right angles from the trunk line every few miles, at about 10,000 volts, and tied to the trunk line by step-up transformers, and branch collecting lines issuing from the main collecting lines every few hundred feet at right angles, thus about parallel to the trunk line. All these collecting lines would be tower lines and each tower contains a wind-wheel with low-voltage induction generator and a step-up transformer, connecting into the collecting line. Possibly some simple cutout device would also be required, connecting the wind-

wheel-induction generator-transformer into the collecting line by centrifugal force and disconnecting it by reverse-power relay.

I believe such an arrangement would be economically feasible, that is, the value of the collected wind-power would pay the interest on the investment and maintenance and a fair profit, even to-day in some localities, where fairly sustained wind velocities exist, power is valuable, and the cost of fuel high.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

BY V. KARAPETOFF

ABSTRACT OF PAPER

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

IN A paper presented before the Institution of Electrical Engineers (British) in February 1916, Mr. O. L. Record described Delon's apparatus for testing the insulation of a high-tension cable.[1] While endeavoring to make clear to himself the theory of this ingenious device the present writer has investigated in general the action of a pre-charged condenser used as a booster in series with some source of e.m.f. for charging another condenser. This led him further to deduce the equations of a general network of such condensers. Before giving these more general relationships it is of interest to explain the principle of Delon's apparatus in a simple numerical example.

The diagram of connections is shown in Fig. 1, in which C is a two-conductor cable under test, T is the testing transformer, R is a synchronous revolving rectifier arm, and C_1, C_2 are two "booster" condensers. The theory of the apparatus, confirmed by actual experience, shows that the cable is subjected to a continuous voltage which is numerically equal to twice the ampli-

1. Journal Inst. Elec. Engrs. Vol. 54 (1916), p. 610; abstracted in the *Electrical World*, Vol. 67 (1916), p. 665.

tude of the transformer voltage. Thus a cable can be readily tested at quite a high voltage, without having a source of considerable reactive kilovolt-amperes, and without subjecting the cable to dielectric hysteresis or dangerous oscillations.

Theory of Delon's Apparatus. Let the capacities of condensers C_1 and C_2 each be equal to one quarter of that of cable C, and let the rectifying arm occupy the position shown in Fig. 1 at the instant when the lower terminal of the transformer is positive, and the voltage is at a maximum. Let the instantaneous value of this maximum voltage be 100 kilovolts and let the two condensers and the cable not be previously charged at all. Then at the instant under consideration condenser C_1 is subjected to the full potential difference of 100 kv. between points A_1 and B, and is being "pre-charged" for the next half of the cycle. For the sake of simplicity we shall assume that the duration of the contact is sufficient to charge C_1 to 100 kv.

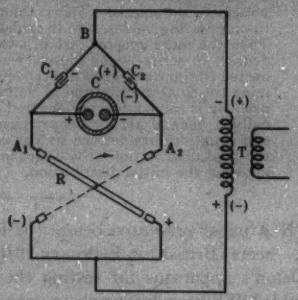

FIG. 1—DIAGRAM OF CONNECTIONS FOR TESTING TWO-CONDUCTOR CABLE

At the same instant the cable C is in series with condenser C_2 and the two together are subjected to 100 kv. Since the capacities C and C_2 are in the ratio of 4 to 1, the voltage across C is 20 kv., that across C_2 is 80 kv. It is important to note that the polarities marked at condensers C_1 and C_2 are those during the process of pre-charging. The polarity of C_2 during the first charge just described is opposite of that shown in the figure.

Half a cycle later the synchronous arm touches contact A_2 and the lower transformer terminal is negative. The corresponding polarities are shown in parentheses. The pre-charged condenser C_1 now serves as a booster in series with the transformer and helps to raise the charge on the cable, while condenser C_2 is being "pre-charged" between A_2 and B for the next impulse. We shall now determine the voltage to which the cable is charged after the second impulse.

Since the capacities C and C_1 are in the ratio of 4 to 1, any gain in voltage X across C corresponds to a loss of $4X$ across

C_1. In other words, if the new voltage across C is $(20 + X)$, that across C_1 is $(100 - 4X)$. But the total instantaneous voltage between A_2 and B is 100 kv., so that

$$(20 + X) - (100 - 4X) = 100,$$

from which $X = 36$ kv. Thus, after the second contact the cable is charged to 56 kv., while the original voltage across C_1 has dropped to -44 kv.

When the synchronous arm comes again in position A_1, the cable is connected in series with condenser C_2 which in the meanwhile has been charged to 100 kv. We thus have the equation

$$(56 + X) - (100 - 4X) = 100$$

from which $X = 28.8$ kv., so that the cable is now charged to 84.8 kv.

Carrying this reasoning further one readily finds that the voltage across the cable indefinitely approaches the value 200 kv. which is the double of the amplitude of the transformer voltage. Only after this value has been reached do the booster condensers and the transformer cease to charge the cable further. This is because the total voltage of the transformer plus that of a booster condenser is equal to 200 kv., and a permanent equilibrium is established. Let the cable voltage after the n – th charge be E_n. Then, by analogy with the foregoing expressions, we have for the $(n + 1)$ th application of voltage

$$(E_n + X) - (100 - 4X) = 100$$

or

$$X = (200 - E_n)/5 \tag{1}$$

so that

$$E_{n+1} = E_n + X = 40 + 0.8E_n \tag{2}$$

It will thus again be seen that the charging will stop or X will become zero when $E_n = 200$ kv.

Applying formula (2) to the first few charges (except the initial one) we get the following values:

$$20;\ 56;\ 84.8;\ 107.8;\ 126.3;\ 141;\ 152.8 \text{ kv.}$$

Here the first value, 20 kv., does not satisfy the left-hand side of equation (2) when on the right-hand side one puts $E_n = 0$. However, it must be remembered that when the voltage is applied for the first time the booster condenser is not pre-charged so that equation (1) does not hold true either. Both equations hold true for every impulse except the first one.

One could imagine n condensers in place of each booster condenser, and with a proper commutating arrangement these

condensers could be pre-charged in parallel and discharged in series. In this case the cable could be subjected to a voltage equal to $n + 1$ times that of the amplitude of the transformer voltage. The idea of charging condensers in parallel and discharging them in series is not new and has been used for other purposes.

Two Condensers in Series. Let us now consider a more general case (Fig. 2) of a condenser of capacity C pre-charged to a voltage E_0 and of a booster condenser C' pre-charged to a voltage E_0'. Let the corresponding charges or displacements of electricity in the condensers be Q_0 and Q_0'. The operations will consist in charging C to a higher voltage by means of pre-charged C' in series with a battery e, then pre-charging C' again, raising the voltage of C, and so forth. The problem is to determine the consecutive values of the voltage across C. These values will be denoted by E_0, E_1, E_2, $\ldots E_n$, while the corresponding pre-charge voltages of the booster condenser will be called E_0', E_1', $E_2' \ldots E_n'$.

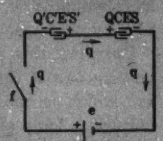

FIG. 2—TWO CONDENSERS IN SERIES

We have the fundamental relations

$$Q_0 = C\,E_0 \tag{3}$$

$$Q_0' = C'\,E_0' \tag{4}$$

Since the two condensers are connected in series, it is more convenient to use the reciprocal of capacity namely the so-called *elastance* of a condenser.[2] Introducing the elastances

$$\left.\begin{aligned} S &= 1/C \\ S' &= 1/C \end{aligned}\right\} \tag{5}$$

we get

$$\left.\begin{aligned} E_0 &= Q_0\,S \\ E_0' &= Q_0'\,S' \end{aligned}\right\} \tag{6}$$

Let now condenser C' be connected in series with a constant-voltage battery e, and the switch f closed. With the polarities shown in the sketch the condenser C' assists the battery in further charging condenser C, and a quantity of electricity q

2. When a capacity is expressed in microfarads, the corresponding elastance is measured in mega-darafs. See the author's "Electric Circuit" (McGraw-Hill, 1912), p. 148; also his paper "Sur quelques calculs pratiques des champs électrostatiques," *Trans. Congresso Internazionale delle Applicazioni Elettriche*, Torino, 1911.

is displaced through the circuit. The charge on C is increased to $(Q + q)$, that on C' is reduced to $(Q' - q)$. The corresponding voltages now are

$$E_1 = (Q_0 + q)\, S \tag{7}$$

$$E_1' = (Q_0' - q)\, S' \tag{8}$$

An equilibrium is established when

$$e + E_1' = E_1 \tag{9}$$

Substituting in this equation the values from equations (6), (7) and (8) we get

$$e + E_0' - q\, S' = E_0 + q\, S,$$

or

$$e + E_0' - E_0 = q\,(S + S') \tag{10}$$

so that

$$q = (e + E_0' - E_0)/(S + S') \tag{11}$$

All the quantities on the right hand side of this expression are known and q may be computed. Substituting this value of q in equation (7) we get

$$E_1 = E_0 + (e + E_0' - E_0)[S/(S + S')] \tag{12}$$

This expression permits one to determine the new voltage of condenser C knowing the initial voltages and the elastances of the condensers.

The larger the booster condenser the more it helps in charging the other condenser, but on the other hand the more expensive it is. Since the result depends only upon the ratio of the elastances we shall denote

$$S'/(S + S') = \alpha \tag{13}$$

$$S/(S + S') = 1 - \alpha \tag{14}$$

In the most efficient and most expensive case of an infinitely large auxiliary condenser ($S' = 0$) we have $\alpha = 0$. In the other extreme, when it is impossible to charge C because $C' = 0$ or $S' = \infty$, the ratio is $\alpha = 1$. In all cases α is a positive regular fraction, and may be called "slack factor." Equation (12) becomes

$$E_1 = (1 - \alpha)\,(e + E_0') + \alpha\, E_0 \tag{15}$$

or

$$E_1 = (1 - \alpha)\, V_1 + \alpha\, E_0 \tag{16}$$

Here

$$V_1 = e + E_0' \tag{17}$$

is the total charging voltage applied to condenser C at the instant of closing switch f.

Let now the switch be opened, the condenser C' pre-charged

to some other voltage E_1' and the switch closed again. By analogy with equation (**16**), the new voltage to which condenser C will be charged is

$$E_2 = (1-\alpha)\ V_2 + \alpha E_1,$$

where $V_2 = e + E_1'$ is the new total charging voltage which in a general case may be different from V_1. If this charging process be repeated n times we get

$$\left.\begin{aligned}
&E_1 = (1-\alpha)\ V_1 + \alpha E_0 \\
&E_2 = (1-\alpha)\ V_2 + \alpha E_1 = (1-\alpha)\ (V_2 + \alpha\ V_1) + \alpha^2 E_0 \\
&E_3 = (1-\alpha)\ V_3 + \alpha E_2 = (1-\alpha)\ (V_3 + \alpha\ V_2 + \alpha^2\ V_1) \\
&\qquad\qquad + \alpha^3 E_0 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&E_n = (1-\alpha)\ (V_n + \alpha\ V_{n-1} + \alpha^2\ V_{n-2} + \ldots + \alpha^{n-1}\ V_1) \\
&\qquad\qquad + \alpha^n E_0
\end{aligned}\right\} \quad (18)$$

No matter how many times the charging process be repeated the voltage across C remains finite because it cannot possibly exceed the highest charging voltage V applied to it.

In practise, the condenser C' would usually be pre-charged every time to the same constant value, say E', so that

$$V_1 = V_2 = \ldots = V_n = V = e + E' \qquad (19)$$

The preceding expression is then simplified to

$$E_n = (1-\alpha)\ (e + E')\ (1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1}) + \alpha^n E_0$$

or

$$E_n = (1-\alpha^n)\ V + \alpha^n E_0 \qquad (20)$$

Compare this expression with equation (**16**). After an infinite number of applications of voltage, that is $n = \infty$,

$$\alpha^n = 0, \text{ and}$$

$$E_\infty = e + E' \qquad (21)$$

independent of α or E_0. However, the values of α and E_0 determine the law according to which the voltage E approaches its ultimate value. With a small slack factor, α, that is with an expensive auxiliary condenser, the final value is practically reached after a few charges, while with an α near unity it may require many thousands of applications.

In the simplest case the same battery e would be used for pre-charging condenser C', so that $E' = e$. Equations (**20**) and (**21**) become

$$E_n = 2\,e\ (1-\alpha^n) + \alpha^n E_0 \qquad (22)$$

$$E_\infty = 2\,e \qquad (23)$$

These two relationships apply in Delon apparatus.

Several Condensers in Series. We shall now consider a still more general case of several pre-charged condensers in series. Let the initial voltage, charge, and elastance of the $k-$th condenser be E_{k0}, Q_k and S_k respectively, so that originally

$$E_{k0} = Q_k S_k \tag{24}$$

Let all these condensers be connected in series and also in series with a source of constant voltage e, say a battery, and the switch closed. It is required to find the new voltages across the individual condensers.

We shall consider as positive those voltages E_{k0} which "buck" or oppose the battery voltage, while the voltages which help the battery will be considered negative. Let q be the displacement of electricity or the charge which passes through the circuit when the switch is closed. The new charge in the $k-$th condenser is $Q_k + q$, and the new voltage

$$E_{k1} = (Q_k + q) S_k = E_{k0} + q S_k \tag{25}$$

But when the equilibrium is established

$$\Sigma E_{k1} = e \tag{26}$$

or

$$\Sigma E_{k0} + q \Sigma S_k = e \tag{27}$$

Hence, the additional charge

$$q = (e - \Sigma E_{k0})/\Sigma S_k \tag{28}$$

But $e - \Sigma E_{k0}$ is the *net* voltage before the switch is closed, and ΣS_k is the *equivalent* elastance of the whole circuit. We denote

$$e - \Sigma E_{k0} = e_n \tag{29}$$

$$\Sigma S_k = S_{eq} \tag{30}$$

The preceding expression for q becomes

$$q = e_n/S_{eq} \tag{31}$$

and the new voltage across the $k-$th condenser

$$E_{k1} = E_{k0} + e_n(S_k/S_{eq}.) \tag{32}$$

Condensers in Parallel. The case of several pre-charged condensers in parallel allows of a very simple solution. Let condensers of capacity $C_1, C_2 \ldots . C_k \ldots . C_n$ be pre-charged to voltages $E_1, E_2 \ldots . E_k \ldots . E_n$ respectively and then put in parallel. Let it be required to find the common voltage E after the equalization of the charges. The $k-$th condenser has lost voltage $E_k - E$, which means that it has lost a charge equal to $(E_k - E)$ C_k. But in the absence of a source of e.m.f. the sum of the

charges after the equalization is the same as before, so that the sum of the changes of charge must be zero. We thus have the equation

$$\Sigma\,(E_k - E)\,C_k = 0 \tag{33}$$

or

$$\Sigma\,E_k\,C_k - E\,\Sigma\,C_k = 0$$

from which

$$E = \Sigma\,E_k\,C_k/\Sigma\,C_k \tag{34}$$

When using this equation one has to be careful regarding the sign of each individual E_k. If all the positive terminals are connected together and all the negative ones together, then all the E_k's should enter in the equation with the same sign, say plus. But if some positive and other negative terminals are connected together, one should select a positive direction of e.m.f. and enter the individual E_k's in equation (34) with the sign plus or minus accordingly. As a practical application of pre-charged condensers in parallel the "method of mixtures" may be mentioned for measuring small capacities.[3]

Network of Condensers. As a most general case treated in this article, let a number of pre-charged condensers be connected in any arbitrary manner so as to form a network similar to a network of conductors. Let the initial voltage of the $k-$ th condenser be E_{k0}, the corresponding charge Q_{k0} and the elastance S_k, so that

$$E_{k0} = Q_{k0}\,S_k \tag{35}$$

Let various sources of d-c. voltage, e_1, e_2, e_3, etc. be distributed throughout the network. In the beginning a sufficient number of switches are supposed to be opened to prevent an equalization of charges and voltages. Then all these switches are closed, and it is required to find the new voltages and charges on the condensers.

The problem is solved in a manner similar to that in which currents are found in a complicated network of conductors, namely by applying equations analogous to Kirchoff's laws. As soon as the switches have been closed, the distribution of charges on the condensers changes, the $k-$ th condenser receiving an additional charge say q_k. Let q_k be considered positive when a positive displacement moves away from a junction point of three or more condensers. Since electricity behaves like an

3. See for example V. Karapetoff, Experimental Electrical Engineering, (Wiley) Vol. II, p. 9.

incompressible fluid, we have a relationship similar to the first Kirchoff law, namely

$$\Sigma\, q_k = 0 \tag{36}$$

An independent equation of this kind may be written for each junction point of the network but one.

For each dielectric circuit that can be traced within the network the second Kirchoff law may be applied which in this case simply becomes

$$\Sigma\, e = \Sigma\, E_{k1} = \Sigma\, (Q_{k0} + q_k)\, S_k.$$

But originally we had $Q_{k0}\, S_k = E_{k0}$, so that the preceding expression becomes

$$\Sigma\, e = \Sigma\, E_{k0} + \Sigma\, q_k\, S_k \tag{37}$$

A sufficient number of equations of this form can be written down, together with equations of the kind (**36**), to enable one to solve them as simultaneous equations for the unknown quantities q_k. Knowing a q_k, the corresponding new voltage across the condenser may be computed from the relationship

$$E_{k1} = (Q_{k0} + q_k)\, S_k = E_{k0} + q_k\, S_k \tag{38}$$

In the beginning the directions of q_k may be assumed arbitrarily; then those which actually take place in the opposite direction will come out negative, from the solution of equations (**36**) and (**37**).

DISCUSSION ON "PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL" (KARAPETOFF), ATLANTIC CITY, N. J., JUNE 28, 1918.

C. O. Mailloux: I wish to say I saw the apparatus perform at the Electrical Exposition at Versailles in 1908, and, of course, it was at that time one of the most interesting features of the exposition.

L. W. Chubb: Prof. Karapetoff's problem becomes more interesting when resistance load instead of static charges are considered. It is perfectly feasible, instead of using condensers in this apparatus, to use batteries or direct-current generators, in which case the fluctuation of energy can be made very great, and power can actually be utilized at a potential higher than the potential of the source. We have used in certain cases a limited power from transformers giving approximately half of the voltage required, and in the case of one or two hundred kv. it is a great advantage to have a transformer, say, at 100 kv. for a load at 175 kv. When these loads are applied the equations for the apparatus become very much more complicated, because while the connections with the pre-charged condensers are being made, there is, of course, a leakage dependent upon the nature of the load and the time that it is on.

W. V. Lyon (Read by B. A. Behrend): The solution for the distribution of electricity among a network of condensers is, as Prof. Karapetoff points out, readily obtained by applying principles similar to those commonly known as "Kirchoff's Laws". The writer has long used problems involving such networks to emphasize the generality of these principles.*

Another point of view should be both interesting and instructive. The current supplied to a single conductor or to any network of resistances, in which there are no electromotive forces acting, will so distribute itself that the total joule heating loss is a minimum. Mathematically this reduces to the statement that the sum of the resistance drops around a closed circuit is zero.

When a number of condensers that have been pre-charged are reconnected in any form of network and allowed to come to an unrestrained equilibrium†, the charges will so redistribute themselves that the total stored energy is a minimum. Mathematically this is also equivalent to the statement that the voltage drop around any closed circuit is zero. If this principle is combined with a second, namely, the algebraic sum of the charges at any insulated junction is constant, a solution for the redistributed charge is at once obtained. If an electromotive force is *inserted in* any branch of this stabilized network there will be produced an *additional charge* on each condenser equal to that which would have been produced had the con-

*Problems in Electrical Engineering (pp. 96, 97, 98) McGraw-Hill Book Co., 1908.

†No electromotive forces acting in the network.

densers originally been discharged. The resultant effect of more than one such electromotive force inserted in the network is determined by calculating the effect of each electromotive force as if it acted alone, all of the others being short-circuited. The net additional charge on any condenser is equal to the algebraic sum of the charges due to the component electromotive forces. The *total* charge on any condenser is the algebraic sum of the stabilized charge and the net additional charge. The stabilized charge should always be calculated for the condition that all of the electromotive forces are short-circuited.

As a means of solution there is little to commend this method. As a method of analysis, however, it emphasizes two important principles. These are the principles concerning the dissipation of energy in a system, and the combination of component effects in producing a resultant condition.

V. Karapetoff: Perhaps one of the reasons why Delon apparatus has not been used in this country is that we have now the possibility of obtaining high voltages of a unidirectional character by means of hot cathode valves, and I hope that a method will be developed, if it has not been already developed, using these hot cathode valves, for producing high unidirectional voltage for testing cables.

Regarding my nomenclature, I do not like the word daraf, but I wanted to create a disturbance among our rigorists in order to induce the Committee on Nomenclature to take some action. To make the situation particularly objectionable, I also spelled the word henry backward, making it yrneh, which word is used in my "Magnetic Circuit". It is not for an individual to coin a new word, and I felt that I was more consistent in following the established practise (ohm, mho) thanin inventing a new word. I will be the first one to adopt a new word for reluctance, the reciprocal of henry, and for the daraf, the reciprocal for farad, as soon as an international action has been taken.

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METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

BY C. L. FORTESCUE

Abstract of Paper

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical co-ordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Introduction

IN THE latter part of 1913 the writer had occasion to investigate mathematically the operation of induction motors under unbalanced conditions. The work was first carried out, having particularly in mind the determination of the operating characteristics of phase converters which may be considered as a particular case of unbalanced motor operation, but the scope of the subject broadened out very quickly and the writer undertook this paper in the belief that the subject would be of interest to many.

The most striking thing about the results obtained was their symmetry; the solution always reduced to the sum of two or more symmetrical solutions. The writer was then led to inquire if there were no general principles by which the solution of unbalanced polyphase systems could be reduced to the solu-

tion of two or more balanced cases. The present paper is an endeavor to present a general method of solving polyphase network which has peculiar advantages when applied to the type of polyphase networks which include rotating machines.

In physical investigations success depends often on a happy choice of co-ordinates. An electrical network being a dynamic system should also be aided by the selection of a suitable system of co-ordinates. The co-ordinates of a system are quantities which when given, completely define the system. Thus a system of three co-planar concurrent vectors are defined when their magnitude and their angular position with respect to some fixed direction are given. Such a system may be said to have six degrees of freedom, for each vector may vary in magnitude and phase position without regard to the others. If, however, we impose the condition that the vector sum of these vectors shall be zero, we find that with the direction of one vector given, the other two vectors are completely defined when their magnitude alone is given, the system has therefore lost two degrees of freedom by imposing the above condition which in dynamical theory is termed a "constraint". If we impose a further condition that the vectors be symmetrically disposed about their common origin this system will now have but two degrees of freedom.

It is evident from the above definition that a system of n coplanar concurrent vectors may have $2n$ degrees of freedom and that a system of n symmetrically spaced vectors of equal magnitude has but two degrees of freedom. It should be possible then by a simple transformation to define the system of n arbitrary congruent vectors by n other systems of concurrent vectors which are symmetrical and have a common point. The n symmetrical systems so obtained are the symmetrical co-ordinates of the given system of vectors and completely define it.

This method of representing polyphase systems has been employed in the past to a limited extent, but up to the present time there has been as far as the author is aware no systematic presentation of the method. The writer hopes by this paper to interest others in the application of the method, which will be found to be a valuable instrument for the solution of certain classes of polyphase networks.

In dealing with alternating currents in this paper, use is made of the complex variable which in its most general form

may be represented as a vector of variable length rotating about a given point at variable angular velocity or better as the resultant of a number of vectors each of constant length rotating at different angular velocities in the same direction about a given point. This vector is represented in the text by $\check{I}$, $\check{E}$, etc., and the conjugate vector which rotates at the same speed in the opposite direction is represented by $\hat{I}$, $\hat{E}$, etc. The effective value of the vector is represented by the symbol without the distinguishing mark as I, E, etc. The impedances Z_a, Z_b, Z_{ab}, etc., are generalized expressions for the self and mutual impedances. For a circuit A the self-impedance operator will be denoted by Z_{aa} or Z_a. In the case of two circuits A and B the self impedance operators would be Z_{aa} Z_{bb} and the mutual impedance operator Z_{ab}. The subletters denote the circuits to which the operators apply. These operators are generally functions of the operator, $D = \frac{d}{dt}$ and the characteristics of the circuit; these characteristics are constants only when there is no physical motion. It will therefore be necessary to carefully distinguish between $Z_a \check{I}_a$ and $\check{I}_a Z_a$ when Z_a has the form of a differential operator. In the first case a differential operation is carried out on the time variable $\check{I}_a$ in the second case the differential operator is merely multiplied by $\check{I}_a$.

The most general expression for a simple harmonic quantity e is

$$e = A \cos pt - B \sin pt$$

in exponential form this becomes

$$e = \frac{A + jB}{2} \epsilon^{jpt} + \frac{A - jB}{2} \epsilon^{-jpt}$$

$(A + jB)\,\epsilon^{jpt}$ represents a vector of length $\sqrt{A^2 + B^2}$ rotating in the positive direction with angular velocity p while $(A - jB)\,\epsilon^{-jpt}$ is the conjugate vector rotating at the same angular velocity in the opposite direction. Since ϵ^{jpt} is equal to $\cos pt + j \sin pt$, the positively rotating vector $\check{E} = (A + jB)\,\epsilon^{jpt}$ will be

$$\check{E} = A \cos pt - B \sin pt + j\,(A \sin pt + B \cos pt)$$

or the real part of $\check{E}$ which is its projection on a given axis is equal to e and therefore $\check{E}$ may be taken to represent e in phase and magnitude. It should be noted that the conjugate vector $\hat{E}$ is equally available, but it is not so convenient since the

operation $\frac{d}{dt}\epsilon^{-jpt}$ gives $-jp\epsilon^{-jpt}$ and the imaginary part of the impedance operator will have a negative sign.

The complex roots of unity will be referred to from time to time in the paper. Thus the complete solution of the equation $x^n - 1 = 0$ requires n different values of x, only one of which is real when n is an odd integer. To obtain the other roots we have the relation

$$\begin{aligned} 1 &= \cos 2\pi r + j \sin 2\pi r \\ &= \epsilon^{j2\pi r} \end{aligned}$$

Where r is any integer. We have therefore

$$1^{\frac{1}{n}} = \epsilon^{j\frac{2\pi r}{n}}$$

and by giving successive integral values to r from 1 to n, all the n roots of $x^n - 1 = 0$ are obtained namely,

$$a_1 = \epsilon^{j\frac{2\pi}{n}} = \cos\frac{2\pi}{n} + j\sin\frac{2\pi}{n}$$

$$a_2 = \epsilon^{j\frac{4\pi}{n}} = \cos\frac{4\pi}{n} + j\sin\frac{4\pi}{n}$$

$$a_3 = \epsilon^{j\frac{6\pi}{n}} = \cos\frac{6\pi}{n} + j\sin\frac{6\pi}{n}$$

$$a_n = \epsilon^{j2\pi} = 1$$

It will be observed that $a_2\, a_3 \ldots a_n$ are respectively equal to $a_1^2\, a_1^3 \ldots a_1^{(n-1)}$.

When there is relative motion between the different parts of a circuit as for example in rotating machinery, the mutual inductances enter into the equation as time variables and when the motion is angular the quantities ϵ^{jwt} and ϵ^{-jwt} will appear in the operators. In this case we do not reject the portion of the operator having ϵ^{-jwt} as a factor, because the equations require that each vector shall be operated on by the operator as a whole which when it takes the form of a harmonic time function will contain terms with ϵ^{jwt} and ϵ^{-jwt} in conjugate relation. In some cases as a result of this, solutions will appear with indices of ϵ which are negative time variables; in such cases in the final statement the vectors with negative index should be replaced by their conjugates which rotate in the positive direction.

This paper is subdivided as follows:

Part I.—"The Method of Symmetrical Co-ordinates." Deals with the theory of the method, and its application to simple polyphase circuits.

Part II.—Application to Symmetrical Machines on Unbalanced Polyphase Circuits. Takes up Induction Motors, Generator and Synchronous Motor, Phase Balancers and Phase Convertors.

Part III. Application to Machines having Unsymmetrical Windings.

In the Appendix the mathematical representation of field forms and the derivation of the constants of different forms of networks is taken up.

The portions of Part I dealing with unsymmetrical windings are not required for the applications taken up in Part II and may be deferred to a later reading. The greater part of Part I is taken up in deriving formulas for special cases from the general formulas (**30**) and (**33**), and the reading of the text following these equations may be confined to the special cases of immediate interest.

I wish to express my appreciation of the valuable help and suggestions that have been given me in the preparation of this paper by Prof. Karapetoff who suggested that the subject be presented in a mathematical paper and by Dr. J. Slepian to whom I am indebted for the idea of sequence operators and by others who have been interested in the paper.

PART I

Method of Symmetrical Generalized Co-ordinates

Resolution of Unbalanced Systems of Vectors and Operators

The complex time function $\check{E}$ may be used instead of the harmonic time function e in any equation algebraic or differential in which it appears linearly. The reason of this is because if any linear operation is performed on $\check{E}$ the same operation performed on its conjugate $\hat{E}$ will give a result which is conjugate to that obtained from $\check{E}$, and the sum of the two results obtained is a solution of the same operation performed on $\check{E} + \hat{E}$, or $2\,e$.

It is customary to interpret $\check{E}$ and $\hat{E}$ as coplanar vectors, rotating about a common point and e as the projection of either vector on a given line, $\check{E}$ being a positively rotating vector and

$\hat{E}$ being a negatively rotating vector, and their projection on the given line being

$$e = \frac{\check{E} + \hat{E}}{2} \tag{1}$$

Obviously if this interpretation is accepted one of the two vectors becomes superfluous and the positively rotating vector $\check{E}$ may be taken to represent the variable "e" and we may define "e" by saying that "e" is the projection of the vector $\check{E}$ on a given line or else by saying that "e" is the real part of the complex variable $\check{E}$.

If (1), a, $a^2 \ldots a^{n-1}$ are the n roots of the equation $x^n - 1 = 0$ a symmetrical polyphase system of n phases may be represented by

$$\left.\begin{aligned} \check{E}_{11} &= \check{E}_{11} \\ \check{E}_{21} &= a\,\check{E}_{11} \\ \check{E}_{31} &= a^2\,\check{E}_{11} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n1} &= a^{n-1}\,\check{E}_{11} \end{aligned}\right\} \tag{2}$$

Another n phase system may be obtained by taking

$$\left.\begin{aligned} \check{E}_{12} &= \check{E}_{12} \\ \check{E}_{22} &= a^2\,\check{E}_{12} \\ \check{E}_{32} &= a^4\,\check{E}_{12} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n2} &= a^{2(n-1)}\check{E}_{12} \end{aligned}\right\} \tag{3}$$

and this also is symmetrical, although it is entirely different from (2).

Since $1 + a + a^2 + a^{n-1} = 0$, the sum of all the vectors of a symmetrical polyphase system is zero.

If $\check{E}_1\ \check{E}_2\ \check{E}_3 \ldots \check{E}_n$ be a system of n vectors, the following identities may be proved by inspection:

$$
\left.
\begin{aligned}
\check{E}_1 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots \check{E}_n}{n} \\
& + \frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots a^{n-1}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\check{E}_3 + \ldots a^{2(n-1)}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + \ldots a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + \ldots \frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots a^{-(n-1)}\,\check{E}_n}{n} \\
\check{E}_2 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots \check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_3 + \ldots a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots a^{-(n-1)}\,\check{E}_n}{n} \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\check{E}_n \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots \check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2(n-1)}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_4 + \ldots a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + \ldots a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots a^{-(n-1)}\,\check{E}_n}{n}
\end{aligned}
\right\} \quad (4)
$$

It will be noted that in the expression for $\check{E}_1$ in the above formulas if the first term of each component is taken the result is $n\,\frac{\check{E}_1}{n}$ or $\check{E}_1^*$. If the succeeding terms of each component involving

$\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, are taken separately they add up to expressions of the form $\frac{\check{E}_r}{n}\,(1+a+a^2+\ldots a^{n-1})$ which are all equal to zero since $(1+a+a^2+\ldots a^{n-1})$ is equal to zero. In like manner in the expression for $\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, all the terms of the components involving each of the quantities $\check{E}_1\,\check{E}_2\,\check{E}_3 \ldots$ etc. excepting the terms involving that one of which the components are to be determined add up to expressions of the form $\frac{\check{E}_r}{n}$ $(1+a+a^2+\ldots a^{n-1})$ all of which are equal to zero, the remaining terms add up to $\check{E}_2\,\check{E}_3 \ldots E_n$ respectively. It will now be apparent that (4), is true whatever may be the nature of $\check{E}_1\,\check{E}_2$ etc., and therefore it is true of all numbers, real complex or imaginary, whatever they may represent and therefore similar relations may be obtained for current vectors and they may be extended to include not only vectors but also the operators.

In order to simplify the expressions which become unwieldy when applied to the general n-phase system, let us consider a three-phase system of vectors $\check{E}_a\,\check{E}_b\,\check{E}_c$. Then we have the following identities:

$$\left.\begin{aligned}
\check{E}_a &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + \frac{E_a+a\,\check{E}_b+a^2\,E_c}{3} + \frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3}\\
\check{E}_b &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a^2\,\frac{\check{E}_a+a\,\check{E}_b+a^2\,E_c}{3} + a\,\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3}\\
\check{E}_c &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a\,\frac{\check{E}_a+a\,\check{E}_b+a^2\,\check{E}_c}{3} + a^2\,\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3}
\end{aligned}\right\}\quad(5)$$

(4) states the law that a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining $(n-1)$ systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms

an equal number of symmetrical n-phase systems. When n is not prime some of the n-phase systems degenerate into repetitions of systems having numbers of phases corresponding to the factors of n.

Equation (5) states that any three vectors $\check{E}_a$ $\check{E}_b$ $\check{E}_c$ may be resolved into a system of three equal vectors $\check{E}_{a0}$ $\check{E}_{a0}$ $\check{E}_{a0}$ and two symmetrical three-phase systems $\check{E}_{a1}$, $a^2\,\check{E}_{a1}$, $a\,\check{E}_{a1}$, and $\check{E}_{a2}$,

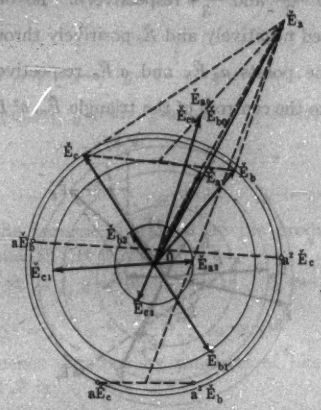

FIG. 1—GRAPHICAL REPRESENTATION OF EQUATION 5.

$a\,\check{E}_{a2}$, $a^2\,\check{E}_{a2}$, the first of which is of positive phase sequence and the second of negative phase sequence, or

$$\left.\begin{aligned}\check{E}_a &= \check{E}_{a0} + \check{E}_{a1} + \check{E}_{a2}\\ \check{E}_b &= \check{E}_{a0} + a^2\,\check{E}_{a1} + a\,\check{E}_{a2}\\ \check{E}_c &= \check{E}_{a0} + a\,\check{E}_{a1} + a^2\,\check{E}_{a2}\end{aligned}\right\} \qquad (6)$$

Similarly

$$\left.\begin{aligned}\check{I}_a &= \check{I}_{a0} + \check{I}_{a1} + \check{I}_{a2}\\ \check{I}_b &= \check{I}_{a0} + a_2\,\check{I}_{a1} + a\,\check{I}_{a2}\\ \check{I}_c &= \check{I}_{a0} + a\,\check{I}_{a1} + a^2\,\check{I}_{a2}\end{aligned}\right\} \qquad (7)$$

Figs. (1) and (2) show a graphical method of resolving three vectors $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ into their symmetrical three-phase components corresponding to equations (5). The construction is as follows:—$\check{E}_{a0}$ $\check{E}_{b0}$, $\check{E}_{c0}$ are obtained by drawing a line from 0 to

the centroid of the triangle $\check{E}_a\,\check{E}_b\,\check{E}_c$. $\check{E}_{a1}\,\check{E}_{b1}$, $\check{E}_{c1}$ are obtained by rotating $\check{E}_b$ positively through an angle $\frac{2\pi}{3}$, and $\check{E}_c$ negatively through the same angle giving the points $a\,\check{E}_b$ and $a^2\,\check{E}_c$ respectively. $\check{E}_{a1}$ is the vector obtained by a line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a\,\check{E}_b$, $a^2\,\check{E}_c$; and $\check{E}_{b1}$ and $\check{E}_{c1}$ *lag* this vector by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively. To obtain $\check{E}_{a2}\,\check{E}_{b2}$ $\check{E}_{c2}$, $\check{E}_b$ is rotated negatively and $\check{E}_c$ positively through the angle $\frac{2\pi}{3}$ giving the points $a_2\,\check{E}_b$ and $a\,\check{E}_c$ respectively; the line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a^2\,\check{E}_b$, $a\,\check{E}_c$ is the

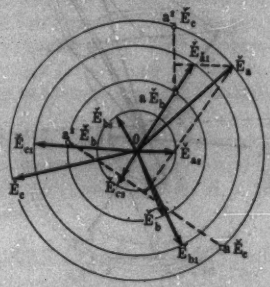

FIG. 2—GRAPHICAL REPRESENTATION OF EQUATION 5.

vector $\check{E}_{a2}$, $\check{E}_{b2}$ and $\check{E}_{c2}$ *lead* this vector by the angles $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively.

The system of operators $Z_{aa}\,Z_{bb}\,Z_{cc}\,Z_{ab}\,Z_{bc}\,Z_{ca}$ may be resolved in a similar manner into symmetrical groups,

$$\left.\begin{aligned} Z_{aa} &= Z_{aa0} + Z_{aa1} + Z_{aa2} \\ Z_{bb} &= Z_{aa0} + a^2\,Z_{aa1} + a\,Z_{aa2} \\ Z_{cc} &= Z_{aa0} + a\,Z_{aa1} + a^2\,Z_{aa2} \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} Z_{ab} &= Z_{ab0} + Z_{ab1} + Z_{ab2} \\ Z_{bc} &= Z_{ab0} + a^2\,Z_{ab1} + a\,Z_{ab2} \\ Z_{ca} &= Z_{ab0} + a\,Z_{ab1} + a^2\,Z_{ab2} \end{aligned}\right\} \tag{9}$$

There are similar relations for n-phase systems.

EXPLANATION OF THEORY AND USE OF SEQUENCE OPERATOR

Let us define the symmetrical sequences of nth roots of unity in the following manner:

$$\left.\begin{array}{l} S^0 = 1,\ \ 1,\ \ 1 \ldots 1 \\ S^1 = 1,\ \ a^{-1},\ \ a^{-2} \ldots a^{-(n-1)} \\ S^2 = 1,\ \ a^{-2},\ \ a^{-4} \ldots a^{-2(n-1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ S^r = 1,\ \ a^{-r},\ \ a^{-2r} \ldots a^{-(n-1)r} \\ S^{(r+1)} = 1,\ a^{-(r+1)},\ \ a^{-2(r+1)} \ldots a^{-(n-1)(r+1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ S^{(n-1)} = 1,\ a^{-(n-1)},\ \ a^{-2(n-1)} \ldots a^{-(n-1)^2} \end{array}\right\} \tag{10}$$

Consider the sequence obtained by the products of similar terms of S^r and S^1. It will be

$$S^{(r+1)} = 1,\ a^{-(r+1)},\ \ a^{-2(r+1)} \ldots a^{-(n-1)(r+1)} \tag{11}$$

Similarly

$$S^k = 1,\ \ a^{-k},\ \ a^{-2k} \ldots a^{-(n-1)k} \tag{12}$$

and the sequence obtained by products of like terms of this sequence and S^r is

$$S^{(r+k)} = 1,\ a^{-(r+k)},\ \ a^{-2(r+k)} \ldots a^{-(n-1)(r+k)} \tag{13}$$

We may therefore apply the law of indices to the products of sequences to obtain the resulting sequence.

In the case of the three-phase system we shall have the following sequences only to consider, viz.:

$$\left.\begin{array}{l} S^0 = 1,\ \ 1,\ \ 1 \\ S^1 = 1,\ \ a^2,\ \ a \\ S^2 = 1,\ \ a,\ \ a^2 \end{array}\right\} \tag{14}$$

The complete system of currents $\bar{I}_a\ \bar{I}_b\ \bar{I}_c$ are defined by

$$S\,(\bar{I}_a) = S^0\,\bar{I}_{a0} + S^1\,\bar{I}_{a1} + S^2\,\bar{I}_{a2} \tag{15}$$

Similarly the impedances $Z_{aa}\ Z_{bb}\ Z_{cc}$ may be expressed in symmetrical form

$$S\,(Z_{aa}) \equiv S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2} \tag{16}$$

and the mutual impedances Z_{ab}, Z_{bc}, Z_{ca} are expressed by

$$S\,(Z_{ab}) \equiv S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2} \tag{17}$$

Attention is called to the importance of preserving the cyclic order of self and mutual impedances, otherwise the rule for the sequence operator will not hold. Thus, Z_{ab}, Z_{bc} and Z_{ca} are in proper sequence as also are Z_{ca}, Z_{ab}, Z_{bc}.

When it is desired to change the first term in the sequence of polyphase vectors the resulting expression will be

$$\left.\begin{aligned} S(\check{I}_b) &= S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_{a1} + S^2\,a\,\check{I}_{a2} \\ S(\check{I}_c) &= S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2} \end{aligned}\right\} \tag{18}$$

Similarly in the case of the operators $S(Z_{ab})$ we have

$$\left.\begin{aligned} S(Z_{bc}) &= S^0\,Z_{ab0} + S^1\,a^2\,Z_{ab1} + S^2\,a\,Z_{ab2} \\ S(Z_{ca}) &= S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2} \end{aligned}\right\} \tag{19}$$

Similar rules apply to the e.m.fs. E_a E_b E_c

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2} \\ S(\check{E}_b) &= S^0\,\check{E}_{a0} + S^1\,a^2\,\check{E}_{a1} + S^2\,a\,\check{E}_{a2} \\ S(\check{E}_c) &= S^0\,\check{E}_{a0} + S^1\,a\,E_{a1} + S^2\,a^2\,\check{E}_{a2} \end{aligned}\right\} \tag{20}$$

It should be kept in mind that any one of the several expressions $S(\check{I}_a)$ $S(\check{I}_b)$ $S(\check{I}_c)$, etc., completely specifies the system, and each of the members of the groups of equations given above is a complete statement of the system of vectors or operators and their relation.

Application to Self and Mutual Impedance Operations

We may now proceed with the current systems $S(\check{I}_a)$, $S(\check{I}_b)$, $S(\check{I}_c)$ and the operating groups $S(Z_{aa})$ $S(Z_{bb})$ $S(Z_{cc})$ etc. and the electromotive forces in exactly the same manner as for simple a-c. circuits. Thus,

$$\begin{aligned} S(\check{E}_a) &= S(Z_{aa})\,S(\check{I}_a) + S(Z_{ab})\,S(\check{I}_b) + S(Z_{ca})\,S(\check{I}_c) \qquad (21) \\ &= (S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2})\,(S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,I_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_a + S^2\,a\,\check{I}_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2}) \\ &= S^0\,(Z_{aa0} + 2\,Z_{ab0})\,\check{I}_{a0} + S^0\,\{Z_{aa2} + (1 + a^2)\,Z_{ab2}\}\,\check{I}_{a1} \end{aligned}$$

$$
\begin{aligned}
&+ S^0 \{Z_{aa1} + (1 + a) Z_{ab1}\} \check{I}_{a2} \\
&+ S^1 \{Z_{aa1} + (1 + a) Z_{ab1}\} \check{I}_{a0} \\
&+ S^1 \{Z_{aa0} + (a + a^2) Z_{ab0}\} \check{I}_{a1} \\
&+ S^1 \{Z_{aa2} + 2\, a\, Z_{ab2}\} \check{I}_{a2} \\
&+ S^2 \{Z_{aa2} + (1 + a^2) Z_{ab2}\} \check{I}_{a0} \\
&+ S^2 \{Z_{aa1} + 2\, a^2 Z_{ab1}\} \check{I}_{a1} \\
&+ S^2 \{Z_{aa0} + (a + a^2) Z_{ab0}\} \check{I}_{a2} \qquad (22)
\end{aligned}
$$

Or since $1 + a + a^2 = 0$, $1 + a = -a^2$, $1 + a^2 = -a$ and $a + a^2 = -1$

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{ab0}) \check{I}_{a0} + S^0 (Z_{aa2} - a\, Z_{ab2}) \check{I}_{a1} \\
&+ S^0 (Z_{aa1} - a^2 Z_{ab1}) \check{I}_{a2} + S^1 (Z_{aa1} - a^2 Z_{ab1}) \check{I}_{a0} \\
&+ S^1 (Z_{aa0} - Z_{ab0}) \check{I}_{a1} + S^1 (Z_{aa2} + 2\, a\, Z_{ab2}) \check{I}_{a2} \\
&+ S^2 (Z_{aa2} - a\, Z_{ab2}) \check{I}_{a0} + S^2 (Z_{aa1} + 2\, a^2 Z_{ab1}) \check{I}_{a1} \\
&+ S^2 (Z_{aa0} - Z_{ab0}) \check{I}_{a2} \qquad (23)
\end{aligned}
$$

Or since

$$
\begin{aligned}
S(Z_{bc}) &= S^0 Z_{bc0} + S^1 Z_{bc1} + S^2 Z_{bc2} \\
&= S^0 Z_{ab0} + S^1 a^2 Z_{ab1} + S^2 a\, Z_{ab2}
\end{aligned}
$$

we may write (23) in the form

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{bc0}) \check{I}_{a0} + S^0 (Z_{aa2} - Z_{bc2}) \check{I}_{a1} \\
&+ S^0 (Z_{aa1} - Z_{bc1}) \check{I}_{a2} + S^1 (Z_{aa1} - Z_{bc1}) \check{I}_{a0} \\
&+ S^1 (Z_{aa0} - Z_{bc0}) \check{I}_{a1} + S^1 (Z_{aa2} + 2\, Z_{bc2}) \check{I}_{a2} \\
&+ S^2 (Z_{aa2} - Z_{bc2}) \check{I}_{a0} + S^2 (Z_{aa1} + 2\, Z_{bc1}) \check{I}_{a1} \\
&+ S^2 (Z_{aa0} - Z_{bc0}) \check{I}_{a2} \qquad (24)
\end{aligned}
$$

which is the more symmetrical form. We have therefore from (24) by expressing $S(\check{E}_a)$ in terms of symmetrical co-ordinates the three symmetrical equations

$$
\left.
\begin{aligned}
S^0 \check{E}_{a0} &= S^0 \{(Z_{aa0} + 2\, Z_{bc0}) \check{I}_{a0} + (Z_{aa2} - Z_{bc2}) \check{I}_{a1} \\
&\qquad + (Z_{aa1} - Z_{bc1}) \check{I}_{a2}\} \\
S^1 E_{a1} &= S^1 \{(Z_{aa1} - Z_{bc1}) \check{I}_{a0} + (Z_{aa0} - Z_{bc0}) \check{I}_{a1} \\
&\qquad + (Z_{aa2} + 2\, Z_{bc2}) \check{I}_{a2}\} \\
S^2 E_{a2} &= S^2 \{(Z_{aa2} - Z_{bc2}) \check{I}_{a0} + (Z_{aa1} + 2\, Z_{bc1}) \check{I}_{a1} \\
&\qquad + (Z_{aa0} - Z_{bc0}) \check{I}_{a2}\}
\end{aligned}
\right\} \qquad (25)
$$

An important case to which we must next give consideration is that of mutual inductance between a primary polyphase circuit and a secondary polyphase circuit. The mutual impedances may be arranged in three sets. Let the currents in the secondary windings be I_u I_v and I_w, we may then express the generalized mutual impedances as follows:

$$\left.\begin{array}{l} \text{(I) } Z_{au}\ Z_{bv}\ Z_{cw} \\ \text{(II) } Z_{bw}\ Z_{cu}\ Z_{av} \\ \text{(III) } Z_{cv}\ Z_{aw}\ Z_{bu} \end{array}\right\} \tag{26}$$

Each set may be resolved into three symmetrical groups, so that

$$\left.\begin{array}{l} S\,(Z_{au}) = S^0\,Z_{au0} + S^1\,Z_{au1} + S^2\,Z_{au2} \\ S\,(Z_{bw}) = S^0\,Z_{bw0} + S^1\,Z_{bw1} + S^2\,Z_{bw2} \\ S\,(Z_{cv}) = S^0\,Z_{cv0} + S^1\,Z_{cv1} + S^2\,Z_{cv2} \end{array}\right\} \tag{27}$$

and we have for $S\,(\check{E}_a)$ the primary induced e.m.f. due to the secondary currents $S\,(\check{I}_u)$

$$S\,(\check{E}_a) = S\,(Z_{au})\,S\,(\check{I}_u) + S\,(Z_{av})\,S\,(\check{I}_v) + S\,(Z_{aw})\,S\,(\check{I}_w) \tag{28}$$

Substituting for $S\,(\check{I}_u)$, $S\,(\check{I}_v)$ and $S\,(\check{I}_w)$ and $S\,(Z_{au})$, $S\,(Z_{av})$ $S\,(Z_{aw})$ their symmetrical equivalents we have

$$\begin{aligned} S\,(\check{E}_a) = {} & S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,I_{u0} \\ & + S^0\,(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{u1} \\ & + S^0\,(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{u2} \\ & + S^1\,(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{u0} \\ & + S^1\,(Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{u1} \\ & + S^1\,(Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{u2} \\ & + S^2\,(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{v2})\,\check{I}_{u0} \\ & + S^2\,(Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{u1} \\ & + S^2\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{u2} \end{aligned} \tag{29}$$

On expressing $S(\check{E}_a)$ in symmetrical form we have the following three symmetrical equations

$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= S^0\,\{(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{u0} \\
&\quad + (Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{u1} \\
&\quad + (Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{u2}\} \\
S^1\,\check{E}_{a1} &= S^1\,\{(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{u0} \\
&\quad + (Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{u1} \\
&\quad + (Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{u2}\} \\
S^2\,E_{a2} &= S^2\,\{(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_u{}^0 \\
&\quad + (Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{u1} \\
&\quad + (Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{u2}\}
\end{aligned}\right\} \qquad (30)$$

For the e.m.f. $S(\check{E}_u)$ induced in the secondary by the primary currents $S(\check{I}_a)$ we have

$$S(\check{E}_u) = S(Z_{au})\,S(\check{I}_a) + S(Z_{bu})\,S(\check{I}_b) + S(Z_{cu})\,S(\check{I}_c) \qquad (31)$$

Since $S(Z_{bu})$ bears the same relation to $S(Z_{cv})$ as $S(Z_{av})$ does to $S(Z_{bw})$ and $S(Z_{au})$ bears the same relation to $S(Z_{bw})$ as $S(Z_{aw})$ does to $S(Z_{cv})$ to obtain $S(\check{E}_u)$ all that will be necessary will be to interchange Z_{bw} and Z_{cv} in (29) and change $\check{I}_{u0}\,\check{I}_{u1}\,\check{I}_{u2}$ to $\check{I}_{a0}\,I_{a1}$ and $\check{I}_{a2}$ respectively, this gives

$$\begin{aligned}
S(\check{E}_u) &= S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0} \\
&\quad + S^0\,(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_{a1} \\
&\quad + S^0\,(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{a2} \\
&\quad + S^1\,(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{a0} \\
&\quad + S^1\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1} \\
&\quad + S^1\,(Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{a2} \\
&\quad + S^2\,(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{a0} \\
&\quad + S^2\,(Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{a1} \\
&\quad + S^2\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a2}
\end{aligned} \qquad (32)$$

and the three symmetrical equations will be

$$\left.\begin{aligned}
S^0\,\check{E}_{u0} &= S^0\,\{(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0} \\
&\quad + (Z_{au2} + a^2\,Z_{bw2} + a\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_{a1} \\
&\quad + (Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{a2}\} \\
S^1\,\check{E}_{a1} &= S^1\,\{(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{a0} \\
&\quad + (Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1} \\
&\quad + (Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{a2}\} \\
S^2\,\check{E}_{u2} &= S^2\,\{(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{a0} \\
&\quad + (Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{a1} \\
&\quad + (Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{a2}\}
\end{aligned}\right\} \quad (33)$$

The same methods may be applied to polyphase systems of any number of phases. When the number of phases is not prime the system may sometimes be dealt with as a number of polyphase systems having mutual inductance between them:—For example, a nine-phase system may be treated as three three-phase systems, a twelve-phase system as three four-phase or four three-phase systems. In certain forms of dissymmetry this method is of great practical value, and its application will be taken up later.

For the present part of the paper we shall confine ourselves to the three-phase system, and dissymmetries of several different kinds.

The operators Z_{au} Z_{aa}, etc., must be interpreted in the broadest sense. They may be simple complex quantities or they may be functions of the differential operator $\frac{d}{dt}$. For if

$$i = \Sigma\,(A_n \cos n\,w\,t + B_n \sin n\,w\,t)$$

it may be expressed in the form

$$\left.\begin{aligned}
i &= \Sigma\left(\frac{A_n - j\,B_n}{2}\,\epsilon^{jnwt} + \frac{A_n + j\,B_n}{2}\,\epsilon^{-jnwt}\right) \\
&= \frac{\check{I}}{2} + \frac{\check{I}}{2} \\
&= \text{real part of } \check{I}
\end{aligned}\right\} \quad (34)$$

and any linear algebraic operation performed on $\check{I}/2$ will give a result which will be conjugate to that obtained by carrying out the same operation on $\check{I}/2$ and since the true solution is the sum of these results, it may also be obtained by taking the real part of the result of performing the operation on $\check{I}$.

Modification of the General Case Met With in Practical Networks

Several symmetrical arrangements of the operator Z_{au} etc.[1] are frequently met with in practical networks which result in a much simpler system of equations than those obtained for the general case as in equations (**29**) to (**33**). Thus for example if all the operators in (**26**) are equal, all the operators in (**27**), except $S^0 Z_{au0}$ $S^0 Z_{bw0}$ and $S^0 Z_{cv0}$ are equal to zero, and these three quantities are also equal to one another so that equation (**30**) becomes

$$\left.\begin{aligned} S^0 \check{E}_{u0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{u0} \\ S^1 \check{E}_{u1} &= 0 \\ S^2 E_{u2} &= 0 \end{aligned}\right\} \qquad (35)$$

and equation (**33**)

$$\left.\begin{aligned} S^0 \check{E}_{u0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{u0} \\ S^1 E_{u1} &= 0 \\ S^2 E_{u2} &= 0 \end{aligned}\right\} \qquad (36)$$

This is the statement in symmetrical co-ordinates that a symmetrically disposed polyphase transmission line will produce no electromagnetic induction in a second similar polyphase system so disposed with respect to the first that mutual inductions between all phases of the two are equal except that due to single-phase currents passing through the conductors.

If in (**26**) the quantities in each group only are equal, equations (**30**) and (**33**) become

$$\left.\begin{aligned} S^0 \check{E}_{u0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{u0} \\ S^1 \check{E}_{u1} &= S^1 (Z_{au0} + a^2 Z_{bw0} + a Z_{cv0}) \check{I}_{u1} \\ S^2 \check{E}_{u2} &= S^2 (Z_{au0} + a Z_{bw0} + a^2 Z_{cv0}) \check{I}_{u2} \end{aligned}\right\} \qquad (37)$$

$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0} \\ S^1\,\check{E}_{a1} &= S^1\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1} \\ S^2\,\check{E}_{a2} &= S^1\,(Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{a2} \end{aligned}\right\} \quad (38)$$

Symmetrical Forms of Common Occurrence

A symmetrical form which is of importance because it is of frequent occurrence in practical polyphase networks has the terms in group (I) equation (**26**) all equal and those in group (II) $\cos\frac{2\pi}{3}$ times those in group (I) and those in group (III) $\cos\frac{4\pi}{3}$ times those in group (I).

Since $\cos\frac{2\pi}{3} = \frac{a + a^2}{2} = \cos\frac{4\pi}{3}$ we have on substituting the values of the impedances in this case,

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= S^0\,\{Z_{au0}\,(1 + a + a^2)\}\,\check{I}_{u0} = 0 \\ S^1\,\check{E}_{a1} &= S^1\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{u1} \\ S^2\,E_{a2} &= S^2\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{u2} \end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned} S^0\,E_{u0} &= S^0\,\{Z_{au0}\,(1 + a + a^2)\}\,\check{I}_{a0} = 0 \\ S^1\,\check{E}_{u1} &= S^1\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{a1} \\ S^2\,\check{E}_{u2} &= S^2\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{a2} \end{aligned}\right\} \quad (40)$$

The elements in group I may be unequal but groups II and III may be obtained from group I by multiplying by $\cos\frac{4\pi}{3}$ and $\cos\frac{2\pi}{3}$ respectively.

The members of the three groups will then be related as follows, the same sequence being used as before,

$$\left.\begin{aligned} &\text{(I)} \quad Z_{au},\ Z_{bv},\ Z_{cw} \\ &\text{(II)} \quad \frac{a + a^2}{2}\,Z_{cw},\ \frac{a + a^2}{2}\,Z_{au},\ \frac{a + a^2}{2}\,Z_{bv} \\ &\text{(III)} \quad \frac{a + a^2}{2}\,Z_{bv},\ \frac{a + a^2}{2}\,Z_{cv},\ \frac{a + a^2}{2}\,Z_{au} \end{aligned}\right\} \quad (41)$$

Consequently the following relations are true:

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \frac{a + a^2}{2} S^0 Z_{au0} \\
S^0 Z_{cv0} &= \frac{a + a^2}{2} S^0 Z_{au0} \\
S^1 Z_{bw1} &= \frac{1 + a^2}{2} S^1 Z_{au1} \\
S^2 Z_{bw2} &= \frac{1 + a}{2} S^2 Z_{au2} \\
S^1 Z_{cv1} &= \frac{1 + a}{2} S^1 Z_{au1} \\
S^2 Z_{cv2} &= \frac{1 + a^2}{2} S^2 Z_{au2}
\end{aligned}\right\} \tag{42}$$

Substituting these relations in (30) and (33) we have for this system of mutual impedances

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{cv0} \\
Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{aw0}
\end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0
\end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\
Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0 \\
Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}
\end{aligned}\right\} \tag{45}$$

which on substitution in (30) and (33) gives

$$\left.\begin{aligned}
S^0 \check{E}_{u0} &= 0 \\
S^1 \check{E}_{u1} &= S^1 \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\
S^2 \check{E}_{u2} &= S^2 \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\}
\end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned}
S^0 \check{E}_{u0} &= S^0 \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u2}\} \\
S^1 \check{E}_{u1} &= S^1 \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\
S^2 \check{E}_{u2} &= S^2 \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\}
\end{aligned}\right\} \tag{47}$$

The above symmetrical forms in which the factors $\cos \frac{2\pi}{3}$ and $\cos \frac{4\pi}{3}$ occur apply particularly to electromagnetic induction between windings distributed over the surfaces of co-axial cylinders; where if the plane of symmetry of one winding be taken as the datum plane, the mutual impedance between this winding and any other is a harmonic function of the angle between its plane of symmetry and the datum plane. In other words, the mutual impedances are functions of position on the circumference of a circle and may therefore be expanded by Fourier's theorem in a series of integral harmonics of the angle made by the planes of symmetry with the datum plane. Since the same procedure applies to all the terms of the expansion it is necessary only to consider the simple harmonic case. In the partially symmetrical cases of mutual induction, such as that taken up in the preceding discussion, there will be a difference between two possible cases, viz:—Symmetrical primary, unsymmetrical secondary, which is the case just considered, and unsymmetrical primary and symmetrical secondary in which the impedances of (**26**) will have the following values

$$\left.\begin{array}{ll} \text{(I)} & Z_{au},\quad Z_{bc},\quad Z_{cw} \\ \text{(II)} & \frac{a+a^2}{2} Z_{bc},\quad \frac{a+a^2}{2} Z_{cw},\quad \frac{a+a^2}{2} Z_{au} \\ \text{(III)} & \frac{a+a^2}{2} Z_{cw},\quad \frac{a+a^2}{2} Z_{au},\quad \frac{a+a^2}{2} Z_{bc} \end{array}\right\} \qquad (48)$$

The results may be immediately set down by symmetry from equations (**46**) and (**47**), but the difference between the two cases will be better appreciated by setting down the component symmetrical impedances, thus we have

$$\left.\begin{array}{l} S^0 Z_{bw0} = \frac{a+a^2}{2} S^0 Z_{au0} \\ S^0 Z_{cv0} = \frac{a+a^2}{2} S^0 Z_{au0} \\ S^1 Z_{bw1} = \frac{1+a}{2} S^1 Z_{au1} \\ S^2 Z_{bw2} = \frac{1+a^2}{2} S^2 Z_{au2} \\ S^1 Z_{cv1} = \frac{1+a^2}{2} S^1 Z_{au1} \\ S^2 Z_{cv2} = \frac{1+a}{2} S^2 Z_{au2} \end{array}\right\} \qquad (49)$$

Substituting these relations in the impedances used in (30) and (33) they become

$$\left.\begin{aligned} &Z_{au0} + Z_{bw0} + Z_{cv0} = 0 \\ &Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} = 1\tfrac{1}{2}\, Z_{au0} \\ &Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} = 1\tfrac{1}{2}\, Z_{au0} \end{aligned}\right\} \quad (50)$$

$$\left.\begin{aligned} &Z_{au1} + Z_{bw1} + Z_{cv1} = 1\tfrac{1}{2}\, Z_{au1} \\ &Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} = 0 \\ &Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} = 1\tfrac{1}{2}\, Z_{au1} \end{aligned}\right\} \quad (51)$$

$$\left.\begin{aligned} &Z_{au2} + Z_{bw2} + Z_{cv2} = 1\tfrac{1}{2}\, Z_{au2} \\ &Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} = 1\tfrac{1}{2}\, Z_{au2} \\ &Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} = 0 \end{aligned}\right\} \quad (52)$$

And we have from (30) and (33), or by symmetry

$$\left.\begin{aligned} &S^0\, \check{E}_{a0} = S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u2}\} \\ &S^1\, \check{E}_{a1} = S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ &S^2\, \check{E}_{a2} = S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \quad (53)$$

$$\left.\begin{aligned} &S^0\, \check{E}_{u0} = 0 \\ &S^1\, \check{E}_{u1} = S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ &S^2\, \check{E}_{u2} = S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\} \end{aligned}\right\} \quad (54)$$

If the angle between the planes of symmetry of the coils and the datum plane are subject to changes, $\cos\frac{2\pi}{3}$ and $\cos\frac{4\pi}{3}$ in the preceding discussion must be replaced by

$$\left.\begin{aligned} &\cos\left(\frac{2\pi}{3} + \theta\right) = \frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta} \\ &\cos\left(\frac{4\pi}{3} + \theta\right) = \frac{a}{2}\, \epsilon^{-j\theta} + \frac{a^2}{2}\, \epsilon^{j\theta} \end{aligned}\right\} \quad (55)$$

where θ is measured from the datum plane

In the strictly symmetrical case of co-axial cylindrical surface windings in which the members of each group of mutual

impedances are equal, the result of substituting (55) in the equations for induced e.m.f. will be

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= 0 \\ S^1\,\check{E}_{a1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\,\check{I}_{u1}) \\ S^2\,\check{E}_{a2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{u2}) \end{aligned}\right\} \tag{56}$$

$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= 0 \\ S^1\,\check{E}_{u1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{a1}) \\ S^2\,\check{E}_{u2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\,\check{I}_{a2}) \end{aligned}\right\} \tag{57}$$

In the case having symmetrical primary and unsymmetrical secondary in which members of each group are different, but in which there are harmonic relations between corresponding members of the different groups, the impedances are

$$\left.\begin{aligned} &\text{(I) } Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &\text{(II) } \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{cw} \\ &\left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\epsilon^{-j\theta}\right)\,Z_{au},\ \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{bv} \\ &\text{(III) } \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{bv}, \\ &\left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) Z_{cw},\ \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) Z_{au} \end{aligned}\right\} \tag{58}$$

The symmetrical component mutual impedances will have the following values in terms of $Z_{au0}\ Z_{au1}\ Z_{au2}$

$$\left.\begin{aligned} S^0\,Z_{bw0} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^0\,Z_{cv0} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^1\,Z_{bw1} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{bw2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^2\,Z_{au2} \\ S^1\,Z_{cv1} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{cv2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^2\,Z_{au2} \end{aligned}\right\} \tag{59}$$

Substituting these relations in the impedances of equations (30) and (33) they become

$$\left.\begin{aligned} &Z_{au0} + Z_{bw0} + Z_{cv0} = 0 \\ &Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} = 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta} \\ &Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} = 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta} \end{aligned}\right\} \tag{60}$$

$$\left.\begin{aligned} &Z_{au1} + Z_{bw1} + Z_{cv1} = 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta} \\ &Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} = 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta} \\ &Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} = 0 \end{aligned}\right\} \tag{61}$$

$$\left.\begin{aligned} &Z_{au2} + Z_{bw2} + Z_{cv2} = 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta} \\ &Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} = 0 \\ &Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} = 1\tfrac{1}{2}\, Z_{au}\, \epsilon^{-j\theta} \end{aligned}\right\} \tag{62}$$

which on substitution in (30) and (33) give

$$\left.\begin{aligned} &S^0\, \check{E}_{a0} = 0 \\ &S^1\, \check{E}_{a1} = S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{u2}\} \\ &S^2\, \check{E}_{a2} = S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{u2}\} \end{aligned}\right\} \tag{63}$$

$$\left.\begin{aligned} &S^0\, \check{E}_{u0} = S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ &S^1\, \check{E}_{u1} = S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ &S^2\, \check{E}_{u2} = S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \end{aligned}\right\} \tag{64}$$

In the case of unsymmetrical primary and symmetrical secondary, we have for the value of the impedance in terms of Z_{au0} Z_{au1} and Z_{au2}

$$\left.\begin{aligned} &\text{(I) } Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &\text{(II) } \left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{bc}, \\ &\left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{cw},\ \left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{au} \\ &\text{(III) } \left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{cw}, \\ &\left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{au},\ \left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{bc} \end{aligned}\right\} \tag{65}$$

The symmetrical component mutual impedances in terms of Z_{au0}, Z_{au1}, Z_{au2} are

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^0 Z_{cv0} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^1 Z_{bw1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{bw2} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2} \\
S^1 Z_{cv1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{cv2} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2}
\end{aligned}\right\} \tag{66}$$

And the impedances of equations (30) and (33) become

$$\left.\begin{aligned}
&Z_{au0} + Z_{bw0} + Z_{cv0} = 0 \\
&Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0} = 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta} \\
&Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0} = 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}
\end{aligned}\right\} \tag{67}$$

$$\left.\begin{aligned}
&Z_{au1} + Z_{bw1} + Z_{cv1} = 1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{j\theta} \\
&Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1} = 0 \\
&Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1} = 1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{-j\theta}
\end{aligned}\right\} \tag{68}$$

$$\left.\begin{aligned}
&Z_{au2} + Z_{bw2} + Z_{cv2} = 1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{-j\theta} \\
&Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2} = 1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{j\theta} \\
&Z_{au2} + a^2\,Z_{bw} + a\,Z_{cv2} = 0
\end{aligned}\right\} \tag{69}$$

And on substitution in (30) and (33), or by symmetry from (63) and (64), we have

$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= S^0\,\{1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{j\theta}\,\check{I}_{u1} + 1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{-j\theta}\,\check{I}_{u2}\} \\
S^1\,\check{E}_{a1} &= S^1\,\{1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\,\check{I}_{u1} + 1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{-j\theta}\,\check{I}_{u2}\} \\
S^2\,\check{E}_{a2} &= S^2\,\{1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{j\theta}\,\check{I}_{u1} + 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{u2}
\end{aligned}\right\} \tag{70}$$

$$\left.\begin{aligned}
S^0\,\bar{E}_{u0} &= 0\\
S^1\,\bar{E}_{u1} &= S^1\,\{1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{-j\theta}\,\bar{I}_{a0} + 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\bar{I}_{a1}\\
&\qquad + 1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{-j\theta}\,\bar{I}_{a2}\}\\
S^2\,\bar{E}_{u2} &= S^2\,\{1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{j\theta}\,\bar{I}_{a0} + 1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{j\theta}\,\bar{I}_{a1}\\
&\qquad + 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\,\bar{I}_{a2}\}
\end{aligned}\right\} \tag{71}$$

A fuller discussion of self and mutual impedances of co-axial cylindrical windings will be found in the Appendix. It will be sufficient to note here that in the case of self inductance and mutual inductance of stationary windings symmetrically disposed if they are equal

$$\left.\begin{aligned}
M_{ab} = M_{bc} = M_{ca} &= \Sigma\left(A_n \cos\frac{2\,n\,\pi}{3}\right)\\
L_{aa} = L_{bb} = L_{cc} = M_{aa} = M_{bb} &= M_{cc} = \Sigma\,A_n
\end{aligned}\right\} \tag{72}$$

If the windings are symmetrically disposed but have different number of turns

$$\left.\begin{aligned}
L_{aa} &= M_{aa} = \Sigma\,A_n\\
L_{bb} &= M_{bb} = \Sigma\,B_n\\
L_{cc} &= M_{cc} = \Sigma\,C_n
\end{aligned}\right\} \tag{73}$$

$$\left.\begin{aligned}
M_{ab} &= \Sigma\left(\sqrt{A_n\,B_n}\cos\frac{2\,n\,\pi}{3}\right)\\
M_{bc} &= \Sigma\left(\sqrt{B_n\,C_n}\cos\frac{2\,n\,\pi}{3}\right)\\
M_{ca} &= \Sigma\left(\sqrt{C_n\,A_n}\cos\frac{2\,n\,\pi}{3}\right)
\end{aligned}\right\} \tag{74}$$

If the coils are alike but unsymmetrically spaced L_{aa} L_{bb} L_{cc} have the same values, namely $\Sigma\,A_n$ and

$$\left.\begin{aligned}
M_{ab} &= \Sigma\left\{(A_n \cos n\,\theta_1)\cos\frac{2\,n\,\pi}{3}\right.\\
&\qquad\qquad \left. + (A_n \sin n\,\theta_1)\sin\frac{2\,n\,\pi}{3}\right\}\\
M_{bc} &= \Sigma\left\{(A_n \cos n\,\theta_2)\cos\frac{2\,n\,\pi}{3}\right.\\
&\qquad\qquad \left. + (A_n \sin n\,\theta_2)\sin\frac{2\,n\,\pi}{3}\right\}\\
M_{ca} &= \Sigma\left\{(A_n \cos n\,\theta_3)\cos\frac{2\,n\,\pi}{3}\right.\\
&\qquad\qquad \left. + (A_n \sin n\,\theta_3)\sin\frac{2\,n\,\pi}{3}\right\}
\end{aligned}\right\} \tag{75}$$

If they are unequal as well as unsymmetrically disposed but are otherwise similar L_{aa} L_{bb} L_{cc} have values as in (**64**) and

$$\left.\begin{aligned} M_{ab} &= \Sigma \left\{ (\sqrt{A_n B_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\ &\qquad \left. + (\sqrt{A_n B_n} \sin n\,\theta_1) \sin \frac{2\,n\,\pi}{3} \right\} \\ M_{bc} &= \Sigma \left\{ (\sqrt{B_n C_n} \cos n\,\theta_2) \cos \frac{2\,n\,\pi}{3} \right. \\ &\qquad \left. + (\sqrt{B_n C_n} \sin n\,\theta_2) \sin \frac{2\,n\,\pi}{3} \right\} \\ M_{ca} &= \Sigma \left\{ (\sqrt{C_n A_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\ &\qquad \left. + (\sqrt{C_n A_n} \sin n\,\theta_3) \sin \frac{2\,n\,\pi}{3} \right\} \end{aligned}\right\} \tag{76}$$

Where the windings are dissimilar in every respect the expressions become more complicated. A short outline of this subject is given in the Appendix.

In the case of mutual inductance between two coaxial cylindrical systems, one of which A, B, C is the primary and the other U, V, W the secondary, the following conventions should be followed:

(a) All angles are measured, taking the primary planes of symmetry as data in a positive direction.

(b) The datum plane for all windings is the plane of symmetry of the primary A phase.

(c) All mechanical motions unless otherwise stated shall be considered as positive rotations of the secondary cylinder about its axis.

(d) The conventional disposition of the phases and the direction of rotation of the secondary winding are indicated in Fig. 3.

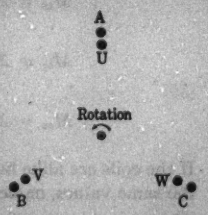

FIG. 3—CONVENTIONAL DISPOSITION OF PHASES AND DIRECTION OF ROTATION.

We shall consider five cases; Case 1 being the completely symmetrical case and the rest being symmetrical in one winding, the other winding being unsymmetrical in magnitude and phase, or both, but all windings having the same form and distribution of coils.

Case I. All Windings Symmetrical.

$$\left.\begin{aligned} M_{au} &= M_{bv} = M_{cw} = \Sigma\, A_n \cos n\,\theta \\ M_{bw} &= M_{cu} = M_{av} = \Sigma\, A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\ M_{cv} &= M_{aw} = M_{bu} = \Sigma\, A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right) \end{aligned}\right\} \quad (77)$$

Case II. Primary Windings equal and Symmetrical, Secondary Windings unequal but otherwise Symmetrical.

$$\left.\begin{aligned} M_{au} &= \Sigma\, A_n \cos n\,\theta,\ M_{bv} = \Sigma\, B_n \cos n\,\theta, \\ &\qquad M_{cw} = \Sigma\, C_n \cos n\,\theta \\ M_{bw} &= \Sigma\, C_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\ M_{cu} &= \Sigma\, A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\ M_{av} &= \Sigma\, B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\ M_{cv} &= \Sigma\, B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\ M_{aw} &= \Sigma\, C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\ M_{bu} &= \Sigma\, A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right) \end{aligned}\right\} \quad (78)$$

Case III. Primary Windings Unequal but Otherwise Symmetrical, Secondary Winding Equal and Symmetrical.

$$\left.\begin{aligned} M_{au} &= \Sigma\, A_n \cos n\,\theta, M_{bv} = \Sigma\, B_n \cos n\,\theta, M_{cw} = \Sigma\, C_n \cos n\theta \\ M_{bw} &= \Sigma\, B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\ M_{cu} &= \Sigma\, C_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\ M_{av} &= \Sigma\, A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\ M_{cv} &= \Sigma\, C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\ M_{aw} &= \Sigma\, A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\ M_{bu} &= \Sigma\, B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right) \end{aligned}\right\} \quad (79)$$

Case IV. Same as Case II except in addition to inequality Secondary Windings are Displaced from Symmetry by angles α_1 α_2 and α_3 whose sum is zero.

$$
\left.
\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta) \\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta) \\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta) \\
M_{bw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{cu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{av} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{cv} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\} \\
M_{aw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\} \\
M_{bu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\}
\end{aligned}
\right\} \quad (80)
$$

Case V. Same as Case III except that the Primary Windings are Unsymmetrically disposed with respect to one another as well as being unequal.

$$
\left.
\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta) \\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta) \\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta) \\
M_{bw} &= \Sigma\,\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{cu} &= \Sigma\,\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{av} &= \Sigma\,\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{2\pi}{3} + \theta\right)\right\} \\
M_{cv} &= \Sigma\,\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\} \\
M_{aw} &= \Sigma\,\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\} \\
M_{bu} &= \Sigma\,\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{4\pi}{3} + \theta\right)\right\}
\end{aligned}
\right\} \quad (81)
$$

The expressions for dissymmetry in both windings and for unsymmetrically wound coils, etc., are more complicated and will be dealt with in the Appendix.

The impedances Z_{aa} Z_{bb}, etc., Z_{au} Z_{bv}, etc., are functions of M_{aa} M_{bb}, etc., M_{au} M_{bv}, etc., and the resistances of the system. The component of e. m. f. proportional to the current due to

mutual impedance is so small that it may generally be neglected so that Z_{au} becomes $\frac{d}{dt} M_{au}$, $Z_{bv} = \frac{d}{dt} M_{bv}$ and so forth.

If the secondary winding is rotating at an angular velocity α, θ in equation (55) becomes αt and the operators Z_{aa}, etc. operate on such products as $e^{j\alpha t} \check{I}_{u1}$ $e^{j\alpha t} \check{I}_{u2}$ where $\check{I}_{u1}$ and $\check{I}_{u2}$ are the variables.

The following relations will be found useful in the application of the method in actual examples.

If D denotes the operator $\frac{d}{dx}$ and $\varphi(Z)$ is a rational algebraic function of Z

$$\left.\begin{aligned} \varphi(D)\, e^{ax} &= \varphi(a)\, e^{ax} \\ \varphi(D)\, \{e^{ax}\, X\} &= e^{ax}\, \varphi(D + a)\, X \\ \varphi(D)\, Y &= e^{ax} \varphi(D + a)\, Y\, e^{-ax} \end{aligned}\right\} \quad (82)$$

Where X and Y may be any function of x.

Star and Delta e.m.fs. and Currents in Terms of Symmetrical Components

It has been shown in the preceding portion of this paper that the e. m. fs. $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ and the currents $\check{I}_a$ $\check{I}_b$ and $\check{I}_c$ whatever their distortion, may be represented by the sum of symmetrical systems of e. m. fs. or currents so that the two expressions

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\, \check{E}_{a0} + S^1\, \check{E}_{a1} + S^2\, \check{E}_{a2} \\ S(\check{I}_a) &= S^0\, \check{I}_{a0} + S^1\, \check{I}_{a1} + S^2\, \check{I}_{a2} \end{aligned}\right\} \quad (83)$$

completely define these two systems.

If we take the delta e. m. fs. and currents corresponding to $S^0\, \check{E}_{a0}$, $S^1\, \check{E}_{a1}$ and $S^2\, \check{E}_{a2}$, $S^1\, \check{I}_{a1}$, $S^2\, \check{I}_{a2}$, we have, since $\check{E}_{bc1}$ leads $\check{E}_{a1}$ by $\frac{\pi}{2}$ and $\check{E}_{bc2}$ lags behind $\check{E}_{a2}$ by the same angle

$$\left.\begin{aligned} S^0\, \check{E}_{bc0} &= 0 \\ S^1\, \check{E}_{bc1} &= j\sqrt{3}\, S^1\, \check{E}_{a1} \\ S^2\, \check{E}_{bc2} &= -j\sqrt{3}\, S^2\, \check{E}_{a2} \\ S^0\, \check{I}_{bc0} &= \text{indeterminate from } S(\check{I}_a) \\ S^1\, \check{I}_{bc1} &= j\, \frac{1}{\sqrt{3}}\, S^1\, \check{I}_{a1} \\ S^2\, \check{I}_{bc2} &= -j\, \frac{1}{\sqrt{3}}\, S^2\, \check{I}_{a2} \end{aligned}\right\} \quad (84)$$

And therefore if we take $\check{E}_{ab}$ as the principal vector

$$\left.\begin{aligned} S^0\,\check{E}_{ab0} &= 0 \\ S^1\,E_{ab1} &= j\,a\,s^1\,\sqrt{3}\,\check{E}_{a1} \\ S^2\,\check{E}_{ab2} &= -j\,a^2\,s^2\,\sqrt{3}\,\check{E}_{a2} \\ S\,(\check{E}_{ab}) &= S^1\,\check{E}_{ab1} + S^2\,\check{E}_{ab2} \end{aligned}\right\} \tag{85}$$

The last equation of group (85) when expanded gives

$$\left.\begin{aligned} \check{E}_{ab} &= j\,\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ \check{E}_{bc} &= j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{E}_{ca} &= j\,\sqrt{3}\,(a^2\,\check{E}_{a1} - a\,\check{E}_{a2}) \end{aligned}\right\} \tag{86}$$

which may also be obtained direct from (83) by means of the relations

$$\check{E}_{ab} = \check{E}_b - \check{E}_a$$

$$\check{E}_{bc} = \check{E}_c - \check{E}_b$$

$$\check{E}_{ca} = \check{E}_a - \check{E}_0$$

Similarly

$$S^0\,\check{I}_{ab} = \text{indeterminate from } S\,(\check{I}_a)$$

$$S^1\,\check{I}_{ab1} = j\,a\,\frac{1}{\sqrt{3}}\,\check{I}_{a1}$$

$$S^2\,\check{I}_{ab2} = j\,a^2\,\frac{1}{\sqrt{3}}\,\check{I}_{a2} \tag{87}$$

$$S\,(\check{I}_{ab}) = S^0\,\check{I}_{ab0} + S^1\,I_{ab1} + S^2\,\check{I}_{ab2}$$

with similar expression for $\check{I}_{ab}$ $\check{I}_{bc}$ and $\check{I}_{ca}$ which may be verified by means of the relations

$$\check{I}_a = \check{I}_{ca} - \check{I}_{ab} + \check{I}_{a0}$$

$$\check{I}_b = \check{I}_{ab} - \check{I}_{bc} + \check{I}_{a0}$$

$$\check{I}_c = \check{I}_{bc} - \check{I}_{ca} + \check{I}_{a0}$$

Conversely to (84) we have the following relations

$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= \text{indeterminate from } S\,(\check{E}_{ab}) \\
S^1\,\check{E}_{a1} &= -\,j\frac{1}{\sqrt{3}}\,S^1\,\check{E}_{bc1} = -j\,\frac{a^2}{\sqrt{3}}\,S^1\,\check{E}_{ab1} \\
S^2\,\check{E}_{a2} &= \;\;j\,\frac{1}{\sqrt{3}}\,S^2\,E_{bc2} = \;\;j\,\frac{a}{\sqrt{3}}\,S^2\,E_{ab2} \\
S^0\,\check{I}_{a0} &= \text{indeterminate from } S\,(\check{I}_{ab}) \\
S^1\,\check{I}_{a1} &= -\,j\,\sqrt{3}\,S^1\,I_{bc1} = -\,j\,a^2\,\sqrt{3}\,S^1\,\check{I}_{ab1} \\
S^2\,\check{I}_{a2} &= \;\;j\,\sqrt{3}\,S^2\,\check{I}_{bc} = \;\;j\,a\,\sqrt{3}\,S^2\,\check{I}_{ab2}
\end{aligned}\right\} \quad (88)$$

It will be sufficient in order to illustrate the application of the principle of symmetrical coordinates to simple circuits to apply it to a few simple cases of transformer connections before proceeding to its application to rotating polyphase systems to which it is particularly adapted.

Unsymmetrical Bank of Delta-Delta Transformers Operating on a Symmetrical Circuit Supplying a Balanced System

Let the transformer effective impedances be Z_{AB} Z_{BC} Z_{CA} and let the secondary load currents be $\check{I}_U$ $\check{I}_V$ and $\check{I}_W$ and let the star load impedance be Z. One to one ratio of transformation will be assumed, and the effect of the magnetizing current will be neglected. The symmetrical equations are

$$\left.\begin{aligned}
O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2}) \\
S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2}) \\
S^2\,\check{E}_{uv2} &= 0 - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2}) \\
S^0\,\check{I}_{u0} &= 0 \\
S^1\,Z\,\check{I}_{u1} &= \check{E}_{u1} \\
S^2\,Z\,\check{I}_{u2} &= \check{E}_{u2}
\end{aligned}\right\} \quad (89)$$

Since the transformation ratio is unity and the effects of magnetizing currents are negligible $S^1\,\check{I}_{ab1} = S^1\,\check{I}_{UV1}$, $S^2\,\check{I}_{ab2} = S^2\,\check{I}_{UV2}$. And therefore by means of the relations (85), the last two equations may be expressed

$$\left.\begin{aligned} S^1 \check{E}_{uv1} &= S^1\, 3\, Z\, \check{I}_{ab1} \\ S^2 \check{E}_{uv2} &= S^2\, 3\, Z\, \check{I}_{ab2} \end{aligned}\right\} \quad (90)$$

in other words, the symmetrical components appear in the secondary as independent systems, $3\,Z$ being the delta load impedance equivalent to the star impedance Z.

Substituting from (90) in the second and third equation and eliminating $\check{I}_{ab0}$ by means of the first equation, and we have

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \left\{ \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\qquad \left. + \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2 O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\qquad \left. + \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned}\right\} \quad (91)$$

which, when S^1 and S^2 are removed, give two simultaneous equations in $\check{I}_{ab1}$ and $\check{I}_{ab2}$.

A modification of the problem may occur even when the load impedances are symmetrical, as they may have symmetrical but unequal impedances Z_1 and Z_2, to the two components $\check{I}_{u1}$ and $\check{I}_{u2}$ respectively, as in the case of a load consisting of a symmetrical rotating machine. The equations corresponding to (89), (90) and (91) then become

$$\left.\begin{aligned} O &= S^0 \left(Z_{AB0}\, \check{I}_{ab0} + Z_{AB2}\, \check{I}_{ab1} + Z_{AB1}\, \check{I}_{ab2} \right) \\ S^1 \check{E}_{uv1} &= S^1 \check{E}_{ab1} - S^1 \left(Z_{AB1}\, \check{I}_{ab0} + Z_{AB0}\, \check{I}_{ab1} + Z_{AB2}\, \check{I}_{ab2} \right) \\ S^2 E_{uv2} &= O - S^2 \left(Z_{AB2}\, \check{I}_{ab0} + Z_{AB1}\, \check{I}_{ab1} + Z_{AB0}\, \check{I}_{ab2} \right) \\ S^0 \check{I}_{u0} &= O \\ S^1 Z_1\, \check{I}_{u1} &= \check{E}_{u1} \\ S^2 Z_2\, \check{I}_{u2} &= \check{E}_{u2} \end{aligned}\right\} \quad (92)$$

$$\left.\begin{aligned} S^1 \check{E}_{uv1} &= S^1\, 3\, Z_1\, \check{I}_{ab1} \\ S^2 \check{E}_{uv2} &= S^2\, 3\, Z_2\, \check{I}_{ab2} \end{aligned}\right\} \quad (93)$$

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \left\{ \left(3 Z_1 + Z_{AB0} - \frac{Z_{AB1} Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\quad \left. \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2 O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\quad \left. \left(3 Z_2 + Z_{AB0} - \frac{Z_{AB1} Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned}\right\} \quad (94)$$

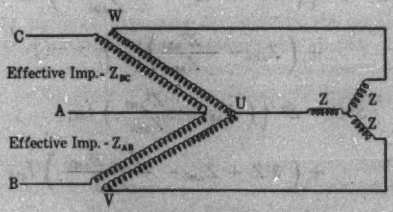

FIG. 4—OPEN DELTA OR V CONNECTION.

In an open delta system $Z_{AB1} = Z_{AB2} = Z_{AB0} - Z_{AB}$ the transformers in this case being both the same. Equation (91) becomes in this particular case where Z_{AB0} is infinite

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \{ (3 Z + 2 Z_{AB}) \check{I}_{ab} + Z_{AB} \check{I}_{ab2} \} \\ S^2 O &= S^2 \{ Z_{AB} \check{I}_{ab1} + (3 Z + 2 Z_{AB}) \check{I}_{ab2} \} \end{aligned}\right\} \quad (95)$$

and we have

$$\check{I}_{ab0} = -\check{I}_{ab1} - I_{ab2} \quad (96)$$

Similarly, instead of (94) we have

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S \{ (3 Z_1 + 2 Z_{AB}) \check{I}_{ab1} + Z_{AB} \check{I}_{ab2} \} \\ S^2 O &= S^2 \{ Z_{AB} \check{I}_{ab1} + (3 Z_2 + 2 Z_{AB}) \check{I}_{ab2} \} \end{aligned}\right\} \quad (97)$$

The secondary voltages are obtained from (90) and (93) for this latter case.

The solution of (95) gives

$$\left.\begin{aligned} \check{I}_{ab1} &= \frac{3\,Z_1 + 2\,Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\,(3\,Z_1 + Z_{AB})}\,\check{E}_{ab} \\ \check{I}_{ab2} &= -\,\frac{Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\,(3\,Z_1 + Z_{AB})}\,\check{E}_{ab} \\ \check{I}_{ab0} &= -\,\frac{1}{3\,Z_1 + 3\,Z_{AB}}\,\check{E}_{ab} \end{aligned}\right\} \quad (98)$$

And we have

$$\left.\begin{aligned} S^1\,\check{I}_{a1} &= S^1\,\frac{3\,Z_1 + 2\,Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_a \\ S^2\,\check{I}_{a2} &= S^2\,\frac{Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_b \end{aligned}\right\} \quad (99)$$

And therefore

$$\left.\begin{aligned} \check{I}_a &= \frac{\check{E}_a}{Z_1 + \dfrac{Z_{AB}}{3}} + \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_{ab} \\ \check{I}_b &= \frac{\check{E}_b}{Z_1 + \dfrac{Z_{AB}}{3}} - \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_{ab} \\ \check{I}_c &= \frac{\check{E}_c}{Z_1 + \dfrac{Z_{AB}}{3}} \end{aligned}\right\} \quad (100)$$

Three-Phase System with Symmetrical Waves Having Harmonics

We may express $\check{E}_a$ in the following form:

$$\left.\begin{aligned} \check{E}_a &= E_1\,\epsilon^{jwt} + E_2\,\epsilon^{j2wt} + E_3\,\epsilon^{j3wt} + \cdots \\ &= \Sigma\,E_n\,\epsilon^{jnwt} \end{aligned}\right\} \quad (101)$$

where E_n is in general a complex number.

If the system is symmetrical three-phase $\check{E}_b$ is obtained by displacing the complete wave by the angle $-\,\dfrac{2\,\pi}{3}$ or

$$\check{E}_b = \epsilon^{-j\frac{2\pi}{3}} E_1 \epsilon^{jwt} + \epsilon^{-j\frac{4\pi}{3}} E_2 \epsilon^{j2wt} + \epsilon^{-j\frac{6\pi}{3}} E_3 \epsilon^{j3wt} + \ldots$$

$$E_c = \epsilon^{j\frac{2\pi}{3}} E_1 \epsilon^{jwt} + \epsilon^{j\frac{4\pi}{3}} E_2 \epsilon^{j2wt} \epsilon^{j\frac{6\pi}{3}} E_3 \epsilon^{j3wt} + \ldots$$

or since $\epsilon^{-j\frac{2\pi}{3}} = a^2$, $\epsilon^{j\frac{2\pi}{3}} = a$ etc.

$$\left.\begin{aligned} \check{E}_a &= E_1 \epsilon^{jwt} + E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \\ \check{E}_b &= a^2 E_1 \epsilon^{jwt} + a E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \\ \check{E}_c &= a E_1 \epsilon^{jwt} + a^2 E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \end{aligned}\right\} \quad (102)$$

or

$$\left.\begin{aligned} S(\check{E}_a) = S^0 \{E_3 \epsilon^{j3wt} + E_6 \epsilon^{j6wt} + E_9 \epsilon^{j9wt} + \ldots\} \\ + S^1 \{E_1 \epsilon^{jwt} + E_4 \epsilon^{j4wt} + E_7 \epsilon^{j7wt} + \ldots\} \\ + S^2 \{E_2 \epsilon^{j2wt} + E_5 \epsilon^{j5wt} + E_8 \epsilon^{j8wt} + \ldots\} \end{aligned}\right\} \quad (103)$$

$$S(\check{E}_a) = S^0 \Sigma (E_{3n} \epsilon^{j3nwt}) + S^1 \Sigma (E_{3n-2} \epsilon^{j(3n-2)wt}) + S^2 \Sigma (E_{2n-1} \epsilon^{j(3n-1)wt}) \quad (104)$$

This shows that a symmetrical three-phase system having harmonics is made up of positive and negative phase sequence harmonic systems and others of zero phase sequence, that is to say of the same phase in all windings, which comprise the group of third harmonics. These facts are not generally appreciated though they are factors that may have an appreciable influence in the performance of commercial machines. It should be particularly noted that in three-phase generators provided with dampers the fifth, eleventh, seventeenth, and twenty-third harmonics produce currents in the damper windings.

In dealing with the complex variable it will be convenient to use for the amplitude the root mean square value for each harmonic. When instantaneous values are required, the real part of the complex variable should be multiplied by $\sqrt{2}$. In the remainder of this paper this convention will be adopted.

Power Representation in Symmetrical Co-ordinates

Since the power in an alternating-current system is also a harmonically varying scalar quantity, it may therefore be represented in the same manner as the current or electromotive force,

that is to say by a complex variable which we shall denote by $(P + j\,Q) + (P_{\text{II}} + j\,Q_{\text{II}})$, $P + j\,Q$ being the mean value, is the term of the complex-variable of zero frequency, P representing the real power and Q the wattless power, $\sqrt{P^2 + Q^2}$ will be the volt-amperes.

The value of the complex variable $(P + j\,Q) + (P_{\text{II}} + j\,Q_{\text{II}})$ may be taken as

$$(P + j\,Q) + (P_{\text{II}} + j\,Q_{\text{II}}) = \check{E}\,\hat{I} + \check{E}\,\check{I} \tag{105}$$

with the provision that for all terms having negative indices the conjugate terms must be substituted, these terms being present in the product $\hat{E}\,\check{I} + \hat{E}\,\hat{I}$, which is the conjugate of the product (105). A similar rule holds good for the symmetrical vector system

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + \;.\;.\;.\;.\; + S^{n-1}\,\check{E}_{a(n-1)} \\ S(\check{I}_a) &= S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + \;.\;.\;.\;.\; + S^{n-1}\,\check{I}_{a(n-1)} \end{aligned}\right\} \tag{106}$$

The conjugate of $S\,\check{I}_a$ is

$$S(\hat{I}_a) = S^0\,\hat{I}_{a0} + S^{(n-1)}\,\hat{I}_{a1} + \;.\;.\;.\;.\; + S^1\,\hat{I}_{a(n-1)} \tag{107}$$

and the power is represented by

$$(P + P_b) + j\,(Q + Q_b) = \Sigma\,\{S(\check{E}_a)\,S(\hat{I}_a) + S(\check{E}_a)\,S(\hat{I}_a)\} \tag{108}$$

with the same provision for terms having negative indices. The sign Σ signifies that all the products in each sequence are added together.

$$\left.\begin{aligned} \Sigma\,\{S(\hat{I}_a)\,S(\check{E}_a)\} &= \Sigma\,S^0\,\{\hat{I}_{a0}\,\check{E}_{a0} + \hat{I}_{a1}\,\check{E}_{a1} + \;.\;.\;.\; \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-1)}\} \\ &\quad + \Sigma\,S^1\,\{\hat{I}_{a0}\,\check{E}_{a1} + \hat{I}_{a1}\,\check{E}_{a2} + \hat{I}_{a2}\,\check{E}_{a3} + \;.\;.\;.\; \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a0}\} \\ &\quad + \Sigma\,S^2\,\{\hat{I}_{a0}\,\check{E}_{a2} + \hat{I}_{a1}\,\check{E}_{a3} + \hat{I}_{a2}\,\check{E}_4 + \;.\;.\;.\; \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a1}\} \\ &\;.\;.\;.

The terms prefixed by S^1, S^2, S^3 . . . $S^{(n-1)}$ all become zero and since S^0 becomes n

$$\Sigma S(\hat{I}_a) S(\check{E}_a) = n \{\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \ldots + \hat{I}_{a(n-1)} \check{E}_{a(n-1)}\} \tag{110}$$

In a similar manner it may be shown that

$$\Sigma S(\check{I}_a) S(\check{E}_a) = n \{\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a(n-1)} + \check{I}_{a2} \check{E}_{a(n-2)} + \ldots + \check{I}_{a(n-1)} \check{E}_{a1}\} \tag{111}$$

and therefore

$$\begin{aligned}(P + jQ) + (P_H + jQ_H) &= n \{\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \ldots + \hat{I}_{a(n-1)} \check{E}_{a(n-1)}\} \\ &+ n \{\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a(n-1)} + \ldots + \check{I}_{a(n-1)} \check{E}_{a1}\}\end{aligned} \tag{112}$$

For a three-phase system the expression reduces to

$$\begin{aligned}(P + jQ) + (P_H + jQ_b) &= 3 (\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \hat{I}_{a2} \check{E}_{a2}) \\ &+ 3 (\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a2} + \check{I}_{a2} \check{E}_{a1})\end{aligned} \tag{113}$$

In the above expression $P + P_H$ is the value of the instantaneous power on the system, P being the mean value and P_H the harmonic portion. When the currents are simple sine waves, Q may be interpreted to be the mean wattless power of the circuit or the sum of the wattless volt-amperes of each circuit. In rotating machinery since the coefficients of mutual induction may be complex harmonic functions of the angular velocity, this is not strictly true for all cases; but if the effective impedances to the various frequencies of the component currents be used, it will be found to be equal to the mean wattless volt-amperes of the system with each harmonic considered independent.

In a balanced polyphase system P_H and Q_H both become zero.

The instantaneous power is a quantity of great importance in polyphase systems because the instantaneous torque is proportional to it and this quantity enters into the problem of vibrations which is at times a matter of great importance, especially when caused by unbalanced e. m. fs. A system of currents and e. m. fs. may be transformed to balanced polyphase by means of transformers alone, provided that the value of P_H is zero, while on the other hand polyphase power cannot be supplied from a pulsating power system without means for

supplying the necessary storage to make a continuous flow of energy.

PART II

Application of the Method to Rotating Polyphase Networks

The methods of determining the constants Z_a Z_u, M, etc., of co-axial cylindrical networks is taken up in Appendix I of this paper. It will be assumed that the reader has familiarized himself with these quantities and understands their significance. We shall first consider the case of symmetrically wound machines taking up the simple cases first and proceeding to more complex ones.

Symmetrically Wound Induction Motor Operating on Unsymmetrical Polyphase Circuit

Denoting the pole-pitch angle by π let the synchronous angular velocity be ω_0 and let the angular slip velocity be ω_1. And let $S^1 E_{a1}$ $S^2 E_{a2}$ be the symmetrical components of impressed polyphase e. m. f. Let R_a be the primary resistance and R_u the secondary resistance. The primary self-inductance being M_{aa}, that of the secondary being M_{uu} and corresponding symbols being used to denote the mutual inductances between the different pairs of windings. Then by means of (**39**), (**40**), (**56**) and (**57**)

$$\left.\begin{aligned}
S^1 \check{E}_{a1} &= S^1 \left\{ R_a \check{I}_{a1} + 1\tfrac{1}{2} M_{aa} \frac{d}{dt} \check{I}_{a1} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{j(w_0 - w_1)t} \check{I}_{u1} \right\} \\
S^2 \check{E}_{a2} &= S^2 \left\{ R_a \check{I}_{a2} + 1\tfrac{1}{2} M_{aa} \frac{d}{dt} \check{I}_{a2} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{-j(w_0 - w_1)t} \check{I}_{u2} \right\} \\
S^1 \check{E}_{u1} = O &= S^1 \left\{ R_u \check{I}_{u1} + 1\tfrac{1}{2} M_{uu} \frac{d}{dt} \check{I}_{u1} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{-j(w_0 - w_1)t} \check{I}_{a1} \right\} \\
S^2 \check{E}_{u2} = O &= S^2 \left\{ R_u \check{I}_{u2} + 1\tfrac{1}{2} M_{uu} \frac{d}{dt} \check{I}_{u2} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{j(w_0 - w_1)t} \check{I}_{a2} \right\}
\end{aligned}\right\} \quad (114)$$

denote $1\frac{1}{2}\ M_{aa}$ by L_a and $1\frac{1}{2}\ M_{uu}$ by L_u, $1\frac{1}{2}\ M_{au}$ by M, the equations (1) become

$$\left.\begin{aligned}
S^1\,\breve{E}_{a1} &= S^1\left\{\left(R_a + L_a\frac{d}{dt}\right)\breve{I}_{a1}\right.\\
&\qquad\left. + M\frac{d}{dt}\,\epsilon^{j(w_0-w_1)t}\,\breve{I}_{u1}\right\}\\
S^2\,\breve{E}_{a2} &= S^2\left\{\left(R_a + L_a\frac{d}{dt}\right)\breve{I}_{a2}\right.\\
&\qquad\left. + M\frac{d}{dt}\,\epsilon^{-j(w_0-w_1)t}\,\breve{I}_{u2}\right\}\\
S^1\,\breve{E}_{u1} = O &= S^1\left\{\left(R_u + L_u\frac{d}{dt}\right)\breve{I}_{u1}\right.\\
&\qquad\left. + M\frac{d}{dt}\,\epsilon^{-j(w_0-w_1)t}\,\breve{I}_{a1}\right\}\\
S^2\,\breve{E}_{u2} = O &= S^2\left\{\left(R_u + L_u\frac{d}{dt}\right)\breve{I}_{u2}\right.\\
&\qquad\left. + M\frac{d}{dt}\,\epsilon^{j(w_0-w_1)t}\,\breve{I}_{a2}\right\}
\end{aligned}\right\}\tag{115}$$

From the last two equations we have

$$\breve{I}_{u1} = -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{-j(w_0-w_1)t}\,\breve{I}_{a1}\tag{116}$$

$$\breve{I}_{u2} = -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{j(w_0-w_1)t}\,\breve{I}_{a2}\tag{117}$$

Substituting these in the first two equations of (115) we obtain

$$S^1\,\breve{E}_{a1} = S^1\left[\left(R_a + L_a\frac{d}{dt}\right) - \frac{M^2\dfrac{d}{dt}\left\{\dfrac{d}{dt} - j\,(w_0-w_1)\right\}}{R_u + L_u\left\{\dfrac{d}{dt} - j(w_0-w_1)\right\}}\right]\breve{I}_{a1}\tag{118}$$

$$S^2 \check{E}_{a2} = S^2 \left[\left(R_a + L_a \frac{d}{d\,t} \right) - \frac{M^2 \frac{d}{d\,t} \left\{ \frac{d}{d\,t} + (j\,w_0 - w_1) \right\}}{R_u + L_u \left\{ \frac{d}{d\,t} - j\,(w_0 - w_1) \right\}} \right] \check{I}_{a2} \quad (119)$$

If $\check{E}_{a1} = E_{a1}\,\epsilon^{jwt}$ and $\check{E}_{a2} = E_{a2}\,\epsilon^{jwt}$ the solution for $\check{I}_{a1}$ and $\check{I}_{a2}$ will be

$$\check{I}_{a1} = \frac{\check{E}_{a1}}{Z_1} \quad (120)$$

$$\check{I}_{a2} = \frac{\check{E}_{a2}}{Z_2} \quad (121)$$

Where

$$Z_1 = R_a + j\,w_0\,L_a + \frac{w_0\,w_1\,M^2}{R_u^2 + w_1^2\,L_u^2}\,(R_u - j\,w_1\,L_u) \quad (122)$$

$$Z_2 = R_a + j\,w_0\,L_a + \frac{w_0\,(2\,w_0 - w_1)\,M^2}{R_u^2 + (2\,w_0 - w_1)^2\,L_u^2}\,\{R_u - j\,(2\,w_0 - w_2)\,L_u\} \quad (123)$$

The impedances Z_1 and Z_2 will be found more convenient to use in the form

$$Z_1 = (R_a + K_1^2\,R_u) + j\,w_0\,(L_a - K_1^2\,L_u) + \frac{w_0 - w_1}{w_1}\,K_1^2\,R_u \quad (124)$$

$$Z_2 = (R_a + K_2^2\,R_u) + j\,w_0\,(L_a - K_2^2\,L_u) - \frac{w_0 - w_1}{Z\,w_0 - w_1}\,K_2^2\,R_u \quad (125)$$

Where, as we will see later, K_1^2 and K_2^2 are the squares of the transformation ratios between primary and secondary currents of positive and negative phase sequence.

The last real term in each expression is the virtual resistance due to mechanical rotation and when combined with the mean square current represents mechanical work performed, the positive sign representing work performed and the negative sign work required.

Thus, for example, to enable the currents $S^2\,\check{I}_{a2}$ to flow, the mechanical work $3\,I_{a2}^2\,\frac{w_0 - w_1}{2\,w_0 - w_1}\,K_1^2\,R_u$ must be applied to the shaft of the motor.

The phase angles of the symmetrical systems $S^1 \check{I}_{a1}$ $S^2 \check{I}_{a2}$ with respect to their impressed e. m. f., $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$ are given by these impedances so that the complete solution of the primary circuit is thus obtained.

The secondary currents are given by equations (116) and (117) and are

$$\check{I}_{u1} = -\frac{j\,w_1\,M}{R_u + j\,w_1\,L_u}\,I_{a1}\,\epsilon^{jw_1t} = \check{K}_1\,I_{a1}\epsilon^{jw_1t} \tag{126}$$

$$\check{I}_{u2} = -\frac{j\,(2\,w_0 - w_1)\,M}{R_u + j\,(2\,w_0 - w_1)\,L_u}\,I_{a2}\,\epsilon^{j(2w_0 - w_1)t} = \check{K}_2\,I_{a2}\,\epsilon^{j(2w_0 - w_1)t} \tag{127}$$

In the results just given, M is not the maximum value of mutual inductance between a pair of primary and secondary windings but is equal to the total mutual inductance due to a current passing through the two coils W and V through the coil

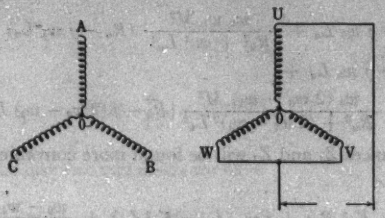

FIG. 5

U as shown in the sketch Fig 5 and the winding "A" when A and U have their planes of symmetry coincident.

Where the windings are symmetrical the induced e. m. f. is independent of the division of current between W and V, but this quantity must not be used in unsymmetrical windings, or with star windings having a neutral point connection so that $\check{I}_{a0}$ is not zero.

The appearance of M in this equation follows from the equation

$$\check{I}_u + \check{I}_v + \check{I}_w = O$$

so that

$$\check{I}_u = -(\check{I}_v + \check{I}_w)$$

The power delivered by the motor is

$$P_o = 3\left\{\frac{w_0 - w_1}{w_1}\,K_1^2\,I_{a1}^2\,R_v - \frac{w_0 - w_1}{2\,w_0 - w_1}\,K_2^2\,I_{a2}^2\,R_u\right\} \tag{128}$$

The copper losses are given by

$$P_L = 3\,\{I_{a1}^2\,(R_p + K_1^2\,R_u) + I_{a2}^2\,(R_p + K_2^2\,R_u)\} \qquad (129)$$

The iron loss is independent of the copper loss and power output. The iron loss and windage may be taken as

$$P_y = \text{Iron loss and windage} \qquad (130)$$

The power input as

$$P_1 = P_0 + P_L + P_y \qquad (131)$$

The mechanical power output is P_0 less friction and windage losses.

$$\text{Torque} = 3\left\{\frac{1}{w_1}\,K_1^2\,I_{a1}^2\,R_u - \frac{1}{2\,w_0 - w_1}\,K_2^2\,I_{a2}^2\,R_u\right\} \times 10^7 \text{ dyne-cm.} \qquad (132)$$

The kv-a. at the terminals is

$$\sqrt{P_1^2 + Q_1^2} = \text{The effective value of } 3\,(E_{a1}\,I_{a1} + E_{a2}\,I_{a2}) \qquad (133)$$

This last result may be arrived at in the following way

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2}) \\ S\,(\hat{I}_a) &= S^2\,\hat{I}_{a1} + S^1\,\hat{I}_{a2} \end{aligned}\right\} \qquad (134)$$

Since $S^2\hat{I}_{a1}$ is conjugate to $S^1\,\check{I}_{a1}$, etc.

The product of $\check{E}_{a1}$ and $\hat{I}_{a0}$ is the power product of the two vectors, $S\,(\check{E}_a)$ and $S\,(\check{I}_a)$ and omits the harmonic variation as a double frequency quantity, the average wattless appears as an imaginary non-harmonic quantity.

$$P_1 + j\,Q_1 = \Sigma\,(S^0\,\check{E}_{a1}\,\hat{I}_{a1} + S^0\,\check{E}_{a2}\,\hat{I}_{a2} + S^1\,\check{E}_{a2}\,\hat{I}_{a1} + S^2\,\check{E}_{a1}\,\hat{I}_{a2}) \qquad (135)$$

The S^1 and S^2 products have zero values, since the sum of the terms of each sequence is zero, hence—

$$P_1 + j\,Q_1 = 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad (136)$$

$$\sqrt{P_1^2 + Q_1^2} = \text{The effective value of } 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad (137)$$

The solution for the general case of symmetrical motor operating on an unsymmetrical circuit is not of as much interest as

certain special cases depending thereon. Some of the most important of these will be taken up in the following paragraphs.

Case I. Single-phase e. m. f. impressed across one phase of three-phase motor.

Assuming the single-phase voltage to be $\check{E}_{bc}$ impressed across the terminals B C. The known data or constraints are

$$\left.\begin{aligned} \check{E}_{bc} &= j\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{I}_a &= O,\ \check{I}_b = -\check{I}_c \end{aligned}\right\} \tag{138}$$

and therefore

$$\check{I}_{a1} = -\check{I}_{a2} \tag{139}$$

$$\frac{\check{E}_{a1}}{Z_1} = -\frac{\check{E}_{a2}}{Z_2}$$

$$\check{E}_{a2} = -\frac{Z_2}{Z_1}\check{E}_{a1} \tag{140}$$

Substituting in (138)

$$\left.\begin{aligned} \check{E}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{Z_1}{Z_1+Z_2} \\ \check{E}_{a2} &= \quad j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{Z_2}{Z_1+Z_2} \end{aligned}\right\} \tag{141}$$

and therefore

$$\left.\begin{aligned} \check{I}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1+Z_2} \\ \check{I}_{a2} &= \quad j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1+Z_2} \end{aligned}\right\} \tag{142}$$

Since $\check{I}_b = \check{I}_{b1} + \check{I}_{b2} = a^2\check{I}_{a1} + a\check{I}_{a2}$

$$\check{I}_b = -I_c = -\frac{\check{E}_{bc}}{Z_1+Z_2} \tag{143}$$

$$P_0 = \left(\frac{w_0 - w_1}{w_1}K_2^2 R_u - \frac{w_0 - w_1}{2w_0 - w_1}K_2^2 R_u\right)I_0^2 \tag{144}$$

$$P_1 + jQ_1 = I_b^2(Z_1 + Z_2) + P_r \tag{145}$$

The power factor is obtained from (145) by the formula

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \tag{146}$$

Substituting from (**142**) in equation (**126**) and (**127**) of the general case we obtain for the secondary currents

$$\left.\begin{aligned} \check{I}_{u1} &= -j\,\check{K}_1 \frac{E_{bc}}{Z_1 + Z_2}\, \epsilon^{j w_1 t} \\ \check{I}_{u2} &= \quad j\,\check{K}_2 \frac{E_{bc}}{Z_1 + Z_2}\, \epsilon^{j(2w_0 - w_1)t} \end{aligned}\right\} \quad (\mathbf{147})$$

Many unsymmetrical cases may be expressed in terms of the operation of coupled symmetrical motors operating on symmetrical systems. This is invariably the case with symmetrical polyphase motors operating on single-phase circuits. Since the physical interpretations are useful in impressing the facts on ones memory they will be given whenever they appear to be useful.

Equations (**141**) and (**142**) show that single-phase operation is exactly equivalent to operating two duplicate motors in series with a symmetrical polyphase e. m. f. $S^1\, E_{ab}$ impressed across one motor, the other being connected in series with the first but with phase sequence reversed, the two motors being directly coupled.

Case II. B and C connected together e. m. f. impressed across A B.

The data given by the conditions of constraint are

$$\left.\begin{aligned} \check{E}_{ab} &= -\,\check{E}_{ca} \\ \check{E}_{bc} &= O = j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \end{aligned}\right\} \quad (\mathbf{148})$$

We therefore have

$$\check{E}_{a1} = \check{E}_{a2} = -\,\frac{\check{E}_{ab}}{3} \quad (\mathbf{149})$$

and

$$\left.\begin{aligned} \check{I}_{a1} &= -\,\frac{\check{E}_{ab}}{3\,Z_1} \\ \check{I}_{a2} &= -\,\frac{\check{E}_{ab}}{3\,Z_2} \end{aligned}\right\} \quad (\mathbf{150})$$

The remainder follows from the general solution and need not be repeated here.

(**150**) shows that a motor operated in this manner is the exact equivalent in all respects to two duplicate mechanically coupled polyphase motors, one of which has sequence reversed, operating in parallel on a balanced three-phase circuit of e. m. f. $S^1\,\frac{E_{ab}}{\sqrt{3}}$.

The secondary currents follow from substitution of (**150**) in equations (**126**) and (**127**) of the general case.

Case III. B and C connected together by the terminals of a balance coil, the impressed e. m. f. E_{AD} applied between A and the middle point of the balance coil. Resistance and reactance of balance coil negligible.

The data furnished by the connection in this case is

$$\check{I}_b = \check{I}_c = -\frac{\check{I}_a}{2} \tag{151}$$

and therefore

$$\check{I}_{a1} = \frac{\check{I}_a - a\,\frac{\check{I}_a}{2} - a^2\,\frac{\check{I}_a}{2}}{3} = \frac{\check{I}_a}{2}$$

$$\check{I}_{a2} = \check{I}_{a1} = \frac{\check{I}_a}{2}$$

We therefore have

$$\left.\begin{aligned} \check{E}_{a1} &= \frac{Z_1\,\check{I}_a}{2} \\ \check{E}_{a2} &= \frac{Z_2\,\check{I}_a}{2} \end{aligned}\right\} \tag{152}$$

we have

$$\begin{aligned} \check{E}_{ab} &= j\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,(a\,Z_1 - a^2\,Z_2) \end{aligned}$$

$$\check{E}_{bc} = j\sqrt{3}\,\frac{\check{I}_a}{2}\,(Z_1 - Z_2)$$

$$\left.\begin{aligned} \check{E}_{ad} &= \left(\check{E}_{ab} + \frac{\check{E}_{bc}}{2}\right) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,\{(a + \tfrac{1}{2})\,Z_1 - (a^2 + \tfrac{1}{2})\,Z_2\} \\ &= -\tfrac{3}{4}\,\check{I}_a\,(Z_1 + Z_2) \end{aligned}\right\} \tag{153}$$

and therefore,

$$\check{I}_a = -1\tfrac{1}{3}\,\frac{\check{E}_{ad}}{Z_1 + Z_2} \tag{154}$$

$$P_0 = \frac{3}{4}\left\{\frac{w_0 - w_1}{w_1} K_1^2 - \frac{w_0 - w_1}{2\,w_0 - w_1} K_2^2\right\} I_a^2 R_a \tag{155}$$

$$P_1 + j\,Q_1 = \frac{3}{4} I_a^2 (Z_1 + Z_2) + P_p \tag{156}$$

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \tag{157}$$

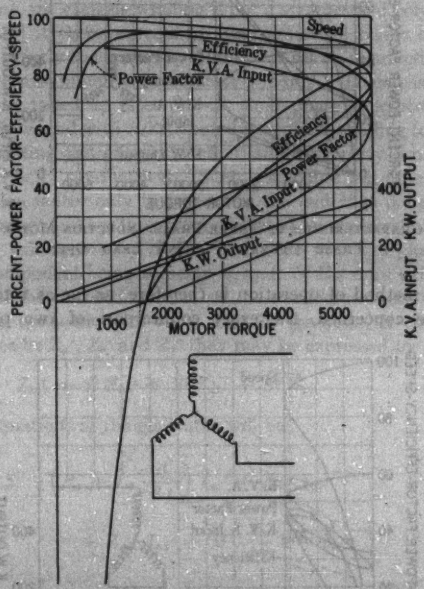

FIG. 6—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—BALANCED THREE-PHASE

Evidently (155), (156) and (157) are identical to (144), (145) and (146) if I_a is equal to $I_b + \frac{\sqrt{3}}{2}$. This will be the case if the value of $E_{ad} = \frac{\sqrt{3}}{2}$ times that of E_{bc}. The total heating of

the motors will be the same in each case but the heating in one phase for Case III will be one-third greater than for Case I.

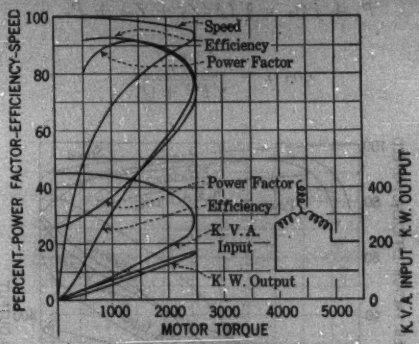

FIG. 7—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION —ONE LEAD OPEN

This method of operation is therefore, as far as total losses, etc. are concerned, the exact counterpart of two polyphase

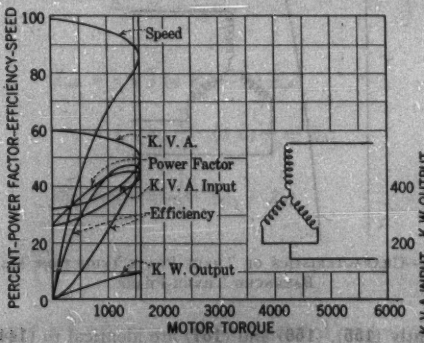

FIG. 8—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION IN MANNER INDICATED

motors connected in series with shafts mechanically connected, one of which has its phase sequence reversed.

Figs. 6, 7 and 8 show characteristic curves of a three-phase

induction motor operating respectively on a symmetrical circuit, according to Case I and according to Case II.

Synchronous Machinery

The Symmetrical Three-Phase Generator Operating on Unsymmetrically Loaded Circuit

The polyphase salient pole generator is not strictly a symmetrical machine, the exciting winding is not a symmetrical polyphase winding and it therefore sets up unsymmetrical trains of harmonics in exactly the same way as they are set up in an induction motor with unsymmetrical secondary winding. These cases will therefore be taken up later on. A three-phase generator may however be wound with a distributed polyphase winding to serve both as exciting and damper winding and if properly connected will be perfectly symmetrical. Such a machine will differ from an induction motor only in respect to the fact that it operates in synchronism and has internally generated symmetrical e. m. fs. which we will denote by $S^1 \check{E}_{a1}$, $S^2 \check{E}_{a2}$ the negative phase sequence component being zero; an e. m. f. $S^0 \check{E}_{a0}$ may exist but since in all the connections that will be considered there will be no neutral connection its value may be ignored. If the load impedances be Z_a', Z_b' and Z_c' they may be expressed by

$$Z_{aa}^1 = S^0 Z_{a0}^1 + S^1 Z_{a1}' + S^2 Z_{a2}'$$

and the equations of the generator will be

$$\left.\begin{aligned}
S^1 \check{E}_{a1} &= S^1 \left\{ \left(R_a + L_a \frac{d}{dt}\right) \check{I}_{a1}' + Z_{a0}' I_{a1}' \right. \\
&\qquad \left. + Z_{a2}' I_{a2}' + M \frac{d}{dt} \epsilon^{jw_1 t} \check{I}_{u1}' \right\} \\
O &= S^2 \left\{ \left(R_a + L_a \frac{d}{dt}\right) \check{I}_{a2}' + Z_{a0}' I_{a2}' \right. \\
&\qquad \left. + Z_{a1}^1 \check{I}_{a1}' + M \frac{d}{dt} \epsilon^{-jw_1 t} \check{I}_{u2}' \right\} \\
O &= \left(R_u + L_u \frac{d}{dt}\right) \check{I}_{u1}' + M \frac{d}{dt} \epsilon^{-jw_1 t} \check{I}_{a1}' \\
O &= \left(R_u + L_u \frac{d}{dt}\right) \check{I}_{u2}' + M \frac{d}{dt} \epsilon^{jw_1 t} \check{I}_{a2}'
\end{aligned}\right\} \quad (158)$$

The last two equations give

$$\left.\begin{aligned} I_{u1}' &= -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{-jw_0t}\,\check{I}_{a1}' \\ I_{u2}' &= -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{jw_0t}\,I_{a2}' \end{aligned}\right\} \tag{159}$$

which on substitution in the first two equations of (158) give the equations

$$\left.\begin{aligned} &\left\{R_a + L_a\frac{d}{dt} - \frac{M^2\dfrac{d}{dt}\left(\dfrac{d}{dt} - j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{dt} - j\,w_0\right)}\right\}\check{I}_{a1}' \\ &\qquad + Z_{a0}^1\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}' = \check{E}_{a1} \\ &Z_{a1}^1\,\check{I}_{a1}' + \left\{R_a + L_a\frac{d}{dt}\right. \\ &\left. - \frac{M^2\dfrac{d}{dt}\left(\dfrac{d}{dt} + j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{dt} + j\,w_0\right)}\right\}\check{I}_{a2}' + Z_{a0}^1\,\check{I}_{a2}' = 0 \end{aligned}\right\} \tag{160}$$

or if

$$\check{E}_{a1} = E_{a1}\,\epsilon^{jw_0t} \tag{161}$$

the impedances Z_{a0}, Z_{a1}, Z_{a2} become ordinary impedance for an electrical angular velocity w_0 and equations (160) become

$$\left.\begin{aligned} &(R_a + j\,w\,L_a + Z_{a0}^1)\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}' = \check{E}_{a1} \\ &Z_{a1}'\,\check{I}_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \\ &\qquad\qquad \tfrac{1}{2}\,K_2^2\,R_u\}\,\check{I}_{a2}' = 0 \end{aligned}\right\} \tag{162}$$

It is apparent that in the generator the impedances

$$R_a + j\,w_0\,L_a = Z_1'$$

and $\{(R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \tfrac{1}{2}\,K^2R_u\} = Z_2'$

take the place of Z_1 and Z_2 in the symmetrical induction motor operating on an unsymmetrical circuit, and we may express equation (**162**)

$$\left.\begin{aligned} (Z_{a0}' + Z_1')\, \check{I}_{a1}' + Z_{a2}'\, \check{I}_{a2}' &= \check{E}_{a1} \\ Z_a'\, \check{I}_{a1}' + (Z_{a0}' + Z_2')\, I_{a2}' &= O \end{aligned}\right\} \quad (\mathbf{163})$$

which gives

$$\check{I}_{a2}' = -\frac{Z_{a1}'}{Z_{a0} + Z_2'}\, \check{I}_{a1}'$$

$$\check{I}_{a1}' = \frac{\check{E}_{a1}}{(Z_{a0}' + Z_1') - \dfrac{Z_{a1}'\, Z_{a2}'}{Z_{a0}' + Z_2'}}$$

Or in more symmetrical form

$$\left.\begin{aligned} \check{I}_{a1}' &= \frac{Z_{a0}' + Z_2'}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1} \\ \check{I}_{a2}' &= -\frac{Z_{a1}{}^1}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1} \end{aligned}\right\} \quad (\mathbf{164})$$

From (**159**) we have for the damper currents

$$\left.\begin{aligned} \check{I}_{u1}' &= O \text{ if } R_u > O \\ \check{I}_{u2}' &= -\check{K}_2\, I_{a2}\, \epsilon^{j\,2wt} \\ \text{where } \check{K}_2 &= j\,\frac{2\, w_0\, M}{R_u + j\, 2\, w_0\, L_u} \end{aligned}\right\} \quad (\mathbf{165})$$

A particular case of interest is when the load is a Synchronous Motor or Induction Motor with unsymmetrical line impedances in series—Equation (**163**) becomes

$$\left.\begin{aligned} (Z_{a0}' + Z_1' + Z_1)\, \check{I}_{a1}' + Z_{a2}'\, I_{a2}' &= \check{E}_{a2} \\ Z_{a1}\, \check{I}_{a1}' + (Z_{a0}' + Z_2' + Z_2)\, I_{a2}' &= O \\ I_{a1}' &= \frac{Z_{a0}' + Z_2' + Z_2}{(Z_{a0}' + Z_1' + Z_1)\,(Z_{a0}' + Z_2' + Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1} \\ I_{a2}' &= \frac{Z_{a1}}{(Z_{a0}' + Z_1' + Z_1)\,(Z_{a0}' + Z_2'\, Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1} \end{aligned}\right\} \quad (\mathbf{166})$$

An important case is that of a generator feeding into a symmetrical motor and an unsymmetrical load. Let the motor currents be

$\check{I}_a$, $\check{I}_b$, $\check{I}_c$, those of the load I_a', I_b', I_c' and the load impedances Z_a', Z_b', Z_c'. The equations of this system will be

$$\left.\begin{aligned} S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}')+Z_{a0}'\,\check{I}_{a1}'+Z_{a2}'\,\check{I}_{a2}'\} \\ S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}')+Z_1\,\check{I}_{a1}\} \\ S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}')+Z_{a0}'\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}'\} \\ S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}')+Z_2\,\check{I}_{a2}\} \end{aligned}\right\} \quad (167)$$

Or, omitting the sequence symbols and re-arranging—

$$\left.\begin{aligned} \check{E}_{a1} &= Z_1'\,\check{I}_{a1}+(Z_1'+Z_{a0}')\,\check{I}_{a1}'+Z_{a2}'\,\check{I}_{a2}' \\ \check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1}+Z_1'\,\check{I}_{a1}^1 \\ O &= Z_2'\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}^1+(Z_2'+Z_{a0}')\,\check{I}_{a2}' \\ O &= (Z_2'+Z_2)\,\check{I}_{a2}+Z_2'\,\check{I}_{a2}' \end{aligned}\right\} \quad (168)$$

These equations can be further simplified as follows:

$$\left.\begin{aligned} O &= (Z_2'+Z_2)\,\check{I}_{a2}+Z_2'\,\check{I}_{a2}' \\ O &= -Z_2\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}'+Z_{a0}'\,\check{I}_{a2}' \\ O &= -Z_1\,\check{I}_{a1}+Z_{a0}'\,\check{I}_{a0}'+Z_{a2}'\,I_{a2}' \\ \check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1}+Z_1'\,\check{I}_{a1}' \end{aligned}\right\} \quad (169)$$

A set of simultaneous equations which may be easily solved.

The Single-Phase Generator is an Important Case of the Three-Phase Generator Operated on an Unbalanced Load

Let the impedance of the single-phase load be Z and let us suppose it to be made up of three star connected impedances

$$Z_a' = 3\,Z_s + \frac{Z}{2}$$

$$Z_b' = \frac{Z}{2}$$

$$Z_c' = \frac{Z}{2}$$

the value of Z_x in the limit being infinity. Then we have

$$\left.\begin{aligned} Z_{a0}' &= Z_x + \frac{Z}{2} \\ Z_{a1}' &= Z_x \\ Z_{a2}' &= Z_x \end{aligned}\right\} \quad (170)$$

Equation (164) in the limit when Z_x becomes infinite reduces to

$$\begin{aligned} \check{I}_{a1}' &= \frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ \check{I}'_{a2} &= -\frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \end{aligned} \quad (171)$$

The single-phase load being across the phase $B\,C$, the single-phase current I will therefore be equal to $\check{I}_c$ or

$$\left.\begin{aligned} \check{I} &= \frac{j\sqrt{3}\,\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ &= \frac{\check{E}_{bc}}{Z + Z_1' + Z_2'} \end{aligned}\right\} \quad (172)$$

$$\left.\begin{aligned} \check{I}_{u1} &= O \text{ if } R_u > O \\ \check{I}_{u2} &= -j\,\frac{1}{\sqrt{3}}\,\check{K}_2\,\check{I}\,\epsilon^{jwt} \\ \check{I}_{u2} &= -\frac{j\,\check{K}_2}{\sqrt{3}}\,I\,\epsilon^{j2wt} \end{aligned}\right\} \quad (173)$$

$\check{I}_{u2}$ is double normal frequency

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,I^2\,Z \\ P_L + j\,Q_L &= 3\,I^2\,(Z_1' + Z_2') \\ (P + j\,Q) + (P_H + j\,Q_H) &= 3\,\check{E}_{bc}\,(\check{I} + \check{I}) \end{aligned}\right\} \quad (174)$$

In the case of the generally unbalanced three-phase load

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,\{(I_{a1}^2 + I_{a2}^2)\,Z_{a0}' \\ &\qquad + \check{I}_{a1}\,\check{I}_{a2}\,Z_{a2}' + \check{I}_{a1}\,\check{I}_{a2}\,Z_{a1}'\} \\ P_L + j\,Q_L &= 3\,\{I_{a1}^2\,Z_1' + I_{a2}^2\,Z_2'\} \\ (P + j\,Q) + (P_H + j\,Q_H) &= 3\,\check{E}_{a1}\,(\check{I}_{a2} + \check{I}_{a2}) \end{aligned}\right\} \quad (175)$$

When the generator has harmonics in its wave form equations (**162**) must be written

$$\left.\begin{aligned}(R_a + j\,w\,L_a + Z_{a0}')\,\check{I}_{a1}' + Z_{a2}'\,I_{a2}' &= \check{E}_{a1}\\ Z_{a1}'\,I_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u)\\ + j\,2\,w\,(L_a - K_2^2\,L_u) - \tfrac{1}{2}\,a^2\,R_u\}\,I_{a2}' &= \check{E}_{a2}\end{aligned}\right\} \quad (176)$$

Where $\check{E}_{a1}$ is finite, $\check{E}_{a2}$ is zero and vice versa, the frequencies being different in each case, we have therefore a solution for each frequency depending on the phase and amplitude and phase sequence of the e. m. f. of this frequency generated. Of course the values of Z_1' and Z_2' change with each frequency on account of the change in the reactance with frequency, and a value must be taken for w conforming with the frequency of the harmonic under consideration.

Symmetrical Synchronous Motor, Synchronous Condenser, Etc.

As in the case of the generator, the synchronous motor has two impedances, one to the positive phase sequence current of a given frequency and the other to the negative phase sequence current of the same frequency. But, since there is no quantity in the positive phase sequence impedance corresponding to the virtual resistance which indicates mechanical work in an induction motor, its equivalent is furnished by the excitation of the field. Let us denote the e. m. f. due to the field excitation by $S^1\,\check{E}_{a1}'$ assuming it to be for the present a simple harmonic three-phase system. Let P_0 be the output of the motor which will include the windage and iron losses assumed to be constant. Then for the synchronous motor on a balanced circuit of e. m. f. $S^1\,\check{E}_{a1}$ we have

$$S^1\,\check{E}_{a1} = S^1\,\{\check{I}_{a1}'(R_a' + j\,w\,L_a') + \check{E}_{a1}'\} \quad (177)$$

$$S^0\,\check{E}_{a1}\,\check{I}_{a1} = S^0\left\{I_{a1}^2\,(R_a' + j\,w\,L_a') + \frac{P_0}{3} - j\,\frac{Q_0}{3}\right\} \quad (178)$$

Where Q_0 is the imaginary part of the product, $\check{E}_{a1}'\,\check{I}_{a1}$. (178) reduces to

$$E_{a1}\,I_{a1}\cos\alpha = I_{a1}^2\,R_a' + \frac{P_0}{3} \quad (179)$$

Where $\cos\alpha$ is the required operating power factor. Solving for I_{a1}

$$I_{a1} = \frac{E_{a1} \cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} \tag{180}$$

$$\bar{I}_{a1} = \bar{E}_{a1} \frac{\cos \alpha}{2 R_{a1}} \left\{ 1 \pm \sqrt{1 - \frac{4 R_{a1}' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) \tag{181}$$

The apparent impedance of the motor is

$$\frac{2 R_1 \sec \alpha}{1 \pm \sqrt{1 - \frac{4 P_0}{3 E_a^2 \cos^2 \alpha}}} (\cos \alpha + j \sin \alpha) \tag{182}$$

and

$$\bar{E}_{a1}^1 = \bar{E}_{a1} \left[1 - \frac{\cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) (R_a' + j w L_a') \right] \tag{183}$$

The same equations apply to the case of the synchronous condenser with the difference that the mechanical work is that required to overcome the iron and windage losses only.

If we take

$$\left.\begin{aligned} \bar{E}_{a1} &= E_{a1} (\cos \alpha + j \sin \alpha) \epsilon^{jwt} = (A_1 + j B_1) \epsilon^{jwt} \\ \bar{E}_{a1}' &= (A_1' + j B_1') \epsilon^{jwt} \end{aligned}\right\} \tag{184}$$

we have

$$\bar{I}_{a1} = \frac{A_1}{2 R_a^1} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \epsilon^{jwt} \tag{185}$$

$$A_1' = \frac{A_1}{2} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \epsilon^{jwt} \tag{186}$$

$$B_1' = \left\{ B_1 - \frac{j w L_a' A_1}{2 R_a'} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \right\} \epsilon^{jwt} \tag{187}$$

Since α may be a positive or negative angle, the sine may be positive or negative for a positive cosine, and therefore the power factor will be leading or lagging accordingly as B_1 is negative or positive respectively. The double signs throughout are due to the fact that for any given load and power factor there are always two theoretically possible running conditions. However, since

we are concerned only with that one which will give the max. operating efficiency, that is the condition that gives I_{a1} the lesser value, for a given value of P_0 the equations may be written

$$\left.\begin{aligned} I_{a1} &= \frac{A_1}{2\,R_a^1}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{jwt} \\ A_1' &= \frac{A_1}{2}\left(1+\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{jwt} \\ B_1' &= \left\{B_1-\frac{j\,w_0\,L_a'\,A_1}{2\,R_a'}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\right\} \end{aligned}\right\} \quad (188)$$

And corresponding values for (**180**), (**181**), (**182**) and (**183**) may be obtained by omitting the positive sign in these equations.

Another condition of operation is obtained by inspection of (**180**), due to the fact that I_{a1} must be a real quantity

$$\frac{4\;R_a'\;P_0}{3\,E_{a1}^2\cos^2\alpha} \text{ must be} > 1 \quad (189)$$

this is the condition of stability. In terms of (**184**) it becomes

$$\frac{4\,R_a'\,P_0}{3\,A_1^2} \text{ must be} > 1 \quad (190)$$

The same conditions apply to the synchronous condenser, the total mechanical load in this case being the iron loss and windage and friction losses.

Proceeding now to operation with unbalanced circuits having sine waves the motor also having a sine wave. In addition to equation (**177**) we shall have

$$S^2\,\check{E}_{a2} = S^2\,Z_2'\,\check{I}_{a2} \quad (191)$$

The mechanical power delivered through the operation of this negative phase sequence e. m. f. is given by P_N where

$$P_N = -\,3\,I_{a2}^2\,\frac{R_a'}{2} \quad (192)$$

this quantity must therefore be subtracted from the value of P_0 in all the equations in which P_0 appears when unbalanced circuits are used in connection with equations (**177**) to (**190**) inclusive. These equations, however, give the conditions for maintaining a given mechanical load and a given power factor in the positive phase sequence component, but in practise what is re-

quired is the combined power factor of the whole system, or the conditions to give a certain combined factor while delivering a given mechanical load; this may be obtained as follows:

The negative phase sequence component is a perfectly definite impedance and is independent of the load, and therefore the zero frequency part of the product $E_{a2}\, I_{a2}$ may be set down as

$$\bar{E}_{a2}\, \bar{I}_{a2} = \frac{P_2}{3} + j\,\frac{Q_2}{3} \tag{193}$$

we have also for the positive phase sequence power delivered

$$(A_1 + j\,B_1)\; I_{a1} = I_{a1}^2\, R_a' + \frac{P_0}{3} - \frac{P_N}{3} + j\,(w\, I_{a1}\, L_a' + B_1^1)\; I_{a1} \tag{194}$$

And the power factor is given by $\cos\alpha$, where

$$\tan\alpha = \frac{I_{a1}\, B_1 + \dfrac{Q_2}{3}}{I_{a1}\, A_1 + \dfrac{P_2}{3}} \tag{195}$$

From (**194**) we have

$$A_1\, I_{a1} = I_{a1}^2\, R_a^1 + \frac{P_0}{3} - \frac{P_N}{3} \tag{196}$$

$$B_1 = w\, I_{a1}\, L_a' + B_1' \tag{197}$$

$$A_1^2 + B_1^2 = E_{a1}^2 \tag{198}$$

The simplest method of solving these equations is by means of curves. Taking arbitrary values of I_{a1}, B_1 and A_1 are chosen consistent with (**198**) so as to satisfy (**195**), $\frac{P_0}{3}$ A_1' and B_1' are then obtained from (**196**) and (**197**). If there are harmonics in the impressed e. m. f. but there are none in the wave form of the machine, the machine will have a definite impedance to the positive and negative phase sequence components of each harmonic, so that there will be a definite amount of mechanical work contributed by each harmonic which must be subtracted from the total work to be done to give the amount of work contributed by the positive phase sequence fundamental component, the equations will be identical to (**193**), (**194**), (**195**), (**196**),

(197) and (198), if we take P_W to mean the total mechanical work done by the harmonics both positive and negative phase sequence and P_2 and Q_2 to represent the products

$$\Sigma\ ({}_n\check{E}_{a1}\ \ {}_n\check{I}_{a1} + {}_n\check{E}_{a2}\ \ {}_n\check{I}_{a2})$$

the zero frequency part only being taken into account.

When harmonics are present both in the impressed wave and in the generated wave, the problem becomes too complicated to treat generally, but specific cases can be worked out without much difficulty.

Phase Converters and Balancers

The phase converter is a machine to transform energy from single-phase or pulsating form to polyphase or non-pulsating form or vice versa to transform energy from polyphase to single-phase. The transformation may not be complete, that is to say, the polyphase system may not be perfectly balanced when supplied from a single-phase source through the medium of a phase converter. Phase converters may be roughly divided into two classes, namely—shunt type and series type.

Induction Motor or Synchronous Condenser Operating as a Phase Converter of the Shunt Type to Supply a Symmetrical Induction Motor or Synchronous Motor

Let Z_1 and Z_2 be the positive and negative phase sequence impedances of the motor, Z_1', Z_2' those of the phase converter. Let $S^1\ \check{E}_{a1}$ and $S^2\ \check{E}_{a2}$ be the positive and negative phase sequence components of the star e. m. f. impressed on the motor as a result of the operation. The single-phase supply will be one side of the delta e. m. f. $S\ \check{E}_{bc}$ which has positive and negative phase sequence components $S^1\ E_{bc1}$ and $S^2\ E_{bc2}$ the single-phase supply being $\check{E}_{bc} = \check{E}_{bc1} + \check{E}_{bc2}$.

The value of Z_2' may be considered fixed for all practical purposes and since in the induction motor phase converter the speed is practically no-load speed, Z' is practically the no-load impedance plus a real part obtained by increasing the real part of the no-load impedance by the ratio of the normal no-load losses to these same losses plus $\frac{1}{2}$ the secondary losses due to the phase converter currents. The latter may be calculated roughly as even a large error in its value will have an inappreciable effect on the actual results. We have therefore

$$\left.\begin{aligned} S^1 \check{E}_{a1} &= -\, S^1 j \frac{\check{E}_{bc}}{\sqrt{3}} \\ S^2 \check{E}_{a2} &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}} \end{aligned}\right\} \quad (199)$$

$$\left.\begin{aligned} S_1{}^1 \check{I}_{a1}' &= -\, S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1'} \\ S^1 \check{I}_{a1} &= -\, S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1} \end{aligned}\right\} \quad (200)$$

$$\left.\begin{aligned} S^2 \check{I}_{a2}' &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2'} \\ S^2 \check{I}_{a2} &= \quad S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2} \end{aligned}\right\} \quad (201)$$

In the common lead of motor and converter we have

$$\check{I}_{a1}' + \check{I}_{a2}' + \check{I}_{a1} + \check{I}_{a2} = 0 \quad (202)$$

or, substituting from (200) and (201)

$$\check{E}_{bc2}\left(\frac{1}{Z_2'} + \frac{1}{Z_2}\right) = \check{E}_{bc1}\left(\frac{1}{Z_1'} + \frac{1}{Z_1}\right) \quad (203)$$

$$\frac{\check{E}_{bc1}}{\check{E}_{bc2}} = \frac{\dfrac{1}{Z_2'} + \dfrac{1}{Z_2}}{\dfrac{1}{Z_1'} + \dfrac{1}{Z_1}} \quad (204)$$

$$\check{E}_{bc1} = \frac{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\, \check{E}_{bc} \quad (205)$$

$$\check{E}_{bc2} = \frac{\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\, \check{E}_{bc} \quad (206)$$

which give the complete solution for all the quantities required with the aid of equations (200) and (201). For the supply current $\check{I}$

$$\left.\begin{aligned} \check{I} &= \check{I}_{bc1} + \check{I}_{bc2} + I_{bc1}' + I_{bc2}' \\ S\,\check{I}_{bc} &= S^1\,\check{I}_{bc1} + S^2\,\check{I}_{bc2} \\ S\,\check{E}_{bc} &= S^1\,\check{E}_{bc1} + S^2\,E_{bc2} \end{aligned}\right\} \quad (207)$$

$$P_1 + j\,Q_1 = \check{E}_{bc}\,\check{I} \quad (208)$$

In order to obtain a perfect balance we may consider the addition of an e. m. f. $S^2\,j\,\frac{E_{x2}}{\sqrt{3}}$ in series with the phase converter whose value must be a function of the load and the phase converter impedances, and therefore equation (**201**) will be replaced by

$$S^2\,\check{I}_{a2}' = S^2\left(j\,\frac{\check{E}_{bc2}}{\sqrt{3}\,Z_2'} + j\,\frac{\check{E}_{x2}}{\sqrt{3}\,Z_2'}\right) \quad (209)$$

$$S^2\,\check{I}_{a2} = S^2\,j\,\frac{\check{E}_{bc}}{\sqrt{3}\,Z_2}$$

and since the balance is perfect $\check{E}_{bc2}$ is zero, and therefore

$$S^2\,j\,\frac{\check{E}_{x2}}{\sqrt{3}} = S^2\,Z_2'\,\check{I}_{a2}' \quad (210)$$

An e. m. f. equal and of opposite phase to the negative phase sequence drop through the phase converter is required to produce a perfect balance.

Carrying out the solution in the same manner as in the imperfect converter, we obtain

$$\check{E}_{bc2} = \frac{\frac{1}{Z_1} + \frac{1}{Z_1'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,E_{bc} - \frac{\frac{1}{Z_2'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,\check{E}_{x2} \quad (211)$$

and since $\check{E}_{bc2}$ is zero and $\check{E}_{bc1} = \check{E}_{bc}$ the single-phase impressed e. m. f., we obtain

$$\check{E}_{x2} = Z_2'\left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right)\check{E}_{bc} \quad (212)$$

and therefore from (**210**)

$$S^2\,\check{I}_{a2}' = S^2\,j\left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right)\frac{\check{E}_{bc}}{\sqrt{3}} \quad (213)$$

$$S^1 \dot{I}_{a1}' = - S^1 j \frac{\dot{E}_{bc}}{\sqrt{3} Z_1'} \tag{214}$$

$$S^2 \dot{I}_{a2} = O \tag{215}$$

$$S^1 \dot{I}_{a1} = - S^1 j \frac{\dot{E}_{bc}}{\sqrt{3}\, Z_1} \tag{216}$$

Figs. 9, 10, 11 and 12 are vector diagrams of some of the principal compensated shunt-type phase converters. There will be no difficulty in following out these diagrams if the principles of this paper have been grasped.

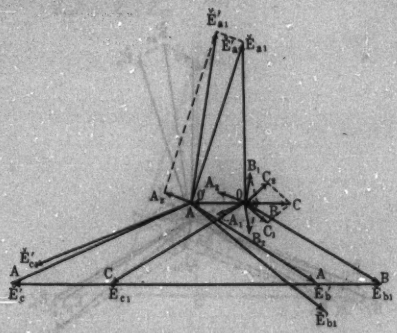

FIG. 9—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER OPERATED FROM TRANSFORMER SO AS TO DELIVER BALANCED CURRENTS

Terminal voltages of phase converter $S\dot{E}'_a$

Terminal voltages of motor $S^1\dot{E}_{a1}$

Negative phase sequence e.m.fs. in phase converter $S^2\,(OA_2)$

The Phase Balancer is a device to maintain symmetry of e. m. fs. at a given point in a polyphase system. It may consist of an induction motor or synchronous condenser with an auxiliary machine connected in series to supply an e. m. f. always proportional to the product of the negative phase sequence current passing through the machine and the negative phase sequence impedance of the balancer. It therefore has the effect of annulling the impedance of the machine to the flow of negative phase sequence current. Thus, in a symmetrical polyphase network, where we have an unbalanced system of currents due to certain conditions

$$S\,\dot{I}_a = S^1\,\dot{I}_{a1} + S^2\,\dot{I}_{a2} \tag{217}$$

If a balancer be placed at the proper point the component $S^2 \check{I}_{a2}$ will circulate between the loads and the phase balancer, the other component $S^1 I_{a1}$ being furnished from the power house. On the other hand, if there be a dissymmetry in the impedance of the system up to the phase balancer, the latter will draw a negative phase sequence current sufficient to counteract the unbalance due to any symmetrical load by causing the proper amount of negative phase sequence current to flow to produce a balance.

The balancer may be made inherently self-balancing by inserting in series with it a machine which is self-exciting and is able

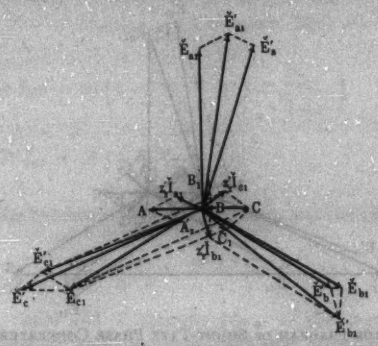

FIG. 10—VECTOR DIAGRAM SHOWING RELATIONS BETWEEN MOTOR TERMINAL E.M.F'S., CONVERTER TERMINAL E.M.FS., AND SYMMETRICAL GENERATED E.M.F'S., SAME CONNECTION AS FOR FIG. 9.

Negative phase sequence drops in phase converter $S^2 Z_2' \check{I}_{a1}$
Conjugate positive phase sequence e.m.fs. $S^1(ABC)$

to furnish an e. m. f. equal to the negative phase sequence impedance drop. The combination thus has zero impedance to negative phase sequence currents. If in the neighborhood of a phase balancer the loads have impedances

$$S\,Z_a = S^0\,Z_{a0} + S^1\,Z_{a1} + S^2\,Z_{a2}$$

The equations of the system are

$$\left.\begin{aligned} S^1\,\check{E}_{a1} &= S^1\,Z_{a0}\,\check{I}_{a1} + S^1\,Z_{a2}\,\check{I}_{a2} \\ S^2\,E_{a2} &= O = S^2\,Z_{a0}\,\check{I}_{a2} + S^2\,Z_{a1}\,\check{I}_{a1} \end{aligned}\right\} \quad (218)$$

The currents in the phase balancer are

$$-S^2 \check{I}_{a2} \text{ and } S^1 \frac{\check{E}_{a1}}{Z_1'}$$

The solution of (**218**) gives $S^2 \check{I}_{a2}$ and $S^1 \check{I}_{a1}$, the former of which are the phase balancer currents. The solution is

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{Z_{a0}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \\ \check{I}_{a2} &= -\frac{Z_{a1}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \end{aligned}\right\} \quad (\mathbf{219})$$

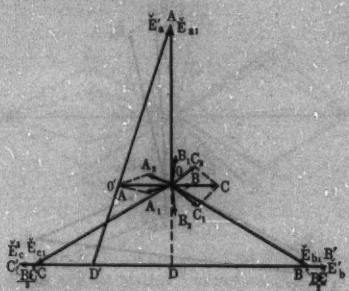

FIG. 11—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER SCOTT CONNECTED WITH COMPENSATION BY TRANSFORMER TAPS

Terminal voltages of converter $D'A$ and $B'C'$

Terminal voltages of motor $S^1\check{E}_{a1}$

The phase balancer is a voltage balancer and will maintain balanced e. m. f. for any condition of impedance, and if the impedance of the mains is unsymmetrical it will draw a sufficient amount of wattless negative phase sequence current through these mains to produce an e. m. f. balance at its terminals. Hence the complete solution requires consideration of all the connections in the network between the supply point and the balancer. Two equations for each mesh and connection are required, one of the positive phase sequence e. m. fs. and the other of the negative phase sequence e. m. f., and these equations may be solved in the usual way.

Series Phase Converter. In discussing the various reactions in rotating machines we have made use of the terms "positive phase

sequence impedance" and "negative phase sequence impedance." These terms are definite enough when dealing with relations between machines whose generated e. m. fs. all have the same phase sequence, but require further definition when we are dealing with relations between machines whose e. m. fs. have different phase sequence. We shall retain the symbols Z_1 and Z_2 for the values of the positive and negative phase sequence impedances, depending upon the sequence symbol S to define whether these impedances apply to a negative or positive phase sequence current. Thus, the phase sequence of the currents and

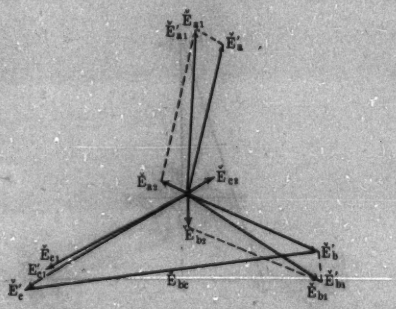

FIG. 12—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER WITH AUXILIARY ROTATING COMPENSATOR TO EFFECT A PERFECT BALANCE

Terminal voltages of phase converter $S\,\check{E}_a'$
Terminal voltages of motor $S^1\check{E}_{a1}$
Terminal voltages of compensator $S^2\check{E}_{a2}$

e. m. f. will be defined by the apparatus supplying and receiving power and the impedances of the transmitting apparatus will be defined in relation to these currents. As an example a motor series connected in counter phase sequence relation in a circuit and driven in a positive direction will have impedances

$$\begin{matrix}\text{positive phase sequence } Z_2 \\ \text{negative phase sequence } Z_1\end{matrix} \tag{220}$$

Where an auxiliary machine is defined as being of negative phase sequence relation to other machines, it will have imped-

ances as given above to the positive and negative phase sequence currents passing through the other machines.

A single-phase transformer winding tapped at the middle point may be regarded as an unbalanced three-phase system where

$$\check{E}_a = O, \check{E}_b = + \check{E}_s, \check{E}_c = - \check{E}_s$$

$2\check{E}_s$ being the single-phase e. m. f The system may be represented by the equation

$$S\check{E}_a = S^1\check{E}_{a1} + S^2\check{E}_{a2}$$

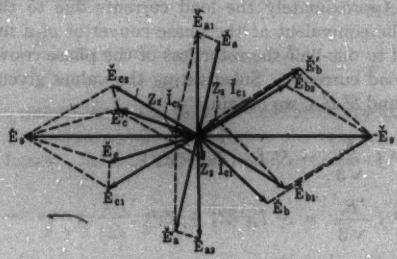

FIG. 13—VECTOR DIAGRAM OF SERIES-TYPE CONVERTER.
NO LOAD E.M.F'S. ACROSS MOTOR TERMINALS $S_1\check{E}_{a1}$
NO LOAD E.M.F'S. ACROSS CONVERTER TERMINALS $S^2\check{E}_{a2}$
SINGLE-PHASE E.M.F'S. $2\check{E}_s$
E.M.F. ACROSS TERMINAL OF MOTOR UNDER LOAD $\check{E}_a\check{E}_b\check{E}_c$
E.M.F. ACROSS TERMINAL OF CONVERTER UNDER LOAD $\check{E}'_a\check{E}'_b\check{E}'_c$

$$\left.\begin{aligned} \text{where } \check{E}_{a1} &= j\,\frac{\check{E}_s}{\sqrt{3}} \\ \check{E}_{a2} &= -j\,\frac{\check{E}_s}{\sqrt{3}} \end{aligned}\right\} \quad (221)$$

If, therefore between the single-phase source of power and the load we interpose a polyphase machine with e. m. f. $- S^2\check{E}_{a2}$, we shall have at the load terminals the e. m. f. $S^1\check{E}_{a1}$. If we use an induction-type phase converter it will have impedances to motor currents as follows

$$\left.\begin{aligned} &\text{To positive phase sequence } Z_2' \\ &\text{To negative phase sequence } Z_1' \end{aligned}\right\} \quad (222)$$

we therefore have the relations

$$S^1 \check{E}_{a1} = S^1 \check{I}_{a1} (Z_1 + Z_2') \tag{223}$$

$$S^2 \check{E}_{a2} = S^2 \check{I}_{a2} (Z_2 + Z_1') \tag{224}$$

If the converter is doing no mechanical work, Z_1' is large compared with Z_2' or Z_2, and therefore the component of negative phase sequence is small in the motor. The value of Z_1' depends upon the slip of the phase converter which will depend on the mechanical load it carries as well as on the load carried by the motors. Approximately the load currents due to the motors produce the equivalent at the phase converter of a mechanical load equal to one-half the rotor loss of the phase converter due to these load currents. Substituting the values given in (221) for $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$, we obtain

$$\left.\begin{aligned} S^1 j \frac{\check{E}_s}{\sqrt{3}} &= S^1 \check{I}_{a1} (Z_1 + Z_2') \\ - S^2 j \frac{E_s}{\sqrt{3}} &= S^2 \check{I}_{a2} (Z_2 + Z_1') \end{aligned}\right\} \tag{225}$$

$$\begin{aligned} S^1 \check{I}_{a1} &= \quad S^1 j \frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \\ S^2 \check{I}_{a2} &= - S^2 j \frac{\check{E}_s}{\sqrt{3}\,(Z_2 + Z_1')} \end{aligned} \tag{226}$$

If instead of an induction-type phase converter a synchronous phase converter is used an e. m. f. of negative phase sequence $S^2 \check{E}_{a2}$ the generated e. m. f. of the phase converter must be introduced in equations (224) *and* (225) *and the value and phase of these e. m. fs. will depend upon the load on the phase converter shaft as well as the load carried by the motors.* The equations will be

$$S^1 \check{E}_{a1} = S^1 \check{I}_{a1} (Z_1 + Z_2') \tag{227}$$

$$S^2 \check{E}_{a2} = S^2 \check{I}_{a2} (Z_2 + Z_1') + S^2 \check{E}_{a2}' \tag{228}$$

or

$$\left.\begin{aligned} S^1 j \frac{\check{E}_s}{\sqrt{3}} &= S^1 \check{I}_{a1} (Z_1 + Z_2') \\ - S^2 j \frac{\check{E}_s}{\sqrt{3}} &= S^2 \check{I}_{a2} (Z_2 + Z_1') + S^2 \check{E}_{a2}' \end{aligned}\right\} \tag{229}$$

The last member of equations (229) is the equation of a synchronous condenser. Assuming its windage, iron loss and increased losses due to secondary reactions to be P_0, we have by equation (160) of the Section on Synchronous Motors

$$\frac{E_s}{\sqrt{3}} I_{a2} \cos \alpha = I_{a2}^2 (R_2 + R_1') + \frac{P_0}{3} \tag{230}$$

Let

$$\dot{I}_{a2} = a_2 + j\, b_2 \tag{231}$$

then (230) becomes

$$\frac{E_s}{\sqrt{3}} a_2 = (a_2^2 + b_2^2)(R_2 + R_1') + \frac{P_0}{3} \tag{232}$$

Of the two quantities a_2 and b_2, b_2 alone is arbitrary and depends upon the excitation, a_2 will depend upon the value of b_2 and also upon the losses. Solving therefore for a_2 in terms of b_2, we have

$$a_2 = \frac{E_s}{2\sqrt{3}(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1')\{3 b_2^2 (R_2 + R_1') + P_0\}}{E_s^2}} \right\} \tag{233}$$

Since b_2 is arbitrary we may now determine $\cos \alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$ and the value of $\dot{I}_{a2}$ in terms of the impressed e. m. f. will be by (181) of Section on Synchronous Motors

$$S^2 \dot{I}_{a2} = -S^2 \left[j \frac{\dot{E}_s}{\sqrt{3}} \frac{\cos \alpha_2}{2(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \epsilon^{j\alpha} \right] \tag{234}$$

The effective value of $\dot{I}_{a2}$ in terms of the effective value of $\dot{E}_s$ will then be

$$I_{a2} = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \tag{235}$$

and since the component of the e.m.f. generated in phase with the current is determined only by the magnitude of $\check{I}_{a2}$ and the motor losses, if we define its value by A_2' the quadrature component being B_2' we shall have

$$A_2' = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{236}$$

and

$$B_2' = -\frac{E_s}{\sqrt{3}} \sin \alpha_2 - \frac{w\,(L_2 + L_2')}{A_2^1} \tag{237}$$

$$= -\frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \tag{238}$$

and therefore we have

$$\check{E}_2' = -j \frac{E_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1') \cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \right] (\epsilon^{jwt}) \tag{239}$$

The impedance of the phase converter to the flow of negative phase sequence current is

$$\frac{2\,(R_2 + R_1') \sec \alpha}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \tag{240}$$

The balance will be at its best when $\check{I}_{a2}$ is a minimum with $\cos \alpha_2$ as the independent variable. This will be the case when $\cos \alpha_2$ is unity; that is to say when b_2 is zero.

Recapitulating the results given above, we have for the general case taking the single-phase e. m. f. $\check{E}_s$ as reference

$$S^1 \check{I}_{a1} = S^1 j \frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \tag{241}$$

$$S^2 \check{I}_{a2} = -j\,(a_2 + j\,b_2) \tag{242}$$

where b_2 is arbitrary and

$$a_2 = \frac{E_s}{2\sqrt{3}\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,\{3\,b_2^2\,(R_2 + R_1') + P_0\}}{E_s^2}} \right\} \tag{243}$$

Since b_2 is arbitrary $\cos\alpha_2$ is determined by

$$\cos\alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \tag{244}$$

we may express $\check{I}_{a2}$ in terms of $\check{E}_s$ by

$$S^2 \check{I}_{a2} = -S^2 j \frac{\check{E}_s}{\sqrt{3}} \frac{\cos\alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2\alpha_2}} \right\} \epsilon^{j\alpha_2} \tag{245}$$

The effective value of $\check{I}_{a2}$ will be

$$I_{a2} = \sqrt{a_2^2 + b_2^2} = \frac{E_s}{\sqrt{3}} \frac{\cos\alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2\alpha_2}} \right\} \tag{246}$$

If A_2' and B_2' are components of $\check{E}_{a2}'$ these being the generated e. m. f. in phase and in quadrature with the current $\check{I}_{a2}$ we shall have

$$\check{E}_{a2}' = -j\,(A_2' + j\,B_2') \tag{247}$$

and A_2' and B_2' will have the following values

$$A_2' = \frac{E_s}{\sqrt{3}} \; \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{248}$$

$$B_2' = -\frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + Q_1')\,P_0}{E_s^2 \cos \alpha^2}}\right) \right\} \tag{249}$$

and $\check{E}_{a2}'$ expressed in terms of $\check{E}_s$ becomes

$$\check{E}_2' = -j\,\frac{\check{E}_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L')\cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \right] \tag{250}$$

The effective impedance of the phase converter to the flow of negative phase sequence currents is

$$\frac{2\,(R_2 + R_1')\sec \alpha_2}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \,(\cos \alpha_2 - j \sin \alpha_2) \tag{251}$$

or

$$\frac{E_s^2}{P_0} \cdot \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \epsilon^{-j\alpha_2} \tag{252}$$

In the above equations $\cos \alpha_2$ is arbitrary or b_2 may be considered arbitrary and $\cos \alpha_2$ will then be determined.

Minimum Unbalance is obtained when $\cos \alpha_2$ is made unity or when b_2 is made zero in equations (**241**) and (**252**).

Perfect Balance is obtained by driving the phase converter mechanically so as to supply the mechanical power P_0 from a separate or symmetrical source. Under this condition a_2 and b_2 both become zero when $\cos \alpha_2$ is unity. The only equation of the system is then (**241**).

Currents and Power Factor in the Single-Phase Supply Circuit of Series Phase Converter.

The e. m. f. is $2\,\check{E}_s$ and the current supplied is

$$\check{I}_s = \frac{\check{I}_b - \check{I}_c}{2}$$

$$= \frac{\check{I}_{b1} - \check{I}_{c1}}{2} + \frac{\check{I}_{b2} - \check{I}_{c2}}{2} \tag{253}$$

If we take

$$S^1\,\check{I}_{a1} = S^1 j\,(a_1 - j\,b_1) \tag{254}$$

$$\frac{\check{I}_{b1} - \check{I}_{c1}}{2} = \frac{\sqrt{3}}{2}\,(a_1 - j\,b_1) \tag{255}$$

Similarly, since under the same conditions

$$S^2\,\check{I}_{c2} = -\,S^2 j\,(a_2 + j\,b_2) \tag{256}$$

$$\frac{\check{I}_{b2} - \check{I}_{c2}}{2} = \frac{\sqrt{3}}{2}\,(a_2 + j\,b_2) \tag{257}$$

and therefore

$$\check{I}_s = \frac{\sqrt{3}}{2}\,\{(a_1 + a_2) - j\,(b_1 - b_2)\} \tag{258}$$

where a_1, b_1, a_2, b_2 are to be obtained by means of equations (**243**) to (**254**). The single-phase power factor is given by

$$\tan\,\theta = \frac{b_1 - b_2}{a_1 + a_2} \tag{259}$$

of these quantities a_2 is usually the smallest and its value may be obtained approximately by assigning to b_2 a value which will make the ratio $\dfrac{b_1 - b_2}{a_1}$ equal to $\tan\,\theta$, and obtaining the corresponding value of a_2 by (**242**), the value of b_2 may then be recalculated from (**259**) by substituting the tentative value obtained for a_2. This procedure may be repeated until sufficient accuracy has been obtained.

Single-Phase Power Factor in Shunt-Type Phase Converter.

The simplest procedure is to obtain a curve of admittances for varying excitation of the converter and plot the power factor obtained by varying the admittance with a fixed load. The true

and wattless power is obtained easily by means of (208) whether the system is balanced or unbalanced.

Figs. 14, 15, 16 and 17 are vector diagrams of several methods of using phase converters to supply a balanced 3-phase e. m. f. to a symmetrical load such as an induction motor. The diagrams are all based on a main machine having the same negative phase sequence impedance and the system in each case is

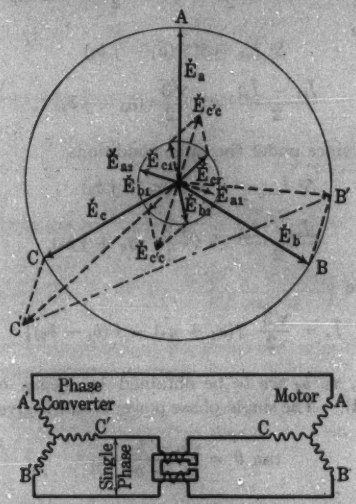

FIG. 14

SINGLE-PHASE IMPRESSED E.M.F. $= B'C'$
MOTOR E.M.F. $= BC$
NEGATIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a1}\check{E}_{b1}\check{E}_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB'C'$

delivering the same amount of power at the same voltage and 3-phase power factor without supplying any wattless power. It will be noted that the scheme Fig. 14 has the lowest single-phase power factor, Fig. 16 the highest and the rest arcing alike. It may be remarked, however, that with the shunt-type schemes adjustments can be made for power factor correction which will result also in better regulation.

APPENDIX I

Cylindrical Fields in Fourier Harmonics

When we have a diametrical coil around a cylinder concentric with another cylinder which forms the return magnetic path, and the length of the gap is uniform and the coil dimensions are very small, the field across the gap takes the form of a square topped

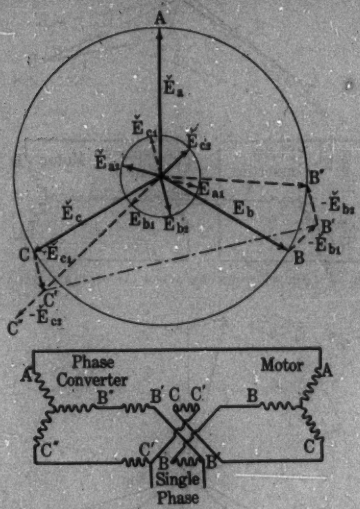

FIG. 15

SINGLE-PHASE IMPRESSED E.M.F. $= B'C'$
MOTOR E.M.F. $= BC$
PHASE CONVERTER E.M.F. $= B''C''$
NEGATIVE PHASE SEQUENCE E.M.F $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.F. $\check{E}_a{}^1E_{b1}E_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB''C''$

wave, which may be expressed in the form of a Fourier series with the plane of symmetry of the coil as reference plane, and its Fourier expansion is

$$\mathfrak{G} = \frac{4B}{\pi}\left(\cos\theta - \tfrac{1}{3}\cos 3\theta + \frac{1}{5}\cos S\theta - \ldots + \ldots\right) \qquad (1)$$

where B is the average induction in the air gap.

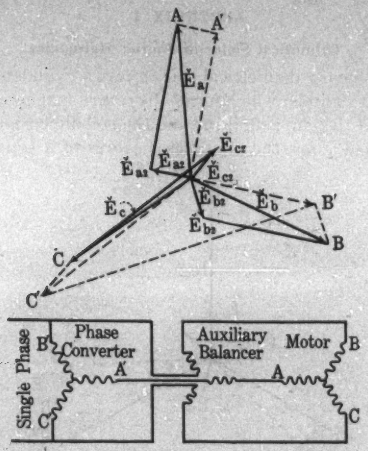

FIG. 16—PHASE CONVERTER WITH AUXILIARY BALANCER.

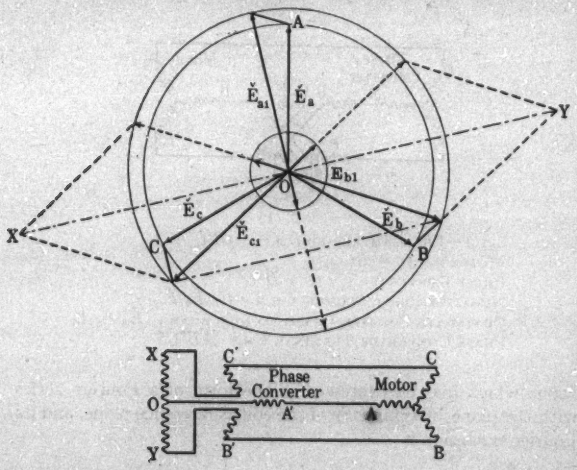

FIG. 17

SINGLE-PHASE IMPRESSED E.M.F. = XY

MOTOR E.M.F. = ABC

THERE IS A 2 TO 1 TRANSFORMATION OF E.M.F. FROM SINGLE-PHASE TO THREE-PHASE IN THIS CONNECTION

With pitch less than π the curve will have a different form, the amplitude being greater on one side of the plane of the coils than on the other, the areas of each wave will remain the same and second harmonic terms will appear. Let $2\,m_0\,\pi$ be the new pitch then the average amplitude of the induction will be the same as before, namely B, and the value on one side of the coil will be $2\,(1-m_0)B$ and on the other side $2\,m_0\,B$ so that the total flux will be the same on either side. To obtain the values of the coefficients we have

$$2\,(1-m_0\,B\int_0^{m_0\pi}\cos n\,\theta\,d\,\theta+2\,m_0\,B\int_{m_0\pi}^{2\pi}\cos n\,\theta\,d\,\theta=\frac{\pi}{2}\,A_n$$

$$2\,(1-m_0)\,B\left[\frac{1}{n}\sin n\,\theta\right]_0^{m_0\pi}-2\,m_0\,B\left[\frac{1}{n}\sin n\,\theta\right]_{m_0\pi}^{2\pi}=\frac{\pi}{2}\,A_n$$

$$A_n=\frac{4\,B}{\pi}\left\{\frac{(1-m_0)+m_0}{n}\sin n\,m_0\,\pi\right\}$$

$$A_n=\frac{4\,B}{\pi}\left(\frac{1}{n}\sin n\,m_0\,\pi_1\right) \tag{2}$$

Let $2\,m_0\,\pi=\frac{2}{3}\,\pi$, then $(1-m_0)\,\pi=\frac{2}{3}\,\pi$ and

$$\mathfrak{B}=\frac{2\sqrt{3}\,B}{\pi}\left(\cos\theta+\frac{1}{2}\cos 2\,\theta-\frac{1}{4}\cos 4\,\theta-\frac{1}{5}\cos 5\,\theta\right.$$

$$\left.+\frac{1}{7}\cos 7\,\theta+\frac{1}{8}\cos 8\,\theta-\frac{1}{10}\cos 10\,\theta\;.\;.\;.\right) \tag{3}$$

A general expression for $\mathfrak{B}$ where B is the average of the positive and negative, maximum value for any pitch coil would be

$$\mathfrak{B}=\frac{4\,B}{\pi}\,\Sigma\left(\frac{1}{n}\sin n\,m_0\,\pi\cos n\,\theta\right) \tag{4}$$

and includes all possible coil pitches. If the number of teeth in a pole pitch be n_τ; in addition to the average induction as indicated by (4), there will also be a tooth ripple of flux, the maximum value of which will depend upon the average value of the induction at each point. The value of m_0 must be a fraction having n_τ as denominator and an integral numerator. The

value of the integral numerator is therefore always $m_0 n_\tau$. The correct value for the max. induction will therefore be

$$\mathfrak{B}_m = \left\{ \frac{4B}{\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \pi \cos n\, \theta \right) \right\} (1 - (-1)^{m_0 n_\tau} K_\tau \cos n_\tau \theta) \qquad (5)$$

where K_τ is the ratio of the average to the min. air gap. "m_0" must always be chosen so that $m_0 n_\tau$ is an integer.

If the length of the average effective air gap in centimeters be d the value of B is given by

$$B = \frac{4\pi}{10} \frac{I N}{2 d} \text{ gauss}$$

where I is the maximum value of the current in the coil and N is the number of turns. If d is given in inches we may write

$$B = \frac{4\pi}{10} \frac{I N}{2 d} \times 2.54 \text{ maxwells per square inch.}$$

If we integrate (5) between the limits $(\theta - m_0 \pi)$ and $(\theta + m_0 \pi)$ we shall have the total flux φ through the coil

$$\varphi = \frac{4 B r l}{\pi} \int_{\theta - m_0 \pi}^{\theta + m_0 \pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \pi \cos n\, \theta \right) d\theta$$

$$- \frac{4 B r l}{\pi} (-1)^{m_0 n_\tau} \int_{\theta - m_0 \pi}^{\theta + m_0 \pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \cos n\, \theta \right) K_\tau \cos n_\tau \theta\, d\, 0$$

$$= \frac{4 B r e}{\pi} \left[\frac{1}{n^2} \sin n\, m_0 \pi \sin n\, \theta \right]_{\theta - m_0 \pi}^{\theta + m_0 \pi}$$

$$- \frac{4 B r l}{\pi} (-1)^{m_0 n_\tau} K_\tau \Sigma \frac{1}{n} \sin m_0 n \pi \left[\frac{\sin (n - n_\tau) \theta}{2 (n - n_\tau)} + \frac{\sin (n + n_\tau \theta)}{2 (n + n_\tau)} \right]_{\theta - m_0 \pi}^{\theta + m_0 \pi} \qquad (6)$$

The second expression is zero for all values of θ which are integral multiples of the tooth pitch angle, so long as $m_0 n$ is also an integer and therefore it is zero for all mutual inductive relations of similar coils on a symmetrical toothed core we therefore have:

The induction through a coil displaced an angle θ from the axis of a similar coil carrying a current giving a mean induction B both coils being wound on the same symmetrical toothed core is

$$\varphi = \frac{8 B r l}{\pi} \Sigma \left(\frac{1}{n^2} \sin^2 n\, m_0\, \pi \cos n\, \theta \right) \tag{7}$$

The second term in equation (**6**) also becomes zero when n_τ becomes infinite independent of the value of θ. We may therefore safely make use of an imaginary uniformly distributed winding when considering self and mutual impedances. It will also be shown later on, that with certain groupings of windings the second term may be reduced to zero for every value of θ.

If N_1 be the total number of complete loops in one complete pole pitch, we may take $\frac{N_1}{2\pi}$ as the density of winding per unit angle of the complete pole pitch. The mutual induction per turn in a coil angularly displaced an angle θ from another coil of winding density $\frac{N_1}{2\pi}$ with an effective total air gap $2d$ and with windings subtending an angle $2 m_1 \pi$ is given by

$$M_1 = \frac{8 N_1 r l}{10^9 \pi d} \int_{-m_1 \pi}^{+m_1 \pi} \Sigma \left\{ \frac{1}{n^2} \sin^2 n\, m_0 \pi \cos n(\theta + \theta') \right\} d\theta' \text{ henrys} \tag{8}$$

$$= \frac{8 N_1 r l}{10^9 \pi d} \Sigma \frac{1}{n^3} \sin^2 n\, m_0\, \pi \left[\sin n\, (\theta + \theta)' \right]_{\theta' = -m_1 \pi}^{\theta' = m_1 \pi} \text{ henrys}$$

$$M_1 = \frac{16 N_1 r l}{10^9 \pi d} \Sigma \left(\frac{1}{n^3} \sin^2 n\, m_0 \pi \sin n\, m_1 \pi \cos n\, \theta \right) \text{ henrys} \tag{9}$$

Next, if the loop of which M_1 is the mutual inductance is part of a winding having distribution density of winding $\frac{N_2}{2\pi}$ and subtending an angle $2 m_2 \pi$ its mutual inductance with the other winding will be

$$M_{12} = \frac{8 N_1 N_2 r l}{10^9 \pi^2 d} \int_{-m_2 \pi}^{m_2 \pi} \Sigma \frac{1}{n^3} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, (\theta + \theta')\, d\theta' \text{ henry} \tag{10}$$

$$= \frac{8 N_1 N_2 r l}{10^9 \pi^2 d} \Sigma \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \left[\sin n\, (\theta + \theta') \right]_{\theta' = -m_2 \pi}^{\theta' = m_2 \pi} \text{ henrys}$$

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (11)$$

This is the general expression for the mutual inductance between two groups of connected coils of like form on the same cylindrical core. It should be noted how much the harmonics have been reduced due to grouping.

When the coils are not of like design as in the case of a rotor and stator and the pitch of the coils is different in one from the other, $\sin n\, m_0\, \pi$ will not appear twice in the equation but one of its values must be replaced by $\sin n\, m_x\, \pi$ where $2\, m_x\, \pi$ is the pitch of the new coil. Equation (**11**) then becomes

$$M_{1x} = \frac{16\, N_1\, N_x\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin n\, m_0\, \pi \sin n\, m_x\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (12)$$

This formula is strictly correct when m_x is an integer and when θ is an integral multiple of the tooth pitch. It is true for all values of θ if either m_0 or m_x or both are unity.

By considering the axes of two similar groups of coils as coincident we obtain the value of $\Delta_1\, L_1$ which is part of the self inductance of the group, thus

$$\Delta_1\, L_1 = \frac{16\, N_1^2\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right) \qquad (13)$$

The other factor that enters into the self inductance is the slot leakage inductance which depends upon the number of turns in a coil, the number of coils in a group and the width and depth of the slot and the length of the air gap. Since with the value of $\Delta_1\, L_1$ all the field which links the secondary winding has been included, only the portion of the slot leakage which does not link all the turns in the opposed secondary coil should be considered. No hard and fast rule can be made for determining this quantity since it depends upon the shape of the slots, there should be little trouble in making the calculation when the data are given. Denoting this quantity by $\Delta_2\, L_1$ we have

$$L_1 = \Delta_1\, L_1 + \Delta_2\, L_1 \qquad (14)$$

Symmetrically Grouped Windings. The above formulas give the mutual impedance between groups of coils, each group of which may be unsymmetrical. Generally machines are designed so that, although the individual groups of coils due to fractional pitch may be unsymmetrical, the complete winding is symmetrical. When two coils are together in a slot this may be done by connecting one group of coils opposite the north pole in series with the corresponding group opposite the south pole; that is to say, the group displaced electrically by the angle π. If therefore we take equation (**11**) and consider the mutual induction as due to a group having axis at θ = zero and another having its axis at $\theta = \pi$ with a similarly arranged group of coils having its axis at θ, we find that (**11**) becomes

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (\mathbf{15})$$

Similarly

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin n\, m_1\, \pi \sin n\, m_x\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi \sin n\, m_a\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (\mathbf{16})$$

Since $1 - \cos n\pi$ is zero for all even values of n it is evident that (**15**) and (**16**) contain no even harmonics, moreover the above formulas give the mutual induction between two similarly connected groups of windings, but if $(1 - \cos n\, \pi)$ is used only with the first power these formulas give the mutual impedance between one pair of such symmetrically grouped windings and another single group with axis inclined at an angle θ.

The value of self induction is

$$\Delta_1 L_1 = \frac{16\, N_1^2\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right.$$

$$\left. (1 - \cos n\, \pi)^2 \right\} \qquad (\mathbf{17})$$

$\Delta_2 L_1$ is found in the same manner as before

$$L_1 = \Delta_1 L_1 + \Delta_2 L_1 \qquad (\mathbf{18})$$

It is obvious from (**15**) and (**16**) that the effect of dissymmetry is to introduce more or less double frequency into the wave form of generated e. m. f.

It will be seen from an examination of (**15**) and (**17**) that, for example, a winding of pitch $\frac{2\pi}{3}$ and subtending an angle $\frac{\pi}{3}$ when connected in a symmetrical group of two has the same field form and characteristics as a full pitch winding of the same number of turns subtending an angle $\frac{2\pi}{3}$.

There are many symmetrical forms of winding but all will be found to be covered by the formulas (**15**) and (**16**).

Unsymmetrical Windings. These may take many forms which may be classified:

(1) Dissymmetry of flux form due to even harmonics.

(2) Dissymmetry in axial position of polyphase groups.

(3) Dissymmetry in windings due to incorrect grouping of coils.

(4) Dissymmetry due to unsymmetrical magnetic characteristics of the iron.

Of these various forms of dissymmetry the most common is a combination of (**1**), (**2**) and (**3**). These forms of unsymmetrical windings may all be calculated by the formulas (**11**) to (**16**).

It is to be noted that the mutual inductance between a symmetrical and an unsymmetrical winding is harmonically symmetrical. Hence, if the field of a machine is harmonically symmetrical, the e. m. f. generated will be also harmonically symmetrical whatever may be the form of the windings.

The reciprocal nature of M is fully established by its form, for it is immaterial in obtaining (**16**) whether we start out with the winding whose pitch is m_z or with that whose pitch is m_0, the result will be the same. The effect of saturation will be to tend to alter the values of the coefficients of M but the general form will not vary appreciably. We shall now consider some standard windings of generators and motors.

Three-Phase Symmetrical Full Pitch. Here m_0, m_1 and m_2 are 0.5, 0.1666 and 0.1666 respectively. Using formula (**15**) all the even harmonics disappear and $(1 - \cos n\pi)^2$ is equal to 4 or zero.

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\ \pi^2\, d} \left(\cos\theta + \frac{4}{81}\cos 3\theta + \frac{1}{625}\cos 5\theta + \frac{1}{2401}\cos 7\theta + \frac{4}{6561}\cos 9\theta + \ldots\right) \qquad (19)$$

Theoretical Symmetrical Three-Phase Winding. Here $m_0 = 0.5, m_1 = m_2 = 0.333$. Using formula (**11**)

$$M_{12} = \frac{3}{4} \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d} \left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2401}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \tag{20}$$

Here the third group of harmonics is entirely eliminated.

Three-Phase Symmetrical $\frac{2\pi}{3}$ *Pitch Winding.* Here $m_0 = 0.333, m_1 = m_2 = 0.166$. Using formula (**15**)

$$M = \frac{3}{4} \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, a} \left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2461}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \tag{21}$$

which gives the same result as (**20**).

Formulas for Salient Pole Machines

The formulas given in the preceding discussion are appropriate for distributed winding and non-salient poles. Where salient poles are used the field form due to the poles with a given winding will be arbitrary so that with the polar axis as reference we shall have

$$\mathfrak{B} = \frac{2\,\pi\, N_a\, I_a}{d}\, \Sigma\,(A_n \cos n\,\theta) \tag{22}$$

Where $\mathfrak{B}$ is the induction through the armature or stator. When the poles are symmetrical $A_n \cos n\,\theta$ might be chosen at once for this condition and in this case we do not require coefficients of mutual induction between pole windings, since the value of $\mathfrak{B}$ is obtained by considering the mutual reaction between pole windings to be such as will produce symmetry. We may however assume $\mathfrak{B}$ to be perfectly general in form in which case the flux through a coil of pitch $2\, m_0\, \pi$ is

$$\varphi = \frac{4\,\pi\, N_a\, I_a\, r\, l}{10\, d}\, \Sigma \left(\frac{A_n}{n} \sin n\, m_0\, \pi \cos n\,\theta\right) \tag{23}$$

We have therefore for the mutual induction between one pole and a group of coils at an angle θ and subtending an angle $2\, m_1\, \pi$

$$M_{a1} = \frac{4\ N_a\ N_1\ r\ l}{10^9\ d}\ \Sigma\left(\frac{A_n}{n^2}\ \sin n\ m_0\ \pi \sin n\ m_1\ \pi \cos n\ \theta\right) \quad (24)$$

and where there is symmetry due to grouping of windings, we have

$$M_{a1} = \frac{4\ N_a\ N_1\ r\ l}{10^9\ d}\ \Sigma\left\{\frac{A_n}{n^2}\ \sin n\ m_0\ \pi \sin n\ m_1\ \pi\right.$$

$$\left.(1 - \cos n\ \pi)^2 \cos n\ \theta\right\} \quad (25)$$

where N_a is the number of turns for one pole and (**25**) applies to one pair of poles and the corresponding group of coils. When there are more than one pair of poles in series and the corresponding groups of winding are also in series, if it is desired to consider the mutual inductance of the complete winding, the result given above must be multiplied by the number of pairs of poles.

If in equation (**16**) we take

$$\left.\begin{aligned} &\frac{N_a}{2\ \pi}\ \frac{1}{n}\ \sin n\ m_a\ \pi = N_a \\ \text{and}\qquad &\frac{1}{\pi\ n} = B_n \end{aligned}\right\} \quad (26)$$

it becomes

$$M_{1a} = \frac{32\ N_1\ N_a\ r\ l}{10^9\ d}\ \Sigma\left\{\frac{B_n}{n^2}\ \sin n\ m_x\ \pi \sin n\ m_0\ \pi\right.$$

$$\left.\sin n\ m_1\ \pi\ (1 - \cos n\ \pi)^2 \cos n\ \theta\right\} \quad (27)$$

which is the expression corresponding to (**25**) starting with the winding flux form. (**25**) and (**27**) must therefore be identical and we have

$$\frac{32\ N_1\ N_a\ r\ e}{10^9\ d}\ B_n \sin n\ m_x\ \pi - \frac{4\ N_a\ N_1\ r\ e}{10^9\ d}\ A_n$$

or

$$\mathfrak{B}_n = \frac{A_n}{8 \sin n\ m_x\ \pi} \quad (28)$$

and

$$\mathfrak{B}_1 = \frac{2\ \pi\ I_1}{10\ d}\ \Sigma\ (B_n \sin n\ m_0\ \pi \cos n\ \theta) \quad (29)$$

and is the induction wave form for a single turn of the winding.

The expression for the mutual inductance between windings of the same core for salient poles is obtained in terms of the pole flux wave form by substituting in the formulas $\frac{A_n}{8 \sin n\, m_x \pi}$ for $\frac{1}{n\,\pi}$. We have therefore the following formulas for salient poles.

General expression considering only one pole and one group of coils.

$$\mathfrak{B}_a = \frac{2\,\pi\, N_a\, I_a}{10\, d}\, \Sigma\, (A_n \cos n\, \theta) \tag{a}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left(A_n\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_a\, \pi} \cos n\, \theta\right) \tag{b}$$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta\right) \tag{c}$$

$$M_{12} = \frac{2\, N_1\, N_2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n^2}\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_x\, \pi} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. \sin n\, m_2\, \pi \cos n\, \theta\right) \tag{d}$$

$$\Delta_1 L_a = \frac{4\, \pi\, N_a^2\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n} \sin n\, m_x\, \pi\right) \tag{e}$$

$$\Delta_1 L_1 = \frac{2\, N_1^2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n}\, \frac{\sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi}{\sin n\, m_x\, \pi}\right) \tag{f}$$

General expressions considering only poles to be symmetrical. Considered on the basis of two poles, N_a being turns on one pole.

$$\mathfrak{B}_a = \frac{2\, \pi\, N_a\, I_a}{10\, d}\, \Sigma\, \{A_n\, (1 - \cos n\, \pi) \cos n\, \theta\} \tag{a'}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left\{A_n\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_x\, \pi}\, (1 - \cos n\, \pi) \cos n\, \theta\right\} \tag{b'}$$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left\{\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. (1 - \cos n\, \pi) \cos n\, \theta\right\} \tag{c'}$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^3}\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\,\cos n\,\theta\right\}\text{ henrys}\qquad(\mathbf{d}')$$

$$\Delta_1 L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)\right\}\qquad(\mathbf{e}')$$

$$\Delta_1 L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\,\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right\}\qquad(\mathbf{f}')$$

General expression with both polar and winding symmetry.

$$\mathfrak{B}_a = \frac{2\,\pi\,N_a\,I_a}{10\,d}\,\Sigma\,\{A_n\,(1-\cos n\,\pi)\cos n\,\theta)\qquad(\mathbf{a}'')$$

$$\mathfrak{B}_1 = \frac{\pi\,I_1}{20\,d}\,\Sigma\left\{A_n\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,(1-\cos n\,\pi)\cos n\,\theta\right\}\qquad(\mathbf{b}'')$$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,(1-\cos n\,\pi)^2\right.$$

$$\left.\cos n\,\theta\right\}\qquad(\mathbf{c}'')$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^3}\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\,(1-\cos n\,\pi)^2\cos n\,\theta\right\}\qquad(\mathbf{d}'')$$

$$\Delta_1 L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)^2\right\}\qquad(\mathbf{e}'')$$

$$\Delta_1 L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\,\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right.$$

$$\left.(1-\cos n\,\pi)^2\right\}\qquad(\mathbf{f}'')$$

In using any of the formulas given above for machines having more than two poles, it must be divided by the number of pairs of poles and likewise the expression for M or $\Delta_1 L$ must be multiplied by the number of pairs of poles, which leaves the formula for these quantities unchanged.

Let us next consider the actual induction in the air gap with a distributed winding operating with three-phase currents. Let i_{m1} be the magnetizing current of the first phase i_{m2} and i_{m3} those of the other phases. The induction due to one group of coils of phase 1 is

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,\cos n\,\theta\right\} \tag{30}$$

and if the phase displacement of 2 and 3 from 1 be φ_{12} and φ_{13}

$$\mathfrak{B}_2 = \frac{8\,N_2\,i_{m2}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_2\,\pi\,\cos\,(n\theta - \varphi_{12})\right\} \tag{31}$$

$$\mathfrak{B}_3 = \frac{8\,N_3\,i_{m3}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_2\,\pi\,\cos\,(n\,\theta - \varphi_{13})\right\} \tag{32}$$

For symmetrically grouped coils the formulas become

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,(1 - \cos n\,\pi)\right.$$
$$\left.\cos n\,\theta\right\} \tag{33}$$

$$\mathfrak{B}_2 = \frac{8\,N_2\,i_{m2}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_2\,\pi(1 - \cos n\,\pi)\right.$$
$$\left.\cos m\,(\theta - \phi_{12})\right\} \tag{34}$$

$$\mathfrak{B}_3 = \frac{8\,N_3\,i_{m3}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_3\,\pi\,(1 - \cos n\,\pi)\cos m\right.$$
$$\left.(\theta - \varphi_{13})\right\} \tag{35}$$

For a symmetrical three-phase motor with full pitch coils $m_0 = 0.5$, $m_1 = m_2 = m_3 = 0.166$ (**33**), (**39**) and (**35**) become of the four

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\left\{\cos\theta - \frac{2}{9}\cos 3\,\theta + \frac{1}{25}\cos 5\,\theta + \frac{1}{49}\cos 7\,\theta\right.$$
$$\left. - \frac{2}{81}\cos 9\,\theta + \frac{1}{121}\cos 11\,\theta + \frac{1}{169}\cos 13\,\theta + \right\} \tag{36}$$

which is the field due to one group of coils alone. The wave is flattened by the third group of harmonics but all the other harmonics are peaking values. There is therefore a decided gain in such a wave form of flux since it permits of high fundamental flux density.

The maximum value of flux is approximately

$$B_{max} = 0.823 \cdot \frac{8\, N_1 i_m}{10\, \pi\, d} \text{ gaus} \tag{37}$$

where d is given in centimeters.

$$B_{max} = \frac{1.67\, N_1 i_m}{\pi\, d} \quad \text{maxwells per square inch,}$$

with d given in inches.

For the total winding the resultant induction will be the sum of B_1, B_2, and B. If we take the symmetrical winding with angles between planes of symmetry

$\varphi_{12} = \dfrac{2\,\pi}{3}$ and $\varphi_{13} = \dfrac{4\,\pi}{3}$, we have

$$\left.\begin{aligned} \cos n\,\theta &= \frac{\epsilon^{jn\theta}}{2} + \frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{2\,\pi}{3}\right) &= a^{-n}\,\frac{\epsilon^{jn\theta}}{2} + a^{n}\,\frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{4\,\pi}{3}\right) &= a^{n}\,\frac{\epsilon^{jn\theta}}{2} + a^{-n}\,\frac{\epsilon^{-jn\theta}}{2} \end{aligned}\right\} \tag{38}$$

If we multiply these three quantities successively by $\check{I}_{m1}$, $a^2\check{I}_{m1}$, $a\check{I}_{m1}$ and add, we have

$$\check{I}_{m1}\left\{\frac{\epsilon^{jn\theta}}{2}\,(1 + a^{-(n-2)} + a^{(n+1)})\left(+\frac{\epsilon^{-jn\theta}}{2}\right.\right.$$
$$\left.\left.\times\,(1 + a^{n+2} + a^{-(n-1)})\right)\right\} \tag{39}$$

and giving n successive odd values from 1 up, we find for (39) the following values

$n = 1$ (39) becomes $\dfrac{3}{2}\,\check{I}_{m1}\,\epsilon^{-j\theta}$

$n = 3$ " " 0

$$n = 5 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{j5\theta}$$

$$n = 7 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{-j7\theta}$$

$$n = 7 \quad " \quad " \quad 0$$

$$n = 11 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{j11\theta}$$

$$n = n \quad " \quad " \quad 2 \check{I}_{m1} \sin^2 \frac{2 n \pi}{3} \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta}$$

We may therefore express $\mathfrak{B}$ by

$\mathfrak{B}$ = real part of

$$\frac{16 N_1 \check{I}_{m1}}{10 \pi d} \Sigma \left\{ \frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \times \sin^2 \frac{2 n \pi}{3} \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta} \right\} \tag{40}$$

It will be obvious that if we proceed around the cylinder in the negative direction of rotation at an angular speed w and $\check{I}_{m1}$ is equal to $I_{m1} \epsilon^{jwt}$, for $n = 1$ the value of B_1 will remain constant and real, hence B_1 must be a constant field rotating at angular velocity w in the negative direction. The value of B may be expressed in harmonic form, but in this form it does not illustrate the rotating field theory so aptly. The harmonic form is given below and is simpler in appearance than (**40**).

$$\mathfrak{B} = \frac{16 N_1 i_{m1}}{10 \pi d} \Sigma \left(\frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \sin^2 \frac{2 n \pi}{3} \cos n \theta \right) \tag{41}$$

For a symmetrical three-phase motor with full pitch coil ($m_0 = 0.5$ $m_1 = 0.166$) $\mathfrak{B}$ becomes

$$\mathfrak{B} = \frac{12 N_1 i_{m1}}{10 \pi d} \Sigma \left\{ \cos \theta + \frac{1}{25} \cos 5 \theta + \frac{1}{49} \cos 7 \theta + \frac{1}{121} \cos 11 \theta + \frac{1}{169} \cos 13 \theta + \ldots \right\} \tag{42}$$

This gives for the maximum induction approximately

$$\mathcal{B}_{max} = \frac{1.075 \times 12\, N_1\, i_{m1}}{10\, \pi\, d} = \frac{1.29\, N_1\, i_m}{\pi\, d} \text{ gauss} \tag{43}$$

where d is measured in centimeters.

$$\mathcal{B}_{max} = \frac{3.28 \times N_1\, i_{m1}}{\pi\, d} \text{ maxwell per square inch} \tag{44}$$

where d is measured in inches and N is the total number of turns per pair of poles.

APPENDIX II

Graphical Construction for Obtaining Symmetrical Components

The graphica method for finding the symmetrical components of $S\,(\check{E}_a)$ given in the text serves as a geometrical interpretation

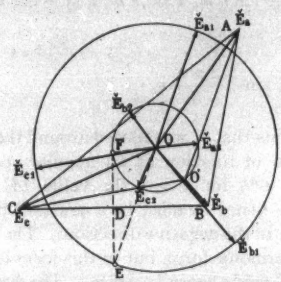

FIG. 18—GRAPHICAL METHOD FOR OBTAINING THE SYMMETRICAL COMPONENTS OF THE THREE-PHASE VECTORS $\check{E}_a, \check{E}_b, \check{E}_c$

of equation (5), but the graphical method shown in Fig. 18 is much simpler and more convenient, the construction is as follows. Find E and F the centroids of the two equilateral triangles with $B\,C$ as base: With 0 the centroid of triangle $A\,B\,C$ as centre describe the two circles passing through E and F then; $E\,O$ extended till it touches the circle through E at the opposite end of the diameter gives $\check{E}_{a1}$; $\check{E}_{b1}$ and $\check{E}_{c1}$ are obtained by laying of points on the circle 60 degrees from E. Similarly if $F\,O$ is extended to meet the circle through F at the opposite end of the diameter we obtain $\check{E}_{a2}$; and $\check{E}_{b2}$ and $\check{E}_{c2}$ are obtained by the same construction as before.

The proof of this construction is as follows: If G and H are

the apexes (not shown in figure) of the equilateral triangles having $B\,C$ as base

$$O\,E = \frac{O\,B + O\,C + O\,G}{3} = \frac{-\,O\,A + O\,G}{3} = \frac{-\,\check{E}_a + O\,G}{3}$$

$$O\,F = \frac{O\,B + O\,C + O\,H}{3} = \frac{-\,O\,A + O\,H}{3} = \frac{-\,\check{E}_a + O\,H}{3}$$

$$-\,a^2\,.\,O\,C = -\,a^2\,(O\,D + D\,C)$$

$$-\,a\,.\,O\,B = -\,a\,(O\,D + D\,B) = -\,a\,(O\,D - D\,C)$$

$$\begin{aligned} -\,a^2\,.\,O\,C - a\,.\,O\,B &= -\,(a + a^2)\,O\,D + (a - a^2)\,.\,D\,C \\ &= O\,D + j\sqrt{3}\,D\,C \\ &= O\,G \end{aligned}$$

That is

$$O\,G = -\,(a\,\check{E}_b + a^2\,\check{E}_c)$$

Similarly

$$O\,H = -\,(a^2\,\check{E}_b + a\,\check{E}_c)$$

and therefor

$$O\,E = -\,\frac{\check{E}_a + a\,\check{E}_b + a^2\,\check{E}_c}{3} = -\,\check{E}_{a1}$$

and

$$O\,F = -\,\frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3} = -\,\check{E}_{a2}$$

The construction when $\check{E}_a + \check{E}_b + \check{E}_c$ is not zero is so obvious that it is not necessary to show it here.

If lines be drawn from E_a to $\check{E}_{a2}$, E_b to $\check{E}_{b2}$, E_c to $\check{E}_{c2}$ these lines will be parallel to $\check{E}_a'$ $\check{E}_b'$ and $\check{E}_c'$ respectively and will meet at a point O'. The vectors $O'\,E_a$ $O'\,E_b$ and $O'\,E_c$ give the values of the e. m. f. across each member of a star delta bank of transformers when operated on the three-phase circuit $S\,(\check{E}_a)$ with ratios changed so as to give a balanced secondary triangle of e. m. f's.

DISCUSSION ON "METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS" (FORTESCUE), ATLANTIC CITY, N. J., JUNE 28, 1918.

J. Slepian (by letter): During the past eighteen months it has been my very good fortune to have been in close contact with Mr. Fortescue and to have had many interesting discussions on the ideas embodied in this paper. Since I have had so long to think over and digest these ideas, I think I may be pardoned for going at great length here into the viewpoint I have reached. I feel all the greater need for a long discussion, because on reading the paper I see that the great wealth of material to be presented in a limited space has crowded out much detailed explanation and appeal to analogy, which in discussions with Mr. Fortescue have contributed so much to my clearer conception of the ideas presented here.

The method given here had its origin in considering the operation of balanced induction machines under unbalanced conditions. Using the "coordinates" proposed here, the theory of these machines may be given with beautiful simplicity. I think I am right in saying that the utility of the method is practically entirely limited to the case of rotating induction machines. Purely static apparatus treated in this way does not show any simplification. When one considers, however, that almost every practical alternating-current circuit contains at least one rotating machine, namely the generator, the broad field of application of the method becomes apparent.

The root of the ideas given here is old and was given early in treatments of single-phase motors. The simple constant rotating nature of the flux in a balanced polyphase motor was well known. It was discovered that the more complicated flux in a single-phase motor could be resolved into the sum of two such constant rotating fluxes of opposite rotations, with the magnitude of these fluxes not necessarily equal. It was observed also, that if the balanced polyphase states, which would give independently each of these rotating fields, were superimposed, the resultant state would give correctly the currents. voltages, torques, etc., of the single-phase motor. For a recent discussion of this, see Mr. Lamme's paper, page 627, Volume I, TRANSACTIONS 1918.

Mr. Fortescue has generalized this method of resolution of the flux in a polyphase machine under unbalanced conditions into the sum of fluxes, each corresponding to a balanced condition, to a similar resolution of an unbalanced system of any polyphase quantities whatever.

In my discussion here I shall, for simplicity, confine myself to the three-phase case. To illustrate the resolution of a set of unbalanced three-phase currents into symmetrical or balanced components, consider three arbitrary currents I_a I_b I_c flowing into the terminals of a three-phase, star-connected apparatus. (Fig. 1). Mr. Fortescue shows that this set of currents may be had by adding together the following three sets of currents.

Method of Symmetrical Coordinates. First a current I_{a0} in each phase. These three currents then are all equal and all in phase. They combine to give a current in the neutral. Second: a current I_{a0} in phase a, a current $a^2 I_{a1}$, in phase b, and a current aI_{a1} in phase c, where

$$a = -\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3}$$

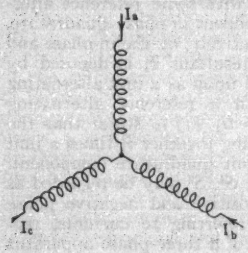

FIG. 1

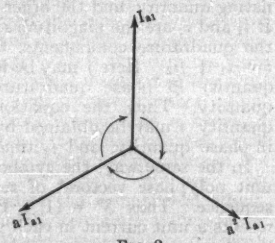

FIG. 2

is a cube root of unity. This set of currents is clearly a balanced polyphase set of what is called positive sequence. (Fig. 2). Third a current I_{a2} in phase a, a current $a\ I_{a2}$ in phase B and a current $a^2\ I_{a2}$ in phase c. (Fig. 3.). This set of currents is clearly a balanced polyphase set of what is called negative phase sequence. If these three sets of currents be made to flow simultaneously in the three-phase apparatus the resultant current in

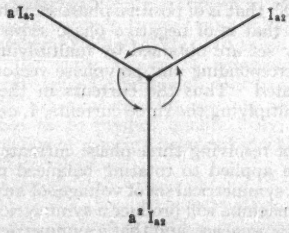

FIG. 3

each phase will be respectively I_a, I_b, I_c. The values of I_{a0}, I_{a1}, I_{a2} are obtained from equations (**5**). They are:

$$I_{a0} = 1/3\ (I_a + I_b + I_c).$$
$$I_{a1} = 1/3\ (I_a + aI_b + a^2\ I_c).$$
$$I_{a2} = 1/3\ (I_a + a^2 I_b + aI_c).$$

In a similar way, any set of three voltages to neutral acting on the three-phase apparatus can be resolved into the sum of

three sets of balanced voltages of zero phase sequence, positive phase sequence, and negative phase sequence, respectively.

The sequence operators, S^0, S^1 and S^2, which Mr. Fortescue introduces, have a close analogy, it seems to me, with the j used in the usual treatment of alternating currents. In the elementary theory, it is found useful to resolve an alternating quantity into two components, one in phase with some reference alternating quantity, and the other component in phase quadrature. If i_1 and i_2 are the magnitudes respectively, of the in-phase and the quadrature components, their resultant I, is denoted by $i = i_1 + ji_2$. Here j may be looked upon as a unit alternating quantity in phase quadrature with a reference alternating quantity. Thus, the equation $i = li_1 + j\, i_2$ states that the quantity i may be obtained by adding together i_1 times a unit in phase quantity, and i_2 times a unit quadrature component.

In the same way, the symbols S^0, S^1, S^2 may be regarded as unit polyphase vectors* of zero, positive and negative phase sequence. Thus $S^0 = (1, 1, 1)$, if referring to currents, represents a unit current in each phase of a three-phase apparatus the three currents being in phase with some reference alternating quantity. Similarly, $S^1 = (1, a^2, a)$ represents a unit balanced, three-phase current, of positive phase sequence, in the three-phase apparatus, the current in the first phase being in phase with the reference alternating quantity. Likewise $S^2 = (1, a, a^2)$ represents a unit balanced, three-phase current of negative phase sequence. Thus the equation (**15**) $S\,(I_a) = S^0\, I_0 + S^1\, I_{a1} + S^2\, I_{a2}$ states that a system of three currents $S\,(I_a)$ is equal to the sum of three sets of currents, the first set being of the type S^0, that is of zero phase sequence, the second set of the type S^1, that is of positive phase sequence, and the third set of type S^2, that is of negative phase sequence. The three currents of any set are obtained by multiplying the three currents of the corresponding unit polyphase vector by the complex quantity indicated. Thus the currents in the second set are obtained by multiplying the three currents, 1, a_2, a, respectively, by I_{a1}.

The utility of resolving three-phase currents and voltages in this way, when applied to rotating balanced machines, lies in this fact, that a symmetrical set of voltages of any phase sequence applied to the machine will produce a symmetrical set of currents of the *same phase sequence*, and that a symmetrical set of currents of any phase sequence flowing into the machine will produce a symmetrical set of terminal voltages of the same phase sequence. This fact is well known in the theory usually given of balanced polyphase apparatus, although it is seldom explicitly stated. Other methods of dividing a set of unbalanced three-phase quantities acting on a balanced machine into components would

*This very fortunate term is due to Mr. C. T. Allcutt and expresses the idea of the paragraph most succinctly.

not have this simplicity. For example: suppose the three actual currents in the three phases, and the three actual voltages of the terminals relative to neutral were taken as the components or coordinate of the currents and voltages. Then one component of current alone, that is a current in one phase only, would not produce only the corresponding component of voltage, but would produce voltages in all three phases.

Neglecting saturation, for a given rotor speed, the symmetrical currents produced in a balanced machine by symmetrical applied voltage, are proportional and in a definite phase relation to the symmetrical applied voltage. Thus the currents may be obtained by multiplying the individual members of the symmetrical set of applied voltages by some complex number. Similarly, a symmetrical set of currents flowing into a balanced machine produces a symmetrical set of terminal voltages which may be obtained by multiplying the individual members of the symmetrical set of currents by some complex number. The first complex number could be called the symmetrical admittance of the machine, and the second the symmetrical impedance.

This symmetrical impedance and admittance will be different, of course, for symmetrical components of different phase sequence. Thus for a star-connected, ungrounded neutral machine, the admittance for symmetrical voltage of zero phase sequence will be zero; if the neutral is grounded, the impedance to zero phase sequence current will be principally the leakage reactance between phases; if the rotor is running near synchronism in the sense of positive phase sequence, the impedance Z_1 to positive phase sequence current will be large, while the impedance Z_2 to negative phase sequence will be small. The complete expressions for these impedances Z_1, Z_2, are given in equations (**122**), (**123**). The relations between current and voltage components are given in equations (**120**) and (**121**). In these four equations is concentrated the whole theory of symmetrical machines operating under unbalanced conditions. But so simple are these equations and their physical meaning so clear, that once understood, they enable us to predict qualitatively, without computation, the behavior of rotating balanced machines under any unbalanced condition whatever.

Consider, for example, the simple picture of the action of a phase balancer which the above treatment gives. The balancer is merely a balanced machine across the polyphase line running near synchronism. It offers high impedance for the symmetrical voltage of normal or positive phase sequence, but offers a very low impedance for any negative phase sequence component of voltage. Thus the negative phase sequence voltage is "short-circuited out," and balance on the line is preserved.

The harmful effects of slight unbalance in the terminal voltage of a polyphase machine upon the machine's rating is also clearly shown. Since the negative sequence impedance is very low, a small negative phase sequence voltage will produce large negative

phase sequence currents with their attendant heating. This suggests a better quantity to denote the degree of unbalance of a line than the one commonly used. It is the ratio of the negative phase sequence component of the line voltage to the positive phase sequence component.

Balanced stationary apparatus also enjoys the simplicity pointed out in balanced rotating machines, namely that symmetrical voltages of any phase sequence produce symmetrical currents of the same phase sequence, but here the impedances for positive and negative phase sequence are always the same. The zero phase sequence impedance is generally different.

When we pass to unbalanced apparatus, things become more complicated. Symmetrical currents of one phase sequence no longer produce e. m. fs. of that phase sequence alone, but the total e. m. fs. are unsymmetrical and contain components of other phase sequences. Let us consider in detail an unbalanced star impedance, with grounded neutral. Let the phase impedances be Z_a, Z_b, Z_c.

Let us first study the e. m. fs. to neutral produced by a zero phase sequence current. If I_{ao}, I_{ao}, I_{ao}, are the three currents, the three e. m. fs. are Z_aI_{ao}, Z_bI_{ao}, Z_aI_{ao}. Resolving these three e. m. fs. into symmetrical components, we find by equations (5) that the zeroth order component is:

$$\tfrac{1}{3}\,(Z_aI_{ao} + Z_bI_{ao} + Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}$$

The positive phase sequence components will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + aZ_bI_{ao} + a^2Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)\,I_{ao}$$

The negative phase sequence component will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + a^2Z_bI_{ao} + aZ_cI_{ao}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{ao}$$

Thus the total e. m. f. using the symbols $S^0 S'$ S^2 would be written:

$$S^0\,[\tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}] + S^1\,[\tfrac{1}{3}(Z_a + aZ_b + a^2Z_c)I_{ao}] + S^2\,[\tfrac{1}{3}(Z_a + a^2Z_b + aZ_c]\,I_{ao} = S^0\,(Z_{ao}I_{ao}) + S'\,(Z_{a1}I_{ao}) + S^2\,(Z_{a2}I_{ao}).$$

Where
$$Z_{ao} = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)$$
$$Z_{a1} = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)$$
$$Z_{a2} = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)$$

Now consider the e. m. f's. given by the positive phase sequence currents, I_a, a^2I_{a1}, aI_{a1}. They will be Z_aI_{a1}, $Z_ba^2I_{a1}$ Z_caI_{a1}.

Resolve the system of em. f's. into symmetrical components. We find for the zero phase sequence component:

$$\tfrac{1}{3}\,(Z_aI_{a1} + Z_ba^2I_{a1} + Z_caI_{a1}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{a1} = Z_{a2}I_{a1}$$

For the positive phase sequence component we find

$$\tfrac{1}{3}\,(Z_a I_{a1} + aZ_b a^2 I_{a1} + a^2 Z_c a I_{a1}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\, I_{a1} = Z_{a0} I_{a1}$$

For the negative phase sequence component we find

$$\tfrac{1}{3}\,(Z_a I_{a1} + a^2 Z_b a^2 I_{a1} + aZ_c a I_{a1}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2 Z_c)\, I_{a1} = Z_{a1} I_{a1}$$

Lastly, consider the e. m. f.'s produced by negative phase sequence currents, I_{a2}, $a\ I_{a2}$, $a^2 I_{a2}$. The zero phase sequence component will be

$$\tfrac{1}{3}\,(Z_a I_{a2} + Z_b a I_{a2} + Z_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2 Z_c)\, I_{a2} = Z_{a1} I_{a2}.$$

The positive phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(Z a I_{a2} + aZ_b a I_{a2} + a^2 Z_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + a^2 Z_b + aZ_c)\, I_{a2} = Z_{a2} I_{a2}.$$

The negative phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(Z_a I_{a2} + a^2 Z_b a I_{a2} + aZ_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\, I_{a2} = Z_{a0} I_{a2}$$

The components of voltage are then expressed easily in terms of the quantities Z_{a0}, Z_{a1}, Z_{a2}, defined in equations (8).

Let us bring these results together in tabular form where the relations between them can be observed.

Currents	Voltage Components		
	S^0 or zero phase sequence	S^1 or positive phase sequence	S^2 or negative phase sequence
Zero phase sequence, $S^0\ (I_{a0}) = (I_{a0}\ I_{a0}\ I_{a0})$	$Z_{a0} I_{a0}$	$Z_{a1} I_{a0}$	$Z_{a2} I_{a0}$
Positive phase sequence, $S^1\ (I_{a1}) = (I_{a1},\ a^2 I_{a1},\ a I_{a1})$	$Z_{a2} I_{a1}$	$Z_{a0} I_{a1}$	$Z_{a1} I_{a1}$
Negative phase sequence $S^2\ (I_{a2}) = (I_{a2},\ a I_{a2},\ a^2 I_{a2})$	$Z_{a1} I_{a2}$	$Z_{a2} I_{a2}$	$Z_{a0} I_{a0}$

Studying the table above, we see that each symmetrical component of current gives rise to symmetrical voltages of all three phase sequences. We notice first the e. m. f. component which is of the same phase sequence as the current by which it is produced, is obtained by multiplying the producing currents by Z_{a0}. That is the unbalanced apparatus may be said to have one component of impedance, Z_{a0}, which gives e. m. f. 's of the same phase sequence as the currents. This component of impedance does not change the exponent of the S symbol defining the phase sequence of the currents. Thus Z_{a0} alone corresponding to the currents $S^0\ (I_{a0})$ gives the e. m. fs. $S^0\ (Z_{a1} I_{a0})$; corresponding to the

Currents S^1 (I_{a1}) gives the e. m. fs. S^1 $(Z_{ao}I_{a1})$; and corresponding to the

Currents S^2 (I_{a2}) gives the e. m. fs. S^2 $(Z_2\, I_{a2})$.

Now let us examine how Z_{a1} enters in the above table. Corresponding to the zero phase sequence.

Currents S^0 (I_{ao}) we find the e. m. fs. S^1 $(Z_{a1}I_{ao})$ of positive phase sequence. Corresponding to the positive phase sequence.

Currents S^1 (I_{a1}) we find the e. m. fs. S^2 $(Z_{a1}I_{a1})$ of negative phase sequence. Corresponding to the negative phase sequence.

Currents S^2 (I_{a2}) we find the e. m. fs. S^0 $(Z_{a1}I_{a2})$ of zero phase sequence.

If we agree now to the following definitions of the symbols S when affected with higher exponents:

$$S^3 = S^0;\ S^4 = S^1;\ S^5 = S^2;\ S^6 = S^0;\ S^7 = S^1;\ \text{etc.}$$

We see that the way in which Z_{a1} terms enter can be summarized in this way:

The unbalanced apparatus has a component of impedance, Z_{a1}, which increases the exponent of the phase sequence symbol by unity.

Lastly, let us see how Z_{a2} enters the above table. We find that:

Currents S^0 (I_{ao}) give e. m. fs. S^2 $(Z_{a2}I_{ao})$.
Currents S^1 (I_{a1}) give e. m. fs. S^0 $(Z_{a2}I_{a1})$.
Currents S^2 (I_{a2}) give e. m. fs. S^1 $(Z_{a2}I_{a2})$.

This may be summarized by saying that the unbalanced apparatus has a component of impedance Z_{a2} which increases the exponent of the sequence symbol by two.

All these results will be obtained automatically if we suppose that symbols S^0, S^1, S^2, are attached respectively to Z_{ao}, Z_{a1}, Z_{a2}, and when multiplying the current components, S^0 (I_{ao}), S^1 (I_{a1}), S^2 (I_{a2}), the exponential laws hold. Thus:

$$\begin{aligned}
S^0\,(Z_{ao})\,S^0\,(I_{ao}) &= S^0\,(Z_{ao}\,I_{ao});\ S^1\,(Z_{a1})\,S^0\,(I_{ao}) = S^1\,(Z_{a1}I_{ao}).\\
S^0\,(Z_{ao})\,S^1\,(I_{a1}) &= S^1\,(Z_{ao}\,I_{a1});\ S^1\,(Z_{a1})\,S^0\,(I_{a1}) = S^2\,(Z_{a1}\,I_{a1}).\\
S^0\,(Z_{ao})\,S^2\,(I_{a2}) &= S^2\,(Z_{ao}\,I_{a2});\ S^1\,(Z_{a1})\,S^2\,(I_{ao}) = S^3\,(Z_{a1}\,I_{a2}).\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad = S^0\,(Z_{a1}\,I_{a2}).
\end{aligned}$$

$$\begin{aligned}
S^2\,(Z_{a2})\,S^0(I_{ao}) &= S^2\,(Z_{a2}I_{ao})\\
S^2\,(Z_{a2})\,S^1(I_{a1}) &= S^3\,(Z_{a2}I_{a1}) = S^0(Z_{a2}I_{a1})\\
S^2\,(Z_{a2})\,S^2\,(I_{a2}) &= S^4\,(Z_{a2}I_{a2}) = S^1\,(Z_{a2}I_{a2})
\end{aligned}$$

The whole result can be expressed by writing the total impedance, $S(Z_a) = S^0\,Z_{ao} + S^1\,Z_{a1} + S^2\,Z_{a2}$.

To get the e. m. fs. corresponding to any currents, we multiply together the total impedance by the total current, following merely the rules of algebra, and interpreting higher powers of S as described above. Thus we get:

$$\begin{aligned}
S(E_a) &= S(Z_a)\,S(I_a)\\
&= (S^0Z_{ao} + S^1Z_{a1} + S^2Z_{a2})\,(S^0I_{ao} + S^1I_{a1} + S^2I_{a2})\\
&= S^0(Z_{ao}I_{ao} + Z_{a1}I_{a2} + Z_{a2}I_{a1}) + S^1(Z_{ao}\,I_{a1}\\
&\qquad + Z_{a1}I_{ao} + Z_{a2}I_{a2}) + S^2\,(Z_{ao}I_{a2} + Z_{a1}I_{a1} + Z_{a2}I_{ao})
\end{aligned}$$

We get here the foundation of a complex three-phase algebra, which performs the same functions for three-phase systems as the usual complex quantity does for simple alternating current.

This leads to another interpretation or mode of viewing the sequence symbols, S^0, S^1, S^2, and here again we may profit by analogy with the j in the complex algebra of simple alternating quantities. Suppose i is a current in phase with some reference alternating quantity. If this current flows through a resistance r. the resulting e. m. f., ri is also in phase with reference alternating quantity. If the resistance is of unit value the resulting e. m. f. is i. Suppose, however, that the current flows through a reactance whose impedance is x ohms. The resulting e. m. f. will be $x\,I$ in magnitude, but it will be in phase quadrature with the reference alternating quantity. This is taken care of in the usual theory by affecting the expression for the reactance with a j, thus xj, and making j have the property that when multiplying a vector or alternating quantity it does not change the value of the vector but merely advances its phase by ninety degrees This is a different meaning from what was given before. j here no longer represents a unit vector or alternating quantity. It now represents an operator, or a symbol for advancing the phase of any vector which it multiplies. Where j by itself is referred to as a vector, it should be understood that ji is meant; where i is a unit vector in phase with the reference vector.

It is clear that operating twice successively by j upon a vector merely reverses the phase of that vector. Thus $j\,(j.i) = -1$, which leads to the rule of multiplication $J^2 = -1$. Lastly, we see that the e. m. f. induced by a current flowing through an impedance may be obtained correctly both in phase and magnitude by multiplying the current by a number of the form $r + jx$, which represents the complete impedance. Thus the complex algebra of simple alternating quantities is born.

We may attach significance as operators in a similar way to the symbols S^0, S^1, S^2. Now they shall no longer represent polyphase vectors, but shall merely be operators which when written next to a polyphase vector, change it into a polyphase vector of another phase sequence. The quantities which the S's affect shall be considered as polyphase vectors. Thus the symbol I shall stand for three currents, each equal to I. Operating on I with S^0 means multiplying each of these currents by unity. Thus S^0 is a unity operator and does not change the polyphase vector upon which it operates. A separate symbol for it might have been omitted.

$S^1\,I$ or S^1 operating on I, means that the three currents I, I, I, are to be multiplied by 1, a^2, a, respectively; or that the first current is to be unchanged, the second to be advanced in phase by 120 deg. and the third by 240 deg. S' by itself shall not mean anything unless it is understood to be followed by 1 in which case it stands for the unit polyphase vector of positive, rotation obtained by operating with S^1 on the three currents (1, 1, 1).

Similarly, S^2I or S^2 operating on I, means that the three currents I, I, I, are to be multiplied by I, a, a^2, respectively; or that the first current is to be unchanged, the second advanced in phase by 240 deg. and the third by 120 deg. S^2 by itself shall not mean anything unless it is understood to be I, in which case it stands for the unit polyphase vector of negative phase rotation obtained by operating upon the polyphase quantity (I, I, I).

With the meaning now given to S^0, S^1, S^2, it at once follows that they satisfy the law of exponents. For example, S^1 (S^1I) means leave the first current I unchanged; advance the second current I, in phase by 120 deg., and then again by 120 deg; advance the third current, I, by 240 deg. and than again by 240 deg. It is clear that the final results are exactly the same as the results of operating with S^2 on I. Also it is clear that multiplication by an operators and a constant Z is commutative. Thus $ZS^1I = S^1Z_{a1}I$. This serves as the foundation of the complex three-phase algebra.

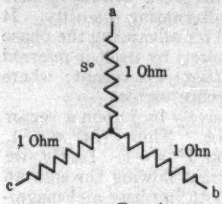

FIG. 4

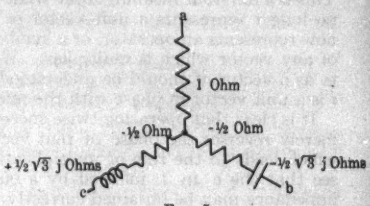

FIG. 5

In the complex algebra of simple alternating quantities, it is easy to give an illustration of the operator J as in impedance. In fact, a reactive impedance of one ohm gives for any current an e. m. f. which may be obtained from the current by multiplying by J. Can we similarly illustrate the three operators S^0, S^1, S^2.

S^0, of course, is easy. Take a grounded star of resistances, each of one ohm. Then we find for the currents S^0 $(I) = (I, I, I)$ the voltages, $(I, I, I) = S^0 I$; for the currents S^2 $(I) = (I, aI, a^2I)$ the voltages $(I, aI, a^2I) = S^2I$.

Now for S^1. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3J})$ ohms $= a^2$ ohms, and of phase c, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms $= a$ ohms. Now we find for the currents S^0 $(I) = (I, I, I)$, the e. m. fs. $(I, a^2I, aI) = S^1I = S^1$ (S^0I) for the currents S^1 $(I) = (I, a^2I, aI)$, the e. m. fs. $(I, aI, a^2I) = S^2I = S'$ $(S'I)$; for the currents S^2 $(I) = (I, aI, a^2I)$, the e. m. fs. $(I, I, I) = S^0I = S^1$ (S^2I). Fig.5.

Lastly, S^2. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms, and of

phase c, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3}j)$ ohms. Then we find for the currents $S^0\ (I) = (I, I, I)$, the e. m. fs. $(I, aI, a^2I) = S^2I = S^2S^0I$; for the currents $S'\ (I) = (I, a^2I, aI)$ the e. m. fs. $(I, I, I) = S^0I = S^2S'I$; for the currents $S^2\ (I) = (I, aI, a^2I)$ the e. m. fs. $(I, a^2I, aI) = S^1I = S^2S^2I$. Fig. 6.

In these examples, negative resistances appear. This is not surprising, as it is clear that unbalanced apparatus, when current of one phase sequence flows, may feed energy into impressed

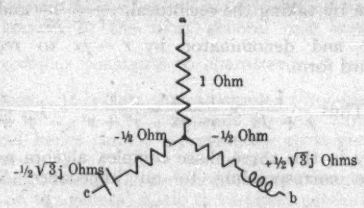

FIG. 6

e. m. fs. of another phase sequence. The negative resistance may be represented physically by a series commutator a-c. motor, driven at constant speed in the opposite of its motoring direction of rotation.

The resolution of an arbitrary three-phase star impedance into its three-phase components is merely the problem of finding three three-phase impedances of the S^0, S^1 and S^2 type, respectively, which put in series as in Fig. 7 will reproduce the given three-phase star.

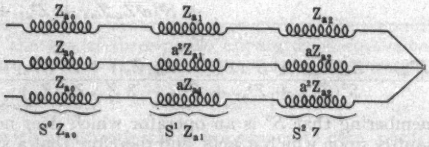

FIG. 7

It is clear that admittances may be treated in the same way as impedances above. Thus the general three-phase admittance may be written in terms of its components, thus:

$$S\ (Y_a) = S^0Y_{a0} + S^1Y_{a1} + S^2Y_{a2}$$

The currents produced by a general three-phase voltage $S\ (E_a) = S^0E_{a0} + S^1E_{a1} + S^2E_{a2}$ will be:

$$S(I_a) = (S^\circ Y_{ao} + S^1 Y_{a1} + S^2 Y_{a2}) \quad (S^\circ E_{ao} + S^1 E_{a1} + S^2 E_{a2}).$$

$$= S^\circ (Y\ E_{ao} + Y_{a1} E_{a2} + Y_{a2} E_{a1})$$
$$+ S^1 (Y_{ao} E_{a1} + Y_{a1} E_{ao} + Y_{a2} E_{a2})$$
$$+ S^2 (Y_{ao} E_{a2} + Y_{a1} E_{a1} + Y_{a2} E_{ao}).$$

In the complex algebra of simple alternating currents we pass formally from an impedance $R + jx$ to the corresponding admittance by taking the reciprocal, $\frac{l}{r + jx}$ and multiplying numerator and denominator by $r - jx$ to reduce it to the standard form.

$$\text{Thus } \frac{l}{r + Jx} \times \frac{r - jx}{r - jx} = \frac{r}{r^2 + x^2} - j\frac{x}{r^2 + x^2}$$

Similarly in this three-phase complex algebra we obtain the admittance corresponding to an impedance $S^\circ Z_{ao} + S^1 Z_{a1} + S\,Z_{a2}$ by taking the reciprocal $\frac{1}{S^\circ Z_{ao} + S^1 Z_{a1} + SZ_{a2}}$. But now to reduce to standard form we must multiply numerator and denominator by

$$S^\circ (Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2})\ (S^\circ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}).$$

Thus:

$$\frac{1}{S^\circ Z_{ao} + S^1 Z_{a1} + S^2 Z_{a2}} \cdot \frac{S^\circ Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2}}{S^\circ Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2}} \cdot \frac{S^\circ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}}{S^\circ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}}$$

$$= \frac{S^\circ (Z_{ao}^2 + a^2 Z_{a1} Z_{a2} + aZ_{a1} Z_{a2}) + S^1 (a^2 Z_{ao} Z_{a1} + aZ_{ao} Z_{a1} + Z^2_{a2}) + S^2 (a^2 Z_{ao} Z_{a2} + Z^2_{a1} + aZ_{ao} Z_{a2})}{D}$$

$$= \frac{S^\circ (Z^2_{ao} - Z_{a1} Z_{a2}) + S^1 (Z^2_{a2} - Z_{ao} Z_{a1}) + S^2 (Z^2_{a1} - Z_{ao} Z_{a2})}{S^\circ (Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\, Z_{ao} Z_{a1} Z_{a2})}$$

Remembering that S° is an operator which does not change the quantity upon which it acts, and therefore that a symbol for it might have been omitted, we may leave off the S° in the denominator and get for the final admittance:

$$S(Y_a) = S^\circ \frac{Z^2_{ao} - Z_{a1} Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\, Z_{ao} Z_{a1} Z_{a2}}$$
$$+ S^1 \frac{Z^2_{a2} - Z_{ao} Z_{a1}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\, Z_{ao} Z_{a1} Z_{a2}}$$
$$+ S^2 \frac{Z^2_{a1} - Z_{ao} Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\, Z_{ao} Z_{a1} Z_{a2}}$$

The principles just enunciated enable us to put together three-phase apparatus and calculate the resulting three-phase components just as is done in single-phase apparatus. For apparatus, in parallel, we add admittances; for apparatus in series we add impedances.

So far, in the static apparatus considered, the mutual inductance between "legs" of the three-phase apparatus has been assumed zero. When these mutual inductances are not zero, things become more complicated. A three-phase apparatus with grounded neutral is, of course, a four terminal network, and as is well known, in the most general case would require $\frac{4 \times 3}{2} = 6$ complex constants to characterize it under steady state conditions. It is clear that a complex three-phase expression $S^0 Z_{a0} + S^1 Z_{a1} + S^2 Z_{a2}$ depends on only three complex constants, and hence cannot represent the most general three-phase apparatus.

The relations between the symmetrical components of voltage and current on the general static three-phase network are given in equations (**25**). One way of summarizing these equations would be to say that the three-phase apparatus has three different impedances for the three phase-sequence currents. Thus the impedance to zero phase sequence current as given by equations (**25**) is:

$$S^0 (Z_{aa0} + 2 Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

the impedance to positive phase sequence currents is

$$S^0 (Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} + 2 Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

and the impedance to negative phase sequence currents is

$$S^0 (Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} + Z_{bc2}),$$

Another way of summarizing the equations (**25**) would be to say that the actual three-phase apparatus is equivalent to a three-phase network having the three-phase impedance

$$S^0(Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2}),$$

in series with a network whose impedance to zero, positive and negative phase sequence current are respectively, $S^0 3 Z_{bc0}$; $S^1 3 Z_{bc1}$; $S^2 3 Z_{bc2}$.

With the ideas developed in this paper, solutions of problems on symmetrical rotating machines with unbalanced static apparatus may be worked with comparative ease. As an example I shall consider the case of a generator, with ungrounded neutral, acting on a given star load, of leg impedances, Z_a, Z_b, Z_c. Suppose the machine is symmetrical, *i. e.*, non-salient poles, and a damping squirrel cage on the field. In this case the generator has an impedance Z_1 to positive phase sequence current, and a

small impedance Z_2 to negative phase sequence currents. The positive phase sequence impedance Z_1 is what has been called the synchronous impedance of the machine such that if E is the no load voltage, $E - Z_1 I$ is the voltage on a positive sequence balanced load I. The problem is to find the currents and voltages on the load, and the voltage between load neutral and generator neutral.

We have given that $I_{ao} = O$.

We calculate the impedance drops of positive phase sequence and put their sum equal to the generated positive sequence voltage. Thus:

$$Z_1 I_{a1} + Z_{ao} I_{a1} + Z_{a2} I_{a2} = E.$$

Similarly, since the generated negative phase sequence voltage is zero,

$$Z_2 I_{a2} + Z_{a1} I_{a1} + Z_{ao} I_{a2} = 0.$$

Solving for I_{a1} and I_{a2}

$$I_{a1} = E \frac{Z_2 + Z_{ao}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

$$I_{a2} = E \frac{- Z_{a1}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

The terminal voltages will be:

$$E_{a1} = E - Z_1 I_{a1} = Z_{ao} I_{a1} + Z_{a2} I_{a2} = E \cdot \frac{Z^2_{ao} - Z_{a1} Z_{a2} + Z_2 Z_{ao}}{0}$$

$$E_{a2} = - Z_2 I_{a2} = Z_{a1} I_{a1} + Z_{ao} I_{a2} = E \frac{Z_2 Z_{a1}}{0}$$

The voltage between load neutral and generator neutral will be:

$$E_{ao} = Z_{a1} I_{a2} + Z_{a2} I_{a1} = E \cdot \frac{- Z^2_{a1} + Z_{ao} Z_{a2} + Z_2 Z_{a2}}{D}$$

A special case of interest is where $Z_b - Z_c$ are zero, that is, a single-phase short circuit. We then have:

$$Z_{ao} = \tfrac{1}{3} Z_a \qquad Z_{a1} = \tfrac{1}{3} Z_{a1} \qquad Z_{a2} = \tfrac{1}{3} Z_a$$

$$I_{a1} = E \frac{(Z_a + 3 Z_2)}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$I_{a2} = E \frac{-Za}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{a1} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{a2} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{a0} = E \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

If just previous to the single-phase short circuit the generator was unloaded, Z_a becomes infinite and our final results are:

$$I_{a1} = - I_{a2} = \frac{E}{Z_1 + Z_2}$$

$$E_{a1} = E_{a2} = E_{a0} = E \frac{Z_2}{Z_1 + Z_2}.$$

If the generator above did not have a damper winding on the field, it would be an unsymmetrical machine having a single-phase rotor. As we know, the theory of unsymmetrical rotating machines is of considerable complication. Mr. Fortescue has promised a treatment by these methods of the general unsymmetrical rotating machine in a future paper, which I look forward to with great interest.

C. P. Steinmetz: In dealing with calculations or investigations of polyphase systems, or, as usually the case, three-phase systems, the difficulties which we meet are not so much mathematical difficulties, but are what I may call mechanical difficulties. The equations, while mathematically not complicated, lead to expressions which are so complicated and extensive in form as to make any such calculations extremely difficult.

In dealing with the balanced polyphase system, this difficulty has been overcome by the introduction of the equivalent single-phase system, by considering the polyphase system as resolved into a number of single-phase systems, each comprising the circuit from one of the phase wires to the neutral point of the system. This method however, fails in the unbalanced polyphase system—and naturally practically all existing commercial polyphase systems are more or less unbalanced—and the theory of the vector method given to us today in a more extensive description by Mr. Fortescue, gives the solution by showing us in the case of the general three-phase system that it can be resolved into two balanced three-phase systems of opposite phase rotation. We can apply the same plan to other polyphase systems.

V. Karapetoff: Mr. Fortescue deserves the gratitude of the profession for bringing out a new method for numerical compu-

tations in unsymmetrical polyphase systems, and also for applying the method to a number of practical cases.

Expressions (4) represent the result of solution of certain equations, which equations unfortunately are not given in the paper. The expressions quoted are unnecessarily involved, and it is difficult to see their method of derivation. Moreover, it may be shown that they do not represent the most general case of resolution of an unsymmetrical system of vectors. The following procedure would seem to be preferable.

Definition. A multiple-angle symmetrical polyphase system of vectors is defined as one in which the vectors are numbered not consecutively, but skipping a certain number of vectors. Thus, Fig. 8 represents a triple-angle seven-phase clockwise system, because three angles are comprised between phase 1 and phase 2, and also between phase 2 and 3, etc. Such a generalization of the concept of polyphase systems is useful when the number of phases and the number of angles skipped are prime numbers, so that all the phases may be numbered consecutively, without omitting or repeating any. If the total number of phases n is a prime number, then the total number of possible combinations or multiple systems is $(n-1)$, the angles between the consecutive phases being $\frac{2\pi}{n}$, $2 \times \frac{2\pi}{n}$, . . . $(n-1)\ \frac{2\pi}{n}$. It is assumed that the numbering in *all* the systems is either clockwise or counter-clockwise, so that there is no confusion between a positive and a negative phase sequence.

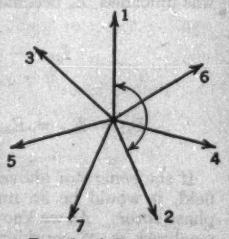

FIG. 8—A TRIPLE-ANGLE SEVEN-PHASE SYMMETRICAL SYSTEM

Theorem 1. An arbitrary system of n unequal and unsymmetrical vectors, *without residue*, may be represented by $(n-1)$ symmetrical multiple n-phase systems. A system without residue is defined as one in which the sum of the vectors is equal to zero. Let the given vectors be $E_1, E_2, \ldots\ldots E_n$, and let the $(n-1)$ unknown systems be denoted by $A, B, \ldots M$, where A is a single-angle system, B is a double-angle system, etc. It is required to prove that the following equations are consistent and may be solved for $A, B, \ldots\ldots M$:

$$\left.\begin{array}{l} E_1 = A_1 + B_1 + \ldots + M_1 \\ E_2 = A_2 + B_2 + \ldots + M_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = A_n + B_n + \ldots + M_n \end{array}\right\} \qquad (1)$$

These equations correspond to eqs. (4) in Mr. Fortescue's paper when $\Sigma E = 0$. In order to preserve the same conventions as in

the original paper, the component systems are assumed to be numbered clockwise (though the vectors are rotating counter-clockwise). Then if a is an operator which rotates a vector by an angle $2\pi/n$ counter-clockwise, the preceding equations become

$$\left.\begin{array}{l} E_1 = A_1 + B_1 + \ldots + M_1 \\ E_2 = a^{-1} A_1 + a^{-2} B_1 + \ldots + a^{-(n-1)} M_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = a^{-(n-1)} A_1 + a^{-2(n-1)} B_1 + \ldots a^{-(n-1)(n-1)} M_1 \end{array}\right\} \quad (2)$$

These n equations contain $(n-1)$ unknown vectors $A_1, B_1, \ldots M_1$, but only $(n-1)$ equations are independent of each other, because of the condition $\Sigma E = 0$. To solve for A_1 multiply the second equation by a, the third by a^2, etc. and add them together. The result is

$$n A_1 = E_1 + a E_2 + \ldots + a^{n-1} E_n \quad (3)$$

This checks with the second line of Mr. Fortescue's equations (4) By analogy we obtain

$$n B_1 = E_1 + a^2 E_2 + \ldots\ldots + a^{2(n-1)} E_n \ldots\ldots \quad (4)$$

and

$$n M_1 = E_1 + a^{n-1} E_2 + \ldots\ldots a^{(n-1)(n-1)} E_n \quad (5)$$

The theorem is thus proved because equation (2) can be solved for the $(n-1)$ unknown vectors in terms of the given E's.

Theorem 2. Referring to theorem 1, if the given system of vectors has a *residue*, say, equal to a vector R, this residue may be split into n arbitrary vectors, $\rho_1, \rho_2, \ldots \rho_n$, in phase or out of phase with one another, and equations (1) become

$$\left.\begin{array}{l} E_1 = \rho_1 + A_1 + B_1 + \ldots\ldots + M_1 \\ E_2 = \rho_2 + A_2 + B_2 + \ldots\ldots + M_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = \rho_n + A_n + B_n + \ldots\ldots + M^2 \end{array}\right\} \quad (6)$$

This proposition follows at once from the fact that the vectors $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. form a system without residue, so that theorem 1 applies to them. Mr. Fortescue considers only a particular case when

$$\begin{aligned} \rho_1 = \rho_2 = \ldots\ldots = \rho_n &= R/n \\ &= (E_1 + E_2 + \ldots E_n)\ /n \end{aligned} \quad (7)$$

and writes this expression as the first term on the right-hand side in all the formulas of his equations (4). This is perfectly justifiable in so far as he is after a symmetrical solution, but there may be practical cases in which a more general representation according to equations (6) is preferable. In this case the expressions $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. must be used in equations (3) to (5), instead of E_1, E_2, etc.

The net result is that only a system without residue may be resolved in a perfectly definite manner, while a system with a residue first must be converted into one without residue by

subtracting arbitrary parts of the whole residue from each vector.

With a three-phase system the double-angle component simply becomes a three-phase system of an opposite phase rotation, so that it is not necessary to introduce the question of multiple-angle systems. From a practical point of view the paper would gain much if the author would begin directly with the three-phase system on the basis of his equations (**6**), and leave the general consideration of n-phase systems for the end of the paper. For reference purposes one would not have to study the general theory and perhaps get discouraged before obtaining the required bit of specific information.

The introduction of the sequence operator S does not seem to be necessary, at least for the three-phase system. A proper cyclic notation should accomplish the same purpose with much less theory and much less writing. As a matter of fact, the index of S may be obtained correctly from the subscripts in the expression to which it refers. For example, in Mr. Fortescue's formula (**22**) on the first line the sum of the subscripts of $Z\,I$ is $1 + 2 = 3$, and the order of S is 3 or, which is the same, 0. On the second line the sum of the subscripts of $Z\,I$ is $1 + 0 = 1$, and the order of S is 1. The same is true for all the fundamental formulas on pp. 1038 to 1042 as well as in applications. Therefore, it would seem that the operator S ought to be dropped, and the formulas so rewritten that they would apply to any phase of the system in cyclic rotation.

The fundamental formula (**21**) will then simply become

$$E = Z\,I + W_+ I_+ + W_- I_- \tag{8}$$

Here E, Z, and I apply to any of the three phases, I_+ is the current in the next consecutive phase, and I_- is the current in the preceding phase. For example, if E, Z, and I refer to phase b, I_+ refers to phase c and I_- refers to phase a. At the same time the equation is so written that any of the three phases may be taken as the phase under consideration. W_+ and W_- are the corresponding mutual reactances.

When the mutual inductance between a primary and a secondary polyphase circuit must be considered, Fortescue's equation (**28**) applies, and in the simplified cyclic notation it becomes

$$E = X\,J + X_+ J_+ + X_- J_- \tag{9}$$

where J, J_+ and J_- are the secondary currents in the three phases. The letter X is used instead of W to indicate that the mutual inductance is between the phases of the primary and the secondary circuit. X without subscript covers the combinations of the "corresponding" phases $a - u$, $b - v$, and $c - w$, while X_+ refers to the combinations $a - v$, $b - w$, $c - u$ of the phases "tied one forward", and X_- refers to the remaining combination of the phases "tied two forward", or "tied one backward".

Each of the quantities in equations (**8**) and (**9**) may be rep-

resented by means of symmetrical coordinates in a general manner, without specifying phases a, b and c. Thus,

$$\left.\begin{aligned} I &= I_0 + I_p + I_n \\ I_+ &= I_0 + a\,I_p + a^{-1}\,I_n \\ I_- &= I_0 + a^{-1}\,I_p + a\,I_n \end{aligned}\right\} \qquad (10)$$

Similarly,

$$W_+ = W_0 + a\,W_p + a^{-1}\,W_n \qquad (11)$$

The first equation (**10**) refers to any phase, and states that the current in that phase is a sum of the residue current I_0, plus the vectors I_p and I_n of the symmetrical coordinates *of the same phase.* The subscripts p and n refer to the systems of positive and negative phase rotation respectively. In an n-phase system subscripts 1, 2, 3, etc. should be used in place of $_p$ and $_n$, to indicate single-angle, double-angle, triple-angle, etc. coordinates.

Substituting expressions such as (**10**) and (**11**) into equations (**8**) and (**9**) one obtains comparatively simple polynomials with three groups of terms: those without a, those multiplied by a, and those multiplied by a^{-1}. These final expressions give the coordinates of E, and are the fundamental equations upon which all applications should be based, without any need for the sequence operator S.

Incidentally, I believe that the expression "symmetrical components" is a more correct and descriptive term of the method than the term "symmetrical coordinates", and I should like to see it so changed while it is not too late.

I also suggest that the term "mutual reactance" be used throughout the paper in place of the "mutual impedance". The latter term implies a combined resistance and inductance action, while the formulas in the paper seem to refer to the magnetic inductive action exclusively. If the resistance component is intended to be taken into account, a mathematical definition of the mutual impedance becomes necessary, because two coils are presupposed to have different resistances, and it is not clear how these resistances enter into the expression for mutual impedance.

It may also be mentioned here that there are cases of unsymmetrical polyphase connections which can be solved more readily without resolving the given electrical quantities into their symmetrical components. Several such cases are considered in the present writer's work entitled "Ueber Mehrphasige Stromsysteme bei Ungleichmaessiger Belastung (Enke, Stuttgart, 1900). For example, in the solution of any arbitrary n-phase star-connected system with given voltages E and phase admittances Y, it is convenient to take the voltage e between the two neutral points as the independent variable. Then, for each phase the current $I = Y(E - e)$, and according to the first Kirchhoff law

$$\Sigma\, I = \Sigma\, Y(E - e) = 0,$$

so that

$$e = (\Sigma\, Y E) / \Sigma\, Y.$$

Knowing e, the currents in the individual phases are computed from the first equation. In the most general case of a complicated network with a-c. voltages inserted at different places, the Kirchhoff equations of two kinds, written for the vectors of currents and voltages, furnish a complete solution, and one has to consider in individual applications whether or not the resolution into symmetrical components is desirable or not.

A. M. Dudley: Attention has been called to the practical applications of this solution, and one or two may be mentioned here. It has been long known that in the ordinary induction motor in addition to the main rotating working field there may exist several other fields of different amplitude and frequency and phase rotation and that the existence of such fields is probably responsible for the operating freaks sometimes noticed in motors. For example, a motor is sometimes found which will not reverse its mechanical direction of rotation when running light even though the leads be reversed so as to give the opposite direction of phase rotation; or, a two-phase motor will be found which when running light takes power from the line on one phase and returns power to the line on the other, or a, squirrel-cage motor will start from rest with considerably more torque than it is capable of accelerating up to full speed so that it apparently has a so-called sub-synchronous speed. These are actual practical operating conditions often mentioned but never determined quantitatively. Mr. Fortescue's analysis offers the means of such quantitative study.

As a further practical instance may be mentioned the two cases of single-phase connections referred to in the analysis of equations (**141**), (**142**) and (**150**), one of which is equivalent to two coupled polyphase motors connected in parallel and the other to two coupled polyphase motors connected in series. This was fully discussed from another view point in Mr. Lamme's April paper before the Institute and the identity of the two conclusions is a check on Mr. Fortescue's method.

To those who have expressed the idea that the paper is hard to read I would commend Dr. Slepian's discussion also printed herewith as illuminating both to the method and the results. He makes the suggestion that the method as outlined may constitute the basis of a new complex polyphase algebra which will greatly simplify the study of polyphase network problems of all kinds.

Dr. Steinmetz and Prof. Karapetoff have told us that as a mathematical achievement and as a demonstration of theory this paper is a masterpiece. I should like to venture the prediction that as a practical working tool it will eventually come into the greatest usefulness.

Let us then pay our respects to Mr. Fortescue both as a great mathematician and as a scientist and also as the developer of a practical tool which shall make easier the daily task of the engineer in the ranks.

Charles F. Scott: Mathematics and engineering have their varying conditions, and they have their supporters and their critics. To a certain class of engineers the mathematician is an abstract man who starts somewhere and finds something that can lead to consecutive conclusions. On the other hand, a practical man, using that term in the extreme, is apt to rely wholly on experience and to have no use for the mathematics he does not understand. We all know that the really valuable position is the mean between these two, in which the theoretical and the practical do go together.

The reference to papers of a mathematical and explanatory character, and their relations to engineering reminded me somehow of the first Institute paper I came in contact with. It was through the pages of the *Electrical World*. I had been experimenting with alternating currents and was associated with men who knew only slightly more about it than I did. Many things seemed mysterious. Then the paper by Mr. William Stanley on "Alternating-Current Phenomena," appeared, and that mysterious thing which was the stumbling block of about everybody who came up against alternating-current, the simple single-phase phenomenon of current and voltage out of phase, was cleared up. Mr. Stanley drew some triangles and explained a few things. That put me on a new basis. I could see how the mathematical explanation, the physical phenomena, were related, and it gave me a real understanding of the alternating current. A little later came the polyphase, and it was a step from that to the work on the unbalanced polyphase system.

As we advance then, getting into more and more complicated problems, these papers of solution and explanation come to guide the engineer in his work.

What is this paper? When you look at a paper of this sort it looks as if a mathematician had produced it, leading on and on to interesting formulas, one after another. Now, to bring out my point, I want to emphasize and say I do not think this is a product of the mathematician, but of the engineer. Mr. Fortescue did not work these things out from the theoretical, mathematical standpoint, but as an engineer who was confronted with problems in which unbalanced polyphase systems were concerned. It may have been transmission or some kind of particular machinery which involved problems which he did not have the proper tools to solve.

Therefore, in studying the problems and seeing the inadequacy of the mathematical tools at hand, he went to work and constructed tools to perform that work. Mr. Fortescue has done just as some other engineers do, who work first with experiments, then go to mathematics, and then back to the experiments. That combination of first theory and then practise, back and forth, constitutes the work of the engineer who can be constructive and do pioneer work. I think I am right in saying that this is not a paper of a theoretical mathematician, but a

paper of a practical engineer, who is developing a new tool for himself and offering it to others.

C. O. Mailloux: This is the second great paper which Mr. Fortescue has presented before this Institute. Much that I had to say in the way of friendly criticism has already been anticipated by Prof. Karapetoff. I sympathize with Prof. Karapetoff's views, and I share them, in regard to the possibilities of simplification of this paper. I thank Mr. Fortescue and many other members of the Institute will also thank him, for having given an introduction to the paper, because that introduction is really the mathematical specifications in accordance with which the paper has been prepared; and to me it was more valuable, as a criterion of the value of the paper, than the paper itself, because the paper itself cannot be read offhand. The bewildering exhibit of subscripts to be found in it is something that will, well, make one pause; but the introduction, to any one who is at all familiar with mathematics, tells exactly the basis on which the paper is written, what it aims to do, and substantially the method by which it accomplished its aim. I am very thankful, indeed, personally, that that was done.

Part I is also very useful, but if it had followed the method of presentation suggested by Prof. Karapetoff, it would have been much simplified. The paper is so valuable, that it would be well worth while to rewrite it for the purpose of making it more easily digestible and more easily accessible to the great body of practical men. It appears to have been written too much from the point of view of the professed mathematician, who seeks first to make a generalization, and then proceeds to experimentation to find practical and special cases. It would have been a better arrangement if that generalization had been put in an appendix, and if the statement had been made that from the general case is derived the particular case which interests us, namely, the case of the three-phase current. I think that to discuss *n-phases* is of academic interest only at present, even though it is possible that there may be some day when it will be of practical importance. Until that seemingly remote day comes, a generalization which includes *n*-phases might as well lie in an appendix, out of harm's way, and especially where it would not encumber the rest of the discussion. If Mr. Fortescue had written the paper by putting into the appendix, the generalization, the thing which makes it complete and comprehensive, he would have simplified it and made it more useful.

It is because I think so highly of the paper, I would like to see it made clearer. It is not a difficult paper, the mathematics are simple, but they look complex, because one is bewildered by a maze of subscripts and by many "operators." The fundamental idea of the operator is a splendid one, but it is just as well to put that explanation in an appendix, in all its generality, and then use it in the simple form that Prof. Karapetoff suggests. Those who deal with the matter practically, would not then

have so many equations to analyze and not so many symbols to deal with.

The analysis of the actions and reactions of the various forces present in electrical circuits, and the interpretation of the equations devised for expressing these actions and reactions, under different conditions, whether they occur in balanced or in unbalanced systems, may be facilitated by remembering that all physical forces and all forms of energy are subject to the same general principles of conservation and equilibrium, and, especially, that Newton's principle of dynamic equilibrium,—namely, that under all conditions and at all times, action and reaction are opposed and equal to each other, exactly—, applies to electrical, as well as to mechanical, forces, powers, and energies.

Analogy requires that, in any electrical circuit, at any instant of time whatever, the forces of action and those of reaction should always be opposing and balancing each other exactly, just as they do in mechanical systems; and it is known that they always do balance exactly. Analogy also requires that, for electrical circuits, when dealing with instantaneous values, the equations of electrodynamic equilibrium should have the same general characteristics as the general equation of dynamic equilibrium or balance for mechanical forces. The latter, it is well known, reduces to an algebraical sum of four terms, representing four distinct kinds or amounts of force, or power, or energy, according to the case, whose resultant, when a state of equilibrium occurs, is equal to zero.

The four kinds or amounts of force, power, or energy, in terms of which every dynamic reaction whatever, in mechanics, can be completely expressed, find exact parallels in electrodynamics. Here, also, we find four kinds of force, power, or energy; we have reversible forces of two kinds, corresponding to those which, in mechanics, produce or result from changes in kinetic energy, or changes in potential energy; we have the dissipated force, which is expended in overcoming ohmic resistance, corresponding to the force lost in overcoming friction in mechanics, and we have, for the fourth kind, the force, power, or energy representing the balancing actions or reactions which are necessary to maintain the equilibrium that must exist at every instant of time between action and reaction. The first two kinds represent energy which is not immediately dissipated but is displaced and stored in the system; the third kind represents energy which is immediately consumed in the system; and the fourth kind represents energy which is put into the system from an outside source, or else which is taken out of the system by overflow into another system or circuit, as may be required to maintain equilibrium.

These considerations show that even in an unbalanced electrical system, so-called, there must still be dynamic balance or equilibrium. The balancing force, power, or energy, according to the case, appears in the fourth term, where it represents the compensating force, power, or energy by which equilibrium is maintained.

Charles L. Fortescue: I maintain that the paper is quite simple. The mathematical portion is not difficult to understand, anybody can follow it if he takes the pains. I admit that the appearance of the equations is cumbersome, but that is almost impossible to overcome. The nature of the subject makes the equations cumbersome.

Prof. Karapetoff in his discussion used the word "stimulation". I wish to say that necessarily in a paper of this kind there are many sources of stimulation, as I pointed out in my introduction, many of the ideas are not new. The idea of symmetrical component three-phase systems is being used more or less by others, but the theory has never been presented systematically, and I think the idea of symmetrical operators is new.

Dr. Steinmetz's and Prof. Karapetoff's discussions are what I may term conjugate discussions. Dr. Steinmetz discusses the paper purely from the practical point of view. He thinks that the system is capable of practical application, and will be of great use for that purpose. Dr. Karapetoff points out the possibility, from a theoretical point of view, of the n-phase system. I wish to say nothing would have given me greater pleasure than to go into that purely theoretical matter much more extensively. It is very fascinating and has great promise, but I felt that there was needed, in this paper, some practical justification for presenting it. The theoretical part was so long drawn out, that I felt it was necessary to show by practical illustrations why I presented it.

I felt that the presentation of mathematical solutions did not alone afford a justification for the paper, but that there must also be a good practical reason for it, and I felt if I went into all the theoretical ramifications of this very interesting subject people would ask: "What is he about? What does he mean by driving us through all this painful stuff without giving us a good reason?" So I thought that the presentation of the theory should be as concise and short as possible, and for that reason I left out a good deal of explanation that some people think ought to be there.

Prof. Karapetoff has apparently lost sight of the fact that unless his arbitrary vectors are all equal to $\frac{\dot{E}_1 + \dot{E}_2 + \ldots \dot{E}_n}{n}$ they in turn may be resolved into $(n - 1)$ symmetrical systems and the "residue" will eventually reduce to the same value as that given by me, namely n equal vectors of value

$$\frac{\dot{E}_1 + \dot{E}_2 + \ldots \dot{E}_n}{n}.$$

The form in which the theorem is presented does not preclude the adoption of any artifice such as the subtraction of n arbitrary vectors to make the vector sum equal to zero, when any material advantage is gained thereby. Such an artifice, in fact, is gener-

ally adopted in the case of Y-connected circuits where the vector sum is rarely zero on account of the presence of the third harmonic. Furthermore, in any symmetrical system there is a definite impedance to each component symmetrical current including the "residue" as Prof. Karapetoff has termed the constant vector, this is not true if an arbitrary "residue" is adopted.

Prof. Karapetoff brings up another point, the use of sequence symbols. In three-phase systems, once a person has become familiar with the use of this method, it becomes quite a natural system. He thinks quite naturally in terms of symmetrical components, and he usually omits the use of the symbols. In other words, the symbols are not absolutely necessary, but they simply afford a sure method of keeping track of the mechanical part of the mathematical work. If one makes a mistake in carrying out the mathematical work, one can always go back to the sequence symbols and check it up. If a quantity looks unsymmetrical, and a mechanical error is suspected, one can always follow it out with the sequence symbols and be sure of getting the correct result. Furthermore, in dealing with a system of more than three phases, the sequence operator becomes important, because there are so many cross mutual inductances that the formulas become very much involved, and it is necessary to have some reliable mechanical device that will keep one in the right path; the sequence operators are such guides. They are almost indispensable when dealing with power both in three-phase and n-phase systems. Those are the principal reasons for their use.

Prof. Karapetoff brings up the question—What is the "mutual impedance?" I have not used the term perhaps quite correctly as I have applied it to cases in which the dissipative forces are not strictly proportional to the velocities or currents. Mutual impedance may be defined in this manner: If we have two terminals of a circuit carrying a given current, and two other terminals of another circuit, the electromotive force produced across the second pair of terminals, given a sine-wave current of a given frequency is, the product of the mutual impedance between the two circuits and the current in the first pair of terminals. In some cases the current follows paths common to both circuits, so that the mutual impedance may have a real component; in other cases the energy component may be due to eddy currents or losses in subsidiary circuits common to both.

Mr. Dudley has pointed out that the practical engineer must always keep track of the theoretical side of his work. The object of all mathematical investigation and theory should be to carry our knowledge of operating conditions further, and to investigate such obscure phenomena as arise from time to time. Very often we find that a new tool has to be devised in order to enable us to carry out our theoretical investigations without becoming involved in too great complication.

I feel that a paper presented before a scientific body that cannot be read by every member is hardly justified. I believe that all can follow this paper if they sincerely try to do so, it may take a little time, but the mathematics is not difficultt.

V. Karapetoff (by letter): In my Theorem 2, ρ_2, ρ_2 etc. are arbitrary vectors such that $\Sigma\ \rho = R$, where R is the total residue of the given system of vectors. Each ρ may be resolved geometrically into a vector R/n, in phase with total R, and another vector, say ρ, so that $\rho = \rho\ + (R/n)$. Hence $\Sigma\ \rho + (R/n) = R$, or $\Sigma\ \rho' = 0$. Thus, the system of vectors is without residue and consequently may be resolved into $(n - 1)$ symmetrical systems. The system of n vectors each equal to R/n, in phase with total R, is an *irreducible residue*.

When it is desired to sift out all the symmetrical components from the given vectors, the form R/n for the residue should be used. However, there may be conceivable cases in which there might be some advantage in splitting the residue into certain unequal or unsymmetrical parts, without attempting to extract all of its symmetrical components. For this reason, Theorem 2 is stated in the most general form. I am glad to have my attention called to the fact that only in the case where each ρ is equal to R/n, we have the *true* or the *irreducible* residue.

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SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

BY N. S. DIAMANT

Abstract of Paper

It is shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s.s.c. fundamental. See Fig 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Attention is called to the following facts for which explanations and theoretical proofs are offered.

(1) *The ripples at the crest of the e. m. f.* waves of the stator coils are due not so much to the flux pulsations as to (a), the to-and-fro flux swing across the pole, and (b) the cutting of a *B*-curve similar to wave (b), Fig. 6. See equations 22, and 23; also Table I and Note A, at the end. Compare Figs. 7 and 2nd wave Fig. 4.

(2) *The most important oscillations set up by the armature reaction* are 4 and 6 times machine frequency for two- and three-phase machines respectively. See supplementary Notes B and C; also Figs 8A, 8B wave lengths marked *B* and *D*, and Fig. 5, wave length marked *B*, and Table I.

(3) *Eccentric rotor sets up pulsations* once in every revolution, under load or at no-load, but none under s.s.c. See Fig. 4, 2nd wave, Fig. 5 wave length marked *A*, and Figs. 8A, 8B, and 8C. Also see Table I, column 4. For a tentative explanation see p 1183.

(4) *The e. m. fs. and fluxes* for coils No. 10 at the top of a tooth are very much larger than the e. m. fs. and fluxes for coil No. 9, at the top of a wedge. See Tables II and III; also Figs. 10, 11, 12.

(5) *The spacing of the ripples of the no-load e. m. fs.* is unequal, contrary to theory. See Fig. 13 and Table IV. Also equations (22) and (23) p. 1195.

(6) *The e. m. fs. and fluxes* for search coil No. 5, at the bottom of the slot, are larger than the e. m. fs. and fluxes for the coil No. 1 at the top of the slot. See Fig. 7 and Tables V, VI, and XV.

(7) *The third harmonic e. m. fs.* of the stator search coils increase with the increasing excitation. See Table IX.

(8) *Under s.s.c. there is present some cross-magnetizing armature reaction* which (a), makes crests 1 and 3 Fig. 27, unequal. See Table XV. (b), it shifts the axis of the flux towards the larger crest, Figs. 23, 27, etc., and thus makes angle *B*, Fig. 27, less than (1/2) *A*. See Table XIV.

(9) *The agreement between the armature reaction* obtained by subtracting e_r from e_{fo}, (see Figs. 24, 25, 31 and Figs. 16, 27, etc.) and that obtained by subtracting the equation of the s. s. c. waves as given in Tables XI and XII, from corresponding open-circuit e. m. f. equations as given in Tables VII and VIII, is fairly satisfactory. See last column of Table XIII.

(10) *The direct component of the armature reaction of an imperfect two-phase machine* consists of (a) a regular demagnetizing component, plus (b), a transverse or cross magnetizing component in quadrature to (a). See equations 56, 59, etc.

(11) *The analysis of the no-load field form* gives the 3rd, 5th, and 7th harmonics as 5 to 15 per cent but the $(2q \pm 1)$ ths seem to be small.

(12) *The armature current under s.s.c.* is very nearly sinusoidal. See Figs. 15 and 16. Also Table X. The *H*-curve of armature reaction set up by such a current will be sinusoidal, see equations 32, 33, etc. and succeeding two paragraphs. The *B*-curve, however, will not be sinusoidal for lack of constancy and uniformity of the permeability at different points of the magnetic circuit, and on account of saturation. See sections III and IV.

I. Introduction

In a paper dealing with the sudden short circuits of alternators, presented at the Panama-Pacific Convention, in passing, the author called attention to the following[1]: First, that the voltages induced in full-pitch stator exploring coils under sudden short circuits "are not of as simple a form as theory would indicate." Second, that the flux distribution within about one or two cycles, after the short circuit takes the general shape which it has under steady or sustained short circuit. The e.m.fs. induced in exploring coils placed at the top, middle and bottom of different slots, all, very shortly after the sudden short circuit, assume a very distorted wave form which persists for the whole length of the film, *i.e.*, about 10 or 15 cycles. The above are clearly seen from Figs. 1 and 2[2] which represent the two extreme cases of symmetrical and totally unsymmetrical armature currents.

The ordinary theories of armature reaction or of short-circuit impedance tests, would hardly lead one to expect results which these oscillograms seem to indicate. A number of valuable investigations on the flux distribution[3] at various loads and power

1. Trans., A. I. E. E., 1915. Vol. 34, part II, p. 2259.
2. See Table III, p. 2255.
3. *Journal, I. E. E.*, Vol. 37, p. 148. G. W. Worrall and F. T. Wall. See Figs. 2 to 7 inclusive.

factors have given results quite consistent with those deduced from the usual theory. The flux distribution was in no case sinusoidal, to be sure, but it never deviated greatly from the no-load field form usually met with in practise. This is clearly

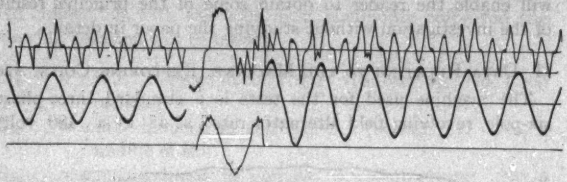


FIG. 1

Oscillogram No. 101. Table III, footnote (2). Machine series Y-connected and suddenly short-circuited at its terminals. Field excitation = 5.1 amperes. Speed = 1200 rev. per min. First wave: E_{1T} = e.m.f. induced in coil No. 1, at top of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 293 amperes.

brought out in a series of interesting oscillograms given by W. J. Foster.[4]

In view of the above and the entire lack of reliable data, as to the detailed analysis of sustained short-circuit (s.s.c.) phenomena, the writer could make no more definite statement in the paper referred to, than, that "it would seem that a good way

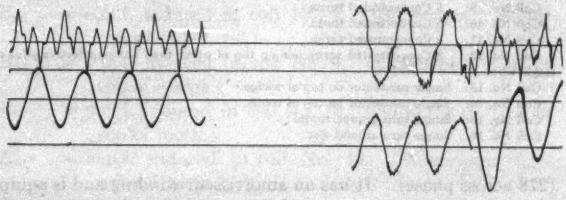


FIG. 2

Oscillogram No. 102. Table III, footnote (2). Machine series Y connected and suddenly short-circuited at its terminals. Field excitation = 5.05 amperes. Normal speed. First wave: E_{1M} = e.m.f. induced in full-pitch coil No. 3 at middle of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 297 amperes.

to study sudden short circuits was to begin by studying s.s.c. in detail. Although a great deal of work has been done on s.s.c. it is believed that so far little or no attention has been given to the question of flux distribution under such conditions."

4. TRANS., A. I. E. E., Part I, 1913, p. 749. See Curves 31, 34, 37, 38, 40, 48, etc.

In the following pages an attempt is made to show that all the complicated phenomena of s.s.c. can be explained by taking proper account of certain details and secondary effects. At the end a critical resumé and discussion of results is given which will enable the reader to obtain some of the principal results of the investigation without studying the paper in detail.

II. Data Relating to the Machine, Exploring Coils, etc.

The machine used for the tests is a standard three-phase, six-pole, revolving field alternator rated at 45 kv-a., 480 volts.

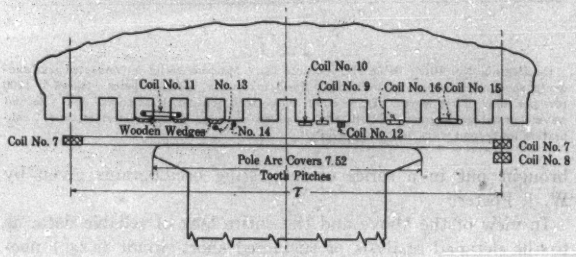


Fig. 3

Coil No. 7: 40 Full pitch concentrated turns.
Coil No. 8: 20 Concentrated turns.
Coil No. 9: 5 Concentrated turns.
Coil No. 10: 5 Concentrated turns.
Coil No. 11: 5 Concentrated turns.
Coil No. 12: 2 Concentrated turns, one on top of other thus having effective pitch of single turn.
Coil No. 13: Single conductor on top of wedge.
Coil No. 14: Single conductor on top of tooth.
Coil No. 15: Single turn around tooth.
Coil No. 16: Single turn around slot.

(278 across phase). It has an amortisseur winding and is equipped with a set of search coils as follows:

Coil No. 1. Full-pitch, 5 concentrated turns at top of slot, right under wooden wedge.

Coil No. 2. Same as No. 1, except that it is $\frac{2}{3}$ pitch.

Coil No. 3. Similar to No. 1, but placed at the middle of the slot.

Coil No. 4. Same as No. 3, except that it is $\frac{2}{3}$ pitch.

Coil No. 5. Similar to No. 1 but placed at the bottom of the slot.

Coil No. 6. Same as No. 5 except that it is $\frac{2}{3}$ pitch.

These coils were placed in different slots. The other search coils are schematically shown and described in Figs. 3 and 3A. The armature winding is of the ordinary double-layer type with a $\frac{2}{3}$ pitch, and it has two circuits which may be connected either in parallel or in series.

In connection with the coils most commonly employed the following notation will be used:

$*E_{1T}$ = e.m.f. induced in coil No. 1. (Subscripts: No. 1 top of slot.)

$*E_{3M}$ = e.m.f. induced in coil No. 3. (Subscripts: No. 3 middle of slot.)

$*E_{5B}$ = e.m.f. induced in coil No. 5. (Subscripts: No. 5 bottom of slot.)

$*E_{RF}$ = e.m.f. induced in coil No. 7, (Subscripts: Rotor full-pitch coil.)

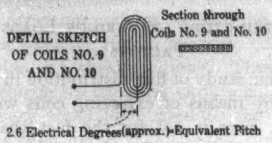


FIG. 3A

$*E_{RC}$ = e.m.f. induced in coil No. 8. (Subscripts: Rotor concentrated coil.)

$*E_{TW}$ = e.m.f. induced in coil No. 9. (Subscripts: Coil at top of wedge.)

$*E_{TT}$ = e.m.f. induced in coil No. 10. (Subscripts: Coil at top of tooth.)

$*E_{AT}$ = e.m.f. induced in coil No. 11. (Subscripts: Coil around tooth.)

$*E_{TT2}$ = e.m.f. induced in coil No. 12. (Subscripts: 2 turns on top of tooth.)

$*E_{CTS}$ = e.m.f. induced in coil No. 13. (Subscripts: Conductor on top of slot.)

$*E_{CTT}$ = e.m.f. induced in coil No. 14. (Subscripts: Conductor on top of tooth.)

B = Flux density as further specified by its subscript.

f = (actual) machine-frequency in cycles per second = r.p.s. x p.

*All under conditions to be specified.

k = Order of space harmonic. See p. 1164 for explanation.
p = Number of pairs of poles.
p_k = Number of pairs of poles of kth space harmonic $= k\,p$
q = Number of slots per pole.
S.S.C. = sustained short circuit.
τ = Pole pitch in centimeters or inches.
τ_k = Pole pitch of kth space harmonic $= \tau/k$.
$e_{rot.st.}$ = Instantaneous value of rotational e.m.f. induced in stator coil.
$e_{pul.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to pulsations of flux. (In magnitude only.)
$e_{sw.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to to-and-fro swing of flux. (Without change in magnitude.)
$e_{pul.r.}$ = Same as $e_{pul.st.}$ except that it refers to coil on rotor.
$e_{sw.r.}$ = Same as $e_{sw.st.}$ except that it refers to coil on rotor.

III. Flux Distribution at Various Loads and Power Factors

The oscillographic study of flux distribution in alternators and other apparatus by means of exploring coils would have been extremely simple, were it not for the fact that the quantity recorded on the film is not the flux wave in which we are interested but a quantity proportional to its *time rate of change.*

A conductor moving across a magnetic field, constant both in time and space, *i.e.*, a field constant in magnitude and stationary with regard to the pole axis, will have an e.m.f. induced in it which may be designated as motional or rotational e.m.f.; this will be maximum when an element $d\,l$, of the conductor under consideration, its velocity v, and the flux density B_m, all three, are mutually perpendicular. However, without any relative motion between flux and conductor, if the flux enclosed by the circuit varies for any cause and in any manner whatsoever, there will be an e.m.f. induced proportional to the rate of change of flux. This may be called static or variational e.m.f.

Thus in the most general case,[5] which is the one met with in practise, the e.m.f. induced in a conductor or coil is:

$$e_{\text{total}} = e_{\text{motional}} + e_{\text{variational}},$$

where e_{total} is the quantity recorded on the film.

5. See, in this connection, A. Blondel, *Sur l'énoncé le plus général des lois de l' induction.* Compt. Rend. 1914, Vol. 160, Nos. 20 and 22.

The application of the above to the analysis and interpretation of the complex waves obtained under s.s.c. or even the simpler phenomena of open circuit is difficult. Therefore, it will be well to call attention at first, to some general facts which though not entirely new do not seem to be generally known.

It is clear that the e.m.f. induced in a conductor or a number of concentrated turns (on the armature) will be a true reproduction of the B-curve under the pole, provided the variational e.m.f. were zero—which is seldom the case. The variational or static e.m.f. will depend upon the manner in which B varies with respect to time only. As a result of some valuable researches[6] it has been found that B may vary with respect to time in two ways: First, due to what may be termed the inductor alternator action of the teeth, there may be a *variation in the amount of reluctance* of the magnetic circuit of the machine. This will cause pulsations in the intensity of B and thus an e.m.f. will be induced in the armature coils as well as the pole pieces, yoke, etc. The latter will give rise to parasitic eddy currents which will tend to wipe out the pulsations. Thus, as indicated by theory and as shown by test, the variational e.m.f. due to this cause will be very small (compared to the rotational e.m.f.). Secondly, it has been found that the flux density at any point under the pole may vary due to a *to-and-fro swing* of the flux-wave with respect to the (geometrical) pole-axis. It has been shown by Worrall[6] by trimming the pole-shoes and varying the number of slot-pitches covered by it, that the pulsation is maximum when the pole-arc covers a whole number of slot-pitches, while the flux-swing is maximum when the pole-arc covers (an integer $+ \frac{1}{2}$) slot-pitches. This neglects fringing and in the experiments referred to, it seems to hold remarkably well. However, it seems to the writer that fringing should not be neglected since it is not the geometrical tooth or slot but the manner in which the flux is distributed in these that is responsible for the static e.m.f. Thus, his conclusions seem to be correct, except that it would be more rational to consider the effective teeth and slots (so far as flux is concerned).

The study of the experimental results as well as the quantitative expressions developed in the supplementary notes show that the e.m.f. waves of coils No. 7 and No. 8 give chiefly the

6. G. W. Worrall, *Jour., Inst. E. E.*, 1907, Vol. 39, p. 206. Also 1908, Vol. 40, p. 413. Guery, *L' Eclairage Electrique*, 1903, Vol. 36, p. 51. Arnold and La Cour, *Samn. Elek. Vort.*, 1901, Vol. 3, p. 58. K. Simon, *E. T. Z.*, 1908, Vol. 27, p. 631. *Electrician*, Vol. 57, p. 581.

e.m.f. due to flux pulsations. See second wave Fig. 4 and Fig. 5. The e.m.f. due to the to-and-fro motion of the flux is given by: (for $x_1 = 0$ see Fig. 6 and note A)

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1 \sin(4\,q\,\omega\,t) \tag{22}$$

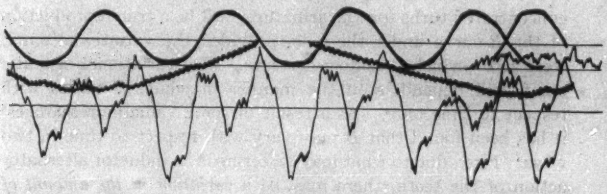


FIG. 4

Oscillogram No. O A 84. Test No. 196. First wave: armature current under s.s.c. at 4.96 amperes field excitation, normal speed and machine connected 2-phase. Third wave: E_{RF} = e.m.f. in full-pitch 40-turn rotor coil No. 7 under same conditions. Second wave: E_{RF} at same excitation and speed at no-load.

and that due to the pulsations of flux by:

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1 \sin(2\,q\,\omega\,t) \tag{23}$$

where η and ϵ are small fractions denoting the amplitude of the variations of flux, in each case. The former would be expected to be small as it involves ϵ^2 and oscillograms give no indication of the existence of ripples of ($4\,q \times$ machine-frequency), as required by (**22**). As seen from Fig. 4, second wave, and Fig. 5, the little kinks in the E_{RF} wave (due to the pulsations

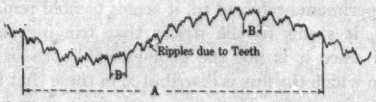


FIG. 5

Oscillogram No. O A 21. Test No. 91. Table I. Machine parallel Y-connected and run as synchronous motor. Field current = 4.4 amperes. Line current = 15.5 amperes. Line voltage = 220. E_{RF} = e.m.f. induced in rotor coil No. 7. Note: A = 1080 deg. B = 60 deg.

of flux) under synchronous motor and no-load operation, are fairly small and have a frequency of $2\,q\,f$, *i.e.*, show $2\,q = 24$ ripples per cycle. See Table I, column 6. This is what would be expected from (**23**).

While it is clear that the kinks in the E_{RF} wave, Figs. 4 and 5, are due to the teeth, the long flat superimposed wave having a period A, is believed to be due to the rotor being eccentric with respect to the stator. This explanation seems the more plausible when it is considered that A, Fig. 5, is found to be 1080 electrical degrees, as indicated in Table I column 4;

TABLE I.

Data relating to the e.m.fs. induced in rotor exploring coils under conditions given below.

No. of oscill. / No. of test	E_{RF} / E_{RC}	Field current in amps.	A in degrees see Fig. 5	B in degrees see Fig. 8	No. of ripples per cycle	Freq. in cycles per sec.	Amplitude in max. volts see Figs. 8a, b, c pos.	neg.	Line current in amps.	Approx. power factor
1	2	3	4	5	6	7	8	9	10	11
				Synchronous Motor Operation						
O A 21/91	E_{RF}	4.4	1080	54.8	24	59.5	0.69	0.82	15.5	1
O A 22/92	E_{AP}*	4.4	1080	60		59.5	0.66		15.2	1
O A 39/115	E_{AP}*	1.0	1080		24	59.5	1.12	mean	79	†0.27 lg.
O A 40/116	E_{RC}	8.3	1080	61.3		59.5	1.12	mean	76	0.28 ld.
O A 36/118	E_{RC}			61	24	59.5	0.037	0.053	85.5	0.25 lg.
O A 37/109	E_{RC}	8		60.6	24	59.5	0.037	0.053	71.2	0.24 ld.
				Sustained Short Circuit. Series Y Connection						
–/164	E_{RF}	5			24	54	0.97 mean			
–/165	E_{RC}	5			24	50	0.03 mean			
–/166	E_{RC}	3.16			24	52	0.027 mean			
–/167	E_{RF}	3.16			24	52	0.71 mean			
226/–	E_{RF}	5		59.2	24	60				
				Open-Circuit Operation						
–/163	E_{RF}	5				60	2.46			
–/162	E_{RF}	3.16				60	1.94			
–/161	E_{RF}	5				54	2.31			
–/160	E_{RF}	3.16				54	1.98			
O A 24/94	E_{RF}	5	1080	60	24					

*E_{AP} = e.m.f. induced in search coil around pole, right under pole-shoe; this gave practically same results as full-pitch coil No. 7 and therefore it was disconnected from slip-ring and not used.

†lg. = lagging. ld. = leading.

i.e., the superimposed wave is found to have under load or at no-load a frequency $(\frac{1}{4})\ f = (1/p)\ f =$ one cycle per revolution.

For a conductor placed at the middle of the pole-arc, *i.e.*, at $x_1 = \tau/2$, Fig. 6, $e_{sw.r.}$ would be maximum and $e_{pul.r.}$[7] would be zero; *i.e.*, such a conductor would give ripples due to the

7. See equations (21) and (20) respectively in supplementary notes.

flux-swing only. Attempts were made to take advantage of this but owing to the high centrifugal force it was found difficult to fasten down securely wires at the mid-pole position.

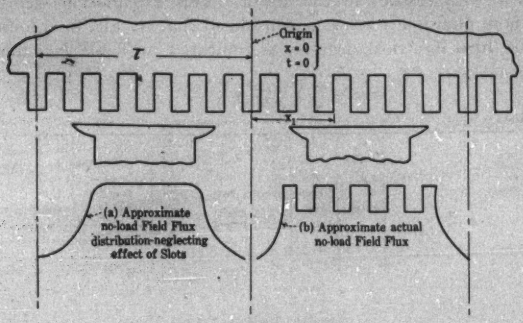


FIG. 6

However, a very good idea of the relative magnitude of the e.m.f. due to flux-swing, can be obtained by studying the e.m.f. waves induced in the stator coils. These will consist of (e_{rot}

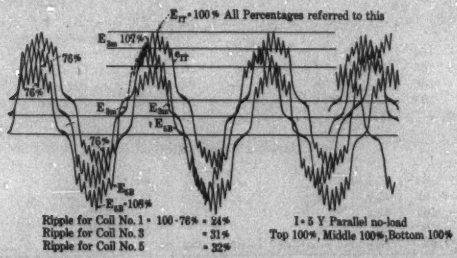


FIG. 7

Oscillogram No. O A 38. Test No. 110. Tables IV, V and VI. Machine run on open-circuit. First wave: E_{1T} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3, at middle of slot. E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. Speed normal, *i.e.*, corresponding to 60. Excitation = 5 amperes. Same vibrator and resistance in vibrator circuit used for all waves. Compare to Fig. 22.

$+ e_{pul} + e_{sw}$). As indicated by equations (7), (13) and (18) Note A, e_{pul} will be negligible for a rotor coil placed at the middle of the pole-shoe, *i.e.*, at $x = \tau/2$, or for a stator coil

whenever $x = \tau/2$, see Fig. 6. In other words, e_{pul} will be negligibly small *whenever* $e_{rot.st.}$ is maximum. Thus the ripples at the crest of the e.m.f. wave ($x = \tau/2$) of the stator coils 1, 3 and 5 are due mainly to flux-swing and a comparison of the ripples in them, Fig. 7, with the kinks of the E_{RF} wave, second wave Fig. 4 and Fig. 5, and E_{RC}, Fig. 8a, will show at once that the kinks (due to pulsation) are quite negligible compared to the ripples (due to flux-swing)—especially when it is remembered that coils No. 8 and 7 have many more turns than the stator coils and the volts per centimeter for the calibration of the former is about ten per cent of that for the latter.

The subject of tooth ripples is of great practical importance,

FIG. 8A

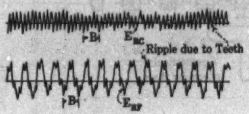


FIG. 8B

FIG. 8C

Fig. 8a Oscillogram No. O A 36. Table I. Machine run as synchronous motor. Line current 85.5 amperes lag. Line voltage = 200. E_{RC} = e.m.f. induced in coil No. 8. Exactly same wave shape and amplitude obtained in oscillogram No. O A 37 with 71 amperes of leading line current and 238 line voltage.

Fig. 8b Oscillogram No. O A 49. First wave E_{RC} = e.m.f. induced in coil No. 8. B = 60 deg. Second wave: E_{RF} = e.m.f. induced in coil No. 7. B = 60 deg. Machine series Y-connected and short-circuited at its terminals. Field current = 5 amperes. Frequency = 54 cycles per second. Resistance in vibrator circuit = 0 ohms and gold-leaf fuses short-circuited.

Fig. 8c Oscillogram No. O A 50. Similar to O A 49 in every respect, except field excitation = 3.16 amperes. Same vibrators used.

not only in case of accurate tests and measurements but also in connection with the operation of large alternators, induction motors, etc. Though effective methods have been developed for their elimination there seems to be lack of general concensus of opinion[8] as to their origin and detailed analysis. In this connection it seems pertinent to ask, whether the ripples are entirely variational in their origin or are they partly motional. The actual flux density in the air-gap has the general form indicated by curve (*b*) rather than curve (*a*) Fig. 6. Since the motional

8. S. P. Smith and R. H. Boulding, *Jour., I. E. E.*, 1915, Vol. 53, p. 205. Discussion p. 238. Also see correspondence *Electrician*, June, 1914. Further, see references given under foot-note (6).

e.m.f. will be duplicate of the B-curve, it is clear that the tooth ripples will partly have motional origin, *i.e.*, will be partly due to the cutting of a B-curve like that shown in (*b*) Fig. 6.

A general idea of the relative importance of the ***tooth kinks in the B-curve*** may be gained from Fig. 9 which is self-explanatory. The flux measurements were obtained by pulling out of the air gap an exploring coil fastened on a piece of cardboard and having a length approximately equal to the axial thickness of the armature core, and a width about 30 per cent of that of a tooth. In the test recorded in Fig. 9, the machine was stationary; in the next one, the rotor was excited and slowly rotated and the following maximum throws of the Grassot fluxmeter were observed.

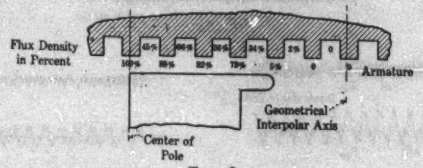


FIG. 9

Flux distribution with the machine stationary as measured by Grassot fluxmeter.

Coil No. 10—coil at top of tooth—maximum deflection = 100 per cent.

Coil No. 9—coil at top of wedge—maximum deflection = 41 per cent.

Coil No. 8—concentrated coil—maximum deflection = 8 per cent

The actual flux density under running conditions was further investigated by means of the exploring coils No. 9, 10 and 11, Figs. 3 and 3A, by taking oscillographic records of the e.m.fs. induced in them under no-load, s.s.c. and synchronous motor operation. The data obtained in this series of tests are brought together in Table II. As seen from Figs. 10, 11 and 12 the E_{AT}, E_{T1} and E_{TW} waves have two prominent crests above, and two below, the zero line, per cycle. These are quite characteristic and are designated as 1st and 2nd positive and 1st and 2nd negative. See Table II, columns 8 and 11. The angular distance between successive + or − crests is designated by α and that between successive positive and negative ones by β. See Fig. 10 and Table II columns 6 and 7.

TABLE II.—Data relating to the e.m.fs. induced in Exploring Coils No. 9, No. 10 and No. 11 (Fig. 3) under open-, sustained short-circuit and synchronous motor operation.

No. of oscillograms / No. of test	$E_{A.T.}$ $E_{T.T.}$ $E_{T.W.}$	No load, s.s.c. or synch. mot. / field amperes	Line current in amperes	Approximate power factor leading or lagging	β (in degrees, See Figs. 10, 12, etc.)	α (in degrees, See Figs. 10, 12, etc.)	Consecutive crest values in max. volts per turn: Positive crests 1st	Positive crests 2nd	Negative crests 1st	Negative crests 2nd
1	2	3	4	5	6	7	8	9	10	11
O A 25/95	$E_{A.T.}$	N. L./5	0		118°	61°	1.47	1.39	1.49	1.27
O A 61/172	$E_{T.T.}$	N. L./4.95	0		120.5°	59.5°	0.508	0.422	0.466	0.416
O A 20/90	$E_{A.T.}$	S. M./4.49	15.5	0.95 lg.	117°	59.9°	1.18	1.24	1.02	1.23
O A 27/97	$E_{T.T.}$	S. M./4.24	15.5	1.0	116.5°	62.3°	0.33	0.33	0.38	0.34
O A 27/97	$T_{T.W.}$	S. M./4.24	15.5	1.0	119°	59.4°	0.15	0.16	0.15	0.15 +
O A 23/93	$E_{A.T.}$	S. M./3	18.5	0.81 lg.	113°	65.1°	1.19	1.07	1.05	1.19
O A 28/98	$E_{T.T.}$	S. M./2.9	39.5	0.56 lg.	119°	62.2°	0.24	0.28	0.24	0.29
O A 28/98	$E_{T.W.}$	S. M./2.9	39.5	0.56 lg.	115°	60.9°	0.12	0.14	0.12 +	0.14 +
O A 26/96	$E_{A.T.}$	S. M./	40.7	0.4 ld.	124.8°	58.5°	1.41		1.44	1.47
O A 29/99	$E_{T.T.}$	S. M./5.4	39.5	0.45 ld.	118.4°	62.15°	0.47	0.48	0.48 +	0.48 −
O A 29/99	$E_{T.W.}$	S. M./6.4	39.5	0.45 ld.	119.5°	61.7°	0.19	0.196	0.194	0.194
O A 37/109	$E_{A.T.}$	S. M./8	71.2	0.24 ld.	121.5°	57.3°	1.47	1.47	1.51	1.51
O A 36/108	$E_{T.T.}$	S. M./−	85.5	0.25 lg.	117.5°	63.3°	0.71 +	0.94	0.71 +	1.0 −
O A 41/117	E_{TT2}	N. L./ −	0		117°	62.4°	0.017	0.014		
O A 41/117	E_{TT2}	S. M./ −	24.7	0.93 lg.	116°	61.4°	0.041	0.034		
O A 40/116	E_{TT2}	S. M./−	76	0.26 ld.	120°	61.5°	0.045	0.047	0.045	0.047
O A 39/115	E_{TT2}	S. M./−	79	0.27 lg.	111.5°	62.6°	0.022	0.033	0.022	0.033
		Field amperes	Kind of s.s.c.	Frequency						
O A 61/172	E_{TT}	4.95	Y series	60	118.4°	61.6°	0.23	0.187	0.25	0.187
O A 59/180	T_{TW}	5.0	Y series	60	119°	60.1°	0.18	0.12	0.16	0.12

NOTE: α and β are in terms of machine-frequency. Thus if one cycle at machine-frequency occupies m cms. on the film and α is n cms., then $\alpha = \frac{n}{m} \times 360$ deg.

The peculiar shape of these curves may be explained as follows: As a concentrated coil like No. 9 or No. 10, moves past a (stationary) field pole, the amount of change in the enclosed flux will depend upon the tooth kinks of the B-curve, (*b*) Fig. 6. If the B-curve were flat topped, see (*a*) Fig. 6, there would be no change of flux through coils No. 9 or 10 while under the pole-arc. Thus the small ripples in the portion of the B-curve marked β, Fig. 10, are due partly to the bunching of the flux at the teeth and partly due to the flux variations (= pulsations in magnitude, and to-and-fro swing), as already explained. As the coil emerges from under the horn of, say, a north pole and begins to enter the

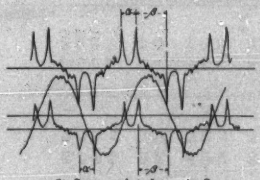


FIG. 10

Oscillogram No. O A 28. Test No. 98, Table II. First wave: E_{AT} = e.m.f. induced in coil No. 11 around tooth. Second wave: volts across the terminals of the machine. Third wave: E_{TW} = e.m.f. induced in coil No. 9 on top of wedge. Machine parallel Y-connected and operated as a synchronous motor. Line current = 39.5 amperes. Line voltage = 212 volts. Power factor = 56 deg. lag. α corresponds to interpolar space. β corresponds to space under pole-arc.

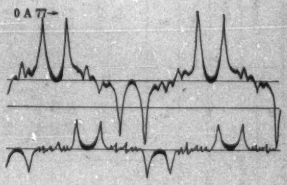


FIG. 11

Oscillogram No. O A 77. Test No. 189. First wave: E_{TT} = e.m.f. induced in search coil No. 10 at top of tooth. Second wave: E_{TW} = e.m.f. induced in exploring coil No. 9 at top of wedge. Machine run at open-circuit and same vibrator and same resistance in vibrator circuit for both waves.

interpolar region and then the tip of the south pole, there will be, two, comparatively large changes in the flux enclosed; these flux changes will give rise to two peaks whose angular spacing α, as seen from Table II is about 60 elec. deg. *i.e.*, α corresponds to the angular distance between two consecutive pole-horns. Evidently these crests should be on the same side of the zero line since one is due to a decreasing flux issuing from a north pole, while the other is due to an increasing flux of opposite polarity.[9]

9. See curve 32 given by W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 754.

At first sight it might be expected that the e.m.fs. induced in coils No. 9 and 10 should be similar in shape and equal in amount since the only difference between them is that one is placed on top of a tooth, while the other is on top of a wedge. However, as seen from Table II this is by no means the case; further as

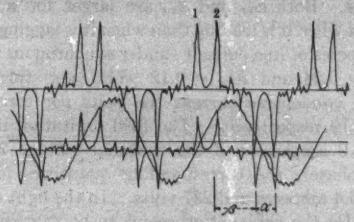


FIG. 12

Oscillogram No. O A 29. Test No. 99. Table II. Machine Y parallel connected and run as a synchronous motor. Field excitation, 6.4 amperes. Line voltage = 237 volts Line current = 39.5. Approximate power factor = 0.45 leading. First wave: E_{TT} = e.m.f. induced in coil No. 10 at top of tooth. Second wave: Line voltage. Third wave: E_{TW} = e.m.f. induced in coil No. 9 at top of wedge.

seen from Table III the area enclosed by E_{TT} for the coil at the top of a tooth, is very much larger than E_{TW} for the coil at the top of a wedge. This can be explained on the assumption that a given peak is due to the change of the enclosed flux to nearly zero, from the value it had just before the coil left the

TABLE III.

Data relating to synchronous motor operation of machine and giving the area per one-half cycle per turn enclosed by the e.m.f. waves induced in coils No. 9 and No. 10.

No. of oscill.	Field amps.	Line amps.	Line volts	Power factor	Maxwells or volts per turn per ½ cycle
O A 29	6.4	39.5	237	0.45 ld.	45,000, coil No. 10
O A 29	6.4	39.5	237	0.45 ld.	12,036, coil No. 9
O A 27	4.24	15.5	218	1.00	59,400, coil No. 10
O A 27	4.24	15.5	218	1.00	29,800, coil No. 9

horn of the pole. This change will be much larger for a tooth than for a slot. The above, in connection with Tables II and III, give a fairly good idea of the flux distribution and flux pulsations under actual running conditions, in contrast to the phenomena which obtain with the machine stationary and which are easily investigated, as indicated in Fig. 9.

Study of Table II and the oscillograms given in connection therewith reveals several interesting facts. A comparison of Figs. 10 and 12 will show that the kinks between a + crest and a − crest, *i.e.*, under the pole-arc, are larger when the armature current is leading or demagnetizing than when it is lagging or magnetizing. Both E_{TT} and E_{TW} are larger for a given armature current when it is leading than when it is lagging; for example at 39.5 amperes of line current (under synchronous motor operation) $E_{TT} = 0.24$ and $E_{TW} = 0.12$ volts per turn. At 39.5 amperes of condensive current the volts per turn increase to 0.47 and 0.19 respectively. The field excitation in the former case was 2.9 amperes and the terminal voltage 212 (effective volts per phase); the corresponding quantities for the latter case being 6.4 amperes and 237 volts. In the light of the above, (see Table II) the explanation seems to be that in spite of the

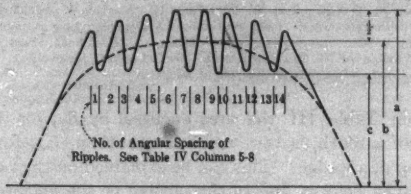


FIG. 13

Exaggerated figure showing e.m.f. induced in exploring coils at no-load and unequal spacing of tooth ripples.

demagnetizing action of the leading currents the flux density in the gap increases on account of the higher excitation and voltage.

Under s.s.c. the crest voltages E_{TT} and E_{TW} do not differ as widely which is no doubt due to the very peculiar and distorted flux distribution as will be explained in the next section.

In connection with the subject of tooth ripples, attention may be called to several other peculiarities. As seen from Table IV and indicated in Fig. 13, the angular spacing of the tooth ripples from crest to crest is not uniform as ordinary theory would indicate. (See equations **13** and **14**). In the first place, the angular spacing (1 + 2), (3 + 4), etc. Fig. 13, is slightly less than π/q, or in this case 15 electrical deg. Secondly the angular spaces 2, 4, 6, 11, and 13 are larger than 1, 3, 5, etc.; this is probably due to the hysteresis effect in the teeth. Thirdly, the amplitude d, Fig. 13 is distinctly smaller for E_{1T} than for E_{5T}

this is easily seen from Fig. 7 in which the waves were taken under the same operating conditions, using the same vibrator

TABLE IV.

Angular spacing in degrees (electrical) of the maximum and minimum crests of the tooth ripples of the no-load search coil e.m.fs.

No. of oscill.	Search coil	Field amperes	Mean angular spacing of (see Fig. 13)			
			1, 3, 5 & 7	2, 4 & 6	(1 + 2), (3 + 4), etc.	(2 + 3), (4 + 5), etc.
1	2	3	4	5	6	7
272	No. 1, top of slot	8.8	6	8.5		
O A 35	No 1, top of slot		7.15	7.35	14.62	13.29
O A 35	No.2, midd. of slot		6.46	7.81	13.9	14
O A 35	No. 5, bott. of slot		6.94	7.21	14	14.2
O A 38	No. 1, top of slot	5	6.64	7.45	14.18	14.4
O A 38	No.3, midd. of slot	5	6.64	7.08	13.77	14
O A 38	No. 5, bott. of slot	5	6.84	8.03		
260	No. 1, top of slot	5	6.41	9.13		
263	No. 3, midd. of slot	4	6.03	8.43		
268	No.5, bott. of slot	5	6.37	8		
Mean:			6.55	7.89	14.09	13.18

and the same vibrator resistance. A quantitative idea of the variations may be gained from Table V.

In Table VI are given the areas enclosed per $\frac{1}{2}$ cycle by E_{1T},

TABLE V.

No. of oscillograms	E.m.f. of search coil	$\frac{d}{2} \times \frac{1}{b} \times 100$ See Fig. 13	Field amperes	Frequency cycles per second
272	E_{1T}	7.5%	8.8 appr.	50
272	E_{5B}	8.3%	8.8	50
O A 38	E_{1T}	7.2%	5	60
O A 38	E_{3M}	7.9%	5	60
O A 38	E_{5B}	9.2%	5	60

E_{3M} and E_{5B}, and it is seen that for the bottom coil No. 5 the flux is 107.5 per cent of that enclosed by the top coil. Under synchronous motor operation the percentage is smaller, about

103 per cent for leading, *i.e.* demagnetizing, and larger, about 113 per cent for lagging currents.

These phenomena may be explained on the basis of the following assumptions. Consider two surfaces, one passing through the top and the other through the bottom of the armature slots.

TABLE VI.

Mean areas of consecutive positive and negative half-waves of e.m.f. induced in full-pitch, 5-turn exploring coils at top, middle, and bottom of slots = E_{1T}, E_{3M} and E_{5B} respectively.

No. of oscill. test No.	E_{1T} E_{3M} or E_{5B}	Pre-quency in cycles per sec.	Synchr. motor, short-circuit or no load oper-ation	Approx. power factor	Line current V	$(\phi_1-\phi_2)$ in mega-maxwells or volts per turn per cycle	$(\phi_1-\phi_2)$ in per cent	Field amps.
1	2	3	4	5	6	7 8	9	10
O A 32/104	E_{3M}	59.5	S. M.	0.42 ld.	45.2 ld.	2.31	100	
O A 32/104	E_{5B}	59.5	S. M.	0.42 ld	45.2 ld.	2.40	104	
O A 34/106	E_{1T}	59.5	S. M.	0.41 lg.	52.6 lg.	1.95	100	
O A 34/106	E_{3M}	59.5	S. M.	0.41 lg.	52.6 lg.	2.05	105	
O A 35/107	E_{1T}	59.5	S. M.	0.37 ld.	52.6 ld.	2.22	100	
O A 35/107	E_{3M}	59.5	S. M.	0.37 ld.	52.6 ld.	2.235	101.5	
O A 35/107	E_{5B}	59.5	S. M.	0.37 ld.	52.6 ld.	2.281	103	
O A 33/105	E_{1T}	59.5	S. M.	lag.	60 appro	1.93	100	
O A 35/105	E_{3M}	59.5	S. M.	lag.	60 appro	2.0	103.7	
O A 35/105	E_{5B}	59.5	S. M.	lag.	60 appro	2.18	113	
O A 38/110	E_{1T}	60	no load			2.316	100	5
O A 38/110	E_{3M}	60	no load			2.38	103.0	5
O A 44	E_{1T}	54	s.s.c. connection Y series			0.415	100	5
O A 72	E_{5B}	54	s.s.c. connection Y series			0.282	70.3	5
O A 51	E_{1T}	52	s.s.c. connection Y series			0.3095	100	3.16
O A 54	E_{3M}	52	s.s.c. connection Y series			0.245	79.2	3.16
O A 67	E_{5B}	52	s.s.c. connection Y series			0.0969	34.4	3.16
O A 81	E_{1T}	60	s.s.c. connection 2-phase			0.1348	100	4.96
O A 82	E_{5B}	60	s.s.c. connection 2-phase			0.1056	78.3	4.96

Assume that the flux density in the slots is smaller and in the teeth is larger and more nearly perpendicular[10] to the direction of rotation of the armature conductors, for the second surface than for the first. Further, if the armature were revolving, the

10. Compare equations (4) and (4a) in supplementary notes.

TABLE VII.

Order of harmonics	Wave analysis No.: (I − 2) No. of oscillogram: 206 Field current, amperes: 6.5 Frequency, cycles per sec.: 60 Exploring coil: No. 1, top of slot			(I − 4) 201 6 60 No. 1, top of slot			(I − 5) 206 5 60 No. 1, top of slot		
	A, volts B, volts	C, volts (C/C_1) 100	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$
1	28.82	28.9	99.7%	27.93	27.99	99.85%	25.3	25.3	100%
	− 0.62	100%	− 2.14%	1.39	100%	4.97%	0.17	100%	0.66%
3	− 0.73	1.08	− 2.53%	− 1.03	1.05	− 3.68%	− 0.88	0.95	− 3.44%
	0.805	3.77%	2.78%	− 0.21	3.75%	− 0.75%	0.35	3.75%	1.36%
5	− 2.22	2.24	− 7.65%	− 2.41	2.42	− 8.61%	− 2.18	2.19	− 8.63%
	0.33	7.76%	1.12%	− 0.24	8.65%	− 0.86%	− 0.14	8.65%	0.56%
7	− 2.28	2.28	− 7.89%	− 2.09	2.55	− 0.75%	− 2.02	2.05	− 7.9%
	0.12	7.89%	0.403%	− 1.47	0.91%	− 5.26%	− 0.33	8.09%	− 1.3%
9	− 0.81	0.94	− 2.79%	0.02	0.09	0.07%	2.68	2.7	10.64%
	− 0.48	3.25%	− 1.67%	− 0.09	0.03%	− 0.03%	− 0.00	10.65%	− 0.02%
11	0.35	0.39	1.22%	0.14	0.64	0.05%	0.62	0.64	2.42%
	− 0.17	1.35%	− 0.58%	0.63	2.29%	2.25%	0.15	2.5%	0.59%
13	1.15	1.16	3.98%	0.44	0.72	1.57%	0.34	0.38	1.31%
	− 0.15	4.02%	− 0.52%	0.58	2.57%	2.07%	0.20	1.5%	0.78%
15	0.30	0.56	1.04%				0.13	0.26	0.50%
	0.47	1.93%	1.63%				0.22	1.02%	0.89%
17	− 0.26	0.26	− 0.88%				0.40	0.52	1.60%
	− 0.03	0.9%	− 0.12%				0.32	2.04%	1.26%
23	1.14	1.17	3.95%	− 0.29	0.87	− 1.04%	0.79	1.12	3.14%
	− 0.27	4.05%	− 0.9%	0.81	3.1%	2.93%	0.78	4.43%	3.09%
25	− 0.87	0.88	− 3%	0.49	1.09	1.75%	− 0.65	0.77	− 2.57%
	0.14	3.04%	4.95%	− 0.97	3.89%	1.39%	− 0.42	3.04%	− 1.66%

NOTE: Wave analysis, I − etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\theta +$ etc. $+ B_1 \cos \theta + B_3 \cos 3\theta +$ etc. The second column gives the coefficients of the same wave when expressed as: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + 3\varphi_3} +$ etc. A, B and C are also given in per cent of C_1 in second and third columns.

TABLE VIII.

Order of harmonics	Wave analysis No.: (I − 10) No. of oscillogram: 206 Field amperes: 2.75 Frequency: 60 Exploring coil, No. 1, top of slot		(I − 13) 263 4 60 No. 3, middle of slot		(I − 15) 263 3 60 No. 3, middle of slot		(I − 21) 201 8 60 No. 5, bottom of slot		(I − 25) 228 3 60 No. 5, bottom of slot	
	A, volts B, volts	C, volts (C/C_1) 100	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$
1	16.75 0.97	16.8 100%	22.2 −0.15	2.22 100%	17.2 0.05	17.2 100%	28.9 0.72	28.9 100%	17.4 0.002	17.43 100%
3	−0.66 0.35	0.74 4.4%	−1.0 0.28	1.09 4.91%	−0.9 0.36	−0.97 5.64%	−1.89 0.05	−1.89 6.54%	−0.62 0.5	0.79 4.55%
5	−1.58 −0.8	1.76 10.59%	−2.4 −0.11	2.41 10.85%	−2.12 −0.14	2.12 1.23%	−3.66 −0.8	3.75 12.96%	−0.66 −0.5	0.83 4.76%
7	−0.89 −0.88	1.25 7.44%	−1.08 −0.28	1.12 5.04%	−1.27 −0.26	1.29 7.50%	−1.67 −0.35	1.67 5.78%	−1.93 −2.07	2.83 16.23%
9	0.05 −0.08	0.09 0.56%	1.01 0.03	1.01 4.55%	0.06 −0.13	0.15 8.55%	0.39 −0.56	0.68 2.35%	−1.31 −0.58	1.43 8.90%
11	0.11 0.57	0.58 3.45%	0.96 0.26	0.80 3.60%	−0.68 0.097	0.68 3.95%	0.92 0.57	1.08 3.74%	0.14 1.64	1.85 9.47%
13	−0.1 0.47	0.48 2.85%					0.35 0.36	0.5 1.73%		
15	0.75 −0.16	0.17 1.01%					−0.24 0.1	0.26 0.9%		
17	−0.44 1.04	1.13 6.71%					−0.36 −3.96	3.98 13.76%		
23	−0.34 −0.00	−0.34 2.02					0.67 1.77	1.02 3.53%		
25	0.47 0.067	0.47 2.8					−0.72 −1.02	1.25 4.33%	0.48 0.26%	0.55 3.17%

NOTE: Wave analysis I − etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\theta$ etc. $+ B_1 \cos \theta + B_3 \cos 3\theta$ etc. Second column gives coefficients of the same wave when transformed into: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + \varphi_3}$ etc. Coefficients C in per cent of fundamentals C_1 are also given in column (2).

peripheral velocity would have been larger for the second surface. These assumptions will partly at least explain the above and they seem reasonable as may be seen by examining the approximate flux distribution[11] in slotted armatures under different load conditions.

Tables VII and VIII give wave analyses of the e.m.fs. induced at no-load in different exploring coils. It will be seen that for a given field current, for example 6 amperes, wave analysis

TABLE IX.

Showing increase of 3rd harmonic of e.m.f. induced in stator exploring coils, with increasing excitation.

Excitation in amps. = I_f	Third harmonic e.m.fs. induced in stator full pitch exploring coils			Per cent values of columns				No. of oscillogram from which columns		
								No. 2	No. 3	No. 4
	No. 1 E_{1T}	No. 3 E_{3M}	No. 5 E_{5B}	No. 1 I_f	No. 2 E_{1T}	No. 3 E_{3M}	No. 4 E_{5B}	were calculated		
1	2	3	4	5	6	7	8	9	10	11
				No-load operation						
8.8	2.21		3.94	320	298		532	272		272
6.5	1.08			236	146			206		
6.0	1.05		1.89	218	142		255	201		201
5.0	0.95	1.67	1.28	182	128	225	173	206	261	266
4.0		1.09		145.5		147			263	
3.0	0.75	0.97	0.79	109	101	131	107	228	263	228
2.75	0.74			100	100			206		
				Y series sustained short circuit						
5	3.99		7.52	182	532		1000	202		286
4			5.12	145.5			684			286
3	2.46		4.04	109	328		538	229		286

NOTE: The sustained s.c. percentages are in terms of 0.75 volts, induced in top search coil No. 1 at no-load and 3 amperes excitation.

$I - 4$, the amplitudes in maximum volts for the 1st, 3rd and 5th harmonics are respectively, 27.99, 1.05 and 2.42 for coil No. 1. The corresponding values from wave analysis $I - 21$, Table VIII, for coil No. 5 at the bottom of the slot, are: 28.9, 1.89

11. See for example, Figs. 110 and 111 p. 282 and p. 283, Principles of Electrical Design, by A. Still. *Jour., I. E. E.*, Magnetic flux distribution in toothed core armatures. H. S. Hele-Shaw, A. Hay and P. H. Powell. Figs. 183, 223 etc. pp. 165 and 220, in S. P. Thompson's Polyphase Electric Currents, Second Edition.

and 3.75. As the third harmonic is of special interest in three-phase work Table IX has also been prepared; inspection of this table shows that taking the e.m.f. induced in coil No. 1, at the top of the slot, at 2.75 amperes of field excitation, as 100 per cent, the third harmonic of e.m.f. increases for coil No. 1 with increasing excitation until it reaches 298 per cent at 8.8 amperes of field current. Under s.s.c. we have a similar condition; the third harmonic for coil No. 5 reaches 1000 per cent while the corresponding value for coil No. 2 is 532 per cent at 5 amperes.[12]

In Fig. 14 the e.m.fs. E_{CTT} and E_{CTS} induced in a single

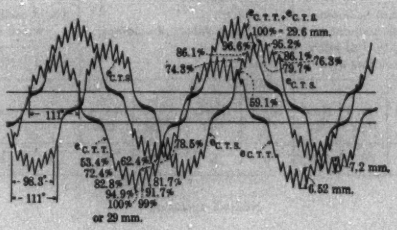


FIG. 14

Oscillogram No. O A 75. Test No. 187. Open-circuit test at 5 amperes field excitation and normal speed. First wave: (e.m.f. induced in single conductor No. 14 at top of tooth) + (e.m.f. induced in single conductor No. 13 at top of immediately succeeding wedge) = $E_{CCTT} + E_{CTS}$. Second wave: E_{CTS}. Third wave: E_{CTT}. Same vibrator used for all exposures. 30 ohms resistance in vibrator for $E_{CTT} + E_{CTS}$; 10 ohms for other two waves.

conductor placed on top of a tooth and on top of a slot respectively (see coils No. 13 and 14, Fig. 3) show certain peculiar differences in that E_{CTT} has nine distinct kinks while E_{CTS} has eight. This is easily explained by following step by step the variational e.m.f. induced in a conductor on top of a tooth and one on top of a wedge, as the armature slots move with respect to the field poles.

12. For a tentative explanation of the fact observed, namely that the third harmonic of e.m.f. for a given exploring coil will increase with excitation see, P. Janet, *Lecons d' électrotechnique générale*, T. II, p. 191. However, note further that not only the 3rd harmonic increases for a given coil with increase of field current but it seems to increase faster for a search coil at the bottom of slot than one at the top of a slot.

IV. Experimental Investigation of the Direct Component of Armature Reaction and the Resultant Flux Distribution Under Sustained Short-Circuit Conditions

The armature current under s.s.c. has been found to be very closely sinusoidal, as can be seen by inspecting first two waves in Fig. 15 and fifth wave in Fig. 16, which are for Y and delta

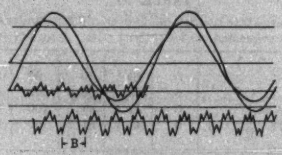


Fig. 15

Oscillogram No. 281. Table X. Sustained short circuit with machine series delta connected. Second straight line; zero line for first two waves representing armature phase current at 3.27 and 4.1 amperes field excitation respectively. Third and fourth waves give e.m.f. induced in 29-turn full-pitch rotor exploring coil No. 7, under same conditions.

connections. A quantitative idea of the approach of these waves to a sine wave may be gained from Table X.

Throughout this paper the armature current under s.s.c. *will be assumed* to be sinusoidal and if the armature winding were

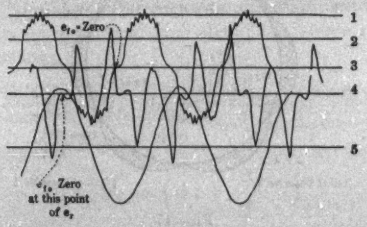


Fig. 16

Oscillogram No. O A 44. Test No. 148, Table XV. *Exposure No.* 1. Open-circuit test at 5 amperes field excitation and 54 cycles per second. First wave: contactor. Third wave: E_{1T} = e.m.f. induced in coil No. 1. *Exposure No.* 2. Sustained short-circuit test with machine series Y connected. Same excitation and speed as before. Fourth wave: E_{1T}. Fifth wave: line current. Second wave: contactor.

sinusoidally distributed, see Fig. 17, the armature m.m.f. would be sinusoidal. However, it can be shown (see supplementary notes, equation **30**) than an ordinary armature winding with N conductors per slot and one slot per pole per phase, see Fig.

18, supplied with a sinusoidal current will be exactly equivalent to: (1) a winding sinusoidally distributed (in space) having a pitch of τ cm. or inches. (2) a winding sinusoidally distributed having a pitch of $\tau/3$ and $\frac{1}{3}$ as many turns as the first, etc., etc. In other words, imagine a set of sinusoidally distributed windings, of pitch τ/k, where k is the order of the space harmonic,

TABLE X.

Per cent amplitude of armature current, i_a, at every 30 deg., under s.s.c. and the machine Y-series and delta-series connected.

θ in degrees	0	30	60	90	120	150	180
$\sin\theta \times 100$	0	50	86.6	100	86.6	50	0
i_a, Y conn. (oscill. O A 44)	0	51.1	84.5	100	85.4	50	0
i_a, $\triangle$ conn. (oscill.281)	0	48.7	81.2	100	85.1	48.7	0

all placed in series and carrying the same sinusoidal current (with respect to time): These will produce the same H-curve as the actual non-sinusoidal winding carrying the sine-wave current of fundamental (with respect to time) frequency.

Without going further into the analysis of this, suffice to

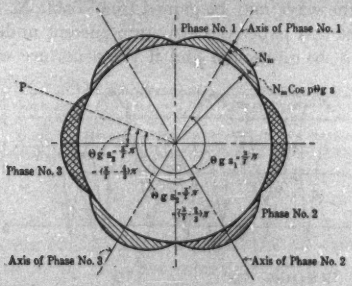


FIG. 17

Schematic representation of three-phase two-pole armature winding which is sinusoidally distributed (in space).

state[13] that for a polyphase alternator, with a sine wave (with respect to time) armature current and any ordinary type (non-

13. See A. Potier, *Journal de Physique*, 1897, 3me serie, T. VI, p.341 and 483. A. Blondel, *L' Eclairage Electrique*, 1895, T. IV, pp. 241, 308 et 358. T. V., p. 268. See also P. Janet, *Lecons d' Electrotechnique générale*, T. III, Chp. V. Alex. Russell, Altern. Currents Vol. II, Chp. XIV.

sinusoidally distributed, in space) winding, the resultant m.m.f. curve can be considered as consisting of a series of H-curves, all of fundamental frequency (in time) but having a pole pitch τ/k and an amplitude equal to:

$$H_a = \frac{N_{ph}}{2} \cdot \frac{H_m}{k} S \cdot D_f \cdot S_{p.f.} \tag{31}$$

where N_{ph} = number of phases.

H_m = $(4/\pi)\ Y_m$. See Fig. 18.

S = slots per pole per phase.

D_f = distribution factor = $\dfrac{\sin \frac{1}{2} S \frac{\lambda}{\tau} \pi}{S \sin \frac{1}{2} \cdot \frac{\lambda}{\tau} \pi}$

λ = slot pitch.

$S_{p.f.}$ = short-pitch factor = $\cos \epsilon k \pi$, where $(1 - \epsilon)\tau$ = winding pitch.

For a three-phase alternator the first, seventh, thirteenth etc., or $(6n + 1)\ th$ space harmonics, ($n = 0, 1, 2, 3$ etc.), will rotate in the same direction at a speed of $2\pi f/k$ or at an angular velocity ω/pk. The $(6n - 1)\ th$ harmonics will rotate in the opposite direction with the same velocity. In mathematical short-hand the above may be expressed as follows:

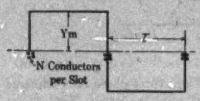


FIG. 18

$$H_1 = H_a \cos\left(\omega t - \frac{x}{\tau}\pi\right) \tag{32}$$

$$H_3 = 0 \tag{33}$$

$$H_5 = H_a \cos\left(\omega t + \frac{5x}{\tau}\pi\right) \tag{34}$$

$$H_7 = H_a \cos\left(\omega t - \frac{7x}{\tau}\pi\right), \text{ etc.} \tag{35}$$

where H_a is given by (31).

In case of two-phase machines the first, fifth etc., or $(4n + 1)\ th$ space harmonics will travel in one direction while the third, seventh etc. or $(4n - 1)\ th$ space harmonics will travel in the opposite direction. The expressions for these will be similar to the ones given above and their amplitude will be given by (31)

The m.m.f. per centimeter at any point x of the stator periphery will be $(4\,\pi.\ 10^{-1}\,H)$ where,

$H = H_1 + H_5 + H_7$ etc.

= sum of series of sinusoidal H-waves, all of fundamental frequency and having phase angles, $(-\,6\,x_1\,\pi/\tau)$, $(+\,6\,x_1\,\pi/\tau)$, $(-\,12\,x_1\,\pi/\tau)$, etc., with respect to the fundamental whose phase angle $(-\,x_1\,\pi/\tau)$ is assumed to be zero. See Notes B and C.

This leads to the following important proposition: With polyphase balanced alternating currents varying according to a sine law (in time) and flowing through ordinary non-sinusoidally distributed windings, (in space) the H-curve will be sinusoidal (*i.e.*, will be the sum of a series of sine waves). Thus, under load

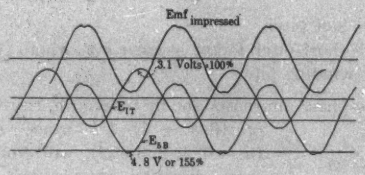


FIG. 19

Oscillogram No. O A 13. Test No. 32. Rotor removed and three-phase 85.8 amperes (158 per cent normal current) introduced into the stator from outside source. First wave: e.m.f. impressed on stator. Second wave: E_{1T} = induced in full pitch search coil No. 1 Third wave: E_{5B} = e.m.f. induced in full pitch exploring coil No. 5. Second and fou.th lines: calibrations for E_{1T} and E_{5B} respectively. Note E_{1T} and E_{5B} are due to the rotating field set up by three-phase currents.

or s.s.c., in case of induction motors or alternators, the H-curve due to sinusoidal (in time) armature currents will be sinusoidal. Consequently if the permeability be constant, and uniform across the pole-face, the B-curve due to armature reaction will also be sinusoidal. However, if we find the e.m.fs. induced in exploring coils due to armature reaction to be distorted this must be charged to the permeability μ, of the circuit. The importance of this result will be appreciated when we attempt to explain certain phenomena to be considered presently.

Fig. 19 gives E_{1T} and E_{5B} when the stator was supplied with 3-phase currents from an outside source and the rotor removed. It is seen that the impressed e.m.f. does not deviate greatly from a sine wave. But the e.m.f. E_{5B}, induced in the coil at the bottom of the slot is peaked and over 150 per cent of E_{1T} = e.m.f. induced in coil on top of slot. The waves given in Fig. 20 were taken with the rotor in place and running without

excitation at a speed corresponding exactly to that of the gliding field produced by the three-phase stator currents. In this case the current supplied was only 28 per cent of full-load current. As in Fig. 19, E_{5B} is more peaked than E_{1T}; $E_{5B} = 1.17\ E_{1T}$.

Assuming the m.m.f. per centimeter to be the same at the top and bottom of the slot, it is clear that the inequality between E_{1T} and E_{5B} must be charged, at least partly, to the difference in permeance between the points considered. Generalizing the above it can be stated that although the H-curve due to armature currents may be sinusoidal the B-curve will differ from the former for two reasons: First, the lack of constancy and uniformity in μ across the pole face; further the B-curve at the bottom of the slot may differ from that at the top owing to the difference in the permeance per unit area at these two points.

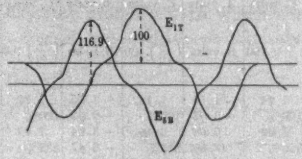


FIG. 20

Oscillogram O A 66. Test 177. First wave: E_{1T} = e.m.f. induced in full-pitch search coil at top of slot. Second wave: E_{5B} = e.m.f. induced in full-pitch search coil at bottom of slot. A-c. supplied from outside source = 15.1 amperes. Two exposures taken under identical conditions. Rotor not excited and run at synchronous speed. Same vibrator used for both exposures.

Second, according to the degree of saturation the B-curve will ' e more or less flat topped; this of course is well known especially in connection with induction motor design.

It is possible for the B-curve due to armature reaction alone, to undergo certain oscillations, similar to those described in Section III. However, a little study of Table I, columns 8 and 9 and of Figs. 19, 20 and of others to be given later will show that these secondary effects may safely be neglected.

Summarizing then we can state that the experimental and theoretical studies given herein and by others show that the no-load flux may and does deviate from a sine wave to the extent of containing the 3rd, 5th and 7th harmonics to the amount of 10 per cent or thereabouts. The field-form of most modern machines, at no load or full load approaches reasonably well the general shape of a sine wave[14]

14. W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 749. L. A. Herdt, TRANS., A. I. E. E., 1902, Vol. 19, p. 1093.

and is symmetrical with respect to the $\pi/2$ or the (geometrical) pole-axis except in case of loads having considerable cross-magnetizing effect.

As explained before the H-curve due to armature reaction will be sinusoidal while the B-curve will show no *extreme* deviations from a sine wave. This, rather theoretical result, is

TABLE XI

Wave analysis No.:	*II* − 1		*II* − 2		*II* − 3		*II* − 4	
No. of oscillogram:	202		229		286		286	
Field excit. amperes:	4.95		3		5.02		3	
Expl. coil, No. 1, top of slot			No. 1, top of slot		No. 5, bottom of slot		No. 5, bottom of slot	
Order of harmonics	*A*, volts *B*, volts	*C*, volts $\frac{C}{C_1} \times 100$	*A*, volts *B*, volts	*C*, volts $\left(\frac{C}{C_1}\right) 100$	*A*, volts *B*, volts	*C*, volts $\left(\frac{C}{C_1}\right) 100$	*A*, volts *B*, volts	*C*, volts $\left(\frac{C}{C_1}\right) 100$
1	1.24	1.24	1.16	1.16	− 1.05	1.39	0.85	0.86
	− 0.004	100%	− 0.03	100%	− 0.92	100%	0.13	100%
3	1.96	3.99	0.98	2.46	− 3.64	7.42	1.97	4.24
	3.32	323%	2.26	213%	− 6.47	532%	3.74	490.5%
5	1.06	2.14	0.17	1.77	1.77	2.24	− 1.15	1.49
	− 1.86	172.5%	− 1.16	152.5%	− 1.37	160.5%	− 0.96	173%
7	0.91	1.45	0.67	1.1	− 1.05	2.7	1.25	2.04
	− 1.12	116.8%	− 0.78	95%	− 2.47	193%	− 1.61	236%
9	0.45	0.45	0.03	0.03	− 1.18	1.38	1.51	1.75
	0.027	36.5%	− 0.001	2.8%	0.72	98.9%	0.87	203%
11	− 0.31	0.32	− 0.24	0.03	− 0.11	0.11	− 0.18	0.38
	− 0.097	25.86%	− 0.01	2.24%	− 0.02	81%	− 0.34	44.4%
13	− 0.193	0.39	− 0.01	0.27	− 0.88	1.29	− 0.33	0.71
	− 0.34	31.7%	− 0.02	2.29%	0.95	92%	0.63	82.5%
15	0.128	0.15	0.004	0.004				
	0.086	12.5%	0.003	0.4%				
17	− 0.056	0.12						
	0.10	9.5%						
23	0.005	0.12						
	0.12	9.5%						
25	0.004	0.006						
	0.004	0.5%						

NOTE: Wave analysis, II-etc. refers to analysis of waves obtained under s.c.c. with the machine series Y connected. See also note, Table VIII, as to meaning of *A*, *B*, *C*.

further corroborated by the oscillograms given in Figs. 19 and 20. According to these, either with the rotor removed or in place and running at synchronous speed, the e.m.fs. induced in stator search coils, due to stator currents alone, are fairly smooth. Furthermore the flux pulsations as indicated by the rotor coil e.m.fs., E_{RF}, and E_{RC} (under s.s.c.) were found to be extremely small.

The resultant of two or more sine waves is, of course, a sine wave, and therefore it might be expected, at first sight, that the resultant of the B-curve due to the field and that due to the armature reaction will not deviate greatly from a sine wave, since neither of them is very distorted to begin with. However, turning to oscillograms obtained under s.s.c. we find indications of an extremely distorted flux distribution. A glance at Figs. 23, (fourth wave) 16, (fourth wave) etc. and Table XI which gives the coefficients of the harmonics of the waves obtained under s.s.c., show that the 3rd harmonic of e.m.f. induced in the stator search coils is several hundred per cent of the fundamental and that a number of other harmonics are quite comparable to it.

In order to avoid any false deductions and detect any peculiar defects in the machine under test to which the distorted s.s.c. waves might be due it was considered advisable to investigate

FIG. 21

Oscillogram No. O A 40. Test No. 116. Tables I and II. First wave: line current Second wave: E_{TT_2} = e.m.f. induced in coil No. 12. Third wave: E_{RF} = e.m.f induced in full-pitch rotor coil No. 7. Synchronous motor operation. Line current = 76 amperes lead. Line voltage = 241 volts. Power factor = 0.28 ld.

briefly the B-curves obtained under different loads. For this purpose the machine was run as a synchronous motor and a number of oscillograms taken, a few of which have been included in the paper.

As seen from Figs. 21, 22 etc., the waves E_{1T}, E_{3M}, E_{RF}, E_{RC} etc. are not unlike those obtained at no-load with small variations which might have been expected. It will be observed by comparing Fig. 22 and Fig. 7 that the tooth ripples are smaller under synchronous motor operation. One of the important results of this series of tests was that even with heavy magnetizing or demagnetizing armature reactions, (no field excitation, or 150 per cent or more excitation) the flux distribution did not in any way approach the B-curves obtained under s.s.c.

Other points of interest have already been considered in the previous section and as the subject is not new it will not be

considered any further. However, attention may be called to the e.m.f. induced in the rotor search coils No. 7 and 8. The e.m.f. waves of these coils show $2\,q = 24$ small indentations per cycle which are due to the teeth. The e.m.f. of full-pitch coil No. 7, shows a large superimposed oscillation which recurs every B deg., see Fig. 21, 3rd wave.

As shown clearly in the supplementary notes this is due to the fifth and seventh space harmonics of armature reaction and as indicated in Table I column 5, $B = 60$ elec. deg., Fig. 21, *i.e.*, the periodicity of the superimposed wave is $360/60 = 6$ times machine-frequency, as we would expect. The long flat oscillations which repeat themselves every $A = 1080$ elec. deg., see Fig. 5, and Table I column 4, are due to eccentric rotor, as explained before.

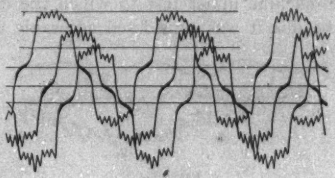


FIG. 22

Oscillogram No. O A 35. Test No. 106. Table VI. Machine Y parallel connected and run as synchronous motor. First wave: E_{1Y} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3 at middle of slot. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5 at bottom of slot. Line current = 52.6 lead. Power factor = 0.37 approx. Compare to Fig. 7.

To determine the armature reaction under s.s.c., *experimentally*, in terms of the voltages induced in the stator coils the writer resorted to the following method. A contactor[15] having a fixed space relation with respect to the poles was placed on the machine. This closed a direct-current circuit of very little inductance so that the rise of current was quite abrupt and the instant of make well defined. See Fig. 23. An exposure was first made giving the contactor wave and the open circuit e.m.f. induced in one of the search coils. The zero lines of both waves were then moved, in the same direction, and a second exposure was made giving the contactor and search coil e.m.f. under the same operating conditions except that the machine was short-circuited at its terminals.

15. The contactor used is similar to the one described in TRANS., A. I. E. E., Vol. 34, p. 2254, except for slight changes in the design of the brush rigging.

Obviously it is thus possible to determine accurately the point on the s.s.c. wave corresponding to any point on the open-circuit wave. If we designate by e_{fo} the e.m.f. induced in one of the stator coils No. 1, 3, or 5 at no-load and given field current

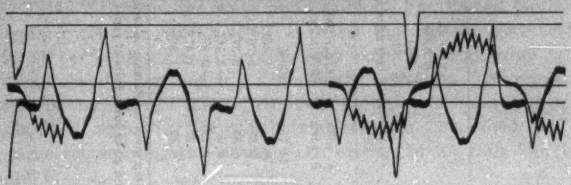


FIG. 23

Oscillogram No. O A 72. Test No. 184. Table XV. *Exposure No.* 1. Open-circuit run at 5 amperes field current and 54 cycles per second. First wave: contactor. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. *Exposure No.* 2 Sustained short circuit; series Y connection and under same conditions. Second wave Contactor. Fourth wave: E_{5B}.

and speed, and by e_r the e.m.f. induced in the same coil under the same conditions of speed and excitation but with the machine short-circuited, we can, as a fair approximation assume that e_{fo}

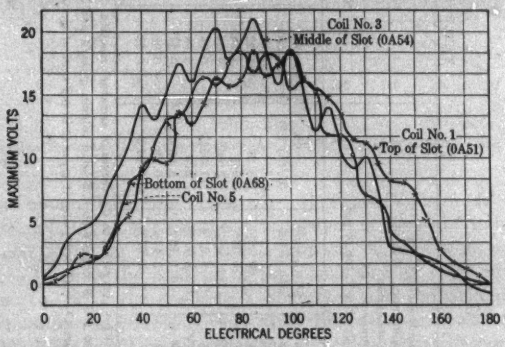


FIG. 24

E_5 curves = difference between no-load and s.s.c. e.m.fs. induced in top, bottom and middle of slot full-pitch exploring coils, Nos. 1, 3 and 5 = $(E_{fo} - E_r)$—field current = 3.16 amperes — periodicity = 52 cycles per second — Y series connection.

plus a certain e.m.f. e_a, which we can charge to armature reaction, give us the resultant e.m.f. e_r.

Fig. 24 shows $e_a = (e_r - e_{fo})$ for the three full-pitch stator exploring coils taken at a field excitation of 3.16 amperes and a

TABLE XII.

Wave analysis: VI – 1 No. of oscillogram: O A 44 Field excitation: 5 Frequency: 54 Exploring coil: No. 1, top of slot				(VI – 2) O A 51 3.16 52 No. 1, top of slot			(VI – 3) O A 68 3.1 52 No. 5, bottom of slot		
Order of harmonics	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100
1	– 23.02	– 99.8%	23.07	– 15.22	– 99.4%	15.31	14.75	100%	14.75
	1.16	5.03%	100%	1.64	10.73%	100%	0.2	1.35%	100%
3	4.87	21.0%	5.06	2.09	13.62%	2.14	0.85	5.76%	0.92
	– 1.38	– 6.01%	21.05%	– 0.65	– 4.27%	14%	0.35	2.37%	6.24%
5	– 0.87	– 3.77%	0.9	0.73			– 0.6	– 4.1%	0.62
	– 0.25	– 1.10%	3.93%	– 0.34			– 0.12	– 0.8%	4.18%
7	0.56	2.44%	0.68	– 0.08			0.22	1.47%	0.35
	0.38	1.67%	2.94%	– 0.52			0.27	1.8%	2.35%

NOTE: Wave analysis VI – etc. refers to C_3 curves of armature reaction See Figs. 25 and 2_4. First column gives coefficients of series: $A_1 \sin \theta + A_3 \sin 3\theta$ etc. $+ B_3 \cos \theta + B \cos 3\theta$ etc. Second column gives A and B in per cent of C_1. Third column gives coefficients of series $C_1 \sin \theta + \psi_1 + C_3 \sin 3\theta + \psi_3$ etc. where $C^2 = A^2 + B^2$; also it gives C in per cent of C_1

frequency of 52 cycles per second; 3.16 amperes being the field excitation necessary to send full-load current under s.s.c. Fig. 25 gives e_a for a frequency of 54 cycles per second and a field current of five amperes.

Table XII gives the wave analysis of e_a, Fig. 25 coil No. 1, and Fig. 24 coils No. 1 and 5. It is seen that the 3rd harmonic is 21 per cent for five amperes field excitation and drops to about 5 and 6 per cent at 3.1 amperes of excitation. A general idea of the shape of these e_a curves of armature reaction may be obtained from Fig. 24A which is an average "smoothed out" curve representing the waves given in Fig. 24 for coils No. 1 and 5.

The last two columns of Table XIII give the wave analysis of

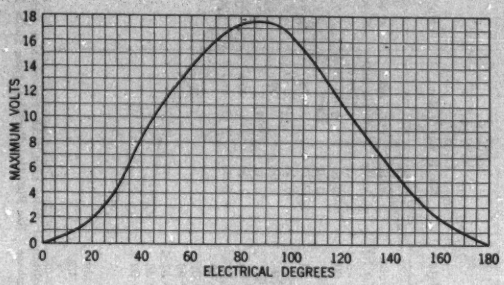


FIG. 24A

Average "Smoothed Out" curve representing e.m.f., E_a induced in coils No. 1 and No. 5 Fig. 24.

e_a as obtained by subtracting $(1-5)$ Table IV, from $(II-1)$ Table XI. $(I-5)$ gives the wave analysis of the e.m.f. induced in coil No. 1 at no-load and 5 amperes of excitation, while $(II-1)$ gives the analysis of the corresponding wave of the same coil with the machine Y connected and short-circuited. Comparing $(II-1)-(I-5)$ with the wave analysis of e_a, Table XII, it will be seen that the agreement is satisfactory.

The important result derived from the above is that the s.s.c. flux distribution is badly distorted primarily on account of the fact that the fundamental of the resultant flux is so greatly reduced that the higher harmonics become quite predominant. For example, from Table VII, wave analysis $(I-5)$, the 3rd is seen to be 0.95 volts which is 3.75 per cent of the no-load fundamental, which is 25.3 volts; but it is $(0.95/1.24) \times 100 = 67$ per cent

TABLE XIII.

Wave analysis No. III – 2 No. of oscillogram: 225 Field excitation amperes: 5.05 Exploring coil. No. 1, top of slot			(III – 4) 227 3 No. 1, top of slot		(V – 4) O A 81 5 No. 1, top of slot		(V – 5) O A 82 5 No. 5, bottom of slot		(II – 1) – (I – 5) 5 No. 1, top of slot	
Order of harmonic	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C* volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	$(A/25.3)$ 100 $(B/25.3)$ 100
1	– 0.75	1.21	0.525	1.00	– 2.65	2.65	– 2.19	2.64	– 24.06	95.2
	0.96	100%	0.86	100%	0.05	100%	1.48	100%	– 0.18	0.71
3	1.74	3.54	1.17	2.13	– 3.77	3.78	– 4.9	4.96	2.84	– 11.21
	3.09	291.5%	1.78	213%	0.28	144%	0.82	188	2.97	– 11.74
5	– 1.4	1.76	– 0.94	1.0	1.93	1.99	1.6	2.15	3.24	– 12.8
	– 1.07	144.8%	0.38	100%	– 0.47	75%	– 1.44	81.5	– 1.72	6.8
7	0.97	0.99	0.38	0.64	1.22	1.375	1.29	1.9	2.93	– 11.58
	– 0.20	81.6%	0.51	63.7%	– 0.63	51.8	– 1.4	71.9	– 0.79	3.12
9	0.89	0.94	0.19	0.39					– 2.23	8.82
	– 0.28	77.1%	– 0.35	39.3%					0.03	
11	– 0.47	0.94	0.048	0.99					– 0.93	3.7
	0.82	77.4%	– 0.08	9.57%					– 0.25	0.99
13	– 0.25	0.40	0.15	0.32					– 0.53	2
	– 0.31	33.1%	– 0.28	0.32%					– 0.54	2.13
15	0.25	0.29							0	
	0.01	23.8%							– 0.31	1.23
17			0.07	0.25					– 0.46	1.82
			– 0.24	25.1%					– 0.22	0.87

NOTE: Wave analysis III etc. refers to analysis of waves obtained under s.s.c. with the machine series Y connected and neutral in. V etc. refers to waves obtained under s.s.c. with the machine connected two-phase. See Fig. 31. For II – 1 see Table XI; for I – 5 see Table VII. As to meaning of *A*, *B* and *C* see Table XII.

of the s.s.c. fundamental. From wave analysis ($II - 1$) Table XI, the third harmonic under s.s.c. is 3.99 volts *i.e.*, (3.99/25.3) $\times$ 100 = 16 per cent of the no-load fundamental, but (3.94/1.24) $\times$ 100 = 323 per cent of the s.s.c. fundamental. The increase from 0.95 volts to 3.99, on the other hand, must be charged mainly to the B-curve of armature reaction which will be distorted, unlike the H-curve producing it, which will be sinusoidal

Turning to Table XII, consider wave analysis No. ($V\ I - 1$) which gives the coefficients for E_{IT} at five amperes excitation and 54 cycles per second. The fundamental is seen to be 23

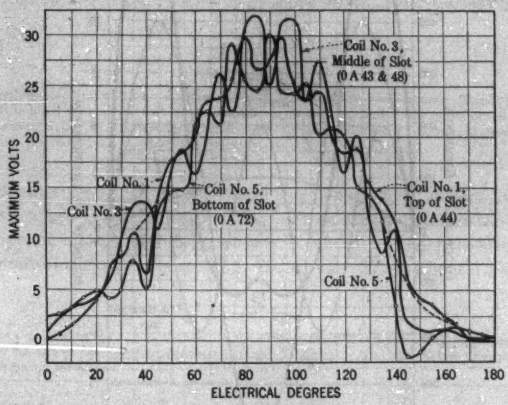


FIG. 25

E_s curves = $E_{fo} - E_r$ = difference between open-circuit and s.s.c. e.m.fs. induced in full-pitch exploring coils Nos. 1, 3 and 5 — field current = 5 amperes—periodicity = 54 cycles per second—Y series connection.

volts; the 3rd is 21 per cent of e_a but (60/54) (5.05/1.24) $\times$100 equals 450 per cent of the s.s.c. fundamental.

In general, then, if both the flux distribution at no-load and the flux wave due to armature reaction are distorted to the extent of containing 10 to 20 per cent 3rd and 5th harmonics, the resultant s.s.c. flux distortion will be badly distorted, because the above harmonics will amount to several hundred per cent of the fundamental, of the s.s.c. wave. Assume, for example, that e_{fo} and e_a consist of:

	e_{fo}	e_a
1st harmonic	100 per cent	− 95 per cent
3rd "	10 " "	10 " "
5th "	5 " "	5 " "

These values are quite conservative as inspection of Tables VII, VIII, and XII will show, and neither of the waves can be said to be very distorted. The resultant s.s.c. flux distribution will be equal to $e_r = (e_{fo} + e_a)$ and the amplitudes of its 1st, 3rd, and 5th harmonics will be 5, 20 and 10 or 100, 400 and 200 per cent respectively, in terms of the s.s.c. fundamental. These

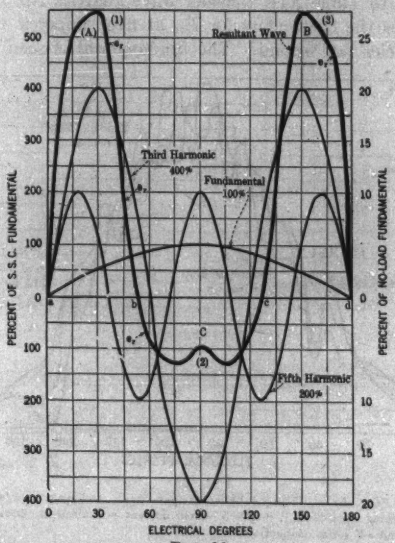


FIG. 26

Illustrating how the extremely distorted resultant wave forms are produced under sustained short circuit.

are drawn in Fig. 26 and their sum, e_r is seen to be extremely distorted and not unlike the waves obtained experimentally under s.s.c.

It is clear now why the flux distribution waves under synchronous motor operation deviate comparatively little from the no-load curve. Obviously at very low voltages such as are obtained under s.s.c., the harmonics of the field and armature reaction

will assume a predominant rôle (unless they happen to be out of phase by ½ of their own period and thus neutralize each other).

In Fig. 27 the angular spacings between different crests are

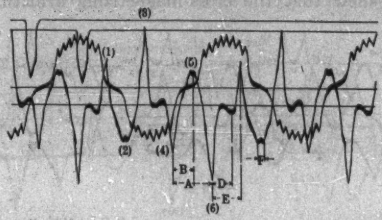


FIG. 27

Oscillogram No. O A 68. *Exposure No.* 1. First wave: contactor. Third wave e.m.f. induced in coil No. 5 at no load, 3.1 amperes field excitation, 52 cycles per second. *Exposure No.* 2: Machine series Y connected and short-circuited at same excitation and speed. Second wave: contactor. Fourth wave: E_{AB}. See Fig. 24.

designated, A, B, D, E and F and their values for different kinds of s.s.c. are given in Table XIV. From this it is seen that A is approximately 105 deg., or nearly equal to the pole-arc, as it would be expected—see Fig. 26; E, Fig. 27, is about 75 deg.

TABLE XIV.

Data relating to the e.m.fs. induced in the coils at the top and bottom of different slots under s.s.c. and the machine connected, series Y, series Δ and two-phase. Frequency = 60 cycles per second.

No. of oscill.	Field amps.	E_{1T} E_{AB}	Kinds of short-circuit	A	B	D	E	F
				in degrees see Figs. 27, 34, etc.				
202	4.95	E_{1T}	Y	105	51.3	40	75.5	19
O A 88	5	E_{1T}	Y	106.3	48	35.6	65.2	10
226	5	E_{1T}	Y neutral in	109	51	37	70	
O A 89	5	E_{AB}	Y	103.2	50.4	37.1	73.6	10
O A 88	5	E_{1T}	Δ	104.5	49.7	41.8	75.4	10
275	4.9	E_{AB}	Δ	106.2	50.8	39.2	73.5	
O A 89	5	E_{AB}	Δ	104.5	49.3	36.8	72.0	
O A 81	4.96	E_{1T}	two-phase	105	51.1	36.8	72.3	17.4
O A 86	4.96	E_{1T}	two-phase	105.3	47.8	48.2	76.1	15
O A 82	4.96	E_{AB}	two-phase	104.9	52.4	42.9	79.2	10.1

so that $(A + E) = 180$ deg. D is nearly $(1/2\,E)$ but B is less than $(1/2\,A)$; referring to the theoretical assumed curve e_r of Fig. 26, it will be easily seen that if the transverse armature reaction (due to the active or in-phase component of armature

current with respect to the geometrical pole-axis) is not entirely negligible, the peaks (1) and (3) will be unequal, as it is the case in actual oscillograms Figs. 23 and 27, etc. A little consideration will show that the cross-magnetizing armature reaction

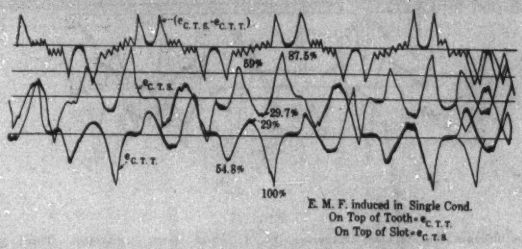


FIG. 27A

Oscillogram No. O A 76. Test No. 188. *Exposure No.* 1. Open circuit run at 5 amperes field excitation and normal speed. First wave = (E_{CTT} − E_{CTS}) = (e.m.f. induced in single conductor at center of tooth, No. 14, Fig. 3) − (e.m.f. induced in single conductor at top of succeeding wedge, No. 13, Fig. 3). This should be compared with Fig. 14. *Exposure No.* 2. s.s.c. run under same conditions. Second wave: E_{CTS}. Third wave: E_{CTT}. These two waves should be compared with Figs. 23, 27, 28a, etc., which give the e.m.fs. induced in full-pitch exploring coils.

will not only reduce, say crest (3), Fig. 26, but will shift crest (2) towards (3). Thus the angle B, between (2) and (3), Fig. 26, will be less than $(1/2\,A)$ = angle between crests (1) and (3), Fig. 27.

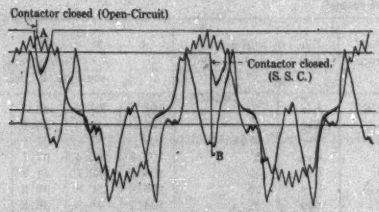


FIG. 28

Oscillogram No. O A 81. Test No. 193. Tables XIV and XV. First wave: Contactor. Third wave: E_{1T} = e.m.f. of coil No. 1 at 4.96 amperes of field current, 60 cycles per second and no-load. Second wave: contactor. Fourth wave: E_{1T} under same conditions except machine short-circuited and connected two-phase.

In Fig. 27 the different crests are designated 1, 2, 3, etc. and their values in maximum volts for different kinds of s.s.c. are given in Table XV. Study of this table shows that for three-phase Y or delta s.s.c. the peaks on the same side of the zero line,

i.e., (1), (3) or (4), (6) are quite unequal which might have been expected according to the preceding paragraph. Compare also Fig. 27A. In case of two-phase s.s.c., however, the corresponding crests (1), (3) or (4), (6) are very nearly equal. See Fig. 28.

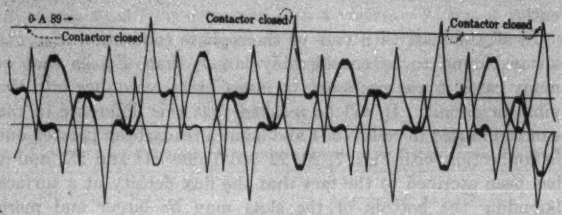
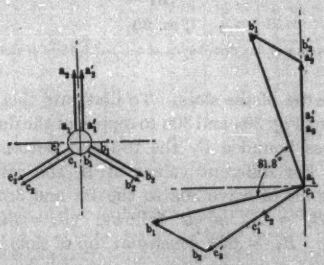


FIG. 28A

Oscillogram No. O A 89. Test No. 201. Table XIV and XV. First wave: Contactor. Third wave: E_{2B} = e.m.f. induced in search coil No. 5 under sustained short circuit at 5 amperes field excitation, 60 cycles per second and series delta connection. Second wave Contactor. Fourth wave: E_{2B} under same conditions, using same vibrator, etc., except machine series Y connected.

Before comparing further the s.s.c. phenomena with the alternator connected Y and delta, see Fig. 28A, also Y with the neutral in, and two-phase, attention should be called to the fact that the two-phase connection was only an imperfect one, the angle



Three-phase two-circuit armature. Two-phase connection giving 81.8 degrees between phases

FIG. 29

between the phases being 81.8 deg. instead of 90. This is clearly shown in Fig. 29 which is self-explanatory. Further as stated in the beginning of this section the armature current under three-phase s.s.c. is sinusoidal, which is not true in this case as may be seen from Fig. 4, first wave.

In view of these two discrepancies it seems rather remarkable that the general wave shapes of the e.m.fs. induced in the search coils under two-phase short circuit, Fig. 28, are so similar to those obtained with three-phase connection, see Figs. 23, 27 etc., except for some small differences which will be considered presently.

In all the tests with two- or three-phase connection E_{5B} was always found to be considerably larger than E_{1T} so that in many cases it was necessary to insert extra resistance into the vibrator circuit. It will be recalled that this difference in the e.m.fs. induced in coils No. 1 and 5 has already been pointed out in connection with Figs. 7, 20, 22 and Table VII and IX, and it has been ascribed to the fact that the flux density at a surface bounding the bottom of the slots may be larger and more nearly perpendicular to the search coil conductors, than the flux

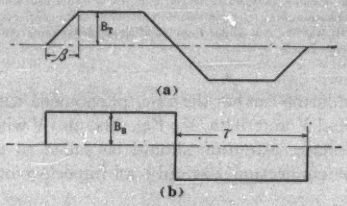


FIG. 30

Assumed flux distribution at top and bottom of slot.

density at the top of the slots. To illustrate this point quantitatively, assume Fig. 30A and 30B to represent the flux distribution at two surfaces bounded by the top and bottom of the slots respectively. For different values of β, the ratio of the 1st and 3rd harmonic of wave 30B to the 1st and 3rd harmonic of Fig. 30A, if $B_T = B_B$, is as follows; (B_B = flux density at bottom of slot. B_T = flux density at top of slot).

β in degrees	1	5	10	20	30
Ratio of firsts	1.022	1.005	1	1.02	1.11
Ratio of thirds	1.005	1.01	1.005	1.205	1.575

If $B_B = (1.09\ B_T)$ which is a fair assumption (see Fig. 20) the ratios are as follows:

β in degrees	1	5	10	20	30
Ratio of firsts	1.115	1.09	1.09	1.1	1.21
Ratio of thirds	1.092	1.103	1.37	1.31	1.91

Further, with all connections the crests (2) and (5) columns (7) and (10) Table XV, Figs. 23 and 27 are distinctly larger for E_{3B} than for E_{1T}. Again, the amplitudes of these crests in volts do not differ greatly for delta or for Y connection but are smaller for two-phase connection. For example, crest (2) for E_{1T} is about 3.7 for delta and it is 4 volts for Y, but it is only about 1.8 or 2.6 for two-phase connection. The reason for this will be seen from Fig. 26 which shows that crest (2) is due

TABLE XV.

Data relating to the crest values of the e.m.fs. induced in the stator search coils No. 1, No. 3 and No. 5 under s.s.c. with the machine connected series Y, series Δ and two-phase.

No. of oscill.	Field amps.	Kind of s.s.c.	Frequency	Coil e.m.f.	Amplitudes in max. volts of: Crest (1)	Crest (2)	Crest (3)	Crest (4)	Crest (5)	Crest (6)
					See Figs. 27, 28, etc.					
1	2	3	4	5	6	7	8	9	10	11
202	4.95	Y	60	E_{1T}	6.62	4.01	7.88	6.4	3.7	7.56
O A 44	5	Y	54	E_{1T}	5.45	3.25	7.12	5.6	3.25	7.49
O A 88	5	Y	60	E_{1T}	7.21	6.12	11.24	7.17	6.21	11.24
O A 43	5	Y	54	E_{3M}	4.24	4.24	9.48	4.24	4.69	9.48
O A 72	5	Y	54	E_{3B}	6.43	5.01	10.62	6.3	5.28	10.43
O A 89	5	Y	60	E_{3B}	7.47	5.66	11.75	6.89	5.89	11.64
275	4.64	Δ	60	E_{1T}	7.86	3.73	9.59	8.06	3.73	9.45
275	4.9	Δ	60	E_{3B}	8.34	6.12	12.32	7.92	6.53	12.67
O A 88	5	Δ	60	E_{1T}	6.20	3.77	8.05	6.2	3.69	8.05
O A 89	5	Δ	60	E_{3B}	7.37	5.98	11.32	6.83	5.98	11.54
205	5 +	two-phase	60	E_{1T}	8.23	2.62	8.11	8.11	3.08	8.11
O A 81	4.96	two-phase	60	E_{1T}	7.89	1.85	8.32	7.68	1.85	7.89
O A 86	4.96	two-phase	60	E_{1T}	9.42	1.95	9.74	9 42	1.95	9.74
O A 52	3 16	Y	52	E_{1T}	3.1	1.94	4.08	3.1	1.75	4.37
O A 54	3.16	Y	52	E_{3M}	2.52	2.81	5.91	2.71	2.47	5.91

to the amplitude of the armature reaction which is the same for delta or Y but it is smaller for two-phase connection (imperfect) on account of the distribution factor of the winding.

Taking everything into consideration, it is found that the e.m.fs. induced in rotor coils with two-phase connection are fairly regular; this is further illustrated by wave analysis $V-4$ and $V-5$. Table XIII and also the e_s curves given in Fig. 31.

However, leaving aside, for a moment, all secondary effects it may be well to refer to a graphical explanation given by A.

E. Clayton.[16] He considers an ideal alternator with no magnetic oscillations or secondary effects, no fringing, etc.; the no-load field form may then be assumed to be rectangular and the armature reaction to be portion of a sine wave as indicated in Figs. 32A and 32B. The resultant curve is given in Fig. 32c which resembles in general the waves obtained under s.s.c. conditions.

The rotor exploring coils under s.s.c. show $2\,q = 24$ ripples per cycle due to the teeth. See Table I, column 6 and Figs. 8B and 8c. They also indicate oscillations which repeat themselves

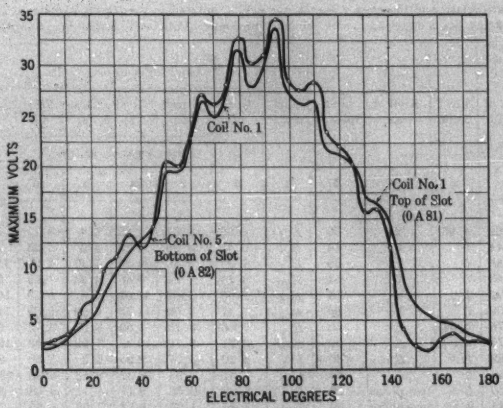


FIG. 31

E_a curves = $E_{fo} - E_r$ = difference between open circuit and s.s.c. e.m.fs. induced in full-pitch top and bottom search coils—field excitation = 5 amperes—periodicity = 60 cycles per second—connection = two-phase.

every 60 deg. See column 5. These as already explained, are due to armature reaction and their jagged unsymmetrical shape may be accounted for as suggested in Fig. 33 where A and B are due to armature reaction and the teeth, and have a frequency of $6\,f$ and $24\,f = 2\,q\,f$, respectively. Their resultant is seen to be similar to the E_{RF} waves Figs. 8B and 8c. In this connection it may be noted that the long flat oscillations which have been

16. The wave shapes obtained with a-c. generators working under steady short-circuit conditions. *Journal, Inst. E. E.*, (London), 1916, Vol. 54, p. 34. This is the first paper, so far as I am aware, to throw any light on this complex subject.

ascribed to the eccentricity of the rotor disappear from the E_{fr} wave under s.s.c. Compare Figs. 4, 5 and 8. This may be explained if the peculiar flux distribution under s.s.c. be considered; taking this to be approximately as represented in Fig. 26 e_r wave, assume that at the position of minimum reluctance the area designated by (A), under crest (1) between $a\,b$, $= 83 = B$ and $C = 66$ = area under crest (2) between $b\,c$; thus the total net flux per pole under s.s.c. will be $166 - 66 = 100$ per cent. Next, at the position of minimum permeance, assume $A = 75 = B$ and $C = 50$; the net flux per pole is again $150 - 50 = 100$ per cent. Obviously under these conditions the e.m.f. due to the eccentricity of the rotor would disappear. However, the areas A, B and C when actually measured were not in the proportion assumed; but it is

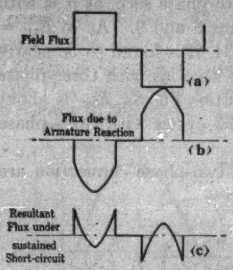


FIG. 32

Flux distribution curves of ideal salient-pole alternator under sustained short circuit

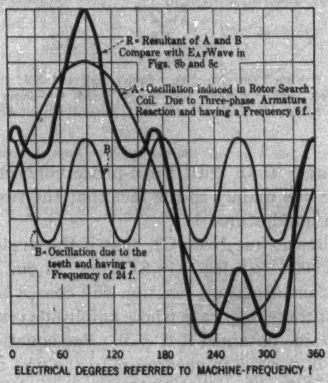


FIG. 33

Theoretical explanation of the production of the peculiar e.m.f. waves shown in Figs. 8B and 8C.

believed that some such explanation as the one just given is responsible for the disappearance of these oscillations which occur once in every revolution under no-load or synchronous motor operation.

The e.m.fs. induced in rotor coils No. 8 and 7 with two-phase connection are given in Fig. 4 (third wave) and Fig. 34 (first wave) respectively. E_{RF} = e.m.f. induced in coil No. 7, is nearly 3 times (2.7) as large under two-phase s.s.c. as it is with Y connection. See Table I, columns 8 and 9. As to E_{RC} = e.m.f. induced in coil No. 8, see Fig. 34, it is found to be nearly 6 times (5.7) as large with two-phase s.s.c. as with three-phase s.s.c. Under synchronous motor operation Fig. 8A, E_{RC} is about 0.053 at 59.5 cycles against 0.03 at 50 cycles with three-phase s.s.c. Table I, Test No. 165.

In short, the flux oscillations with two-phase connection are

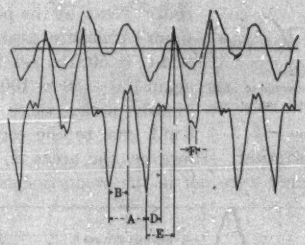


FIG. 34

Oscillogram No. O A 86. Test No. 198. Tables XIV and XV. Sustained short circuit at 4.96 amperes field excitation, and normal speed, two-phase connection. First wave: E_{RC} = e.m.f. induced in rotor coil No. 8. Second wave: E_{1T} = e.m.f. induced in coil No. 1.

very much larger than with three-phase connection. This is believed to be due mainly to the fact that the two phases were not exactly in quadrature.

It can be shown, (see Note C), that if the angle between the two phases is $(90 - \alpha)$ deg., (= 81.8 deg. in our case, see Fig. 29), the resultant armature reaction will consist approximately of (*a*) a regular travelling wave whose amplitude is approximately equal to the value obtained if $\alpha = 0$. See equation (**56**). (*b*), it will consist of a wave in *quadrature* to the first and travelling in the opposite direction and at the same speed but having an amplitude $(\sin 2\alpha)/2$ of (*a*). Hence it will be seen, that neglecting other effects, this second rotating field, which is *in quadrature to the first and maximum at* $x_1 = 0$, *i.e., at the coil-side of coils No.* 7 *and* 8, will be responsible for the e.m.fs. induced in them under two-phase s.s.c. Further it will account for the peculiarly distorted shape of the E_{RF} wave. This is illustrated

in Fig. 35 where the e.m.f. induced in the rotor coil No. 7 is assumed to be due to H_{3R}, H_3^{α} and H_{5R}, *i.e.*, regular 3rd space harmonic travelling wave, 3rd space harmonic due to lack of quadrature and regular 5th space harmonic travelling wave. See equations (**59**) and (**62**). The amplitude of these waves is assumed to be 100, 27 and 66 per cent respectively, *i.e.*, $H_3^{\alpha} = 0.27\, H_{3R} = (\frac{1}{2} \sin 4\,\alpha)$, approximately. $H_{5R} = H_{3R}\,(4f/6f) = 0.66\, H_{3R}$, where (6 : 4) = ratio of their relative velocities with respect to the rotor. Thus it is seen that, Fig. 35 explains

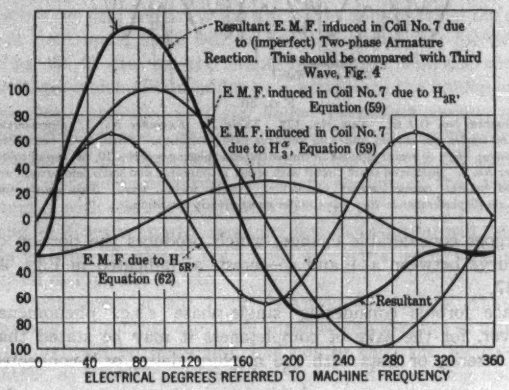


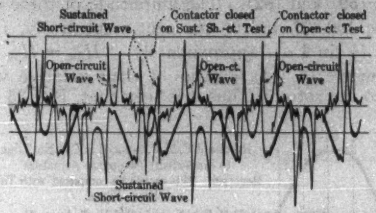
FIG. 35

Theoretical explanation of production of the peculiar E_{RF} wave under 2-phase s. s. c. See third wave Fig. 4.

approximately the peculiar shape of the E_{rf} wave given in Figs. 4 and 34.

Finally considering the e.m.fs. induced under s.s.c. with Y connection, in coils No. 9 and 10, Figs 3 and 3A, we find these to conform to the general wave shape we would expect, according to the explanation given in Section III. In Fig. 36, E_{TT} at no-load and at s.s.c. is given and it is seen that both are characterized by two prominent peaks, and they are quite similar to each other except for the decided difference in shape between a + crest and a − crest. The reason for this will be clear if it be recalled, as explained in detail in Section III, that the portion of the wave between a + and a − crest is produced while the coil is under the pole-arc; on open-circuit the portion of the *B*-curve under the pole is more or less toothed and it gives rise

to little kinks see curve (*b*), Fig. 6. Under s.s.c., however, the *B*-curve is very small or even negative at the middle of the pole as indicated by crest (2) Figs. 26, 28, 37 etc. Consequently, as the coil passes from one pole tip to the next its enclosed flux



F G. 36

Oscillogram No. O A 61. Test No. 172. Table II. *Exposure No.* 1. Open-circuit run at 60 cycles per second and 4.95 amperes field excitation. First wave: contactor Third wave: E_{TT} = e.m.f. induced in concentrated exploring coil No. 10, on top of tooth. *Exposure No.* 2. Sustained short circuit with series Y connected and same field current and speed as before. Second wave: contactor. Fourth wave: E_{TT}: This shows that there is no displacement in E_{TT} wave under short-circuit condition.

undergoes considerable change, which produces the portion of the curve between a + and a − crest, as indicated in Figs. 36 and 37.

Space forbids considering single-phase s.s.c. phenomena; however, for the sake of completeness it may be stated that from a series of tests with the machine delta- or Y-connected

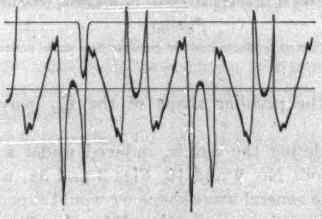


FIG. 37

Oscillogram O A 69. Test No. 180. Sustained short circuit at 5 amperes field excitation, and frequency of 60 cycles per second. Y series connection. First wave: E_{TT} = e.m.f. induced in coil No. 10 on top of tooth.

and short-circuited between two terminals, or one terminal and the neutral, (in case of Y) the exploring coil e.m.fs. were found to be fairly similar to those already described. The similarity is rather striking when the presence of complex secondary effects and pulsations of single-phase alternators is considered.

V. Critical Resume and Conclusions

It seems fair to conclude from the above that the sustained short-circuit (s.s.c.) phenomena of alternators in spite of their complexity and the extremely distorted wave-forms to which they give rise, are amenable to the ordinary theories of synchronous generators as given by Blondel, Potier and others. It is only necessary to restrict the usual sweeping simplifying assumptions of so-called text-books in order to arrive at some of the general results given in the preceding pages. It has been shown that unless the field and armature flux waves are sinusoidal the resultant field under s.s.c. will be badly distorted, see for example Figs. 23, 27 etc., for the simple reason that with the very low voltages which obtain under s.s.c. conditions the fundamental is so greatly reduced that the higher harmonics assume a very predominant role. Starting with a field flux-wave consisting of, 1st 100 per cent, 3rd + 10 per cent, 5th + 5 per cent, and a corresponding armature flux curve containing the same harmonics with relative amplitudes of − 95 per cent, + 10 per cent and + 5 per cent, the resultant flux distribution will be given by e_r, Fig. 26; this is not unlike the actual s.s.c. oscillograms given in Figs. 23, 28, etc.

Furthermore it has been shown in Section IV, that with sinusoidal (in time) armature currents the H-curve of armature reaction will be sinusoidal, but not so with the B-curve; the latter, as is well known will be flat-topped depending upon the degree of saturation, and will further differ from the H-curve in that it will be toothed and have different values, and directions with respect to the armature conductors, at a surface bounded by the top of the slots than at a surface bounded by the bottom of the slots. Thus it is clear that from the view-point of s.s.c. phenomena from among the different schemes used for obtaining "sine-wave alternators" the best is the one which attempts to get to the root of the evil and obtain a sinusoidal flux distribution, under the poles to begin with. However, even then harmonics may and will be introduced by armature reaction. It is well in this connection to emphasize the fact that the B-curve of armature reaction will differ more or less from the H-curve producing it for the reasons just given. Consequently the elimination of the higher harmonics from the B-curve (of armature reaction) is very difficult, if not impossible, since even such devices as fractional slots per pole will avail little unless the permeance per unit area across the pole face is made uniform, constant

and free from extreme effects of saturation. However, with a thorough understanding of the subject it should be possible to reduce the effects of the B-curve to a negligible quantity.

The magnetic oscillations under load, no-load and s.s.c. conditions are studied not only by means of full-pitch exploring coils placed on the stator and the rotor but also concentrated coils placed on top of a wedge or a tooth and having a pitch equal to a slot-pitch or fraction thereof. See Figs. 3 and 3A.

The principal oscillations observed are:

(1) *Those due to the teeth*; taking the amplitude (a), Fig. 13 as 100 per cent for the coil No. 1 on top of the slot, we find (a) to be 107 per cent for coil No. 3 and 108 per cent for coil No. 5. On the same basis of percentages, (d), Fig. 13, is 24, 31 and 32 per cent respectively for the same coils. See Fig. 7. These oscillations (due to the teeth) decrease under synchronous motor operation. Compare Figs. 7 and 22.

(2) *Oscillations due to the eccentricity of the rotor*. These have a frequency of $(1/p)$ machine-frequency, or one cycle per revolution.

(3) *Oscillations due to armature reaction* and having a frequency of 6 times and 4 times machine-periodicity in case of three-phase and two-phase connection. It seems rather remarkable that oscillations (2) disappears under s.s.c.; an explanation of this is given on p. 1183.

(4) *Oscillations due to the pulsations of magnetic reluctance* caused by the salient-pole construction of the machine. These are best illustrated by means of exploring coils placed on top of a slot and on top of a tooth and having a pitch equal to that of a tooth. See Figs. 10, 11, etc. These oscillations are analyzed in detail in Table II and it is seen that the e.m.f. induced in a concentrated coil on top of a tooth is about twice as large as the e.m.f. induced in an exactly similar coil placed on top of a wedge: These oscillations are studied in detail and they give a clear idea of the flux distribution under *actual running conditions*, which are, of course very important from the point of view of iron losses etc. In this connection it is well to call attention to the methods of analysis and investigation followed in this research; guided by a simplified general mathematical theory the flux pulsations and oscillations were separated and analyzed by taking oscillograms of the e. m. f.'s induced in different stator and rotor exploring coils as explained in detail in Section I.

Certain peculiarities and differences between the e.m.fs. induced in search coils at the top and bottom of slots are considered and, at least, tentative explanations are given. For these the reader can consult sections III and IV.

In regard to different kinds of short circuits it is shown that there is practically no difference between s.s.c. phenomena with the machine connected delta, Y, and Y with the neutral in. This might have been expected considering the elimination of triple frequency circulating currents due to the $\frac{2}{3}$ pitch armature winding. In case of two-phase connection, however, the armature current is not as sinusoidal as with three-phase connection; the variational e.m.fs. induced in the rotor coils under s.s.c. show very peculiar wave distortion as seen from Fig. 34, first wave, etc. An explanation of this is indicated in Fig. 35.

Before concluding these remarks it may be well to call attention to the bearing of the results given in the paper upon what are considered standard methods of testing large alternators. The thorough understanding of the underlying facts and the collection of quantitative data on the subject for salient as well as non-salient pole generators will throw a great deal of light on efficiency tests, cyclic heat run tests, short-circuit load loss tests, phase-shifting or feeding back method of load tests, etc. In such tests the flux distribution and the flux density at different points of the magnetic circuit is of considerable importance. It seems to the writer that, in such cases, even if it be extremely difficult to devise tests and means of predetermining by calculation certain characteristics of the machine, without actually loading it, it is very desirable to know and understand, as well as possible, the internal reactions so that proper rational allowances may be made. In any case, the detailed analysis of these phenomena seems to be a step in the right direction, and although the application of such information to design and test problems does not fall within the scope of this paper, it may be well to give a simple quantitative illustration of it.

Assume that the open-circuit core loss of an alternator is 2.8 per cent of its rating and that half of this can be charged to hysteresis and the other half to eddy current loss; let the s.s.c. test loss, excluding friction and windage, be 1.5 per cent. According to the ordinary assumption that the flux distribution under s.s.c. is sinusoidal and 10 per cent of the open-circuit value the hysteresis loss, P_h and the eddy current loss, P_e will be:

$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,B}{B}\right)^{1.6} \cdot \left(\frac{f}{f}\right)$$

$$= 0.035 \text{ per cent}$$

$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,Bf}{Bf}\right)^2$$

$$= 0.014 \text{ per cent}$$

Consequently, $P_h + P_e = 0.049$ per cent.

Next assume that the fundamental is 5 per cent, the third harmonic is 15 per cent and the fifth 10 per cent; these are fair and very conservative values according to the results of this investigation. Then,

$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,B}{B}\right)^{1.6} \cdot \left(\frac{f}{f}\right) + \left(\frac{0.15\,B}{B}\right)^{1.6} \cdot \left(\frac{3f}{f}\right) + \left(\frac{0.10\,B}{B}\right)^{1.6} \cdot \left(\frac{5f}{f}\right)$$

$$= 0.114 \text{ per cent}$$

and,

$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,Bf}{B.f}\right)^2 + \left(\frac{0.15\,B.3f}{Bf}\right)^2 + \left(\frac{0.10\,B.5f}{B.f}\right)^2$$

$$= 0.45 \text{ per cent}$$

hence, $(P_h + P_e) = 0.564$ per cent.

Thus according to the usual assumption $(P_h + P_e)$ under s.s.c. tests is negligible, being $(0.049/1.5) \cdot 100 = 3.3$ per cent, nearly, in this example. But if we take the actual flux distribution under a s.s.c. test into consideration, and assume the very conservative values taken above for the 3rd and 5th harmonics of the flux-wave, we find that the eddy current and hysteresis losses amount to, $(0.564/1.5) \cdot 100 = 38$ per cent, nearly, of the total s.s.c. test losses, *i.e.*, the iron losses are not necessarily negligible.

Finally it seems almost unnecessary to call attention to the necessity of standardizing our terminology. In specifications or technical papers and discussions it is only fair that in case of least ambiguity some qualifying term be used to specify the kind of short circuit implied. The words "sudden" and "sustained" seem suitable. However, the question of importance is that we should appreciate the great difference between these

two important and complex phenomena of sudden and sustained short circuits and never leave any doubt as to which one is meant.

In conclusion it is a pleasure to the author to acknowledge his indebtedness to the authorities of the Rice Institute for the facilities afforded and particularly the moral support they have lent him. He is also glad to take this opportunity and acknowledge the conscientious and very able assistance of Mr. J. S. Waters, Jr., of the Class of 1917.

SUPPLEMENTARY NOTES

Note A. General E.M.F. Equations

1. *E.M.F. Induced in Stator Search Coils.* As explained in the e.m.f. equations.

$$e = -N\frac{d\phi}{dt} \tag{1}$$

ϕ may be and is usually, a function of the time t, as well as the position x of the coil with respect to the pole. Thus,

$$\frac{d}{dt}(\phi_{x,t}) = \frac{\partial\phi}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\phi}{\partial t}\cdot\frac{dt}{dt}$$

$$= v\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t} \tag{2}$$

But,

$$\phi_{(x,t)} = \int B_{(x,t)}\cdot l\overset{s}{dx} \tag{3}$$

whence,

$$e = -N\frac{d\phi}{dt} = -Nl\left[v\frac{\partial}{\partial x}\left(\int B\,dx\right) + \frac{\partial}{\partial t}(B\,dx)\right] \tag{4}$$

$$= e_{\text{motional}} + e_{\text{variational}}$$

Strictly speaking, $\phi = \int B\,da.\cos(B, da)$, where $da = ldx$ = element of area at which the flux density is B, and $\cos(B, da) = \cos\theta$, is the angle between the vector B and the normal to the elementary surface da; $\cos\theta$ is, of course, ordinarily neglected, except where methods of vector analysis are used. In our attempt to explain certain phenomena, we shall find it necessary to take account of this—which at first may

seem an unnecessary refinement. Thus strictly (4) should be written:

$$e = -N l v \frac{\partial}{\partial x}\left[\int B\,dx \cos\theta\right] - N l \frac{\partial}{\partial t}\left[\int B\,dx \cos\theta\right] \tag{4a}$$

$$= e_{\text{rotational}} + e_{\text{variational}}$$ [17]

By means of (4) or (4a), which are quite general, it is easy to obtain expressions for the rotational and variational e.m.fs. induced in any of the stator or rotor coils. Assuming the flux under the pole to be symmetrical with respect to the interpolar axis, we can write,

$$B = \left[B_{1m} \sin\left(\pi \frac{x}{\tau}\right) + B_{3m} \sin\left(3\pi \frac{x}{\tau}\right) + \text{etc.}\right.$$

$$\left. + B_{1m} \cos\left(\pi \frac{x}{\tau}\right) + B_{3m} \cos\left(3\pi \frac{x}{\tau}\right) + \text{etc.}\right] \tag{5}$$

The rotational e.m.f. induced in a stator coil of N concentrated turns and having an active length of l cm. will be:

$$e_{rot.st.} = -N l v \frac{d}{dx} \int_{x_1}^{x_1+\tau} (B\,dx) \tag{6}$$

$$= 2 N l v B = 2 N l v \left(B_{1m} \cdot \sin\left(\frac{x}{\tau}\pi\right) + \text{etc.}\right) \tag{7}$$

since according to the assumption made in (5) even harmonics are absent, and in case of a full-pitch coil B at $x_1 = -B$ at $(x_1 + \tau)$. For a short-pitch coil (7) must be multiplied by the pitch factor.

In order to find expressions for the variational e.m.fs. let us make the assumption that the *flux variations are sinusoidal.* This is a very convenient assumption, since our equations already involve sine functions, and it is reasonably correct, as indicated by experience, because we are dealing with a *correction term* to be *added* to the comparatively large rotational e.m.f. Thus if $2q$ = number of slots per pair of poles, p, and (f/p) = speed of the machine in rev. per sec., obviously, $2q\,(f/p)$ will be the number of slots per second passing a given point x_1; this will set

17. S. J. Barnett, Electromagnetic Theory, pp. 333 to 340.

up an oscillation of small amplitude and of frequency $2qf$, which will be expressed in mathematical symbolism by:

$$B_{pul} = B\,(\eta \cos (2\,q\,\omega\,t)) \tag{8}$$

Neglecting the higher harmonics of the B curve, see (**5**), we get,

$$B_{pul} = \eta\,B_1 \left(\sin\left(\frac{x}{\tau}\,\pi\right)\right)(\cos\,(2\,q\,\omega\,t)) \tag{9}$$

whence,

$$\frac{\partial}{\partial t}\,(B_{pul}) = -\,\eta\,B_1\,2\,q\,\omega \left(\sin\left(\frac{x}{\tau}\,\pi\right)\right)(\sin\,(2\,q\,\omega\,t)) \tag{10}$$

and according to the second term of the right hand member of (**4**) the variational e.m.f. is proportional to:

$$\int_x^{x+\tau} \frac{\partial B_{pul}}{\partial t}\,dx = -\,\eta\,B_1\,4\,q\,\omega\,\frac{\tau}{\pi}\left(\cos\frac{x}{\tau}\,\pi\right)(\sin\,2\,q\,\omega\,t) \tag{11}$$

To find the e.m.f. induced in any armature conductor it must be remembered that owing to the relative movement between armature and field,

$$x = v\,t = (2\,\pi\,r)\ (\text{r.p.s.})\ t$$

$$= 2\left(\frac{2\,\pi\,r}{2\,p}\right)(\text{r.p.s.})\ t = (2\tau f)\,t = \left(\omega\,\frac{\tau}{\pi}\right)t \tag{12}$$

where r = radius of rotor, so that,

$$e_{pul.st.} = -\,N\,l\,\frac{\partial}{\partial t}\int B\,dx$$

$$= 4\,N\,l\,\frac{\tau}{\pi}\,q\,\omega\cdot\eta\,B_1\,(\cos\,\omega\,t)\,(\sin\,2\,q\,\omega\,t) \tag{13}$$

$$= 4\,N\,l\,q\,\omega\,\frac{\tau}{\pi}\,\eta\,B_1\,[\sin\,(2\,q\,\omega + 1)\,\omega\,t + \sin\,(2\,q - 1)\,\omega\,t] \tag{14}$$

To find analytical expressions for the static or variational e.m.f. in an armature conductor due to the to-and-fro swing of the flux, we shall assume that the B-curve is constant in magnitude, but it moves across the pole-face according to simple harmonic motion, at a frequency of $2\,q\,f$. Hence neglecting the higher harmonics of (**5**) we may write,

$$B_{sw} = B_1\left[\sin\frac{\pi}{\tau}\,(x - \epsilon\,\sin\,2\,q\,\omega\,t)\right] \tag{15}$$

where $2\,q\,\omega$ is the angular velocity and ϵ the amplitude of the variations.

Differentiating (15) with respect to t and making the reasonable assumption that,

$$\cos\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) = 1, \text{ and}$$

$$\sin\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) \doteq \frac{\pi}{\tau}\ \epsilon\,(\sin 2\,q\,\omega\,t)$$

we obtain:

$$\frac{\partial B_{sw}}{\partial t} = -\,B_1\left[\cos\frac{\pi}{\tau}\,(x - \epsilon \sin 2\,q\,\omega\,t)\right]$$

$$\left[2\,\epsilon\,q\,\omega\,\frac{\pi}{\tau}\,\cos 2\,q\,\omega\,t\right] \tag{16}$$

and

$$\int_{x}^{x+\tau} \frac{\partial B_{sw}}{\partial t}\,d\,x = [4\,\epsilon\,B_1\,q\,\omega\cos(2\,q\,\omega\,t)]$$

$$\left[\sin\frac{\pi}{\tau}\,(x - \epsilon \sin 2\,q\,\omega\,t)\cos\frac{x}{\tau}\,\pi\right] \tag{17}$$

As before, to find the variational e.m.f. in a given armature conductor due to the flux-swing, substitute,

$$x = (2\,f\,\tau\,t) = \left(\omega\,\frac{\tau}{\pi}\,t\right)$$

Thus,

$$e_{sw.st.} = -\,4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\,(\cos 2\,q\,\omega\,t)$$
$$[\sin(\omega\,t - \epsilon \sin 2\,q\,\omega\,t)\cos\omega\,t]$$

$$= -\,4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\left[\{\sin(2\,q + l)\,\omega\,t - \sin(2\,q - l)\,\omega\,t\}\right.$$

$$\left. - \left\{\frac{\epsilon}{2}\sin(4\,q + 1)\,\omega\,t + \sin(4\,q - 1)\,\omega\,t\right\}\right] \tag{18}$$

2. *E.M.Fs. Induced in Field Exploring Coils.* Equation (7) gives the rotational e.m.f. induced in a coil of N concentrated turns having a relative speed v with respect to B. Evidently this is zero for a coil placed on the rotor, and therefore,

$$e_{mot.r.} = 0 \tag{19}$$

To find the variational e.m.f. due to flux pulsations in a field (rotor) search coil whose coil-side is at any point x_1, simply substitute $x = x_1$ in (**13**). Thus,

$$e_{\overline{pul.r}} = 4\,N\,l\,q\,\omega\,(n\,B_1)\cos\left(\frac{\pi}{\tau}\,x_1\right)(\sin 2\,q\,\omega\,t) \qquad (\mathbf{20})$$

Similarly from (**17**) and (**4**) we get,

$$\begin{aligned} e_{sw.r.} &= -\,N\,l\,\frac{\partial}{\partial t}\int B\cdot d\,x \\ &= 4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\,(\cos 2\,q\,\omega\,t) \\ &\quad \left[\left\{\sin\left(\frac{\pi}{\tau}\,x_1 - (\epsilon\sin 2\,q\,\omega\,t)\right)\right\}\cos\frac{\pi}{\tau}\,x_1\right] \end{aligned} \qquad (\mathbf{21})$$

For coils No. 7 and 8, $x_1 = 0$, whence we obtain directly (**22**) and (**23**) from (**20**) and (**21**) respectively, as given in the first part of the paper:

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1\sin(4\,q\,\omega\,t) \qquad (\mathbf{22})$$

and,

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1\sin(2\,q\,\omega\,t) \qquad (\mathbf{23})$$

Note B

1. *Theoretical Investigation of Armature Reaction.* Make the reasonable assumption (as shown on p. 1163) that the armature current under s.s.c. is sinusoidal (with respect to time). Further assume that the armature winding of say, a three-phase alternator is sinusoidally distributed (in space) as schematically represented in Fig. 17. If one of the phase windings were supplied with direct-current the magnetic field intensity H, at any point θ_{gsr}, geometrical space radians or x centimeters, from the axis of reference, see Fig. 17, would be,

$$H_1' = \gamma\,I_{d.c.}\,(N_{max}\cos p\,\theta_{gsr}) = \gamma\,I_{d.c.}\,N_{max}\cos\left(\frac{x}{\tau}\pi\right) \qquad (\mathbf{24})$$

where, γ = constant, and $\left(\frac{x}{\tau}\,\pi\right) = p\,\theta_{gsr}$ = electrical space radians. Similarly for the other phases:

$$\left.\begin{aligned} H_1'' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x'}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau} - \frac{2}{3}\right)\pi \\ H_1''' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x''}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau} - \frac{4}{3}\right)\pi \end{aligned}\right\} \qquad (\mathbf{25})$$

When these windings are supplied with balanced three-phase currents the resultant field intensity due to armature reaction will be:

$$H_1' = \gamma\,[I_m \cos \omega t]\left[N_m \cos \frac{x}{\tau}\,\pi\right] \tag{26}$$

$$H_1'' = \gamma\left[I_m \cos\left(\omega t - \frac{2}{3}\,\pi\right)\right]\left[N_m \cos\left(\frac{x}{\tau} - \frac{2}{3}\,\pi\right)\right] \tag{27}$$

$$H_1''' = \gamma\left[I_m \cos\left(\omega t - \frac{4}{3}\,\pi\right)\right]\left[N_m \cos\left(\frac{x}{\tau} - \frac{4}{3}\,\pi\right)\right] \tag{28}$$

whence by trigonometry,

$$\begin{aligned} H = H_1' + H_1'' + H_1''' &= \frac{3}{2}\,H_m \cos\left(\omega t - \frac{x}{\tau}\pi\right) \\ &= \frac{3}{2}\,\gamma\, I_m \cos\left(\omega t - \frac{x}{\tau}\,\pi\right) \end{aligned} \tag{29}$$

This gives the resultant field intensity due to armature reaction and it will be recognized to be the well known equation of a travelling wave which is a function of the time t, as well as the space position x, and whose maximum, H_m, occurs when $\omega t = \left(\frac{x}{\tau}\,\pi\right)$; in other words, whose maximum H_m travels at a speed $(x/t) = (\omega\,\tau/\pi) = (2\,f\,\tau)$, which is the peripheral speed of the rotor. See (**12**). Next, consider the case of N conductors per slot and one slot per pole per phase carrying I_{dc}. amperes. Evidently, as indicated in Fig. 18 the field intensity H, will be rectangular in shape and may be represented by the series:

$$H = \frac{4}{\pi}\,Y_m\left[\sin\left(\frac{x}{\tau}\,\pi\right) + \frac{1}{3}\,\sin\left(3\,\frac{x}{\tau}\,\pi\right) + \ldots\right] \tag{30}$$

Equation (**30**) takes the place of (**24**) and similar expressions can be written out for the other phases, corresponding to (**25**) and (**26**). However, without going any further into this, suffice to call attention to the references given in foot-note (**13**) and state that if the non-sinusoidally distributed winding represented in Fig. 18, be supplied with balanced alternating-current instead of direct-current, expressions exactly analogous to (**26**), (**27**) etc. will be obtained. Combining these trigonometrically, we get (**32**), (**33**) etc. which were given in the first part, in section IV.

2. *E.M.F. Induced in Rotor and Stator Exploring Coils by the Armature Reaction.* It is a simple matter, now to obtain general expressions for the e.m.fs. induced in the different search coils, if we assume that the B-curve is sinusoidal; the rotational e.m.f. of a stator coil whose coil-side is at a fixed distance x_1 from the origin, see Fig. 6 will be according to (**4**):

$$e_{rot.st} = -N l v \frac{\partial}{\partial x} \int B\, d x \tag{36}$$

$$= -N l (2 f \tau) \frac{\partial}{\partial x} \int \frac{3}{2} B_a \cos\left(\omega t - \frac{x_1}{\tau}\pi\right) \cdot d x \tag{37}$$

$$= -N l (2 f \tau) \left(\frac{3}{2} B_a\right) \cos\left(\omega t - \frac{x_1}{\tau}\pi\right) \tag{38}$$

This is a sinusoidal e.m.f. of fundamental frequency and having a phase angle $\left(\frac{x_1}{\tau}\pi\right)$ with respect to the origin.

The fifth space harmonic of armature reaction will give:

$$e_{rot.st.} = -N l \frac{2 f \tau}{5} \cdot \frac{\partial}{\partial x} \int \frac{3}{2} B_a \cos\left(\omega t + \frac{5 x_1}{\tau}\pi\right) \tag{39}$$

$$= -N l \frac{2 f \tau}{5} \cdot \frac{3}{2} B_a \cos\left(\omega t + \frac{5 x_1}{\tau}\pi\right) \tag{40}$$

i.e., an e.m.f. of fundamental machine frequency and having a phase angle $(6\, x_1/\tau)\, \pi$ with respect to the origin. Similar expressions may be derived for all other harmonics.

However, as already explained, the sinusoidal H-curve produced by the armature reaction will give rise to a distorted B-curve. This will produce in the stator coils a motional e.m.f. which will contain higher *time harmonics*. To find analytical expressions for these e.m.fs. (due to a distorted gliding or rotating B-curve) it will be convenient to assume the B-curve as stationary and the exploring coils revolving at the same speed in the opposite direction. In this manner the problem is reduced to the one already considered in Note A, and it seems hardly necessary to derive equations for them here.

As to the e.m.fs. induced in the rotor search coils (due to armature reaction), it is clear that the fundamental travelling

wave given by (**32**), will induce no e.m.f. The 3rd harmonic will, of course, be zero for a three-phase machine. See (**33**).

To find the motional e.m.f. induced in a rotor coil due to the fifth space harmonic B_5,

$$B_5 = (1/2)\, B_a \cos\,(\omega\, t + 5\, x\, \pi/\tau) \tag{41}$$

it must be remembered that x increases at the rate of $2\, f\, \tau$ with respect to the rotor. Thus substituting in (**41**),

$$\omega\, t = \pi\, x/\tau = (\pi/\tau)\, 2\, f\, \tau\, t \tag{42}$$

we obtain,

$$B_5 = (3/2)\, B_a\, (\cos\, 6\, \omega\, t) \tag{43}$$

Similarly,

$$B_7 = (3/2)\, B_a\, (\cos\, 6\, \omega\, t) \tag{44}$$

That is, the 5th and 7th space harmonics though of fundamental frequency with respect to the stator are of sextuple frequency with respect to the rotor. This will be clear when it is considered that the 5th space harmonic glides backward at a speed $(2\, f\, \tau/5)$ while the seventh travels forward at a speed $(2\, f\, \tau/7)$. Assuming for convenience the rotor as stationary, it is evident, that the relative speed between the rotor and these gliding fields will be:

$$(2\, f\, \tau) - \left(-\,\frac{2\, f\, \tau}{5}\right) = 6\left(\frac{2\, f\, \tau}{5}\right) \tag{45}$$

and

$$2\, f\, \tau - \left(+\,\frac{2\, f\, \tau}{7}\right) = 6\left(\frac{2\, f\, \tau}{7}\right) \tag{46}$$

respectively. The pole pitches τ_5 and τ_7 of the fifth and seventh fields are $(\tau/5)$ and $(\tau/7)$, and the corresponding number of pairs of poles are $5\, p$ and $7\, p$ respectively. Consequenty the frequency of the e.m.fs. induced in a rotor coil by them will be:

$$f_5 = (\text{r.p.s.})_5\, (p_5) = (\text{speed})_5 \left(\frac{1}{2\, \tau\, p}\right) (5\, p) \tag{47}$$

$$= 6\left(\frac{2\, \tau\, f}{5}\right) \cdot \left(\frac{1}{2\, \tau\, p}\right) (5\, p) = 6 f \tag{48}$$

Similarly for the 7th space harmonic,

$$f_7 = 6\left(\frac{2\, \tau\, f}{7}\right)\left(\frac{1}{2\, \tau\, p}\right) 7\, p = 6 f \tag{49}$$

In general for the $(6\, n - 1)\, th$ space harmonic which travels backward with respect to the fundamental,

$$f_{(6n-1)} = 6\,n\left(2\tau f/(6\,n-1)\right)\left((6\,n-1)\,p/2\tau\,p\right) = 6\,n f \qquad (50)$$

and for the $(6\,n+1)\,t\,h$ harmonic,

$$f_{(6n+1)} = 6\,n\left(2\tau f/(6\,n+1)\right)\left((6\,n+1)\,p/2\tau\,p\right) = 6\,n f \qquad (51)$$

Thus the 11th and 13th will give rise to rotor e.m.fs. having periodicities of $12\,f$, and so on for the others.

In case of two-phase alternators the 3rd and 5th will give rise to e.m.fs. of (4. machine frequency). The seventh and ninth will produce e.m.fs. of (8. machine frequency), etc.

However, as shown by the oscillograms given before, the rotor search coils indicate only pulsations having periodicities of $6\,f$ in case of three-phase connection and $4\,f$ in case of two-phase connection. In concluding this note attention may be called to the fact that the above analysis of armature reaction and the e.m.fs. induced by it in the stator and rotor coils is applicable to induction motors; as a matter of fact it is a little difficult for the writer to understand why standard texts do not treat armature reaction in alternators etc. in exactly the same way as the rotating fields of induction motors, as it has been done above, because it is more convenient and lends itself better to the derivation of quantitative expressions for the e.m.fs. induced in the stator and rotor by armature reaction alone.

Note C. Direct Component of Armature Reaction of an Imperfect Two-Phase Alternator

As already explained in detail the resultant magnetic field intensity due to balanced two-phase currents flowing through sinusoidally distributed two-phase windings will be given by:

$$H_1' = H_1' + H_1'' = H_m\left[\left\{\cos\omega t\right\}\left\{\cos\left(\frac{x}{\tau}\pi\right)\right\} + \left\{\cos\left(\omega t + \frac{\pi}{2}\right)\right\}\left\{\cos\left(\frac{x}{\tau}\pi + \frac{\pi}{2}\right)\right\}\right] \qquad (52)$$

Now in case of an imperfect machine which has a displacement between phases of $\left(\frac{\pi}{2} - \alpha\right)$ deg. instead of $\frac{\pi}{2}$, the sum of H_1' and H_1'' will be:

$$H_1 = H_m \left[\{\cos \omega t\} \left\{ \cos \left(\frac{x}{\tau} \pi \right) \right\} \right.$$

$$\left. + \left\{ \cos \left(\omega t + \frac{\pi}{2} - \alpha \right) \right\} \left\{ \cos \left(\frac{x}{\tau} \pi + \frac{\pi}{2} - \alpha \right) \right\} \right] \quad (53)$$

$$= H_m \left[\frac{1}{2} \cos \left(\omega t + \frac{x}{\tau} \pi \right) + \frac{1}{2} \cos \left(\omega t - \frac{x}{\tau} \pi \right) - \right.$$

$$\left. \frac{1}{2} \cos \left(\omega t + \frac{x}{\tau} \pi - 2 \alpha \right) + \frac{1}{2} \cos \left(\omega t - \frac{x}{\tau} \pi \right) \right] \quad (54)$$

$$= H_m \left[\left\{ \cos \left(\omega t + \frac{x}{\tau} \pi \right) \right\} \left\{ \frac{1}{2} - \frac{\cos 2\alpha}{2} \right\} \right.$$

$$\left. - \left\{ \frac{\sin 2 \alpha}{2} \right\} \left\{ \sin \left(\omega t + \frac{x}{\tau} \pi \right) \right\} + \left\{ \cos \left(\omega t - \frac{x}{\tau} \pi \right) \right\} \right] \quad (55)$$

or approximately,

$$H_1 = H_m \left[\cos \left(\omega t - \frac{x}{\tau} \pi \right) - \left(\frac{\sin 2 \alpha}{2} \right) \left(\sin \left(\omega t + \frac{x}{\tau} \pi \right) \right) \right] \quad (56)$$

= regular fundamental (space) wave gliding forward + a second wave due to lack of quadrature, and gliding backwards.

Further it can be shown, as it was explained in Note B, that with ordinary non-sinusoidally distributed windings, supplied with alternating current of fundamental frequency, (with respect to time) there will be a 3rd space harmonic of the form, $H_3 = H_3' + H_3''$,

$$H_3 = H_m \left[\{\cos (\omega t)\} \left\{ \cos \left(\frac{3 x}{\tau} \pi \right) \right] \right.$$

$$\left. + \left\{ \cos \left(\omega t + \frac{\pi}{2} - \alpha \right) \right\} \left\{ \cos \left(\frac{3 x}{\tau} \pi + \frac{3 \pi}{2} - 3 \alpha \right) \right\} \right] \quad (57)$$

Translating (57) into words we shall find that it simply represents the field intensity due to a current of fundamental pulsation ω flowing through the 3rd harmonic of a non-sinusoidal winding. Simplifying this equation, we get,

$$H_3 = H_a \Big[\left\{ \cos\left(\omega t + \frac{3x}{x}\pi\right) \right\} \left\{ \left(\frac{1}{2} + \frac{\cos 4\alpha}{2}\right) \right\}$$

$$+ \left\{ \cos\left(\omega t - \frac{3x}{\tau}\pi\right) \right\} \left\{ \left(\frac{1}{2} - \frac{\cos 2\alpha}{2}\right) \right\}$$

$$+ \left\{ \sin\left(\omega t + \frac{3x}{\tau}\pi\right) \right\} \left\{ \left(\frac{\sin 4\alpha}{2}\right) \right\}$$

$$- \left\{ \sin\left(\omega t - \frac{3x}{\tau}\pi\right) \right\} \left\{ \frac{\sin 2\alpha}{2} \right\} \Big] \tag{58}$$

or approximately,

$$H_3 = H_a \Big[\cos\left(\omega t + \frac{3x}{\tau}\pi\right)$$

$$+ \left(\sin\left(\omega t + \frac{3x}{\tau}\pi\right)\right)\left(\frac{1}{2}\sin 4\alpha\right)$$

$$+ \left(\sin\left(\omega t - \frac{3x}{\tau}\pi\right)\right)\left(\frac{1}{2}\sin 2\alpha\right)\Big] \tag{59}$$

$= H_{3R} + H_3^{\alpha} + H_{31}^{\alpha}$

= regular third space harmonic travelling backward at a speed $2f\tau/3$. + a second wave due to lack of quadrature and in quadrature to the first and travelling in the same direction and at the same speed as the first. + a third wave due to lack of quadrature and in phase with the second but travelling forwards, at the same speed as the first two.

If desired the third wave in (59), H_{31}^{α}, may be neglected, and H_3 be taken equal to H_{3R} and H_3^{α}.

Exactly as before for the 5th space harmonic, due to a sine wave of current of fundamental pulsation ($\omega = 2\pi f$), we have,

$$H_5 = H_a \Big[\{\cos(\omega t)\} \left\{ \cos\left(\frac{5x}{\tau}\pi\right) \right\}$$

$$+ \left\{ \cos\left(\omega t + \frac{\pi}{2} - \alpha\right) \right\} \left\{ \cos\left(\frac{5x}{\tau}\pi + \frac{5\pi}{2} - 5\alpha\right) \right\} \Big] \tag{60}$$

Simplifying this, we find,

$$H_5 = H_a \Big[\Big\{ \cos\Big(\omega t + \frac{5x}{\tau}\pi\Big) \Big\} \Big\{ \frac{1}{2} - \frac{\cos 6\alpha}{2} \Big\}$$

$$+ \Big\{ \cos\Big(\omega t - \frac{5x}{\tau}\pi\Big) \Big\} \Big\{ \frac{1}{2} + \frac{\cos 4\alpha}{2} \Big\}$$

$$- \Big\{ \sin\Big(\omega t + \frac{5x}{\tau}\pi\Big) \Big\} \Big\{ \frac{1}{2} \cdot \sin 6\alpha \Big\}$$

$$- \Big\{ \sin\Big(\omega t - \frac{5x}{\tau}\pi\Big) \Big\} \Big\{ \frac{1}{2} \cdot \sin 4\alpha \Big\} \Big] \qquad (61)$$

or approximately,

$$H_5 = H_a \Big[\cos\Big(\omega t - \frac{5x}{\tau}\pi\Big)$$

$$- \Big(\sin\Big(\omega t + \frac{5x}{\tau}\pi\Big)\Big) \frac{1}{2} \cdot \sin 6\alpha$$

$$- \Big(\sin\Big(\omega t - \frac{5x}{\tau}\pi\Big)\Big) \frac{1}{2} \cdot \sin 4\alpha \Big] \qquad (62)$$

$= H_{5R} + H_5{}^{\alpha}\, H_{51}{}^{\alpha}$

= a regular fifth space harmonic, travelling forward at a speed $2f\tau/5$. + a second wave in quadrature to the first and travelling backward at the same speed. + a third wave in quadrature to the first and gliding forward at the same speed.

As in (59) H_5 may be taken equal to $(H_{5R} + H_5{}^{\alpha})$ and the third wave be neglected. Similar expressions for other space harmonics may be derived by exactly the same procedure. Furthermore, in all the above we have assumed the current to be of the form $I_m \cos \omega t$. If the current contains higher time harmonics the resultant armature reaction may be found in a similar manner.

DISCUSSION ON "SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS" (DIAMANT), ATLANTIC CITY, N. J., JUNE 28, 1918.

F. D. Newbury: To a considerable extent, this paper appears to cover the same ground as the paper by A. E. Clayton on "Wave Shapes of A-C. Generators with Steady Short-Circuit Conditions," presented before the British Institution in 1915. The latter paper has an important advantage in that it shows the current waves and flux distributions of a number of different generators having a considerable range in size and of commercial design. This English paper treats this complex subject in a very clear and simple manner by showing the close correspondence between theoretical curves derived by graphical methods and the oscillograms.

The greater part of Prof. Diamant's paper is concerned with the results from a single 45-kv-a. generator, whose design proportions are not given. In considering these results, it cannot be too strongly emphasized that the significance of the various curves depends entirely upon the detail design proportions of the generator, and the location and spread of the search coils. For example, the double peaks in the curves of Fig. 11 are caused, as the author points out, simply by the reduction of flux density in the space between poles, and thus indicate a very familiar fact by an unfamiliar method.

It is dangerous to draw general conclusions from the results obtained from a single generator. I have in mind the results obtained from a 1500-kv-a., 12,000-volt synchronous motor, having very wide armature slots in comparison with the air gap. The ratio between armature slot opening and air gap was something over 3. There was a very pronounced ripple in both the field form and in the no-voltage e. m. f. wave form. A search coil of one tooth pitch indicated curves very similar to those shown by the author in Fig. 11. While there were only six slots per pole, the wave forms indicated a much higher frequency disturbance and analysis of the e. m. f. wave showed a large twenty-third harmonic. Further investigation of the design developed the fact that partially closed damper slots in the rotor had a pitch only slightly greater than half the slot pitch of the stator, and this conjunction of stator and rotor slots was the cause of the voltage pulsation. This conclusion was confirmed later by building another motor using the same stator punchings, but with a larger tooth pitch for the damper winding, in which the wave form had a relatively small ripple, in spite of the large ratio between stator slot opening and air gap. While this example illustrates a well-known cause of wave-shape distortion, it nevertheless emphasizes the point that the higher harmonics sometimes found in commercial generators are due to a variety of obscure causes, and the search coil e. m. f. waves shown in the paper may or may not have general significance.

There are a number of specific points in the paper to which attention should be called.

It is stated that the subject of tooth ripples is of great practical importance. Tooth ripples are important in synchronous generators and motors only in connection with questions of telephone interference, and then only in connection with salient-pole alternators, in which air gaps are inherently small. One is apt to obtain an exaggerated idea of the importance of these variations in air gap reduction from this and other statements made by the author. For example, in Fig. 9 the variations in air gap flux were obtained with the rotor at rest, or moving slowly. Under these conditions, much greater variations are obtained than are obtained during normal operation when eddy currents very largely reduce the flux variations that tend to exist. In Fig. 9, the flux opposite a slot is 45 per cent of the flux opposite a tooth. In Fig. 7, obtained presumably from the same generator, this ratio for coil No. 1 is 76 per cent. In generators of commercial design, the ripples rarely, if ever, produce greater variations than those shown in Fig. 7.

Throughout the discussion of armature reaction, it is assumed that the space distribution of m. m. f. in a single concentrated armature coil is sinusoidal. At any instant of time, the space distribution of m. m. f. and flux, assuming uniform reluctance, is rectangular, as illustrated in Fig. 18. At equal intervals of time, the m. m. f. and flux may be represented by a series of rectangles, such as Fig. 18, the ordinates of which vary sinusoidally. This is important, because it means that the space distribution of the armature reaction flux depends on the space distribution of the armature coils, and the latter should be taken into account in all calculations concerning armature reaction.

This assumption of sine-shaped armature reaction flux, and the assumption recorded that the field form of most modern machines approaches the sine shape, leads to the statement that the resultant of the main field and armature field under sustained short-circuit conditions must also be approximately sine shaped. These assumptions are immediately shown to be false, by the statement that "we find indications of an extremely distorted flux distribution." The reason for this distortion is explained by Fig. 32, taken from Clayton's British paper, in which the main flux is assumed to be rectangular, and the armature flux a segment of a sine curve. It seems hardly worth while to build up a theory upon assumptions that are immediately shown to be wide of the fact.

In general, harmonics in the distribution curve of flux have a considerably reduced effect on the voltage generated in the armature winding, because the voltages generated by any particular harmonic of the flux distribution curve are in most cases (except the third and multiples) opposed in different conductors connected in series. Furthermore, the current on short circuit

is still closer to a sine wave on account of the increasing reactance of the winding for the higher harmonics. Therefore, while the flux distribution on short circuit, as shown in the paper, varies widely from the flux distribution at normal voltage and no load, the generated voltage on short circuit and the current resulting therefrom, depart less and less from the sine form.

One matter of practical importance in connection with the sustained short-circuit operation is this distortion of the flux, due to the relatively weak field excitation under this condition.

This matter has been quite completely treated in Clayton's paper previously referred to, and is moreover quite simply treated graphically, even when actual flux distributions are taken into account. One important result of this distorted flux is the increase in losses under short-circuit conditions in some generators. The short-circuit losses of large two and four-pole turbo-generators, when operated on short circuit, are very much larger than can be accounted for by the losses in the armature winding and the core loss corresponding to the internal voltage. The departure of the flux distribution from the sine increases rapidly as the ratio of field to armature m. m. f. decreases, and with the ratio of unity or less, now commonly used, it is quite probable that the large short-circuit losses in these machines are largely due to the flux distortion produced by the armature reaction. This is a matter of considerable practical importance, as the present Institute Standardization Rules require that the total short-circuit losses be included in the loss existing under normal operating conditions. There is opportunity here for experimental and analytical work that will enable us to more completely determine the source of loss under the condition of sustained short circuit.

R. E. Doherty: I wish to agree with practically all that Mr. Newbury has said in regard to this paper, but there is one point which I think he has put a little too strong. I think it is not Mr. Diamant's intention to give quantitative results to be used as a guide in designing the machines, but rather to give a qualitative explanation of some of the curious wave irregularities that we all recognize as existing in sustained short-circuits. Of this he has given the very clear explanation that while under sustained short-circuit the fundamental is practically annihilated, the harmonics are not very much affected and, therefore, the relative importance of the harmonics as compared to the fundamental has been increased hundreds of times in some cases. This produces those curious waves shown on the oscillogram of coil voltage at sustained short circuit.

William F. Dawson: In the average modern turbo-alternator the field form is trapezoidal until saturation is reached (in the field teeth nearest the pole) when the corners of the trapesoid are rounded off. The trapezoid is a close approach to a sine wave and is more so when rounded at the corners by saturation.

With a satisfactory flux wave as the foundation, conditions are further improved by a generous number of slots per phase and pole, and by the practical elimination of the third harmonic by making the armature coil pitch approximately two thirds, the resulting e. m. f. and current waves at the armature terminal are very close to true sine waves at all loads. Considerations of ventilation, regulation and the minimizing of core loss insure such large air gaps as compared with slot widths that while tooth ripples still exist their amplitude is too small to create any objectionable disturbance even under sustained short circuit.

C. M. Davis: I wish to emphasize what Mr. Doherty has said, that this paper presents the results only of tests upon a particular machine, and it is not the author's intention in any way to imply that the case mentioned is a general one. He is merely seeking a way of explaining, to his own satisfaction, some of the actions which are not evident on first examination.

N. S. Diamant: In answer to Mr. Newbury's remarks, may I say that I am rather disappointed in that I was expecting to hear something worth while and profitable considering his experience which I am sure is very wide. I was expecting to have him tell us something about the more complex aspects of the subject, which are carefully taken up in my paper.

As Mr. Newbury does not seem to be sure to what extent my paper and that of A. E. Clayton cover the same ground, I may add that reference is made to A. E. Clayton's 1916 paper on 1181 and 1182. There, in a general way Clayton's explanation is given, and a further note made that his paper; namely, Clayton's, is the first one, as far as I am aware, to throw any light on this complex subject. As explained in my introduction, I had started this work before Clayton's paper appeared. The explanation and theory of Clayton, which is graphical, is very good; so good in fact that I have included it in my paper. My explanation of the same phenomenon is a little different as may be seen by referring to Figs. 24, 24A, 25 and 31, also oscillograms Figs. 23, 27 etc., and Tables VII, VIII and XI.

Furthermore, I have taken questions of flux distribution under load and no-load conditions as well as under sustained short circuit. It is disappointing that Mr. Newbury, with his wide experience, doesn't tell us more definitely in regard to several points, at least twelve of which are very specifically referred to in the abstract and very carefully treated, in the paper itself. For example, Mr. Newbury states that the "paper is concerned with results from a single 45-kv-a. generator", and that it is dangerous to draw general conclusions from the results obtained from a single machine. This may be true enough but it would be very much more useful to me and to the reader for Mr. Newbury, instead of making these general remarks which are of no practical value, to tell us more specifically just what points in the paper he thinks could be generalized and what points should not. Frankly, it is not quite clear to me just what is the

relation between the example he gives about a 1500-kv-a. machine and the danger of generalizing, unless it be to show that it is a rather extravagant way of investigating flux pulsations, "by building another motor using the same stator punchings but with a larger tooth-pitch for the damper winding." To be more specific, may I ask Mr. Newbury to tell us what he thinks is the origin and cause of to-and-fro flux oscillations and pulsations. From his experience is the general theory given by Worall and referred to in my paper acceptable to him. What does Mr. Newbury think about:

1. My method of determining experimentally the armature reaction by subtracting no-load and sustained short-circuit exploring coil voltages, as given in Figs. 23, 27, etc.

2. What does he think is the explanation of the fact that the third harmonic seems to increase with increasing excitation as given in Table IX.

3. What does Mr. Newbury think about my experimental and mathematical analysis of armature reaction of imperfect two-phase alternator as given in Fig. 4, Fig. 35 and the supplementary note C page 1199.

(A tentative explanation of this is referred to in foot note 12.)

I don't quite agree with his statement that "tooth ripples are important only in connection with questions of telephone interference, and then only in connection with salient-pole alternators." Is not the question of tooth ripples and higher harmonics important in connection with iron losses and sometimes dielectric losses also. However, I may add that my paper did not attempt to state just how important higher harmonics may become in certain cases, or of how little importance they may be in other cases. The object is more to consider the fundamental theory of the origin and production of ripples in m. m. f. curves, in flux waves and induced voltages, and I, for one, will appreciate it if I could have Mr. Newbury's ideas on the subject.

Mr. Newbury states that the assumption of sine-shaped armation reaction flux is made in the paper. This is not quite so. As stated on p. 1163, the armature current under short-circuit conditions is assumed to be sinusoidal and then very clear distinction is made as to whether the m. m. f. will be sinusoidal or not and whether the flux, due to the m. m. f. will be sine-shaped or not. If I am not mistaken, in most discussions this distinction, namely, whether a sinusoidal m. m. f. may or may not give a sinusoidal flux, is not quite as clearly adhered to as it is in the paper.

Although outside of the scope of the paper, in connection with the "critical resume conclusions" attention is called to the practical bearing of the results of the paper on standard methods of testing alternators by means of cyclic heat run tests, short-circuit load loss tests, phase-shifting load tests etc. It will be interesting to hear something in regard to this.

There is little to add to Mr. Doherty's discussion except that as in Mr. Newbury's case, considering his wide experience covering a great many types and sizes of machines, it will be of considerable interest to the profession to have his views on the many different specific points in connection with flux distribution that are taken up and at least tentatively explained in the paper.

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REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

BY R. E. DOHERTY AND O. E. SHIRLEY

Abstract of Paper

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, Figs. 20A, B, C, making the calculation from design sheet data a matter of a few minutes. Table I shows a comparison of calculated and test values (obtained from saturation and synchronous impedance curves) for 138 machines, ranging from high-speed turbine generators to the low-speed engine type.

Three points were brought out during the investigation:

(1) That in polyphase machines, the armature self-inductive reactance, just as the armature reaction, is a polyphase, not a single-phase phenomenon, and therefore the mutual induction of phases in a three-phase machine increases the effective self-induction of each phase by approximately 50 per cent over the single phase value, while in two-phase machines, in which the mutual induction of phases is zero, the effective self-induction of the phase is the same for two-phase or single-phase operation.

(2) That the variation of armature reactance during the cycle, due to salient-pole construction, is practically eliminated in Y-connected, three-phase machines for the reason that the variation, consisting almost entirely of a third harmonic, is cancelled in such machines. This leaves, in effect, a uniform reluctance for the leakage flux emanating from the tooth tips.

(3) That in the familiar method of obtaining the armature self-induction from the saturation and synchronous impedance curves (*i.e.*, by subtracting the armature reaction, that is, the demagnetizing ampere turns of normal current under sustained short circuit, from the corresponding field ampere turns), a very large error in this test value of self-induction may occur, if, as is usually done, the armature reaction is calculated on sine wave assumptions. A set of curves shown in Fig. 20, which are plotted from results derived in Appendices A and B, give values of the correction factor which applies to calculations based on sine wave.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by

other authors, *transient reactance*. Table IV shows a comparison on eleven machines of the actual short-circuit currents as determined by oscillograph, and as calculated by methods proposed in the paper.

The calculations of *transient reactance* apply strictly to salient laminated pole alternators without amortisseur winding. However, such experience as is recorded in Table IV, if analyzed in connection with theoretical considerations, affords a basis for estimating the *transient reactance* of turbine generators with massive steel rotors and of salient-pole machines with amortisseur windings. These points are summed up in "Summary and Conclusions".

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit. This may be (if the rotor is entirely laminated) 30 or 40 per cent of normal flux, and apparently explains why, in machines with solid steel rotor rims, the field attenuation factor, α_f, changes radically after the first few cycles after short circuit.

Introduction

As the march of time obliges designers to become ever more efficient in the use of materials, it becomes more and more necessary that they should extend design calculations to further completeness. The self-inductive reactance of synchronous machines stands in very important relation to this problem. In the predetermination of field excitation, of voltage regulation, and of the initial short-circuit current, the reactance is, of course a very essential factor.

Three years ago the authors undertook a survey of the work which had been done on the calculation of armature reactance in the hope of finding, if not in any particular existing formula, at least in a combination of different ones, an expression for reactance that would give reliable calculations—reliable, we mean when applied to the entire range of machines as built by one of the large manufacturing companies. The result was probably very much the same as that which others have obtained who may have made a similar attempt. Particular expressions applied well to particular classes of machines, but none was found, and no combination of existing expressions was found, that would apply generally. Moreover, on account of the many factors in the problem, the equation for reactance is a very clumsy thing to handle—too clumsy, indeed, to make practical the calculation of reactance on each new, proposed design. It is nevertheless desirable to have this calculation. By methods described later an expression was finally arrived at which does apply generally

to all classes of machines from two-pole turbine-generators to 136-pole water-wheel driven generators, of any voltage and frequency, three-phase, two-phase or single-phase, and the results are put in convenient shape for the designer's use, thereby making the calculation for each machine a practical matter. A tabulated comparison of calculated and test results (taken from ordinary commercial tests for open and short-circuit characteristics) is shown in Table I.

The importance of predetermining the initial short-circuit current of synchronous machines need not be dwelled upon. Mechanical stress involved, the rupturing capacity of circuit breakers, heating, etc., all make it a matter of supreme importance. The theory underlying short circuits has been thoroughly worked out by Steinmetz, Berg and Bouchert; and Diamant[1] has extended the mathematics involved and investigated the possibility of determining experimentally the attenuation factors, r/L, which determine the rate of decrease of the transients. That is, the complete equation for the instantaneous values of short-circuit current has been available for some time. The difficulty in its use has been that the factors involved could not be predetermined, and, to some extent, there has been uncertainty regarding what constitutes the reactance which determines the short-circuit current. The authors will show that this reactance includes the field self-induction as well as that of the armature, and will derive a formula by which the total reactance may be calculated. To the author's knowledge this has not been previously worked out. Table IV gives a comparison of calculated and actual values of short-circuit current.

In the process of obtaining the above results, another closely related problem was incidentally solved, namely, the effect of the harmonics in the no-load flux wave upon the field excitation required for normal voltage and also upon the armature reaction. This is given in Appendices A and B.

The object of the paper, therefore, is (1) to present a reliable, general formula for the calculation of self-inductive armature reactance of synchronous machines. (Given in Part I).

(2) To give a simplification of the general equation, permitted by standard construction, accompanied by working curves making it possible to calculate the per cent armature reactance from the design sheet in less than five minutes. (Given in Part I).

1. TRANS., A. I. E. E. 1915, Vol. XXXIV, p. 2237.

TABLE I

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{ps}	
						Calc.	Test
1	33	3	60	1200	440	13.4	8.06
2	50	3	60	300	2300	16.0	13.9
3	125	3	62½	750	2300	13.5	11.0
4	180	3	60	900	220	10.9	12.3
5	240	3	60	150	440	17.1	15.2
6	265	3	25	150	400	28.1	34.8
7	300	3	60	1200	4000/2300	15.6	13.0
8	320	3	50	200	5000	12.5	15.5
9	345	3	60	720	4000	13.1	10.7
10	370	3	50	187½	500	14.5	20.0
11	375	3	60	150	2300	14.7	13.2
12	375	3	60	150	3100	14.4	11.4
13	400	3	25	300	2300	15.2	21.0
14	400	3	60	300	4000	27.5	30.1
15	400	3	60	200	600	18.9	19.3
16	400	3	60	150	6600	24.6	21.4
17	450	3	25	750	13200	8.8	3.1
18	475	3	50	214	440	16.4	15.3
19	500	3	60	600	2300	7.6	7.5
20	500	3	60	150	2300	16.0	13.7
21	500	3	60	100	240	14.4	13.9
22	540	3	60	164	6600	31.0	27.3
23	560	3	25	750	480	6.6	5.8
24	560	3	60	720	13200	15.1	12.0
25	560	2	60	720	2400	18.6	23.2
26	570	3	25	750	13200	9.0	8.9
27	580	3	60	720	2300	13.4	15.6
28	585	3	60	90	2000	14.4	15.6
29	605	3	60	150	2200	16.8	18.1
30	625	3	60	150	2300	10.7	10.0
31	625	3	60	100	480	21.1	15.8
32	675	3	60	90	6600	36.8	33.5
33	700	3	25	750	2200	10.1	6.00
34	700	3	25	750	13200	13.2	16
35	700	3	25	500	6600	11.4	7.3
36	700	3	60	900	4000	13.8	12.7
37	700	3	40	300	600	10.5	10.2
38	750	3	60	300	600	22.5	23.6
39	750	3	60	150	2300	18.4	19.5
40	780	3	60	120	2300	21.7	21.0
41	800	3	60	90	2300	41.9	44.0
42	850	3	25	500	6600	5.6	8.4
43	900	3	60	360	600	20.5	18.6
44	900	3	60	150	2300	22.1	18.2
45	1000	3	60	450	2300	22.5	19.8
46	1000	3	60	200	600	21.2	16.8
47	1200	3	25	500	4400	16.3	16.9
48	1250	3	25	750	13200	10.5	10.5
49	1250	3	50	500	5500	12.3	12.8
50	1250	3	60	300	2300	12.0	10.5
51	1250	2	62.5	375	5400	12.1	10.2

TABLE I. (*Continued*)

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa} Calc.	x_{pa} Test
52	1350	3	25	375	6600	11.3	12.8
53	1400	3	25	500	2200	10.0	8.9
54	1400	3	25	375	13200	11.5	13.8
55	1400	3	60	514	13200	10.8	10.9
56	1400	3	60	514	13200	12.9	11.9
57	1420	3	25	300	2300	11.0	5.8
58	1425	3	60	240	2200	21.1	22.0
59	1450	3	60	720	2300	11.3	8.6
60	1500	3	25	500	2300	7.9	10.2
61	1500	3	60	514	2200	14.5	15.0
62	1500	2	62.5	750	2300	14.3	20.0
63	1625	3	25	750	3300	10.5	10.5
64	1800	3	60	300	2300	10.6	12.6
65	2000	3	60	720	2200	13.8	14.0
66	2000	3	60	100	2300	8.4	8.0
67	2000	2	60	360	2400	9.6	7.2
68	2100	3	60	514	4600	15.7	15.7
69	2125	3	40	200	600	15.3	14.1
70	2200	3	25	750	3300	9.1	15.0
71	2200	3	25	375	2300	9.1	9.7
72	2250	3	25	214	2300	5.9	4.4
73	2250	3	60	514	6000	15.3	16.9
74	2250	3	60	450	3450	17.8	17.1
75	2500	3	25	250	2200	5.8	4.0
76	2800	3	25	300	2300	5.5	4.9
77	2000	3	60	514	13200	18.9	19.2
78	2800	3	60	450	4000	15.0	18.0
79	3000	3	50	500	6600	13.1	17.4
80	3000	3	60	450	2300	8.25	11.0
81	3125	3	25	750	14000	16.1	15.8
82	3500	3	50	500	11500	19.2	20.0
83	4000	3	60	600	2300	13.8	11.2
84	5000	3	25	300	2300	8.9	10.2
85	5000	3	25	88.3	6600	13.9	13.8
86	5650	3	60	360	6600	19.8	21.3
87	6000	3	50	500	3450	7.65	6.1
88	6000	3	50	500	16500	21.6	26.5
89	6600	3	25	375	14000	9.5	11.0
90	6250	3	60	600	6600	16.8	13.9
91	6750	3	50	600	11000	19.3	25.6
92	7050	3	40	400	600	6.6	8.5
93	7050	3	50	375	6600	13.9	13.0
94	7500	3	60	600	13800	13.3	16.9
95	8750	3	50	500	600	15.1	19.4
96	8750	3	50	500	6600	6.9	7.1
97	9000	3	25	57.7	11000	13.9	16.4
98	10000	3	60	514	6600	6.81	7.9
99	10000	3	62.5	55.6	6600	20.4	19.4
100	12000	3	25	375	6600	13.2	12.6
101	15000	3	50	375	6600	10.2	12.3

TABLE I. (*Continued*)

TURBINE ALTERNATORS							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
1	3750	3	60	3600	2300	9	14.5
2	4375	3	60	1800	2400	5.9	2.4
3	4375	3	60	1800	4500	6.7	3.9
4	4375	3	60	1800	2300	7.7	8.2
5	4375	2	60	1800	2300	6.25	5.1
6	5000	3	25	1500	5500	10.6	10.2
7	5000	3	25	1500	6600	6.4	6.8
8	5000	3	25	1500	6600	7.3	5.5
9	5000	3	60	3600	2300	9.0	8.9
10	5625	3	25	1500	2300	6.1	3.3
11	6250	3	25	1500	6600	10.1	10.6
12	6250	3	60	3600	2300	8.6	12.2
13	6250	3	60	1800	6600	8.6	7.8
14	7500	3	25	1500	2300	7.1	6.9
15	7500	3	25	1500	6600	10.5	7.1
16	7500	3	25	1500	6600	10.5	11.8
17	7500	3	60	1800	4000/2300	9.4	5.0
18	7500	3	60	1800	4160/2300	9.1	8.2
19	7500	3	60	1800	2300	7.6	9.1
20	7812	3	25	1500	2300	7.7	7.8
21	7812	3	60	1800	2300	10.1	9.5
22	7812	3	60	1800	4000/2300	10.6	8.6
23	7812	3	60	1800	4000/2300	11.8	10.9
24	9375	3	25	1500	13200	8.9	9.3
25	9375	3	60	1800	4000/2300	8.0	8.6
26	9375	3	60	1800	5000	11.0	9.2
27	10000	3	25	1500	6600	9.9	9.8
28	10000	3	40	2400	10000	12.5	13.8
29	10000	3	60	1800	6600	14.2	9.2
30	12500	3	25	1500	11000	8.4	8.2
31	12500	3	60	1800	4000/2300	9.1	7.4
32	12500	3	60	1800	11000	13.0	17.1
33	12500	3	60	1800	13200	19.2	13.5
34	12500	3	60	1800	2300	10.2	8.6
35	18333	2	37.3	2240	4469	6.3	7.4
36	18750	3	60	1800	2300	13.8	12.0
37	25000	3	60	1800	13200	14.4	15.4

(3) To derive a formula for calculating the field self-induction which combines with the armature self-induction to determine the initial short-circuit current; and further to describe in part the physical phenomena which exist in the machine at short circuit. (Given in Part II).

(4) To derive a formula for calculating armature reaction, taking into account the effect of field distribution. (It is necessary to determine the armature reaction accurately, before the armature self-induction can be determined from standard open and short-circuit tests). (Given in Appendix B).

(5) To show the effect of flux distribution (*i.e.* the effect of the harmonics in the no-load flux wave) on the field excitation. (Given in Appendix A).

(6) To give a simplified method of calculating field excitation which has worked out well in connection with the *per cent* armature reactance. (Given in Appendix C).

PART I.

Armature Reactance

Of all the work which had been done on armature reactance, that of Gray[2] was, in our opinion, the most complete. While it involved certain familiar factors, such as that for the slot reactance and the leakage from tooth tip to tooth tip which have appeared in other reactance formulas, it yet took into account for the first time, we believe, the variation of tooth-tip reactance in salient-pole machines for different positions of the pole, which is very important. Fechheimer introduced a factor to account for the effect of fractional pitch armature coils. The results given in this paper have been obtained by a modification and extension of the work done by Gray, Fechheimer[3] and others. A bibliography which may be of historical interest is included at the end of the paper.

There were three difficulties of fundamental importance which it was necessary to overcome before consistent results were obtained. The first was the influence of the harmonics in the no-load flux wave upon the effective armature magnetomotive force or armature reaction. The armature self-inductive reactance, as obtained from the saturation and synchronous impedance curves (the open-circuit and short-circuit characteristics) involves the difference between the actual field m.m.f. on short circuit and the effective armature m.m.f. for the corresponding current. Since the armature reaction is calculated, this difference must also, to that extent, be considered as calculated. The familiar, approximate formula usually employed in determining effective armature m.m.f. per pole,

$$A = \frac{2.12\, N_q\, I}{K_p\, K_d} \quad \text{(three-phase)}$$

is based upon sinusoidal distribution of flux and armature m.m.f.

2. Gray's Elec. Machine Design, 1st Edition, page 215.
3. Trans. A. I. E. E., Vol. 31, p. 578.

N_e = series turns per pole per phase
I = amperes per turn
K_p and K_d = coefficients, greater than unity, which take account of the effect of fractional pitch coils and the distribution of coils respectively.

Often the flux distribution is very different from a sine wave. In such cases the actual effective armature m.m.f. may be different by 8 to 10 per cent from the value based on sine-wave assumptions—*i.e.*, as calculated from the above formula. This error in armature m.m.f., of course causes a much larger percentage

FIG. 1—TOOTH-TIP LEAKAGE FLUX

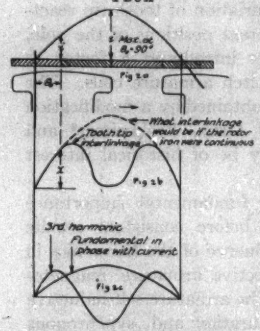


FIG. 2—TOOTH-TIP INTERLINKAGES AT ZERO POWER FACTOR

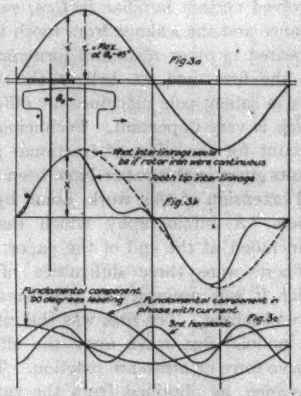


FIG. 3—TOOTH-TIP INTERLINKAGES AT 80 PERCENT POWER FACTOR

error in the difference between field and armature m.m.f's. on short circuit, which difference, correctly determined, represents the armature self-induction. It is therefore obvious that the effect of flux distribution must be taken into account. This has been done by a single factor, K_ϕ, which is derived in Appendix A, and is given in Fig. 20.

The second difficulty was met in accounting correctly for the, variation in the permeance of the "tooth-tip" leakage path for different positions of the pole. Gray's method of accounting for this neglected certain serious factors, as will be shown later. Fig. 1 shows *tooth-tip* leakage of one-phase belt at a time when

the pole-face iron forms part of the leakage path. It is obvious from Fig 1, that as the pole moves on, a part, and finally all, of the path between tooth-tips will be air. At zero power factor the current in any particular phase group is a maximum when that phase group is half way between poles, when the angle, θ_0, between the center of the phase group and the center of the pole Fig. 2A, is 90 deg.; and is zero when the group is over the middle of the pole, when θ_0 is zero. This produces a curve of leakage interlinkages shown as x in Fig. 2B. The dotted curve indicates what the leakage curve would be if the iron of the rotor were continuous. Now for any other power factor, say $\cos \theta = 0.7$, if the distortion of the magnetic field is neglected, the current will be a maximum when

$$\theta_0 = \theta = 45 \text{ deg.}$$

and zero when $\theta_0 = -45$ deg. But distortion can not be neglected. By shifting the flux toward the trailing tip (in a generator) the position of the pole is relatively advanced, thereby increasing the total angle between the center line of the pole and the position of the phase group when the current is a maximum, to

$$\theta_0 = \theta + \theta_d$$

where θ_d is the distortion angle, shown in Fig. 18. For illustration let the current be a maximum when θ_0 equals 45 deg. Fig. 3A. This would correspond in certain machines to a power factor of about 80 per cent and would produce a curve of leakage interlinkages, x, shown in Fig. 3B. The dotted curve indicates the leakage which would exist if the rotor iron were continuous.

An analysis of the curves of interlinkages shown in Figs. 2B and 3B discloses an interesting and very important fact: that these curves as shown in Figs. 2C and 3C, are made up principally of a fundamental and a large third harmonic, all other harmonics being negligible; and that since in a Y-connected 3-phase machine the third is eliminated, the fundamental alone remains for consideration in such machines—which probably comprise 95 per cent of all that are now being manufactured. And even in two-phase or Δ-connected three-phase machines the effective results are much the same. Experience has been that the short-circuit characteristic and saturation curves show practically the same value of reactance in the same machine

whether it is connected Y or Δ. The probable explanation of this is that in Δ-connected or two-phase machines the current, especially under short circuit, is not a sine wave, as it is for Y-connected machines, but instead, contains a negative third harmonic, as shown in Fig. 4. This operates to lower the peaks in Fig. 2B, that is, to decrease the third harmonic in the reactance. And although, of course, the effect of the remaining, decreased third harmonic in the reactance voltage is to restrict the flow of current at certain points of the cycle, and therefore produce a current of smaller average value than would exist if the third harmonic were eliminated, nevertheless the form factor of the resulting peaked wave shown in Fig. 4, is correspondingly higher, that is, the ammeter will read correspondingly higher, and therefore show in this case about the same reading for sustained short-circuit current as would be shown in the case of a Y-connected machine. In general, then, it is necessary to consider only the fundamental in the tooth-tip leakage.

The salient pole construction, which is the cause of the large third harmonic mentioned above, is therefore, in effect replaced by a continuous rotor of uniform reluctance, which is greater than that of iron but lower than that of air, and which is different for different power factors, being lower at high power factor. This is obvious from a comparison of Figs. 2B and 3B.

At high power factor, an interesting circumstance arises. In addition to that component of the fundamental of the tooth-tip leakage which is in phase with the current and which therefore produces a reactive voltage lagging behind the current by 90 deg., there is also a cosine term, leading the current by 90 deg., which produces a reactive voltage in phase with the current. This has important significance in the matter of field excitation required under load, as pointed out later.

Gray's value for the tooth-tip factor is based on a root-mean-square of instantaneous values of a curve somewhat similar to that shown in Fig. 2B., the principal difference in the curves being that Fig. 2B., takes account of the distribution of the coils in the phase belt. Being based on the root-mean-square and neglecting the spread of the phase belt, Gray's value for this factor is approximately 100 per cent higher than that given in this paper, and does not take account of the in-phase component of reactive voltage, or of the effect of the distortion of the magnetic field—both of which occur under watt load.

The third difficulty involved the conception that on a polyphase machine, the self-induction of the armature is a polyphase, not a single-phase phenomenon; and that therefore the reactance per phase of the same machine, connected three phase in one case, two phase in another, would be approximately 1.5 times the single-phase value in the former, 1.0 times the single-phase value in the latter. In other words the magnetic field of self-induction is a rotating polyphase field just as is the field of armature reaction, and the resulting overlapping of phase produces greater leakage interlinkages in any particular phase of a three-phase machine than would exist by the m.m.f. of the phase alone. For instance, in a three-phase machine, when the

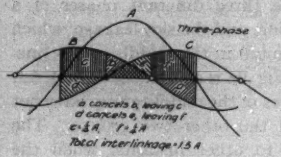


FIG. 5A

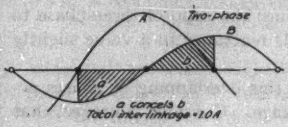


FIG. 5B
MUTUAL INTERLINKAGES—POLYPHASE MACHINE

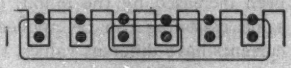


FIG. 6 A

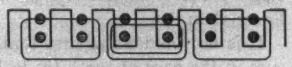


FIG. 6B
SLOT LEAKAGE IN THREE-PHASE MACHINE

current is maximum in phase A, it is one-half maximum in B and C. The increase of interlinkages in A, due to B and C, is shown in Fig. 5A. For two-phase, the current is zero in B when a maximum in A, and moreover they are displaced in space so that the mutual interlinkages must be zero at all times as shown in Fig. 5B.

Unless this fact is taken into account when considering three-phase machines, the calculated value, which is the reactance per leg, will be only two-thirds as large as the test value, unless some of the factors in the reactance equation are overburdened to make up the difference. Then, obviously, the formula would not apply to two-phase machines; it would give calculated results 50 per cent too high. This is just what had happened

during this investigation. By using Gray's formula modified by Fechheimer's "Short Pitch Factor", calculations checked fairly well on certain classes of three-phase machines, but not on two-phase. The factor of tooth-tip leakage being 100 per cent too high and the other factors being approximately right, made up for neglecting the increase of 50 per cent on the total, and naturally made two-phase calculations too high. By using the proper value of tooth-tip leakage, and allowing for the polyphase action, the equation applies equally well to three-phase, two-phase and single-phase machines.

There is, however, one term of the reactance equation which is not affected by the mutual induction of phases, namely, the slot leakage. Fig. 6B shows the three different phases of a two-slot per pole per phase machine, and the slot leakage which exists when the current is a maximum in the middle group, one-half maximum in the other two. Fig. 6A is equivalent to Fig. 6B, and it is clear that the interlinkages of the middle group have not been increased by the m.m.f. of the other two groups. The same reasoning can be applied to other instants, when the currents have different values, to show that there is no mutual induction. This operates to reduce the ratio of three-phase to single-phase (terminal to neutral) reactance to a value slightly lower than 1.5.

It is not obvious at first that this overlapping of phases occurs in the case of tooth-tip leakage. Consider, however, that the elimination of the third harmonic in this term, in effect produces a path of uniform reluctance (similar to a path of air only having lower reluctance) for the tooth-tip leakage. In other words, the case is similar to that of the end winding whose leakage path is air, or to that of the induction motor rotating main field. And, as in the two latter cases, the polyphase field is greater than that produced by one phase alone by about 50 per cent, as shown in Fig. 5A.

Turning to the question of applying the reactance in the calculation of field excitation required under load, experience has demonstrated that the use of the zero power factor value of reactance (that is, the value determined from open-circuit and short-circuit test) gives reasonably reliable calculation of field current for loads of any power factor. Why this could be true when the reactance at 80 per cent power factor is, by reason of the different pole position when the current is a maximum, about 60 per cent higher than that at zero power factor, was a perplex-

ing question. It is answered by the appearance of the in-phase component of reactive voltage in the tooth-tip factor. The rI drop is in phase opposition to the current; and the voltage consumed by it is in phase with the current. The $x_1 I$ drop, produced by leakage interlinkages in phase with the current, is 90 deg. behind the current; and the voltage consumed by it is 90 deg. ahead of the current. The $x_2 I$ drop produced by leakage interlinkages 90 deg. ahead of the current, as shown in Fig. 3c, is in phase with the current; and the voltage which it consumes is in phase opposition to the current. Fig. 7 shows these relations for approximately 80 per cent power factor, and makes clear why the internal voltage E_1', determined by the use of the zero power factor reactance, is practically the same as E_1 which is actually required by the much higher reactance, corresponding to 80 per cent power factor, the effect of the latter being mitigated by the in-phase component $x_2 I$. There is a difference in phase between E_1' and E_1 which, by using the zero power factor value of reactance, will give a calculated excitation slightly too low, but for practical purposes the difference is considered negligible.

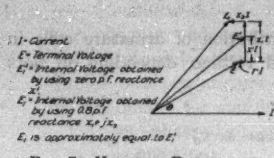


FIG. 7—VOLTAGE DIAGRAM AT 80 PERCENT POWER FACTOR

In the interest of simplicity and convenience in calculation, therefore, it is proposed that for the present at least, the zero power factor value of reactance be used. The method of calculating field excitation which is given in this paper follows that assumption. In the future it may become desirable to make correction for the distortion angle and power factor; hence the problem remains open to this further refinement.

CALCULATION OF ARMATURE REACTANCE

A = effective armature reaction in ampere turns per pole

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

C = air-gap coefficient based on Carter's fringing coefficient.

c = number of circuits in multiple in armature winding.

C_e = reduction factor for Φ_e at zero power factor, depending on ratio of pole arc to pole pitch.

C_i = reduction factor for Φ_i at zero power factor, depending on ratio of pole arc to pole pitch.

C_1 = 1.5 C_i for three-phase
= C_i for two-phase.
C_2 = 1.5 C_a for three-phase
= C_a for two-phase.
d_1 = distance from top of copper to bottom of copper in the armature winding Fig. 8.
d_2 = distance from top of copper to armature face Fig. 8.
e = voltage per phase
f = frequency in cycles per second.
g = average air gap.
I_n = normal current per phase in amperes
i = instantaneous value of current in conductor
K = reduction factor for Φ_s Φ_i and Φ_a depending on pitch of armature coils.
K_d = factor for effect of distribution of armature coils in calculation of induced voltage, depending on slots per pole and phase.

TABLE II—VALUES OF K_d

s		1	2	3	4	5	6	7	8up
K_d	Three-phase	1.0	1.035	1.043	1.044	1.045	1.046	1.047	1.047
K_d	Two-phase	1.0	1.083	1.098	1.103	1.106	1.108	1.109	1.11

K_i = factor for Φ_i depending on slots per pole and phase.
K_p = factor for effect of short pitch of armature coils in calculation of induced voltage.

$$K_p = \frac{1}{\text{Sin } (90^\circ \times p)}$$

where p is pitch of coil in per cent of full pole pitch.

K_ϕ = factor for effect of form of flux wave shape in calculation of induced voltage. Values of K_ϕ are given in Fig. 20.
L_e = length of one phase belt of end connections.
l = gross stacked length of armature core (iron + ducts)
N_s = total number of stator coils
n = number of phases.
p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)
q = number of poles
s = slots per pole and phase.
w_s = width of slot

w_t = width of tooth at armature face.
For w_s and w_t see Fig. 8.
x_a = reactance in ohms
x_{pa} = armature reactance in per cent.
α = ratio of pole arc to pole pitch
τ = pole pitch at armature core surface.
Φ = flux per pole in maxwells.
Φ_a = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is between the poles.
ϕ_a = effective instantaneous value of Φ_a.
Φ_e = effective end connection interlinkages per phase belt per ampere conductor for unit length of phase belt of end connections.

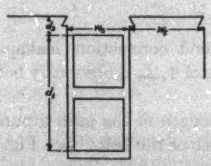


FIG. 8—SLOT DIMENSIONS

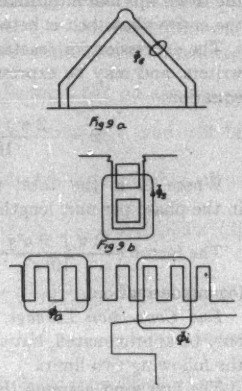


FIG. 9C
ARMATURE LEAKAGE FLUX DIAGRAMS

Φ_i = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is under a pole.
ϕ_i = effective instantaneous value of Φ_i.
Φ_s = effective slot interlinkages per phase belt per ampere conductor for unit length of armature.
ψ = electrical angle by which the armature coil span is less than 180 deg.

Theory. The armature leakage flux in an alternating-current generator may be divided into three factors.

(1) End connection leakage flux, represented by $\Phi_e L_e$, the effective flux per ampere conductor interlinking one phase belt of end connections. (Fig. 9 A).

(2) Slot leakage flux represented by Φ_s, the effective flux per ampere conductor through the slot interlinking unit length of one phase belt. (Fig. 9 B).

(3) Tooth-tip leakage flux represented by

(a) Φ_t, the effective flux per ampere conductor, crossing the air gap into the pole face and returning across the gap and interlinking unit length of one phase belt, when the entire phase belt is under the pole. (Fig. 9C.)

(b) Φ_n, the effective flux per ampere conductor emerging from the tooth tips and interlinking unit length of one phase belt when the entire phase belt is between the poles. (Fig. 9c.)

The expression for reactance has been derived by a number of writers, and may be expressed in our notation by the following equation:

$$x = \frac{2\,\pi\,f\,s^2\,a^2\,q\,l}{10^8\,c^2}\,L_0 \qquad (1)$$

Where L_0 is the total interlinkages per ampere conductor in the phase per unit length of the armature[4].

The factor $\frac{2\,\pi\,f\,s^2\,a^2 q\,l}{10^8\,c^2}$ will be replaced by M in the following derivation.

End Connection Leakage. The end connection leakage can only be approximated, but the value of $\Phi_e\,L_e$ is obviously between the following two limits.

The first limit assumes that the length of the path around the phase belt varies directly as the width of the belt. See Fig. 9A.

Then

$$\Phi_e = K_1\,\frac{n}{\tau}$$

where K_1 is a constant.

The second limit assumes that the length of the path is independent of the width of the belt.

Then Φ_e would be constant.

Obviously Φ_e can not change directly as $\frac{n}{\tau}$ because the depth of coil is a factor of the length of path; on the other hand, the length of path *must* increase some as τ increases, that is, it can not be independent of τ. Hence the variation of Φ_e must be between the first power of $\frac{n}{\tau}$ and the zero power of $\frac{n}{\tau}$. The

4. Electrical Machine Design, Gray Edition 1913, p. 21.

analysis of tests for entire armature reactance on over 100 machines, as well as exploring coil tests for end leakage reactance on a 125-kv-a. generator indicates that Φ_e varies approximately as $\sqrt{\frac{n}{\tau}}$ that is

$$\Phi_e = K_2 \sqrt{\frac{n}{\tau}}$$

The length of the phase belt $L_e = p\,\tau\,K_3$.

Then $$\Phi_e\,L_e = K_4 \sqrt{n}\,p\,\sqrt{\tau}$$

For a three-phase double-layer winding, and dimensions in centimeters, $K_4\sqrt{n} = 2.8$ was found to give satisfactory results.

The factor for a two-phase two-layer winding will be reduced in the ratio of $\frac{\sqrt{2}}{\sqrt{3}}$ and also by the factor 0.667 on account of zero mutual induction between phases in this case, and will be 1.6 in round numbers.

The end connection reactance for a double-layer winding will be

$$x = \frac{2.8\,M\,p\,\sqrt{\tau}}{l} \qquad \text{Three-phase} \quad (2)$$

$$= \frac{1.6\,M\,p\,\sqrt{\tau}}{l} \qquad \text{Two-phase} \quad (3)$$

For a single-layer winding the value of $\Phi_e\,L_e$ will be doubled as proposed by Gray.

Slot Leakage. Three-phase coil pitch between 66.66 and 100 per cent. The expression for slot reactance with short-pitch armature winding was derived by Fechheimer[5], and is used with a correction in the number of A and B coils, which is $\frac{N_s\,\psi}{180}$ instead of $\frac{N_s\,\psi}{2 \times 180}$ as given in his paper.

Omitting the factors for the top of the partially closed slot and modifying for our notation, the expression for inductance of open slots becomes

$$L = 0.4\,\pi\,a^2 \left[\left(\frac{19}{96}\frac{d_1}{w_s} + \frac{3}{8}\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \left(\frac{7}{96}\frac{d_1}{w_s} + \frac{3}{8}\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \frac{N_s}{3}\left(1 - \frac{\psi}{60}\right)\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right)\right] \qquad (4)$$

5. Fechheimer "Synchronous Motors" A. I. E. E., Vol. XXXI, 1912, p. 580.

Simplifying

$$L = 0.4\,\pi\,a^2\,N_s\left[\left(1-\frac{\psi}{320}\right)\frac{d_1}{3\,w_s}+\left(1-\frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \tag{5}$$

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1-\frac{\psi}{320}\right)\frac{d_1}{3\,w_s}+\left(1-\frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \tag{6}$$

The derivation of this expression in the above paper omits the allowance for the out of phase component of the reactive voltage induced in one phase by the half coils of another phase in the same slot. The final expression however is correct as it can readily be seen by reference to Fig. 10, the reactance component of E_2, the voltage induced in phase one by phase two,

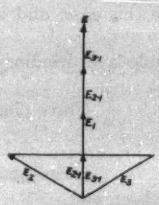


FIG. 10—MUTUAL INDUCTION IN SHORT-PITCH ARMATURE WINDINGS

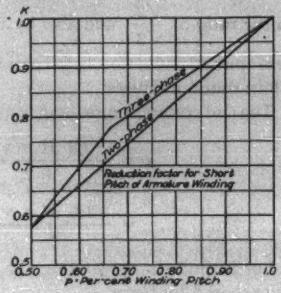


FIG. 11

will be exactly balanced by the reactive component of E_3 the voltage induced in phase one by phase three.

For simplicity and with very little decrease in accuracy, we may replace the two short-pitch factors in equation (6) by an average value K from Fig. 11.

Then $$x = M\,\frac{0.4\,\pi}{s}\,K\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right) \tag{7}$$

Pitch below $66\frac{2}{3}$ per cent.

Following Fechheimer's method with the correction for number of half coils, the expression for reactance where the pitch is less than $66\frac{2}{3}$ per cent will be

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1.187-\frac{\psi}{160}\right)\frac{d_1}{3\,w_s}+\left(1.25-\frac{\psi}{120}\right)\frac{d_2}{w_s}\right] \tag{8}$$

Replacing the two factors by an average value K from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{9}$$

Two phase.

Pitch between 50 and 100 per cent.

The expression for two-phase slot reactance with short pitch derived by a similar process is

$$x = M \frac{0.4\,\pi}{s} \left[\left(1 - \frac{\psi}{240}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{180}\right)\frac{d_2}{w_s}\right] \tag{10}$$

Using an average value of reduction factor from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{11}$$

Tooth-Tip Leakage. The tooth-tip leakage may be most conveniently calculated by dividing it into two components; the effective leakage flux around the part of the phase belt opposite the iron of the pole face (maximum value Φ_i); and the effective leakage flux around the part of the phase belt between the poles (maximum value Φ_o).

The values of Φ_i and Φ_o for a complete phase belt were determined by Gray.[6] His method has been followed for Φ_i except in determining the area of the path across the air gap. The leakage flux will fringe from the teeth in almost exactly the same way as does the no-load flux, hence the area of the path across the air gap is the length of armature times the tooth pitch divided by the air gap coefficient, Fig. 12B.

For one and two slots per pole and phase,

$$\Phi_i = 0.4\,\pi \frac{w_s + w_i}{2\,g\,C} \tag{12}$$

Following Gray's derivation, it will be found that for higher numbers of slots per pole and phase

$$\Phi_i = 0.4\,\pi\,K_i \left(\frac{w_s + w_i}{2\,g\,C}\right) \tag{13}$$

A curve for K_i plotted against slots per pole and phase is given in Fig. 12A.

Gray's expression for Φ_o has been adopted without change,

6. Electrical Machine Design, Gray Edition, 1913, p. 220.

and simplified by assuming $w_s = w_t$, which gives a definite value of Φ_a for any given value of slots per pole and phase.

Φ_a plotted against slot per pole and phase is shown in Fig. 12.

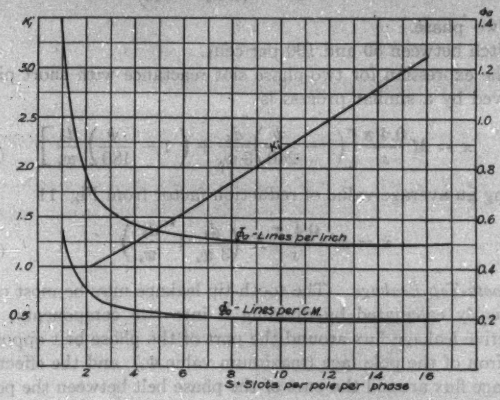


FIG. 12A

Modification of Φ_t and Φ_a by Definite Pole Construction. It should be noted that Φ_t, would be the leakage flux per ampere conductor in the phase belt, if the phase belt were opposite the

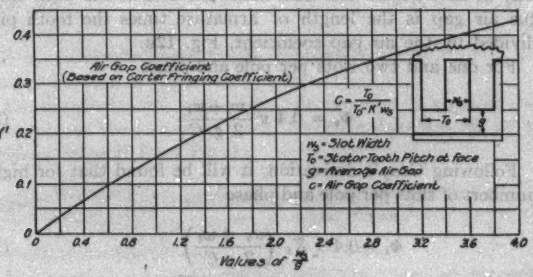


FIG. 12B

pole face iron throughout the cycle. Also Φ_a would be this leakage flux if there were no iron in the tooth-tip leakage path. The actual condition is a path partly iron and partly air and varying

in proportion of air and iron during the cycle. This condition may be approximated by calculating the interlinkages in the iron part of the path, and also the interlinkages in the air part of the path. The ratio of the former to Φ_i, and of the latter to Φ_a will be reduction factors to be applied to Φ_i and Φ_a in calculating the reactances due to tooth-tip leakage. The actual tooth-tip leakage will be obtained by adding the two parts as calculated in this way.

The general theory of the variation of Φ_i and Φ_a at zero power factor has been discussed in the first part of this paper, and it will now be applied to the actual calculation.

The instantaneous values of leakage flux $\phi_i\, i$ vary from zero when the current is zero (Fig. 13A position *a*) to a maximum approximately where the pole begins to leave the phase belt (Fig. 13A position *b*) and then decreases to zero where the phase

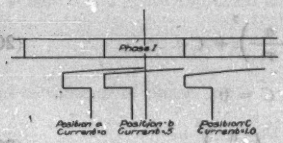


FIG. 13A—DIAGRAM OF POLE POSITIONS

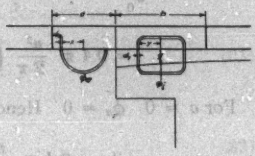


FIG. 13B—TOOTH-TIP LEAKAGE

belt is entirely between the poles. (Fig. 13A position *c*). The instantaneous value of leakage flux $\phi_a\, i$ is zero when the current is zero, and remains zero until the pole begins to leave the phase belt, after which it increases gradually to a maximum where the current is a maximum at 90 deg. Obviously the curves for $\phi_i\, i$ and $\phi_a\, i$ are symmetrical about the 90 deg. axis. Fig. 14.

The values of $\phi_i\, i$ and $\phi_a\, i$ for complete distribution of the armature winding were calculated by the following methods.

Tooth-tip leakage $\phi_a\, i$

For one ampere in the phase belt and unit length of phase belt the m. m. f. at x (See Fig. 13B) is

$$\text{m. m. f.} = \frac{2\,x}{\dfrac{\tau}{n}} = \frac{2\,n\,x}{\tau} \tag{14}$$

where n = number of phases

and τ = pole pitch

$$d\,\phi = 0.4\,\pi\,\frac{2\,n\,x}{\tau}\;\frac{d\,x}{\pi\,x} \tag{15}$$

$$= 0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{16}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_a$

Then
$$d\,\phi_a = \frac{2\,x}{\frac{\tau}{n}}\,d\,\phi \tag{17}$$

$$= \frac{2\,n\,x}{\tau}\,0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{18}$$

$$\phi_a = \int_0^{a/2} d\,\phi_a = 0.4\,\pi \times \frac{2\,n^2}{\pi}\;\frac{x}{\tau^2} + C\Big]_0^{a/2} \tag{19}$$

$$= 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 + C \tag{20}$$

For $a = 0$ $\phi_a = 0$ Hence $C = 0$

$$\phi_a = 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{21}$$

for one centimeter of phase belt.

$$\phi_a = 3.2\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{22}$$

for one inch of phase belt.

For the special case of three-phase and where a is expressed as a per cent. of τ.

$$\phi_a = 4.6\,a^2 \text{ for one inch of phase belt.} \tag{23}$$

Tooth-tip Leakage $\phi_i\,i$

Referring to Fig. 13B

$$\text{m. m. f.} = \frac{2\,y\,n}{\tau} \tag{24}$$

Let g_b = average air gap over b.

$$d_\phi = 0.4\,\pi\,\frac{2\,y\,n}{\tau}\;\frac{d\,x}{2\,g_b} \tag{25}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_i$

$$d\,\phi_i = \frac{2\,y\,n}{\tau}\,d\,\phi \tag{26}$$

$$= \frac{0.4\,\pi\,2\,n^2\,y^2}{g_b\,\tau^2}\,d\,y \tag{27}$$

$$\phi_i = 0.4\,\pi\,\frac{2\,n^2}{g_b\,\tau^2}\int_0^{b/2} y^2\,d\,y \tag{28}$$

$$= 0.4\,\pi\,\frac{2\,n^2}{3\,g_b\,\tau^2}\,y^3 + c\Big]_0^{b/2} \tag{29}$$

At $y = 0 \quad \phi_i = 0$ hence $c = 0$

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b\,\tau^2} \tag{30}$$

Where b and g_b are expressed as a per cent. of τ

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b} \tag{31}$$

per centimeter of phase belt.

$$= 3.2\,\frac{n^2\,b^3}{12\,g_b} \tag{32}$$

per inch of phase belt.

For three-phase and one-inch length of phase belt.

$$\phi_i = 2.4\,\frac{b^3}{g_b} \tag{33}$$

The area of leakage path across the air gap in actual machines will be reduced due to the effect of armature slots and this will be allowed for by the introduction of the air gap coefficient, C, given in Fig. 12B.

$$\phi_i = 2.4\,\frac{b^3}{C\,g_b} \tag{34}$$

The derivation of the actual values of $\phi_i\,i$ and $\phi_a\,i$ will be illustrated by an example for three-phase where α is $66\frac{2}{3}$ per cent.; the minimum gap is 1.5 per cent of the pole pitch; and the ratio of maximum to minimum gap is 2 to 1, giving an average gap of approximately 2 per cent of the pole pitch. Average values of air gap coefficient will be assumed. The armature winding will be assumed to be completely distributed.

Table III gives the calculation of $\phi_i\, i$ and $\phi_a\, i$ as the current varies through 90 deg. calculated from the formulas.

$$\phi_i = 2.4\,\frac{b^3}{C\,g_b} \qquad \text{from } (33)$$

$$\phi_a = 4.6\,a^2 \qquad \text{from } (23)$$

TABLE III.

θ	b	g_b	C	b^3	Cg_b	ϕ_i	a	a^2	ϕ_a	i	$\phi_i\, i$	$\phi_a\, i$
0	0.333	0.0165	1.09	0.036	0.018	4.76	0	0	0	0	0	0
15	0.333	0.0177	1.09	0.036	0.0193	4.43	0	0	0	0.26	1.15	0
30	0.333	0.0188	1.08	0.036	0.204	4.2	0	0	0	0.50	2.1	0
45	0.25	0.0196	1.08	0.0156	0.0212	1.75	0.083	0.007	0.032	0.71	1.24	0.023
60	0.167	0.020	1.08	0.0047	0.0216	0.52	0.167	0.028	0.128	0.87	0.45	0.111
75	0.083	0.023	1.07	0.006	0.0246	0.06	0.250	0.063	0.29	0.97	0.06	0.28
90	0	0.027	1.07	0	..	0	0.333	0.110	0.505	1.0	..	0.505

Dimensions are expressed as per cent of pole pitch.

Average gap = 0.02

$$\Phi_i = 2.4\,\frac{0.333^3}{1.08 \times 0.02} = 4.08$$

$$\Phi_a = 4.6 \times 0.333^2 = 0.505$$

Fig. 14 shows the above values of $\phi_i i$ and $\phi_a i$, and the total tooth-tip leakage over 180 deg. These waves were analyzed for harmonics and the equation is given in the figure. The fundamentals only are plotted.

It will be noted that the third is the most important harmonic, and since the third harmonics cancel in the terminal voltages of star connected three-phase machines, the effect of only the fundamental needs to be considered. It is obvious from the equation in Fig. 14 that the higher harmonics may be neglected without introducing any appreciable error.

C_i is the ratio of the maximum value of the fundamental of $\phi_i\, i$ Fig. 14 to the value of Φ_i that is,

$$C_i = \frac{0.89}{4.08} = 0.218$$

Similarly

$$C_a = \frac{0.22}{0.505} = 0.435$$

In the derivation of C_i and C_a a sine wave of armature current has been assumed. This is very nearly the case with a star-

connected machine, and, as has been explained, in a Δ-connection the short-circuit current as read on an ammeter is practically equal to that with star-connection for the same field current. The values of C_i and C_a as derived for star connection may therefore be used for delta connection, even though the current is not a sine wave in the latter case.

The values of C_i and C_a were calculated for complete distribution of the armature winding with various ratios of pole arc to

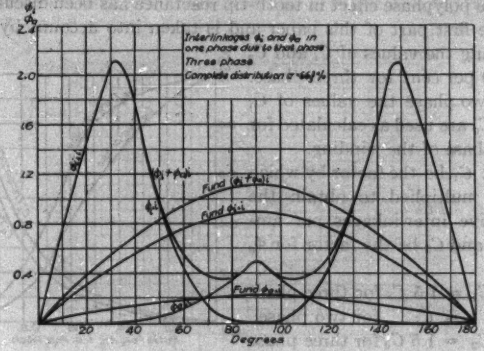


FIG. 14

$$\phi_i = .89 \sin \theta + 1.22 \sin 3\theta + .29 \sin 5\theta - .15 \sin 7\theta - .18 \sin 9\theta - .06 \sin 11\theta$$

$$\phi_a = .22 \sin \theta - .13 \sin 3\theta + .06 \sin 5\theta - .04 \sin 7\theta + .04 \sin 9\theta - .01 \sin 11\theta$$

$$\phi_i + \phi_a = 1.11 \sin \theta + 1.09 \sin 3\theta + .35 \sin 5\theta - .19 \sin 7\theta - .14 \sin 9\theta - .07 \sin 11\theta$$

pole pitch for three phase and two phase by the method shown in Table III, and are given in Fig. 15.

The variation of C_i and C_a with distribution of the armature winding was investigated and it was found the results obtained by the application of these factors C_i and C_a to Φ_i and Φ_a would give practically correct results when applied to the case of one slot per pole and phase. It is evident that there is just one leakage path around the coil in the case of one slot per pole and phase, and this path will be partly through the pole face iron and partly through air, but the length of path in each will vary as the pole moves along. The total interlinkages were calculated over 90 deg. for this condition and the waves analyzed. It was found

that the ratio of the fundamental of the total interlinkage wave in this case to the value of $\Phi_i + \Phi_a$ was practically the same as the ratio of $C_i \Phi_i + C_a \Phi_a$ to $\Phi_i + \Phi_a$, that is the values of C_i and C_a derived for complete distribution apply also to one slot per pole and phase.

It is very important to note that while the values of C_i and C_a do not change with distribution, Φ_i and Φ_a do show a large variation.

The polyphase effect in tooth-tip reactance has been discussed in the first part of this paper and is taken into account by increasing the values of C_i and C_a by 50 per cent for three phase. For two phase the values of C_i and C_a are used as calculated for one phase of the winding.

Let C_1 be the factor by which Φ_i is multiplied to obtain the effective interlinkages per ampere, and C_2 be the factor for Φ_a. Then

$$C_1 = 1.5\ C_i \text{ for three phase}$$
$$= C_i \quad \text{for two phase}$$
$$C_2 = 1.5\ C_a \text{ for three phase}$$
$$= C_a \text{ for two phase}$$

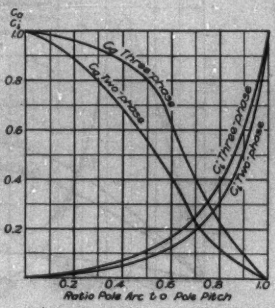


FIG. 15—VALUES OF C_i AND C_a

It will be noted that the values of C_i and C_a are not the same for two phase as for three phase.

The factors C_1 and C_2 for both three phase and two phase are plotted against ratio of pole arc to pole pitch in Fig. 16.

The reduction factor for tooth-tip reactance accounting for short pitch coils may be taken as the average factor K from Fig. 11.

Reactance Formulas. The various factors of the reactance formula may be combined to give the following equation for a double-layer armature winding.

For dimension in centimeters.

Three Phase.

$$x_a = \frac{2\pi f\, s^2 a^2\, q\, l}{10^8\, c^2} \left\{ \overbrace{\frac{2.8\, p\sqrt{\tau}}{l}}^{\text{End}} + \right.$$

$$\left. K \left[\overbrace{\frac{0.4\pi}{s}\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right)}^{\text{Slot}} + \overbrace{0.4\pi\, C_1 K_i \frac{w_s + w_t}{2gC} + C_2 \Phi_a}^{\text{Tooth Tip}} \right] \right\} \quad (35)$$

Two Phase.

$$x_a = \frac{2\ \pi f\ s^2\ a^2\ q\ l}{10^2\ c^2} \left\{ \overset{\text{End}}{\frac{1.6\ p\ \sqrt{\tau}}{l}} + \right.$$

$$\left. \overset{\text{Slot}}{K\left[\frac{0.4\ \pi}{s}\left(\frac{d_1}{3\ w_s}+\frac{d_2}{w_s}\right)+0.4\ \pi\ C_1\ K_i\ \frac{w_s+w_t}{2\ g\ C}\right.} \overset{\text{Tooth Tip}}{+C_2\Phi_a\Big]}\right\} \quad (36)$$

For single layer armature winding, multiply end connection factor by 2.0 and use $K=1.0$.

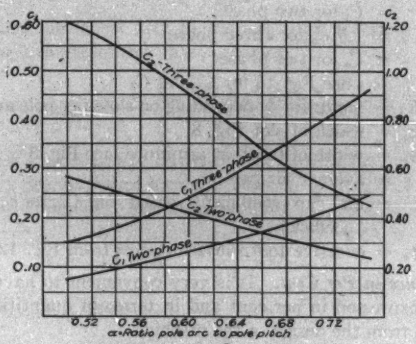


FIG. 16—VALUES OF C_1 AND C_2

REDUCTION FACTORS FOR TOOTH-TIP LEAKAGE

$C_1 = 1.5\ C_i$ for three phase

$= C_i$ for two phase.

$C_2 = 1.5\ C_\alpha$ for three phase

$= C_\alpha$ for two phase.

For dimensions in inches.

Substitute 4.5 for 2.8 in factor for three-phase end leakage; 2.5 for 1.6 in case of two-phase: and 3.2 for 0.4π in slot leakage and first factor of tooth-tip leakage. Use Φ_a in lines per inch.

Where x_a = armature reactance per phase in ohms.

f = frequency in cycles per second

s = slots per pole and phase

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

q = number of poles

l = gross stacked length of armature core (iron + ducts)

c = number of circuits in multiple in armature winding
p = per cent pitch of armature winding (=0.8 for 80 per cent pitch)
τ = pole pitch at armature core face
K = reduction factor depending on pitch of armature coils
d_1 = distance from top of copper to bottom of copper in the armature winding. Fig. 8
d_2 = distance from top of copper to armature face Fig. 8
C_1 = 1.5 C_i for three phase
= C_i for two phase
C_2 = 1.5 C_a for three phase
= C_a for two phase
See Fig. 16 for C_1 and C_2
K_i = factor for Φ_i depending on slots per pole and phase
w_s = width of slot Fig. 8
w_t = width of tooth at armature face Fig. 8
g = average air gap
C = air gap coefficient based on Carter's fringing coefficient
Φ_a = effective tooth interlinkages from Fig. 12

Reactance in Per Cent. It is very convenient to have the reactance expressed in per cent and in terms of quantities easily available from the design of the machine.

Three Phase

Per cent reactance is given by $x_{pa} = \dfrac{I_n x_a}{e}$ (37)

(x_{pa} = 0.20 means 20 per cent reactance)

Where I_n = normal current per phase
x_a = reactance per phase in ohms
e = voltage per phase

$$\text{Armature reaction } A = \frac{2.12\ s\ a\ I_n}{2\ K_p\ K_d\ K_\phi\ c} \tag{38}$$

$$\text{Flux per pole } \Phi = \frac{e \times 10^8\ K_p\ K_d\ K_\phi\ c}{2.22\ s\ a\ q\ f} \tag{39}$$

Let L_0 = bracket expression from reactance formulas (35) or (36)

$$x_a = \frac{2\ \pi\ f\ \ s^2\ a^2\ q}{10^8\ c^2}\ L_0 \tag{40}$$

From (38)

$$I = \frac{2\ K_p\ K_d\ K_\phi\ A}{2.12\ s^1\ a} \tag{41}$$

and from (39)

$$e = \frac{2.22\ s\ a\ q\ f}{10^8\ K_p\ K_d\ K_\phi\ c}\ \Phi \tag{42}$$

$$x_{pa} = \frac{I\ x_a}{e} = \frac{2.67\ K_p^2\ K_d^2\ K_\phi^2 A\ l}{\Phi}\ L_0 \tag{43}$$

Two Phase

Replacing the factor 2.12 in the armature reaction formula by 1.41 the formula for two phase becomes

$$x_{pa} = \frac{4.\ K_p^2\ K_d^2\ K_\phi^2\ A\ l}{\Phi}\ L_0 \tag{44}$$

Where K_d, and L_0 are based on the two-phase curves and formulas. See reactance formula (36).

Working Formulas. The expression for

$$\Phi_i = 0.4\ \pi\ K_i\ \frac{w_s + w_i}{2\ g\ C}$$

for dimensions in centimeters and

$$3.2\ K_i\ \frac{w_s + w_i}{2\ g\ C}$$

for dimensions in inches may be simplified by assuming $w_s = w_i$ and then a series of curves for Φ_i plotted against the values of $\frac{w_s}{g}$ may be derived for different values of slots per pole and phase. The curves are given for dimensions in inches in Fig. 17. The working formulas for reactance of double layer windings with lengths expressed in centimeters are as follows:

Three Phase

$$x_{pa} = \frac{2.67\ (K_p\ K_d\ K_\phi)^2\ A\ l}{\Phi}\ \left\{ \overset{\text{End}}{\frac{2.8\ p\ \sqrt{\tau}}{l}} + \right.$$

$$\left. \overset{\text{Slot}}{K\left[\frac{0.4\pi}{s}\left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s}\right)\right.} + \overset{\text{Tooth Tip}}{\left.\left. 0.39\ (C_1\ \Phi_i + C_2\ \Phi_s)\right]\right\}} \tag{45}$$

Two Phase

$$x_{pa} = \frac{4.\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{1.6\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{0.4\pi}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{0.39\,(C_1\,\Phi_i + C_2\,\phi_a)}\right]\right\} \qquad (46)$$

Note that factor 0.39 is introduced in the tooth-tip leakage because Φ_i and Φ_a in the curves are expressed in leakage lines per inch length of core.

For dimensions in inches.

Three Phase.

$$x_{pa} = \frac{2.67\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{4.5\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \qquad (47)$$

Two Phase

$$x_{pa} = \frac{4.0\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{2.5\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \qquad (48)$$

See Fig. 11 for K, Fig. 17 for Φ_i, Fig. 12 for ϕ_a and Fig. 16 for C_1 and C_2. ($x_{pa}=0.2$ means 20 per cent reactance.)

Reactance Curves. The formula may be still further simplified by assuming average values of d_1, d_2 and w_s for a certain narrow range of slot pitches. A series of five curves covering the ordinary ranges of slot pitches, and giving values of end, slot and tooth-tip factors as a function of the slots per pole and phase are given in Figs. 20A, 20B, etc. The curves will give the accuracy necessary for most calculations of reactances and make the work quite simple.

Reactance of Non-Salient Pole Generators. The reactance of turbine generators, the most common type of non-salient pole machines, was calculated by assuming the ratio of pole arc to

pole pitch to be the ratio of the unslotted pole centre to pole pitch. Since practically all machines of this type now have five or more slots per pole and phase, the expression for Φ_i was simplified by substituting in equations (**35**) and (**36**) the value of K_i expressed as a function of the slots per pole and phase.

That is $$K_i = 0.625 + 0.156\, s \text{ (above } s=4) \tag{49}$$

With a ratio of pole arc to pole pitch of 33 per cent, an approximate average for turbine generators, the value of C_1 is 0.075 for three phase and 0.035 for two phase, and C_2 is 1.32 for three phase and 0.60 for two phase.

Three Phase

$$C_1\, \Phi_i = 0.075\, (0.625 + 0.156\, s)\; 0.4\, \pi\, \frac{w_s + w_t}{2\, g\, C} \tag{50}$$

in lines per centimeter.

$$C_1\, \Phi_i = 0.075\, (0.625 + 0.156\, s)\; 3.2\, \frac{w_s + w_t}{2\, g\, C} \tag{51}$$

in lines per ampere cond'r per inch.

Allowing the approximation $w_s = w_t$ and simplifying we have

$$C_1\, \Phi_i = (0.059 + 0.015\, s)\, \frac{w_s}{g\, C} \tag{52}$$

in lines per centimeter.

$$C_1\, \Phi_i = (0.15 + 0.038\, s)\, \frac{w_s}{g\, C} \tag{53}$$

in lines per inch.

$$C_2\, \phi_a = 1.32 \times 0.198 = 0.26 \text{ line per centimeter}$$
$$= 1.32 \times 0.50 = 0.66 \text{ line per inch.}$$

Two Phase

Similarly

$$C_1\, \Phi_i = (0.0275 + 0.007\, s)\, \frac{w_s}{g\, C} \tag{54}$$

lines per centimeter.

$$C_1 \Phi_i = (0.07 + 0.018\, s)\ \frac{w_s}{g\, C} \qquad (55)$$

lines per inch.

$C_2 \Phi_e = 0.12$ line per centimeter.

$= 0.3$ line per inch

Substituting these values in equations (**45**), (**46**), (**47**), and (**48**) will give the formulas for turbine generator reactances.

Application of Formula. The use of the reactance formulas will be illustrated by the application of equation (**47**) to a 6600-kv-a. three-phase 25-cycle 14,000-volt generator.

Example:

$s = 5.25 \quad p = 82.5\% = 0.825 \quad A = 11{,}000 \quad g = 0.925$

$\tau = 36.5 \quad \alpha = 0.698 \qquad w_s = 1.125$

$d_1 = 3.2 \quad d_2 = 1.05 \qquad \Phi = 47.8 \times 10^6$

$K_p = 1.04 \quad K_d = 1.046 \qquad K_\phi = 1.016$

$$\frac{2.67\,(1.04 \times 1.046 \times 1.016)^2\ 11{,}000 \times 38}{47.8 \times 10^6} = 0.0283$$

$$\frac{4.5\, p\, \sqrt{\tau}}{l} = \frac{4.5 \times 0.825\ \sqrt{36.5}}{38} = 0.59$$

$$\frac{3.2}{s}\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right) = \frac{3.2}{5.25}\left(\frac{3.2}{3.375} + \frac{1.05}{1.125}\right) = 1.15$$

$$K = 0.885 \qquad K \times 1.15 = 1.02$$

$$\frac{w_s}{g} = \frac{1.125}{0.925} = 1.22 \qquad \Phi_i = 5.1 \qquad C_1 = 0.37$$

$$K\, \Phi_i\, C_1 = 5.1 \times 0.37 \times 0.885 = 1.67$$

$$\Phi_e = 0.54 \qquad C_2 = 0.57$$

$$K\, \Phi_e\, C_2 = 0.54 \times 0.57 \times 0.885 = 0.27$$

$$3.55$$

$$x_{pa} = 0.0283 \times 3.55 = 0.10$$

$$= 10 \text{ per cent}$$

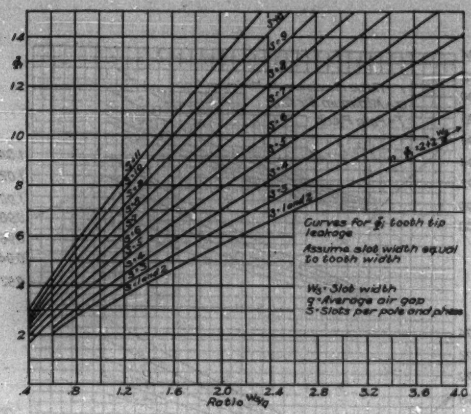


FIG. 17

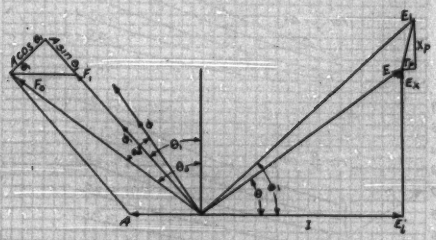


FIG. 18—EXCITATION DIAGRAM

FIG. 19A—M. M. F. DIAGRAM FOR OVER-EXCITED MACHINE

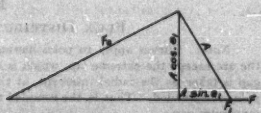


FIG. 19B—M. M. F. DIAGRAM FOR UNDER-EXCITED MACHINE

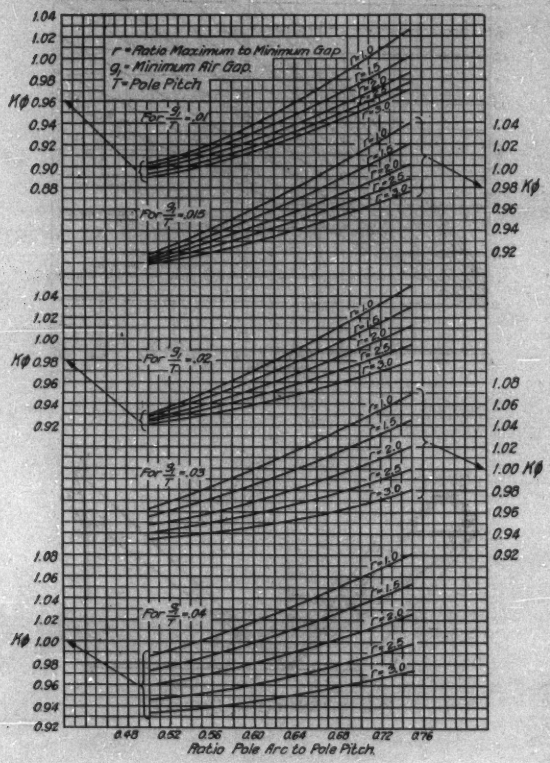


FIG. 20

FLUX DISTRIBUTION COEFFICIENT, K_ϕ

Note: Curves apply to poles having a constant radius of pole arc for all points of the arc except the extreme tip, which is rounded off by small radius. The maximum air gap is taken as the radial intercept, at the pole edge, between the armature face and the pole arc extended. That is, the rounded tip of small radius is neglected in determining the maximum gap.

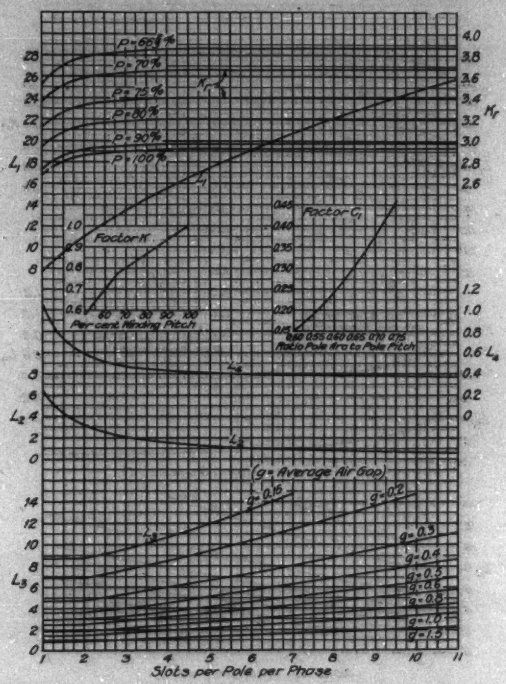


FIG. 20 A

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r K_\phi^3 A l}{\Phi}\left[\frac{p L_1}{l} + K (L_2 + C_1 L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 0.9 to 1.1. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use K = 1.0.

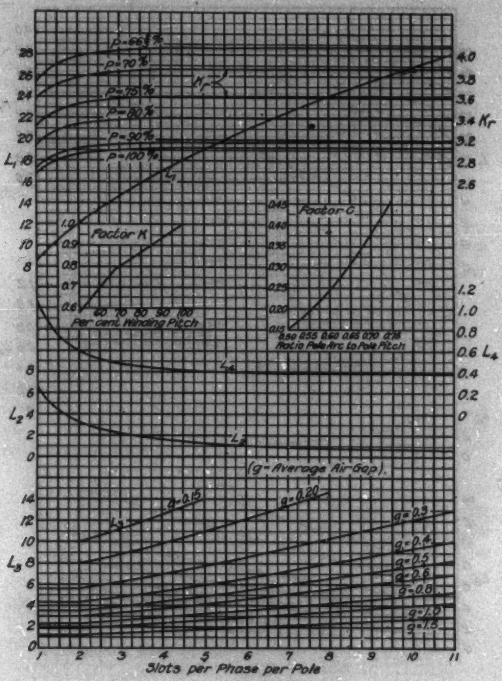


FIG. 20 B

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi{}^3\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.1 to 1.3. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use K = 1.0.

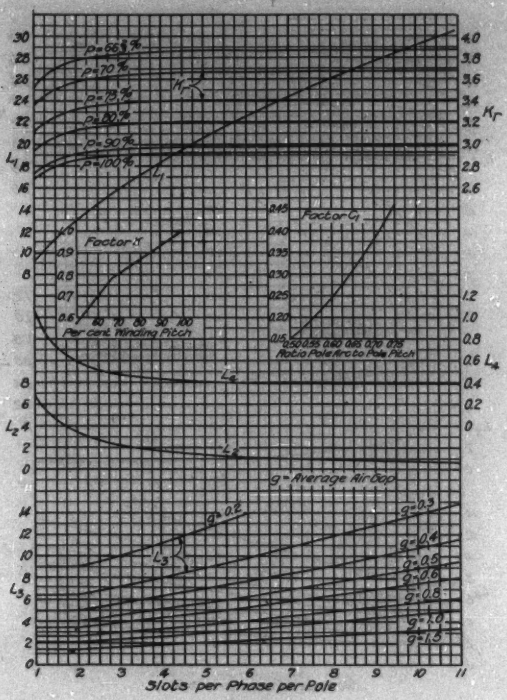


FIG. 20C

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r K_\phi^2 A l}{\Phi}\left[\frac{p L_1}{l} + K (L_2 + C_1 L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.3 to 1.5. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

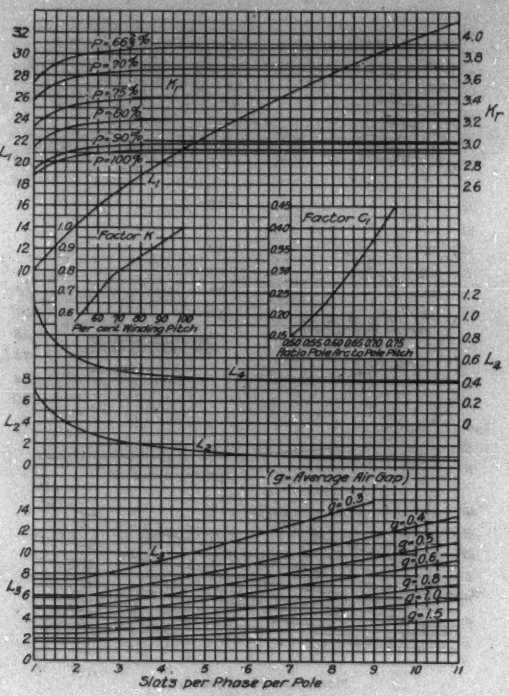


FIG. 20D

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r K \phi^3 A l}{\Phi}\left[\frac{p L_1}{l} + K (L_2 + C_1 L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.5 to 1.9. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

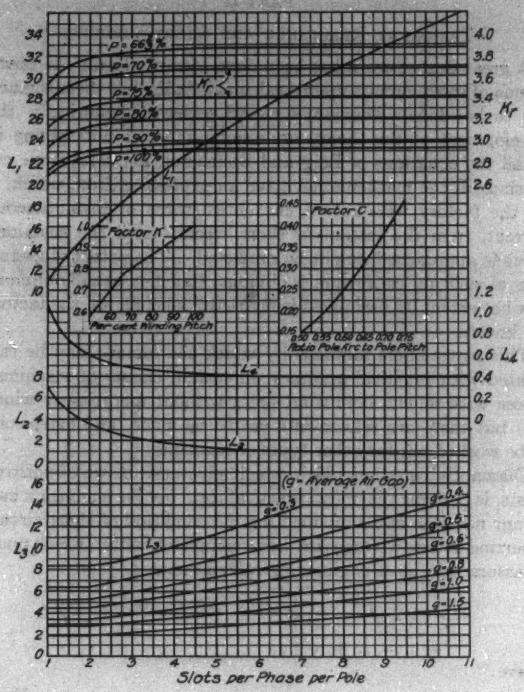


FIG. 20E

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\,T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80%)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.9 to 2.1. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

PART II

Transient Reactance and Short Circuits

A great deal of work has been done toward placing the phenomena of short circuits upon a calculable basis, but there is much yet to be done. Diamant's paper[7] reviewed historically the development of the theory underlying short circuits giving in detail the equations of Berg and Boucherot, and with some extensions in the mathematics, gave an expression from which, if all the factors were known, the entire wave of instantaneous current, from the instant of short circuit until the sustained value is approached, might be predicted. The potential usefulness of such an expression is obvious; but it is equally obvious to what extent its usefulness is restricted until all of the factors are known.

This paper offers methods by which the factors may be calculated with practical accuracy on certain classes of machines, (those with salient, laminated poles without amortisseur winding) but there are refinements yet to be made, and extensions to be worked out for other types of machines.

Diamant's equation **(26)**[7], which gives the curve of positive crests, is perhaps the most practical form for the engineer's use. In our notation and simplified for the condition of short circuit occurring at zero voltage (*i.e.* wave completely offset, which must be assumed in most engineering calculations) is,

$$i_c = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t} + \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t} + \sqrt{2}\,I_s$$
$$= i_1 + i_2 + \sqrt{2}\,I_s$$

where

I_0 = *alternating component* of short-circuit current $= \dfrac{E}{x_0}$

E = voltage per leg.

x_0 = total reactance effective on sudden short circuit. This should be called *transient reactance*, as suggested by Durgin and Whitehead[9] and by F. D. Newbury[10].

I_s = value of sustained short-circuit current.

α_f = field attenuation factor, defined later.

7. Trans. A. I. E. E., 1915, Vol. 34, p. 2237.

8. Term proposed by Hewlett, Mahoney and Burnham, Trans. A. I. E. E., 1918, Vol. XXXVII, p. 128.

9. Trans. A. I. E. E., Vol 31, 1912, p. 1662,

10. *Elec. Journal*, April 1914, p. 196.

α_a = armature attenuation factor, defined later.
t = seconds after short circuit.

For calculation, a convenient form is:

1st term, $$i_1 = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t}$$

$$\log_e \frac{\sqrt{2}\,(I_0 - I_s)}{i_1} = \alpha_f t$$

2d term, $$i_2 = \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t}$$

$$\log_e \frac{\sqrt{2}\,I_0}{i_2} = \alpha_a t$$

Fig. 21 shows the skeleton thus derived. Beyond the ordinate A, when i_2 is practically zero, the wave is symmetrical about

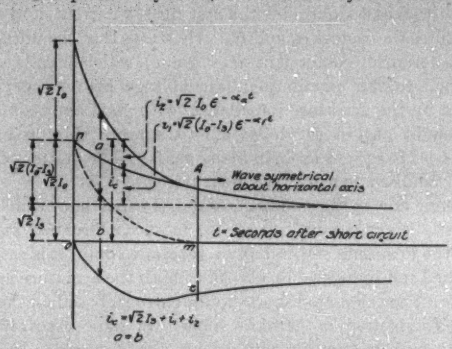


FIG. 21—SHORT-CIRCUIT ARMATURE CURRENT

the horizontal axis. The curve of negative crests between o and c can be obtained by plotting i_2 from the zero line as shown by dotted curve $n\ m$. This may be then taken as the zero of the alternating component; *i.e.* make $a = b$ at all points between o and m.

With the curve shown in Fig. 21, the three questions which the designing engineer is asked, can be more accurately answered than in the past; namely, what is the first rush of current; what is the current at, say, ¼ or ½ second after short circuit, this interval depending usually upon the operating time of the circuit breaker; and what is the sustained value.

The sustained value is easily taken from the design sheet by the well known method. The value of I_0 is determined by the

transient reactance, x_0. And it will be shown later that α_f and α_a depend upon x_0 and the resistance of field and armature circuits.

In the past, the rate of decrease of the current has been predetermined by empirical curves, based upon oscillographic data, such as given by Hewlett, Mahoney and Burnham[11]. After all, for commercial engineering work, such curves will still be used; but in their preparation or in the investigation of special cases, the above equation will be very useful. But before those curves can be used, the generator transient reactance which adds directly to the external reactance, must be known.

The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper, of the factors which determine the first rush of current, and of what constitutes the factors α_f and α_a. He states that the initial value of the alternating component (crest value), which he calls $(A+B)$, "is equal to the maximum phase voltage before short circuit divided by the armature impedance, with good approximation." As brought out in the following paragraph, this assumption would lead to serious error in many cases. The matter of α_f and α_a will be discussed in subsequent pages.

Turning now to the problem in hand, the reactance which limits the initial short-circuit current in most synchronous machines is not the armature self-inductive reactance only; it is rather the combined self-inductive reactance of both the armature and field circuits. The calculated short-circuit current, based on the armature self-induction, is therefore higher, in some cases 40 to 50 per cent higher, than the actual short-circuit current. In salient, laminated pole alternators, in which the field leakage flux, that is, the self-induction, is large and in which rotor eddy currents are practically negligible, this difference is more pronounced than in the continuous rotor construction of turbine generators, in which the effective field self-induction is largely reduced by the distribution of the field winding and also by the solid steel rotor,[12] or in salient pole machines with low resistance amortisseur winding. Hence in the latter two cases, as might be expected and as experience indicates, the armature self-inductive reactance is a reasonably accurate measure of the short-circuit current; and it is proposed, for the present at least, that this basis be used for

11. Trans. A. I. E. E., 1918, Vol. XXXVII, p. 130.
12. A. B. Field, Trans. A. I. E. E., Vol. 31, Part 2, p. 1651.
13. Trans. A. I. E. E., Vol. 31, Part 2, p. 1657.

these cases. This is not the point of view taken by Durgin and Whitehead[13] in 1912. Their conclusion, based on tests of a 12,000-kw., 9000-volt, 25-cycle turbo-generator in the Fisk Street Station of the Commonwealth Edison Company, was that "the short-circuit currents of alternators are limited by reactance much more complex and much higher than the self-inductive reactance of the armature." While the authors agree, as stated above, that this is true for alternators with salient, laminated poles, they submit that it is not true for many turbo-generators, as now ordinarily constructed, or in the case of salient pole machines with low resistance amortisseur windings. Tests in Table IV, confirm this.

In order to establish a conception of the effect of armature and field self-induction, consider in part the phenomena which occur in an alternator under short circuit, and the fundamental principle involved. *A closed electric circuit, without resistance, must persist magnetically in the same condition as at the instant of closing; that is, must contain, so long as it is closed, the same number of magnetic interlinkages.* If the flux tends to increase or decrease, this will produce a current of sufficient magnitude to maintain the interlinkages which existed when the circuit was closed. This law determines, in a large measure, what happens under short circuit; because, although both the armature and field circuits actually contain some resistance, and the currents therefore die down in the familiar transients, Fig. 22, yet its effect during the first cycle or so after short circuit is, for qualitative considerations at least, negligible.

It follows that the flux linked with the field winding before short circuit must persist after short circuit; that likewise whatever flux is caught in the armature circuit at the instant of short circuit, must there persist; and that the flux from the rotating field poles can not now enter the armature, *i. e.*, can not change the magnetic interlinkages of the armature circuit.

Consider a simple case of closed circuit in Fig. 23. The closed-ring conductor, without resistance, is moved from position *a* to the center of the magnetic field, ϕ, in *b*. This causes a current I_0 in the ring—a current sufficient to produce around the leakage paths of the ring a flux equal to $-\phi$. This is illustrated hypothetically in *c*. But I_0 in the ring in *b* tends to demagnetize the field. This requires a corresponding increase in m.m.f. of the field coil, to maintain the flux ϕ linked with the coil. The result, as shown on *b*, is to force the flux around the ring.

Suppose the ring is closed around the flux as shown in *d*. Then if the ring is moved to position in *f*, the result will be as shown: the magnetic flux ϕ will remain linked with the field

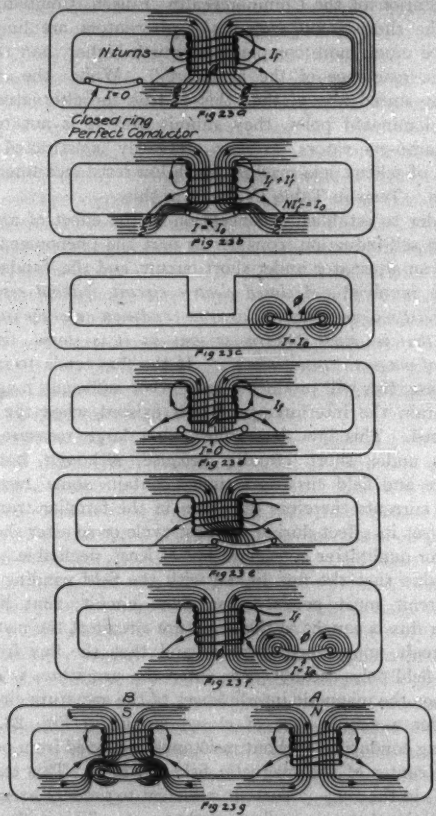


FIG. 23—DIAGRAMMATIC REPRESENTATION OF SHORT-CIRCUIT FLUXES

coil, as in *d*, and in addition, another magnetic field of equal value will be maintained around the ring by a current I_0, the same value, (assuming equal reluctance of leakage path),

FIG. 4

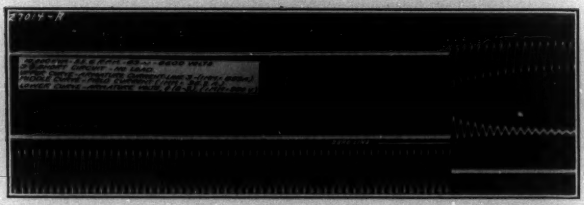
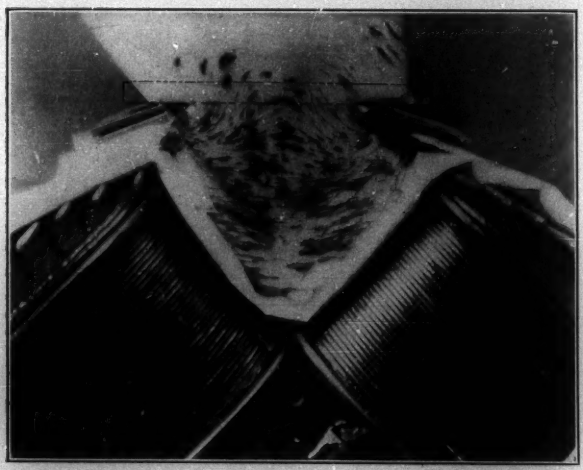


FIG. 22



[DOHERTY AND SHIRLEY]

FIG. 25B.—ACTUAL MAGNETIC DISTRIBUTION OF A FOUR-POLE FIELD—DERIVED DISTRIBUTION SHOWN IN FIG. 25A

which was necessary in *b*. Force is required to move the ring from *d* to *f*, as shown by the distortion in *e*. The work done in moving the ring is the energy stored in the new magnetic field around the ring in *f*. If the ring containing zero flux, as in *a*, were moved along in *g*, the current would rise to I_0 when under A, decrease to zero between A and B, and rise to $-I_0$ under B; in other words, an alternating current would be produced. If, however, the ring, enclosing the flux ϕ and containing the direct current I_0, as in *f*, were moved as above, the current would decrease to zero under A, rise again to I_0 between A and B, and reach $2I_0$ under B. In this case, the resultant current, varying from zero to $2I_0$ is made up of an *alternating component* and a *direct component*, the latter being equal to the maximum of the former.

The events occurring in an alternator immediately after a no-load single-phase short circuit have thus been illustrated. The short circuit at maximum voltage is represented by the ring enclosing zero flux, the short circuit at zero voltage, by the ring enclosing the flux ϕ. Fig. 23e illustrates how a large alternating torque is produced when the short circuit occurs at zero voltage.

A three-phase short circuit at no load is different from the single-phase short circuit in this important respect, that it is impossible, three-phase, to have the short circuit occur when zero flux is enclosed in the short-circuited armature winding. The entire flux is at all times enclosed by some parts of the armature winding, and therefore in whatever position the field structure happens to place the flux when the short circuit occurs, there it must remain, a series of polar regions, stationary in space, while each field pole with its equal value of flux moves on These stationary polar regions of normal intensity, of course generate in the rotating field winding an alternating current of normal frequency, and of a maximum value in m.m.f. equal to the direct current which maintains the stator polar regions. That is, the flux, once intact through the entire normal magnetic path is now literally torn apart at the air gap, and at 180 electrical degrees from the instant of short circuit, forms two separate magnetic circuits, closing through the leakage paths between armature and field windings. At the end of one cycle the flux is again intact through the normal path. All the work which, during the first half cycle, was done on the field in tearing it apart, in actually establishing normal interlinkages in circuits of much lower permeance than that of the normal circuit, is returned to the rotor during the last half cycle. The torque

which accompanies this transfer of energy has been calculated by S. H. Weaver, *G. E. Review*, Nov., 1915.

Return to the question of the self-inductive reactance existing at sudden short circuit. It has been generally understood

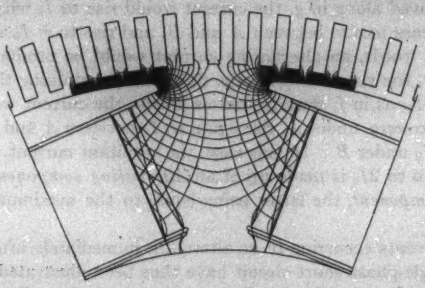


FIG. 24A—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

how the armature self-induction limits the initial short-circuit current; that is, how, if a machine had 10 per cent armature self-induction, normal armature current would cause flux through

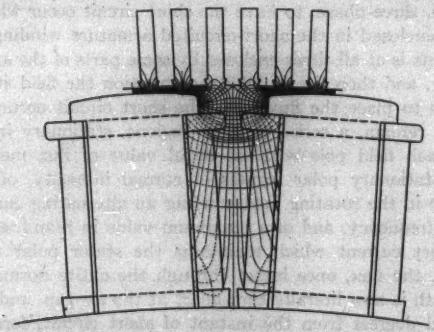


FIG. 24B—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

the armature leakage paths equal to 10 per cent of normal flux, and that therefore on sudden short circuit, when (neglecting field self-induction) the total normal flux must be absorbed in the leakage paths, the armature current would have to rise to

ten times normal. It may not have been so clear how and to what extent the field self-induction affects the short-circuit current. From foregoing considerations and from an approximate knowledge of the field leakage paths, it is possible to determine this point. The leakage paths have been approximately determined by plotting equi-magnetic potential lines as shown in Figs. 24A and 24B. The flux must be at right angles to these lines. The entire flux emanating from field poles of different proportions is thus illustrated in these diagrams. Figs. 25A and 25B show the graphically determined and actual magnetic distribution of a 4-pole rotor with an iron plate bridging the poles, wooden blocks supporting the ends and forming an air gap. It will be noted that a large portion of the leakage flux emanates

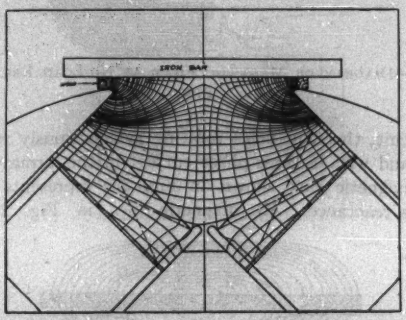


FIG. 25A—DERIVED DIAGRAMS OF MAGNETIC DISTRIBUTION OF THE FOUR-POLE FIELD SHOWN IN FIG. 25B

from the under side of the pole tip, which, we believe has not generally been considered in leakage calculations. In Fig. 25B the paths as indicated by the iron filings can not be traced to the pole body and to all of the under surface of the pole tip, on account of the field winding. But it is clear from the direction of the paths at the edge of the field winding that they will terminate approximately as shown in Fig. 25A. These figures of course do not show quite what the distribution would be if the rotor were placed in the stator. Then, both the derived and actual distribution would be different, but not greatly different. This experiment was made to confirm the method. Fig. 26 shows the distribution on a machine with a large number of poles.

Fig. 27A represents the distribution of flux at no load on a three-phase generator. *A*, *B* and *C* represent the different phase belts. Fig. 27B represents the same machine with open

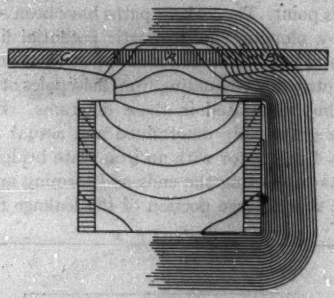


FIG. 27A—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD EXCITATION ON FIELD

field circuit, the field poles rotating synchronously in position shown, and with a three-phase current in the armature. The entire magnetic flux shown in Fig. 27B represents the synchronous reactance. All magnetic lines in Fig. 27B which

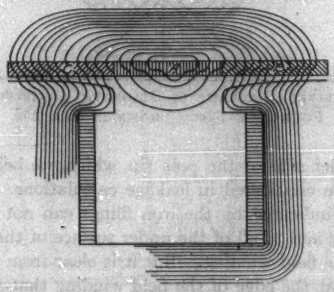


FIG. 27B—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD—THREE-PHASE EXCITATION ON ARMATURE FIELD REVOLVING AT SYNCHRONOUS SPEED

do not link with the field winding represent the armature self-induction, or armature reactance. Likewise in Fig. 27A, that part of the field flux which does not link with the armature winding represents field self-induction, or field reactance. Just

as on sustained short circuit, the field current causes the armature current to flow, and must therefore be greater in m.m.f. than the armature current, by the armature self-induction, that is greater in the ratio,

$$\frac{\text{synchronous reactance } AT}{\text{effective armature reaction}}$$

$$= \frac{\text{total flux linked with armature, Fig. 27B}}{\text{mutual flux, Fig. 27B}}$$

so also on sudden short circuit, when the armature current causes the rise in field current, the armature m.m.f. must be

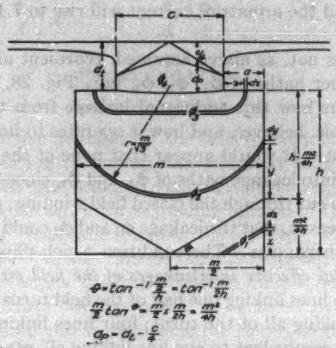


FIG. 28—DIAGRAM FOR CALCULATING FIELD LEAKAGE

greater than the rise in field m.m.f. by the field self-induction, that is greater in the ratio,

$$\frac{\text{total field winding interlinkages, Fig. 27A}}{\text{mutual flux interlinkages, Fig. 27A}}.$$

For convenience in developing the point, assume that all of the field leakage is concentrated between pole tips shown as ϕ_4 in Fig. 28. On sudden short circuit, since the normal flux which is linked with the field winding and which before short circuit entered the armature, can now neither enter the armature nor die out through the field winding, it must be forced through the leakage paths. At the instant at which the armature current in its rise at short circuit passes through normal value, the ac-

companying rise in field m.m.f. will be approximately equal (actually less) to the effective armature reaction, A, corresponding to normal current. This increase in field m.m.f. must cause a corresponding increase in ϕ_4, Fig. 28; but since ϕ_4 links with the field circuit, and the field interlinkages can not increase, it follows that the increase in ϕ_4 represents the portion of the normal flux Φ, which is absorbed on short circuit by the field self-induction when the field and armature m.m.f's. have risen to A. As an illustration, let the flux ϕ_4 corresponding to a field m.m.f., A, be 4 per cent, and let the armature self-induction be 10 per cent. Then as the armature current passes through normal value, 14 per cent of normal flux Φ, will have been absorbed in the leakage paths; and the armature current will rise to 7.15 times normal value.

But ϕ_4 does not, as above assumed, represent all of the field leakage. Other paths of ϕ_1, ϕ_2, ϕ_3, etc., Fig. 28, exist. It is not so obvious how this additional leakage from the sides and ends of the pole, behaves, and how it operates to limit the short-circuit current. It would appear that none of the normal flux, Φ, could sink into leakage paths of ϕ_1, and ϕ_2, since in doing so it would have to cut through the closed field winding; and also, and for the same reason, that the leakage ϕ_1 and ϕ_2 could not increase. Actually, both happen. The condition which must be fulfilled is that *the total effective interlinkages of the field circuit shall not change.* Two lines linking one half of the field turns is equivalent to one line linking all of the turns; four lines linking one fourth of the turns is equivalent to one line linking all, etc., etc. What actually happens, therefore, is that as the short-circuit current rises, some of the normal flux, Φ, sinks into the paths of ϕ_1 and ϕ_2; *and to just the extent to which interlinkages are decreased in this process at the top of the winding, new flux, compensating for the loss, appears at the bottom of the winding.* In machines which have rotors entirely laminated, this increase in flux may be 30 to 40 per cent. So that the total field leakage flux, Φ_l, existing under sudden short circuit, is the same (or nearly so) as would exist on open circuit with the same field m.m.f.; and the effective interlinkages which Φ_l represents is the portion of the normal flux Φ, which would be absorbed in the total field self-induction on short circuit.

Let ϕ'_{lm} be the effective field leakage interlinkages per unit length of machine, corresponding to a m.m.f. on the field equal to normal armature reaction, A. Then, *if* a rise to normal

current in the armature would cause a rise in field m.m.f. equal to A, the ratio

$$x'_{pf} = \frac{l\phi'_{ln}}{\Phi} \tag{1}$$

would represent the per cent field self-induction. Actually the rise in field m.m.f. corresponding to normal armature current is less than A, as already pointed out, by the ratio

$$\frac{\text{mutual flux interlinkages,} \quad \text{Fig. 27A}}{\text{total field winding interlinkages,} \quad \text{Fig. 27A}}$$

$$= \frac{\Phi}{\Phi + \frac{F}{A}\, l\, \phi'_{ln}} = \frac{1}{1 + \frac{F}{A}\frac{l\,\phi'_{ln}}{\Phi}} = \frac{1}{1 + \frac{F}{A} x'_{p_f}} \tag{2}$$

where

F = no-load field m.m.f.

l = length of machine

It is still further reduced by the field self-induction in another way: the portion of normal flux which at short circuit is absorbed in the field leakage paths no longer requires, as it did before short circuit, a corresponding portion of the no-load field m.m.f. F to maintain it across the air gap. But all of F remains at short circuit. Hence the difference between F and the portion of F which is actually needed for the flux which is not absorbed in field leakage paths, is balanced by a correspondingly decreased rise in field m.m.f. Hence, a rise in armature m.m.f. equal to A, produces a rise in field m.m.f. equal to

$$F_a = \frac{A}{1 + \frac{F}{A} x'_{p_f}} - x_{p_f} F \tag{3}$$

Where X_{pf} represents the per cent actual field self-induction or per cent field reactance. This is

$$x_{p_f} = \frac{F_a}{A} x'_{p_f} \tag{4}$$

But

$$x'_{p_f} = \frac{l\,\phi'_{ln}}{\Phi}$$

and $\phi'_{ln} = A\,L$ (from equation **35**)

Hence

$$x'_{p_f} = \frac{A\,l\,L}{\Phi} \tag{5}$$

Substituting (3) and (5) in (4) and simplifying

$$x_{p_f} = \frac{A}{F} \frac{\frac{F\,l\,L}{\Phi}}{\left(1 + \frac{F\,l\,L}{\Phi}\right)^2} \tag{6}$$

Where

A = effective polyphase armature reaction AT corresponding to normal current I_n

F = no-load field AT

Φ = normal flux per pole (in maxwells) entering the armature

$$L = \left[1.42 \frac{h}{m} + 0.16 \left(\frac{m}{h}\right)^5 + 1.6 \log\left(1 + \pi \frac{a}{c}\right) + 5.0 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log\left(1 + \frac{\pi}{4} \frac{b}{c}\right)\right\} + 0.55 \frac{d}{l} \right]$$

If length is in inches, the expression for L becomes

$$L = \left[3.6 \frac{h}{m} + 0.04 \left(\frac{m}{h}\right)^5 + 4 \log\left(1 + \pi \frac{a}{c}\right) + 12.8 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log\left(1 + \frac{\pi}{4} \frac{b}{c}\right)\right\} + 1.4 \frac{d}{l} \right]$$

a, b, c, d, h, m, d_p, see Fig. 28 and Fig. 34

m = distance between pole cores at $h \div 2$

l = gross stacked length of machine (core + ducts)

d = width of pole core

L = applies for all values of $\frac{h}{m}$ above 0.5, and is the effective leakage lines per unit length of machine, linking the entire field winding when the field m.m.f. is one ampere turn. The expression for L is derived in the following pages.

The terms $4 \log\left(1 + \pi \frac{a}{c}\right)$ and $0.62 \log\left(1 + \frac{\pi}{4} \frac{b}{c}\right)$ are plotted in Fig. 29.

The total per cent reactance therefore which determines the initial short-circuit current (that is, the transient reactance) is given by

$$x_{p_0} = x_{p_a} + x_{p_f} \tag{7}$$

where x_{p_a} = per cent armature self-inductive reactance (or armature reactance) calculated by equation (**47**) Part I.

x_{pf} = per cent field self-inductive reactance (or field reactance) calculated by equation (**6**) Part II

(x_{p_0} = 0.2, means 20 per cent reactance)

Hence the effective value of the alternating component of the initial short-circuit current is

$$I_0 \frac{I_n}{x_{p_0}} \tag{8}$$

where I_m = normal current.

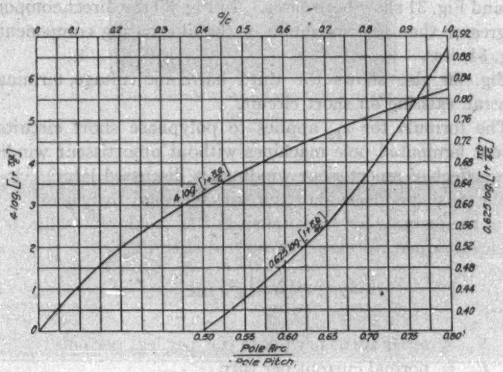


FIG. 29

The above discussion has considered only the alternating component of the initial short-circuit current. Actually a direct component of armature current, approximately equal in m.m.f. to the alternating component of armature current must always exist on a three-phase short circuit, and *may* exist on a single-phase short circuit, as already mentioned. That is at 180 degrees after short circuit, it is necessary, in such a case, for twice normal flux to exist across the leakage paths of the armature and field. It is obvious that this additional flux of normal value would be absorbed in the leakage paths in the same manner as above described.

Assuming a sine distribution of both the field flux Fig. 27A and

of the armature flux Fig. 27B, the direct component of current would be equal to the crest of the alternating component; that is

$$I_{dc} = \sqrt{2}\, I_0 \tag{9}$$

The distribution is, however, usually not sinusoidal. Since the effect of the important harmonics on short-circuit current (alternating component) is practically eliminated, as discussed in Part I, it follows that the magnitude of the alternating component is determined by the fundamental of the flux wave. The direct component, however, is determined by the total flux, that is by the entire flux which the armature encloses, and may therefore be greater or less than $\sqrt{2}\, I_0$ by a small amount. Fig. 30 and Fig. 31 show both cases. In Fig. 30 the direct component is greater than the maximum of the alternating component; in Fig. 31, less.

Fig. 30 also shows the third harmonic voltage, terminal to neutral, existing on short circuit.

The formula for x_{p_0} applies to polyphase short circuits on salient laminated pole machines without amortisseur windings. (The effect of amortisseur windings is discussed later). It will apply to single-phase short circuit, terminal to neutral, if: (1) A, in the formula for x_{pf} is taken as the maximum value of the pulsating armature m.m.f. of normal current, *i.e.*

$$A_1 = \sqrt{2}\, K'\, N_q\, I_n \tag{10}$$

where

N_q = series turns per phase (*i.e.* per leg) per pole.

I_n = normal current per turn

K' = factor accounting for short coil pitch, coil distribution, and flux distribution. (2) the armature reactance is taken equal to two thirds of the three-phase value since as pointed out in Part I, three-phase reactance is approximately 50 per cent higher than the single-phase. That is, for single-phase short circuit, terminal to neutral,

$$x_{p_{01}} = \frac{2}{3}\, x_{p_a} + x_{p_{f1}} \tag{11}$$

where x_{pf1} is field reactance for single-phase short circuit. But since the three-phase armature reaction is

$$A_3 = 1.5\, \sqrt{2}\, N_q\, I_n\, K' \tag{12}$$

FIG. 26—ACTUAL MAGNETIC DISTRIBUTION

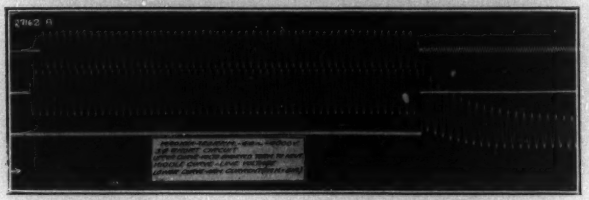


FIG. 30

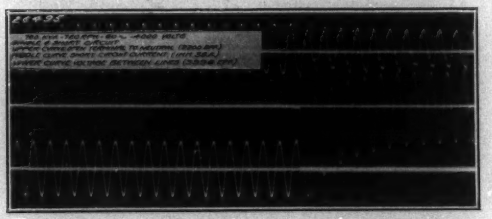


FIG. 31 [DOHERTY AND SHIRLEY]

FIG. 32A

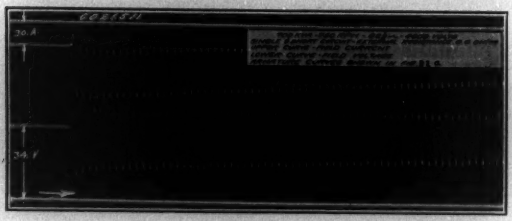


FIG. 32B

[DOHERTY AND SHIRLEY]

FIG. 35B—FIELD LEAKAGE PATH AT END OF POLE

the equation for x_{p01} can be written

$$x_{p_{01}} = \frac{2}{3}\left(x_{pa} + x_{pf}\right) = \frac{2}{3}x_{p_0} \tag{13}$$

That is, on a single-phase short circuit, terminal to neutral, the current should be about 50 per cent larger than on a three-phase short circuit. The authors have not made tests to confirm this point.

The action of the field and armature on single-phase short circuit is as follows: if the short circuit occurs when the armature encloses zero flux, that is, at maximum voltage, Figs. 32A and 32B, the pulsating armature reaction produces in the field, for well known reasons, a direct m.m.f. equal to about one-half (actually less) of the maximum armature m.m.f., and a double frequency m.m.f. whose maximum value equals the direct m.m.f. The double frequency m.m.f. is of course at negative maximum at instant of short circuit, so that at the end of the first half cycle the total field m.m.f. is about equal to the maximum armature m.m.f.; that is, the armature and field absorb in their leakage paths at this instant, the total normal flux in proportion to their respective self-inductions.

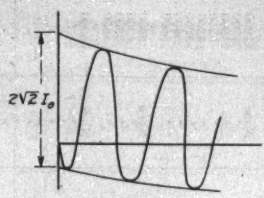


FIG. 33—SHORT CIRCUIT ARMATURE CURRENT

A single-phase short circuit between terminals gives, in most cases, practically the same current as a three-phase short circuit, as experience has shown, and may therefore be calculated by the formula for three-phase until a more accurate method is worked out.

The total self-inductive reactance effective at short circuit expressed in ohms, that is, the transient reactance, is

$$x_0 = \frac{x_{p_0} \times \text{normal voltage per leg}}{\text{normal current per leg}} \tag{14}$$

Table IV shows a comparison of calculated and actual values of the alternating component of short-circuit current in terms of normal current. The actual values were determined by projecting to the axis of zero time the curve of positive crests and also the curve of negative crests, Fig. 33. The intercept on the vertical axis represents $2\sqrt{2}\,I_0$, *i.e.* two times the crest value of the alternating component.

TABLE IV—SALIENT-POLE MACHINES

Machine					Conditions of short circuit			Type of rotor	Amortisseur winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a.	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
10000	3	63	55.6	6600	3	None	6600	Laminated salient pole	None	0.27	0.25
"	"	"	"	"	3	None	"	"	"	"	0.25‡
"	"	"	"	"	3	None	"	"	"	"	0.26‡
"	"	"	"	"	3	None	"	"	"	"	0.25
"	"	"	"	"	3	7500kw	"	"	"	"	0.22
"	"	"	"	"	3	7500kw	"	"	"		0.24
9000	3	25	57.7	11000	3	None	11000	"	None	0.22	0.26
"	3	"	"	"	3	"	11000	"	"	"	0.26
"	"	"	"	"	3	"	11000	"	"	"	0.25
"	"	"	"	"	3	"	8250	"	"	"	0.25
"	"	"	"	"	1	"	8250	"	"	"	0.26
"	"	"	"	"	1	"	8250	"	"	"	0.29
6600	3	25	375	14000	3	None	16000	"	None	0.15	0.12
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	8000	"	"	"	0.14‡
4300	1	25	300	11000	1	None	11000	"	Low-Res.	0.19*	0.20
"	"	"	"	"	1	"	11000	"	"	"	0.22
"	"	"	"	"	1	"	11000	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	0.20
"	"	"	"	"	1	"	8250	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	0.23
"	"	"	"	"	1	"	5500	"	"	"	0.23
"	"	"	"	"	1	"	5500	"	"	"	0.23
1500	3	50	500	6600	3	None	6600	"	None	0.16	0.12
1500	3	50	500	6600	3	"	4950	"	"	"	0.13
1500	3	50	500	6600	3	"	3300	"	"	"	0.14
1500	3	50	500	6600	3	"	1650	"	"	"	0.15
1400	3	60	720	2300/4000	3	None	4000	"	High-Res.	0.18	0.21
"	"	"	"	"	3	"	"	"	"	"	0.16
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.18
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.22
"	"	"	"	"	1	"	"	"	"	"	0.20
"	"	"	"	"	1	"	3000	"	"	"	0.21

SALIENT-POLE MACHINES (*Continued*)

Machine					Condition of short circuit			Type of rotor	Amortissuer Winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
700	3	60	720	2300/4000	1	None	3996	Laminated salient pole	High Res	0.18	0.20
"	"	"	"	"	1	"	3996	"	"	"	0.23
"	"	"	"	"	1	"	3996	"	"	"	0.16
"	"	"	"	"	1	"	3036	"	"	"	0.18
"	"	"	"	"	1	"	3036	"	"	"	0.20
"	"	"	"	"	1	"	2016	"	"	"	0.17
"	"	"	"	"	1	"	2016	"	"	"	0.19
"	"	"	"	"	3	"	3996	"	"	"	0.15
"	"	"	"	"	3	"	3996	"	"	"	0.14
"	"	"	"	"	3	"	3036	"	"	"	0.15
"	"	"	"	"	3	"	2016	"	"	"	0.17
"	"	"	"	"	3	"	2016	"	"	"	0.15
125	3	60	900	2300	3	None	2360	"	High-Res	0.18	0.19
"	"	"	"	"	3	"	2300	"	High-Res.	"	0.19
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.17
"	"	"	"	"	3	"	1150	"	None	"	0.20
"	"	"	"	"	3	"	1150	"	None	"	0.21

TURBINE ALTERNATORS

10000	3	40	2400	10000	3	None	12000	Continuous solid steel	None	0.12*	0.12‡
"	"	"	"	"	"	"	10000	"	"	"	0.13‡
"	"	"	"	"	"	"	5000	"	"	"	0.16‡
"	"	"	"	"	"	Full Load 10 P. F.	10000	"	"	"	0.15‡
"	"	"	"	"	"	"	10000	"	"	"	0.15‡
"	"	"	"	"	"	Full Load O P. F.	10000	"	"	"	0.12‡
6250	3	60	3600	2300	3	None	2300	"	"	0.10*	0.09
3750	"	60	3600	2300/4000	3	None	4000	"	None	0.09*	0.10
"	"	"	"	"	"	"	4000	"	"	"	0.08
"	"	"	"	"	"	"	3000	"	"	"	0.11
"	"	"	"	"	"	"	3000	"	"	"	0.09
"	"	"	"	"	"	"	2000	"	"	"	0.12

†In order to have ratios on same basis, normal current is decreased or increased according as voltage at short circuit is less or greater than normal.
*Transient reactance taken equal to armature reactance.
‡Average from three waves on one film.

Example: 6600-kv-a., three-phase, 25-cycle, 14,000-volt generator.

$$l = 38 \quad A = 10970 \quad \Phi = 47.7 \times 10^6$$

$$F = 15600 \quad \frac{h}{m} = 0.687 \quad \frac{a}{c} = 0.413$$

$d_1 = 2.0$ $\quad d_p = 2.0 - \dfrac{11.5}{4}$ (neglect d_p if it is zero or negative) $\quad \dfrac{d}{l} = 0.407$

Ratio $\dfrac{\text{pole arc}}{\text{pole pitch}} = 0.675 \quad \dfrac{b}{c} = 2.08$

$$L = \left[3.6 \times 0.687 + 0.04 \left(\frac{1}{0.687}\right)^2 + 4 \log (1 + \pi \times 0.413) + 1.4 \times 0.407 \right]$$

$$= [2.47 + 0.26 + 3.3 + 0.57] = 6.6$$

$$x_{p_f} = \frac{10970}{15600} \frac{\dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}}{\left(1 + \dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}\right)^2} = 0.049$$

Armature reactance $= x_{pa} = 0.10$ (see example, Part I)

$$x_{p0} = x_{pa} + x_{pf} = 0.10 + 0.049 = 0.149$$

That is, the three-phase short-circuit current (*a-c.* component) would be

$$I_0 = \frac{1}{0.149} = 6.7 \text{ times normal}$$

Consider now the question of attenuation factors, α_f and α_a. The attenuation factor of a circuit is the ratio of the effective resistance to the (interlinkages per ampere), that is, the inductance. In the field attenuation factor, α_f, the inductance is not as Diamant gives[14] the exciting inductance of the machine, (using his notation)

$$\frac{\Phi N_f}{I_f 10^{-1}}$$

14. TRANS. A. I. E. E., Vol. 34, p. 2247.

where Φ is the flux which the normal field excitation, I_f, would produce in the normal magnetic circuit, as given by the design sheet. It is rather

$$\frac{\Phi' N_f}{I_f 10^{-1}}$$

where Φ' is the flux, linking the field winding, which I_f would produce in the *leakage paths of both field and armature*. The correct attenuation factor of the field to be used in determining the rate of decrease of field flux on short circuit is the ratio of the effective resistance of the circuit to the (transient field interlinkages maintained through the leakage paths of both armature and field on short circuit, per ampere rise in the field circuit). Therefore the question is, what is the value of field interlinkages thus maintained per ampere *rise* in field current on short circuit. It is,

$$\frac{\Phi_t N_f}{I_{f0}} \quad \text{per pole}$$

or

$$\frac{q\,\Phi_t N_f}{I_{f0}} \quad \text{total}$$

That is,

$$\frac{q\,\Phi_t N_f}{10^8 I_{f0}} \quad \text{henrys} \tag{15}$$

Where

Φ_t = transient flux = $\Phi - \Phi_s$

Φ = normal flux per pole

Φ_s = flux per pole, absorbed in armature self-induction on sustained short circuit

$$\Phi_s = \frac{F\Phi}{\dfrac{A}{x_{p_a}} + F_g}$$

x_{p_a} = armature reactance
(x_{pa} = 0.2 means 20 per cent)

F_g = m.m.f. consumed in air gap by normal flux

N_f = turns per pole

q = number of poles

I_{f_0} = *d-c.* component of *rise* in field current at short circuit. This corresponds to the reaction of the *alternating component*

in the armature, and for reasons previously stated is less than the armature m.m.f. as shown by equation (3).

$$I_{f_0} = \frac{F_a}{x_{p_0} N_f} = \frac{F}{x_{p_0} N_f} \left[\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right] \tag{16}$$

Hence the field inductance during transient is,

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi_t\, N_f^2}{10^8\, F} \frac{1}{\left[\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right]}$$

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi\, N_f^2}{10^8\, F} \frac{\left(1 - \frac{F}{\frac{A}{x_{p_a}} + F_g} \right)}{\left(\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right)} \text{ henrys} \tag{17}$$

The field attenuation factor is therefore

$$\alpha_f = \frac{r_f}{L_{f_0}} \tag{18}$$

where r_f = effective resistance of the field circuit. In laminated pole machines, this may be taken in approximate calculations as the ohmic resistance of the field circuit.

The inductance in the armature attenuation factor α_a, which determines the rate of decay of the *direct component*, is determined as follows: the interlinkages of the armature per pole per phase, maintained through the leakage paths of both the armature and the field, per effective ampere turn rise in the armature m.m.f. on short circuit, is

$$\frac{(K' N_q)\, \Phi}{\frac{A}{x_{p_0}}} \tag{19}$$

or, for all poles

$$\frac{q\, x_{p_0}\, \Phi\, (K' N_q)}{A} \tag{20}$$

But A, for three-phase, is the product

$$\sqrt{2}\, I_n \times 1.5\, (K' N_q) \tag{21}$$

where K' is a constant depending upon the pitch and distribution of coils, and upon the flux distribution; I_n = normal rated current; and N_q = series turns per pole per phase. Hence, the inductance, that is the interlinkages with $(K' N_q)$, is

$$L_{ao} = \frac{1.5\, q\, x_{p_0}\, \Phi\, (K' N_q)^2}{10^8} \text{ henrys} \tag{22}$$

The factor 1.5 accounts for the mutual induction of phases. For single phase or two phase this factor would be unity. L_{ao} can be arrived at in another and more convenient way.
It is equal to

$$L_{ao} = \frac{x_0}{2\,\pi\, f} \tag{23}$$

Where x_0 = transient reactance in ohms
f = frequency in cycles per second

$$x_o = \frac{x_{p_0}\, E}{I_n}$$

Hence

$$L_{ao} = \frac{x_{p_0}\, E}{2\,\pi\, f\, I_n} \tag{24}$$

The armature attenuation factor is

$$\alpha_a = \frac{r_a}{L_{ao}} \tag{25}$$

where r_a = effective resistance per phase.

It may seem at first thought that since the sum of the inductances of both the field and armature are involved in the attenuation factors, the sum of resistance of both circuits also should enter. However, this is not the case. The armature and field transients involve magnetic energy storage under unstable conditions. In the case of the field transient, immediately after short circuit the energy stored in the field is the product of normal field flux times the large field m.m.f. which now maintains it. The flux can die down only if its energy is dissipated in the circuit with which it links, because, by the very way it is constituted, the energy can decrease only by a decrease in interlinkages, and this can not happen except by resistance

loss in the linking circuit. From a slightly different viewpoint, suppose the field circuit has no resistance. Then the field flux and current would never die out, regardless of the armature resistance. The armature m.m.f. operates only to increase the reluctance[15] of the path through which the flux must pass on short circuit. The armature resistance operates only to decrease the initial rise in current, that is to decrease the amount of flux which on short circuit must be sent through the paths of increased reluctance, and therefore to increase the sustained value of flux, that is, to decrease the magnitude of the transient, but not its time. The same reasoning will apply to the armature transient.

The relative values of α_f and α_a can be illustrated by taking some rough average figures for salient-pole machines. The ratio, armature reactance to armature resistance is, say, 20; field reactance to field resistance, 80; field reactance to armature reactance, 0.5.

Hence

$$\text{Armature} - \frac{\text{resistance}}{\text{transient reactance}} = \frac{1}{30}$$

$$\alpha_a = \frac{2\pi f}{30}$$

$$\text{field} - \frac{\text{res stance}}{\text{transient reactance}} = \frac{1}{240}$$

$$\alpha_f = \frac{2\pi f}{240}$$

That is, the armature transient will die out eight times as fast as the field, in this case.

Diamant found from oscillographic tests that the field attenuation factor was larger during the first few cycles than during the rest of the transient. The authors have found this to be the case; and agree with Diamant that, until this difference, due probably to saturation and eddy currents, can be further studied, the lower value (which corresponds to our calculation) should be used. This is on the safe side, since by this assumption, at any time after short circuit the current will be lower than the calculated value.

15. An equivalent reluctance could theoretically be produecd, with the same effect on the field transient, by a sudden increase, at open circuit, in full reluctance of the armature iron.

Saturation of the leakage paths at short circuit and the consequent reduction of reactance may cause an increase in α_f, and probably does so in machines which have restricted leakage paths—turbine generators, for instance. In salient-pole machines, however, where tests show the short-circuit current to be practically proportional to the voltage at short circuit, it is probable that the increase is largely due to eddy currents. The increase in flux at the bottom of the field pole—*i. e.* next to the rotor rim—at sudden short circuit, as explained in preceding pages, operates in two ways to increase α_f. If the rotor rim is solid, as is usually the case, this change in flux is largely restricted by eddies in the solid rim. To whatever extent it is

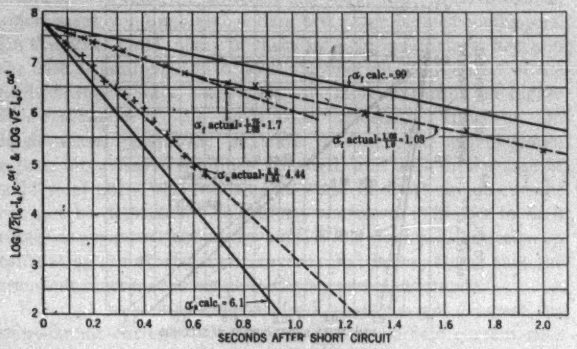


FIG. 33A—ATTENUATION FACTORS—6600 KV-A.—375 REV. PER MIN.—14000-VOLT ALTERNATOR

restricted, the effective field self-induction is reduced, which would increase α_f. But a much larger increase in α_f results from loss of energy due to eddies in the rim, increasing the effective resistance. Since the alternating flux in the rim is caused by the direct component of the armature current, *i. e.* the armature transient, the increased α_f would by this theory, exist only during the armature transient, which lasts about ¼ second. That is what actually happens, as shown in Fig. 33A. The initial value of α_f is 1.7 times the constant value which exists after the termination of the armature transient.

There is further evidence that this theory is correct. If the entire rotor magnetic circuit were laminated the change in flux

at the bottom of the pole could occur without producing serious eddies, and therefore, without appreciable change in α_f. Actual tests on such a machine shows this to be the case. The results are shown in Fig. 33B. There is no appreciable change in slope of the curve.

The question arises, how much will calculations of reactance be thrown out by saturation? From the data available it would seem that in salient-pole machines without amortisseur windings, saturation is not a very serious factor. Although at 180 deg. after a three-phase short circuit there must be two times normal flux through the leakage paths, these paths are widely distributed, and therefore very serious concentration is prevented. Yet there is

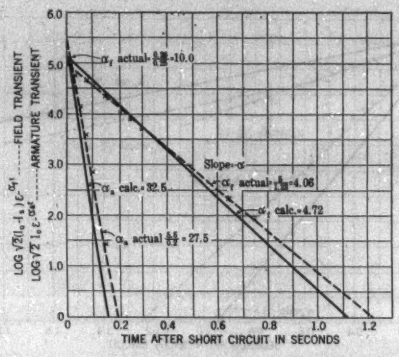


FIG. 33B—ATTENUATION FACTORS—125-KV-A.—900 REV. PER MIN.—60-CYCLE—2300-VOLT ALTERNATOR

some. The smaller the section of the principal leakage paths (the pole tip and teeth) the greater the saturation. It is still further increased by the action of an amortisseur winding, and by reason of the slots that contain it. But we believe it is safe to assume that in the case of most laminated, salient-pole machines, without amortisseur windings, the short-circuit current is practically proportional to the voltage at which short circuit occurs. When thin pole tips and shallow armature slots are used, the short-circuit current may be 10 to 20 per cent greater than indicated by calculations which neglect saturation. In the case of low resistance amortisseur windings, solid steel poles, or in turbine generators, where the leakage paths are much more restricted,

saturation makes a greater difference. Hence, for this, as well as another reason given in the following paragraph, the calculation of such machines should be based upon the armature self-induction only. The matter of saturation of leakage paths is one which should be much further investigated.

Another matter of importance is the effect of amortisseur windings, short-circuiting collars around the pole, solid steel poles, etc. Obviously if the amortisseur winding were without resistance, the flux at short circuit could not change through it and consequently the effect of field self-induction would be nil, since the field m.m.f. would not rise. Moreover, the armature leakage paths would be more restricted. That is the transient reactance would be less than the armature self-induction. But, of course, amortisseur windings and collars are never perfect conductors. On the other hand, many of them are poor conductors; hence in the case of laminated field poles they permit a large part of the field self-induction to exert itself—the more, naturally the higher the resistance of the amortisseur winding. However, in single-phase generators, or such machines, in which heavy copper amortisseur windings are used, and also in the case of solid steel poles, the resulting increase in short-circuit current can not be neglected. For safety, therefore, it is best in such cases, as proposed above, to neglect the field self-induction in calculating the short-circuit current; that is, to assume that the transient reactance is equal to the armature reactance.

External, as well as internal, field reactance will decrease the short-circuit current. The rate at which the field current rises at short circuit will not be impeded by the external reactance, because the new magnetic interlinkages established in the external reactance represent a corresponding decrease in the interlinkages of the field winding itself, since the net total of the circuit can not change appreciably during the first cycle or so. But, by reason of this decrease in interlinkages of the field winding itself, or what is the same thing, the increase in interlinkages in the external reactance, there remains in the field less flux to be forced across the leakage paths in the machine; hence the short-circuit current will be less by just that amount.

With zero external field reactance the rise in field m.m.f. on short circuit is less than the armature m.m.f. in the ratio

$$G = \frac{1}{1+\frac{F\,l\,L}{\Phi}} \text{ by equations } (\mathbf{2}) \text{ and } (\mathbf{5})$$

Or, in terms of total interlinkages in the machine,

$$G = \frac{q\,\Phi\,N_f}{q\,\Phi\,N_f + \frac{F}{N_f}\,L_f\,10^8} = \frac{1}{1 + \frac{F\,L_f\,10^8}{N_f^2\,\Phi\,q}}$$

where L_f = coefficient of self-induction of the field winding.

$$L_f = \frac{q\,l\,L\,N_f^2}{10^8} \text{ henrys}$$

F = no-load field m.m.f.
l = gross length of armature core (iron + ducts)
L = bracket quantity in equation (**33**) or (**33a**)
Φ = normal flux per pole (maxwells)
q = number of poles
N_f = turns per pole on field.

If an external reactance, x_{fz}, (coefficient of self-induction, $= L_{fz}$), is placed in the field circuit, the ratio becomes,

$$G = \frac{1}{1 + \frac{F\,L_f\,10^8}{N_f^2\,\Phi\,q} + \frac{F\,L_{fz}\,10^8}{N_f^2\,\Phi\,q}}$$

$$G = \frac{1}{1 + \frac{F\,10^8}{q\,N_f^2\,\Phi}\,(L_f + L_{fz})}$$

Let x_{pf}'' represent the per cent reactance of the field circuit, assuming the rise in field m.m.f. equals the armature m.m.f. on short circuit. Then

$$x_{pf}'' = \frac{\frac{A}{N_f}L_f\,10^8}{\Phi\,N_f} + \frac{\frac{A}{N_f}L_{fz}\,10^8}{\Phi\,N_f}$$

$$x_{pf}'' = \frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fz})$$

The reactance of the field circuit is

$$x_{pfz} = G\,x''_{pf}$$

$$= \left[\frac{\frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fz})}{1 + \frac{F\,10^8}{N_f^2\,q\,\Phi}\,(L_f + L_{fz})}\right]$$

$$x_{pfz} = \frac{A}{F}\left[\frac{1}{1 + \frac{1}{\frac{F\,10^8}{N_f^2\,\Phi\,q}(L_f + L_{fz})}}\right] \qquad (\mathbf{25a})$$

The total transient reactance of the machine with external field reactance is

$$x_{p0} = x_{pa} + x_{pf_x}$$

where x_{pa} = armature reactance

(x_{p0} = 0.2 means 20 per cent reactance)

The principal objection[16] to the use of reactance in the field circuit is the high voltage which occurs across its terminals at short circuit. With an external reactance in the field circuit equal to the internal field reactance, this voltage may be of the order of 50,000 volts in the case of large generators.

Under load the short-circuit current is slightly higher than at no-load. The alternating component, which depends upon the flux per pole, is greater for short circuit under load than at no-load, by the increase in flux which is required under load to overcome the true impedance of the armature. Hence for the same load current, the alternating component of short-circuit current will be greater, the lower the power factor, until at zero power factor, it will be increased by the per cent armature reactance of the machine—10 per cent reactance, 10 per cent increase in the alternating component. At unity power factor, the 10 per cent being added in quadrature, would not cause appreciable increase. The direct component, depending upon the flux linked with the armature circuit, *i.e.* upon the terminal voltage, is the same for any load or power factor, providing there is no external reactance in the armature circuit. External reactance decreases the direct component because it decreases the net magnetic interlinkages of the armature circuit. With 10 per cent external reactance, zero power factor load, the direct component would be 0.9 as large as if the short circuit occurred at no load, the same terminal voltage. The reverse would be true of leading current. For unity power factor, the difference would be negligible, for reasons mentioned above.

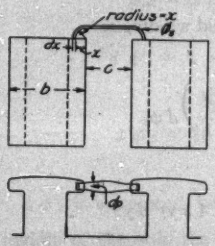


FIG. 34

DERIVATION OF FORMULA FOR CALCULATING FIELD SELF-INDUCTION

The field leakage paths shown in Figs. 24A and 24B are approximated by the assumed paths shown in Figs. 28, 34 and 35A.

16. This point has been observed by K. Ito, *Journal Elec. Society* (Japanese), Dec. 1917, p. 891.

With a m.m.f. equal to NI per pole the effective leakage lines, linking the entire field winding, per unit length of machine will be as follows: the flux element $d\,\phi_1$ is

$$d\,\phi_1 = 2 \times 0.4\,\pi\,\frac{x}{h}\,NI\,\frac{d\,x}{Kx}$$

where $$K = \frac{1}{\tan\theta}\ \text{(approximately)} = \frac{2\,h}{m}$$

$$d\,\phi_1 = 0.4\,\pi\,NI\,\frac{m}{h^2}\,dx$$

Effective flux due to ϕ_1 linking entire winding is

$$\phi_{n1} = \int_0^{\frac{m^2}{4\,h}} \frac{x}{h}\,d\phi_1$$

$$\phi_{n1} = 0.039\,NI\left(\frac{m}{h}\right)^5 \tag{26}$$

$$d\,\phi_2 = 2 \times 0.4\,\pi\left(\frac{\frac{m^2}{4\,h} + y}{h}\right) 2\,NI\,\frac{dy}{\frac{2\,\pi}{3}\,\frac{m}{\sqrt{3}}}$$

$$d\,\phi_2 = 1.05\,\frac{NI}{h^2\,m}\,(m^2 + 4\,h\,y)\,d\,y$$

It is here assumed that the length of path is one third of the arc of a circle whose radius is $\frac{m}{\sqrt{3}}$, that is, the pole sides intercept $\frac{1}{3}$ of the arc.

Effective flux of ϕ_2 linking the entire field winding is

$$\phi_{n2} = \int_0^{h - \frac{m^2}{4h}} \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) d\,\phi_2$$

$$= 0.27\,\frac{N\,I}{h^4\,m}\int_0^{h - \frac{m^2}{4\,h}} (m^2 + 4\,h\,y)^2\,d\,y$$

$$\phi_{n2} = 0.27\,N\,I\left[\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^5\right] \tag{27}$$

$$d\,\phi_3 = 2 \times 0.4\,\pi \times 2\,N\,I \frac{d\,z}{c + \pi\,z}$$

$$d\,\phi_3 = 5\,N\,I\,\frac{d\,z}{c + \pi\,z}$$

All of ϕ_3 is assumed to be linked with the winding. Hence

$$\phi_{n3} = \phi_3 = 5\,NI \int_0^a \frac{d\,z}{c + \pi\,z}$$

$$\phi_{n3} = 1.6\,N\,I \log\left(1 + \pi\,\frac{a}{c}\right) \tag{28}$$

In determining ϕ_4, it is necessary to find the portion of the pole tip from which ϕ_4 emanates, Fig. 28, and further to find what depth of path in the air between tips should be used. By plotting a large number of diagrams, such as Figs. 24A and 24B for different pole proportions, it was found that this average depth of path could be approximated by the expression

$$d_p = d_t - \frac{c}{4} \tag{29}$$

That is the factor may entirely disappear, which happens in the case of thin pole tips and low pole arc ratio. When $\frac{c}{4}$ is greater than d_t, ϕ_{n4} should be neglected.

$$\phi_{n4} = \phi_4 = 2 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p}{c}$$

$$\phi_{n4} = 5\,N\,I\,\frac{d_p}{c} \tag{30}$$

$$d\,\phi'_5 = 4 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p\,d\,x}{\pi\,x + c} \quad \text{(See Fig. 11)}$$

$$d\,\phi'_5 = 10\,N\,I\,d_p \int_0^{b/4} \frac{d\,x}{\pi\,x + c}$$

$d\,\phi'_5$ is integrated between 0 and $\frac{b}{4}$ because toward the middle of the pole the leakage from the end of pole shoe no longer goes from pole to pole, but instead bends upward and enters the stator iron, cutting the armature winding. The

approximation is therefore made that the dividing line falls at $b/4$.

$$\phi_s' = 3.2\, N\, I\, d_p \log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)$$

$$\phi_{ns} = \frac{\phi_s'}{l} = 3.2\, N\, I\, \frac{d_p}{l} \log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right) \tag{31}$$

where l = gross length of armature core (iron + ducts).

The leakage at the end of the pole is in paths practically straight and downward to the field spider rim as shown in Fig. 35A and 35B. At about three-fourths of the way up the pole the leakage bends up and enters the stator iron, part of it, certainly, cutting the armature winding. Of course, depending upon design, the division line may fall at one-half or may be at the top of the pole. But in the authors' judgment, three-fourths represents the average case. On this and the further assumption that the length of path at any height ν, Fig. 35, is 1.3 ν,

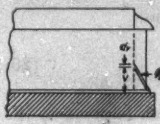


FIG. 35A

$$d\,\phi'_6 = 2 \times 0.4\,\pi\,\frac{\nu}{h}\,NI\,\frac{d\,d\nu}{1.3\,\nu}$$

where h = height of pole core

$$d\,\phi'_6 = 1.95\,\frac{d}{h}\,N\,I\,d\,\nu$$

$$\phi'_{n6} = \int_0^{\frac{3}{4}h} \frac{\nu}{h^2}\,d\,\phi_6'$$

$$= 1.95\,N\,I\,\frac{d}{h^2}\int_0^{\frac{3}{4}h} \nu\,d\,\nu$$

$$\phi_{n6} = \frac{\phi'_{n6}}{l} = 0.55\,N\,I\,\frac{d}{l} \tag{32}$$

While one or two rather arbitrary assumptions were made in deriving the above equations, they were made in the light of an experimental and graphical study of the leakage paths, and with the object of avoiding complicated expressions; and it is believed that the error in the final result will not be serious, one reason being that the assumptions involving the most uncertainties are in the smallest factors, such as ϕ_{n5} and ϕ_{n6}.

Adding equations (26), (27), (28), (30), (31) and (32), the total self-induction of the field is

$$\phi_{ln} = \phi_{n1} + \phi_{n2} + \phi_{n3} + \phi_{n4} + \phi_{n5} + \phi_{n6}$$

For lengths in centimeters,

$$\phi_{ln} = N I \Big[0.039 \left(\frac{m}{h}\right)^5 + 0.27 \left\{ \frac{16}{3} \frac{h}{m} - \frac{1}{12} \left(\frac{m}{h}\right)^5 \right\}$$

$$+ 1.6 \log \left(1 + \pi \frac{a}{c} \right) + 5 \frac{d_p}{c}$$

$$+ 3.2 \frac{d_p}{l} \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right) + 0.55 \frac{d}{l} \Big]$$

$$\phi_{ln} = N I \Big[1.42 \frac{h}{m} + 0.16 \left(\frac{m}{h}\right)^5 + 1.6 \log \left(1 + \pi \frac{a}{c} \right)$$

$$+ 5 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right) \right\} + 0.55 \frac{d}{l} \Big] \quad (33)$$

If inches are used, the equation becomes

$$\phi_{ln} = N I \Big[3.6 \frac{h}{m} + 0.04 \left(\frac{m}{h}\right)^5 + 4 \log \left(1 + \pi \frac{a}{c} \right)$$

$$+ 12.8 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log \left(1 + \frac{\pi b}{4 c} \right) \right\} + 1.4 \frac{d}{l} \Big] \quad (33a)$$

ϕ_{ln} is the effective leakage lines per pole, linking the entire field winding, per unit length of machine, produced by a m.m.f. of NI ampere turns.

The coefficient of self-induction of the field is therefore

$$L_f = \frac{q N_f^2 l L}{10^8} \text{ henrys} \quad (34)$$

where q = number of poles
N_f = turns per pole
l = gross stacked length of core (iron and ducts)
L = bracket quantity in equation (33) or (33a)

For the value of leakage, ϕ'_{ln}, per unit length, corresponding to a m.m.f. on the field equal to normal armature reaction, A, substitute A for $N I$ in equation (33) and (33a).
That is

$$\phi'_{ln} = A L \quad (35)$$

Summary and Conclusions

(1) A reliable prediction of the armature self-inductive reactance of synchronous machines can be made by equations (**45**) to (**48**) inclusive, given in Part I. Moreover, working curves for three-phase, based on those equations and given in Figs. 20_A 20_B, etc., make the complete calculation from the design sheet a matter of four or five minutes. It is proposed that this reactance be called, for brevity, *armature reactance*.

(2) The variation of the effective armature m.m.f. or armature reaction, (under zero power factor conditions) with the no-load flux distribution has been determined. The armature reaction of three-phase machines, as calculated by the familiar formula based on sinusoidal distribution, is the basis of the new calculation. This result is modified by a single factor, K_ϕ given in Fig. 20, and developed in appendices A and B. The correct calculation of armature reaction is very important in its relation to the armature self-inductive reactance, since the latter is determined from test by the difference between the field and armature m.m.f.'s on sustained short circuit.

(3) As a corollary of (2), the effect of the harmonics in the no-load flux distribution upon the generated voltage is accounted for also by K_ϕ. See appendix A.

(4) A simplified method of calculating field excitation that has worked out well in conjunction with the reactance given by Figs. 20_A, 20_B, etc., is shown in appendix C.

(5) The reactance which determines the initial short-circuit current of most synchronous machines is not the armature self-induction only; it is rather the combined self-induction of both the armature and field circuits. The internal field self-induction (which, on short circuit is transient) is often 50 per cent of the armature self-induction; in which case, the calculated short-circuit current, based on the armature self-induction alone, would, of course, be 50 per cent too high.

(6) A method of calculating the transient field self-induction is given in Part II, equation (**6**). It is proposed that this reactance be called, for brevity, *field reactance*.

(7) Equation (**7**), Part II, gives a value, x_{p0} of the total per cent reactance, which determines the alternating component of short-circuit current. It is proposed that this reactance be called *transient reactance*.

(8) The attenuation factors α_f and α_a, which determine the

rate of decrease of the field and armature transients, respectively are given in equations (**18**) and (**25**).

(9) External field reactance reduces the short-circuit current. Its effect can be calculated by equation (**25a**). The high voltage across the field collector rings (which may be 50,000 volts on large generators, assuming external field reactance equal to internal) precludes its use in most cases.

(10) Table IV confirms the following points, which are brought out in the theory given in Part II:

(a) The short-circuit current of synchronous machines can be calculated with practical accuracy.

(b) In the case of turbine generators with solid steel rotors, and also of salient-pole machines with low resistance amortisseur windings, the transient reactance should be taken equal to the armature reactance.

(c) In the case of high resistance amortisseur windings, the same value of transient reactance should be used as for laminated salient-pole machines without amortisseur windings. That is, the value given by equation (**7**) Part II.

(d) While strictly, the complete calculations apply only to salient, laminated pole machines without amortisseur windings, and the proposals in (b) and (c) are only approximations, nevertheless the results shown in Table IV justify the use of these approximations until the method is further extended.

(e) Saturation at short circuit does not practically affect value of short-circuit current of salient, laminated pole machines without amortisseur windings, unless the magnetic densities in the pole tips and teeth are high at normal voltage, as in the case of the 1500-kv-a. generator in Table IV. It does affect the short-circuit current of turbine generators, and, probably, of salient-pole machines with low resistance amortisseur windings.

(f) The short-circuit current is practically the same whether the short circuit on three-phase machines occurs between two terminals or three.

(g) A single-phase short circuit-terminal to neutral, produces a current 50 per cent greater than that of a three-phase short circuit.

In conclusion the authors wish to express their thanks to Dr. C. P. Steinmetz and W. J. Foster for their interest in reviewing the results of the paper, and to Messrs. E.J. Burnham, E. S. Henningsen, H. K. Humphrey, and J. J. Thalheimer for heir assistance.

APPENDIX A

Effect of Harmonics in No-Load Flux Wave

The shape of the wave of flux distribution is determined largely by the shape of the pole face and the ratio of the pole arc to pole pitch. The armature teeth of course add ripples of high order, but usually of small magnitude. The latter are neglected in the following derivation because, even if they are of appreciable magnitude to cause trouble in telephone circuits, etc., they have little effect upon the effective armature m.m.f. or upon the value of terminal voltage. The third, fifth, seventh, and possibly the ninth, all of whose magnitude are determined by the design of the pole, are the terms which introduce errors into calculations based upon fundamental sine functions. To determine the effect of these harmonics, the following equations have been developed. They show that the effect of the harmonics upon the terminal voltage, for a given value of flux per pole, can be taken into account by a single factor, K_ϕ.

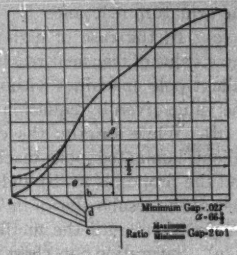


Fig. 36—Flux Distribution Curve

The flux waves of a number of representative machines were plotted by the following method. Refer to Fig. 36.

The flux at the middle of the pole was assumed to be 10, and the flux at any point toward the edge of the pole was taken inversely as the air gap. To obtain the flux beyond the edge of the pole, the distance *a b* was divided into the same number of spaces as the distance *c d* and lines drawn joining corresponding points on these lines. Assuming the flux to be inversely proportional to the length of air gap, the broken curve was derived. Since the effect of the adjacent poles would be to make the flux zero at the point midway between the poles, the solid line was drawn gradually deviating from the broken curve to pass through zero at the middle point.

The calculated flux waves were compared with test flux waves taken by exploring coils on three machines and found to agree very close'y. The effect of saturation of the teeth in the middle of the pole was found to make a slight difference in the flux wave shapes on the same machines at different voltages.

At crdinary tooth densities, however, the variation will make only one or two per cent difference in the value of K_ϕ, for values of voltage from one half to normal.

Flux waves were obtained by the method shown in Fig. 36 for ratios of pole arc to pole pitch of 50, 66⅔ and 75 per cent, for ratios of maximum to minimum air gap from unity to three to one, and for different minimum air gaps varying from one to four per cent of the pole pitch. The flux waves were then analyzed for the harmonics up to the eleventh, and the values of K_ϕ were calculated. These values of K_ϕ are given in Fig. 20.

The factor K_ϕ gives the ratio of the total flux in the irregular wave to the total flux in a sine wave which would give the same effective voltage at the terminals of the machine.

This is exactly true only when the terminal voltage has a form factor equal to that of a sine wave. This is approximately the case with practically all commercial machines as now built, as the distribution of armature winding, short pitch of armature coils and star connection of the machines all operate to eliminate the effect of harmonics in the flux wave.

Let Fig. 36 represent the flux distribution at no load.

$$\beta = (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots) \qquad (1)$$

Instantaneous voltage generated in unit length of one conductor is

$$e' = \frac{\nu\beta}{10^8} = \frac{\nu}{10^8}(\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots) \qquad (2)$$

Where

ν = velocity of flux relative to conductor, in centimeters per second.

β = flux density, lines per sq. cm.

Effective value of voltage is

$$e_{eff.} = \sqrt{\frac{2}{\pi}\int_0^{\pi/2} e'^2 d\,\theta}$$

$$= \frac{\nu}{10^8}\sqrt{\frac{2}{\pi}\int_0^{\pi/2} \beta^2\, d\,\theta} \qquad (3)$$

$$\beta^2 = (\beta_1 \sin\theta + \beta_3 \sin 3\theta + \beta_5 \sin 5\theta + \ldots)^2$$
$$= \beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\theta + \beta_5^2 \sin^2 5\theta + \ldots$$
$$+ 2(\beta_1 \beta_3 \sin\theta \sin 3\theta + \beta_1 \beta_5 \sin\theta \sin 5\theta$$
$$+ \beta_3 \beta_5 \sin 3\theta \sin 5\theta + \ldots)$$

Since $2\int_0^{\pi/2} (\beta_1 \beta_3 \sin\theta \sin 3\theta + \beta_1 \beta_5 \sin\theta \sin 5\theta + \beta_3 \beta_5 \sin 3\theta \sin 5\theta + \ldots)$ is zero.

$$e_{eff.} = \frac{\nu}{10^8} \sqrt{\frac{2}{\pi}\int_0^{\pi/2} (\beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\theta + \beta_5^2 \sin^2 5\theta + \ldots)\, d\theta}$$

$$e_{eff.} = \frac{\nu}{10^8\sqrt{2}} \sqrt{(\beta_1^2 + \beta_3^2 + \beta_5^2 + \ldots)} \tag{4}$$

$$e_{ff.} = \frac{\nu \beta_1}{10^8\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{5}$$

Where

$$k_3 = \beta_3 \div \beta_1$$
$$k_5 = \beta_5 \div \beta_1 \quad \text{etc.}$$
$$\nu = 2\tau f \tag{6}$$

where

τ = pole pitch

f = cycles per second.

$$e_{ff.} = \frac{2\tau f \beta_1}{10^8\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{7}$$

The effective voltage of a complete, full-pitch coil of N_q series turns, (that is, concentrated winding) is

$$E_{eff.} = 2 N_q\, l\, e_{eff.} \tag{8}$$

where

l = gross stacked length of core (iron + ducts)

$$E_{eff.} = \frac{4\tau l f N_q \beta_1}{10^8\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{9}$$

If the coil does not span the full pitch τ, the quantity under the radical is reduced. The voltage of the two coil sides are out of phase by angle $(1-p)\ \pi$, for fundamental and by angle

$$(1 - p)\ n\ \pi$$

for n *th* harmonic,
where,

$$p = \frac{\text{coil pitch}}{\text{pole pitch}}$$

Hence the reduction factor will be the cosine of half the angle, that is,

$$k_p = \cos\ (1-p)\ \frac{\pi}{2} \tag{10}$$

for fundamental and

$$k_{p_n} = \cos\ (1-p)\ \frac{n\ \pi}{2} \tag{11}$$

for n th harmonic.

[17]Adams has given curves for k_{pn}

If the N_q series turns per pole per phase are distributed in two or more slots, there is an additional reduction. This factor k_{d_n} is given in table prepared also by [17]Adams.

If the three-phase machine is connected Y, there is a still further reduction, but since the reduction for fundamental is same as for all harmonics, namely 0.866, except the 3rd and 9th which are entirely eliminated, it is not necessary to take this into account separately, as is done with k_{p_n} and k_{d_n}. [17]Adams gives also a table showing the product

$$k_{p_n}\ k_{d_n} = k_{r_n} \tag{12}$$

for different conditions.

Hence the effective voltage per phase (per leg) is

$$E_{eff.} = \frac{4\ \tau\ l\ f\ N_p\ \beta_1}{10^8\ \sqrt{2}}\ \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5}\ k_5^2 + \ldots} \tag{13}$$

Where

N_p = series turns per phase.

To find value of β_1, refer to equation (1)

$$\beta_1 = \beta_1 \sin\ \theta + \beta_3 \sin\ 3\ \theta + \beta_5 \sin\ 5\ \theta + \ldots \tag{1}$$

17. TRANS. A. I. E. E., Vol 28, Part 2, p. 798, 799, 805.

$$\beta_{ave.} = \frac{2}{\pi}\int_0^{\pi/2} \beta \, d\,\theta$$

$$\beta_{ave.} = \frac{2}{\pi}\left[\beta_1 + \frac{\beta_3}{3} + \frac{\beta_5}{5} + \ldots\right]$$

$$\beta_{ave.} = \frac{2}{\pi}\,\beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{14}$$

Total flux per pole is

$$\Phi = \tau\, l\, \beta_{ave.}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{15}$$

Let

$$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{16}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\, K_\phi$$

$$\beta_1 = \frac{\Phi}{\frac{2}{\pi}\,\tau\, l\, K_\phi} \tag{17}$$

Substituting in (13)

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi}{10^8\, K_\phi}\sqrt{k_r^2 + k_{r3}^2\, k_3^2 + k_{r5}^2\, k_5^2 + \ldots} \tag{18}$$

It is obvious from (18) that if fractional pitch is used and the coils are distributed, k_{rn} becomes practically negligible for the higher harmonics. If the machine is Y-connected, the third and its multiples are eliminated, leaving the value of the radical very near to

$$k_r$$

which is

$$k_p\, k_d$$

In most cases the radical may be taken as $k_r = k_p\, k_d$ with very little error in E_{eff}.

In concentrated windings, full pitch, Δ-connected, the radical cannot be neglected. It can be neglected with little error for machines with two or more slots per pole per phase, or frac-

tional slots per pole per phase, Y-connected armature. In such cases,

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi\, k_p\, k_d}{10^8\, K_\phi} \tag{19}$$

This means that the flux of the fundamental only

$$\Phi_1 = \frac{\Phi}{K_\phi}$$

is useful. That is, the useful flux is

$$\Phi_1 = \int_0^\pi \beta_1 \sin\theta\, d\,\theta \tag{20}$$

This value Φ_1 is what design calculations have usually been based on, *i.e.* calculations which assumed sine distributions of flux.

Therefore, such calculations should be modified only by the factor

$$K_\phi$$

Thus

$$\Phi = K_\phi\, \Phi_1$$

$$\Phi = \frac{E_{eff.} \times K_p \times K_d \times K_\phi \times 10^8}{4.44\, f\, N_p} \tag{21}$$

where

$E_{eff.}$ = effective voltage per phase. *i.e.* per leg

N_p = series turns per phase.

f = frequency.

$K_p = \frac{1}{k_p}$ = fractional pitch coefficient ordinarily used in design

$K_d = \frac{1}{k_d}$ = distributed coefficient ordinarily used in design, and given in Table II.

$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right)$, and is given in Fig. 20.

Where

k_3 = percent third harmonic in flux wave
k_5 = " fifth " " " "
k_7 = " seventh " " " "
etc.

If the radical in equation (**18**) is not negligible, (**21**) becomes

$$\Phi = \frac{E_{ff} \times K_\phi \times 10^8}{4.44 f N_p} \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5}^2 k_5^2 + \ldots} \quad (22)$$

APPENDIX B

Armature Reaction

Much has been written about armature reaction, particularly in Europe and during the last few years[18]. Practically without exception the method of attacking the problem has been to obtain an expression for the wave of armature m.m.f. and an expression for the wave of field m.m.f. and combine the two for the resultant which determines the flux. But at this point difficulty is encountered, especially on salient-pole machines. One must know the reluctance at all points, as well as the m.m.f. before the flux can be obtained. On turbine generators where reluctance is practically uniform over the whole pole pitch, the flux can be assumed to be proportional to the m.m.f., and the problem works out well. However, such a method has not succeeded in solving, without questionable assumptions, the case of salient poles.

The method given in the following pages for solving the case of salient poles is based on the principle that *the effectiveness of an armature ampere turn at any instant and at zero power factor, as compared with a field ampere turn, in establishing flux in the mutual magnetic path, depends upon the percentage of the total useful no-load flux that the armature turn would enclose at that instant on open circuit.* The method was worked out during the investigation of armature self-induction covered in the main body of the paper. The authors were met in this investigation by the necessity of determining more accurately the relative strength of armature and field, since the armature self-induction is tested by taking the difference between these quantities on sustained short circuit. This, as in the case given in Appendix A, is one of the few instances where an apparently hopeless case has

18. See bibliography at end of Appendix B.

worked out to the simplest result: the armature reaction is calculated by the familiar formula based on sine wave assumption, and this is modified by a single factor which may be taken directly from the curve shown in Fig. 20

It was found by methods described in Appendix A that the no-load flux wave could be very closely determined from the dimensions of the machine. The flux-wave shape was thus plotted for the variety of conditions within the ranges of normal design. These waves were expressed as a Fourier's series. With the equation of the wave given, the solution easily followed.

Let Fig. 37A represent the no load flux wave. If one armature ampere turn is placed symmetrically about the field magnetic circuit (*i.e.* directly opposite it, when $\theta = O$ in Fig. 37A), the flux wave will be a replica of that produced by one field ampere

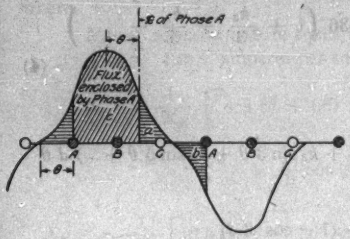


FIG. 37A

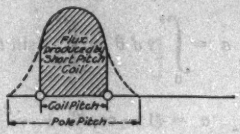


FIG. 37B

turn. If the coil does not span the entire pole pitch, then within the portion which is spanned the flux wave will be a replica of that same portion of a complete wave of a full-pitch turn, as in Fig. 37B.

On sustained short circuit, an armature current is maintained which is just sufficient to create an opposing m.m.f. acting on the mutual magnetic circuit, equal to that of the field; or in other words, sufficient to annihilate the mutual flux. Hence, as mentioned above, the measure of the effectiveness of any turn at any instant is the percentage of the total mutual flux it would enclose on open circuit.

Refer to Fig. 37A. A full-pitch turn, A, encloses all of the flux when in position $\theta = 0$. At any angle θ, observe that the flux enclosed by A is the same as if the two sides of the coil were moved toward the center of the flux wave by the same angle, θ;

because the positive flux, a, is now neutralized by the negative flux b.

The effectiveness of the turns in phase A at the angle θ is

$$\lambda_\theta = \frac{\text{area } c}{\text{area total wave}} = \frac{c}{c + 2a} \tag{1}$$

As in Appendix A, the equation of the wave is

$$y = \sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + \ldots. \tag{2}$$

$$a = \int_0^\theta y\,d\,\theta \tag{3}$$

From equations (14) and (16), Appendix A, it follows that

$$c + 2a = \pi \times \text{average ordinate}$$

$$= \pi \times 0.636 \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right)$$

$$c + 2a = 2\,K_\phi \tag{4}$$

From (3)

$$a = \int_0^\theta y\,d\,\theta = \int_0^\theta (\sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + ..)\,d\,\theta$$

$$a = (1 - \cos\theta) + \frac{k_3}{3}(1 - \cos\,3\,\theta)$$

$$+ \frac{k_5}{5}(1 - \cos 5\,\theta) + \ldots.$$

$$a = K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right) \tag{5}$$

From (4),

$$c = 2\,(K_\phi - a)$$

Hence, from (1)

$$\lambda_\theta = \frac{2\,(K_\phi - a)}{2\,K_\phi}$$

$$\lambda_\theta = \left(1 - \frac{a}{K_\phi}\right) \tag{6}$$

$$= 1 - \frac{K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right)}{K_\phi}$$

$$\lambda_\theta = \frac{1}{K_\phi}\left(\cos\theta + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}\cos 5\theta + \ldots \right) \tag{7}$$

For the case of a three-phase machine with 60-deg. phase belt, and concentrated winding, and assuming a sine wave armature current (which practically exists on sustained short circuit), the effective m.m.f. of phase A, Fig. 37A is

$$\lambda_\theta \sqrt{2}\, N_q\, I \cos\theta \tag{8}$$

of phase B,

$$\underset{(\theta+60)}{\lambda} \sqrt{2}\, N_q\, I \cos(\theta + 60) \tag{9}$$

of phase C,

$$\underset{(\theta+120)}{\lambda} N_q\, I \cos(\theta + 120) \tag{10}$$

where,

N_q = series armature turns per pole per phase.
I = r.m.s. amperes per turn.

$$\underset{(\theta+60)}{\lambda} = \frac{1}{K_\phi}\left[\cos(\theta + 60) + \frac{k_3}{3}\cos(3\theta + 180) + \frac{k_5}{5}\cos(5\theta + 300) + \ldots \right]$$

$$\underset{(\theta+60)}{\lambda} = \frac{1}{K_\phi}\left[(0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}(0.5\cos 5\theta + 0.866\sin 5\theta) + \frac{k_7}{7}(0.5\cos 7\theta - 0.866\sin 7\theta) \right] \tag{11}$$

This neglects harmonics above 7th.
By the same reasoning,

$$\underset{(\theta+120)}{\lambda} = \frac{1}{K_\phi}\left[(-0.5\cos\theta - 0.866\sin\theta) + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}(-0.5\cos 5\theta + 0.866\sin 5\theta) + \frac{k_7}{7}(-0.5\cos 7\theta - 0.866\sin 7\theta) \right] \tag{12}$$

The total m.m.f. per pole (*i.e.* the armature reaction) is

$$A = \lambda_\theta \sqrt{2}\, N_q\, I \cos\theta$$
$$+ \lambda_{(\theta+60)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta - 0.866\sin\theta)$$
$$- \lambda_{(\theta+120)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta + 0.866\sin\theta)$$
$$A = \sqrt{2}\, N_q\, I\,[\lambda_\theta \cos\theta + \lambda_{(\theta+60)}(0.5\cos\theta - 0.866\sin\theta)$$
$$- \lambda_{(\theta+120)}(0.5\cos\theta + 0.866\sin\theta)] \qquad (13)$$

Substituting equations (7), (11), and (12),

$$A = \sqrt{2}\, N_q\, I \left[\frac{\cos\theta}{K_\phi} \left\{ \cos\theta + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}\cos 5\theta + \frac{k_7}{7}\cos 7\theta \right\} \right.$$
$$+ \frac{(0.5\cos\theta - 0.866\sin\theta)}{K_\phi} \left\{ (0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\theta \right.$$
$$\left. + \frac{k_7}{7}(0.5\cos 7\theta - 0.866\sin 7\theta) \right\}$$
$$- \frac{(0.5\cos\theta + 0.866\sin\theta)}{k_\phi} \left\{ (-0.5\cos\theta - 0.866\sin\theta) \right.$$
$$+ \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}(-0.5\cos 5\theta + 0.866\sin 5\theta)$$
$$\left. \left. + \frac{k_7}{7}(-0.5\cos 7\theta - 0.866\sin 7\theta) \right\} \right]$$

Simplifying, this becomes

$$A = \frac{1.5\sqrt{2}\, N_q\, I}{K_\phi} \left[1 + \cos\theta \left(\frac{k_5}{5}\cos 5\theta + \frac{k_7}{7}\cos 7\theta \right) \right.$$
$$\left. + \sin\theta \left(-\frac{k_5}{5}\sin 5\theta - \frac{k_7}{7}\sin 7\theta \right) \right] \qquad (14)$$

If the armature turns are distributed over the phase belt in a number of coils instead of being concentrated in one coil, and if

the coils do not span the full pole pitch, the bracket quantity of equation (**14**) is modified as follows:

1 becomes $k_p\, k_d$

k_5 and k_7 are multiplied by k_{rn}.

Adams[19] has given tables for these reduction factors.

However, since the 5th and 7th are usually not large, and since, as shown by (**14**), their effect is decreased by the order of the harmonic, it may be safely assumed that the further decrease by k_{rn} renders negligible all of the harmonics, in cases where the winding is distributed in more than one slot per pole per phase.

Hence for such cases equation (**14**) becomes,

$$A = \frac{1.5\sqrt{2}\, N_q\, I}{K_\phi\, K_d\, K_p} = \frac{2.12\, N_q\, I}{K_\phi\, K_d\, K_p} \text{ (three-phase)} \qquad (\mathbf{15})$$

where

$$K_p = \frac{1}{k_p} = \frac{1}{\sin\,(p\ 90)}$$

$$K_d = \frac{1}{k_d} \text{ (See Table II, Part I, for value of } K_d.)$$

N_q = series turns per pole per phase

I = current per turn

K_ϕ = flux distribution coefficient given in Fig. 20.

APPENDIX C

Calculation of Excitation

The reactance as calculated by the method developed in this paper may be used in calculating excitation of alternating-current machines. The method used is an application of the generally accepted principles of excitation calculation, which have been developed by a number of writers in slightly different forms.

The diagram, Fig. 18, illustrates the factors used in the calculation for a generator with lagging current output.

I_n = normal armature current.

E = terminal voltage (is taken as unity for calculations on the per cent basis.)

E_i = in-phase component of terminal voltage

= $\cos\,\theta$

19. Trans. A. I. E. E., Vol. 28, Part 2, pp. 798, 799, 850.

E_x = quadrature component of terminal voltage
= $\sin\theta$
$\cos\theta$ = power factor
r_{pa} = voltage required to overcome resistance drop in per cent of E. ($r_{p_a} = 0.01$ means 1 per cent)
x_{pa} = voltage required to overcome reactance drop in per cent of E. ($x_{p_a} = 0.2$ means 20 per cent)
E_1 = internal induced voltage under load in per cent of E.
= $(\cos\theta \pm r_{pa}) + j(\sin\theta \pm x_{pa})$

From the diagram it can be seen that $\cos\theta = \dfrac{E_i + r_{pa}}{E_1}$

Note that for a motor the sign of r_{pa} will be minus.

Also $\sin\theta = \dfrac{E_x + x_{p_a}}{E}$

This expression holds for the over-excited condition, that is, for a generator with lagging current or a motor with leading current, but the sign of x_{p_a} will be minus for the under-excited conditions.

A = ampere turns per pole required to overcome the effective armature reaction.

$$A = \frac{2.12\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for three phase}$$

$$= \frac{1.41\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for two phase}$$

N_q = series armature turns per pole per phase
K_p = fractional pitch coefficient
K_d = armature winding distribution coefficient.
K_ϕ = flux distribution coefficient.
Φ = no load flux per pole for normal voltage E.
Φ_1 = full-load flux per pole for internal induced voltage E_1
Φ_2 = flux for impedance drop.
F_1 = ampere turns from no-load saturation curve required to produce Φ_1.
F_0 = full-load ampere turns per pole.

The m.m.f. diagram is all that is required for the calculation of the field current and this diagram taken from Fig. 18 for the over-excited condition is given in Fig. 19A, and for under-excited condition in 19B. The following relation may be derived from this diagram by inspection.

$$F_0 = (F_1 + A \sin \theta_1) + j\, A \cos \theta_1$$

For the under-excited condition the expression

$$F_0 = (F - A \sin \theta_1) + j\, A \cos \theta_1$$

The application of this theory can best be illustrated by an example. The formulas necessary for the calculations are given below

$$\cos \theta_1 = \frac{\cos \theta \pm r_{pa}}{E_1}$$

Sign is + for generator
Sign is − for motor

$$\sin \theta_1 = \frac{\sin \theta \pm x_{pa}}{E_1}$$

$$F_0 = (F_1 \pm A \sin \theta_1) + j\, A \cos \theta_1$$

Sign is + for generator lagging current or motor leading current.

Sign is − for generator leading current or motor lagging current.

$$E_1 = (\cos \theta \pm r_{pa}) + j\,(\sin \theta \pm X_{pa})$$

Example:

Generator—Lagging current.

Assume $r_{pa} = 0.01 = 1$ per cent
$x_{pa} = 0.15 = 15$ per cent
Power factor $\cos \theta = 0.80$
$\sin \theta = 0.60$
$F = 3800$ ampere turns (not used in calculations)
$F_1 = 4500$ ampere turns.
$A = 3000$ ampere turns.

Then $E_1 = (\cos \theta + r_{pa}) + j\,(\sin \theta + x_{pa})$
$= 0.81 + j\,0.75 = 1.10$

The value of F_1 is obtained from the no-load saturation curve

(test or calculated) at 110 per cent normal terminal voltage and is 4500 ampere turns.

$$\sin \theta_1 = \frac{0.75}{1.10} = 0.68$$

$$\cos \theta_1 = \frac{0.81}{1.10} = 0.735$$

$$\begin{aligned} F_0 &= (4500 + 0.68 \times 3000) + j\,0.735 \times 3000 \\ &= 6900 \text{ ampere turns.} \end{aligned}$$

Motor—Lagging current.

Same data as above except $F_1 = 3400$ ampere turns.

$$\begin{aligned} E &= (\cos \theta - r_{pa}) + j\,(\sin \theta - x_{pa}) \\ &= 0.79 + j\,0.45 = 0.91 \end{aligned}$$

F_1 corresponds to 91 per cent of normal voltage and is 3400 ampere turns.

$$\cos \theta_1 = \frac{0.79}{0.91} = 0.87$$

$$\sin \theta_1 = \frac{0.45}{0.91} = 0.495$$

$$\begin{aligned} F_0 &= (3400 - 0.495 \times 3000) + j\,0.87 \times 3000 \\ &= 3230 \text{ ampere turns.} \end{aligned}$$

The calculation of the above generator may be represented by the following simple calculation.

$$E \left\{ \begin{matrix} 0.81 \\ 0.75 \end{matrix} \right| = 1.10$$

$$F_0 \left\{ \begin{matrix} & & \dfrac{3000}{4500} = A \\ & & \dfrac{}{} = F \\ \dfrac{3000}{1.10} \times 0.75 = & \left.\begin{matrix} 2040 \\ 6540 \end{matrix}\right| & \\ & & = 6900 = F_0 \\ \dfrac{3000}{1.10} \times 0.81 = & 2210 & \end{matrix} \right.$$

Note that $\left.\begin{matrix} 0.81 \\ 0.75 \end{matrix}\right| = 1.10$ and $\left.\begin{matrix} 6540 \\ 2210 \end{matrix}\right| = 6900$

indicate combination of vectors at 90 deg.

This method of calculation of field current is an approximation, and does not allow for the increased field leakage under

load. This is probably compensated for by including the added no-load saturation in the armature magnetic circuit which does not exist under load. The method has been found to give a very close check on test results on a very large number of commercial machines.

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Discussion on "Reactance of Synchronous Machines and its Applications" (Doherty and Shirley), Atlantic City N. J., June 28, 1918.

Carl J. Fechheimer: All who have been identified with the design of a-c. synchronous machinery have appreciated the great importance of internal reactance of the stator. All of us have grappled with the problem of estimating the value of the reactance, and probably there are no two who will agree either on a method of calculating its value or even on its magnitude after the machine is tested. Certainly, we cannot agree upon the value of reactance until we establish what reactance means, and are in accord upon its valuation from test data. Thus, as pointed out by the authors, the reactance determined from instantaneous short circuits may be different than that which obtains during steady operation.

We shall enumerate the most important ways by which the stator reactance can be measured.

(1) With the use of the open-circuit saturation and short-circuit curves, and a knowledge of the proportions of the machine, the ampere turns required to neutralize the armature reaction on sustained short circuit may be deducted from the total ampere turns on short circuit, thus leaving the ampere turns which are needed to drive the flux through the magnetic circuit. From this latter, and with the open-circuit saturation curve, the reactance can be estimated.

(2) From no-load saturation curve, a corrected curve which assumes full-load leakage may be drawn. From this latter curve, and with the use of the load saturation curve at nearly zero power factor, the drop may be broken up into armature reaction and armature reactance by assuming that these are constant along the curve.

(3) The rotor may be removed and single-phase current at normal frequency may be circulated in the stator windings between one terminal and neutral for three-phase star connection. Inasmuch as the resistance drop is usually negligible, we may take the reactance to be the volts from terminal to neutral, divided by the current.

For three-phase delta, circulate current in one leg; for two-phase, circulate current in one phase.

(4) Same as 3, except that instead of single-phase current between neutral and terminal, polyphase current may be circulated between terminals.

(5) Same as 3, except with the rotor in position. (This gives different values for different positions of the rotor.)

(6) Same as 4, except with the rotor in position. (This gives different values for different positions of the rotor.)

(7) A condenser may be connected across the terminals of the machine, the value of the capacity of which may be varied, so that approximate resonance can be obtained. It is then a simple matter to calculate the inductance.

(8) The reactance may be estimated from oscillograms obtained on instantaneous short circuits. In that case, the peaks of the current should be projected back to the ordinates drawn at the instant of short circuit.

These eight methods of measurement give various values of reactance, although two or more methods may be found to check in some machines. What then shall we say is the correct method? The subject is well worthy of the attention of the Standardization Committee of the Institute. We shall attempt to discuss these methods briefly.

No. 1 (determination from short-circuit curve) has two decided objections: (a) It is difficult to estimate with sufficient accuracy the armature reaction, and a relatively small error in

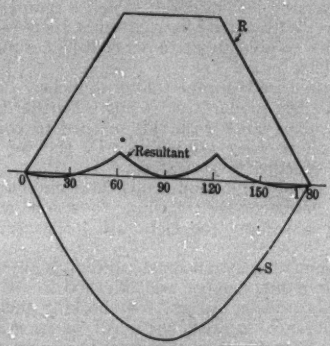

FIG. 1

this estimate will be magnified considerably in the value of the reactance; thus 5 per cent error in estimate of armature reaction may result in 40 per cent error in reactance. (b) The voltage which is induced in the individual conductors on sustained short circuit (and which is entirely consumed by impedance) is in general of badly distorted wave form.

For instance, we are showing in Fig. 1 the ideal field form of the magnetomotive force of the rotor of a turbo-generator R, and an assumed sine wave space distribution of the m. m. f. of the stator current S (ideal conditions). The resultant of these two gives the m. m. f. which drives the flux through the magnetic circuit. Although the higher harmonics which would result from the distortion are pretty well damped out in

the current wave form, they still appear in the wave of induced e. m. f. in the individual conductors. It will therefore readily be seen that reactance computed in this way from tests is liable to be considerably in error. That the distortion is very much reduced at full load, zero power factor, is quite evident.

No. 2 (determination from no-load and zero power factor load curves) has the objection that it involves calculation of the full-load leakage no-load saturation curve, which is subject to error. The proportions of reaction and reactance as finally determined by this method, even if we were able to estimate the full-load leakage curve, are also liable to be in error. Furthermore, saturation in the magnetic circuit may cause the reactance to change in magnitude along the curve.

Nos. 3 to 6, inclusive, (pertaining to circulating current in windings) all give different results, but our observations lead us to believe that No. 3 gives results which are more nearly correct than 4, 5 or 6. It is certain that with the rotor in position the reactance measured is considerably higher than that which actually obtains. With three-phase currents, there is sufficient mutual inductance to give about 15 per cent higher inductance than with single-phase.

No. 7 has the disadvantage that a condenser has not lent itself readily to use in our test rooms. Furthermore, we have had too little experience with this method to compare it with others, but believe there are possibilities and we therefore suggest that it be further investigated.

No. 8 (pertaining to instantaneous short circuit) has the drawback that it subjects the machine to severe strains, and is in general expensive and too elaborate for general commercial use.

Discussing first the sustained reactance, it is our understanding that the authors of the paper used Method No. 1 (short-circuit, and no-load saturation curves) for checking their formula for sustained reactance against test. All of us will agree that an analytical determination of reactance is next to impossible, and we must have recourse to test data, and thereby obtain empirical constants; especially is this true of end connection leakage, and we think that it is essential for tooth-tip leakage also. Inasmuch as the authors used a test method which is subject to an error of the order of 50 per cent, we must be cautious in adopting their formula, and rely upon the accuracy of reactance estimated thereby. Aside from the error pointed out in determining reactance from test, we find in Table I, that 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and the discrepancy in one case is 145 per cent, and in another 185 per cent.

We believe that the authors have allowed too much for increase due to mutual inductance with three-phase over single-phase. Tests which we have made removing the rotor and circulating current through windings, indicate only 16 per

cent higher reactance three-phase (star-connection) than single-phase terminal to neutral.

Turning now to the reactance which obtains in instantaneous short circuit, we are in agreement with the authors that the leakage flux of the rotor plays a considerable part in reducing the rush of current, although we do not view the phenomenon from the same point of view. For instance, we believe that saturation of the rotor poles in conjunction with rotor leakage flux, may have considerable influence in reducing the pole leakage. For example, let us assume that the combined transient reactance is such that the stator current can rise to ten times normal r. m. s. value, and that the total ampere turns in the rotor also rise to ten times the no-load value. Then the difference in magnetic potential between poles, being ten times, the leakage flux will rise to ten times. If the no-load field leakage were 12 per cent, then with ten times field current, there will be $12 \times 10 = 120$ per cent leakage, so that, if the density in the poles at no load were 90,000 lines per square inch, the density would rise to $\left(\frac{1 + 1.2}{1 + 0.12}\right) 90{,}000 = 177{,}000$ lines per sq. in. That the density in the poles does not reach such high value will be quite evident from a glance at the B - H curve for the steel, and the result is that some of the flux which had previously been useful as generating e. m. f. in the stator conductors, "sinks" into leakage flux, so as to maintain substantially constant interlinkages with the conductors in the rotor (these may be field turns, rivets, or any paths for the currents). Saturation of the stator teeth may also play some part, for with the reduction in main flux, more leakage flux may flow, and this may, in some teeth, be of sufficient magnitude to reduce the percentage leakage. With smooth, solid rotors, such as those in turbo-generators, the phenomenon as regards pole saturation is quite different from that which obtains with definite poles, because most of the counter m. m. f. in the rotor is produced by eddy-currents near the rotor surface, and they are not, in turn, productive, of a great percentage increase in leakage on instantaneous short circuit. Although it is very difficult to check with the sustained short circuit and no-load and zero power factor saturation curves, it seems that the transient and sustained reactances in turbo-generators are not in very great disagreement, and we note that the authors are of the same opinion.

Other factors of not negligible importance as affecting the transient reactance might be cited. We shall mention only one: In salient-pole alternators, most of the spiders are solid, and as is well known, the flux (pulsates whether polyphase or single-phase short circuit) and eddy-currents are induced in the spider, which in turn influence the rotor leakage.

As we look over the authors' paper we note that they believe "saturation is not a very serious factor," and they have not

considered eddy currents in the spider. We are very much of the opinion that such factors cannot be ignored if even reasonably approximate accurate results are desired.

Again, eddy-currents and saturation play their parts in estimating the attenuation factors—as both affect resistance and inductance. Owing to the fact that the saturation of the poles decreases with time, due to reduction in m. m. f. and flux, and therefore (1) the distribution of leakage fluxes from pole to pole are altered (assuming salient pole-construction) which in turn affects the interlinkages with the field winding, (thereby changing the coefficient of self-induction of the field circuit); and (2) the percentage of the total flux which enters the stator is automatically increased. Inasmuch as eddy currents are in parallel with the field currents, insofar as net m. m. f. is concerned, the resistance and inductance of their paths are necessarily coupled magnetically with that of the field circuit. It will, therefore, be seen that an analytical determination of the attenuation factors is well nigh impossible, and that these factors may not be constant, but change with time.

The whole problem involves so many difficulties and complications that we believe that we must rely upon empirical data. Our first problem is to find a way of measuring sustained reactance, and then modify the constants in an empirical equation to enable prediction of reactance within a maximum error of, say, 15 to 20 per cent. Then we can multiply this value of reactance by some factor or factors determined from test so as to enable agreement with transient reactance, the factors being of different magnitude for various types of machines. The determination of the field and armature attenuation factors which are probably not constants, must be determined from oscillograms of short circuits, and should be some function of the time.

In regard to the measurement of sustained reactance, we are planning to conduct some tests in line with method No. 7 (involving a condenser) and take oscillograms to determine the magnitude of the fundamental as well as of the harmonics. This method will not, we believe, be suitable for general application, especially not on large machines, because of difficulty in obtaining large condensers, but we hope thereby to establish some constants for modifying measured sustained reactance (say by circulating current in stator with rotor removed) which we can adopt as reliable figures.

I have noted the authors' reference to an error in the writer's paper, and wish to thank them for calling attention to it. I previously noted this mistake about four years ago, but have not called it to the attention of the Institute members.

I believe the authors deserve a great deal of credit for the great amount of work which they have done, for more work of this character would be helpful in cleaning up some of the problems with which we are confronted.

V. Karapetoff: It seems to me that the terms reactance and inductance are used somewhat indiscriminately in this paper. After formula (3) in Part II. the authors say: "The effective field self-induction or field reactance." Reactance is a different concept from self-inductance, and in a paper which is intended for reference purposes it would seem rather unfortunate not to have these terms more definitely discriminated because one has to find out from the context whether the reactance or the inductance is meant. The same criticism applies to other places in the paper.

I should like to see the next administration of the Institute make a strong effort towards improving the "readibility" of the Institute papers. In a mathematical paper like the one under discussion a very careful list of symbols should be demanded of the author, and not left to the author's discretion. It is true that in this particular paper partial lists of symbols are found in two places, but they do not include, by any means, all the symbols, nor do they show in which units these symbols are expressed. I think that this is an important matter, and should not be left to the discretion of the author, but should be made uniform by definite rules of the Editing Committee of the Institute.

Now, as to the method of computation of inductance. A number of years ago our new honorary member, Mr. Oliver Heaviside, showed that inductance and magnetic permeance are almost identical concepts, that the inductance of a coil is equal to the permeance of the magnetic path, times the square of the number of turns. Since the number of turns is a numeric, the physical dimension of inductance is the same as that of magnetic permeance.

The expression for a magnetic permeance is similar to that of the electrical conductance. The permeance of a uniform path is equal to the permeabilty of the medium times the cross-section of the path, and inversely as the length of the path. If the path is not uniform a differential expression for permeance must be properly integrated to cover the whole field. What I am driving at is that computations of inductance should be performed in the same way as computations of the permeance or conductance of an irregular path, that is, by adding the permeances of the individual component paths. Most authors including these authors, compute inductances by taking ratios of magnetomotive forces to the fluxes. That is an unnecessary complication.

Suppose you had an irregular resistance to figure out, you would not assume certain e. m. fs. and currents, and divide one by the other, you would use a formula for an element of resistance, and properly make a summation of such terms for an irregular shape. Why, then, is it necessary in a paper on inductance always to use the ratio of ampere-turns to fluxe ? If you will glance at the formulas in the paper you will find every-

where a ratio of flux to ampere-turns. Why not call this ratio the permeance, and deal with permeances entirely? This procedure has been followed consistently in the chapters on inductance in my book entitled "The Magnetic Circuit."

I notice from the list of references that the authors are more familiar with the English and German literature, and have apparently neglected the French contributions to the subject. While they give credit to Rezelman, through a translation in *The Electrician*, his greater work on leakage reactance in alternators in *La Lumiere Electrique* is not mentioned. Also there is no reference to the latest contributions by André Blondel on the armature reaction in synchronous machines. I believe the paper would gain if more use had been made of the work of the French and Belgian investigators on the subject.

I do not wish to detract from the value of the work done by the authors and the remarkable frankness with which the results are presented, including the test data. I fully appreciate the importance of such a paper for practical designers and students, and my only plea is for a better and clearer presentation of the subject. We are swamped with new reading matter, and yet a day still has only twenty-four hours, so that the skill in presentation almost determines the fate of a paper, that is, whether it will be used or not. If it is not properly presented so that it takes too long to get the meat out of it, many people will not take the trouble to refer to it.

F. D. Newbury: Before we can harmonize various methods of calculating reactance, we must agree upon a method of determining reactance by test; we must define the quantity we are attempting to calculate.

In the alternator, there are two fluxes that are set up by the armature current and the difficulty and confusion in this matter arises from the difficulty in separating one of these fluxes from the other. One produces armature *reaction* and the other produces armature *reactance*.

When the alternator is operated at synchronous speed, on short circuit, and rated current circulates through the armature, the value of field ampere turns is a measure of the m. m. f. necessary to produce both of these fluxes.

The flux of armature *reaction* is that part of the flux set up by the armature current that combines with the field winding flux to produce the effective or resultant exciting flux; the flux of armature *reactance* is that part of the armature flux that is more or less completely localized about the armature winding, and that, by its self-inducing effect, produces a voltage of self-induction in the armature winding that combines with the main induced voltage to produce the terminal voltage. Thus armature reaction may be conveniently treated as a flux phenomenon, as its action does not progress beyond the flux stage; following the same thought, the armature reactance may be considered as a voltage phenomenon. The effect of armature reaction

on the total m. m. f. required for a given voltage and current condition is the same at zero voltage, normal voltage, or any other terminal voltage; the effect of armature reactance on the total m. m. f., on the contrary, varies considerably with the terminal voltage because it appears as a voltage and the m. m. f. to generate a certain additional voltage at low terminal voltages is very much less than the m. m. f. required to generate the same additional voltage at a high terminal voltage, because of magnetic saturation.

This distinction is important because it leads to a fairly satisfactory method of separating these two parts of the armature flux. I refer to the graphical method based on the no-load saturation curve and saturation curve at zero-power factor and full-load current. This method, I believe, was first called to the attention of the Institute some fifteen years ago by Mr. B. A. Behrend. In my experience, it gives consistent results for a wide range of machines. It has the advantage that it is obtained by test under flux and voltage conditions very nearly approaching those existing during normal operation. This cannot be said when the separation is based entirely on the short-circuit current at zero terminal voltage which is the method used by the authors. The authors calculate the armature reaction, which is the larger of the two parts, and subtract it from a test value, representing the total armature flux. This test method is in reality a calculated method, because an error of, say five per cent in the calculation of the armature reaction flux, may result in an error of 20 to 40 per cent in the reactance voltage. The graphical method referred to above also involves a calculated correction but a given error in this has a much smaller effect on the result than does the same percentage error in the calculated part in the authors' method.

Mr. Fechheimer referred, in his communication, to various methods of obtaining the reactance by test. Each of these methods involves different conditions, and no two methods really give the same quantity. The first difficulty in this problem, and the main difficulty, I feel, is that we have not agreed on a method of test for reactance. We have not agreed on the fundamental definition of the quantity we propose to calculate. If we can take that step, I think it is a fairly easy matter to obtain methods of calculating reactance that will check with such test results. It may not be possible to obtain a general formula such as the authors have tried to do, that will hold for all machines of all proportions, but it is not a difficult matter to make up a more or less empirical formula for a given line of machines which will check with test results.

As an example of results from the use of some of the methods cited by Mr. Fechheimer, I give the test results of a 4500-kv-a., 11,000-volts, single-phase, 25-cycles, 500 rev. per min. generator.

Tests were made at 4500 kv-a., single-phase, and at 7800 kv-a., three-phase:

	At 4500-kv-a. 1-phase.	At 7800-kv-a. 3-phase.
From instantaneous short-circuit test, (by oscillograph)	13%	10%
From no-load and zero power factor saturation curves	13	11
From locked saturation test (rotor at rest)	15	12.5
From test without rotor in position	21	18

This machine had a low resistance damper winding on the rotor. From the locked saturation test, that is, with the rotor at rest, the reactance is somewhat higher than during synchronous operation, but still quite close to it. This is reasonable, because although the rotor is at rest, the low resistance damper winding practically wipes out that part of the flux corresponding to armature reaction in synchronous operation, so that there is left practically the same flux that, in synchronous operation, is responsible for armature reactance. The same agreement would not exist if the generator did not have the damper winding. A test without the rotor in position showed the reactance to be about 50 per cent greater, or 21 per cent. This, again, is reasonable, because the entire flux set up by the armature winding is effective in producing reactance voltage.

The second part of the paper, referring to sudden short-circuit phenomena, is extremely important, in a practical way. Reactance of alternator windings is of interest, quantitatively, only to designers, except as applied to sudden short-circuit conditions. In this matter, users generally are interested in the value of reactance since it largely determines the value of transient currents during short circuit.

The authors state that the total reactance effective during sudden short circuit is considerably greater than armature reactance, this difference amounting in some cases to 50 per cent of the armature reactance. The authors' method of obtaining armature reactance from tests is so open to question that this generalization is not warranted. In my own experience the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit. This does not mean that the two reactances are identical; it only means that in the total reactance (effective during short circuit) the increase in reactance due to the field has been neutralized by some other action. What this complete action is, we don't know. The fact remains that in short-circuit tests made on generators varying from 4000 to 20,000 kv-a. in rating, with a wide range in speed, and with laminated and solid rotors, with and without damper windings, the tested

sustained armature reactance has practically the same value as the total reactance obtained from short-circuit oscillograms. Here again, the difference in test results and conclusions may be due to differences and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled.

W. F. Dawson: Mr. Newbury raised the point of the discrepancies between the figures shown by Messrs. Doherty and Shirley in their method, and his own method, and I am moved to ask if he has considered fully the question of end coil support, the nature of the material that supports the end coil, particularly in turbo-alternators, and the material that clamps these coils? I know personally there is a slightly different method right within the General Electric Company's works. The Schenectady engineers employ a different material for coil support, and rather more of it that I do myself, and I believe the Westinghouse support is different again, and the method of clamping the coils is different. There certainly must be a difference in reactance, whether it is transient reactance or the steady reactance, that effects the excitation, due to these several different methods of support. I simply offer this as a suggestion which may explain the discrepancy.

V. Karapetoff: I should like to ask the authors regarding Fig. 25A. The orthogonal rulings shown there indicate magnetic lines of force and equipotential surfaces. This ruling extends into the space occupied by the coils. As far as I am aware, the magnetic flux has no potential in the space occupied by the current, and I do not quite see how equipotential surfaces could consistently be continued into the space occupied by the coils.

In the simplest case of a circular conductor the magnetic lines of force are concentric circles surrounding the conductor, and the flux inside the conductor also consists of concentric circles. The equipotential surfaces outside the conductor are radial planes, but these planes do not extend inside the conductor, because there the difference of magnetic potential between two points depends upon the path selected. I therefore feel that it might be better to omit the orthogonal ruling or at least the equipotential lines, within the space occupied by the coils, so as to avoid a possible criticism on this point.

Charles L. Fortescue: Is not the uncertainty of this method due, as Mr. Newbury says, to its empirical nature? If we consider the dynamic equations of a system like this, we find that we have a set of three or more linear differential equations of the first order. The solution of such a system of equations depends upon another differential equation, which is of the third or fourth order according to the number of equations. That would mean that the actual transient solution would involve three or more arbitrary constants and, instead of one reactance three or four different reactances would enter into the transient

solution in a manner depending on the conditions of the initial short-circuit. It seems to me therefore that it would be very difficult, to express the results in terms of a single compound reactance. We would find possibly that the value of the effective reactance would depend upon at what point on the voltage wave the short circuit took place and that the solution for othcr conditions, would require different values for the effective reactance, or, putting it another way, one would have to have a different effective reactance for each short circuit. The problem is quite complicated, and as I said before, it is possible that the empirical nature of this reactance accounts for the fact that it does not check up under all conditions.

H. R. Summerhayes: This question of reactance is exceedingly important, not only from the standpoint of the designing engineer to ascertain what the machine will do but for the engineer working on the system who wants to know what amount of energy is released when a short circuit occurs. His calculations on the rupturing capacity of switches, the strength of transformers the installation of protective reactance and setting of relays all depend on the short-circuit values of the generator for different time intervals. In order to properly design the system we must know the characteristics of the generator and this work of the authors is a pains taking attempt to outline methods of obtaining these data.

I hope that the paper will lead to a discussion among the engineers and manufacturing companies which will, if it does not bring to standardization all methods of calculation, at least cause them to come to some agreement on the nomenclature, so that, when the reactance of synchronous machines is discussed engineers will talk the same language and will know what kind of reactance is meant and what factors are included. This will go a long way toward the correct determination of the actual and comparative values for different machines.

N. S. Diamant: In order to make the discussion of this paper as brief and as definite as possible let me start by making the following statements, even at the risk of seeming to be a little dogmatic:

A. The writers do not in any way contradict or disprove any theoretical or experimental results given in my 1915 paper on sudden short circuits.

B. It is very gratifying that they corroborate a number of points brought out in the paper referred to and even those which they think they contradict.

C. The writers are laboring under the serious and entirely false impression that *they* for the first time are calling attention to a fundamental fact which is the a, b, c, of sudden short circuits or of any transient phenomenon for that matter. It was clearly explained by Boucherot in 1911; it was referred to by Berg and experimentally illustrated in my paper.

D. The writers are using in their paper, as published at pres-

ent, a terminology entirely different from that of other writers and mine, and for some reason which I cannot quite understand, they have failed to ascribe to the terms self-inductance the proper or at least a possible meaning and thus were led first to believe that they disagree with me in certain points, when they actually do not, and secondly were led to the false impression stated under C.

E. The writers make several valuable contributions: (1) They give an expression for calculating the armature leakage reactance. (2) An expression for calculating the leakage reactance of the field. This so far as I am aware is new, although the leakage of the field in "per cent" is derived and used by designers. (3) They have shown how unpractical and inefficient it is to use terms in a loose and unscientific fashion. (4) They have shown that the bibliography they give at the end of their paper, (to which it would be well to add Boucherot's memoirs and Douglas' paper) if properly studied and used, would have shortened their paper considerably and made it more useful. However, the mental attitude of the writers towards the work of others does not seem to be critical and aggressive enough if one is to judge from their statement—"A bibliography which may be of historical interest is included".

Now I shall proceed to prove the above statements, one by one. I regret that with reference to C and D I shall have to introduce here some elementary text-book material, as follows:

Definitions of Four Kinds of Inductances or Reactances

A straight thin wire carrying ten amperes and having Φ lines set up around it will have a self-inductance L equal to Φ abhenries. In case of a solenoid, etc. exactly the same definition holds, except that the calculation of L becomes more complicated. (See Bulletin Bureau of Standards, Vol. 8, No. 1, where formulas and tables are given for the calculation of L. For experimental measurement of L see Laws, Eric Gerard and other works on electrical measurements.) Now instead of a single isolated coil or system as above we have a second coil, mutually inductively related to the first, we introduce a second concept—that of the mutual inductance M. M is defined in any text in physics or electrical engineering as $\Phi_{12c}\, n_2$; where, Φ_{12c} = flux per abampere (flowing in the primary) which is common to both coils or links both coils, and n_2 = number of turns in secondary. Similarly if we reverse the role of the coils and send 10 amperes through coil No. 2, instead of No. 1 we get $M = \Phi_{21c}\, n_1$ ($= \Phi_{12c}\, n_2$ if μ is constant). For calculation and measurement of M see references given for L.

The above concepts of self- and mutual inductance are fundamental and absolutely necessary both to the physicist and engineer. However, the former seems to find these sufficient while the engineer is forced to introduce two more concepts, namely that of "leakage inductance" of primary, or secondary

or of armature or of field etc. and that of "total leakage inductance reduced to" primary or to secondary or to armature, as the case may be. Let us denote these by α and λ respectively. Then $L =$ leakage inductance of coil No. 1 $= n_1 \Phi_{1l}$ where Φ_{1l} is the leakage flux of coil No. 1, *i. e.* the flux that interlinks with coil No. 1 only. Thus if $\Phi_1 =$ total flux set up by 10 amperes flowing through coil No. 1 $= (\Phi_{12c} + \Phi_{1l})$, we have,

$L_1 = n_1 \Phi_1 =$ self-inductance of coil No. 1

$\alpha_1 = n_1 \Phi_{1l} =$ leakage inductance of coil No. 1

$M = n_2 \Phi_{12c} =$ mutual inductance of coil No. 1

Similarly if we reverse the role of the two coils we get,

$L_2 = n_2 \Phi_2 =$ self-inductance of coil No. 2

$\alpha_2 = n_2 \Phi_{2l} =$ leakage inductance of coil No. 2

$M = n_2 \Phi_{21c} =$ mutual inductance of coil No. 2

From the above it follows directly that,

$$M^2 = (L_1 - \alpha_1)(L_2 - \alpha_2)$$

and if we imagine the leakage of No. 2 to be zero and charge all the leakage to coil No. 1 we get

$$M^2 = (L_1 - \lambda_1) L_2$$

or $\lambda_1 = \left(L_1 - \frac{M^2}{L_2}\right) =$ total leakage inductance reduced to coil No. 1.

Similarly for λ_2.

Simple Application of Above. Consider the case of a single coil or in general the case where the mutually inductive effect of coil No. 2 is nil. Then, if we impress an alternating e. m. f. of frequency f across coil No. 1, $I_1 =$ current in coil No. 1 $= E_1/Z_1$ where $Z_1^2 = r_1^2 + 2\pi f L_1^2$. This of course refers to permanent conditions. Under transient conditions, with either alternating or direct current we will find that the rise or decay of current depends upon $\alpha_1 =$ attenuation factor of coil No. 1 $= r_1/L_1$.

Next let us take the case where $M \pm O$, *i. e.* there is a closed coil No. 2, mutually inductively related to No. 1. If we impress the same e. m. f. as before, we shall find that $I_1 =$ current in coil No. 1 $= E_1/Z_1$ where now Z_1^2 is equal to $(r_1^2 + 2\pi f \lambda_1^2)$ and not $(r_1^2 + 2\pi f L_1^2)$ and the latter may be 5 or 10 or 20 times as large as the former. Furthermore $\alpha_1 =$ attenuation factor of coil No. 1 $= r_1/\lambda_1$ and not r_1/L_1 as before. Thus as we all know, from our study of a choke coil and transformer, it makes all the difference in the world whether there is a secondary coil affecting the primary coil or not. At this point let me call attention to: First, that the matter is simplified like this, if the secondary resistance is negligible; and secondly, that though I have used the general terms coil No. 1 and No. 2 the above apply to any system be it primary and secondary or field and armature or rotor and stator, etc.

In my papers I have used the above terminology and I propose to do the same in this discussion, and for sake of consistency

in quoting from Doherty's and Shirley's paper I shall call the *leakage inductance*, leakage inductance and not self-inductance. It is true that with our typical engineering looseness and crude superficiality some authors use the two terms as synonymous, just like the writers do; but I find it extremely difficult to explain or understand why the writers failed or refused to ascribe to the term self-inductance, as is used in my paper, its usual meaning of self-inductance, at least as a possible interpretation. Instead no matter how clearly I expressed myself they have taken self-inductance to mean leakage inductance, or something akin to it.

With reference to the reactance which affects sudden short circuits, they state that "the authors will show that this reactance includes the field leakage inductance (writers use term, self-induction) as well as that of the armature." Unfortunately for the authors this has been shown by Boucherot in 1911, who discussed the matter, in a very much more thorough and masterly way than the authors have done. It may be well to call attention here to the fact that my discussion of L, α, M and λ is not quite as thorough as Boucherot's who gives seven and one-half pages to this question, from a purely theoretical point of view. Those interested further on this may consult his memoir. However, even if the writers did not consult Boucherot, it is difficult to understand why they overlooked the following statements in my paper in 1915 TRANS., page 2244: ". . . x_f is not the self-inductive reactance of the field spools but the total leakage reactance of the field and armature. Similarly α_a is a complicated quantity . . . The complex nature of these quantities has been recognized by several investigators such as Miles Walker, and Berg who without attempting to define them rigorously (from a mathematical point of view as Boucherot has done) has called them the equivalent resistance and reactance. . . Others, however, have apparently failed to understand the real meaning of these and have attempted either to simplify them too much . . . or complicate them unnecessarily. . ." Following these remarks I described experimental methods of determining α_a and α_f from the rise of direct current in the armature and field respectively. I showed that the attenuation factor α_f obtained with the secondary or armature open-circuited and at rest is quite different from the attenuation obtained with the armature dead short-short-circuited and the machine running at full speed. I went a step further and showed that α_f obtained from the rise of direct current in the field with the armature running at full speed and short-cricuited through a high resistance is practically the same as α_f obtained with the armature open-circuited and at rest. Thus I illustrated fully the effect of mutual inductance which may be negligible even with the secondary closed. Now it is clear that in the 1st and 3rd cases, above $\alpha_f = r_f/L_f$; while in the 2nd case $\alpha_f = r_f/\lambda_f$. I called attention to these facts at the top and bottom of page 2246 TRANS. 1915.

To quote from page 2246 this is what I said: in this case we have "the important difference that x_f is no longer the true self-inductive reactance but the total leakage reactance reduced to the field circuit" I fail to see how one could have expressed the facts more clearly. Again on page 2247 it is stated that "for this particular machine, . . . the ratio of the self inductive reactance to the total leakage reactance reduced to the field is about three." Immediately following this, I state that ordinary methods of calculating x_f from equation (**14**):

$$\frac{N_f \Phi_f}{I_f 10^{-1}} \text{ abhenry, or more accurately}$$

$$\frac{N_f \Phi_f}{I_f 10^{-1}} \text{ (1 + per cent leakage of poles)} \qquad (\mathbf{14})$$

"utterly fail to give correct results" since "it is the attenuation factor involving the leakage reactance that enters the problem." All the above quotations occur within four pages of each other. However in spite of all this, the writers give the foregoing equation and triumphantly state that in the field attenuation factor, the inductance is not as Diamant gives the exciting inductance of the machine . . . but rather $\Phi^1 N_f / I_1 10^{-1}$ where Φ^1 is the flux, linking the field winding, which I_f would produce in the leakage paths of both field and armature." It may be well for the writers to compare this and other long clumsy expressions which they use—see for example (**5**) in their statement of conclusions, etc.—to the standard brief and clear statement: $\Phi^1 N_f / I_f 10^{-1} =$ total leakage inductance reduced to the field.

On page 1250 the writers make the following statement: "The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper . . . ." I trust that it is clear to any one by this time that the authors really do not disagree with any of my conceptions—at least I agree with theirs although I am inclined to think that they are not practical and scientific enough either in expressing their ideas or using the proper terms. I shall take up these points again later, but right here, let me illustrate the truth of my statements: They begin by stating clearly that they do not agree with the *conception* given in my paper and to prove this they quote from my paper that the alternating component "is equal to the maximum phase voltage divided by the armature impedance, with good approximation." Now I fail to see why the writers use such poor logic. In engineering problems we frame a more or less ideal theory and base our conceptions on it; but in practical calculations we are invariably forced to make approximations and the good engineer is the one who can make good approximations. Thus my *approximations* do not necessarily affect my *conceptions*.

However, if we go a step further we shall find the extremely gratifying fact that the writers are entirely in agreement with my "good approximation" which I found to be true for 1 or 2 machines and theoretically sound; while, they have found them to be true from extensive tests on a great many machines. Within one-half page from where they say that they do not agree with me, they state that in case of machines with solid steel rotors or alternators equipped with low resistance amortisseur windings "the armature leakage inductance is a reasonably accurate measure of the sudden short-circuit current." Furthermore they not only thus corroborate my approximation but "propose its use for at least the present." See also (b) and (c) page 1281. In connection with this the writers deserve credit for determing that this approximation is allowable for machines with low resistance amortisseur windings only, as was the case with the machine I used, as stated in my paper.

In 1915 I called attention to the fact that single-phase sudden short circuits are quite complicated and that in case of a short circuit between neutral and line the armature current, for a given set of conditions, is about 140 or 150 per cent higher than in case of three-phase or line to line short circuit. Further I urged care in such cases and state clearly whether by single-phase short circuit reference is made to a short circuit between line and line, or line and neutral.

It is very gratifying that from extensive tests the writers corroborate both of my results, namely, that for a given set of conditions, three-phase or line to line short circuits give about the same rush of current; but short circuit between line and neutral is more serious. They offer an explanation for the latter on the basis that with three-phase currents flowing in the armature the leakage inductance is increased 50 per cent on account of the mutual inductance between phases. (See their paper bottom page 1262 and top of 1263.) Reprints of the paper were available so late that I have been unable to check this explanation. However, I would ask the authors to explain on the same basis why there seems to be practically no difference between a three-phase and line to line sudden short circuit.

I called attention in 1915 to the fact that α_f, when determined, either directly from oscillographic records according to methods developed on pages 2260-2264 or when obtained experimentally from the rise of direct current in the field with the armature dead short-circuited and running at full speed, comes out larger for the first cycle or so of the transient than for subsequent ones. I gave an explanation of this on page 2262 and it is gratifying that the writers not only corroborate this fact but agree with me that until we know more about the subject it is well to use the lower and safer value.

On page 1255 they state that "it may not have been so clear how and to what extent the field leakage inductance affects the short-circuit current." As I have already shown the writers

are simply laboring under the wrong impression, in making a statement like this and I regret to say that their attempt, to show that the quantity to be used is the total leakage inductance reduced either to field or armature is rather crude and superficial compared to the explanation of Boucherot. They devote several pages to this (pages 1251 to 1259) and therefore I may be justified in calling attention to the following. On page 1251 they give the important law that "an electric circuit, without resistance, must so long as it is closed contain the same number of magnetic interlinkages. Now it is really unnecessary to make the ideal assumption of a circuit without resistance." Let me state the law, which as they say *is* very important, in a clear general way as follows. Let the flux in the field, or any electric circuit in general, change from Φ_1 to Φ_2 during the transient interval t; then the e. m. f. induced by this change will be $\frac{\Phi_1 - \Phi_2}{t}$. This transient e. m. f. will produce a quantity of electricity $Q = i \times t$ where i is a decaying direct current. As I pointed out in a recent article in the *Electrical World* this is a well known fundamental law used sometimes in electrical measurements (see *Electrical World*, May 18, 1918 page 1030, foot note 15). Let us apply this principle to the alternator. During sudden short circuit the flux in the field dies down from Φ_f to $\Phi_{f\,sec.}$ This produces a decaying direct current in the field which in turn produces the decaying alternating component of the armature current. Again if for any given armature phase there be a change of flux from Φ_a to $\Phi_{a\,sec}$; this will produce a decaying direct current in the armature etc.

For further details the reader may consult Boucherot or the 1915 paper of mine referred to by the writers or the recent articles in the *Electrical World* of May 18 and June 1. Before leaving this subject which has been well thrashed out in the above references let me call attention to the fact that the writers should be careful not to confuse the explanation of sudden short circuits based on the flux per phase or m. m. f. per phase and another possible explanation based on the total polyphase flux or m. m. f. of the armature circuit. The latter explanation is given in a short article, illustrated by oscillograms, in the *Electrical World*, Dec. 29, 1917, page 1251 and it is not necessary to go into it here.

Another point that I must mention in this connection is this. The statement of the writers on page 1275 that "the alternating component, . . . depends upon the flux per pole, . . . " is entirely wrong. So is the next statement, that the "direct component depends upon the flux linked with the armature circuit." As I have just explained, and as required by the fundamental law given on page 1251 by the writers, and which I have endeavored to generalize, the alternating and direct components depend upon the *difference* between normal and

sustained short-circuit fluxes ($\Phi_f - \Phi_{fsac}$) and ($\Phi_a - \Phi_{asac}$) and not upon the flux per pole or the armature circuit as the writers state. This is a very fundamental point and there is little use for the writers to take several pages to explain things in a crude fashion, compared to Boucherot's explanation or the one given by me in the references already mentioned, and then come back and state that the direct component depends upon the flux linked with the armature circuit. No, first it depends *not* upon the *flux* but the *difference* in flux at the instant of the sudden short circuit and at the end of same; secondly, it depends upon the difference in flux at the start and end of sudden short circuit linking with the phase we are considering and not linking with the armature circuit.

Furthermore it seems to me that their statement that the direct component depends upon the terminal voltage is wrong. I think, it should be internal voltage due to the resultant flux.

There are other points in connection with appendix A and the factor $K\phi$, etc., that I would like to discuss, but as I have taken enough time I shall leave those to others. However, as I am anxious to see some good come out of this discussion, I will make the following definite proposals.

(1) That the qualifying adjectives sudden and sustained or permanent, etc., be used to qualify the kind of short circuit implied whenever there is room for ambiguity. Otherwise the shorter term short-circuit be used.

(2) That the terms leakage inductance and leakage reactance be exclusively used in referring to these quantities.

(3) That the term self-inductance or self-inductive reactance never be used to mean leakage inductance or leakage reactance. This is unscientific and impractical, since no term is left to be used to refer to "self-induction." Furthermore the word leakage is 50 per cent shorter than self-inductive and tells us the whole story.

(4) The term self-inductance be reserved to refer to what is sometimes called coefficient of self-induction. This is the universal term used consistently by physicists, writers on electrical measurements etc. etc.

(5) In conformity with transformer practise the sum of leakage reactances of two systems, be referred to total leakage inductance reduced to primary, or armature etc, as the case may be.

(6) If desired, the term inductance or reactance be referred to any of the above when there is no danger of ambiguity.

(7) According to the above it is the only common sense and practical scientific procedure not to introduce any terms like "transient reactance," "exciting reactance" etc. The x_{pa} of the author's is simply the leakage reactance of the armature; the quantity that one would use in calculating regulation etc. x'_{pf} is simply the leakage reactance of the field. x_{pf} (see equation (4) page 1259) is simply the leakage reactance of the field reduced to the armature, and it is extremely interesting to note the way the

writers express this on page 1259. They say, "the effective field self-induction or field reactance, x_{pf}, expressed as per cent reactance in the same terms as the per cent armature self-inductive reactance." Why not say x_{pf} is the "leakage reactance of the field reduced to the armature?" This is very much more simple and familiar, since we have the same thing in connection with the transformer. The writers, however, do not stop here, they take the sum of the leakage reactance of the armature and leakage reactance of the field $= x_{pa} + x_{pf} = x_o$ and they give to this a new impressive, misleading name of "transient reactance." I say misleading because I wonder how many engineers suspected that this wonderful "transient reactance" is nothing but the "total leakage reactance reduced to the armature."

It is well to note here that the writers in introducing this short term did not even attempt to explain its real meaning or give any reasons for introducing such a term. I am afraid they are thus preparing the ground for further unnecessary complications and misunderstandings; these are very common and annoying in practise and there are cases in which they cost quite a good deal of money and trouble. The point I wish to make is that the loose unscientific terminology of the writers or any engineer is not an academic question but an extremely practical one.

Thus why not call x_o, the total leakage reactance reduced to armature—because that is what it is. The length of the term will not cause half as much trouble as misunderstandings in discussions, letters, contracts and specifications etc., etc. Furthermore if it is necessary to have a short term why not call it "equivalent reactance" this I believe is used in transformer practise in referring to $(x_{\text{primary}} + \alpha^2 x_{\text{sec.}})$ and we ought to try and keep our electrical engineering theory, practise, terminology etc. as a single unit rather than narrow so much that those who specialize in alternators, transformers, electrical measurements etc. etc. should develop theories and terminologies without due regard to the needs of their fellow workers. Personally I care very little what the outcome of the above may be, but I am inclined to think that it is to the best interests of the profession to standardize things in some such way as given above and not introduce any unnecessary weird sounding impressive terms.

In conclusion I would like to ask the opinion of the authors as to the results of a paper which was presented at the Pittsburgh meeting of the A. I. E. E. by Douglas. This paper, represented what seemed to be fairly exhaustive work carried on at Cornell on the question of reluctance and leakage reactance of machines. It gave flux distribution figures similar to 24a, 24b, etc. and checked these experimentally. It stated clearly that present day formulas for calculation of field leakage, etc. are very inaccurate. I hope this paper was of more than "historical interest" and I am interested to know what the writers think about it.

C. M. Laffoon (communicated after adjournment): Additional refinements in the methods of calculating the reactance of syn-

chronous machines, as suggested by the authors, are important on account of the inherent relation of the reactance to the still unsettled questions of field excitation, voltage regulation, and generator protection under short-circuit conditions. Before such refinements, however, can be applied, and the results generally accepted without serious confusion, the following two fundamental conditions should be universally agreed upon and adopted;

(a) A concise interpretation of what is meant by, and included in the term armature reactance under different stable and transient conditions; and,

(b) A standardized reliable test method for checking the calculated values of armature reactance and the determination of constants of reactance formulas.

In extending the refinements to the calculation of the reactance of polyphase synchronous machines, the authors are justified in raising the question whether or not a portion of the armature reactance flux of each phase interlinks part of the conductors of the other phases. Obviously, this is true to a limited extent, with regard to the end connections, in which the reactance is produced by the total flux interlinking this part of the winding; but for the leakage flux, which crosses the slots and leaves the tooth-

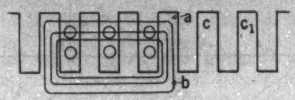

FIG. 2

tips, it is not so apparent that there is mutual reactance effect between the different phases. On polyphase synchronous machines the magnitude of the mutual inductance effect, if any exists, of the different leakage reactance fluxes, can be determined analytically only by actually laying out the flux diagrams produced by all of the electric circuits acting simultaneously (taking into consideration the difference in permeability and the damping currents in the iron portions of the magnetic circuit) and combining the different fluxes in the proper space relations at different instants of time. This is an extremely difficult task to accomplish on account of the varying saturation in the iron, and irregularity or indefiniteness of the flux paths. At best it can only be very roughly approximated, even when wide sweeping simplifying assumptions are made. Since the authors apparently have not followed this procedure in arriving at their conclusions, the writer feels that they have neither established nor justified their major premise that the leakage reactance in three-phase synchronous machines is increased approximately 50 per cent, due to a mutual leakage inductance effect, while in two-phase machines it is negligible.

If the slot leakage flux is assumed, to act independent of other flux interlinking the coils, it is evident from Fig. 2 that the flux

produced by the current in a given phase belt will not cross the slots of the other phases from a to c, c_1, etc., unless there is an appreciable m. m. f. drop from a to b. Then, on the basis of the usual assumptions, wherein the saturation of the teeth is totally neglected, there would be no m. m. f. drop from a to b, and hence no flux from a to c, c_1, etc. But in actuality there must be more or less of an m. m. f. drop from a to b, due to the other flux, which is present, and hence some mutual inductive effect must exist. However, the increase in slot reactance, due to this effect, would in all probability be more than compensated for by considering the total m. m. f. of each phase belt acting across its own slots.

The extent to which the tooth-tip leakage flux produces a mutual inductance effect upon the adjacent phases depends upon the path that it is constrained to take. If the leakage paths were confined to an air circuit, the density of the tooth-tip leakage flux would diminish very rapidly, (assuming as before that this flux is free to act independent of all other fluxes) as the distance from the magnetic center of the phase group around the armature per-

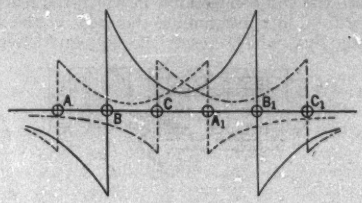

FIG. 3

iphery increases, and consequently the mutual leakage flux would be small, if not inappreciable. However, when iron is in the presence of the teeth, the path of the tooth-tip flux is dependent on the action of the damping currents in the iron. It is the writer's opinion that the authors have idealized the laminated salient-pole rotor, and have not given sufficient consideration to the damping action of the eddy currents which unquestionably circulate in the lamination of the pole faces, rivets, bolts, etc. The cross-section of the tooth-tip flux path must be more or less restricted and confined to the air gap on account of the damping action of these currents. This in turn tends to minimize the mutual effect of this part of the leakage flux. The authors have not made it clear in their presentation how the armature current can force leakage flux across the air gap from the pole face to the teeth of other phases when the same m. m. f. has been used on the same path as a counter m. m. f. to the field m. m. f.

In the case of the end connections, where there is no question concerning the existence of a mutual inductance effect, the flux density diagram produced by each phase will be as in Fig. 3. if the following radical simplifying assumptions are made.

(a) The adjacent iron parts of the machine do not affect the flux interlinking the end connections;

(b) The end connections are rectangular in shape and very long;

(c) The machine has only one slot per phase per pole; and

(d) The equivalent conductors are circular in section.

Under these conditions, it can readily be shown, by combining the respective flux forms, that the mutual inductance effect could at the outset amount to only a few per cent. Moreover, in actual machines the percentage of mutual inductance effect will be still further reduced on account of the fact that in most machines with V-shaped end connections, half of the end connectors of each phase are at approximately right angles to one-half of the end connectors of the other phases, and the mutual flux interlinking the end turns is thereby very appreciably reduced.

At all events, it is certainly evident from the above that the space distribution of the leakage flux of each phase is not sinusoidal and, consequently, in three-phase machines there is not a simple rotating field of leakage flux, as in the case of the work flux of an induction motor, and the mutual inductance effect in no wise amounts to 50 per cent. These conclusions are practically confirmed by test results, when single-phase and polyphase currents are circulated in the stator with the rotor in and removed, as indicated in Mr. Newbury's discussion.

The argument, advanced by the authors, that the mutual inductance effect must be 50 per cent because certain formulas which gave good results on three-phase machines gave results approximately 50 per cent too high on two-phase machines, is not necessarily conclusive, for the fault might have rested elsewhere in the formula. The writer has used certain formulas that did not consider any mutual leakage inductance effect, but which checked test values of reactance equally well on both two and three-phase machines.

Even if it is assumed that the damping currents are negligible, and the leakage flux is free to enter or leave the pole face, the authors' method of calculating the tooth-tip leakage flux is not fundamentally sound, for it assumes that all of the tooth-tip flux passes from teeth to pole face, independent of the ratio of the slot width to the air-gap radial length. It is evident that if the slot is wide, compared to the air gap, as shown in Fig. 4 a most of the tooth-tip leakage flux would cross the gap and then return to the next tooth. However, if the slot width is narrow, compared to air-gap width, as shown in Fig. 4b, the most of the tooth-tip leakage flux will not enter the pole face at all.* Since in actual machines the ratio of slot width to air gap varies through fairly wide limits, the method of calculating tooth-tip leakage must consider the total flux that leaves the tooth tip. That is, it should include the flux that crosses the equipotential

*This argument applies particularly to machines which have few slots per pole per phase.

section *ab-bc* (Fig. 5). The part of the flux that enters *ab* can be found by using Carter's pole fringing constant for different ratios of slot width to air gap; the part that crosses *bc* is more difficult to determine. In determining this flux, the author has apparently used Carter's coefficient for slots and vent ducts, which is based on the assumption that the tips of the teeth are at the same magnetic potential and that there is no tendency for the flux to pass from tooth to tooth. This is evidently incorrect, for it does not include the flux crossing section *bc*. Table I shows the values of reactance as calculated by the authors' formula, and the formulas used by the writer for machines with

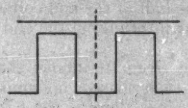

FIG. 4A

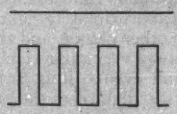

FIG. 4B

different ratios of slot width to air gap. These results indicate that in the case of machines having small gap and wide slots, the calculated values of reactance obtained by the two formulas are in reasonably close agreement, but as the ratio of slot width to air gap decreases, and a smaller percentage of the tooth-tip flux crosses the gap to the pole face, the calculated values of reactance by the two formulas diverge very rapidly.

In this connection, the writer also questions the advisability of using curves which are based on the ratio of the maximum to the minimum air gap, to determine constants for the tooth-tip

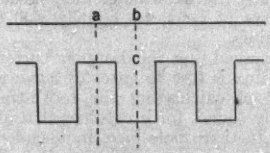

FIG. 5

leakage formula. In the first place, such curves could only be applicable to machines having a given pole-face shape, for the maximum and minimum gaps have little or no relation to the variation of the air gap in the intervening space. In the second place, the maximum gap is not a definite quantity and is subject to considerable variation in interpretation as to where it should be measured. At least, this was the writer's experience in attempting to calculate the reactances of several machines by the authors' formulas. The most natural and logical place to measure the maximum air gap would be along the path that the

no-load flux is assumed to take. It is true that in laying out the no-load flux form, it does not make a great deal of difference how the lines are assumed to pass, so long as a consistent policy is followed. But in the determination of the authors' constants, it makes an appreciable difference, for a small per cent increase in the maximum air gap may mean quite a large variation in the ratio of maximum to minimum gap.

Much of the confusion and difference of opinion concerning the reactance of synchronous machines is due, as has been previously suggested, to the fact that there is no standard and accepted method for determining it experimentally. The test method used by the author is in reality a calculated method, and, as such, is subject to serious limitations, for the armature reaction forms the major portion of the measured field ampere-turns, and a small percentage error in its calculation causes a large percentage error in the test value of the reactance.

In the writer's opinion, the most satisfactory experimental method, at present, of determining the reactance is from the no-load saturation and full-load zero power factor saturation test curves, used in connection with the calculated curve, which shows the increase of ampere turns on the no-load saturation curve due to the full-load leakage. This method also more nearly approaches the actual load conditions than any of the others. It is true that it is subject to limitations in that (a) the increase due to full-load leakage curve is calculated and subject to error, and (b) the reactance is probably not constant for all points on the saturation curve. The latter is not felt to be a serious disturbing limitation, for it is probably of the same order of magnitude as the errors in ordinary commercial tests. Granting the same degree of accuracy in the calculated terms by the two methods, the error in the test reactance obtained from the above method would be considerably less than that obtained by the method used by the authors, (as has been pointed out in Mr. Newbury's discussion). However, in most cases of salient-pole machines, in which the saturation of the pole core and yoke is not carried too high and the percentage field leakage flux is not too large, the increase in the no-load saturation curve, due to full-load leakage, can be calculated with a reasonable degree of accuracy. However, the method of obtaining it is apt to be rather long and tedious, and require painstaking effort. The accuracy of the calculated curve can be verified to a certain extent by checking the no-load and full-load zero power factor saturation test curves by the same method.

In columns 5, 6, and 7 of Table I are tabulated the per cent reactance as obtained by the authors' formula, the formula used by the writer, and by the above mentioned experimental method for a number of two and three-phase laminated salient-pole synchronous machines of quite a wide range in rating. This comparison indicates that there is a very close agreement between the results obtained by the method used by the writer

and the test method for both two and three-phase machines, whereas, as was previously stated, the values obtained by the authors' method check the test results reasonably close for machines which have a large ratio of slot width to air gap, but for machines with a small ratio of slot width to air gap, it gives results from 30 to 50 per cent too low.

It is becoming more generally recognized by different engineers that the maximum value of the initial current delivered by an a-c. generator under short circuit is limited by a reactance more complex than the armature reactance under normal balanced load conditions. For, under short-circuit conditions, the armature reaction pulsates in magnitude, and shifts in position with respect to the field poles, and alternating currents are induced

TABLE I.

Machine	Kv-a.	Phase	Slot width/Air gap	Per cent reactance		Test
				†D. & S.	‡L	
1	500	3	4.94	20.6	22.7	23.1
2	175	2	4.67	..	21.5	22.75
3	1132	2	3.56	..	18.5	19.0
*4	1250	3	3.4	9.6	10.9	..
5	875	3	2.78	13.6	16.7	16.5
*6	700	3	2.4	13.6	22.85	..
7	600	3	2.34	10.2	13.2	..
8	1132	2	1.92	..	15.0	14.1
9	1132	2	1.92	..	17.7	18.1
10	1250	3	1.62	11.7	15.7	..
11	3750	3	1.6	9.36	13.63	13.25
*12	560	3	1.067	17.6	26.0	..
13	12500	3	0.94	13.2	20.2	19.3

*The design data on machines No. 4, 6 & 12 were submitted to writer by Messrs. Doherty & Shirley.

†The values of reactance in column 5 were obtained by Doherty & Shirley formula.

‡The values of reactance in column 6 were obtained by formula used by writer.

in the rotor and field circuits. The magnetic leakage of these circuits, consequently, produces a self-induction which must (after multiplying by the proper ratio of transformation) be added to the armature self-induction in order to give the short-circuit or transient reactance, just as in a transformer.

The authors, in treating this problem, apparently have assumed that the armature component of the transient reactance is the same as the armature reactance under normal load conditions, and have derived their reactance formula to meet this condition. It would be more logical to consider the armature reactance under stable and transient conditions as different phenomena, and treat each case separately.

It has been the writer's experience that the calculated value of the armature reactance does not happen to differ widely from the value of the transient reactance determined from short-

circuit oscillograms. Table II shows the calculated value of the armature reactance and the transient reactance as determined from oscillographic records for several different types of synchronous machines. There does not seem to be any very definite relation between the stator reactance under load and the transient reactance for different classes of machines, and it is our opinion that it is more or less of a coincidence. However, the coincidence can be partially explained on the basis as suggested above, that the armature reactance under short circuit is not the same as under load; that is, the value of the armature reactance under short circuit is decreased by an amount approximately equal to the added equivalent field reactance. This reduction of the armature reactance may be due to (a) the saturation of the iron part of the magnetic circuit of the leakage paths, and (b) the

TABLE II.

No.	Machine.	Kv-a.	Phase.	Number phases short-circuited.	Ohms reactance	
					Calculated. Syn. operation.	From osc. trans. conditions.
1	Turbo-Alt.	15800	3	1	1.622	1.572
2	" "	"	"	3	1.622	1.695
3	" "	15000	"	"	1.09	1.11
4	" "	20000	"	1	1.96	1.87
5	" "	15000	"	3	1.82	1.61
6	Syn. Cond.	4500	"	"	3.84	3.87
7	" "	"	"	1	3.28	3.95
8	Alt. Gen.	12500	"	3	1.22	1.25
9	" "	"	"	"	1.22	1.28
10	Syn. Cond.	15000	"	"	1.066	1.064

NOTE: Machines 1 to 5, inclusive, are of solid cylindrical rotor type; the remainder are of the laminated salient pole type; 6 7 & 10 are equipped with damping windings.

reduction in cross-section of the leakage flux paths by the damping currents in the field poles and adjacent iron parts. The writer is also of the opinion that this damping action is not confined to solid types of rotors, but exists to an appreciable extent in laminated salient-pole machines. However, definite quantitative conclusions, regarding the effect of saturation and the damping currents on the armature reactance under short-circuit conditions, cannot be given until more complete experimental data are available.

R. E. Doherty: If there is a single characterizing feature of all this discussion, it is disagreement. The authors naturally expected much disagreement on a subject of this nature, and of course were not disappointed on that score. They were disappointed, however, at the nature of the criticism. While some of it contains constructive suggestions, much of it is purely destruc-

tive. It says the method, and therefore the calculated result, is wrong; yet it does not show how to make it right, nor does it give adequate reasons for saying the method is wrong. For instance, Messrs. Newbury, Fechheimer and Laffoon, who, as designers, are in possession of data from which to form opinion, apparently all agree that the method of calculation is fundamentally wrong because they believe serious factors are omitted; that even if it were correct, the method employed to arrive at the "test" value is wrong, because, they believe armature reaction can not be calculated with sufficient accuracy: and therefore agreement of "test" and calculated results could not be of much significance. But not a word is said about the theoretical soundness of the method, given in Appendix B of the paper, for more accurate calculation of armature reaction, taking into account the influence of harmonics in the flux wave. This influence has always been the cause of the greatest uncertainty regarding armature reaction calculations. Neither do they show any way of including the factors which they believe we have omitted. They are therefore urged by the weight of that acknowledgment to suggest, in the way of an alternative, the old friend—the expedient of having different formulas and different empirical constants for different lines of machines. That is, they are apparently agreed that this thing which we are trying to do can not be done. Mr. Fortescue subscribes heartily to that viewpoint, giving reasons in mathematical terms which to those even moderately versed in mathematics, can not possibly have much significance, and which to others seem to be evidence of a more thorough knowledge of mathematics than of alternators, inasmuch as he fails to consider that "in a system like this" the integration constants for his different differential equations may turn out to be the same constants for all, and that he may as well have had one equation. This is not the first instance where a voice has been raised that "it can not be done", when at that very moment it was being done. It may not be possible to obtain a single reactance formula, which can be depended upon in practical application to synchronous machines of all ranges in speed, voltage and capacity now being manufactured, but as evidence that it might be possible, we submit the fact that one of the large manufacturing companies has actually used such a formula for about two years, and the predictions based on those calculations have proved to be dependable. And that formula is the one given in the paper.

That sums up in a general way the larger and more serious objections to the paper. There are, of course, several others which, although not of a nature to determine whether the results of the paper shall stand or fall, are yet very important. These, as well as the individual points included in the discussion referred to above, will now be taken up in order.

Prof. Karapetoff offers a number of criticisms. First, he contends that "field self induction" and "field reactance" are

different concepts, and therefore the authors can not properly say "field self-induction or field reactance." If he had read the next line of the same sentence he would have seen that these terms are expressed as a ratio, *i. e.*, percentage, and therefore are numerically identical. This result is what the sentence which he criticizes, refers to.

He believes next that the Institute should demand of the author a list of symbols, instead of allowing him to define them in the context, and thereby increase the "readability." That may be worth while, but there are obvious advantages and disadvantages on both sides, which have been discussed so often we shall not enter it here.

Referring to Fig. 25 A, he asks why flux and equipotential lines are drawn through the field conductor, since he understands that the flux has no potential in space occupied by current. The paper says: "In Fig. 25 B the paths as indicated by the iron filings can not be traced to the pole body on account of the field winding. But it is clear from the direction at the edge of the field winding that they will terminate approximately as shown in Fig. 25 A." Further than that, we can not answer.

Prof. Karapetoff's other criticism is that the authors are guilty, with many other authors, of expressing inductance as a ratio of flux to ampere turns, instead of taking simply the permeance. He then calls attention to the fact that the permeance idea is followed consistently in his book on the "Magnetic Circuit." Our supposition is that because most of us were started off in college from the "interlinkages per ampere" viewpoint, we are inclined to fall back upon that viewpoint both in writing and in our effort to understand what is written. In order, therefore, not to decrease the "readability" of the paper, the authors were less interested in revolutionizing peoples' viewpoint to confirm to Prof. Karapetoff's book than in helping readers understand the problem in hand by using terms familiar to them.

Messrs. Newbury, Laffoon and Fechheimer have raised a fundamentally important question, what is "the quantity we are attempting to calculate." Also how are we to test for that quantity in the actual machine. What the authors consider as the "quantity," must be clear to any one who has studied the paper. They were not aware that there was so much uncertainty regarding what particular reactance should be considered in various circumstances of operation. Since such uncertainty and also the question of testing do exist, these should be cleared away to permit some approach to agreement.

As a starting point toward settling what armature reactance means, take Mr. Newbury's lucid explanation regarding armature reactance and armature reaction. We will take up transient reactance later. With that portion of his discussion (1st four paragraphs) covering the explanation, we are in entire agreement, as may be obvious from reading the paper. Figs. 27 A and 27 B show our conception of the fluxes he describes. There can

be no question, therefore, about what leakage flux is involved when we say *armature reactance*. Fechheimer's discussion breaks the accord at this point, by injecting the old bogy of harmonics which produce distortion and upset all calculations; then he closes by admitting: "that the distortion is very much reduced at full load, zero power factor, is quite evident." Now in order to compose this particular difficulty, let's say we mean the armature reactance which exists under zero power factor condition—*i. e.*, armature and field m. m. f's. in direct opposition. Then the complication of distortion is eliminated, and we can start once more. If it happens, as actually does and as explained in the paper, that this same value may be applied in regulation calculation, etc. under watt load conditions also, that is another matter which has no direct bearing on this present problem of establishing a bench mark to work from. Permit us to say with much emphasis: that is the reactance which is called *armature reactance* in the paper, and which equation (**47**) in Part 1 is intended to calculate. It is the calculated value listed in Table I.

Before taking up the difficulties involved in testing this value consider further Mr. Fechheimer's discussion regarding the influence of the harmonics which remain, even at zero power factor. It is pointed out in the paper that the salient-pole construction causes variation in reluctance of the leakage path, producing principally a third harmonic, which, with other third harmonics, disappears in the terminal voltage of Y-connected machines for well known reasons. There are other higher harmonics naturally but how much influence do they exert in the reactance or impedance, *i. e.*, in determining, in conjunction with the armature reaction, the current which flows under sustained short circuit, where, as already mentioned, the field and armature m.m.f's. are in direct opposition. The armature reactance under this condition, remember, is the reactance we are concerned with. That current, as Fechheimer acknowledges, is substantially the fundamental, free from harmonics. A glance at any oscillogram of this current will confirm that. Whether these harmonics are damped out by eddies alone, as he suggests, or are partly suppressed by the higher reactance to the higher harmonics, matters little. The simple fact is they are not there. If we want to know what determines the current, which is the fundamental, it must be obvious therefore that the fundamental of the reactive voltage is the factor we are after. A fifth harmonic, or any other higher harmonic, may be in the reactive flux or voltage, as he says, but what if it is? If we were studying core loss, that fact might concern us, but not here.

Take up now the several methods of testing reactance which Mr. Fechheimer mentions. This is important, because much of the disagreement centers on the test method. On the same machine, Mr. Newbury obtains from test by Mr. Fechheimer's No. 2 method, which he adopts, a value of armature reactance

different from the value the authors obtain by No. 1 method, which they adopt. Don't misunderstand. We are both after the same thing. The question is, which method of obtaining it by test is right. As we see it, the principle in each method is sound, and by his explanation referred to above, Mr. Newbury agrees. To draw the issue clearly, Messrs. Newbury and Fechheimer think that our respective "test" determinations of armature reactance on the same machine do not check, because they believe we are unable to calculate armature reaction accurately. Hence the difference between that wrong armature reaction and the test value of field m. m. f. (synchronous impedance *A T*) gives the wrong reactance. He agrees, however, that that difference, properly determined, gives *armature reactance*. Our only answer is that we have given in Appendix B a new method of accurately calculating armature reaction, and it seems that they should have felt obliged to show why that method is not right. To simply say it is inaccurate is not at all convincing. Not a word was said about the method in Appendix B; not a word as to *why* armature reaction can not be calculated. We are therefore not in position to know and discuss what is in their minds.

In applying No. 2 method, which Mr. Newbury recommends, one meets a number of difficulties. It involves, first a graphical solution in which a cut-and-try triangle is made to fit in at all points between two curves. One of those curves is calculated, and involves estimates of pole leakage and also estimates of how saturation is changed by having, under load, more flux in the pole body, but practically no more flux in the armature and a varying amount of increase in the teeth. Hence that calculated curve must rest considerably upon judgment. It is important to note that the same objection which was raised to No. 1 method regarding the large error resulting in the difference of two large quantities, from a small error in one of the large ones, applies equally to method No. 2; as does also the objection that one of the large quantities is calculated. It is our opinion, therefore, that method No. 1 involving a simple subtraction of ampere turns armature (in which the amperes are tested, the actual turns are known, and the only calculated factor is "effective" turns) from ampere turns field (in which the amperes are tested and the known turns are 100 per cent effective) is much more reliable than No. 2 method with its difficulties mentioned above; and that the discrepancy, mentioned by Messrs. Newbury and Laffoon, between their test determination and ours is due to their errors in applying No. 2 method. It may be asked: if the "test" value of armature reactance determined in error by No. 2 method is applied in excitation and regulation calculations, why will not these calculations also be in error. Simply because the steps by which the reactance was determined from load excitation curves are retraced in load excitation calculations. This pursuit and follow of one's self about a circle does not apply to the

use of No. 1 method. Therefore, assuming that armature reaction calculations are even approximately correct, then it would seem that to obtain a value from test with which to compare the calculated value, No. 1 method is far superior. It is acknowledged that no method yet proposed, which is based on commercial tests of saturation curves and synchronous impedance curves, can be depended upon, for reasons already mentioned, to give in every case the armature reactance within, say, 10 or 15 per cent. This may mean only 2 or 3 per cent error in the tested curves. If that error is more serious, as sometimes happens, then the reactance error goes skyward. The error in reactance value incident to test error applies alike to both methods referred to. Hence, although accurate tests of course eliminate this point, it is yet true that where, as in our case, the experience available involves such tests, this fact must be weighed in the interpretation of the data. This has important bearing upon the data in Table I, which I shall refer to later.

Considering further Mr. Fechheimer's list of methods, No. 3 and No. 4 give nothing but the impedance of the stator under those test conditions, *i. e.*, with rotor out. It is absurd to take this as the *armature reactance*, since obviously the latter would be different in the same stator with rotors of different pole proportions. Methods 5 and 6, give, as did 3 and 4, the impedance of the machine under the conditions of the test, *i. e.*, with stationary rotor, where any given phase group has a fixed relation to the pole. If there happens to be a secondary winding, *i. e.*, a squirrel-cage, solid-rotor or short-circuiting collars on the poles, the transformer impedance is involved, that is none of these is the *armature reactance* which we have been discussing. This seems too obvious for comment. Method 7 may give some interesting results of the synchronous reactance in Mr. Fechheimer's investigation. However, it is probably as accurate and infinitely easier to read the synchronous reactance directly from the saturation and synchronous impedance curves.

Method 8 gives the *transient reactance* accurately.

Of the eight methods, therefore, either 1 or 2 (if 2 is correctly applied) should be used for obtaining *armature reactance*. Method 8 should of course, be taken for *transient reactance*.

Referring to Table I, Mr. Fechheimer urges caution in adopting the formula, because 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and points out two others in which the difference is 145 per cent and 185 per cent. Please bear in mind these two facts: In that table of 138 machines, all of the bad ones, as well as all of the good ones are there, *i. e.*, all the cards are on the table. Secondly, the "test" values, with few exceptions, were determined from commercial tests of saturation and synchronous impedance, in which circumstance, something would be wrong, as already intimated, if all machines checked within 10 or 15 per cent. Many of those tests are all right; others may be off by 2 or 3

per cent giving 10 or 15 per cent in reactance; others, more. It would be nice to have 138 different, accurate tests to compare with, but they are not available. We are obliged to accept the commercial test records for the mass of test data. Where tests are known to be accurate, no radical disagreement in results occurs: But we submit that knowing the probable error that *may* result from test, and noting the consistency of the majority of the cases of widely different machine characteristics, in falling in line with calculations, the extreme cases mentioned, as well as a variation of 20 per cent in 40 per cent of the cases, are to be expected, and can not be taken, as Mr. Fechheimer suggests, as evidence that the formula is wrong. If the entire table is studied with these points in mind, I think it must be granted that the agreement is most encouraging. If the table contained only eight or ten machines of about the same characteristics, then it could not be offered as evidence that the formula is reliable. But that is not the case. There are 138 of them. We might add that during the seven months which have passed since the paper was written, additional data, including many accurate tests, have been obtained, all confirming the reliability of the formula.

Referring to transient reactance, Mr. Fechheimer submits an ingenius example to show that saturation prevents the field leakage from having much influence in limiting the initial short-circuit current, and says also that eddy currents in the spider should be considered as to their influence on transient reactance and attenuation factors. Now that example showing the tremendous increase in pole density at short circuit is most interesting. We agree that saturation would certainly be a "serious factor" in such a case. But Mr. Fechheimer is confused. It has apparently escaped him that the flux in the pole at short circuit is roughly what it was before short circuit. He adds the 10 times 12 per cent leakage, to the 100 per cent normal flux, giving a total of 220 per cent. He forgets that the field leakage can not possibly be more than a fraction of the original 100 per cent. As a matter of fact, the only difference after short circuit is that *part* of the 100 per cent which before short circuit crossed the air gap into the armature, now crosses the field leakage paths. The rest goes through the armature leakage paths. So you see he has accounted for 220 per cent which, if it were not for saturation, would cross the field leakage paths, and has not yet mentioned the armature paths, which, by the charitable assumption that the field leakage is 50 per cent as great as the armature leakage, and that the ratio of effective field interlinkages to the product of field turns times flux is about $^2/_3$, the armature paths would absorb $^2/_3$ times 440 per cent equals about 300 per cent more. That is, 5.2 times normal flux, which of course is absurd, since it means manifold increase in field interlinkages. To have remembered that these interlinkages can not change significantly at short circuit would surely have prevented such a weird example. A study of Part 2 of the paper will clear up this point.

A glance at Table II may save discussion about the influence of saturation. Short-circuit tests are shown at different voltages. It will be noted that this evidence is in accord with conclusion (**10e**) given at the end of Part 2.

Regarding the effect of eddies in a solid rotor rim upon transient reactance and upon the attenuation factors, this phase of the problem was considered, as mentioned briefly in the last paragraph of the abstract at the beginning of the paper. Specific data regarding this effect was not ready in time for the original copy of the paper, but has been included in the TRANSACTIONS.

In speaking of transient reactance Mr. Newbury states that in his experience "the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit." He grants that they may not be the same reactances, and explains that their numerical equality may be due to the fact that whatever increase may occur in reactance due to the field leakage, is offset by "some other action." From what we have said in previous paragraphs, our explanation of that numerical coincidence is due less to "some other action" in the machine than to the fact that Mr. Newbury draws in the calculated modification of no-load saturation curve too near the no-load curve. It is a hopeful augury that he agrees in his last sentence that it "may be due to difference and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled."

It is very important to note at this point that from an exchange with Mr. Newbury of designs and test data for the purpose of removing all question regarding application of the formulas and interpretation of tests, calculations by equation (**47**) Part 1 and equations (**6**) and (**7**) Part 2 checked the tests on his machines, as shown in a following table, in connection with Mr. Laffoon's discussion. That is, the formulas apply alike to machines of different manufacture, when the calculated result is referred to a "test" value which the formula is intended to calculate, and not to some other "test" value.

Mr. Dawson suggests that the difference between the results given in the verbal discussion by Mr. Newbury, and those in the paper might be explained by different practises in end-coil support. I think we have made clear where the discrepancy lies. I remember a case where the same machine was tested first with steel-ring end-coil support, and then with brass support. The difference in reactance could not be detected. This was a moderate speed, about 1000-kv-a. machine, and the steel ring was about one inch (25.4 mm.) square.

Mr. Laffoon's comments center more upon the details of theory underlying the reactance formulas. But he prefaces those comments with some general objections. His reference to

definition and "standardized reliable test", is repetition of Messrs. Newbury and Fechheimer's discussion of the same points, which has been fully answered.

He rather more fully amplifies their viewpoint regarding the extremely complicated nature of the problem. He believes that the problem is too intricate to treat analytically; that necessary flux diagrams, effect of eddy currents, change in permeability, indefiniteness of the flux paths and a few other things render the calculation hopeless. To take account of all these factors in an equation would be well nigh hopeless, we acknowledge. But if we understand the problem correctly, there is an important distinction to be drawn between a method involving a possible error of 10 per cent or so which for the practical purpose for which the reactance is used, is negligible, and a method required to include all factors to the extent of reducing the possible error to, say, 1 per cent. It is somewhat like the distinction between the method of a house carpenter who works to inches, and the factory mechanic who works to thousandths of an inch. In one the relative error may be many times larger than in the other; but, practically, they are of the same order. Our point is from our understanding of the problem in hand, and from our experience, we believe that practically all of those annoying and confusing factors which Mr. Laffoon refers to, are trifling and may be neglected in obtaining a practical estimate of reactance from the design. Where, for instance a factor can not be neglected, as harmonics in armature reaction, we have taken it into account. But to worry about most of the things he mentions is unwarranted, for reasons given in previous comments.

Regarding slot leakage and his Fig. 2, he reasons that there will be slight mutual action between phases due to saturation of teeth, then decides that there is some other effect that compensates for it. That is, agrees with the authors that there isn't any mutual effect of slot leakage.

Now as to the mutual induction of the end connections and tooth-tip leakage. Mr. Laffoon shows why in the case of the end connections the mutual effect could amount to "only a few per cent:" then shows that it would have to be less than that. Perhaps the shortest answer to his logic is that if exploring coils are put on the end connections following around the V-ends of a phase group, and then from an exterior source, the same current is put through the armature, first single phase and then three phase, the exploring coil voltage will be 50 to 60 per cent more in the latter case than in the former. This ratio for end leakage is the same whether the rotor is in or out. The following Table III gives data from such a test on a 125-kv-a. machine, four slots per pole per phase, and also on coils assembled in a wooden armature dummy.

He believes that the tooth-tip leakage is more complicated still, and also that the mutual induction between phases due to this leakage must be very small. Why? Because damping

currents in the laminations, rivets, etc., etc., prevent the tooth-tip flux from entering the pole face. This restricts this leakage to the path along the air gap. This of course would not only prevent mutual induction with other phases, but also cut down radically the leakage itself. You see he is still allowing harmonics to cause undue confusion. Certainly there are no damping currents to the fundamental which we are after, and which being polyphase is stationary with respect to poles. Instead, the problem is rendered more simple that the harmonics are partially

TABLE III
EXPLORING COIL TEST FOR END LEAKAGE ON 125 KV-A. ALTERNATOR

Test	* Rotor	Phase	† Exp. Coils in series	End Shield	Amps. in Phase group	‡ Exp. coil voltage	Column 7 corrected for 32 amps.	Ratio taken of tests	Ratio
1	Pos. 1	3	4	on	31.2	0.983	1.02	1 & 3	1.69
2	Pos. 2	3	4	on	33.6	1.09	1.04		
3	Pos. 1	1	4	on	31.4	0.595	0.605	2 & 4	1.52
4	Pos. 2	1	4	on	31.4	0.673	0.685		
5	Out	3	4	on	32.4	1.03	1.02	5 & 6	1.6
6	Out	1	4	on	31.5	0.627	0.637		
7	Out	3	4	off	31.3	1.04	1.07		1.67
8	Out	1	4	off	31.4	0.627	0.64		
Similar Test on Armature Dummy									
9		3	2		130	2.99		9 & 10	1.49
10		1	2		130	2.01			

*Position 1, phase group bearing exploring coils, symmetrically surrounding the pole. Position 2, phase group directly over pole face.

† Exploring coils were arranged as shown in Fig. 7, except there were four instead of two slots per pole per phase.

‡ Maximum of fundamental. Fig. 6 shows typical oscillogram of exploring coil voltage.

or totally damped out. We trust that the puzzling features he mentions will clear up if these points are kept in mind.

He adduces Mr. Newbury's test with rotor removed showing 16 per cent increase three-phase over single phase reactance to show that that increase can not be of the order of 50 per cent. At the time the problem was being studied, the authors of course made such tests, and agree that under that condition (rotor out) the ratio is about 1.16. But as we have emphasized before, we are not interested in what it is with the rotor out. What we are after is the reactance with the *rotor in and running in synchronism at zero power factor*. Under which circumstance (and relegating

this damping action of the pole rivets etc., as mentioned above) the iron rotor permits mutual interlinkages by the fundamental with other phases. This idea is fully developed in the paper. In this respect, there is other possible evidence, available to those concerned, that has bearing on this point. Theoretical considerations indicated that this mutual induction factor, three-phase to single phase (terminal to neutral), obtains in the transient as well as the armature reactance. This led to equation (**13**) Part 2. While at the time the paper was printed, no tests were available, a test has since been made which confirms that point of view. It might be instructive for those interested to try this out. The results of that test which was made on a 1000-kv-a., 750 rev. per min., 50-cycle generator are as follows: Short circuit, three-phase, no load gave 1900, 1800, 1900 amperes (a-c. component) on three different tests. Short circuit, one phase, terminal to neutral, no load and same voltage, gave 2950 and 3060 amperes (a-c.

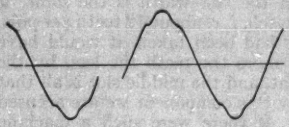

FIG. 6—OSCILLOGRAM OF EXPLORING COIL VOLTAGE OF END CONNECTION LEAKAGE ON 125 KV-A. ALTERNATOR

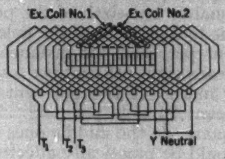

FIG. 7—EXPLORING COIL TEST FOR END LEAKAGE—ARMATURE COILS ASSEMBLED IN DUMMY INDICATED BY HEAVY LINES AT CENTER OF FIGURE

component) on two different tests. Average ratio three phase to single phase is 1.6.

Mr. Laffoon believes we have made an error in the calculation of tooth-tip leakage by neglecting the important component of this leakage which goes along the air gap and does not enter the pole face. Now there may be a number of viewpoints from which the authors have not studied this problem, and there may be possibilities of significant factors which they have failed to search out and weigh, but may we not, in the face of this avalanche of criticism, properly give ourselves credit for having been pretty careful and painstaking to do these things. The question just mentioned about tooth-tip leakage was analyzed and the factor for including this "along-the-air-gap" component was put in the equation; but it was found to be practically negligible, and was omitted. We take it that Mr. Laffoon has not properly made such an analysis, or he would not have brought the question up. The space to discuss this criticism may be worth while, because it is a characteristic type. At first thought, the matter seemed to us very much the same as it still does to Mr. Laffoon. That

is why we worked it out. It seems obvious that with slot width equal to air gap on a one slot per pole per phase, the leakage from the face of one tooth to the face of next tooth, will not all go across the gap to the pole face and return; part will go along the air gap. With two slots per phase group, the path across to the pole face has not changed, but the path of the component along the air gap is doubled. For three slots per group the first path between outside teeth is still the same, the path along the gap is trebled. The middle slot has its own leakage, but as explained by Gray, this contributes only one ninth to the total leakage of the group. And so on; the component along the air gap growing progressively less as the slots per phase group are increased. Then why does not that component have to be considered for one or two slots per pole per phase. Because no one, I think, ever saw an air gap equal to the slot width on such a machine. If one did, one saw a monstrosity. Assuming tooth and slot width equal, and three slots per pole, would mean an air gap equal to one-sixth of the pole pitch. A normal air gap is, say, one to 3 per cent of the pole pitch, possibly four per cent. Take four per cent, for instance. If the slot width is the same, it means eight per cent per tooth pitch, *i. e.* about 12 teeth per pole, or more. If two per cent gap had been taken, it would have meant 24 slots per pole. The more the teeth the less, by the square, do the middle ones count, and the middle slot is all that would be practically affected by this component we are accused of having omitted. Moreover, if there were such a machine where this factor was a significant portion of the total tooth-tip leakage, then the total tooth-tip leakage would be of practically negligible magnitude compared to end leakage and slot leakage; and the error would be of no practical significance in the total reactance. Those are reasons why we omitted that factor. We did not overlook it.

Regarding the application of Carter's Fringing Coefficient to tooth-tip leakage. Take any alternator and lay out the distribution of tooth-tip leakage over the pole. Determine the coefficient to account for the spread of flux. Then look up the value on Carter's curve. It is practically the same, as one might expect, except possibly in some monstrous example as Mr. Laffoon suggests. Even then, for reasons mentioned, the error, is harmless.

The title for the curves based on the ratio maximum to minimum air gap, which assume a constant pole arc radius for any given pole, has been modified to meet his objection regarding the difficulty of applying them. They can not properly be applied to a pole involving different arc radii.

Turn now to Mr. Laffoon's Table I. Before commenting on the comparison it gives, we would refer to the following table, giving the rest of the data which Mr. Laffoon should have felt obliged to include, but did not. We have already referred to an interchange of design and test data with Mr. Newbury for

the purpose of eliminating the possibility of wrong application of formula and of different interpretations from tests. Mr. Laffoon's Table I is supposed to show the results of that interchange. In our Table IV only machine numbers 4, 6, 11, 12 and 13 appear, since we have not data on the others. But sufficient data is in Table IV for interesting comparison with Mr. Laffoon's Table I. These are the main points of interest:

1. Our calculation of *armature reactance* checks reasonably with the test value (No. 1 method), which the formula is intended to calculate, on Mr. Laffoon's machines, as well as on ours. Mr. Laffoon's Table I does not show the "test" value which our formula calculates, but something else.

2. His calculated and test values are, he avers, either *armature*

TABLE IV
PER CENT REACTANCE

Machine	Kv-a.	Phase	* Slot width air gap	Calc. D. & S.		Test †		Calc.	Test ‡
				Arm.	Transient	Arm.	Transient	L	L
D. & S. 4	1250	3	3.4	9.6	13.6	7.8	...	10.9	...
D. & S. 6	700	3	2.4	13.6	19.1	13.0	...	22.85	...
L 11	3750	3	1.6	10.2	14.5	12.9	...	13.63	13.25
D. & S.12	560	3	1.06	17.6	23.0	18.2	...	26.0	...
L 13	12500	3	0.94	13.2	19.0	14.2	20.8	20.2	19.3

* Minimum air gap is used in this ratio.

† *Armature reactance* from saturation and synchronous impedance curves. Method 1 in Mr. Fechheimer's discussion. *Transient reactance* from oscillograph records of short circuit.

‡ Mr. Laffoon's application of test Method 2, as described in Mr. Fechheimer's discussion.

or *transient* reactance, since they are the same. Note that those values agree with our calculated *transient* reactance. Fortunately, on one of the machines he submits, the 12,500-kv-a., results from short-circuit tests are available. In this case, we calculate 13.2 per cent armature, and 19 per cent transient. Test was 14.2 per cent armature, and 20.8 per cent transient reactance. His calculated and "test" values are 20.2 and 19.3 per cent respectively.

3. Our calculation of armature reactance on machine 11, is 10.2 per cent instead of 9.36 per cent given in Laffoon's Table I.

Therefore the alleged agreement of results at high ratio of slot width to air gap, and a gradual taper to radical disagreement at low ratio, due to our omission of the "along-the-air-gap" component of tooth-tip leakage, vaporizes with the rest of the facts and logic which have been submitted to show that the results of

the paper are entirely wrong. Note in Table IV that, instead, the agreement happens to be better at low ratio (see *D* and *S* 12 and *L* 13) than at high rates. The results in these tables simply show again what has been brought out in all this discussion, that, in the first place, the soundness or the theory underlying the method given in the paper has not been disproved; and also that the difficulty Messrs. Newbury, Fechheimer and Laffoon have experienced in adjusting their viewpoints rests on the fact that they have been working on the unsound principle that the transient reactance and armature reactance are the same. And by the treacherous possibility in No. 2 test method of taking in excitation calculation any value one pleases for armature reactance, (so long as that value is applied in the same way it was derived from the zero power factor excitation curve) they have lead themselves to believe that the above principle is right. A different value of reactance obtained by No. 2 method simply means a change in judgment regarding where the calculated saturation curve should be placed. The coincidence of the value of armature and transient reactance probably had its origin in the view that they should be the same, and the full-load leakage correction of the saturation curve adjusted accordingly.

Mr. Diamant's comments center principally on three points: 1. The author's statement of disagreement with certain results of his own paper. 2. Priority claims. 3. Terminology.

Since Mr. Diamant states that "the authors really do not disagree with any of my conceptions, at least I agree with theirs . . . .", the first point would seem at once to be settled. We disagreed because we misunderstood his meaning. We misunderstood not by a lack of diligence in trying to understand, but because he used terms familiar to us, but to which he attached a meaning entirely different from that almost universally understood, and which is uniformly defined in leading text books. Take one glaring instance: by armature self-inductive reactance, he means synchronous reactance. Referring to Fig. 27B in the paper, his meaning of "self-inductive reactance" involves all the flux shown. What is generally meant by self-inductive reactance, involves only that portion of the total flux shown that does not link with the field winding. With the almost generally accepted definitions in one's mind Diamant's statement "the ratio of the true self-inductive reactance to the total leakage reactance reduced to the field is about three," is perplexing indeed. Being familiar with leading texts that define these terms and with the consequent general understanding, and yet not having stated his intention to use a different meaning, he could hardly have expected his words to be understood.

But after definition has been cleared up, one point remains. We obviously can not agree to the general statement in Diamant's paper that the initial short-circuit current is determined "by the armature impedance with good approximation." Although true in some cases, as tacitly granted in our paper,

"this assumption would lead to serious error in many cases." We gather from Mr. Diamant's interesting comments confined to logic, that, after all, he agrees.

Just a word about our reference to his paper. In taking up this problem of reactance and short circuits, we naturally used as a starting point the most recent work on the subject, and that, of course, was Diamant's paper. He had covered the matter historically, which to us, was a considerable factor in the value of his paper, and had offered interesting and valuable test data, and also some extensions in mathematics, one of which was used in our paper, Fig. 21. As stated, there were in his work points with which we did not agree. Hence it was necessary and proper that that disagreement should be stated, in order to clearly proceed to further development; surely not in order that we might "triumphantly state" that some one was wrong. We regret that the latter interpretation was made by Mr. Diamant, because it has done much to spoil what otherwise might have been a masterful criticism.

We are accused of having attempted to offer, for the first time, what Boucherot proposed in 1912. The first paragraph of Part 2 states that Diamant's paper sums up the theories underlying short circuits as developed by Berg and by Boucherot. These are in complicated mathematical terms from which the average engineer finds difficulty in obtaining a physical conception of the phenomena. Moreover, to apply their equations it is necessary to know the reactance. In order to calculate the reactance which limits the initial short-circuit current, it is necessary to establish a physical conception of what happens to the magnetic fields under that condition. It is just as necessary to establish that conception in the reader's mind, if he is to understand clearly what is being calculated. And to the contempt of those who would say everything mathematically, we have described the phenomena "in terms as free as possible from mathematics." It is an interesting circumstance, therefore, that we are now obliged to acknowledge that we did not discover the principles underlying that exposition; nor did Boucherot, or Diamant. Faraday and Maxwell deserve some credit for them. They are stated in any good text on physics. However, there are one or two points which were brought out which we believe were not generally appreciated. To mention one, the increase in flux density at the bottom of the pole at short circuit. This is not only an interesting result, but one that has important bearing upon the variation in attenuation factors, as explained in the paper. This phenomenon is not all obvious from previous literature.

Before referring to terminology, we will answer one or two other points. We are asked to explain why a single-phase short circuit between terminals of a three-phase machine produces the same current as a three-phase short circuit, especially since a terminal to neutral, single-phase short circuit produces 50 per

cent greater current than the three-phase. If it is assumed that the two phases involved may be considered as different phases, as in three-phase, and with the three-phase value of reactance per phase, then the short-circuit current would be

$$I = \frac{1.73\,E}{2\,X} = 0.86\,\frac{E}{X} = 0.86\,I_3$$

that is 86 per cent of three-phase value. That assumption, however, involves the three-phase mutual induction between phases, which is strictly true only at one instant, when the current is a maximum. Then the pole iron is in same position relative to the armature m. m. f. as at one particular moment under three-phase conditions. If that relative position were maintained, then the current would be as given above, *i. e.*, $0.86\,I_3$. However, as the pole moves on, the relation of the iron path to the leakage flux is changed to decrease slightly the fundamental of the leakage flux, and as tests show, this decrease is about 15 per cent.

Mr. Diamant says the following statements in the paper are wrong: "the alternating component depends upon the flux per pole" and "the direct component depends upon the flux linked with the armature circuit." He says those components depend rather upon *the difference between the mutual and sustained flux*. The above quotations obviously follow if one accepts the principle of constant interlinkages in a closed circuit without resistance, and grants what is the generally accepted view that for a practical study of the phenomena, the resistance is negligible for the first cycle or so. But even if one dissents from that view, we will take the magnitude of these components as determined by projection of wave of crests to zero time. But however obvious those statements may seem to those who look at these matters from a viewpoint of facts, it is acknowledged that where one, instead, reasons from somebody's equations, say Boucherot's, instead of from the background of facts Boucherot had in mind, it is possible for one to arrive at a wrong, sometimes dangerous, conclusion. Consider what happens. At short circuit the entire useful flux is absorbed in the armature and field leakage paths in proportion to their respective reactances. There is of course a very small per cent of it taken up in $R\,I$ drop, but this is negligible. As the flux dies down, the relative proportion of it in the armature leakage path at any time is progressively greater, that in the field path, less, until at the sustained value, it is all absorbed in the armature leakage (except of course the original pole leakage due to original exciting field current). Now the practically universal practise of all authors in expressing that fact mathematically (and it is a mathematical convenience not free from inconsistency) is to assume that the sustained value existed all the time, and that the transient condition involves a change from initial flux and current to sustained flux and current. Diamant's opinion probably rests on such

a *formula*, otherwise it is difficult to understand the basis for his opinion that the a-c. component depends upon the difference between initial and sustained flux. But, as intimated above, such a formula does not explain certain facts. If the sustained value is assumed to have existed from the first instant, then it means that by the formula there is a greater percentage of flux taken in the armature paths at the initial moment than actually is taken. As an instance, assume a ratio of armature to field strength equals two. Also assume ten per cent armature, five per cent field leakage. That means 6.66 times normal current at short circuit. Actually at the first moment the division of the *total* flux, which is 110 per cent, is 66.6 per cent in armature, (33.3 per cent + 10 per cent) in field leakage path. At the sustained condition, the current is practically two times normal, that is, the flux division is 20 per cent in armature leakage, 10 per cent in field. By the well known formula referred to, assuming that the sustained values existed from the start, the initial division would be as follows: the sustained value accounts for armature 20 per cent, field 10 per cent. That leaves 80 per cent of the useful no-load flux to be absorbed in armature and field in the ratio of 10 to 5, *i. e.*, 53.3 per cent armature, 26.6 per cent field. That is a total of (20 per cent + 53.3 per cent) in the armature and (26.6 per cent + 10 per cent) in the field. That is, this equation says 73.3 per cent is in the armature leakage, 36.6 per cent in the field leakage; whereas, in fact, the division is 66.6 per cent in armature, 43.3 per cent in the field. Now obviously, by the well known equation referred to, this question would not come up if the total reactance is assumed to be in the armature. In other words, that equation involves the fundamental assumption that the total reactance is in the armature. If, therefore, one thinks in terms of that equation, one may go astray.

Another viewpoint, which seems more rational, yet perhaps less convenient, is that the sustained value of flux is zero at the initial instant, and by an exponential, reaches the sustained value; and at the same time the total initial flux decreases to zero by the same exponential. That is the flux at any time is

$$\phi = \Phi \epsilon^{-\alpha_f t} + \Phi_s \left(1 - \epsilon^{-\alpha_f t}\right)$$

where Φ = initial flux
Φ_s = sustained flux
α_f = field attenuation factor based on the total leakage of armature and field.

The flux which is caught in the armature circuit starts at that value and decreases to zero by

$$\phi_a = \Phi_a \epsilon^{-\alpha_a t}$$

The equation of short-circuit current crests would be, using notation in paper, and assuming wave completely offset,

$$i = \sqrt{2}\left\{ I_0 \epsilon^{-\alpha_f t} + I_s\left(1 - \epsilon^{-\alpha_f t}\right) + I_0 \epsilon^{-\alpha_a t} \right\}$$

These forms are consistent with the facts. They say that the division of flux in the above example is initially 66.6 per cent armature and 43.3 per cent, as facts require; and at sustained value, 20 per cent armature and 10 per cent field leakage. And at any time during the transient, the proper division is obtained from the equation.

While the other form may be used in practical calculation for practical results (and hence the familiar instead of the new forms were used in the paper) it yet becomes a serious difference if generalization is to be made from the formula to the extent of Diamant's statement that the initial a-c. component depends upon the difference between the initial and sustained flux. Think. If at the instant the short circuit occurs, the field terminals were also short-circuited, the initial armature and field current would necessarily be the same as if those terminals had not been short-circuited. In the former case, however, there would be no sustained value. The flux and currents would die to zero. So the difference between initial and sustained flux could not affect the initial current, either the a-c. or d-c. components; unless it is assumed always that the sustained value of flux is zero, and that is assuming the armature leakage to be zero.

Regarding terminology, we are critized for introducing the "new impressive, misleading name of transient reactance." It has escaped his notice that in previous A. I. E. E. papers and in other literature, this term has been proposed; and in the present paper at the beginning of Part 2, the origin of the term is recorded: Durgin and Whitehead in, 1912; F. D. Newbury in 1914. It may sound to Mr. Diamant as unscientific; but sometimes scientific terms are so long that commercial engineers will not use them, and it is necessary to have a shorter, if less accurate term. How many busy engineers could be persuaded to say, "I want 'the total leakage reactance reduced to armature terms.'" While we reaffirm our acceptance of the term, "transient reactance," as practical and descriptive, and also believe the terminology used in leading tests such as Steinmetz, Berg and Lawrence are scientifically sound, we believe that there should be some terminology which would mean the same to everybody, and avoid such confusion as is experienced in reading Diamant's 1915 paper. His discussion devotes considerable space to this matter. His terms are sound and consistent. Whether it is too late to ask the engineering world to change entirely to adopt Diamant's suggestions, or instead to adopt officially what is already generally used, is questionable.

We acknowledge the omission of Douglas' paper in the bibliography. It properly should have been included.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 28, 1918.

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PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

BY J. J. LINEBAUGH AND J. L. BURNHAM

ABSTRACT OF PAPER

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

The protection afforded by the high-speed breaker or barriers is sufficient for most apparatus and service, but *complete* protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

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THE problem of protection from flashing has for many years confronted engineers who build and operate direct-current machines. Numerous schemes and suggestions have been put forward which it was hoped would overcome the tendency to flashover on extra heavy overloads or short circuits. Some time ago it was felt that the subject of prevention and protection from flashing had not received the study and investigation justified by the trouble experienced and it was decided to make a comprehensive study of the entire subject.

Some form of barrier has been the most common protection suggested, and different forms have been tried with a slight degree of success on some machines and absolute failure on others. It was the opinion of many engineers that barriers could not be designed to take care of a short circuit and that their value was doubtful. However, a special form of barrier, which gives the required protection, will be described later.

It was realized that the means for prevention of flashing at the commutator and brushes of direct-current machines must operate to remove the cause very quickly. The use of some form of high-speed device, which would open the circuit or insert re-

sistance before the short-circuit current could reach a value which would cause flashing, seemed the most logical way to solve the problem, although it was appreciated that the action of the device must be much more rapid than any commercial circuit-opening device previously produced. An investigation was conducted along these lines and two distinct types of high-speed breakers developed, which will be described separately.

A flash at the commutator starts from excessive sparking. Sparking is produced by the breaking of current in the coils short-circuited by the brush as each segment of the commutator passes from under the brush. As the coil is inductive, the spark or arc tends to hold and, if the arc is of sufficient volume, the vapor produced thereby forms a low resistance path between segments and from brush to brush or to frame; through which a large current may pass. See Figs. 1 and 2.

Sparking may be prevented by providing a magnetic field of proper strength and distribution to influence the coils during reversal of their current as they pass through short circuit by the brushes. To provide the correct commutating field for all conditions of load has been the object of designers but success has been only partial. At high loads, saturation of magnetic circuits and distorting influence prevent attainment of the desired field, and for sudden changes in load the changes in field cannot be properly synchronized. It is more difficult to avoid sparking with rapidly varying loads than with gradually changing or steady load, but if a sudden load which would cause flashing is of short enough duration, the arcing at brushes may not produce enough conducting vapor to establish an arc supported by the main voltage. *The value of load that causes flashing when applied suddenly (short circuit) is a function of the time required to throw it off.* The quicker the circuit is opened the higher the value of current that will not cause arcing.

With the ordinary circuit breaker which begins to open in about 0.15 second, there is a certain maximum load which cannot be exceeded for each commutating machine without causing flashing. If feeders have sufficient resistance to limit the short-circuit current to this critical value, flashing will occur only on the rare occasion of a short circuit in a feeder itself. See Fig. 3. It has been the standard practise of nearly all manufacturers to recommend tapping the feeders, especially railway feeders, at a sufficient distance from the substation to insure enough resistance in the circuit to limit current in case of short circuit near the station.

Inductance may be added to the circuit to retard the rate of increase of current on short circuit to such an extent that the ordinary breaker will have time to trip before the current in the machine reaches a value that would cause flashing. The amount of inductance required to delay the rise in current sufficiently, however, introduces other disadvantages which make its use undesirable. When the current is interrupted, the increase in voltage from inductive "kick" is difficult for circuit breakers to handle and introduces the possibility of applying dangerous voltage stresses to the apparatus.

Reactors have been tried in a few instances with some success but it has always been a mooted question whether the resistance of the reactor did not give as much or more protection than the inductance of the coil, and if this is the case resistance only would be much cheaper to install. A coil to give the delay required is usually very large and expensive and occupies much valuable space, giving a total cost out of proportion to the cost of the machines protected or the protection obtained.

With special high-speed circuit-opening devices operating in about 0.005 second, the more sensitive machines, such as 60-cycle synchronous converters for railway voltages, may be short-circuited without flashing over, even though the maximum current is of higher value than would cause flashing with suddenly applied load and ordinary circuit-breaker protection.

The speed at which a circuit breaker must operate to prevent flashing depends on the amount of load thrown on the machine but, under worst conditions, our tests seem to confirm that it must be *quicker* than one-half cycle of the machine to be protected. The time of operation of the breaker would be measured between the time that the current reaches the flashing value to the time that the current is again reduced to the same value after the breaker opens. If the arc formed between two segments is not blown out as they pass from one set of brushes to the next and all following segments have similar arcs formed between them, the arc would completely bridge between positive and negative brushes in one-half cycle, which would complete the flashover. Complete flashover might also occur from gases being blown by windage, magnetically, or by expansion, to increase or decrease the half cycle time.

The time of operation of circuit breakers as given herein is measured from the beginning of short circuit to the instant the breaker begins to reduce the current rise.

Investigation covering these several schemes of protection was made, which it is believed will be of interest and will be described briefly with oscillograms, reproductions from photographs, etc., showing behavior under different loads and short-circuit conditions.

All short-circuit tests were made by connecting positive and negative terminals with a 500,000 circular mil cable; the only equipment in the circuit being the necessary current shunt for the oscillograph, a contactor to close the circuit, and a circuit breaker for overload protection, in addition to the protective device being investigated. Power for the 300-kilowatt, 25-cycle and 500-kilowatt, 60-cycle, 600-volt synchronous converters, used in fuse, barrier, reactor, and high-speed circuit breaker tests, was supplied from a 6000-kilowatt frequency changer set only a few feet from the test, so that there was very little drop in the voltage of the generator or from resistance, and the oil switch was set so that it did not trip out.

High-Speed Circuit Breaker

At the time this development was started it was felt that if a circuit breaker could be designed to operate *within* the time required for a commutator bar to pass from one brush to another; that is, within one half cycle, protection would be afforded against practically any short circuit. Designs were therefore begun on a circuit breaker which would open within 0.007 second, which would cover most commercial machines; *i.e.*, for 60 cycles and lower frequency.

High-speed breakers had been suggested and attempts made to produce such devices previous to this time but, as far as the wr ters know, had never been made to obtain as high speed as the discussion shows would be necessary.

Different types of construction were studied and samples of several preliminary models constructed without obtaining the speed desired. One of the most promising types of construction considered consisted of a knurled fly wheel operating continuously with a knurled cam, so designed and located that a current relay would insert a wedge between the wheel and the cam and trip a breaker attached to the cam by suitable toggle mechanism. This preliminary sample indicated that 0.035 second was the best speed that could be attained.

It was then decided to concentrate all energies on a circuit breaker using the well known principle of a latch, heavy spring

FIG. 1

Flashing at brushes on 1000-kw., 1500-volt generator forming part of 2000-kw., 3000-volt motor-generator set at five times load, showing different stages of arc formation.

FIG. 2 [LINEBAUGH AND BURNHAM]

High-speed photograph of flashing on 300-kw., 600-volt, 25-cycle synchronous converter with short circuit on 0.015 ohms additional in the external circuit and standard circuit breaker.

FIG. 3

High speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter with standard circuit breaker.

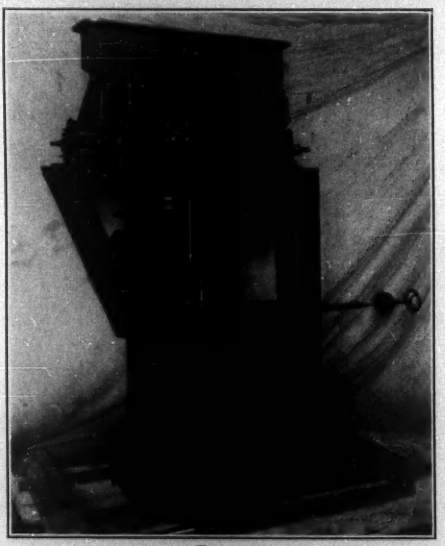

FIG. 4 [LINEBAUGH AND BURNHAM]

3000-ampere, 3600-volt, direct-current high-speed circuit breaker.

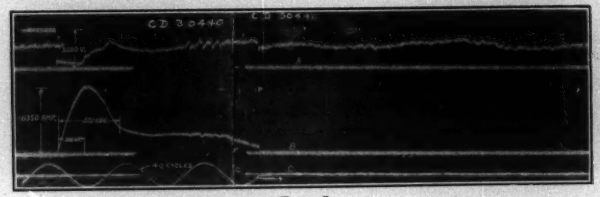

FIG. 5

Direct-current short circuit on 2000-kw., 3000-volt, motor-generator set with high-speed circuit breaker and standard 3000-volt switchboard type circuit breaker.

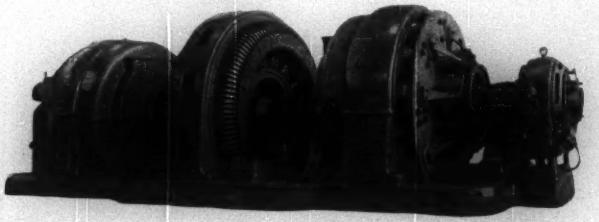

FIG. 6

2000-kw., 3000-volt, direct-current synchronous motor-generator set before assembly of flash-barriers.

[LINEBAUGH AND BURNHAM]

FIG. 7

Type of flash barrier installed on 2000-kw., 3000-volt, synchronous motor-generator set used in connection with high-speed circuit breaker.

FIG. 9
High-speed air cooled fuse holder with magnetic blow-out used in test.

FIG. 10
High-speed oil-cooled fuse holder, used in test.

FIG. 11 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-cooled high-speed fuse. Curve A, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

and series tripping coil, and the high-speed breaker shown in Fig. 4 was finally built.

The problem was to obtain very quick tripping, rapid acceleration of contacts and a sufficient number of ampere turns in the magnetic blowout to insure rapid breaking of the arc. Previous ideas of design had to be abandoned when working for such high speed when a loss of 0.001 second meant a very serious increase in time of operation.

It was found that a series blowout coil had to be used, as sufficient time could not be allowed for the building up of a field after the contacts opened as is ordinarily done in circuit-breaker design, and the strength of this coil must be many times that usually used to rupture the circuit by giving the quick start and acceleration to the arc necessary for the speed desired. The breaker in question has a total of about 150,000 ampere turns at the maximum current obtained.

The moving parts must all be as light as possible, consistent with the great strength required, so that they can be started, accelerated and stopped in a very short space of time and distance. Even with this type of construction, it was found necessary to use somewhat high spring pressure; the spring being compressed to about 8000 pounds when the breaker was closed and ready for tripping.

A very special latch with very small tripping movement was designed somewhat similiar to the hair trigger on a rifle, in connection with a special high-speed tripping coil so that about 0.001 inch movement of the plunger would trip the breaker. It will assist in appreciating the speed attained when it is noted that the breaker must be arranged so that it will not trip under ordinary load condition and must be set above the tripping point of the regular substation breaker so that it will act while the current is increasing from say three and one-half times load to eight times load; current rising at the rate of about 1,000,000 amperes per second. Fig. 5 gives a very good idea of speed and limiting of current, from which it will be seen that the breaker starts to insert resistance in about 0.008 second and the load on the machine is reduced well below the flashing value in 0.020 second after the short circuit was applied.

A breaker was tested very exhaustively in connection with a 2000-kilowatt 3000-volt direct-current synchronous motor-generator set shown in Fig. 6, built for the Chicago, Milwaukee & St. Paul electrification, and found to give complete protection

from damage or burning on short circuit when equipped with barriers shown in Fig. 7.

In connection with the test, it was found that even the speed of 0.008 second obtained would not completely protect machines from flashing on the most severe short circuit, and barriers shown were designed and installed. Tests referred to with high-speed breakers were taken with these barriers, which will be described later.

It is evident that it is preferable in case of short circuit to insert resistance by a high-speed breaker to quickly limit the current to some conservative value and then open the circuit. This type of protection has been adopted as standard. All tests, investigations, et cetera, were based on this theory, although some tests were taken by opening the circuit. It was found that there was a greater tendency for machine to flash if the circuit was opened completely at one time or if too high resistance was inserted, reducing the load to too low a value. For the sake of convenience and comparison, all tests were made throwing short circuit on the machine without load.

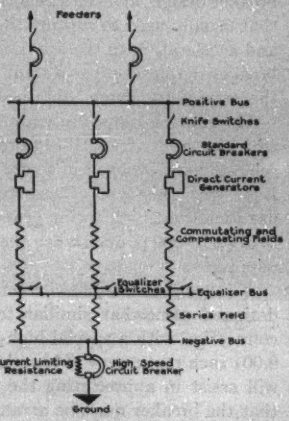

FIG. 8

Diagram of direct-current connections for substation equipped with three motor-generator sets protected by one high-speed circuit breaker connected across limiting resistance

Some of these breakers have been in service since early in 1917 in the substations of the Chicago, Milwaukee & St. Paul Railroad and have amply justified the faith of the railroad company and the designers, as they protect the apparatus from all short circuits experienced, although all feeders are tapped directly to the overhead trolley system immediately at the substation.

One of these breakers is installed in each of the substations connected between the negative bus of the station and the ground or return circuit, as this location gives maximum protection, and one breaker can be used for each machine or one for the entire substation, as shown in Fig. 8.

High-Speed Fuse

It is evident that if a fuse could be developed that would melt at a very small increment of current above normal rating, it might be possible to obtain a speed which would limit the current on a short circuit along the same line as the high-speed circuit breaker just described.

A careful study of all available metals was made by Mr. P. E. Hosegood, who suggested using a silver fuse, and a number of silver fuses of different shapes were tried in the special fuse holders shown in Figs. 9 and 10. The oscillograph record, taken with air break fuse and magnetic blow-out, shown in Fig. 11, indicates that a very high speed is obtained, giving excellent protection and duplicating almost exactly the speed of the high-speed circuit breaker. It was found that a short circuit could be thrown on the 300-kilowatt, 25-cycle, 600-volt synchronous converter without flashing over and with very little sparking at the brushes. The oil-immersed fuse holder without magnetic blow-out gave practically the same result (Fig. 12), the operation being slightly better as far as speed was concerned but the mechanical difficulties of replacing the fuse, etc., being greater.

Reactors

Oscillograph records of short circuit on the 300-kilowatt, 25-cycle, 600-volt synchronous converter show an average initial current rise of about 1,300,000 amperes per second. To protect by reactance, the amount required would depend on the rate of circuit-breaker action. With coils made of 1000 feet of 500,000-circular mil cable, wound on cable reels having an inductance of approximately 0.02 henry in circuit, this particular machine could be short-circuited without flashing when protected by a breaker opening in about 0.15 seconds.

An examination of records, Figs. 13 and 14, will show the severe duty on the circuit breaker and increase in voltage on the apparatus.

It was suggested that shunting the reactor by resistance might reduce duty on the circuit breaker. The coils were shunted by 14 and by 100 ohms and it was impossible to determine from either observation or oscillograph any effect due to the resistance.

The effect of an iron core in a reactor having an inductance of 0.00105 henry is shown in Fig. 15, from which it will be noted that the iron saturated at about 1000 amperes in about 0.007

seconds, after which the current rises abruptly, being limited only by the inductance of the coil as if there were no iron in its magnetic circuit. The delay of about 0.007 seconds due to the prescnce of iron in the eoil, is far less than the time required for the usual breakers, now in use, to open. The weight of this reactor was 7 per cent of the weight of the synchronous converter and would have to be many times larger to give protection with an ordinary breaker.

Second Form of High-Speed Circuit Breaker

Mr. J. F. Tritle has more recently suggested a design for a high-speed circuit breaker which is simple and substantial in construction. This device was built as shown in Fig. 16 and test indicated that the speed was even faster than the large breaker previously described, as will be seen by comparing oscillograms, Fig. 5, on the large breaker, and Fig. 17 taken with the later breaker. This device is essentially a contactor having a laminated structure with electric holding coil and series bucking coil so that it opens when the load current reaches a value sufficient to offset the ampere turns of the holding coil. Tests on the 300-kilowatt, 25-cycle synchronous converter with this device showed that a short circuit could be thrown on the machine without any tendency of the machine to flash over, and the only sparking obtained extended not over one-half inch from the brushes. Similar tests, Fig. 18, on the 60-cycle, 500-kilowatt synchronous converter showed more sparking and, although it protected the machine at times on short circuit, there were other times when the machine flashed over. When the machine was equipped with barriers, dead short circuit could be thrown on with impunity, there being no tendency to flash over and scarcely sufficient sparking to be noticeable.

This later type of high-speed breaker is a part of the more recent equipment being furnished the Chicago, Milwaukee & St. Paul Railway.

Barriers

The barriers shown in Fig. 7 in connection with the description of the high-speed circuit breaker were developed to delay time of flashover, so that the breaker would give complete protection. Such satisfactory and promising results were obtained without the breaker that it was decided to continue investigation to ascertain if it would be possible to devise barriers

FIG. 12

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by oil-cooled high-speed fuse. Curve *A*, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

FIG. 13

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

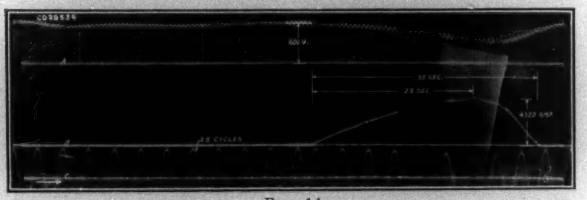

FIG. 14

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker; Curve-*A*, voltage across armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

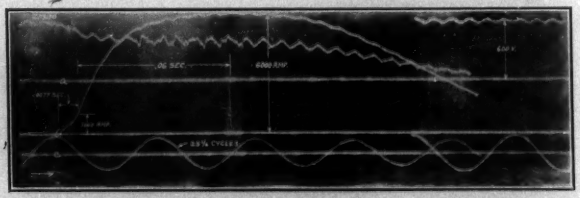

FIG. 15 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter, protected by iron-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across the armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

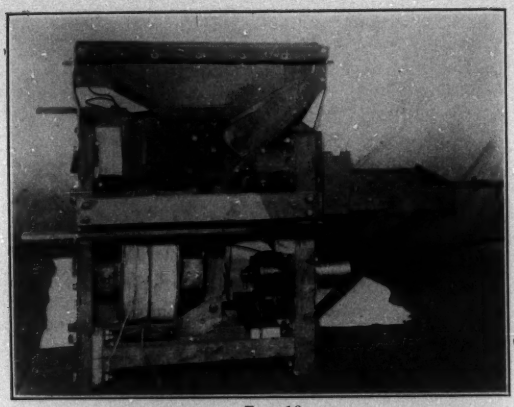

FIG. 16

Second form of high-speed circuit breaker, capacity 1500 amperes, 600 volts.

FIG. 17

Short circuit on 300-kw., 600-volt, 25-cycle synchronous converter protected by second form of high-speed circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

FIG. 18 [LINEBAUGH AND BURNHAM]

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by second form of high-speed circuit breaker.

Left hand curve
Load of 0.03 ohms

Right hand curve
Short circuit.

Curve *A*, armature volts
" *B*, line current
" *C*, collector-ring voltage.

FIG. 19

Final development of flash barriers on 300-kw., 25-cycle, 600-volt synchronous converter.

FIG. 20 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 25-cycle, 600-volt, synchronous converter protected by flash barriers and standard circuit breaker.

FIG. 21

Short circuit on 300-kw., 25-cycle, 600-volt synchronous converter protected by flash barriers and standard circuit breaker.

FIG. 22 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter equipped with flash barriers and standard circuit breaker. Curve *A*, armature volts; Curve *B*, line current; Curve *C*, collector-ring voltage.

FIG. 23

High-speed photograph of short circuit on 300-kw., 600-volt 25-cycle synchronous converter protected by flash barriers and standard breaker.

FIG. 24 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by flash barriers and standard circuit breakers.

FIG. 25

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter without protection.

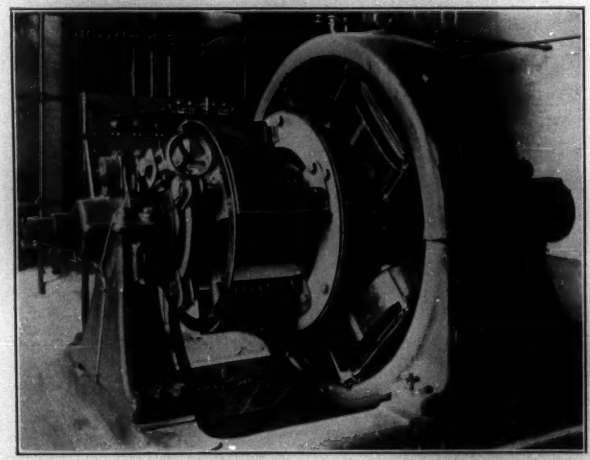

FIG. 26 [LINEBAUGH AND BURNHAM]

500-kw., 25-cycle, 600-volt, synchronous converter, installed in automatic substation, equipped with commercial form of flash barrier.

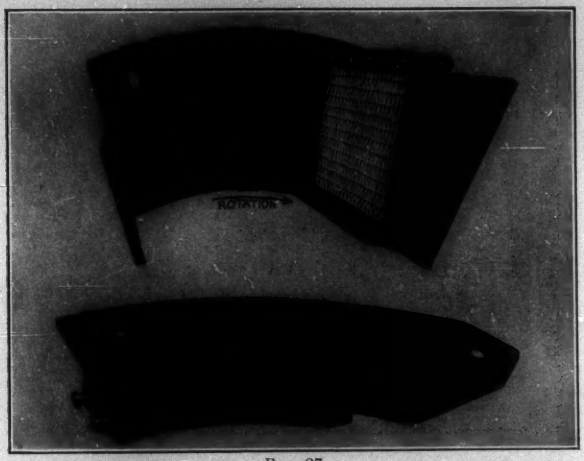

FIG. 27

Flash barrier with front removed to show location and construction of arc scoop and wire-mesh arc coolers.

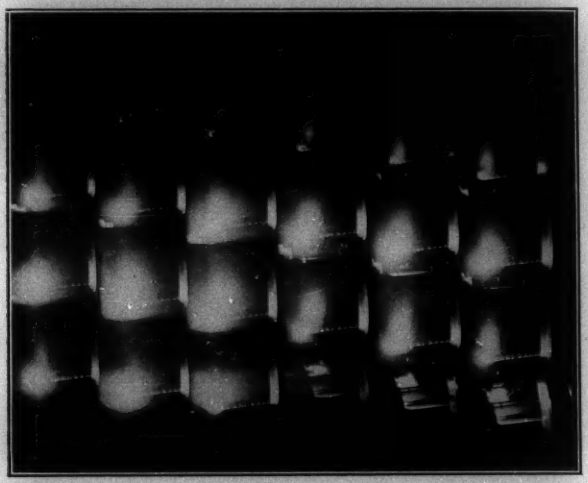

FIG. 28 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of commutator.

FIG. 29

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of brush rigging.

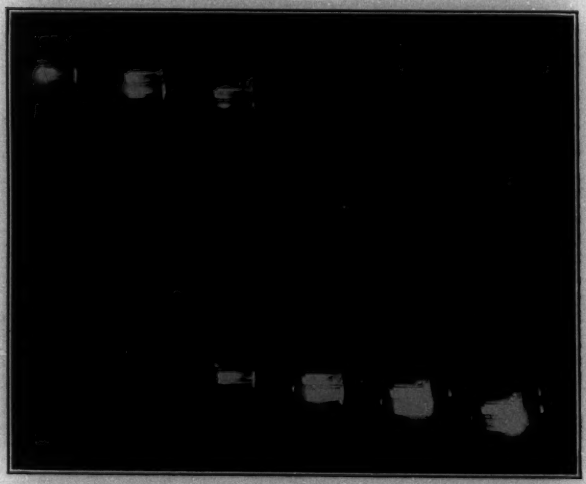

FIG. 30 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging has been changed. Uniform distribution of flashing.

FIG. 31

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging had been changed

FIG. 32

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M* current in middle brush; Curve *I*, current in inside brush.

FIG. 33 [LINEBAUGH AND BURNHAM]

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

FIG. 34

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

FIG. 35 [LINEBAUGH AND BURNHAM]

Current passed through 50-kw., 600 volt generator from an external source.

that would take care of all short circuits experienced in actual service.

Under certain conditions it might be desirable to supplement rather than replace, appliances already installed or to protect from disturbances other than direct-current load which cause flashing. For instance, a synchronous converter could not be protected by a high-speed direct-current circuit breaker if flashing is caused by a.c. phase displacement. For this reason additional protection, such as barriers, to dissipate the arc when started was also needed.

Many different forms of barriers were tried on the 300-kilowatt, 25-cycle, 600-volt synchronous converter, previously mentioned. With increasing success as improvements were made to meet failures, the barriers shown in Fig. 19 were evolved. These barriers gave complete protection from flashover or damage on short circuit. Fig. 20 shows machine on short circuit giving a good idea of flashing and protection afforded, while Fig. 21 shows clearly the small amount of flash which extends beyond the barrier.

About 65 short circuits were thrown on the 300-kilowatt, 600-volt, 25-cycle machine without burning of brushes, brush connections or rigging, or damages of any kind to commutator or machine. Oscillogram, Fig. 22, shows a record of current reaching 34 times full load and gives a good idea of the protection afforded. Many of these short circuits were applied at very short intervals, even as close as one minute apart, without failure to hold and extinguish the arc when the breaker opened the circuit.

Figs. 23 and 24 are very interesting high-speed pictures of the same short circuit analyzed by means of a special high-speed camera devised by Lieut. Chester Lichtenberg, and the successful high-speed pictures we are able to show in this paper are mainly due to his efforts. This camera made it possible to obtain as high as 24 complete pictures of one short circuit, while the best results it was possible to obtain with a moving picture camera were two under-exposed and therefore indistinct pictures.

A little explanation is necessary to read these photographs as, due to the construction of the camera, the lower right hand picture is the first picture of the short circuit; the next picture being the one immediately to the left, and so on to the end of the plate; the first picture at the right of the the next row being

the next picture in the same order and until the end of the plate and the number of rows of pictures. These pictures show very clearly the growth of the arc, disposition on commutator and dissipation of the arc as the regular breaker opens. These permanent records eliminated the personal factors of memory and observation and showed the way for changes to give improvements in barriers. Fig. 25 illustrates very clearly what happens if the machine is short-circuited without protection.

The general arrangement of a successful barrier, Fig. 26, is shown herewith.

A close fitting box of fire-proof insulating material surrounds each set of brushes and is located so as to give a small clearance between the box and the commutator.

On the side of the box towards which the commutator rotates after leaving the brush is fastened a V-shaped "scoop", Fig. 27, of fire-proof insulating material, preferably having good heat conductivity, pointing toward the brush and having small running clearance from the commutator.

Radially above the scoop, about one inch apart, are two metal screens, one coarse and one fine mesh, through which the arc is successively forced and cooled.

It was found that a moderate amount of material is required to give the necessary thermal capacity to prevent an arc from passing beyond a screen of this kind. The scoop running very close to the commutator with narrow edge and small clearance picks up the arc from the commutator and deflects it into the arc coolers which, from their construction, allow free passage of all gases generated by the arc. The cooling and condensing of the arc reduces the gas pressure so that shields at the end of the commutator, to prevent the arc being thrown from the end of the commutator and communicated to pillow block and frame, are permissible. It will be noted from the illustrations that the commutators extend beyond the end of the barrier as it was found that the arc must be prevented from being communicated to the end of the bars.

Investigation was then transferred to a 500-kilowatt, 60-cycle, 600-volt synchronous converter and barriers of similar type, but without continuous end shields, were tried.

Tests showed that these barriers did not give protection on short circuit although they prevented machine from flashing over on very high overload. The high-speed camera record in-

dicated that the arc was being thrown to the outer end of the commutator for some reason, causing such high gas pressure at the outer end of the commutator that the arc was blown under the barrier and the machine flashed over. Figs. 28 and 29.

The differences in performance were ascribed to differences of magnetic fields acting on the arc.

To demonstrate the effect of the magnetic field, various arrangements of connections of brush rigging were made, each to produce a different field where the arcing occurs. The results indicate that it is possible to arrange the brush rigging and connection to make a barrier, as described above, effective on practically all commutating machines and to prevent complete flashover. Figs. 30 and 31 show the effects of change in connection on arc distribution, giving the uniform distribution most favorable to good barrier performance.

Other tests were made to record the simultaneous short-circuit current in the outer, middle and inner brushes by the oscillograph. The records in Figs. 32, 33 and 34 show typical variations of current distribution produced by different connections to brushes. The distribution of current is principally dependent on the magnetic field surrounding the brushes where the arc is formed. To show that differences of impedance have very little influence, record Fig. 35 was taken with current supplied from an exterior source with no flashing. It will be seen that current is practically the same in all brushes. With some connections the deflection of the arc can be plainly seen to follow the well-known relation, as given in Fig. 36, but with the more complicated connections the difficulty of determining resultant field from many sources makes it difficult to determine the direction of deflection of the arc except by experiment.

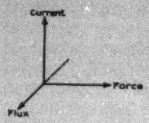

FIG. 36

Direct-current machines for use in * automatic substations are being equipped with these barriers and short-circuit tests at the substations have been taken, indicating that they will take care of any short circuit experienced in actual service. These barriers are in operation and short-circuit tests were taken on a 500-kilowatt, 600-volt, 25-cycle synchronous converter of the Des Moines Electric Railway, Des Moines, Iowa, a 500 kilowatt, 600-volt, 60-cycle synchronous converter of the Coumbus Electric Railway & Light Company, Columbus, Ohio, and a 500-kilowatt,

30-cycle, 1200-volt synchronous converter at Montieth Junction, Michigan, and other installations are now in service.

The investigations and tests indicate that if any commutating machine is equipped with barriers and the last high-speed circuit breaker described, complete protection will be given against external short circuits of all kinds so that interruption to service will not be of any greater duration than necessary for closing the circuit breaker and main switch as in ordinary overload operation.

*See paper by Taylor and Allen, A.I.E.E. TRANSACTIONS Vol. 34, 1915, page 1801.

DISCUSSION ON "PROTECTION FROM FLASHING FOR D-C. APPARATUS" (LINEBAUGH AND BURNHAM), ATLANTIC CITY, N. J., JUNE 28, 1918

Charles L. Fortescue: I would like to congratulate the authors on the fine results they have been able to obtain with a mechanically-operated high-speed breaker. I can assure you, gentlemen, that to attain a speed of 0.005 of a second on a mechanically operated breaker is no small achievement. The very nature of the problem and the materials available set a limitation to the possible speed obtainable.

I am very much interested in the subject, because I have been associated with various men of our company in a similar line of development. In 1911, in association with the late Mr. William Cooper, the problem of preventing flashing of railway motors came to my attention. On one of the elevated lines in New York City, they were having some trouble with flashing of the motors, due to short circuits and it was proposed to prevent it by operating the breakers quickly. Mr. Cooper conceived the idea that we could obtain a quick operating relay, which would operate the valve in the air-operated breakers, depending upon the magnetic energy released due to the short circuit. With this idea, we built a relay-transformer. The transformer had a small air gap in it, and we depended on the discharge of the magnetic energy stored in this gap to operate the relay. With this little device we were able to obtain such satisfactory results that we managed to overcome our troubles on that particular road. We obtained, as I recall, speeds of from 0.025 to 0.030 of a second, and that encouraged us to go a step further, and we decided to design a direct-operated circuit breaker of the carbon type. The second design was a carbon breaker with a magnetic blowout, and it had a rather powerful spring, somewhere around 300 or 400 lb. This breaker proved quite satisfactory. We were able to get speeds down to 0.015 of a second with the latch operated by the transformer device.

Later on I conceived the idea of using a condenser instead of a transformer for storing the energy required to operate the trip, in other words, we placed the condensers in series with the trip coil across the direct-current terminals. When a short circuit occurs, there is an initial fall of potential, and this discharges the condenser through the trip coil and operates the trip magnet. The frequency of the discharge current will depend on the capacity of the condenser and the self-inductance of the magnet. This arrangement was very satisfactory, in fact, with a circuit-breaker of very ordinary design, nothing very special about it, except it had a rather stronger spring than usual, we were able to obtain a speed of 0.011 of a second.

I would like to take exception to a few points in the authors' discussion of the question of reactors used for retarding the growth of current. The problem, as I see it, is not that the

choke coil is expensive, it is a balance between the cost of the choke coil and the high-speed breaker. As it increases in speed the breaker becomes more and more expensive, and as I said before, it is a big problem to produce a mechanical device which will always operate satisfactorily at high speed. There is a chance of breakage and crystallization of parts, the springs are tremendously powerful, in some cases several tons, and the working parts are subject to heavy jarring action, so that the cost and maintenance of a device like that is necessarily very high. By using a choke coil, it is possible to use a lower speed and less costly device. It has been estimated that the economic speed was somewhere around 0.009 of a second.

A description of the early tests referred to in this discussion was published some time around 1913 in "*Electric Traction*" by Mr. Funk. Later on, 1914, Mr. Yardley presented a paper on the "Use of Reactors with Synchronous Converters," in which he showed a set of films that were taken with converters operated with one of these breakers. We also have films showing them successfully operated with 1500-volt direct-current machines.

I might mention that in these mechanically-operated breakers the capacity required to operate the tripping device was quite small. In the first breaker we made we were able to operate very satisfactorily at 500 volts, with a condenser of 1-microfarad capacity and at 1500 volts with a condenser of one-third of a microfarad capacity. A little later we used a stronger operating trip coil, which requires 2 or 3 microfarads at 500 volts and still later we have used a tripping device of very much the same pattern as that which the authors have described.

Another device that we have used to obtain quick operation is a relatively small choke coil, shunted by the trip coil. In this device the impulse current in the trip coil, when short circuit occurs operates the breaker enabling us to get very quick action.

We have had some of the condenser tripped apparatus in actual operation. We have had one installed at the plant of the Youghiogney Coal Company, operating in mine service, and the operation had been very satisfactory. In fact, most of our work has been done with the idea of developing something that could be used for such service and to protect generators and motors from short circuits on feeder circuits. We have tried to obtain something that could be sold at a moderate price, to be used for the protection of motors, etc., in industrial work.

We have also had another circuit breaker installed at the New River Coal Company. I saw a day's record of that system— I think it was last fall—which they say is quite an ordinary record: There were thirty shorts in one day, and in every case the breaker operated so that the motor or generator was protected. In this case we have a small choke coil in series with the machine in addition to the high-speed breakers.

This is a rough summary of the work that we have done in connection with high-speed breakers. We have not reached as high speed as Messrs. Linebaugh and Burnham have obtained with their very special mechanical construction but the results have been very satisfactory with ordinary forms of breakers.

F. C. Hanker: The use of higher direct-current voltages on railway systems has increased the difficulties in connection with the supply of power, particularly with commutation during the short-circuit conditions. In the early days, somewhat similar conditions obtained on 600-volt railways and forced improvements in the commutating characteristics of direct-current generators and converters, so that the modern machines, with commutating poles, and, for more severe service, with pole-face windings, are very rugged and have given satisfactory service.

The advent of the higher voltages has made the problem of the designer more difficult and the resultant increase in flashing troubles has required concerted attention on improvements. The authors have described in detail the manner in which they have attacked the problem and outlined the various forms of breakers and barriers they have developed after extended experimental testing. This method of solving the problem is the logical one, as pointed out by the authors, when we take into consideration that the remedy must be applied to existing installations. In other words, it may be classed as an external attack.

In the form of breaker described in the paper as being installed in the stations of the present electrified zone of the C. M. & St. P., the tripping mechanism is complicated, and extremely heavy spring pressures for rapid opening are used. Such a circuit breaker will be subject to relatively rapid deterioration and, unless kept at the maximum point of efficiency, will slow up in its speed, then if it fails to open the circuit before the flashing starts, it will be too late to stop it.

Another method of solving the problem of flashing that sounded rather revolutionary when it was first proposed was suggested by Mr. N. W. Storer, who is well known for his work in the railway field. It differed radically from the previous methods using circuit opening or limiting schemes and attacked the cause of the flashing. It effectively suppresses it while all other schemes only minimize the possibility of flashing or reduce the resulting effect.

The flash suppressor consists of a circuit closer arranged to short-circuit the armature winding of the direct-current generator through collector rings. It is tripped in a manner similar to circuit breaker practise with the addition of the anticipatory feature developed in previous high-speed circuit breaker investigations. It is also feasible to supplement this method of tripping by taking advantage of the increase in field current under the short-circuit conditions for operating an

auxiliary tripping coil. The moving parts are small and as it is only necessary to move them a relatively short distance the device can easily be closed in from 0.006 to 0.008 sec. without using abnormal spring pressures. Higher speed can be obtained, if necessary, without undue complication, but tests show conclusively that the flashing is suppressed effectively after intervals several times longer, so that it was not considered necessary to secure higher speed of operation.

The flash suppressor really performs a double function; first, short-circuiting the collector rings and thus simultaneously reducing the voltage between commutator bars practically to zero, or, in other words, all of the voltage generated in the winding is absorbed in that winding so that no difference of potential between commutator bars will remain; and second, in reducing the field flux. Short circuits such as are applied by a flashing suppressor, result in a heavy wattless current flowing through the armature winding. The magnetizing force of the armature ampere turns opposes that of the field winding and cuts down the voltage at the maximum safe rate without opening either the field or armature circuit. In this way, the stored energy not only of the line but of the machine itself is harmlessly short-circuited through the machine and dissipated in the armature and field windings.

During the studies made of this method of suppressing flashing numerous tests were made on different types of machines and it may be of interest to review some of the more representative oscillograms.

The first oscillogram Fig. 1 shows a direct-current short circuit limited by resistance that is typical of a standard direct-current generator. The armature current rises to about 13,000 amperes or about eight times full-load current. The actual current obtained was about 60 per cent of the theoretical current that would be obtained by dividing the terminal voltage by the circuit resistance. The initial rate is about 2500 amperes per 0.001 sec. and the current rose to practically its final value in the first 0.010 sec. The shunt field current undergoes an increase in value, due to the transformer action between the armature and field winding.

Fig. 2 shows the effect of the application of the flash suppressor 0.006 sec. after a short circuit was applied to the direct-current side. In this case, the short circuits were made without resistance or reactance of any kind in the circuit. The effect on the commutator was confined to a small explosive puff of very short duration which did no burning on either the brush holders or commutator surface. Many duplicate oscillograph records of combined short circuits were made which established that the suppression of the flashing was independent of the point on the alternating-current wave at which the short circuit occurred and that all armature positions give essentially the same effect. The longer delays resulted in a somewhat heavier

FIG. 1

FIG. 2

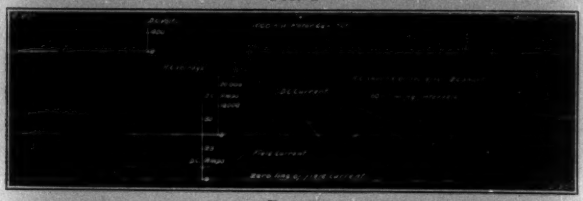

FIG. 3

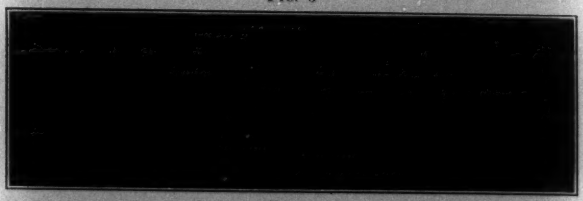

FIG. 4

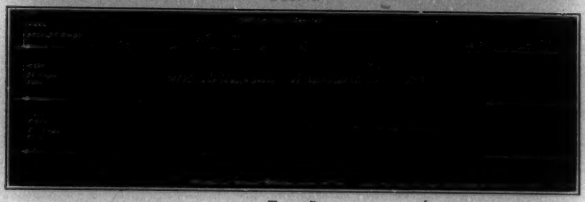

FIG. 5 [LINEBAUGH AND BURNHAM]

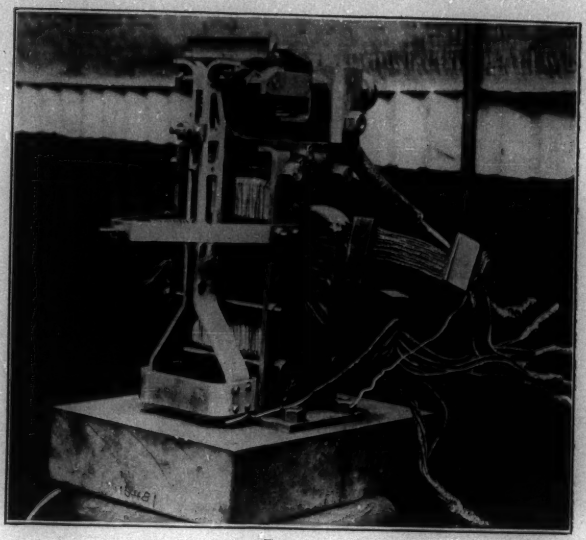

FIG. 6

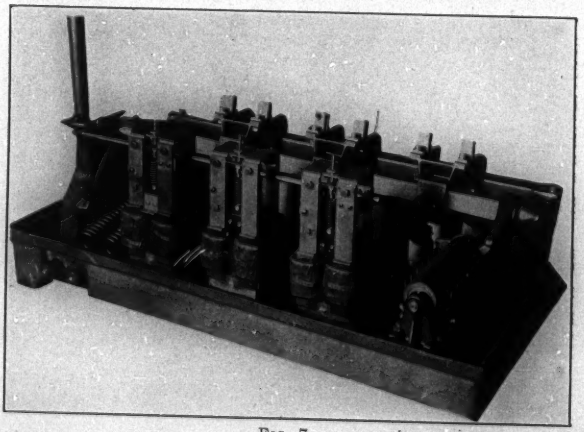

FIG. 7 [LINEBAUGH AND BURNHAM]

flash at the brushes, but none of them of sufficient duration to result in burning.

Fig. 3 is essentially the same as the previous, differing only in the time the operation of the flash suppressor is delayed.

Fig. 4 gives the results of a test with a compound wound generator.

Fig. 5 shows the alternating current in the three phases when short-circuited at full voltage. An analysis of the current of all phases indicates that the resultant current rises uniformly in magnitude until it reaches its maximum value in one-half cycle after the instant of the application of the alternating-current short circuit.

The operation of the suppressor immediately stops any further increase in the direct-current short circuit and in the next half cycle reduces the current to practically zero. The large decrease in the direct current in the entire cycle following the application of the alternating-current short circuit is well understood when it is recognized that an armature winding can deliver only a definite current on short circuit. This current is limited by the electrical constants of the armature winding and is not affected by the number of points at which a short circuit is applied to the winding. This means that the alternating-current short circuit is subtracted directly from the direct-current output and the decrease of direct current corresponds to the building up of the alternating short-circuit current. The alternating component disappears when the direct current finally settles down to a constant value of about 4½ times full load in about 0.075 sec.

In the various tests made by short-circuiting the direct-current side without external resistance, the short-circuit current reaches about 30 per cent of its theoretical short-circuit value. In other words, the normal design of high-speed compensated generator would deliver from twelve to fifteen times or even as high as twenty times full-load current on direct short circuit, that is, the percentage in basing the total on the circuit resistance, including the brush contact.

Fig. 6 is the single-unit type of suppressor consisting essentially of a high-speed circuit-closing contactor. With this design, a closing time of 0.0045 sec. was obtained. Fig. 7 illustrates a design for simultaneously short-circuiting several units, and on test, the closing time was between 0.006 sec. and 0.008 sec. These two photographs illustrate the simplicity of the circuit closing device.

It must be recognized that there are disadvantages in any of these special applications and they are only justified where the complication is necessary, due to the severity of the service. The flashing suppressor will be installed in the substations on the C. M. & St. P. Ry. that are being supplied by the Westinghouse Company while we understand that the new design of high-speed breaker will be installed in the substations that are to be furnished

by the General Electric Company. In this way, both methods will be placed in actual operation on similar service, and a true comparison of the merits of both can be obtained. Engineers, generally, will be interested in watching the operation of these two types and, obviously such a practical test is of far more value than any theoretical discussion.

H. B. Dwight: I would ask the authors if they will say a word or two about the features of design of the brush rigging and brush-holder cross-connections required in order to prevent the troublesome crowding of the flash-over arc toward one end of the commutator, produced under some conditions by the magnetic effect of the current in the brush rigging. They mentioned, in connection with Figs. 30 and 31, that some simple change cured that trouble.

Felix Wunsch: I wish to ask the authors if it would not be practicable to insert blank dividing bars into high-voltage machines to prevent flashing.

F. D. Newbury: The authors make the statement that "reactors" are of questionable benefit. There is no uncertainty as to the effect of reactance. It is simply a question of the reactance voltage generated under the conditions obtaining in a particular case. At the first instant of short circuit, the rate of current increase is high, and a considerable opposing voltage is generated in the reactance. As time progresses, the rate of current increase falls off, and the reactance voltage correspondingly decreases. Finally, when the current reaches its steady value, there is no reactance voltage opposing it. Thus all that reactance does is to reduce the rate of current increase; that is, to reduce the slope of the current curve. In Figs. 13, 14, 15, the current curve has practically reached its steady value before the circuit-breakers started to open. Obviously, in these cases the reactance coil used had no effect at the time of circuit opening except that due to its resistance, a very appreciable effect, however, in Figs. 13 and 14.

In comparison with these results, I wish to show two oscillograms taken from a paper presented before the Institute by Yardley in 1914.* These oscillograms were taken in June, 1912, just six years ago, on a 600-kw., 1500-volt, 60-cycle synchronous converter unit, consisting of two 750-volt converters in series. These machines, by the way, represented the first application of 60-cycle synchronous converters to 1500-volt railway service. They did not have commutating poles which accounts for the relatively low current values at which they flashed over.

The first oscillogram shows results with reactance and a standard circuit-breaker. The current reached its final value of

*"Use of Reactance with Synchronous Converters" by J. L. McK. Yardley, A. I. E. E. TRANS., 1914, Vol. XXXIII, Part II, p. 1521, Figs. 15 and 16.

about 2500 amperes before the breaker opened, and the converter flashed when the circuit-breaker tripped. The second oscillogram shows the same conditions, except that a much quicker acting circuit-breaker was used. The circuit breaker opened in time to limit the current to 1000 amperes and the converter did not "buck."

From these cases it is clear that the function of reactance is to delay the rate of current increase. This is of value only when the circuit breaker opens before the current reaches a steady value at which time the reactance becomes ineffective.

The statement is also made that a generator or converter will often safely carry loads that will cause flashing when the circuit breaker is tripped. This statement has been confirmed in several installations that I am familiar with, and its truth is also shown by the oscillograms in Yardley's paper. The converter did not flash until the circuit breaker tripped. The point of practical importance to the operating engineer is that circuit breakers should be set so that feeder breakers trip before machine breakers, and the latter should trip only when absolutely necessary.

In Fig. 5, the line voltage curve drops to a low value at the instant of short-circuit, and when the current begins to decrease, the voltage begins to rise. So far, the action is reasonable, but beyond this point the voltage does not increase above half normal, even though the current falls to zero. Unless something abnormal happens, the voltage returns to normal when the breaker opens the circuit, as shown in Fig. 14. How is this voltage curve to be explained?

In Fig. 15, there is also a falling voltage curve co-incident with the falling current curve. This also calls for explanation.

The two parts of Fig. 18 are interesting in this same connection. In the left-hand curve, with 0.03 ohm in circuit, the current reached 5640 amperes and only a small temporary dip in armature voltage results. In the right-hand curve, on dead short circuit, the current rises to 7670 amperes, and the voltage does not return to a steady value until a considerable time after the current drops to its minimum value. What caused this voltage disturbance?

Fig. 22 illustrates a short circuit on a 300-kw., 600-volt converter, equipped with flash barriers and standard circuit breaker. The direct-current voltage (after the short circuit) becomes alternating, caused apparently by the converter dropping out of step. Was this the case? The value of current given in this oscillogram also calls for explanation.

In Figs. 13 and 14, the load circuit of the converter is closed through a reactor having a resistance of approximately 0.025 ohm (1000 ft. of 500,000-c.m. cable). The current practically reaches its steady value before the circuit breakers opens, so that only the resistance of the coil is effective. The oscillograms show that the armature voltage (using collector-ring voltage)

drops to about 80 per cent of its initial value at the time of maximum current. Using these data, the resistance of the complete circuit is approximately 0.105 ohm and the resistance of the converter circuit is 0.08 ohm.

In Fig, 22, the converter is short-circuited. The collector-ring voltage falls to approximately 60 per cent of its initial value at the time of maximum current. Using this voltage and the current of 17,100 amperes, the effective resistance would appear to be only 0.02 ohms, or one-fourth the value derived from Figs. 13 and 14.

In Fig. 31, showing the behavior of a 500-kw., 60-cycle converter under the same condition as the 300-kw., 25-cycle converter, Fig. 22, the current is less than in the smaller converter, although it is reasonable to expect that the resistance in the larger machine would be materially lower. Apparently this 500-kw. converter flashed over—the alternating-current voltage as well as the direct-current voltage falling to zero—and the converter apparently dropped out of step.

The various methods of protecting commutating machines against flashing are valuable under conditions justifying their use. Their advantages are not secured without counter-balancing disadvantages. High-speed breakers are expensive, and barriers considerably decrease the accessibility of the commutator and brushes.

In general, higher direct-current voltages—1500-volts and higher—require protection from flashing. I question whether the usual 600-volt installation requires either high-speed breakers or barriers, or that these measures should be employed in preference to the usual protection afforded by a proper disposition of feeder taps. There is no justification, in the usual case, for locating taps immediately at the substation. The increase in losses caused by omitting taps at the substations is negligible.

In this connection, it should be remembered that feeders are usually connected to the trolley wire every thousand feet. Omission of one or two taps nearest the substation only affects appreciably the losses for that part of the energy used in that section, which obviously is a small part of the total energy. The protection provided by a proper disposition of tap is secured at a lower cost, considering cost broadly, than by the methods discussed in the paper. These more elaborate methods, while exceedingly valuable in extreme cases, should be used with discrimination. I would like to have the authors' opinion on this point: Do they advocate the general use of the barriers they have described on 600-volt converters, and are they now indifferent to the location of taps?

W. F. Dawson: The photographs appear to show that all of the flashing is the result of excessive sparking of the brushes, and that it all starts at the edge of the brush, and no flash starts in between the brushes, as might possibly be expected, due to

the distortion of the main flux wave shape, with consequent very high potential between adjacent commutator bars under the edge of the leading pole.

J. F. Tritle: Fig. 8 illustrates the general scheme of operation of the second form of high-speed breaker. This breaker is radically different from the first form in that all mechanical latches and devices of that kind are entirely eliminated and the breaker, instead of being tripped mechanically, is tripped electrically. F^1 and F^2 represent a laminated field structure something like that of an ordinary alternating-current magnet. The poles of F^1 and F^2 are bridged by a very light armature A pivoted at P which is held in contact with the field by a shunt coil S^1 energized from any convenient direct-current source such as the exciter circuit or the main bus. A series bucking bar S^2 which

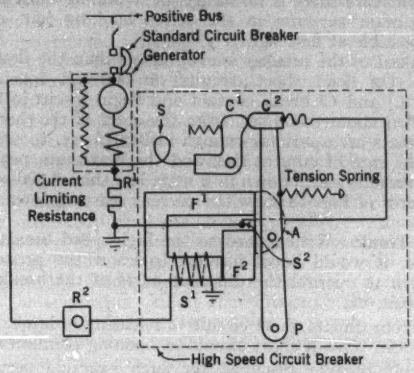

FIG. 8

electrically trips the breaker is located between the poles of the field magnet in a plane perpendicular to the plane of the laminations and in very close proximity to the armature, so that a given current flowing in it produces a maximum change in the armature flux with a minimum change in the flux interlinking the shunt winding S^1. The tension spring attached to the armature provides a means of adjusting the breaker, and also gives the high-speed opening of the contacts. The main contact tips C^1 and C^2 are of the solid copper type used so successfully on railway contactors. The blowout coil S is of the series type and designed to give a very intense magnetic field at the contacts.

Assume that it requires 2000 ampere turns in the holding coil to hold the armature in contact with the field with 0 current in the bucking bar and it is desired to set the breaker to trip on 2000 amperes. The current in the holding coil is simply

adjusted to give approximately 4000 ampere turns by means of the calibrating rheostat R^2, and it is obvious that the armature will release as soon as the flux from the series bucking bar neutralizes or offsets the additional 2000 ampere turns of the shunt coil. If a short circuit occurs on the line or across the terminals of the machine the current may rise to several times the normal tripping point in which case the flux in the armature will be reduced to a very small value or even reversed. This condition gives the maximum speed of operation, because the armature starts to move at the instant the flux is reduced to normal drop-out value and by the time the flux reaches zero the armature is moving at a fairly high rate of speed. Several hundred short-circuit tests on three sizes of machines proved that owing to the steep pull curve and high-speed characteristics of the armature there is no danger of it pulling back in, due to the maximum currents in the series bucking bar, as might appear possible at first sight.

The speed of the breaker is even faster than the first form of breaker. On dead short circuits on 600-volt machines the contacts C^1 and C^2 break contact and begin to cut in the resistance R^1 in about 0.003 sec. from the start of the short circuit. The contacts are apart far enough in 0.0045 sec. to measurably reduce the rate of current rise, and the maximum peak occurs and the current starts down in a little less than 0.006 sec. The oscillograms of Fig. 18 show the current starting down in 0.0053 seconds.

H. E. Trent: With regard to the high-speed breaker shown in Fig. 4, it would be of interest to know the proportion of time taken to operate the different parts of the breaker on a short circuit, viz:

(1) From time of short circuit to releasing of latch.

(2) From time of short circuit to opening of main contacts.

A circuit breaker designed for such exacting service, one imagines, would require fairly frequent renewals with regard to arcing contacts, even taking into consideration the powerful magnetic blow-out provided.

The operation of the silver fuse with magnetic blowout coils is very satisfactory in point of speed, and it would be of interest to know, if information is available, as to how the shape of the fuse influenced the speed of opening the circuit and whether the authors made similar tests with higher d-c. voltage.

J. L. Burnham: Before replying, there is one question I would like to ask Mr. Fortescue, and that is how they measured the time of the three breakers mentioned, *i. e.*, are intervals measured the same as defined in our paper? In the case of the breaker described by Yardley several years ago I remember he said in his paper that the breaker flashed over (probably due to inductance in circuit for protection) and I do not know that the breaker was ever put in commercial operation. In reference to the impulse method of quick tripping, I think you would have

the breaker doing things you did not want it to do, because it would depend on the rate of increase of current rather than actual value of current as to what it would do.

Mr. Hanker mentions possible deterioriation of the first type of circuit breaker we described and stated had been in successful operation over one year. Would add that the maintenance has been very low, and the adjustments have been of very minor importance. In addition severe factory endurance tests were made and none of the performance confirms his prediction. However, the second type of breaker described, which is a later development, is the one that we will now recommend on account of its greater simplicity and ruggedness.

In reference to the short-circuiting scheme of protection for killing the voltage, as described by Mr. Hanker, we still have to learn how this works out in actual service operation. In describing the trial switch he states that it is made up of light parts and has short movement. It may be found that the parts will have to be strengthened or enlarged for greater movement to meet service requirements and the resulting added weight would decrease the speed.

This method of suppressing flash-over has certain objections. The short-circuiting of the machine after every disturbance makes it essentially not universally applicable. That is, it cannot be used on converters as it would short-circuit the a-c. system; it cannot be used on individual feeders; it cannot be used to protect individual machines in parallel,—it must protect the station as a whole, which seems to be its particular application; it is difficult or impossible to apply it to old machines; every operation (additional short circuit) gives a maximum shock to the apparatus; killing the voltage by this method may cause a comparatively long interruption to service; the addition of collector rings and brushes to a direct-current machine involves added risk, especially on the higher voltage machines, and increase in cost of maintenance. There will also be certain losses with these collector brushes, which will decrease the efficiency. A machine equipped with rings is inherently longer. Instead of limiting the rise of current in the armature it adds thereto; regardless of magnitude of short circuit or distance from substation, maximum short circuit is thrown on adjacent substations when suppressor operates; an impulse from a distant short circuit or moderate but sudden increase of load may trip this type of breaker which subjects the apparatus to maximum strains that are not at all commensurate with the original load. It may even be possible for a short circuit in one station to cause adjacent stations to be short-circuited by their flash suppressor and so on until all substations are out of service. For comparison of the two methods of protection would refer you to the last paragraph of our paper.

Answering Mr. Dwight, as to the arrangement of brush holder connections, it involves the question as to whether the brushes

are ahead or behind the stud that carries the current from the brushes, and also whether the connections are taken from one end of the commutator or the other, and I think if he will refer to the part of the paper dealing with the magnetic conditions and forces which affect the arc that it will explain in a general way what he desires to know. There are many combinations of conditions affecting the direction of flashing.

In reference to the placing of alternate "dead" segments between the regular commutator segments, suggested by Mr. Wunsch, it is, of course, equivalent to doubling the width of mica between segments. There may also be some cooling action of the dead segments which would tend to decrease any sparking that occurs. This might increase slightly the load that a machine could carry without flashing over. If this arrangement is effective it must be at the expense of increased diameter of commutator with corresponding increase in speed. The same increase in size of commutator, if possible, would also reduce the tendency to flash if the "dead bars" were not used.

In reference to Mr. Newbury's remarks on the various curves; it will be noted that the curves referred to are made under different conditions, that is, in the case of the generators, we had the voltage killing device operated by a main slow breaker which inserted resistance in series with the field after the slow breaker was tripped. The slow breaker, of course, operated after the fast breaker which explains the decrease in d-c. voltage in Fig. 5. Then, some of the tests were taken with barriers, and others without barriers and some with slow-speed breakers and others with high-speed breakers and also with many combinations of protection.

Fig. 13 was given only to show the rise in voltage across the circuit breaker when reactance is in circuit. The breaker was tripped by hand.

Fig. 15 is a record in which the breaker did not trip. The reactance reduced the synchronizing power of the converter enough to cause it to gradually drop out of synchronism and thus reduce the d-c. voltage. The total time was very short as indicated by the 25½-cycle timing curve.

In Fig. 18 the disturbance in voltage was caused by some tendency to flash, but complete flash-over did not occur as will be seen by the record of current which was reduced by insertion of resistance connected across the high-speed circuit breaker. The complete record shows the voltage back to a steady value in 11 cycles or 11/60 of a second.

The short circuit recorded in Fig. 22 shows that the converter pulsated and reversed polarity to small values. When it became steady the polarity was correct.

Regarding the comparison of short-circuit values of current on the 500-kw. commutating-pole machine and the 300-kw. non-commutating pole machine, I believe there are too many variables in the transient phenomena to arrive at true values of

internal resistances especially when the current is sufficient to throw the machines temporarily out of step as shown in Figs. 22 and 31. Considerable variations in values are obtained on a given machine even under apparently the same conditions.

The apparent discrepancy in resistance values of armature resistance as calculated by Mr. Newbury from data in Figs. 13, 14 and 22, is due to assuming resistance as of one protective coil only. It will be noted that the text refers to coils and there were actually three coils used in the test. This will bring the values of armature resistance into reasonably close agreement considering the rough method of determination.

I endorse Mr. Newbury's suggestion that the operating companies should not tap their feeders into the trolley less than 1000 feet and preferably further from the station. That is the simplest form of protection and in the majority of cases would make the equipment much more reliable. I would much prefer this method to using the other means which have been described, if it proves suitable in any individual case.

Mr. Dawson asked the question whether the arc always starts at the edge of the brushes. The high-speed photographs seem to indicate that this is generally so. In two instances out of many trials a converter flashed back of the brushes at a point where no protection had been provided. This shows, I think, that the conditions back of the brush are favorable for supporting a flash but the direction of motion of the commutator causes the arc to form and be carried around with it from the front of the brush.

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THE AUTOMATIC HYDROELECTRIC PLANT

BY J. M. DRABELLE AND L. B. BONNETT

ABSTRACT OF PAPER

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size, without sacrificing complete control.

This station consists of three 400-kw., 500-kv-a., 60-rev. per min., 2300-volt, vertical generating units, tied in to a system, of which the main generating station contains about 20,000 kv-a. in steam turbo-generators. One striking feature is the entire omission of the usual governors, the waterwheel gates being motor driven and controlled by contact-making ammeters. Each unit has its individual control panel, consisting of the necessary contactors and relays to connect it to the bus at the proper time. A motor-driven drum controller gives the proper time element between the different steps in the operation of placing the generator on the line. Any generator can be started either by a float switch when the pond level reaches the proper height or by a remote control button in the steam station. The starting of the first generator throws on the line the motor of one of the two exciter sets, and the generator cannot be connected to the bus until the excitation voltage has reached the normal value. The waterwheel gates are then partly opened and the generator comes up to approximately normal speed. It is then connected to the bus without field through an iron-core reactance. Then a weak field is applied. Next it is raised to full normal value, and then the reactance is short-circuited. The contact-making ammeter opens the gates to full gate opening and the generator then carries full load in about 40 seconds after either the control button is closed or the float switch is closed.

ON October 2nd there was placed in operation at Cedar Rapids, Iowa, the automatic hydroelectric plant of the Iowa Railway and Light Company. This plant represents a further development of the automatic substation first described in a paper by Messrs. Allen and Taylor in the A.I.E.E. TRANSACTIONS, Volume XXXIV, Part 2, page 1801.

The station operates with no attendant and the different generating units start or stop, depending on the flow of the river or can be started and kept running by remote control from the main steam station, as described in detail later.

The building is 112 ft. (34.13 m.) long, 41 ft. (12.49 m.) wide and has a depth to rock bottom at the bottom of the draft

tubes of 37 ft. (11.27 m.). The substructure is of concrete; the superstructure is 30 ft. (9.14 m.) high above the water table of the building and is constructed of Bedford Limestone. The interior of the building is finished in yellow brick and the floor in red tile. The exterior is shown in Figs. 1 and 2. There are installed three 500-kv-a. 80 per cent power factor, 60-cycle two-phase, 2300-volt, 60-rev. per min. generators, of the vertical type. They are driven by waterwheels rated 540 b. h. p., 60 rev per min. under a 10 ft. (3 m.) head, the operating head varying from 8 ft. (2.43 m.) to 11 ft. (3.35 m.). The runners are of the Francis type, having a diameter of 171 in. (4.33 m.). The rotating element is supported by a thrust bearing at the top, supplied with oil by a pump driven from the generator shaft.

The dam consists of nine spillway sections, each 60 ft. (18.28 m.) long with automatic flashboards, the total over-all length being 600 ft. (182.88 m.). The dam is shown in Fig. 3.

There are two General Electric Induction motor-driven exciter sets rated 100-kw. 125-volts each capable of carrying the entire excitation of the total development, namely, that of four generators. The motors are rated 150-h.p., 2300-volt, two-phase, 60-cycle, 1200-rev. per min. and are designed to be started by the application of full voltage at standstill.

The plant is designed to operate in parallel with the 19,000-kw. steam turbo-generating station of the Iowa Railway and Light Company, the steam plant being located at a distance of 3300 ft. (1005.84 m.) from the hydroelectric plant. The stations are tied together by one underground line consisting of two 600,000-cir. mil, two-conductor, concentric, 7500-volt, varnished cambric, lead-covered cables.

The different generating units may be started and stopped either automatically, depending on the rise or fall of the water level in the river; or by remote control from the steam station when the capacity is needed. To accomplish the latter, a bench board section, shown in Fig. 5, was installed in the steam plant. On this, for each generator, there are provided three single-throw and one double-throw control buttons, the purpose of which will be explained later. All control wires between the stations are bunched in a multi-conductor lead-covered cable consisting of fifty No. 12 and four No. 4 B. & S., rubber-covered wires.

The same benchboard contains for each generator an in-

FIG. 1—EXTERIOR OF STATION—FRONT VIEW

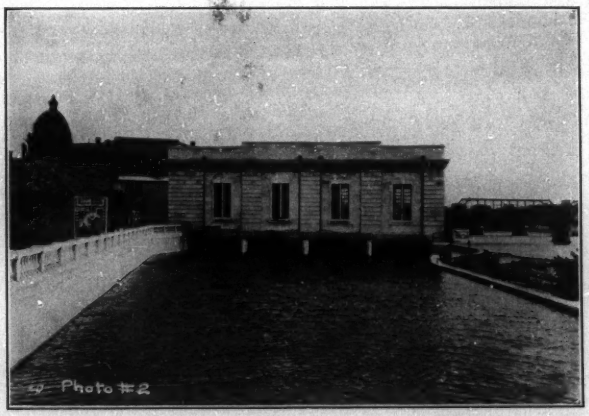

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FIG 2—EXTERIOR OF STATION—REAR VIEW

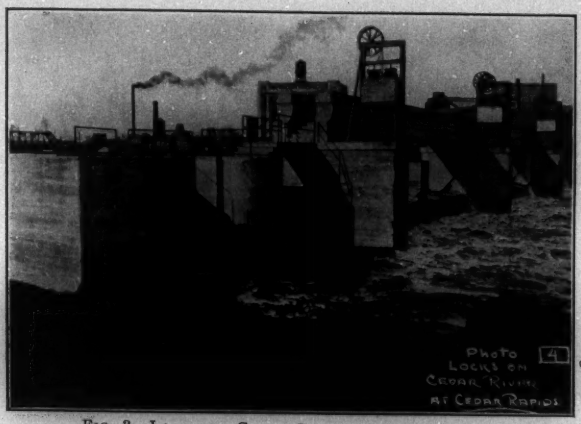

FIG. 3—LOCKS ON CEDAR RIVER AT CEDAR RAPIDS

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FIG. 4—INTERIOR OF STATION

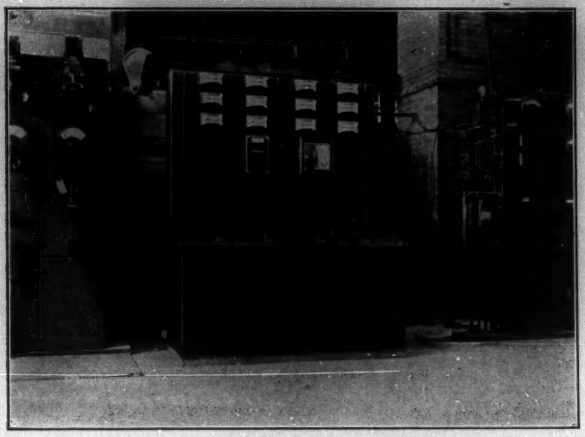

FIG. 5—CONTROL BOARD AT STEAM STATION

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FIG. 6—DRUM CONTROLLER

FIG. 7—RELAY PANEL

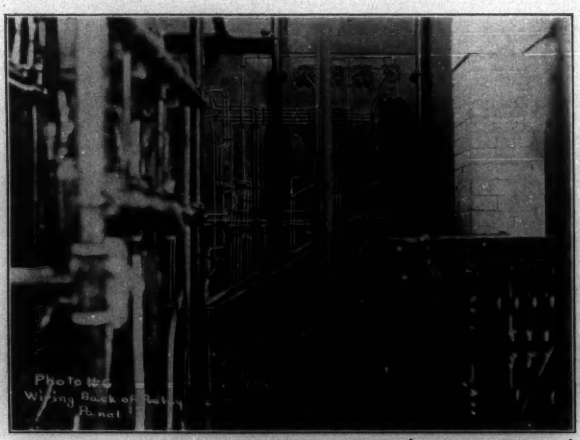

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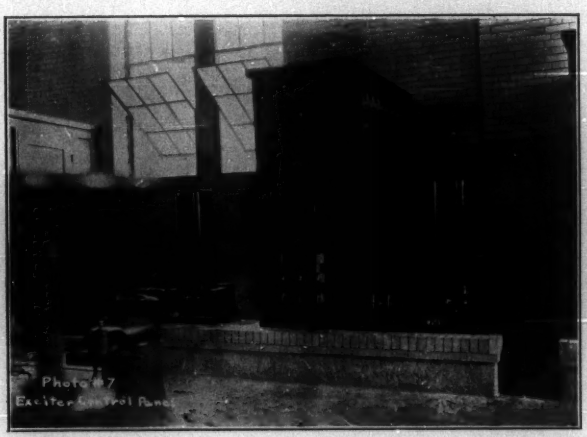
FIG. 8—WIRING BACK OF RELAY PANELS

FIG. 9—CONTACTOR PANEL



(DRABELLE AND BONNET)

FIG. 10—EXCITER CONTROL PANEL

FIG. 11—GATE CONTROL

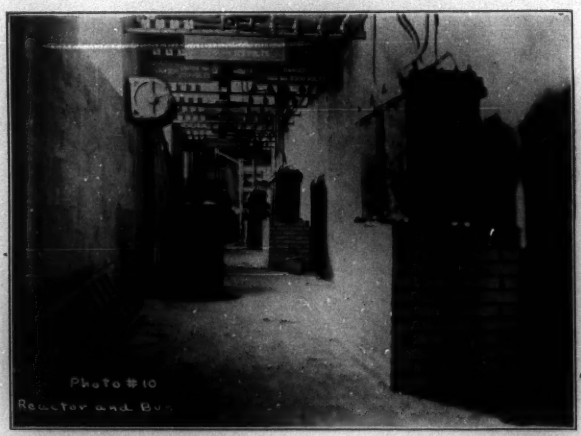


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FIG. 12—REACTOR AND BUS

dicating ammeter (150-ampere scale) and wattmeter (600-kw. scale), there being no instruments in the hydroelectric plant. The leads for these meters from the current transformers in the hydroelectric plant are bunched in an eleven conductor, No. 4, lead-covered cable. These leads are wired through calibrating terminals at the hydroelectric plant so that portable meters can be wired in if desired at any time. The lower row of instruments on the benchboard is a totalizing equipment, reading the total incoming power from the hydroelectric plant. This consists of two ammeters, a power factor indicator, an indicating wattmeter, a curve-drawing wattmeter and a watthour meter.

The waterwheel gates are not supplied with the usual hydraulic governor; but instead the opening and closing of the gates is obtained by a 5 h. p. induction motor through suitable reduction gearing. The gear ratio is such that it requires 25 seconds for the gates to completely open, starting from rest in the closed position. The motor is equipped with a solenoid brake which releases when voltage is applied to the motor. This prevents overshooting by quickly stopping the motor after the power supply is interrupted. Each gate mechanism is equipped with a contact-making device operating indicating lamps on the benchboard at the Sixth Street station, showing the exact amount of gate opening.

The correct sequence of operations in the control of each generator is obtained by properly placed segments on the drum of a controller which is driven by a single-phase one h. p. repulsion-type motor. This controller definitely determines the proper time spacing between the different steps of starting the unit and connecting its generator to the bus. One of these is shown in Fig. 6.

The contactor equipment and protective relays are mounted in cabinets as shown in Figs. 7, 8 and 9. Fig. 8 shows the wiring back of the panels.

For controlling the exciter sets, two oil-immersed contactors are used, also a double-throw, triple-pole switch, which connects either one or the other of the exciters to the exciter bus. The middle blade of this switch selects the proper contactor for starting the exciter set which is connected to the excitation bus. This equipment is shown in Fig. 10. At the top of the cabinet will be noticed a small induction motor. This carries a centrifugal device on the end of the shaft which

in case the frequency of the system rises to 64 cycles, will open, and shut down the plant, thereby preventing runaway.

The units have individual float switches set for slightly different levels. If the water rises to a certain level it closes the float switch of the first unit, and that machine is started and put into service. If the flow of the river is such that the water continues to rise the second float switch closes, starting the second machine. The third switch is set still higher. The machines are shut down in the reverse order as the water level falls.

In general the sequence of operation of each generator is as follows: The closing of the float switch or the proper remote-control button in the steam station, starts the exciter set (provided it has not already been started with another generator), and opens the waterwheel gate. When the generator comes up nearly to synchronous speed it is connected to the bus without field but in series with a 20 per cent reactance. A weak field is then applied pulling the machine into synchronism. The field is then strengthened to full field, the reactance short-circuited and the water wheel gates opened to carry full load on the unit.

The generators operate with a fixed excitation adjusted for normal full-load value. If the water level is so low that with full gate opening normal load cannot be obtained, the generator, due to its high excitation, takes care of part of the wattless kilovolt-amperes of the system. The speed and voltage are, of course, determined by the steam turbine generators.

Details of Operation

Detailed operation of this automatic equipment is as follows, referring to the wiring diagram:

Connected to the 2300-volt bus there are two small transformers stepping down to 220 volts, two-phase, for an operating bus. One phase is taken from this bus through the double pole switch No. 29, for controlling the contactors of each generator equipment. In the line between these transformers and the 220-volt bus there is an emergency throw-over switch which in case of failure of voltage on either phase is thrown over by a spring to a separate source of power brought in from the other station.

The closing of float switch No. 1, as shown on the right-hand side of the drawing, or control button No. 3 in the steam

station, closes contactor No. 4 provided that control button No. 2 is also closed. This energizes segment No. 14 of the controller. Segment No. 17 is in contact with its finger and a circuit is completed through the coil of contactor No. 6, which starts the motor driving the drum controller. A circuit is also made at No. 16 through the contacts of relay No. 41, which is not energized, through the coil of the contactor No. 5, through the contacts of relay *E*, the inverse-time relays No. 5 and the thermostat relay No. 10. The thermostat relay is connected across one phase of the control circuit and is of course, picked up if all the thermostats are closed, indicating that there is no overheating. Relay *E* is connected across the other phase and is picked up unless the frequency of the bus exceeds 64 cycles. Contactor No. 5 now closes energizing segment No. 1 of the controller. A circuit is now made through segment No. 2 picking up relay No. 42. Either one or the other of the two exciters sets can be started, depending on whether the switch *K* is closed up or down. The picking up of relay No. 42, completes the circuit through switches No. 28, which are closed, through the coil of contactor *G*, throwing full 2300 volts on the stator of the induction motor.

Contactor No. 43, with its coil across the armature of the exciter, short circuits with its contacts the field rheostat until the exciter voltage is built up to 75 or 80 volts. The set comes up to speed and the voltage up to normal very quickly and the d-c. voltage relay No. 11 picks up. The holding circuit of contactor No. 5 is now closed through this low-voltage relay No. 11, the lower interlock of contactor No. 5 and through segment No. 15-A, so that No. 5 stays closed after No. 16 has left contact unless the exciter voltage drops.

In the meantime, the controller has been rotating and a circuit is made through finger No. 3 to the middle of point of relay No. 20. As the generator is not connected to the line there is no current flowing in the contact-making ammeter No. 21 and its contacts are closed in the up position. If No.1 button in the steam station is closed the left hand side of relay No. 20 is picked up. The energizing of finger No. 3, therefore, completes the circuit through the coil of contactor No.18-A, which starts the gate motor in the proper direction for opening the gate of the waterwheel. Finger No. 3 is in contact a definite length of time which is sufficient to give a gate opening on the waterwheel of such amount that the no-load constant speed would be about 65 or 70 rev. per min.

The controller motor has been receiving its energy through contact No. 6 held closed by finger No. 17. This finger next comes to the break in its segment and the controller stops in this position, waiting for the generator to come up to speed. On each generator there is a centrifugal device No. 13, set to close at about 55 rev. per min. that is, five rev. per min., below normal. When this switch closes, a connection is made from finger No. 17-A to No. 19 and through the controller to No. 18 to contactor No. 6, starting the controller again.

The gate opening is such that the generator is slowly rising in speed, so that by the time the segment No. 4 on the controller comes into contact, the speed is approximately normal. This segment closes contactor No. 23 connecting the generator to the bus without field and with a reactance in series. Finger No. 5 next closes the field contactor No. 24 putting a weak field on the generator and pulling it into synchronism. Finger No. 6 then closes contactor No. 40, short-circuiting a part of the field rheostat, thus strengthening the field to full normal value. Finger No. 8 closes contactor No. 39, short-circuiting the reactance. In the meantime finger No. 3 has come into contact again and the contact-making ammeter No. 21 opens the gate until the current of the generator is up to normal value.

The generator is now properly connected to the bus and carrying full load. Finger No. 17 comes to the second break in its segment and controller stops in the full running position.

The equipment can be shut down in several different ways. If the pond level falls, due to the fact that the demand for water is greater than the flow of the river; float switch No. 1 will open. If it is desired that the steam station shut down and start from the water level, button No. 4 will be open and the opening of both this button and the float switch will trip contactor No. 4. If it is desired to shut down any generator and prevent its starting again, button No. 2 in the steam station should be opened, thus also tripping contactor No. 4. This de-energizes segment No. 14 of the controller which in turn drops out contactor No. 5. This de-energizes segment No. 1 and drops out all other contactors connecting the machine to the bus. When contactor No. 5 drops out, the upper interlock makes a circuit through segments No. 19 and 18, picking up contactor No. 6 and running the controller to the off position. The middle interlock of contactor No. 5 is also closed, energizing contactor No. 18-B and closing the waterwheel gate. As

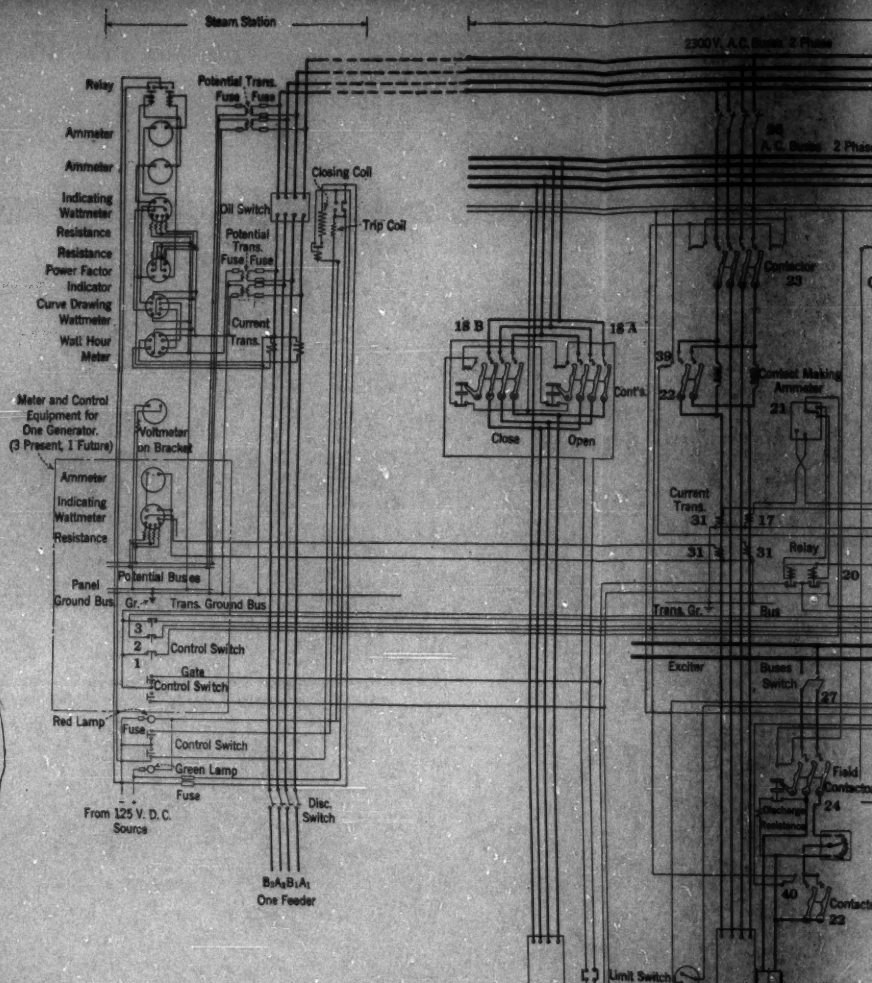
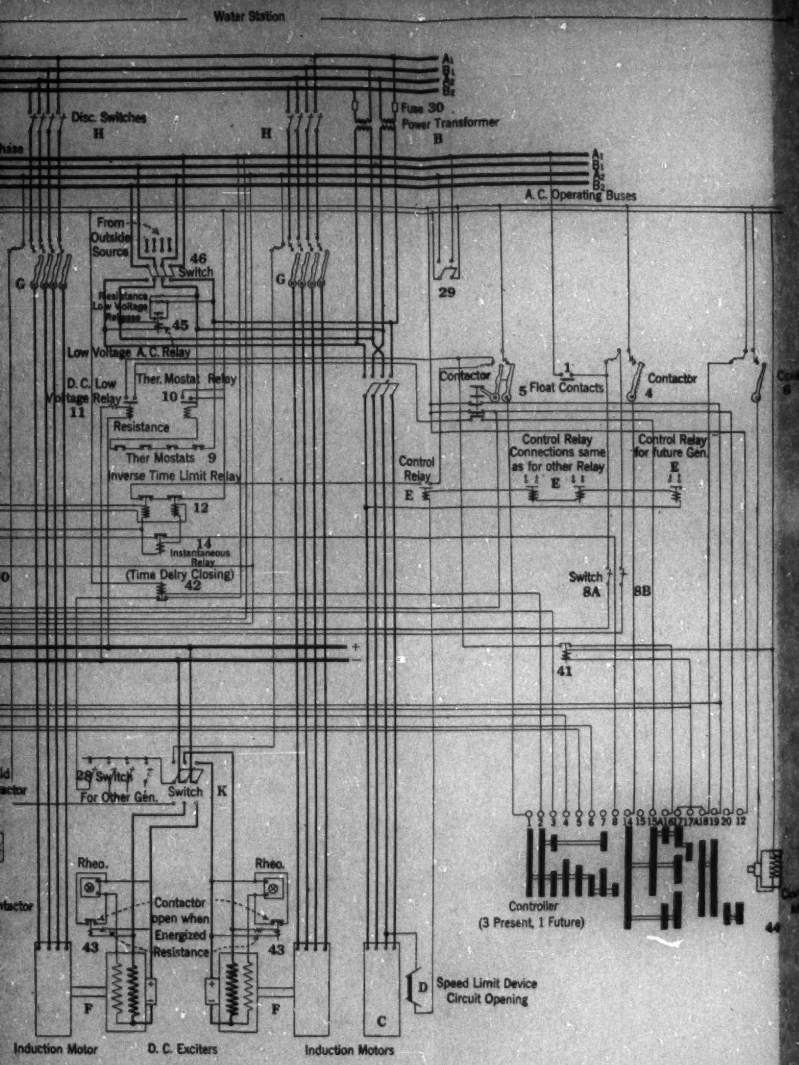


FIG. 13—WIRING DIAGRAM OF AUTO



UTOMATIC HYDROELECTRIC PLANT

[DRABELLE AND BONNETT]

the controller rotates to the off position fingers No. 20 and 21 are bridged. These contactors are in parallel with the middle interlock on contactor No. 5 to give additional contact surface.

Connected to the operating bus is a one-h. p. two-phase motor driving a speed-limit device which opens its contacts when the speed exceeds 64 cycles. If for any reason, the hydroelectric plant becomes separated from the steam station, the generators will, of course, speed up increasing the frequency of the bus. This in turn will trip the speed-limit device *D*, dropping out the four relays *E*, which in turn drop contactors No. 5 on all generators. This drops out all other contactors, and as the bus voltage then drops to zero the automatic throw-over switch No. 46 connects a separate source of power to the control bus thus furnishing energy for closing the gates of the water wheels to shut down. The wiring is so arranged that the relays *E* and No. 10 are connected outside this throw-over switch so that the generators cannot be started until the main cable between the station is re-energized.

If for any reason the generators have gone above speed and the main tie cable has been re-energized, relay No. 41 which is picked up whenever the centrifugal switch on the generator is closed, prevents the picking up of contactor No. 5, until the speed of the generator has dropped to a value somewhat below normal. This prevents the generator from being thrown on the line again when running at a considerable speed above normal.

The foregoing description covers only one generator; but also applies equally to the other generators. Each controller has a relay No. 42 so that the exciter can be started from any controller; that is, any generator can be started first and its controller starts the exciter.

The steam station is equipped with control buttons so that, if desired, the gate opening, and therefore the load on the hydroelectric generators, can be controlled from that place. The ammeter control is cut out by opening button No. 1.

Tests

A considerable number of tests were taken to determine what would happen in the automatic equipment in case of incorrect operation of the control buttons, or failure of some part to operate correctly.

First, one of the No. 1 control buttons was opened and the

load on the generator controlled by hand. It was found impossible to overload the generator, even with the gate opening to the full amount allowed by its limit switch. Button No. 1 was then closed, putting the ammeter again in control, and at the same time the operator tried to control the load from the gate control buttons. It was found that he could increase or decrease the load as desired; but when the hand control button was released the ammeter returned the load to the proper amount determined by its setting.

Buttons No. 2 and 3 were opened and immediately reclosed. The opening of these buttons tripped out all contactors and started the gate motor closing. The reclosing of the buttons caused the controller to rotate but since the generator was running at normal speed or a little above, relay No. 41 was held up and contactor No. 5 could, therefore, not pick up. This prevented any other contactors from closing. The controller made about one and one-half revolutions before the generator speed dropped so that relay No. 41 could reset. The next time around the controller picked up contactor No. 5, and went through the usual sequence of starting. This proved that if the operator accidentally opened one of these buttons and immediately closed it again, the generator would not be thrown immediately back on the bus, but would go through its proper sequence of operations.

In the steam station there is a voltmeter connected on the hydroelectric side of the oil switch. We next tripped this oil switch and found that the voltage dropped to zero in about two seconds, indicating that in this time the contactors in the hydroelectric plant had all tripped out, thus clearing the generators from the bus. Upon tripping of this oil switch the generators lost their load and tended to run away. Within the two seconds mentioned the speed limit device trips relay *E* at 64 cycles and all contactors drop out. The automatic throw-over control switch connects the separate source of power to the control bus and the dropping out of contactors No. 5 closes the water wheel gates. We did not read the maximum speed reached, but it was very apparent from watching the machines that the speed rose to not more than 20 to 25 per cent above normal. The action of the gate is quite rapid and the inertia of the machines is such that the gates closed before the wheels had time to accelerate seriously.

With the generator running under load we shut down the

exciter set. The armature current of the generator immediately increased and the contact making ammeter started to close the gate. The contactors connecting the generator to the bus dropped out, due to the tripping of the direct-current, low-voltage relay No. 11, which dropped because of the failure of the exciter voltage. The generator, after loss of field, broke from synchronism and tended to run above speed, but the contact-making ammeter had already closed the gate far enough to prevent serious overspeed, and the drop out of the contactors completed the closing.

Since the hydroelectric plant has been put into regular operation, the exciter in the main steam generator station failed one day. The load on the system, of course, was much greater than the hydroelectric plant could carry by itself, so that this plant should shut itself down and clear from the steam plant. As soon as the exciter voltage in the steam station began to drop the current of the hydroelectric generators, of course, rapidly rose and the contact-making ammeters started the closing of the waterwheel gates. Before the voltage dropped too low, the ammeters had almost completed the closing of the gates. All contactors dropped out because of low voltage, disconnecting the generators from the bus. The hydroelectric plant was ready to start again upon return of voltage on the tie cable between the two plants.

One of the accompanying illustrations shows an oscillogram taken of the starting conditions. This oscillogram is marked, showing the different contactors operated. The first part of the film shows the generator connected to the line without field. Next a weak field is applied, pulling the machine into step. The field is then strengthened to normal. There now seems to be a tendency for the current to pulsate, presumably caused by the heavy 20 per cent reactance in series with the machine. The short-circuiting of this reactance quickly steadies the current into a stable condition. The wave shown at the end of the film is something less than normal load, as the contact-making ammeter, the day this film was taken, was set for holding less than full load. The time from the throwing on the line to the short-circuiting of the reactance is about eleven seconds.

Although the generators are low speed and have considerable inertia, only 39 seconds are required from beginning of the opening of the gate until the generator is connected to the

bus, with a reactance short-circuited. Within 45 seconds, the generator is carrying full load. In spite of the speed with which these generators are connected to the bus, there is no serious mechanical jar perceptible.

The plant has now been in operation from October 2nd until the present, and so far, no troubles, other than the small ones which always go with any new development, have been experienced, and these have all been of a very minor nature. The changes that have been made, consist essentially of substituting heavier and more reliable relays at one or two points on the control system, and the expectations and hopes of those who have worked on this development have been more than realized. The plant has successfully withstood short circuits, exciter failures in the steam plant, low water and high water, and all of the tests that those in charge of the work could conceive of, wrong operations being brought about artificially. It has also been operated by the regular operating force at the steam plant, and there has been no occasion to keep men, nor have they been kept in the hydroelectric plant to watch its operation.

The automatic development, if it means anything at all, means that it will now be possible to develop a large number of small low-head plants and tie them in on a high-tension system, leaving their operation entirely to the float switch and voltage relays. If there is voltage on all three phases of the high-tension line and water for the turbines, they will start up and go on the line without wrecking themselves or disturbing the operation of the rest of the system. In these days of scarcity of coal and scarcity of labor, together with its high price, the utilization of our water power is of national importance, for, every pound of coal that can be saved by the water of our rivers is just that much more that can be used by our country for the successful prosecution of the War.

With the elimination of the excessive labor cost of operation of a small plant, many water power sites are capable of development as a paying investment.

As this paper goes to press, the station has been in operation about seven months. Some notes as to experience with it might be of interest. During this time, the total flow of the river has been used at all times, and a total of approximately 3,000,000 kw-hr. has been fed into the system. There have, of course, been a few failures to start, as no automatic apparatus can be

FIG. 14—GENERATOR ON LINE WITHOUT FIELD

FIG. 15—GENERATOR WITH WEAK FIELD APPLIED

[DRABELLE AND BONNETT]

FIG. 16—GENERATOR WITH NORMAL FIELD

absolutely infallible. The source of such failures has been easily found and corrected. In this connection the equipment is designed to shut down in case of abnormal conditions, so that all the machines will be protected against injury. It is also of interest to note that owing to the pressure of other work and the shortage of help, this generating station ran continually for ten weeks, with no attention to automatic equipment of any kind. Of course, a more frequent inspection and cleaning of the contacts should be made in order to keep the equipment in the best of condition. There have been several cases of exciter trouble at the main steam station, which resulted in a complete shut down. As soon as bus voltage was restored, the automatic equipment put the hydroelectric plant back on the line without attention of any kind, and with no damage or injury to any of the machines. The expense for operators is a large per cent of the total cost of the operation of small hydroelectric plants. Many plants have considerable trouble in keeping their operating forces, particularly in these days of shortage of labor. With the present high price of coal and the shortage of labor, many small water powers might be developed with a profit as auxiliary plants on a fairly large steam generating system.

DISCUSSION ON "THE AUTOMATIC HYDROELECTRIC PLANT." (DRABELLE AND BONNETT), ATLANTIC CITY, N. J., JUNE 28, 1918.

H. R. Summerhayes: How often is it necessary to send an inspector around? What is the regular routine?

J. J. Linebaugh: The success of this station indicates a means has been found of tying into a large network the small waterpower developments which are available throughout the country, being somewhat along the lines of the paper given by Dr. Steinmetz last evening. The small cost of the automatic equipment makes it possible to use a standard synchronous alternator, for the exciter and auxiliaries, at an automatic substation, without going to the induction generator.

Mr. Bump: I notice the gates were all closed by individual motors on the water wheels. I would like to know what would happen if you had a short-circuit on the line between the waterpower and the power station. How would you ever close those gates?

Oliver C. Traver: I would like to register belief in the possibilities of the automatic station in the future, this faith being based on the results as obtained and as recorded in this paper, and also as observed in other cases. One reason for this faith, however, I will give, and that is, that even with a hand-operated station it is necessary for the operator to rely on some form of indicating device or some signal device by means of which he could be given instructions, and every one of these indicating or signalling devices can be duplicated as an automatic device, to do what the operator would be supposed to do if he had seen the indication or if he had received the instructions, and do it in a more orderly and correct manner than the operator could himself.

J. M. Drabelle: In regard to the question raised about the frequency of the inspection. We have one record we are not very proud of, it came about due to the accumulation of a lot of trouble elsewhere on the system, but this plant ran approximately for ten weeks without an inspection of any of the electrical parts. The trash racks are cleaned once every twenty-four hours by one of the day laborers over at the steam station, and that is all the attention that the plant gets. We try, however, to look at the drum controller and contacts about once a week, or once in two weeks, but the plant did successfully stand ten weeks' operation without an inspection of any kind.

L. B. Bonnett: As to Mr. Bump's question, if you get a dead short-circuit on the line, between the two stations, which I think is meant, there would be no way of reenergizing that line, and you would have to send a man to the station to turn the gates down by hand. There is a hand wheel on the mechanism for doing that. It is a reasonably long job, but it can be done. The generators and water wheels in this particular plant are designed to operate at runaway speed for one hour. That is reasonably simple, on a low-head proposition. These wheels run 60 rev. per min. normally.

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SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

BY H. B. DWIGHT

ABSTRACT OF PAPER

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves (Fig. 3) which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. Although the calculations cannot be carried out for as high frequencies as the calculation for tube, it indicates a method of coordinating the test results for straps, which have been published. A set of empirical curves for straps is given in Fig. 7, from which approximate values of R'/R for any case may be read.

IN any conductor carrying alternating current, the magnetic field around the axis of the conductor produces variations in the current density. In an isolated conductor, the current tends to flow more densely in the outermost parts, farthest from the axis of the conductor. The effect is more pronounced as the frequency becomes high, or as the section of the conductor becomes large. The result with a round wire or tube when the frequency is high, is that the current is concentrated in the outer skin of the conductor—hence the name "skin effect". In an isolated flat strap, the current crowds mainly toward the edges of the strap, and term "edge effect" has been suggested[1], although the general term "skin effect" is often used for all cases of the phenomenon.

The formula for the magnitude of the skin effect in a round, non-magnetic wire may be obtained by assuming an infinite series for the current density at any point of the section[2]. The formula may also be obtained by forming a differential equation connecting the current and the dimensions of the section[3]. An

1. Skin Effect Resistance Measurements, by A. E. Kennelly and H. A. Affel, Proc. Inst. of Radio Engineers, May, 1916.

2. Clerk Maxwell, *Electricity and Magnetism*, Vol. II, para. 689.

3. A. Russell, *Philosophical Mag.*, Vol. 17, 1909, p. 524, and A. E. Kennelly, F. A. Laws and P. H. Pierce, TRANS. A. I. E. E., 1915, p. 1953.

alternative method has been described[4] by which the effect of the magnetic field is traced step by step. Successive increments of current and voltage drop are calculated, to keep the voltage uniform over the section, as of course it really is. The increments become smaller and smaller, forming convergent series. This method gives the usual result for round wire, and is useful for the special cases of tubes and straps, which are calculated in this article.

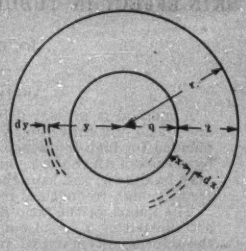

FIG. 1—SECTION OF TUBULAR CONDUCTOR

Assume that a uniform current of density a_0, in absolute electromagnetic units, that is, in absamperes per square centimeter, flows at all parts of the section of the tube indicated in Fig. 1. The current inside the circle dx is

$$I_{(x)} = \pi\, a_0\, (2\, q\, x + x^2) \quad \text{absamperes} \tag{1}$$

The flux density at dx is

$$\frac{2\, I_{(x)}}{q + x}$$

$$= 2\, \pi\, a_0\, q \left[2\, \frac{x}{q} - \frac{x^2}{q^2} \left(1 - \frac{x}{q} + \frac{x^2}{q^2} - \frac{x^3}{q^3} + \ldots \right) \right]$$

$$= 2\, \pi\, a_0\, q \left(2 \frac{x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots \right) \text{ lines per sq. cm.}$$

The flux outside of $d\,x$ and inside the metal of the tube, per centimeter of tube, is

$$\varphi_{(x)} = 2\, \pi\, a_0\, q \int_x^t \left(\frac{2\, x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots \right) d\, x \text{ lines}$$

The reactive drop at $d\,x$ due to a_0 is

$$j\, \omega\, \varphi_{(x)} = j\, \omega\, 2\, \pi\, a_0\, t^2$$

$$\left(1 - \frac{1}{3}\, \frac{t}{q} + \frac{1}{4}\, \frac{t^2}{q^2} - \ldots - \frac{x^2}{t^2} + \frac{1}{3}\, \frac{x^3}{t^2\, q} - \frac{1}{4}\, \frac{x^4}{t^2\, q^2} + \ldots \right)$$

4. Transmission Line Formulas, by H. B. Dwight, 1913, Chap. X.

$$= \frac{j\, m^2\, t^2\, \rho\, a_0}{2} \left(1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} \ldots - \frac{x^2}{t^2} + \frac{1}{3}\,\frac{x^3}{t^2\, q} - \frac{1}{4}\,\frac{x^4}{t^2\, q^2} + \ldots\right) \text{abvolts} \quad (2)$$

putting

$$m^2 = \frac{4\,\pi\,\omega}{\rho} \quad (3)$$

where ω is equal to $2\,\pi$ times the frequency in cycles per second, and where ρ is the specific resistance of the metal in absolute units. The specific resistance in ohms is $\rho \times 10^{-9}$

Let a current of density a_1 flow, such that $a_1\,\rho$ will be equal and opposite to the above terms in x. Then

$$a_1 = \frac{1}{2}\, j\, m^2\, t^2\, a_0 \left(\frac{x^2}{t^2} - \frac{1}{3}\,\frac{x^3}{t^2\, q} + \frac{1}{4}\,\frac{x^4}{t^2\, q^2} - \ldots\right) \text{ absamperes per sq. cm.} \quad (4)$$

By the above process, the total current due to a_1 is found to be

$$\frac{j\, m^2\, t^2\, a_0\, 2\,\pi\, q\, t}{\underline{|3}} \left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{4\times 5}\,\frac{t^2}{q^2} + \ldots\right) \text{ absamperes,} \quad (5)$$

and the reactive drop at $d\,x$, due to flux in the metal caused by a_1, is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, \rho}{\underline{|4}} \left(1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \ldots - \frac{x^4}{t^4} + \frac{2}{5}\,\frac{x^5}{t^4\, q} - \frac{3}{10}\,\frac{x^6}{t^4\, q^2} + \ldots\right) \text{ abvolts} \quad (6)$$

By assuming a current of density a_2, such that $a_2\,\rho$ will neutralize the terms of x in (6), it is found that the total current due to a_2 is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, 2\,\pi\, q\, t}{\underline{|5}} \left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{14}\,\frac{t^2}{q^2} + \ldots\right) \text{ absamperes} \quad (7)$$

and the reactive drop at $d\,x$, due to flux in the metal caused by a_2 is

$$\frac{(j\, m^2\, t^2)^3\, a_0\, \rho}{\underline{|6}} \left(1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} \ldots - \frac{x^6}{t^6} + \frac{3}{7}\,\frac{x^7}{t^6\, q} - \frac{9}{28}\,\frac{x^8}{t^6\, q^2} + \ldots\right) \text{abvolts} \quad (8)$$

By continuing this process, the following results are obtained:

$$I\,Z' = a_0\,\rho\left[1 + b_1\frac{j\,m^2\,t^2}{\underline{|2}} + b_2\frac{(j\,m^2\,t^2)^2}{\underline{|4}} + b_3\frac{(j\,m^2\,t^2)^3}{\underline{|6}} + \ldots\right] \text{ abvolts} \quad (9)$$

where I is the total current in the tube, where Z' is the effective impedance due to resistance and inductance caused by flux inside the metal of the tube, and where

$$b_1 = 1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} - \ldots$$

$$= \frac{1}{2} + \frac{q}{t} - \frac{q^2}{t^2}\,logh\left(1 + \frac{t}{q}\right) \quad (10)$$

$$b_2 = 1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \frac{17}{70}\,\frac{t^3}{q^3} + \ldots \quad (11)$$

$$b_3 = 1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} - \frac{11}{42}\,\frac{t^3}{q^3} + \ldots \quad (12)$$

$$b_4 = 1 - \frac{4}{9}\,\frac{t}{q} + \frac{1}{3}\,\frac{t^2}{q^2} - \ldots \quad (13)$$

$$b_5 = 1 - \frac{5}{11}\,\frac{t}{q} + \frac{15}{44}\,\frac{t^2}{q^2} - \ldots \quad (14)$$

$$b_n = 1 - \frac{n}{2\,n+1}\,\frac{t}{q} + \ldots \quad (15)$$

If R is the resistance of the tube per unit length, in absolute units,

$$I\,R = \frac{I\,\rho}{\pi\,(2\,q\,t + t^2)}$$

$$= \rho\,a_0\left[1 + c_1\,\frac{j\,m^2\,t^2}{\underline{|3}} + c_2\frac{(j\,m^2\,t^2)^2}{\underline{|5}} + c_3\,\frac{(j\,m^2\,t^2)^3}{\underline{|7}} + \ldots\right] \quad (16)$$

where $c_1 = 1 - \frac{1}{20}\,\frac{t^2}{q^2} + \frac{1}{20}\,\frac{t^3}{q^3} - \frac{11}{280}\,\frac{t^4}{q^4} + \ldots$ (17)

$$c_2 = 1 - \frac{1}{14}\,\frac{\ell^2}{q^2} + \frac{1}{14}\,\frac{\ell^4}{q^4} - \ldots \tag{18}$$

$$c_3 = 1 - \frac{1}{12}\,\frac{\ell^2}{q^2} + \ldots \tag{19}$$

and

$$c_4 = 1 - \frac{1}{11}\,\frac{\ell^2}{q^2} + \ldots \tag{20}$$

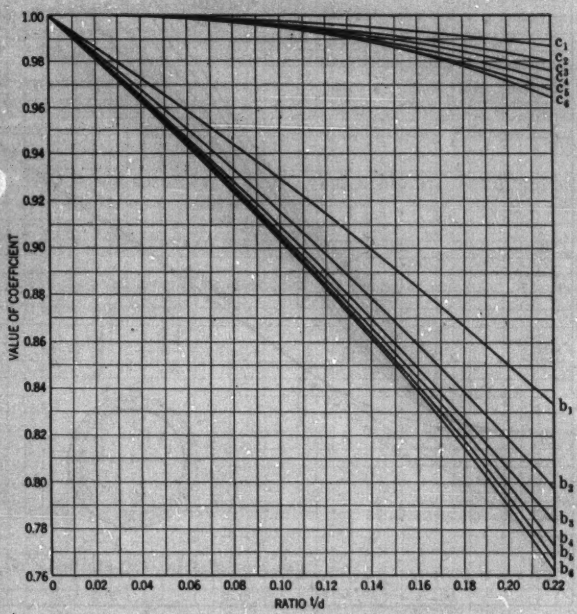

FIG. 2—COEFFICIENTS FOR CALCULATING SKIN EFFECT IN TUBES

The values of the coefficients b_1, b_2, c_1, etc., are shown in Fig. 2. These have been used in calculating the curves of $\frac{R'}{R}$ at the lower frequencies shown by the full lines in Fig. 3. The complex quantity $\frac{Z'}{R}$ is found from (9) and (16). The denominator is rationalized, and then the real part of $\frac{Z'}{R}$ is equal to $\frac{R'}{R}$.

The formula for the skin effect in a tube of infinite radius is of the same form as that for a return circuit of two adjacent straps,[5] but different constants are involved.

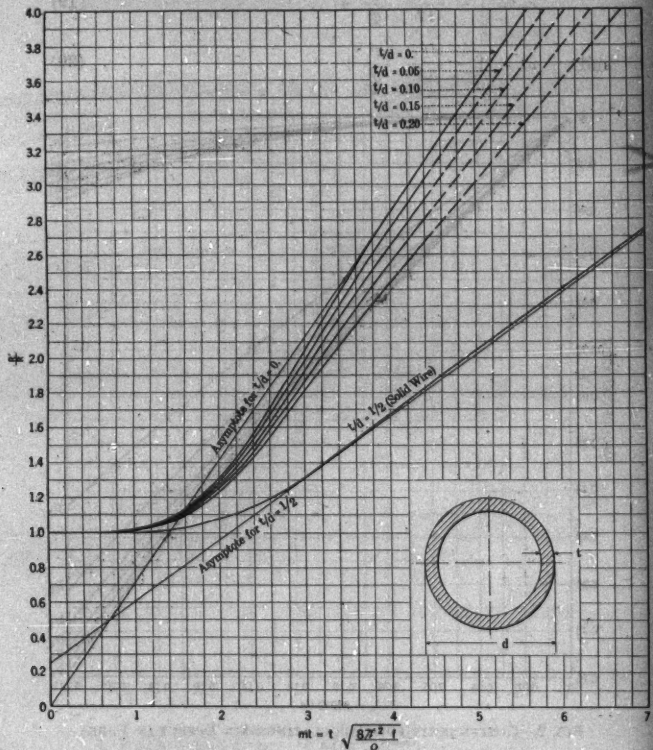

FIG. 3—SKIN EFFECT IN TUBES OF VARIOUS THICKNESSES

The above calculation can be carried to fairly high frequencies and it can be shown by it that the curve of $\frac{R'}{R}$ of a tube be-

5. Skin Effect of a Return Circuit of Two Adjacent Strap Conductors, by H. B. Dwight, *The Electric Journal*, April, 1916.

comes approximately a straight line for high frequencies when $\frac{R'}{R}$ is plotted against the square root of the frequency. The angle of slope of the asymptote, that is, the line which the curve approaches as a tangent, when $\frac{R'}{R}$ is large, may be calculated as follows, in a manner similar to that used by Dr. A. Russell in deriving a method for determining the skin effect of a concentric main, that is, a tube containing an insulated return wire.[6]

The skin effect in an isolated non-magnetic tube, in which the current returns at a considerable distance from the tube, is quite different from the skin effect in a tube containing a return wire, and so a new calculation is required, using different constants.

Let i be the current density at radius y, Fig. 1. The drop per cm. of the tube at radius y is

$$e = \rho\, i + \frac{d\,\varphi}{d\,\tau} \tag{21}$$

where ρ is the specific resistance of the metal in the tube, where τ is time, and where

$$\varphi = \int_y^r \frac{2\,I_{(y)}}{y} d\,y = -\int_r^y \frac{2\,I_{(y)}}{y}\, d\,y \tag{22}$$

$$I_{(y)} = \int_q^y 2\,\pi\, i\, y\, d\,y \tag{23}$$

Therefore, $$\frac{d\,\varphi}{dy} = -\frac{2\,I_{(y)}}{y} = -\frac{2}{y}\int_q^y 2\,\pi\, i\, y\, dy \tag{24}$$

Differentiate (24) with respect to τ

$$\frac{d}{d\,\tau}\,\frac{d\,\varphi}{dy} = \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} = -\frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\, dy$$

Now e is constant over the section. Therefore

$$\frac{d\,e}{dy} = 0 = \rho\,\frac{di}{dy} + \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} \qquad \text{from (21)}$$

$$= \rho\,\frac{d\,i}{dy} - \frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\, dy \tag{25}$$

6. *Philosophical Mag.*, Vol. 17, 1909, p. 524.

When $y = q$, $\quad 0 = \rho \dfrac{di}{dy} - 0$

Therefore, when $y = q$, $\quad \dfrac{di}{dy} = 0 \qquad (26)$

From (25), $\quad y \dfrac{di}{dy} = \dfrac{4\pi}{\rho} \displaystyle\int_q^y y \frac{di}{d\tau} dy \qquad (27)$

Differentiate (27) with respect to y. Then

$$y \frac{d^2 i}{dy^2} + \frac{di}{dy} = \frac{4\pi}{\rho} y \frac{di}{d\tau}$$

$$\text{Put} \quad m^2 = \frac{4\pi\omega}{\rho} = \frac{8\pi^2 f}{\rho} \qquad (28)$$

where f = frequency in cycles per second.

Then $\quad \dfrac{d^2 i}{dy^2} + \dfrac{1}{y} \dfrac{di}{dy} = \dfrac{m^2}{\omega} \dfrac{di}{d\tau}$

and therefore $\dfrac{d^2 i}{dy^2} + \dfrac{1}{y} \dfrac{di}{dy} - j m^2 i = 0 \qquad (29)$

when the current has a sine-wave shape.

The solution of this differential equation may be written

$$i = (A + jB) J_0 (m y \sqrt{-j}) + (C + jD) K_0 (m y \sqrt{-j}) \qquad (30)$$

where $J_0 (my \sqrt{-j})$ is a Bessel function of the first kind and of order zero, where $K_0 (m y \sqrt{-j})$ is a Bessel function of the second kind and of order zero, and where A, B, C, and D are constants to be determined. Thus

$$i = (A + jB)(ber\, my + j\, bei\, my) + (C + jD)(ker\, my + j\, kei\, my) \qquad (31)$$

that is, $\quad i = A\, ber\, my - B\, bei\, my + C\, ker\, my - D\, kei\, my + j(A\, bei\, my + B\, ber\, my + C\, kei\, my + D\, ker\, my)$[7] $\qquad (32)$

From (26), $\quad \dfrac{di}{dy} = 0$ when $y = q$.

7. See equations (29), (44) and (77) of the paper by Kennelly, Laws and Pierce, and the paper by A. Russell, referred to in the second paragraph of this paper, reference 3.

Therefore,

$$A\ ber'\ mq - B\ bei'\ mq + C\ ker'\ mq - D\ kei'\ mq = 0 \qquad (33)$$

and $$A\ bei'\ mq + B\ ber'\ mq + C\ kei'\ mq + D\ ker'\ mq = 0 \qquad (34)$$

Let the total current in the tube be the quantity to which all phase relations are to be referred, and let it be equal to $I + j\,0$. Then, since

$$\int y\ ber\ my\ dy = \frac{y}{m}\ bei'\ my, \quad \int y\ bei\ my\ dy$$

$$= -\frac{y}{m}\ ber'\ my, \quad \int y\ ker\ my\ dy = \frac{y}{m}\ kei'\ my$$

$$\text{and} \int y\ kei\ my\ dy = -\frac{y}{m}\ ker'\ my,$$

we have

$$I = \int_q^r 2\pi\, y\, i\, dy$$

$$= \frac{2\pi}{m}\Big[A\ y\ bei'\ my + B\ y\ ber'\ my + C\ y\ kei'\ my + D\ y\ ker'\ my\Big]_q^r$$

$$+ j\frac{2\pi}{m}\Big[-A\ y\ ber'\ my + B\ y\ bei'\ my - C\ y\ ker'\ my + D\ y\ kei'\ my\Big]_q^r$$

The quantities in the brackets are equal to zero when $y = q$, by (33) and (34). Therefore,

$$I + j\,0 = \frac{2\pi r}{m}\Big[A\ bei'\ mr + B\ ber'\ mr + C\ kei'\ mr + D\ ker'\ mr\Big]$$

$$+ j\frac{2\pi r}{m}\Big[-A\ ber'\ mr + B\ bei'\ mr - C\ ker'\ mr + D\ kei'\ mr\Big]$$

Therefore, $-A\,ber'\,mr + B\,bei'\,mr - C\,ker'\,mr + D\,kei'\,mr = 0$ (35)

and $A\,bei'\,mr + B\,ber'\,mr + C\,kei'\,mr + D\,ker'\,mr = \dfrac{I\,m}{2\,\pi\,r}$ (36)

Let Z' be the effective impedance per centimeter of the tube at a certain frequency due to its effective resistance R' and its inductance caused by flux inside the metal. Since the flux does not cause any drop where $y = r$, we have, from (21),

$$e = I\,Z' = \rho\,i_{(r)}$$

Therefore, by (32),

$$I\,Z' = \rho\left[\,A\,ber\,mr - B\,bei\,mr + C\,ker\,mr - D\,kei\,mr\,\right] + j\,\rho\left[\,A\,bei\,mr + B\,ber\,mr + C\,kei\,mr + D\,ker\,mr\,\right] \tag{37}$$

The drop in phase with the current I is

$$I\,R' = \rho\left[\,A\,ber\,mr - B\,bei\,mr + C\,ker\,mr - D\,kei\,mr\,\right] \tag{38}$$

Now, by (36), $\quad I\,R = \dfrac{I\,\rho}{\pi\,(r^2 - q^2)}$

$$= \frac{2\,\pi\,r\,\rho}{\pi\,(r^2 - q^2)\,m}\left[\,A\,bei'\,mr + B\,ber'\,mr + C\,kei'\,mr + D\,ker'\,mr\,\right] \tag{39}$$

Therefore, $\dfrac{R'}{R}$

$$= \frac{m\,(r^2 - q^2)}{2\,r}\,\frac{(A\,ber\,mr - B\,bei\,mr + C\,ker\,mr - D\,kei\,mr)}{(A\,bei'\,mr + B\,ber'\,mr + C\,kei'\,mr + D\,ker'\,mr)} \tag{40}$$

The four equations (33), (34), (35) and (36) are sufficient to determine the four constants A, B, C, and D.

A similar equation for the skin effect inductance ratio could easily be written down, but would not be of much practical interest in the case of a tube since the flux inside the metal is extremely small compared with the flux outside the tube.

Equation (**40**) could be used to calculate the skin effect resistance ratio of a tube at any frequency if very complete tables of the eight functions, *ber*, etc., were available.

The result is the same as in Dr. Russell's equation (**89**) for a concentric main, except that the inner and outer radii of the tube are interchanged, due to the different position of zero flux. Thus by interchanging the inner and outer radii in Dr. Russell's low-frequency formula (**101**), which is derived from (**89**), a low-frequency formula for an isolated tube would be obtained. In this article, equation (**40**) will be used only for obtaining the slope of the asymptote to the curve of $\frac{R'}{R}$, thus giving approximate values of $\frac{R'}{R}$ for high frequencies by a very simple formula.

For large values of mr, approximate formulas for the eight functions *ber*, etc., are given in Dr. Russell's paper referred to above, of which the following is an example:

$$ber\ mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}}}}{\sqrt{2\,\pi\,mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8}\right) \tag{41}$$

The values given by these approximate formulas are more accurate as mr is larger.

If these formulas are substituted in the well known equation for a solid wire,

$$\frac{R'}{R} = \frac{mr}{2}\,\frac{(ber\ mr\ bei'\ mr - bei\ mr\ ber'\ mr)}{(ber'^2\ mr + bei'^2\ mr)} \tag{42}$$

we obtain

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} \tag{43}$$

which gives the slope of the asymptote of the curve of $\frac{R'}{R}$.

In order to obtain the equation of the asymptote (see Fig. 3), namely,

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} + \frac{1}{4} \tag{44}$$

it is necessary to use values of the form

$$ber\ mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}} + \frac{1}{8\ mr\ \sqrt{2}}}}{\sqrt{2\ \pi\ mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8} - \frac{1}{8\ mr\ \sqrt{2}}\right) \quad (45)$$

given in Dr. Russell's paper. If further terms of the series in $\frac{1}{mr}$ are added, an asymptotic formula for $\frac{R'}{R}$ is obtained, which is very accurate except at low frequencies.

It is thus evident that the substitution of the formulas similar

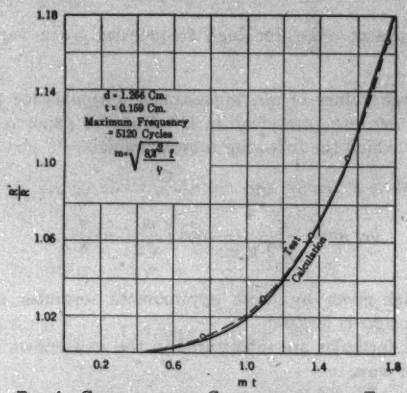

FIG. 4—COMPARISON OF CALCULATION WITH TEST

to (41) in equation (40) will give only the slope of the asymptote of the curve of $\frac{R'}{R}$. The result of the substitution, after a certain amount of trigonometrical work, is

$$\frac{R'}{R} = \frac{mt\ (q + r)}{2\ r\ \sqrt{2}} \frac{\{\sinh\ (mt\ \sqrt{2}) + \sin\ (mt\ \sqrt{2})\}}{\{\cosh\ (mt\ \sqrt{2}) - \cos\ (mt\ \sqrt{2})\}} \quad (46)$$

This reduces to

$$\frac{R'}{R} = \frac{mt\ (q + r)}{2\ r\ \sqrt{2}} \quad (47)$$

which gives the required slope of the asymptote. If the inner

radius $q = 0$, we obtain equation (43) for a solid wire. If q is practically equal to r, (47) becomes

$$\frac{R'}{R} = \frac{mt}{\sqrt{2}} \tag{48}$$

This happens to be the actual equation of the asymptote, which in this case passes through the origin. (See equations (100) and (110) of the paper by Kennelly, Laws and Pierce, reference 3.)

The useful fact is therefore deduced that the asymptote of the curve for $\frac{R'}{R}$ of a thin tube passes close to the origin. It

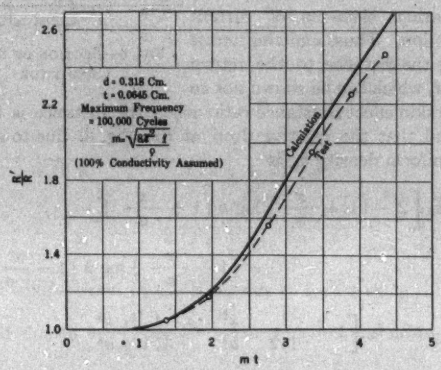

FIG. 5—COMPARISON OF CALCULATION WITH TEST

is therefore possible to plot $\frac{R'}{R}$ closely for tubes at high frequencies, as is done in the dotted lines in Fig. 3. The values found from the high-frequency, Bessel function calculation are in agreement with those found by the low-frequency calculation.

As examples, the results for two tubes tested by Dr. Kennelly, described in the papers mentioned above, are shown in Figs. 4 and 5. From the data given, the d-c. conductivity of the hard-drawn copper tube in Fig. 4 was about 50 per cent, and the curves have been drawn accordingly. The test curve of the soft-drawn copper tube of Fig. 5 was calculated on the basis of 100 per cent conductivity, in the absence of data regarding the

conductivity. If the conductivity were lower, the test curve would be drawn farther to the left.

Skin Effect in Strap. The process of calculating corrective currents to keep the voltage drop uniform over the section of the conductor may be applied to a thin isolated strap as well as to a tube. In this calculation, the strap is assumed to be so thin that the only appreciable action is the crowding of the current to the edges. The test curves shown in Fig. 7 indicate that this assumption is allowable for very thin straps.

Since the magnetic flux around the strap does not lie in circles, the voltage drop at any part of the section is calculated from the effect of the various elements of current in the section. Thus, omitting terms involving the distance to the return conductor, which can be shown not to affect the skin effect resistance ratio when the distance is large, it is found that the reactive drop at dx, Fig. 6, due to a current of uniform density b_0, is

FIG. 6—SECTION OF STRAP CONDUCTOR

$$j\,\omega\,4\,ac\,b_0\left[2-\left(1+\frac{x}{a}\right)\log h\left(1+\frac{x}{a}\right)\right.$$
$$\left.-\left(1-\frac{x}{a}\right)\log h\left(1-\frac{x}{a}\right)\right]$$
$$=\frac{1}{2}\,j\,p^2\,\rho\,b_0\left[1-\frac{1}{1.2}\,\frac{x^2}{a^2}-\frac{1}{3.4}\cdot\frac{x^4}{a^4}\right.$$
$$\left.-\frac{1}{5.6}\,\frac{x^6}{a^6}-\ldots\right] \qquad (49)$$

where $$p^2=\frac{4\,\omega}{\rho}\times 4\,ac=\frac{4\,\omega}{R}$$

and where R is the resistance of the conductor per centimeter in absolute units. The resistance in ohms is $R\times 10^{-9}$.

Also, the reactive drop at dx due to current $b_1\,\dfrac{x^2}{a^2}$

$$=\frac{1}{6}\,j\,p^2\,\rho\,b_1\left[\frac{1}{3}+\frac{x^2}{a^2}\right.$$
$$-\frac{1}{2}\left(1+\frac{x^3}{a^3}\right)\log h\left(1+\frac{x}{a}\right)$$

$$- \frac{1}{2} \left(1 - \frac{x^3}{a^3}\right) \log h \left(1 - \frac{x}{a}\right)\Bigg]$$

$$= \frac{1}{2} j p^2 \rho b_1 \left[\frac{1}{3.3} + \frac{1}{1.2} \frac{x^2}{a^2} - \frac{1}{1.4} \frac{x^4}{a^4} - \frac{1}{3.6} \frac{x^6}{a^6} - \frac{1}{5.8} \frac{x^8}{a^8} - \dots\right] \quad (50)$$

The reactive drop due to current $b_n \frac{x^{2n}}{a^{2n}}$

$$= \frac{1}{2} j p^2 \rho b_n \left[\frac{1}{(2n+1)^2} + \frac{1}{(2n-1)\,2} \frac{x^2}{a^2} + \frac{1}{(2n-3)\,4} \frac{x^4}{a^4} + \dots \right.$$

$$\left. \dots + \frac{1}{1\,(2n)} \frac{x^{2n}}{a^{2n}} - \frac{1}{1\,(2n+2)} \frac{x^{2n+2}}{a^{2n+2}} - \frac{1}{3\,(2n+4)} \frac{x^{2n+4}}{a^{2n+4}} - \dots\right] \quad (51)$$

The total current in the strap due to b_0 is $4\,ac\,b_0$. The total current due to $b_1 \frac{x^2}{a^2}$ is $\frac{1}{3} b_1 4\,ac$ and the total current due to

$$b_n \frac{x^{2n}}{a^{2n}} \text{ is } \frac{1}{2n+1} b_n 4\,ac \quad (52)$$

Assume that a uniform current i_0, flows in the strap. The reactive drop at $d\,x$ is given by (49). Next, assume a current whose resistance drop is equal and opposite to the terms in x of (49). The total current in the strap can be found by applying (52). By repeating the above process, and carrying the various series out to five terms or more in each case, the following results were obtained. Drop in the strap due to alternating current

$$= I\,Z' = i_0\,r\,[1 + j\,\tfrac{1}{2}\,p^2 - 0.0149994\,p^4 + j\,0.003122\,p^6 + 0.000532\,p^8 - j\,0.0000779\,p^{10} - 0.0000096\,p^{12} + j\,0.00000094\,p^{14} + 0.00000005\,p^{16}] \quad (53)$$

Drop in the strap due to a direct current of the same amperage

$$= I R = i_0 r [1 + j\, 0.0965736\, p^2 + 0.0152178\, p^4 - j\, 0.00218\, p^6 - 0.000259\, p^8 + j\, 0.0000237\, p^{10} + 0.00000100\, p^{12} + j\, 0.00000022\, p^{14} + 0.00000008\, p^{16}] \qquad (54)$$

Some of the coefficients of the above series can be expressed exactly by algebraic functions. Thus, in (53) the term in p^4 is

$$-\left(\frac{1}{4}\log h\, 2 - \frac{5}{16} + \frac{\pi^2}{64}\right) p^4 = -\,0.0149994\, p^4$$

In (54) the term in p^2 is

$$j\left(\frac{1}{2}\log h\, 2 - \frac{1}{4}\right) p^2 = j\, 0.0965736\, p^2$$

and the term in p^4 is

$$\left[\frac{1}{8}\log h\, 2 - \frac{1}{4}\left(\log h\, 2\right)^2 + \frac{5\,\pi^2}{192} - \frac{5}{24}\right] p^4$$

$$= 0.0152178\, p^4$$

The fraction $\frac{Z'}{R}$ can be calculated from (53) and (54) for various values of p. Then, after rationalizing the denominator, the real part is equal to $\frac{R'}{R}$, the skin effect resistance ratio.

The values of $\frac{R'}{R}$ for various values of p are plotted in Fig. 7. It is seen from this figure that the curve of $\frac{R'}{R}$ of a strap, when plotted on the square root of the frequency, has the same general shape as the curves for wire, tube, and a return circuit of two adjacent straps, and it seems to approach a straight line as an asymptote in the same way that they do.

The calculated curve has been carried only as far as $p = 2$. The other lines in Fig. 7 are test curves published in the papers by Kennelly and Affel, and by Kennelly, Laws and Pierce, referred to in the first part of this article. The test curves, especially at frequencies up to 70,000 cycles, show approximately straight lines, when plotted on the square root of the frequency. The curve, $\frac{a}{c} = 240$, Fig. 7, has the lowest position of any of

the published curves when they are plotted according to Fig. 7. A conductivity of 100 per cent was assumed for this copper strap. If the conductivity were lower, the curve would be slightly higher. The section of this strap was 0.016 by 3.81 cm.

The curve for a round wire of the same sectional area as the strap lies below the curve for strap up to p equal to about 2.1, but at higher frequencies than that, the wire has a greater ratio than the strap. (See Fig. 8). It is to be expected that the curves for thick straps would lie intermediate between the curve

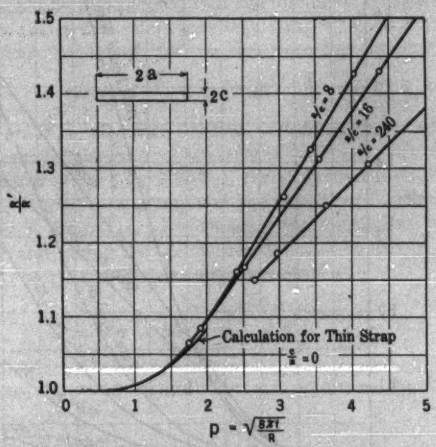

FIG. 7—SKIN EFFECT IN STRAP CONDUCTORS

for thin strap and that for wire. Such curves are represented by test curves $\frac{a}{c} = 16$ (0.158 × 2.52 cm.) and $\frac{a}{c} = 8$ (0.1575 × 1.26 cm.)

The curves of Fig. 7 may be considered as an extension in further detail of the article by the writer in the *Electrical World* of March 11, 1916, page 593, giving an empirical curve for skin effect in straps. The calculated curve now given tends to confirm the theory of the former article.

It would appear from Fig. 7 that rolling a strap thinner and wider, but keeping the same sectional area and conductivity

always reduces the skin effect resistance ratio for values of p greater than 2:1. However, it seems probable that the improvement is inappreciable after $\frac{a}{c}$ is more than about 50, and that the strap then approximates to an infinitely thin strap.

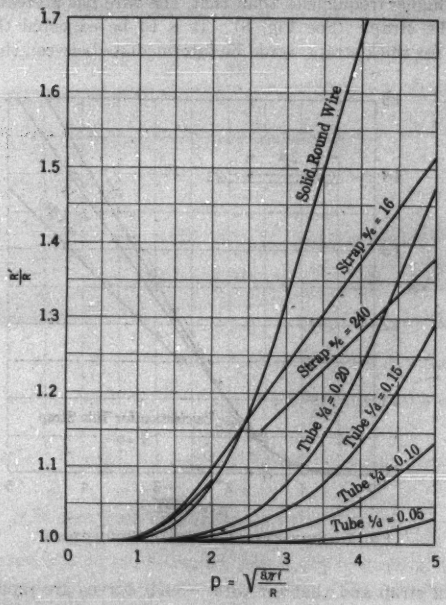

FIG. 8—SKIN EFFECT OF VARIOUS CONDUCTORS OF THE SAME SECTIONAL AREA

Thus a strap a foot wide would have the same skin effect resistance ratio as a thin strap one inch wide, if $\frac{a}{c}$ is in both cases greater than 50, and if both straps have the same resistance per 1000 feet, due to the wide strap being thinner or being made of material of lower conductivity.

By dividing series (53) by (54) a series is obtained which is

convenient for calculations when p is comparatively small. This is

$$\frac{R'}{R} = 1 + 0.0087432\, p^4 - 0.000384\, p^8 + 0.0000189\, p^{12} - \ldots \tag{55}$$

This series is not so convergent as (53) and (54). It should not be used unless p is small enough to make the last term quite small.

It should be remembered that the discussion on skin effect in strap in this article refers only to a strap which is not close to the return conductor, for in that case the proximity effect changes the value of $\frac{R'}{R}$, generally increasing it when the straps are in an edgewise position, and decreasing it when they are in parallel planes.

A copper strap $\frac{1}{8}$ by 5 inches, carrying current at 60 cycles, may be taken as an example of the calculated formula of skin effect in a thin strap. By changing the dimensions to centimeters, and taking the specific resistance of copper in absolute units to be 1724 at 20 deg. cent., we obtain

$$p^2 = 8\,\pi \times 60 \times \frac{1}{8} \times 5 \times \frac{6.45}{1724} = 3.53$$

Therefore $\quad p = 1.88$ and $\frac{R'}{R} = 1.071$, from Fig. 7.

For a strap $\frac{1}{16} \times 10$ inches,

$$p^2 = 8\,\pi \times 60 \times \frac{1}{16} \times 10 \times \frac{6.45}{1724} = 3.53$$

Therefore the two straps which have the same resistance per foot, have the same value of $\frac{R'}{R}$. Both straps can be considered to be practically the same as an infinitely thin strap, since the thicker one has a width 40 times the thickness.

In order to show graphically what types of conductors are most advantageous for high-frequency work, or for heavy currents in electric furnace circuits, the curves of Fig. 8 have been drawn. These show the skin effect to be expected

from using a given weight of copper in the form of wire, strap, or tubes of various proportionate thicknesses. In the case of a tube

$$\frac{8\,\pi\,f\,A}{\rho} = m^2\,t^2\left(\frac{r^2 - q^2}{t^2}\right) \tag{56}$$

where A is the sectional area, and this equation may be used to obtain the curves of Fig. 8 from those of Fig. 3. From the equation of the asymptote to the curve for a tube, it may be found that a tube in which $\frac{t}{d}$ is less than about 0.025 will always have less skin effect than a flat strap of the same sectional area and conductivity. A tube whose proportionate thickness is greater than the above amount may have a greater skin effect resistance ratio than a strap of the same sectional area at very high frequencies.

The curves of Fig. 8 are in agreement with the conclusion expressed in the paper by Kennelly and Affel, that a thin tube is the most economical form of conductor for high-frequency currents, when the return conductor is a considerable distance away.

As a general principle, it may be stated that a conductor, or a combination of conductors, of a certain proportionate shape and a certain value of $\frac{f}{R}$, will have a definite value of $\frac{R'}{R}$. This is true of tubes and round wires, as is shown by Fig. 3, where the skin effect of a tube of a certain ratio of thickness to diameter, but of any size, is given by a single curve. That the principle is true also of any shaped conductor, or any combination of conductors, may be indicated as follows:

Assume that an irregularly shaped conductor carries current at such a frequency, f, that $\frac{R'}{R} = (1 + a)$. The current will be crowded to certain parts of the section of the conductor, and the exact extent of this crowding may be indicated by plotting on the section, the lines for 95 per cent, 100 per cent and 105 per cent of average current density. In the case considered, the increase in resistance by the factor $(1 + a)$ is caused by a certain location of the current density lines.

Suppose that the conductor heats up so that its resistance increases 25 per cent, and suppose that the frequency also in-

creases 25 per cent. The unbalance of current is equivalent to a circulating current which goes with the main current at parts of the section, and returns at other parts of the section. This circulating current is driven by a voltage proportional to the frequency, and its voltage drop is due to resistance and is accordingly proportional to the resistance. Since both the frequency and the resistance have increased 25 per cent, the circulating current will be unchanged. Therefore, the position of the current density lines and the value of $\frac{R'}{R}$ will be unchanged.

Now, instead of the heat in the conductor raising its resistance, let a conductor of the same shape but 80 per cent as large be substituted, and let the frequency still be 25 per cent higher than at first. The resistance of the conductor is 25 per cent higher than at first. Thus the path for circulating current has 25 per cent higher resistance, and the frequency is 25 per cent higher, so that the circulating current is unchanged. Therefore, the relative position of the current density lines will be the same as before, and the value of $\frac{R'}{R}$ will be the same quantity, $(1 + a)$.

Thus in any conductor or combination of conductors or in any return circuit, of a certain proportionate shape, a given value of $\frac{f}{R}$ corresponds to a certain value of $\frac{R'}{R}$. A tested curve of $\frac{R'}{R}$ plotted on $\frac{f}{R}$ will therefore apply to conductors of different size from those tested, when the shape and relative position are the same.

The practical application of this principle is seen by considering that skin effect and proximity effect values are very often desired to be known for very heavy conductors at commercial frequencies. In such cases the currents are too large and the voltage drops are too small, to be conveniently or precisely measured. Precise results could be obtained by constructing miniature conductors to scale, and measuring the skin effect with conveniently small currents at high frequency.

In this way, the current distribution in three-phase circuits of ventilated busbars or of several cables in parallel, could be studied with exactness.

By considering a ventilated busbar to be approximately equivalent to a solid conductor of the same outside dimensions and the same total resistance, a simple extension of Fig. 7, containing curves for various proportionate thicknesses, would show the approximate skin effect of any isolated, rectangular, nonmagnetic conductor, whether solid or ventilated, hot or cold, large or small, and whether copper or aluminum. Such a set of curves would be useful in designing heavy busbars and conductors, not placed close together.

Discussion on "Skin Effect in Tubular and Flat Conductors" (Dwight). (Presented by Publication Only.)

Joseph Slepian: Mr. Dwight has certainly done a very creditable piece of work in solving so completely and in such usable form the skin effect problem in the two important cases of tubular and strap conductors. The paper is, however, really of very much broader scope than its title would indicate, because the method used is one which appears to be applicable to any shape of conductor, and the tube and strap solutions given here are only clearly explained examples of the working of the method. It is probable that with a shape of any great complication the method would become rather laborious, but it would remain simple and straightforward in theory at least and be capable of being carried to a numerical conclusion. The solution by means of expansions in series of orthogonal functions similar to Bessel's functions while in a sense more elegant is of little use for numerical computation and so from the engineer's point of view is not a solution at all.

The similitude principle given on page 1398 is obviously of great importance. It permits the determination of skin effect in large conductors by experiments on small models. A mathematical proof of this principle may be of interest as verifying the informal though convincing proof given.

Suppose a set of parallel conductors given, so that the specific resistance, $\rho\ (x\ y)$, in space is a function of x, y, coordinates in a plane perpendicular to the conductors.

Let $E\ (x, y, t)$ be the impressed e. m. f. per unit length of conductor; let $I\ (x, y, t)$ be the current *density* at any point.

Then we have the relations:

$$E\ (x\ y\ t) = \rho\ (x\ y\ t) \cdot I\ (x\ y\ t) + \frac{\partial A}{\partial t} \tag{1}$$

$$\text{Where } A\ (x\ y\ t) = -2 \int\int I\ (\xi\ \eta\ t) \log r\ \ d\ \xi\ \ d\ \eta \tag{2}$$

$$r = \sqrt{(x - \xi)^2 + (y - \eta)^2}$$

Of course we must assume $\int\int I\, dx\, dy = 0$ if A and E are to remain finite.

If $E\ (x\ y\ t)$ is a given harmonic function of the time then these equations serve to determine uniquely the corresponding steady state of alternating current, $I\ (x\ y\ t)$.

Now consider a new configuration of conductors obtained by expanding linear dimensions of the old configuration in the ratio k to 1. Then for the new configuration the conductivity, $\rho'\ (x\ y)$ is related to the conductivity in the old configuration $\rho\ (x\ y)$ by the equation:

$$\rho'\ (x\ y) = \rho\ (x/k,\ y/k) \tag{3}$$

Now suppose that in the new configuration we have the current density $I'(x, y, t)$ the same as that at corresponding points in the old, except that frequency is reduced in the ratio 1 to l, thus

$$I(x, y, t) = I(x/k, y/k, t/l) \tag{4}$$

What must be the corresponding impressed e. m. f. in the new configuration?

We have:

$$E'(x\,y\,t) = \rho'(x, y)\, I'(x\,y\,t) + \frac{\partial A'}{\partial t} \tag{5}$$

$$A' = -2\iint I'(\xi, \eta, t) \log r\, d\xi\, d\eta \tag{6}$$

Substituting (4) in (6)

$$A'(x\,y\,t) = -2\iint I(\xi/k, \eta/k, t/l) \log r\, d\xi\, d\eta$$

If now we change variables in the integral

$$\xi = k\bar{\xi}$$
$$\eta = k\bar{\eta}$$

$$A'(x\,y\,t) = -2\iint I(\bar{\xi}, \bar{\eta}, t/l) \log\sqrt{(x - k\bar{\xi})^2 + (y - k\bar{\eta})^2}\; k\,d\bar{\xi}\,k\,d\bar{\eta}$$

$$= -2\iint k^2 I(\bar{\xi}, \bar{\eta}\, t/l) \log k\sqrt{(x/k - \bar{\xi})^2 + (y/k - \bar{\eta})_2}\; d\bar{\xi}\,d\bar{\eta}$$

$$= -2k^2\iint I(\bar{\xi}, \bar{\eta}, t/l) \log\sqrt{(x/k - \bar{\xi})^2 + (y/k - \bar{\eta})^2}\; d\bar{\xi}\,d\bar{\eta}$$

$$-2k^2 \log k \iint I(\bar{\xi}, \bar{\eta}, t/l)\, d\bar{\xi}\, d\bar{\eta}$$

Therefore, $A'(x\,y\,t) = k^2 A(x/k, y/k, t/l) - 0$ (7)

Hence, $\dfrac{\partial A}{\partial t}(x\,y\,t) = \dfrac{k^2}{l}\,\dfrac{\partial A}{\partial t}(x/k,\ y/k,\ t/l)$ (8)

Substituting (8), (4) and (3) in (5)

$$E'(x, y, t) = \rho(x/k, y/k) \cdot I(x/k, y/k, t/l) + k^2/l\,\frac{\partial A}{\partial t}(x/k, y/k, t/l) \tag{9}$$

Comparing equation (9) with (1), we see that if $l = k^2$.
so that $k^2/l = 1$,

$$E'(x\,y\,t) = E(x/k, y/k, t/l) \tag{10}$$

Hence in the new configuration, a similar system of impressed e. m. f.'s will produce a similar system of current distribution if the frequency is varied inversely as the square of the linear dimensions. Since, however, the resistance per unit length conductor varies inversely as the square of the linear dimensions, we may say that the frequency must vary directly with the resistance.

H. B. Dwight: Since writing the paper, the author has derived an asymptotic formula for the skin effect in a tube, by using expressions for the *ber* and *ker* functions given by H. G. Savidge.* This formula, which is an extension of equation (**47**), is

$$\frac{R'}{R} = \frac{mt\,(q+r)}{2\,r\,\sqrt{2}}\left(1 + \frac{1}{mr\,\sqrt{2}} + \frac{3}{8m^2\,r^2} + \frac{o}{m^3\,r^3}\right)$$

For values of mt greater than 4, this formula gives curves which are the same as those of Fig. 3. The equation of the asymptote of each curve is

$$\frac{R'}{R} = \frac{mt\,(q+r)}{2\,r\,\sqrt{2}}\left(1 + \frac{1}{mr\,\sqrt{2}}\right)$$

which may be used for obtaining practical values of the skin effect ratio for mt greater than 4. It may be observed that the term in $\frac{1}{m^2\,r^2}$ is positive, and therefore the curves lie a little above their asymptotes when mt is greater than about 4.

To check the above formula, it may be applied to the case of a solid wire by putting $q = 0$ and $t = r$, so that it becomes

$$\frac{R'}{R} = \frac{mr}{2\,\sqrt{2}} + \frac{1}{4} + \frac{3}{16\,mr\,\sqrt{2}} + \frac{0}{m^2\,r^2}$$

This is in agreement with the value of $R'/R = x/2 \times W/Y$ given by equation (**31**) of the paper by Dr. Russell in the *Philosophical Magazine*, Vol. 17, 1909, page 524.

I am much obliged to Mr. Slepian for his contribution. His mathematical proof seems to me to add greatly to the force of the statement made in my paper that skin effect tests made at different frequencies can be corrected mathematically for frequency and compared with each other with confidence that they ought to agree, and in particular, that tests made on miniature conductors at a high frequency such as 5000 cycles can be corrected for frequency so as to be of great use in the design of 60-cycle bus bars and other conductors.

**Philosophical Magazine*, Vol. 19, 1910, page 49.

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A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED

BY S. R. BERGMAN

ABSTRACT OF PAPER

The standing of this problem in the past is briefly reviewed. As far as is known to the author the type of machine described presents a new solution, the advantages of which are shown to be:

1. The machine is self-excited, *i. e.*, regulates independent of any other source of potential.
2. The machine regulates independent of speed and load and may be compounded.
3. The regulation is inherent, *i.e.*, no external regulating device is used.
4. The regulation is instantaneous.
5. The regulation is approximately independent of the heating.

The theory of the machine is described and diagrams of connections given. Performance curves obtained from tests are shown.

A method is described whereby instantaneous regulation of the voltage is obtained, which method also secures approximately a constant voltage independent of the heating.

Finally there follows a discussion of the efficiency of the new machine as compared with a standard machine of the same speed and output.

A PROBLEM in direct-current engineering, especially met with in traction, is that of producing constant voltage at variable speed. Train lighting from axle-driven generators is one of its oldest applications. The transformation from a variable to a constant voltage can be accomplished by revolving machinery provided that the principle of producing constant potential, independent of the speed, is known. Of late years this problem has received new impetus with the introduction of electric starting and lighting of automobiles and a great number of solutions have been offered, most of them depending upon the presence of a storage battery which greatly simplifies the conditions, since the steadying influence of the battery results in a source of nearly constant potential from which a constant excitation may be drawn. The regulation in the great majority of such charging sets depends upon the load current and is spoiled if the load is disconnected; *i. e.*, if the battery is not in the circuit the generator voltage varies greatly with the speed. Other systems exist employing automatic shunt regulators either oper-

ated magnetically or depending on combinations containing resistors with negative temperature coefficients.

In railway and automobile applications it is desirable to avoid all automatic apparatus if possible and it occurred to the writer some years ago that it would be possible to produce a generator which would regulate on constant potential independent of the speed and load, due to inherent properties of the windings, that is, without the use of any kind of regulator. The conditions to be fulfilled appear as follows:

1. The machine should be self-excited, *i. e.*, should regulate independent of any other source of potential.

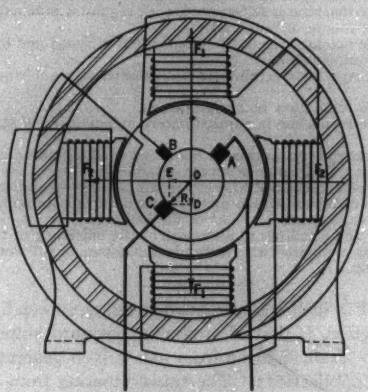

FIG. 1—CONSTANT POTENTIAL GENERATOR

2. The machine should regulate independent of the load and should possess the properties of compounding.

3. The regulation should be inherent.

4. The regulation should be instantaneous, *i. e.*, no over- or under-shooting of voltage should occur at sudden speed variations.

5. The regulation should be independent of the heating, *i. e.*, same potential generated when machine is cold as when hot.

A number of machines fulfilling the above conditions have been built and are now in successful service. In the following pages the principle of operation will be described and illustrated by characteristic curves obtained from tests.

Fig. 1 illustrates, diagramatically, the principle of the machine. The armature is series wound and the field contains twice as many poles as the number of poles for which the armature is wound. In the case illustrated, the armature is wound for two poles and the field contains four poles, symmetrically located. The load is taken from the brushes A and C and in addition to these load brushes there exists a third brush B, placed 90 electrical degrees from the load brushes. The field may be considered

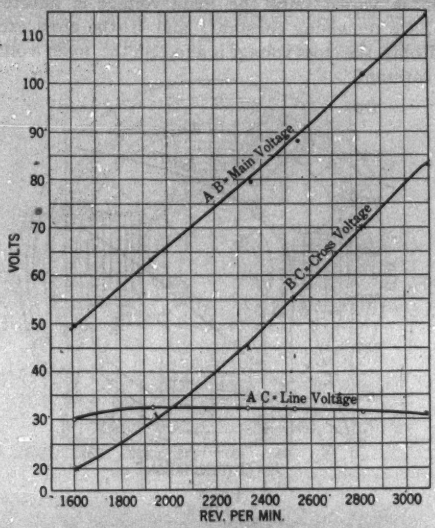

FIG. 2—CONSTANT-POTENTIAL GENERATOR
1.5 KW.—32 VOLTS—1600/3200 REV. PER MIN.

to consist of two independent magnetic circuits, one of these circuits $F1$ is saturated and the corresponding flux ϕ_1, will be called the main flux of the machine. The second magnetic circuit $F2$ is not saturated and the corresponding flux ϕ_2 will be called the cross flux. The flux ϕ_1 generates, between the brushes A and B, an e. m. f. which will be called the main voltage of the machine. The flux ϕ_1 does not generate any e.m.f. between the brushes B and C. Similarly the flux ϕ_2 generates, between the brushes B and C, an e.m.f., which will be called the cross voltage.

This flux does not generate any e.m.f. between the brushes A and B. The excitation of the machine is taken from the brushes A and B for which reason the brush B will be called the exciting brush. The excitation consists, as shown, of two multiple branches, one branch exciting the main poles and the second branch exciting the cross poles. The two fluxes ϕ_1 and ϕ_2 are entirely independent of each other, which may be verified by separately exciting the machine, and tests show that if the main excitation is varied only the main voltage is affected, the cross voltage remaining constant; and if the cross excitation is varied only the cross voltage is affected, the main voltage remaining constant.

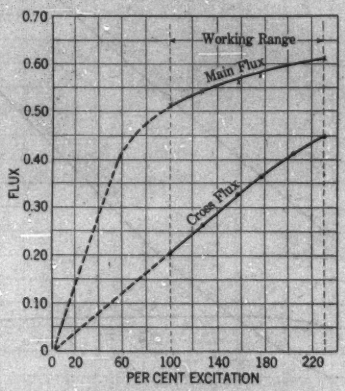

FIG. 3

The directions of the main and the cross fluxes are such that the difference between these fluxes is interlinked, with the line brushes A and C, as shown in the figure, and thus the line voltage AC is the difference between the main voltage AB and the cross voltage BC. Hence, the voltage $AC = AB - BC$. Since the main circuit is saturated the flux ϕ_1 remains constant and the main voltage AB is proportional to the speed. Therefore, the excitation of both the main and the cross fields is proportional to the speed. The cross circuit not being saturated the cross flux ϕ_2 will increase in proportion to the speed and, hence, the cross voltage BC must increase with the square of the speed. As will be shown by saturation curves the cross circuit should not be

entirely unsaturated since then the cross voltage would increase too fast and the machine voltage would decrease with increasing speed. The cross flux should approach saturation and it is possible to choose the saturation of the cross magnetic circuit, so that the line voltage $A\ C$ remains constant. In Fig. 2 is shown the variation of the different voltages with the speed. Since the excitation is taken from the brushes A and B the variation of the exciting ampere-turns follows the main voltage $A\ B$ and may, therefore, in proper scale, be read from the curve $A\ B$. From these curves may be determined the amount of flux in each magnetic circuit for any given excitation and in

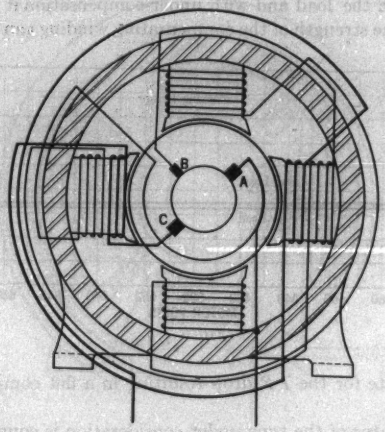

FIG. 4

Fig. 3 are plotted the two fluxes against the excitation. These saturation curves show that over the working range the main magnetic circuit is nearly saturated and the cross magnetic circuit at first is unsaturated but from a certain point, where the curve bends, this circuit starts to approach saturation.

Since the line current is taken from the brushes A and C, Fig. 1, there exists an armature reaction $O\ R$ in the direction of $A\ C$. This armature reaction may be resolved in two components, one $O\ D$ in the direction of the main flux and the other $O\ E$ in the direction of the cross flux. The main magnetic circuit being saturated, the additional excitation, due to the armature re-

action, cannot add anything to the main flux. The component OE in the direction of the cross flux will, however, interfere with this flux and would disturb the regulation of the machine and in order to overcome this influence a series winding is added to the cross poles. This winding should have an equal and opposite strength to the armature reaction working in this direction, and will be called the compensating winding. The location of this winding is shown in Fig. 4. It will be easily understood that by changing the strength of the compensating winding it is possible to obtain either rising or falling machine voltage with increasing load. By over-compensation the voltage will rise with the load and with under-compensation it will fall. Obviously the strength of the compensating winding can be made

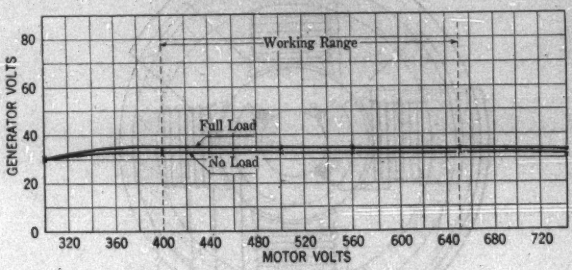

FIG. 5

to compensate for the IR drop resulting in a flat compounded generator.

If a generator of the type under consideration is coupled to a motor such a motor-generator is capable of transforming from a variable voltage to a constant voltage. Fig. 5 shows the performance from test of a motor-generator, the motor voltage varying from 400 to 650 and the generator voltage being maintained constant at 32 volts. As will be seen from Fig. 5 the generator voltage rises slightly with the load showing that the machine is over-compounded.

If the motor of a motor-generator of this type is thrown directly on the line a voltmeter connected to the generator shows that the voltage, due to sudden acceleration, exceeds the rated voltage. The amount of this over-shooting of the voltage depends upon the relative values of the time constants of the

two multiple exciting circuits. For example; assuming that the rate of increase of the exciting current is nearly the same in both circuits then the flux in the unsaturated circuit will appear at a later time than in the saturated circuit. Consequently the negative voltage will not develop fast enough, resulting in the phenomenon which may be called over-shooting of the voltage. It is possible, however, to speed up the rate at which the cross magnetizing current increases by insertion of resistance in this circuit. The complete diagram of connections is shown in Fig. 6 in which the permanent resistance is designated as R. Tests

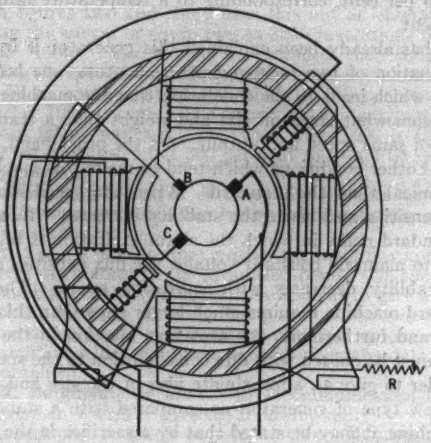

FIG. 6

have demonstrated that if this resistance is properly chosen it is possible to entirely eliminate over-shooting of the voltage. In this figure has also been added commutating poles to assure perfect commutation.

The resistance R also serves another purpose, namely, to keep the regulation of the machine approximately the same when the generator is cold as when hot. Assuming for the moment that the generator was built in accordance with Fig. 4, without any resistance in the cross circuit, it is obvious that when the generator is cold the current in both exciting circuits will be larger than when hot. The main flux will not be influenced by this increased

excitation, since saturation exists. On the other hand, the increased exciting current of the cross circuit will cause an increase in the cross flux resulting in an increase of the cross-voltage. Since this voltage is negative it follows that when the machine is cold the line voltage will be too low. By inserting a resistance of zero temperature coefficient the variation of the cross excitation is limited and tests show that if the resistance R is approximately ⅛ or more of the resistance of the cross exciting circuit, the change of excitation, due to heating, will be limited to such an extent that the variation of voltage is less than 5 per cent, corresponding to a temperature range of 100 deg. cent.

As has already been explained this generator is in reality a combination of two independent generators, one bucking the other, which leads to the conclusion that the machine must become somewhat larger in size and weight than a standard machine of same speed and output. On the other hand, there are several other conditions which tend to make this machine more economical than the standard. When the speed increases all flux densities and hence the stability increases. In a machine of standard make in which the shunt excitation is regulated in order to maintain constant voltage, the flux densities and hence the stability decreases with increasing speed. Therefore, a standard machine requires much larger gaps than this new machine and furthermore, the armature reaction in the standard type must be chosen with due consideration to the speed range. In order to give an approximate idea of the size and weight of this new type of generator, as compared with a standard type of machine, it may be stated that by a sacrifice in the efficiency of approximately 10 per cent, the new machine will have the same weight as a standard type of same rating.

As has already been pointed out this new type of constant potential generator shows some distinct advantages over a standard generator with automatic voltage control. Where reliability of service is concerned this new type is superior since there does not exist automatic devices of any kind for maintaining the regulation. The new machine has also the advantage of being instantaneous in action even at violent and large speed variations. Therefore, it is thought that this new machine will create a field of its own and satisfy a long felt need in certain applications of the electrical industry.

DISCUSSION ON "A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED" (BERGMAN,) (PRESENTED BY PUBLICATION ONLY).

J. G. De Remer: In the paper by S. R. Bergman, attention should be called to an apparent misstatement contained on the lower part of page 1408 thereof.

The reasoning therein given relative to the variation of the main voltage AB and the cross voltage BC is based upon assumptions which do not apply. For if the main voltage varies with the first power of the speed and the cross voltage with the second power of the speed their difference AC can not be constant.

The figures two and three bear this out as observed by extending the curves of figure two to 3200 revolutions per minute (double 1600). The curves so extended then indicate that doubling the speed from 1600 to 3200 increases the main voltages from 50 to approximately 120 which is 140 per cent increase instead of 100 per cent. The cross voltage is increased from 20 to about 90 by increasing the speed from 1600 to 3200 which is 350 per cent instead of 300 per cent as would be required by the square law.

The foregoing is further evidenced by the flux curves shown in Fig. 3 where the main flux is not constant as assumed in the author's statement.

Figs. 2 and 3 indicate that neither the main flux nor the cross flux were maintained at the saturation point. On the contrary, the conditions of the two fluxes were such as to give a resulting flux which decreases with the first power of the speed. Such a flux variation would give constant potential across AC and is the condition which doubtless existed when the data of Fig. 2 were obtained.

S. R. Bergman: In answer to a communication by Mr. J. G. DeRemer, I wish to give the following reply.

The best plan to follow in order to thoroughly understand an electro-magnetic problem is to draw up a picture of the flux distribution. (Fig. 1). If Mr. DeRemer would attempt to do so I predict he will find that the theory he suggests leads nowhere.

The theory which I have advanced in my paper is based on the physical existence of two independent fluxes, the main flux and the cross flux, the distribution of which is shown in Fig. 1.

It is now obvious without further consideration, that the corresponding voltage may be expressed as follows:

$$\left.\begin{aligned} AB &= k \times n \times \Phi_1 \times 10^{-8} \\ BC &= k \times n \times \Phi_2 \times 10^{-8} \end{aligned}\right\}$$

If saturation exists in the main magnetic circuit the main flux is constant and the voltage AB is proportional to the speed as stated in the paper.

Complete saturation never exists in any dynamo electric machine but a slight increase of the flux always takes place when the excitation is increased, as may plainly be seen from Fig. 3

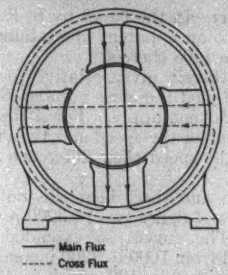

FIG. 1—FLUX DISTRIBUTION. CONSTANT POTENTIAL GENERATOR

in my paper. The main flux is, therefore, in reality not constant but is a function of the excitation and it would not be difficult, although laborious, to solve this problem analytically by aid of

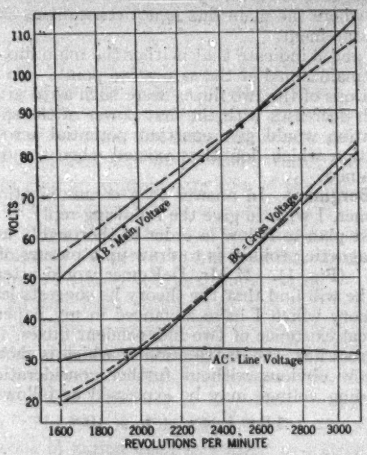

FIG. 2—CONSTANT POTENTIAL GENERATOR 1.5 KW.—32 VOLTS—1600/3200 REV. PER MIN.

the Fröhlich formula—(compare Steinmetz' "Transient Electric Phenomena and Oscillations" page 33.) Such a calculation would so complicate the problem that it would becloud the

physical phenomenon instead of clearing it up. Hence, I prefer in my discussion to state that the saturation keeps the flux constant. Tests justify this assumption with fair approximation as may be seen from Fig. 2 which is a reproduction of Fig. 2 in my paper with the addition of a dotted straight line representing proportionality.

In regard to the cross voltage BC, this voltage increases with the square of the speed if we assume that the main voltage varies directly with the speed, and also, that no saturation exists in the cross circuit. Also, in this case can the problem be solved accurately by aid of the Fröhlich formula. However, the square law holds true with good accuracy as shown in Fig. 2 in which a parabolic curve is shown in dotted lines, representing the square law.

Mr. DeRemer states: "If the main voltage varies with the first power of the speed and the cross voltage with the second power of the speed their difference AC cannot be constant." In order to avoid this very misunderstanding I stated in my paper "As will be shown by saturation curves, the cross circuit should not be entirely unsaturated since then the cross voltage would increase too fast and the machine voltage would decrease with increasing speed. The cross flux should approach saturation and it is possible to choose the saturation of the cross magnetic circuit so that the line voltage AC remains constant."

In order to further emphasize the independence of the two fluxes, I wish to point out that the cross flux branches out from points of equal magnetic potential located on the path of the main magnetic circuit and vice versa, the main flux branches out from points of equal potential on the path of the cross flux.

Very similar conditions exist in a commutating-pole machine, having the same number of commutating poles as main poles. An excellent discussion of the independent existence of the main flux and the commutating flux may be found in Arnold's classical work, "Die Gleichstrom Maschine."

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CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

BY ALEXANDER GRAY AND J. G. PERTSCH, JR.

ABSTRACT OF PAPER

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

ALL of the formulas are based on Maxwell's equation:

$$\text{Pull, in dynes} = \frac{B^2 A}{8\pi}$$

where B = flux density, in lines per sq. cm.

A = cross section, in sq. cm. of the flux path in the gap between the pulling surfaces.

This formula may be established in a simple manner for the case of a saturated magnetic circuit, as follows:

The magnetic circuit shown in Fig. 1 is symmetrical about the axis $O\,O'$, and therefore the action of each pole may be considered separately. With n turns per pole and e volts across each coil, the excitation $n\,i$ produces a magnetic flux of ϕ lines.

In Fig. 2, $O\,a$ shows the magnetization curve for each half of the complete magnetic circuit of Fig. 1 when the air gap clearance between the armature and each pole-face is g cm. If now, the armature is allowed to move by a virtual displacement $\Delta\,g$ under the action of the magnetic pull P, so as to reduce the gap, then the curve for the new magnetic circuit becomes $O\,b$ (Fig. 2).

In order to establish the initial flux ϕ at an excitation $n\,i$, the total energy supplied to the coil after closing the switch $s\,s$, is

$$\int_0 eidt = \int_0^t \left(n\,\frac{d\,\phi}{d\,t}\,.\,10^{-8}\right) idt + \int_0^t iridt$$

In this expression, the stored energy

$$\int_0^{\phi} ni\, d\phi \quad 10^{-8} = \text{Area } Oac \text{ (Fig. 2), in watt-sec.}$$

If, now, the armature is allowed to move under the action of the pull P, so as to reduce the gap by an amount Δg, the excitation $n\,i$ being maintained constant, the flux changes from ϕ to $\phi + \Delta\phi$. The increase in flux $\Delta\phi$ induces a counter e. m. f. in the winding and since, by assumption, the current is held constant, the applied voltage must temporarily increase. The amount of energy so brought into the electromagnet is

$$\int_{\phi}^{\phi+\Delta\phi} \left(n \frac{d\phi}{dt} \cdot 10^{-8}\right) i dt = ni \times \Delta\phi \,.\, 10^{-8} = \text{Area } cabd$$

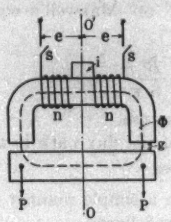

FIG. 1

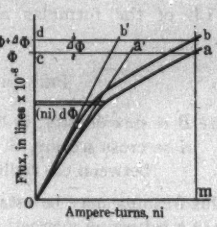

FIG. 2

The energy which is thus supplied by the source goes partly to increase the stored energy and the rest lifts the armature.

The energy stored in the magnetic circuit under the new conditions, with flux $\phi + \Delta\phi$,

$$= \text{Area } Obd.$$

Hence, mechanical work done = initial stored energy + energy supplied from the line − final stored energy

$$\begin{aligned} \text{or } P.\Delta g &= \text{Area } Oac + \text{Area } cabd - \text{Area } Obd \\ &= \text{Area } Oab \\ &= \text{Area } Oa'b' \end{aligned}$$

where Oa' and Ob' in Fig. 2 represent the magnetization curves for the air gaps g and $g - \Delta g$ respectively.

But, for very small values of $\Delta\phi$,

$$\text{Area } Oa'b' = \frac{1}{2} \cdot \text{Ampere-turns for gap } \Delta g \times \phi \cdot 10^{-8} \text{ . . watt-sec.}$$

$$= \frac{1}{2}\left(\frac{10}{4\pi} \cdot \frac{\phi}{A} \cdot \Delta g)\right) \times \phi \,.\, 10^{-8}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta g \times 10^{-7} \text{ . . . . . watt-sec.}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta g = P \times \Delta g \text{ . . . . . ergs.}$$

or, Pull in dynes, $P = \dfrac{\phi^2}{8\pi A} = \dfrac{B^2 A}{8\pi}$

This formula is thus true whether the magnetic circuit is saturated or not. It will also be seen from the above what is meant by A in the formula, and how problems involving slotted armatures are to be solved.

The subject of magnetic pull in dynamo-electric machines has not received much attention in recent years; in fact, it appeared as if the whole field had been covered by 1911.

Fisher-Hinnen[1], in his "Dynamo Design" published in 1899 derives the following formula:

$$\text{Pull, in dynes} = \left(\frac{B^2 A}{25 \times 10^6} \cdot \frac{x}{g\alpha}\right) C$$

where B = density of flux in the air gap, in lines per sq. cm.
A = section of the pole face, in sq. cm.
x = rotor displacement
g = air-gap clearance
α = ratio of the reluctance of the total magnetic circuit to that of the normal air gap alone.
C is a calculated coefficient, given in the following table:

Value of C	Number of poles	Remarks
2	4	Displacement along axis of field magnets.
2.8	4	Displacement along neutral line.
6	6	Displacement along axis of magnet.
9	8	Displacement along neutral line.
15	12	Displacement along neutral line.
20	16	Displacement along neutral line.

1. See Bibliography.

B. A. Behrend[2] in 1900 gives a mathematical discussion of the case of a machine with a very large number of poles and derives the formula

$$\text{Pull} = \frac{B^2S'}{8\pi} \cdot \frac{2x}{g}.$$

where $S' = \pi \times R \times l = \frac{1}{2} \times$ total air-gap area, and shows that this is exactly one-half the value obtained by assuming that half of the poles have a gap $g - x$ and the other half a gap $g + x$.*

If we substitute for $S' = \frac{1}{2} \times A \times$ poles, Behrend's expression becomes

$$\text{Pull} = \frac{B^2A}{8\pi} \cdot \frac{x}{g} \cdot \text{poles}.$$

Edgar Knowlton,[3] about the same time, derives an expression for a machine with a definite number of poles and obtains the formula

$$\text{Pull in lb.} = \left(\frac{B^2A}{77{,}134{,}000} \cdot \frac{x}{g\alpha}\right) C$$

where B = normal gap density in lines per sq. in.
A = area of pole-face in sq. inches
x = displacement
g = normal air gap
α = ratio of the total reluctance of the magnetic circuit to that of the normal gap alone.
C = a coefficient, obtained by calculation
= 2 for 4 poles
= 4.7 for 6 poles
= 7 for 8 poles
= 9.6 for 10 poles

Above 10 poles, C = number of poles, and the formula then checks with that of Behrend.

Knowlton's expression is of exactly the same form as that of Fisher-Hinnen, but the constants given by the two authors for 6 and 8 poles differ considerably. Knowlton, moreover, states that it makes but little difference whether the plane of deflection is taken through a pole or between two poles.

Hans Linsenmann[4], in 1902, uses the saturation curve directly

* Note that in the following paper Rosenberg formulates a similar statement.

(as is done in Fig. 3 of the following article by Rosenberg) for determining, at a given excitation, and with sine distribution of flux under the pole, the variation of the average magnetic pull per pole with the length of airgap. He expresses this variation by an approximate equation and introduces it into the general equations connecting the bending moments of the deflecting forces in large alternators.

J. Rey[5], in 1904, states that Behrend's and Fisher-Hinnen's formulas are not consistent and sets out to derive a new formula especially applicable to induction motors with eccentric rotor. Rey's formula is

$$\text{Pull in dynes} = \frac{1}{8\pi^2}(B_{eff})^2 S \times C$$

where $B_{eff} = B_{av} \times 1.1$ for a sine distribution of flux
S = rotor surface $= 2\pi Rl$
$C = f(\epsilon)$
ϵ = eccentricity, expressed as a fraction, in terms of the single air gap $= x/g$

and for

$\epsilon = x/g =$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
$C =$	0.157	0.319	0.488	0.688	0.866	1.084	1.63	2.415

For an eccentricity less than 20 per cent, this formula reduces to

$$\frac{(B_{eff})^2}{8\pi^2} \times S \times \frac{0.488}{0.15} \times \frac{x}{g}$$

$$= \frac{(B_{eff})^2}{8\pi} \times S \times \frac{x}{g}, \text{ approximate within one per cent,}$$

which is identical with Behrend's formula, because with a sine distribution of flux, as in an induction motor, $(B_{eff})^2$ is the average value of $(B)^2$, where B is the density at any point along the gap.

J. K. Sumec[7] derives a similar formula to that of Behrend and Rey, but by certain transformations reduces it to the form

$$\text{Pull in dynes} = \frac{B^2}{8\pi} \cdot S \frac{x}{g} \frac{1}{[1-(x/g)^2]^{3/2}}$$

This is the same as Behrend's formula except for the last term which for an eccentricity as large as 25 per cent, is equal to 1.1 or a difference of only 10 per cent, while for eccentricities below

10 per cent, the correction term differs from unity by only about one per cent.

Sumec's formula, as might be expected, gives the same values as that derived by Rey. It is to be noted that in most of the formulas derived mathematically, the effect of the reluctance of the iron part of the magnetic circuit is neglected; the assumption made that the flux density is inversely as the airgap is therefore only true in unsaturated machines

In 1905, B. Soschinski[8] gives an account of some tests made to check the formulas of Rey and Sumec. Very good agreement was obtained for small airgaps and iron feebly saturated, taking the area and flux density at the top of the teeth but with increasing saturation, the calculated results (iron reluctance neglected) were higher than experimental ones. With larger values of air gap, however, the test values were throughout higher (up to 100 per cent). The latter result was accounted for by the fact that as the armature moves to one side the lines become concentrated at the tooth tip on the side of the reduced gap and spread out on the side of the increased gap.

In the same year (1905), Niethammer[9], by a transformation of Sumec's expression, obtains the following formula which permits taking into account the reluctance of the iron path by determining the flux density from the saturation curve, as is done by Rosenberg. This expression is regarded by him as giving the most reliable results.

$$\text{Resultant pull, in } kg. = \frac{1}{2}\left\{\left(\frac{B_{max}}{5000}\right)^2 - \left(\frac{B_{min}}{5000}\right)^2\right\} p\,A$$

where p = number of pairs of poles
A = mean effective cross sectional area of gap per pole in sq. cm.

In 1907, Picou[10], starting with the relation for stored energy, derives the expression for magnetic attraction, and gives a modification of the formula due to Sumec for unbalanced pull.

C. R. Moore[12] in 1911, gives a graphical method for studying the unbalanced magnetic pull, by using Maxwell's formula and the given saturation curve of the machine, and summing up the pull for various adjacent halves of adjoining pole faces. He takes the airgap density for these half arcs as corresponding to the average airgap lengths across these faces and to the given excitation. Upon plotting the unbalanced magnetic pull for

various field excitations he establishes the important fact noted by Rosenberg that, for low saturations the pull increases with the excitation; at a critical density, however, corresponding to about the knee of the magnetization curve, the unbalanced pull reaches a maximum for all eccentricities, and for larger excitations then decreases again. Therefore a machine, which normally operates at high saturation, may be subjected to greater stresses while it is building up than under normal field excitation.

Miles Walker[14] in his book on "Specification and Design of Dynamo-Electric Machinery" (1915), points out that the amount of the unbalanced pull for a given displacement will depend on the extent to which the iron parts are saturated, and that the effect of increased saturation is to reduce the pull for a given airgap clearance and magnetic induction. He first assumes that all the ampere-turns are expended in the airgap and by the usual method derives an expression for the unbalanced pull exactly similar to that of Behrend (see above). He then shows how the saturated magnetic circuit may be replaced by an equiva lent airgap obtained by means of a graphical construction on the saturation curve of the machine, and then uses this equivalent airgap in the formula for the unbalanced pull.

R. E. Hellmund[11], Miles Walker[14] and others have pointed out the effect of series and parallel windings on the unbalanced pull and have also discussed how the unbalanced magnetic pull in the induction motor may be reduced to a minimum, by using in the stator winding two paths in parallel which lie on opposite halves of the frame and thus make it impossible for the flux on these two halves to be very unequal.

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MAGNETIC PULL IN ELECTRIC MACHINES

BY E. ROSENBERG

List of Symbols

A	Pole face area in square centimeters
B	Magnetic induction (flux density in the airgap.)
B_1	Reduced induction with increased airgap
B_2	Increased induction with reduced airgap
B_m	Critical induction, causing maximum unbalanced pull
$\frac{dB}{dH}$	Differential quotient of induction and magnetomotive force in lines per sq. cm. and Gilbert's ($4\pi/10$ ampere-turns) respectively
D	Rotor diameter
E	Voltage
$\frac{dE}{di}$	Differential quotient of voltage and exciting current
f	"Gravity deflection"; deflection of the horizontally placed shaft under the static influence of the rotor weight
F_x	Unbalanced magnetic pull of the machine, due to a displacement x
F_f	Unbalanced magnetic pull of the machine, due to a displacement f
$(F_x)_{neutr.}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the neutral diameter.
$(F_x)_{axis}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the field axis
g	"Correct" air gap, taken as an average around the machine
g_1	"Virtual air gap", which, for low induction, would require the same m.m.f. as the magnetic half circuit of the machine
g_2	In two-pole machines: Phantom air gap of the interpole (air gap of an imaginary interpole with same section as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole)

G	Rotor weight
H	Magnetomotive force
i	Exciting current
L	Effective length of rotor (air ducts subtracted)
n	Revolutions per minute
$n_{crit.}$	Critical speed in rev. per. min.
$2p$	Number of poles
q	Ratio between the combined deflection of all the machine parts, caused by unbalanced pull, to the displacement, causing the unbalanced pull
q_r	Ratio between rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull
q_s	Ratio between combined deflection of the other machine parts to the displacement causing the unbalanced pull
$T_{circ.}$	Ampere-turns per pole required to produce a certain induction (within the straight line characteristic) in magnetic circuit of the machine with "correct" air gap
T_x	Ampere turns required to produce the same induction in the air gap x
$\tan \alpha$	Gradient of the magnetization characteristic
x	Displacement of rotor and stator centers
X	Final static displacement
$X_{mom.}$	Final momentary displacement
α	Local angle between magnetization curve and horizontal
θ	Angle between point of periphery and symmetry diameter

NOTE: Lengths are given in centimeters, forces and weights in kilograms, inductions in lines per square centimeter, unless otherwise stated.

Designers who figure in inches and pounds and who use "Kapp Lines per square inch" (one Kapp Line being equal to 6000 c.g.s. lines), substitute for $\left(\frac{B}{5000}\right)^2$ the value $\frac{1}{2} \cdot (B$ Kapplines/sq. inch$)^2$, to obtain the pull in pounds per square inch in formulas (1) to (**10a**).

The critical speed, formula (**11**), changes to

$$n_{crit.} = \sqrt{1 - q} \cdot \frac{188}{\sqrt{f_{inch}}}$$

if f_{inch} denotes the gravity deflection in inches.

Introduction

THE aim of the present paper is to investigate the occurrence and the effect of unbalanced magnetic pull in electric machines and to derive simple practical formulas for the use of the designer. There is an interesting chapter on the subject in Miles Walker's excellent book "Specification and Design of Dynamo Electric Machinery" in which for a given induction the influence of saturation on the unbalanced pull is considered. The present author has endeavored to carry the matter to its logical conclusion and to find, whether in a given machine there is a "critical induction" which gives a higher unbalanced pull than any other, smaller or larger, induction; further, to find a simple rule for determining this critical induction, and to investigate the permissible deflection of the machine parts in connection with the unbalanced pull, and the influence of the latter on the critical speed. Multipolar and bipolar machines on the one hand, and those with salient poles and cylindrical fields with distributed winding on the other hand, show very striking differences although the maximum of the forces occurring can be covered by the same formula.

A. Magnetic Pull of a Pole and of a Series of Poles

The magnetic attraction on a square centimeter of a pole face is

$$\frac{B^2}{8\pi} \text{ dynes}$$

or, with an accuracy of nearly 1 per cent,

$$\left(\frac{B}{5000}\right)^2 \text{ kilograms,}$$

if B, the flux density in the airgaps, immediately adjoining the pole face, is expressed in c.g.s lines per sq. cm. To get the pull per pole, provided the pole extends over a comparatively small part of the periphery, we have to multiply by the area A of the pole face, measured in square centimeters. In general, in electric machines the magnetic pulls of the different poles are equal and are arranged symmetrically around the center. With a stator of the same stiffness in all diameters the magnetic pull would only cause a certain well defined and moderate strain all around the machine. With a split machine, however, especially if the joints are not very well stiffened, the deflection of the frame in the vertical direction will be different from that in the horizontal direction and a distortion may result which actually reduces

sensibly the airgaps in the diameter perpendicular to the plane of the joint. Each half of the stator can be considered as a beam, the support and fixing of the upper half being different from that of the lower half which contains the feet.

It can happen that, through error or accident, only part of the poles are excited. The worst case will be when all the poles in one half of the machine are over-excited, the poles in the other half being without excitation. The mechanical parts of the machine should be strong enough to withstand this condition without being overstrained, although it will not be possible in large machines with small airgaps to prevent entire pulling over of the rotor against the stator core. Considering the resulting force on the one half of the rotor or stator, it is clear that every pole, the center line of which describes an angle θ with the symmetry line (Fig. 1), will contribute a component in the direction of the symmetry line

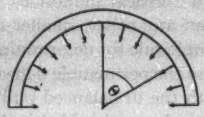

FIG. 1

$$\left(\frac{B}{5000}\right)^2 \cdot A \cdot \cos\theta$$

The average value of the cosine function over a quadrant is $2/\pi$. If therefore the machine has $2p$ poles, only p neighbouring poles being excited, the resulting force, both on the stator and rotor, will be

$$\frac{2}{\pi} \cdot \left(\frac{B}{5000}\right)^2 \cdot A \cdot p \qquad (1)$$

If the air gap density B is not constant over the whole pole face, but graded, the mean value of B^2 should be taken.

The unbalanced pull, which appears when only half of the machine is excited, grows with the second power of the air gap density or of the flux. With an air gap density of 5000 it is one kilogram per sq. cm.; with a density of 10,000 it is four kilograms per sq. cm., and with a density of 12,250, six kilograms per sq. cm. The highest possible, not the normal, saturation has to be considered to provide for this case, for it is likely that the accident or error which causes the non-excitation of one half of the poles, will cause the over-excitation of the other half as for instance, when all the exciting coils are connected in series, and wires leading to opposite coils become accidentally short-circuited. It must be ascertained whether both the shaft and

frame etc. will withstand the force found by formula (1) without overstraining, or whether the rotor core will pull over hard against the stator core with some lower force which is not sufficient to cause overstrain.

B. Unbalanced Pull Due to Eccentricity

I. *Multipolar Salient-Pole Machines.* An entirely different and very general case is that of a machine, the coils of which have an equal excitation, but cause a different flux under the various poles due to differences in the air gap. Many causes contribute to such a condition. Frequently the outer surface of the rotor and the inner surface of the stator are not perfectly cylindrical. Even if they are perfectly cylindrical and concentric while the machine is cold, a noticeable deviation may occur due to the difference in temperature between stator and bedplate when the machine is heated. With a machine of five meters diameter, for instance, a difference of 20 deg. cent. in the average temperatures of frame and bedplate would correspond to approximately one mm. difference in length. If both feet of the hotter frame are rigidly bolted down to points of the cooler bedplate, a distortion of the frame will result; if only one foot is rigidly fixed, a shifting of the stator center will occur.

Another factor is the clearance between shaft and bearings. Even with perfectly new bearings this clearance can be measured by tenths of a millimeter. While the machine is at rest, the oil is squeezed out from underneath the shaft, and all the clearance will be between the top part of the shaft and the upper bearing shell. In this position the machine is erected and centered. When the machine is running, the bearing is flooded with oil and the clearance divides equally around the shaft. Very frequently of course, imperfect erection, a bending of the shaft or slight subsidence of the foundations with consequent distortion of the bedplate may be the cause of a displacement of the rotor and stator centers.

We shall now investigate the case of a rotor and stator each with a cylindrical surface, but with the centers displaced by an amount x (the eccentricity). Without displacement, the radical airgap would have a constant value g. The diameter drawn through the two centers we shall call "symmetry diameter". Assume, first of all, a multipolar machine with salient poles, all the exciting coils giving the same number of ampere-turns. In the symmetry diameter the air gap will in one place be reduced

to $g - x$, in the opposite place increased to $g + x$. At points, the radius through which describes an angle θ with the symmetry diameter, the increase or reduction of the airgap will be, with close approximation, $x \cdot \cos \theta$. At right angles to the symmetry diameter the increase or reduction will be practically zero.

The flux emitted by one field pole returns through the contiguous halves of the neighboring poles. In a bipolar machine the flux of each pole is the same in spite of air gap differences; in a multipolar machine the poles of one half of the machine will carry a greater flux than those of the other half, unless the winding arrangements prevent this (equalizing connections). These apart, the flux of each pole will be different from that of the others, dependent upon the local air gap. Poles in the diameter at right angles to the symmetry diameter will carry the normal flux, for their neighbor poles in the one half (with reduced air gap) will tend to increase their flux just as much as those in the other half (with increased air gap) will tend to reduce it. The result is that in a multipolar machine each pole may be considered as if it would create its own flux and not be dependent upon adjacent poles.

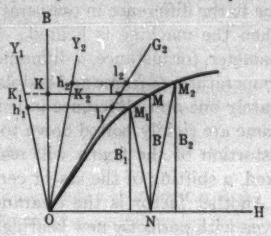

FIG. 2

We assume the machine is excited to give, with the mean air gap, a flux density B lines per sq. cm. of pole face. Where the air gap is reduced to $g - x$, the density will have a greater value B_2, and where the air gap is increased to $g + x$, a smaller value B_1. The first gives a greater magnetic pull in one direction than the second in the opposite direction, and the difference of the two pulls or the "local unbalanced pull" in kilograms per sq. cm. of the pole surface is given by the formula

$$\left(\frac{B_2}{5000}\right)^2 - \left(\frac{B_1}{5000}\right)^2 = \left(\frac{B_2 + B_1}{5000}\right) \cdot \left(\frac{B_2 - B_1}{5000}\right) \qquad (2)$$

It is easy to determine the flux densities B_2 and B_1, if the magnetic characteristic of the machine is given. In Fig. 2 the abscissas H of the curve represent the exciting m.m.f. per pole, while the ordinates B, represent the flux density per sq. cm. of the pole face, with the normal air gap. $O\,N = K\,M$ represents a certain m.m.f., required to produce the induction B. $O\,G$ is

the "air gap line", the portion $K\,L$ representing the m.m.f. required for the magnetization of the air gap g, the part $L\,M$ the m.m.f. for the magnetization of the iron. The real existing air gap, however, is not g but is $g + x$ in one case and $g - x$ in the other. We draw through O two lines $O\,Y_1$ and $O\,Y_2$ such that for any induction $B\ (= N\,M)$, $K_1\,L$ represents the m.m.f. required for the air gap $g + x$ and $K_2\,L$ the m.m.f. for the air gap $g - x$. We then draw through N two lines respectively parallel to these new axes $O\,Y_1$ and $O\,Y_2$. The points M_1 and M_2 where lines so drawn intersect the magnetization curve then give the actual air gap densities B_1 and B_2 under the poles with the increased and with the reduced air gap, for a given excitation $O\,N$, because

$$h_1\,M_1 = h_2\,M_2 = O\,N.$$

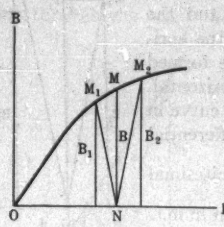

FIG. 3

Fig. 3 is a repetition of Fig. 2 without unnecessary lines.

In general, the abscissas of the lines $O\,Y_1$ and $O\,Y_2$ (Fig. 2) will bear to the abscissa of $O\,G$ for the same flux density very nearly the ratio of the displacement x to the air gap length g. Only if the displacement is very great compared with the average air gap and if the machine has open slots, will the proportionality be markedly disturbed due to the well known crowding of the lines in the air gap near the teeth. Thus, if teeth and slots have equal width and the normal radial air gap is equal to half the width of the slot, then in the extreme case of a machine pulling hard over, the abscissa of the line $O\,Y_1$ will be equal to about 0.85 of the abscissa of the line $O\,Y_2$ which latter one, of course, then coincides with $O\,Y$.

For small displacements, $O\,Y_1$ and $O\,Y_2$ can always be drawn symmetrical with reference to the ordinate, and their m.m.f. for a given induction B is approximately $\pm\,x\,.\,B$, where the displacement x is measured in centimeters and the m.m.f. in gilberts

$$\left(\frac{4\,\pi}{10}\ \text{ampere-turns}\right)$$

In formula (2) there appears the sum and the difference of the two inductions B_2 and B_1. In Fig. 4 the part M_1MM_2 of Fig. 2 is drawn on a greater scale and also the chord M_1M_2 which cuts

the ordinate of point M in m. The chord describes with the horizontal an angle α. The difference $B_2 - B_1$ is in the figure represented by

$$P_1 P_2 = P_1 m + m P_2 = (M_1 P_1 + P_2 M_2) \cdot \tan \alpha$$

$M_1 P_1$ and $P_2 M_2$ are the abscissas of the x characteristics for inductions B_1 and B_2 respectively, are therefore equal to $x B_1$ and $x B_2$. We therefore have

$$B_2 - B_1 = (B_2 + B_1) \cdot x \cdot \tan \alpha$$

and the formula (2) can be written in the form

$$\left(\frac{B_2 + B_1}{5000}\right)^2 \cdot x \cdot \tan \alpha$$

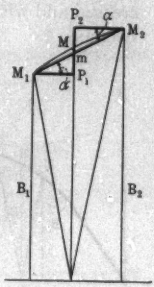

FIG. 4

If x is not very great, the sum $B_2 + B_1$ can be accurately replaced by $2B$, and the angle which the chord forms with the horizontal can be replaced by the angle formed by the tangent in point M with the horizontal. Tan α is then the *gradient* of the curve in point M and can be expressed as a differential quotient $\frac{dB}{dH}$ or the ratio of an infinitesimal increase in induction to the increase in m.m.f. causing it.

The unbalanced pull in kilograms per sq. cm. is, therefore,

$$4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \tan \alpha = 4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \qquad (3)$$

The expression $x \cdot dB$ in formula (3) represents also the m.m.f. necessary to produce an increase dB in induction in an air gap x, while dH represents the m.m.f. per pole necessary to produce in the magnetic half circuit of the machine with the correct air gap the same increase dB in induction. The ratio $\frac{x \cdot dB}{dH}$ is therefore a ratio of magnetomotive forces and can also be replaced by the ratio of the corresponding ampere turns; ampere turns required to drive through the air gap x an infinitesimal increase dB in flux density, divided by the ampere turns per pole required to drive through the magnetic circuit of the machine with the correct air gap the same increase in flux density.

Looking closer into the approximations which we introduced

in formula (3), we find: If x is large and if the magnetization characteristic were a straight line (Fig. 5), the sum $B_2 + B_1$ would actually be greater than $2B$. In Fig. 5 the value $\frac{B_2+B_1}{2}$ is shown dotted in. If, however, the characteristic is strongly curved (Fig. 6), $B_2 + B_1$ will be slightly smaller than $2B$. For hard pulling over, when x reaches the greatest possible value, the saturation will in any case be high enough that the curvature of the characteristic is marked. We are therefore certain that, on this score, formula (3) does not give too low values for the extreme case.

A very important question is, now: How does the expression of formula (3) change with growing excitation? The first part of almost every magnetization curve is a distinct straight line going through the origin of the ordinates. For every straight

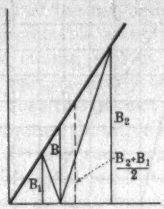

FIG. 5

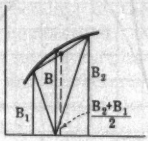

FIG. 6

line characteristic, whether or not going through the origin, the gradient $\frac{dB}{dH}$ is constant, the unbalanced pull therefore will grow with the square of the induction B. As soon, however, as the saturation of the iron parts is noticeable, the quotient $\frac{dB}{dH}$ will constantly diminish, and the question is how to find the induction B for which the product $B^2 \cdot \frac{dB}{dH}$ is a maximum. In Fig. 7 a graphical procedure is shown. For a point M of the saturation curve I the ordinate $B = NM$ is drawn and a line MP perpendicular to the direction of the curve in point M. The angle $N\,M\,P$ is identical with the angle α formed between the curve in point M and the horizontal.

$$\tan \alpha = \frac{dB}{dH}$$

$$N P = N M \tan \alpha = B \tan \alpha$$

and the area of the triangle $N M P$ is

$$\frac{1}{2} B \cdot B \cdot \tan \alpha = \frac{B^2}{2} \cdot \frac{dB}{dH}$$

If such triangles are drawn for different points of the curve, the triangle area will be proportional to the function $B^2 \cdot \frac{dB}{dH}$ for the various points.

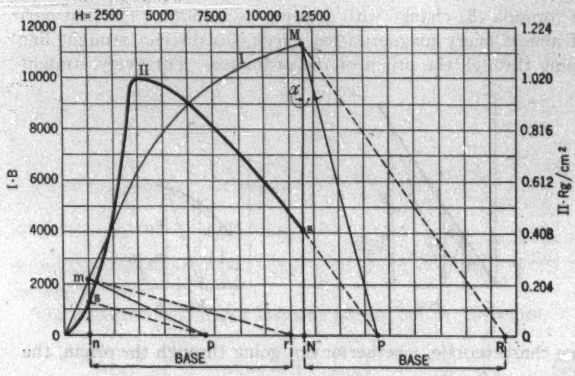

FIG. 7

In order to obtain a curve representing the magnetic pull for every excitation, it is now only necessary to replace all the triangles $N P M$ by others which have a suitable arbitrary base $N R$. Connecting R with M and drawing a line $P S$ parallel to $R M$, it follows from the elements of geometry that the triangles $N P M$ and $N R S$ are equal in area. (The line $R S$ of the latter triangle is not drawn in Fig. 7). $N S$ is therefore a measure of the function $B^2 \cdot \frac{dB}{dH}$. The scale is determined by the base $N R$. If the construction is repeated point for point (for a second point m the construction is shown), we obtain the bold line curve II in Fig. 7, the first part of which, corresponding to the straight

portion of the magnetization characteristic, consists of a parabola with vertical axis. Nearly immediately as the magnetization characteristic begins to bend, the curve of the pull reaches its maximum and from then falls steadily.

This teaches us a thing of the utmost importance. In salient-pole machines which are not *entirely* unsaturated, it is not sufficient to calculate the unbalanced magnetic pull for a high excitation. The "critical" excitation for which the unbalanced magnetic pull is a maximum, is reached at the very beginning of the "knee" in the magnetization curve; as generators and motors must be able to stand abnormal changes in voltage, the unbalanced magnetic pull must be calculated for this critical excitation.

In some machines, exciters for instance, and also turbo-generators with cylindrical fields, very high saturation occurs in a small part of the magnetic circuit (saturation plates or teeth), before appreciable saturation occurs in the other parts of the magnetic circuit, and in this case, after a very short initial straight part, a nearly continuous bending of the saturation curve or even two distinct bends can be observed. At even very small induction the curve bends, (the saturation plates becoming saturated), then there is a straight line characteristic and then there is a second bend, (when the saturation of the iron circuit as a whole becomes marked). In this latter case the straight line characteristic does not go through the origin of the coordinates. The first bend may cause a *local* maximum of the unbalanced pull, that is to say, a point higher than the neighbor points, but the important maximum occurs at the second bend of the characteristic. The "short cut" explained later on for the determination of the "critical induction" does not refer to these cases. The character of the unbalanced pull as shown in the bold curve II of Fig. 7 is obtained for most electric machines.

It is not necessary really to construct the whole curve of unbalanced pull, point by point. A few points at the end of the straight line and at the beginning of the curved characteristic are sufficient. What we are interested in is only the *maximum* value of the unbalanced pull and this is, with such characteristics, obtained immediately after the straight line part of the saturation curve is ended. The only task is, then, to find an official "terminus" of the straight line part.

A rule which the author suggests and which seems to give sufficiently accurate results, is to draw a tangent to the saturation

curve with a gradient equal to 5/6 of the gradient of the straight part of the curve. In Fig. 8 the simple construction is shown for the characteristic of Fig. 7. Through the origin a line $O\,Z$ is drawn which has an ordinate of 5000 for the same abscissa for which the straight line characteristic has an ordinate of 6000. $T\,U$ is the tangent parallel to $O\,Z$; the point of contact with the curve is T. V, the point on the extended straight line, which has the same ordinate B_m as T, is what we will call the end point or terminus of the straight line characteristic.

We now see the two factors which enable us to reduce in a machine with fixed principal dimensions the unbalanced mag-

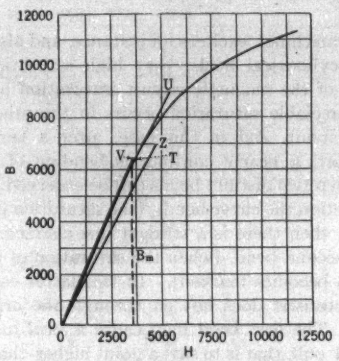

FIG. 8

netic pull caused by a definite displacement. The first is to increase the air gap, because $\frac{dB}{dH}$ for the straight part of the magnetization curve is very nearly in inverse proportion to the air gap. The second means, however, is to reduce the iron section, for example that of poles and teeth, because that causes the saturation characteristic to bend earlier, or in other words it cuts short the straight part of the magnetization curve.

Up to now we have calculated the radial unbalanced pull per sq. cm. for an eccentricity x. The eccentricity changes in a quadrant from x to zero according to the expression $x \cos \theta$.

The radial magnetic pull per sq. cm. in the different parts of a multipolar machine will therefore be very nearly proportional

to $x \cos \theta$ and the component of this pull, working along the "symmetry diameter" will be proportional to $x \cos \theta \cdot \cos \theta = x \cos^2 \theta$.

In machines with a number of poles divisible by 4, we may always consider together two points, separated by a full quadrant one with an angle θ, the other with an angle (90 deg. + θ). The *sum* of the components of these pulls per sq. cm. is

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot [x \cos^2\theta + x \cos^2 (90 \text{ deg.} + \theta)]$$

$$= 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH},$$

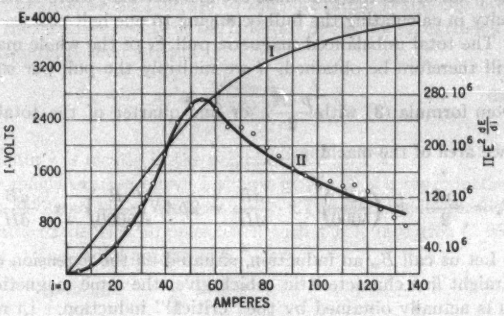

FIG. 9

as cos (90 deg. + θ) = − sin θ and therefore $\cos^2 \theta + \cos^2$ (90 deg. + θ) = 1.

In such machines we can therefore always find two pole pairs which, taken together, develop the same unbalanced pull as a single pole of identical dimensions and induction, the axis of which is parallel and the pole face of which is perpendicular to the symmetry diameter.

In a six-pole machine three pole pairs together develop $1\frac{1}{2}$ times the unbalanced pull of an imaginary pole pair with an axis parallel and a pole face perpendicular to the symmetry diameter, as is proved by the following consideration:

In a six-pole machine we can consider at the same time three

places with angles θ, 60 deg. + θ, 120 deg. + θ, and we have $\cos^2\theta + \cos^2(60 \text{ deg.} + \theta) + \cos^2(120 \text{ deg.} + \theta)$

$$= \frac{1 + \cos 2\theta}{2} + \frac{1 + \cos(120 \text{ deg.} + 2\theta)}{2}$$

$$+ \frac{1 + \cos(240 \text{ deg.} + 2\theta)}{2} = \frac{3}{2}$$

The same investigation for a machine with 10, 14.... poles would confirm that *the total unbalanced pull can be calculated in all multipolar machines, as if the air gap were reduced by an amount x in one quarter of all the poles, and increased by x in another quarter of all the poles, and left alone in the remaining two quarters.* As poles of electric machines are symmetrical, there is no difficulty in calculating the pull belonging to one half pole.

The total unbalanced magnetic pull F_x of the whole machine will therefore be obtained, if we multiply the pull per sq. cm. from formula (3) with $\frac{p \cdot A}{2}$, or one quarter of the total pole face area of the machine.

$$F_x = \frac{pA}{2} \cdot 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} = 2pA \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \quad (4)$$

Let us call B_m an induction, situated on the extension of the straight line characteristic, which gives the same magnetic pull as is actually obtained by the "critical" induction. In reality B_m will be slightly smaller than the critical induction, but we shall speak, with a small inaccuracy, of B_m as critical induction.

For the straight line part of the characteristic, going through the origin, we can replace $\frac{x \cdot dB}{dH}$ by $\frac{x \cdot B}{H}$ or $\frac{T_x}{T_{circ.}}$ if T_x stands for the ampere turns required to produce in an air gap x any flux density within the straight line characteristic and $T_{circ.}$ stands for the ampere turns per pole, required to produce in the magnetic circuit of the machine with the correct air gap the same flux density.

The formula takes therefore the following simple form:

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} \quad (5)$$

or

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6}$$

if g_1 is the "virtual air gap", that is to say, an air gap which would take, for an induction within the straight line characteristic, as many ampere turns as the real air gap and the iron path of the magnetic half circuit together.

For machines with small air gap, as induction motors, the "virtual air gap" is appreciably higher than the real air gap, say 30 per cent higher or more; while in generators, especially turbo-generators, the virtual air gap is nearly equal to the actual one.

Instead of the total pole face area $2pA$, we may substitute the cylindrical field surface $c \cdot \pi \cdot DL$, if c represents the "pole factor" or ratio of the pole arc to the pole pitch, D the diameter and L the effective length of the field (air ducts, if any, excluded). The formula is then written

$$F_x = c \cdot \pi\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

The factor c is mostly in the neighborhood of 2/3.

The magnetization curve I shown in Fig. 7 is a reproduction of Fig. 347 from Miles Walker's book. The machine in question is an alternator with 40 poles, each with a pole face area of 650 sq. cm. The radial air gap is 0.51 cm.

$$2p = 40 \qquad A = 650 \qquad g = 0.51 \text{ cm.}$$

The curve II of the unbalanced pull shows that the maximum is 1.02 kilograms per sq. cm. for a displacement of 0.1 cm. The scale for the "pull curve" II is marked on the right of Fig. 7 and it can be verified by figuring the pull for any particular induction on the straight part of the magnetization characteristic. Taking the simpler form of the determination of the maximum (Fig.8) we obtain the critical induction $B_m = 6300$. The straight part of the line goes through a point with an ordinate $B = 4300$ and an abscissa $H = 2500$. The ratio

$$\frac{x \cdot dB}{dH} = \frac{T_x}{T_{circ.}} = \frac{4300 \cdot x}{2500} = 1.72 \cdot x$$

The maximum radial unbalanced pull per sq. cm. is therefore according to formula (3)

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot x = 4 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot x = 10.8 \cdot x$$

or for a displacement of $x = 0.1$ cm., 1.08 kilograms.
This method of calculation gives here a result 6 per cent high, which is entirely satisfactory. It is, of course, not necessary to calculate at first the force for one sq. cm. Formula (**5**) gives us direct the unbalanced pull for the whole machine

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot 0.1$$
$$= 7100 \text{ kg.}$$

If we had used formula (**6**) instead of formula (**5**), we should have obtained at first from the straight line characteristic

$g_1 = \frac{2500}{4300} = 0.58$ cm. (*i. e.* 0.07 cm., or 14 per cent greater than the actual air gap)

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot \frac{0.1}{0.58}$$
$$= 7100 \text{ kg.}$$

It may be mentioned that Walker figures for the machine in question, which works with a normal air gap density of 9160, an unbalanced pull of 5400 kilograms. If we make the calculation in our way for normal saturation, we should also get approximately the same result. But, as our investigations have shown the normal induction does not give the greatest pull, which must be considered by the designer.

The flux density in the most saturated iron parts (sheet steel) corresponding to an air gap density of 6300 is not more than about 11,000 to 13,000 lines per sq. cm.

A salient-pole machine with given dimensions of the iron parts and with a given air gap experiences for a certain displacement the greatest unbalanced pull with an excitation, which is, as a rule, well below the normal working excitation. There is a passage in Walker's book, which, although it was most likely correctly understood by its author, is apt to be misleading. Walker says on page 56 that in large alternators with a great number of poles and a small air gap, the flux density in the gap must be limited to a moderate value, such as 50 kilolines per square inch (7750 lines per sq. cm.) to keep the unbalanced pull down. Most designers believe indeed that in a given machine

with fixed iron dimensions the unbalanced magnetic pull is reduced by lowering the saturation. This is contrary to the result of our investigation, (unless the induction be reduced below the end point of the straight line characteristic.) As long as the iron dimensions are not changed, the flux density may be increased to any desired value, and the effect will in fact be a reduction of the magnetic pull at the normal voltage, although the *possible* maximum unbalanced pull which has to be considered for the mechanical design remains the same. The unbalanced magnetic pull can only be reduced by a reduction of the flux, if a reduction of the iron sections goes hand in hand with the flux reduction.

II. *Multipolar Machines with Distributed Exciting Winding.* Up to the present we have considered machines with salient poles

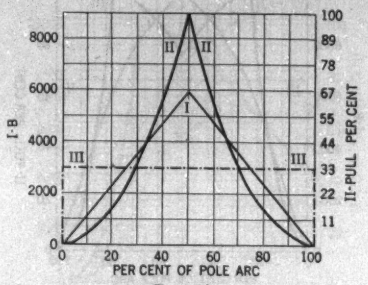

FIG. 10

in which the induction over the whole pole face would be constant but for the displacement of the rotor and stator centres. In the case of induction motors, however, and turbo-alternators, the induction changes gradually from zero to a maximum. At first we shall deal with multipolar machines of this description. The rule already arrived at, that the unbalanced pull is the same as if the air gap had been reduced by the full amount x in one quarter of the poles and increased by x in the opposite quarter holds good, from its derivation, also for machines with distributed winding.

Let us assume that the curve I in Fig. 7 represents the curve of the air gap saturation of such a machine, while the ordinates of the bold line curve II represent the magnetic pull in kilograms per sq. cm. for a displacement of 0.1 cm. The induction over

the pole face may change according to a straight line law, (I in Fig. 10), if the machine is entirely unsaturated and if the exciting winding (in case of turbo-generators for instance) is equally distributed around the whole pole face. These two conditions are hardly ever fulfilled, but the case forms a starting point. For every induction of curve I in Fig. 10 the magnetic pull per sq. cm. can be taken from Fig. 7 and so we obtain as curve representing the local pull over the pole face the bold line parabolas II in Fig. 10. The average value of the pull taken over the whole pole face is in this case exactly 1/3 of the value corresponding to the maximum induction. The area of the dotted rectangle III in Fig. 10 is equal to the area enclosed by the base and the double parabola. This, however, does not represent by any means, the

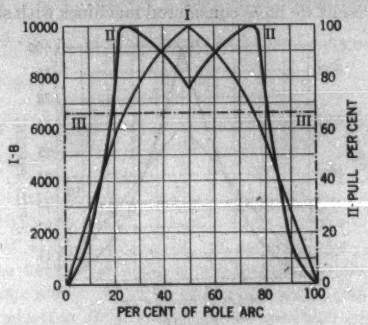

FIG. 11

maximum unbalanced pull possible in this machine. In Fig. 11 the field form I and the "pull curve" II are drawn for the case that the induction in the center of the pole is increased to 10,000. The average value of the pull, line III, is here 66 per cent of the maximum value. A full review of the possible changes in the average value of the pull is given in curve III of Fig. 12. I and II in Fig. 12 are repetitions of the curves I and II in Fig. 7. The ordinate of curve III for any given abscissa gives the average value of all the ordinates of the pull curve II starting from the abscissa *O* up to the abscissa in question. As long as the pull curve II follows the law of a parabola, the ordinates of curve III are exactly 1/3 of those of curve II. The further points are obtained by point to point calculation. With this curve it is quite

easy to give the average value of the pull over one half-pole (and therefore over the whole pole), if the total m.m.f. per pole or the highest induction is known. The curve III has a maximum of about 66.5 per cent of the maximum of curve II for a highest induction of 10,500 lines per sq. cm. The curve III is very much flatter in its upper part than the curve II and its ordinates exceed 60 per cent for abscissas varying from $H = 6250$ to $H = 12,000$. In a machine with distributed winding there is not by any means the marked falling off of the unbalanced magnetic pull, after a certain comparatively low induction is reached. On the contrary, the magnetic pull remains very nearly constant over a wide range of possible inductions.

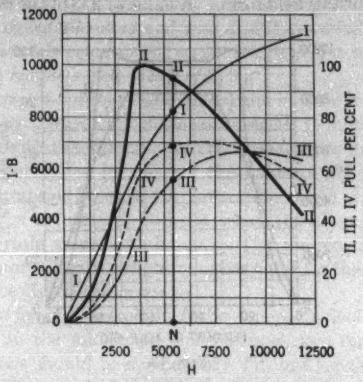

FIG. 12

As a rule, the winding in turbo-generators and induction motors is distributed so as to give a more sinusoidal shape of the flux. For instance, the middle third of a pole face is left without exciting winding, and the winding is only distributed in the first and third part of the pole. If rotor and stator are slotted evenly all around the periphery, as is the case in induction motors, a-c. commutator motors and sometimes also in turbo-generators, the field form can be represented by curve I in Fig. 13, while curve II shows the local pull and the dotted straight line IV the average value of the pull in this case, the area of the dotted rectangle IV being equal to the area enclosed by curve II and the base. It is quite easy to determine the average pull value over the pole

for *any* saturation by combining the curves II and III in Fig. 12. It is shown as an example for the abscissa *ON*, which corresponds to an induction *N* I (curve I). Two-thirds of the pole has an induction gradually rising from zero to the value mentioned before. The average value of the pull in this part is given by the ordinate *N* III. One-third of the pole face has a constant induction *N* I, its pull per sq. cm. being represented by the ordinate *N* II. To get the average over the whole pole, we take 2/3 of *N* III and add 1/3 of *N* II, thus obtaining the ordinate *N* IV. The distance III · IV is one-third of the distance III · II. If we know the maximum value of the induction or the total m.m.f. per pole, the ordinate of curve IV gives at once the average value of the pull for this field form. A value of 2/3 of the peak ordinate

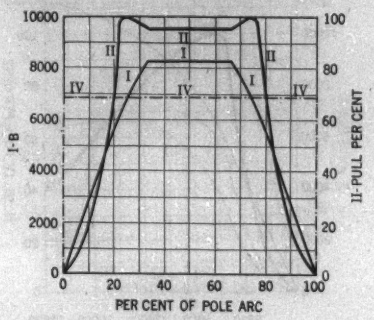

FIG. 13

of curve II is in the example of Fig. 12 already obtained for an induction of approximately 7700, the highest value of IV is 70.5 per cent of the peak value of II and corresponds to an induction of 9300; the line of 2/3 is reached again for an induction of 10,500.

Other magnetization curves were investigated by the author and a slightly higher value was found for the maximum pull of a pole with such distribution of winding.

We can very well say, in a machine with a field slotted all round and a distributed exciting winding, the unbalanced magnetic pull for a given displacement remains very nearly constant with saturations as they are practically applied, and it is approximately 2/3 of the value which would be obtained, if the whole

active cylindrical surface of the rotor were equally excited, so as to give the "critical induction". In semi-closed-slot rotors the active rotor surface is very nearly equal $\pi \cdot DL$, L representing the effective length, after deduction of air ducts. For such machines we can therefore use the formula obtained for salient-pole machines

$$F_x = c \cdot \pi DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

the value of c or "pole factor" being roughly 2/3 for closed-slot rotors. In a salient-pole machine we have called "pole factor" the ratio of the pole arc to the pole pitch and this value as a rule is also about 2/3. *The maximum unbalanced pull in a machine with a field slotted all round and with distributed winding is therefore approximately equal to the maximum pull in a machine with salient poles having a pole factor of* 2/3.

With open-slot rotors (turbo fields) the surface $\pi D L$ should be reduced by the area of the slot openings. It must be born in mind, that, with the reduction of the surface, goes hand in hand a nearly proportional increase of the value of B_m, the critical air gap saturation, because the saturation curve starts to bend when the *iron* saturation, and not when the air gap induction, reaches a certain value. As B_m appears in the second power, an open-slot rotor will give a slightly higher value for the unbalanced pull than the closed-slot rotor.

To get an accurate result in turbo-generator fields which are not slotted in the middle part of the pole, two magnetization characteristics should be considered: The one for the solid part (say 1/3 of the pole width) with the highest induction, and the other for the slotted part, with induction grading down to zero. The critical induction of the slotted part will be appreciably lower than that of the solid part. The unbalanced pull contributed by the slotted parts, will only be about half of that contributed by the solid part. The latter corresponds to a salient pole with a pole factor 1/3. The factor for the whole machine, if B_m *for the solid part only* is put into the formula, will then be about 0.5.

On the other hand, it is possible to adhere to our accustomed pole factor 2/3, if one constructs a *mean* magnetization curve about the average between that of the slotted and that of the plain part and considers the B_m and g_1 of this mean curve.

The case is similar to that of a salient pole with graded airgap.

III. *Salient Pole Machines with Graded Air gap.* Often in salient-pole machines the air gap is not constant but a minimum in the middle of the pole and increased gradually towards the tips. The induction therefore cannot reach the critical value in all parts of the pole face at the same time. The maximum pull will not be reached when the middle part of the pole has the critical induction, because it would mean that all the other parts have a lower induction and therefore contribute an appreciably smaller magnetic pull than the maximum. It will be just as in machines with distributed winding, that the maximum resulting pull is obtained with a higher saturation in the pole center and that the upper part of the curve, representing the magnetic pull dependent on the excitation, is flatter than for a machine with constant induction over the pole face. If in one part of the pole the induction reaches the critical value, this part gives the maximum pull. The parts from there to the corner which have lower values of induction, give a much smaller pull per sq. cm.; the parts towards the center of the pole with higher inductions also give a pull below the maximum, but not falling so rapidly. If the excitation is increased, the point of the critical induction will shift towards the corner, increasing the pull of the corner parts and reducing that of the center parts, so that the total change in pull is not so very marked. If the excitation is lowered, the critical induction will shift more towards the center, increasing the pull there and reducing the pull of the corner parts. In general, it will be possible to change the excitation within comparatively wide limits without very great change of the resulting unbalanced magnetic pull. The calculation of the greatest value of the unbalanced pull can be made, with sufficient accuracy for practical purposes, as if the air gap (without displacement of the rotor center) had a constant mean value.

IVa. *Bipolar Salient-Pole Machines.* In multipolar machines the flux leaving the field at a place where the air gap is reduced by the eccentricity, also returns into the field in a place where the air gap is reduced. The sum total of all the fluxes taken over one-half of the machine will therefore be greater than the sum total of all the fluxes taken over the other half of the machine.

The same possibility exists in a two-pole machine, if the eccentric displacement is at right angles to the field axis. In Fig. 14 this case is shown. A rotor is assumed with salient poles built in the manner of the Siemens *H* armature, with one exciting

coil, the pole face covering a greater arc than is usual in actual machines, but presenting a distinct neutral zone. The field axis $A\ A_1$ has a distance x from the stator center. It is clear that in this machine, lines of force leaving the left corner of the upper pole, marked 1, will after passing through the armature, re-enter the field at the left corner of the lower pole, marked 4, while lines leaving the field at the right hand corner 2 of the upper pole will re-enter at the right hand corner 3 of the lower pole. In the half machine on the left side of the axis $A\ A_1$ the air gaps are throughout smaller than on the right side, and as the two sides represent two parallel magnetic paths with different reluctance, the total flux on the left side will be greater than the total flux on the other side. This case is similar to that of a multipolar machine except for the fact on the one hand that through the neutral zone a

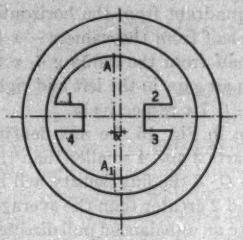

FIG. 14

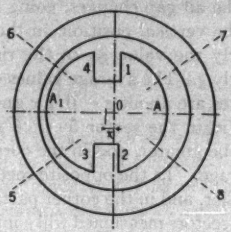

FIG. 15

considerable part of the magnetic pull is cut out, while on the other hand the saturation of the stator and rotor *cores* is hardly influenced by the unequal distribution of the flux in the two machine parts. In order to find out the air gap densities B_2 and B_1 corresponding to a certain displacement x, a magnetic characteristic of the air gap, teeth and pole tips should be drawn, excluding the ampere turns required for the cores, and therefore the critical induction B_m will be higher and the local unbalanced pull resulting from the eccentricity will be greater than it would be if the magnetic characteristic of the whole machine had to be considered.

A different state of affairs exists if the rotor center is displaced with regard to the stator center by an amount x in the direction of the field axis. (Fig. 15). Although the air gap in the left half is smaller than in the right half, the total flux must be equal in both halves, if no flux can return through the neutral zone.

The lines of force pass in series through the smaller and the larger air gaps. The only effect of the eccentricity will be a different distribution of the flux over the pole faces. Let us assume for simplicity's sake that the rotor in a central position the air gap induction over the whole pole face would be equal. Then it is clear, that with the rotor shifted as in Fig. 15 the flux density will be a minimum in the center A of the right pole, where the air gap is a maximum, while it will increase gradually to the corners 1 and 2 of the pole. In the left pole, however, the flux density will be a maximum in the pole center A_1 and will grade down towards the corners 3 and 4. Therefore, considering a square centimeter in the middle A of the right pole and one in A_1 of the left pole, there will indeed act on the rotor an unbalanced magnetic pull directed to the left.

The air gap changes, going in a quadrant from the horizontal to the vertical position, in the left half from the value $g - x$ to nearly the value g; in the right half from the value $g + x$ to nearly g. The average values of the air gaps in the left and right halves are about $g - 0.7\,x$ and $g + 0.7\,x$ respectively.

Within the sector $5 \cdot O \cdot 6$ shown in Fig. 15 the flux density will be greater, and in the corner parts 3 and 4 smaller than the average. In the opposite sector $7 \cdot O \cdot 8$ the flux density will be smaller, and in the corner parts 1 and 2 greater than the average. The sectors mentioned will contribute an unbalanced pull directed to the left, the corner parts a smaller component directed to the right. It is clear that the total unbalanced pull in this case is smaller than in the case of Fig. 14. A two-pole rotating field will therefore, if its center is displaced with regard to the stator center, experience an unbalanced pull which changes twice during a revolution from a maximum to a minimum value.

In practise, a salient-pole of a two-pole machine, covers about 120 deg. In the case corresponding to Fig. 14 (displacement parallel to the neutral diameter) the limits for O from 1 to A are 30 deg. and 90 deg. or $\pi/6$ and $\pi/2$. An element of the surface covering an infinitely small angle $d\,\theta$ has an area $\frac{1}{2}\,DL \cdot d\,\theta$. The component of unbalanced pull in the horizontal direction contributed by this element is

$$\frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot x\,\frac{dB}{dH} \cdot \cos^2\theta$$

$$= \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2\theta$$

The integration is to be made from + 30 deg. to 90 deg and from − 30 deg. to − 90 deg., or the integration made from 30 deg. to 90 deg. must be doubled. This gives

$$4\,DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \cdot d\,\theta$$

The value of the integral is $\frac{\theta}{2} + \frac{\sin 2\,\theta}{4}$ taken between above limits, or $\frac{\pi}{6} - \frac{\sqrt{3}}{8} = 0.307$. Therefore

$$(F_x)_{neutr.} = 1{,}228 \cdot DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

$$= 0.39 \cdot \pi\,DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad \textbf{(7a)}$$

The pole factor is here 0.39 instead of the accustomed value of 2/3 We see that the unbalanced pull in case of a two-pole salient pole turbo-rotor with 120 deg. pole angle is only 59 per cent of that of a multipolar rotor of the same dimensions and total pole area, provided that the critical induction and the virtual air gap are the same in both cases.

IVb. *Bipolar Salient-Pole Machines with Interpoles.* Bipolar salient-pole machines are used mainly for continuous current, while for two-pole turbo-alternators the cylindrical field and distributed winding is the rule.

If there are interpoles in a two-pole constant-current machine, the total flux of one main pole can be different from that of the other main pole, and the case of Fig. 15 requires a revision for such machines. If, for example, the air gap of the north pole is smaller than that of the south pole, there will be a greater flux in the north pole and the difference in flux will return through the iron of the interpoles, creating in the latter ones south magnetism, whereas normally they would be neutral as long as the armature gives no current. The two interpoles together carry the difference between the actual fluxes of the main poles. One interpole carries the "excess flux" viz. the difference between the greater flux and the average flux. In a four-polar machine

which has poles in and at right angles to the symmetry diameter, no m.m.f. is required to drive the excess flux back, for in that case there is, for instance, just as much increase of flux in the upper pole as reduction of normal flux in the lower pole. In the present case, however, a certain amount of m.m.f. is required for the return of the excess flux through the interpole. This will depend mainly upon the section of the interpole and the air gap under the interpole. In a multipolar machine one per cent vertical displacement causes one per cent excess m.m.f., therefore one per cent excess flux in the one pole and likewise one per cent deficit flux in the opposite pole. Here one per cent excess m.m.f. has to drive the excess flux not only through the upper air gap, main pole and half yoke, but also through the interpole. If the magnetic reluctance of the interpole were equal to that of the main pole, the percentage of the excess flux would obviously be one-half of the percentage of excess m.m.f. By introducing

$$\frac{x}{2g_1} \text{ instead of } \frac{x}{g_1} \text{ for } x \cdot \frac{dB}{dH} \text{ in formula } (3).$$

we would obtain the local radial pull for a displacement in the direction of the field axis. In general we may introduce the idea of a "phantom interpole air gap" (Symbol g_2), or the air gap length of an imaginary interpole with the same section and pole tip as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole. While g_1 is only slightly greater than the real main pole air gap g, the phantom gap g_2 will be a multiple of the real interpole air gap, if the section of the interpole is only a fraction of that of the main pole.

Introducing $g_1 + g_2$ instead of g_1, we obtain the element of the unbalanced pull component parallel to the field axis for an induction B_m:

$$\frac{1}{2} DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \frac{x}{g_1 + g_2} \cdot \cos^2 \theta$$

We assume a pole factor of $^2/_3$ or a pole arc of 120 deg. The total unbalanced pull therefore is given by integrating twice between 0 and $\pi/3$

$$(F_x)_{axis} = 4\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \int_0^{\frac{\pi}{3}} \cos^2 \theta \,.\, d\,\theta$$

The solution of the integral is $\frac{\theta}{2} + \frac{\sin 2\,\theta}{4}$, taken between

above limits, that is $\frac{\pi}{6} + \frac{\sqrt{3}}{8} = 0.740$

$$(F_x)_{axis} = 2.96\ DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2}$$

$$= 0.94\ \pi\ DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \qquad \textbf{(7b)}$$

If g_2 is larger than $2g_1$ (generally it *is* larger), the pull calculated according to this formula is smaller than the pull due to a displacement at right angles to the field axis, formula (**7a**), which is for interpole machines the same as for other two-pole machines, as long as the armature gives no current.

V. *Bipolar Machines with Cylindrical Field and Distributed Winding.* The case of bipolar cylindrical fields lends itself to easy mathematical, treatment, if we assume an entirely unsaturated sine-shaped field and a rotor and stator slotted evenly around the periphery. $x \cdot \frac{dB}{dH}$ in formula (**3**) is then constant and can be replaced, for a displacement in the direction of the neutral diameter, by x/g_1. The induction at any point, (starting to count the angle θ from the neutral diameter bb_1 in Fig. 16) is $B \sin \theta$. For a displacement x in the direction of the neutral diameter the unbalanced pull is

$$(F_x)_{neutr.} = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B \sin\theta}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2 \theta$$

$$= 4\ DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta \cdot d\ \theta$$

$$= \frac{1}{4} \cdot \pi\ DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1}$$

For a displacement in the direction of the field axis, the conditions are in principle the same as in two-pole machines with interpoles; the phantom air gap of the interpole can be regarded here as equal to the virtual air gap of the main pole. We write

$2\,g_1$ instead of $g_1 + g_2$ and consider further that, if we start to count the angle from the field axis, the local induction is $B \cos \theta$. The unbalanced pull is then

$$(F_x)_{axis} = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot DL \cdot d\,\theta \cdot 4 \left(\frac{B \cos \theta}{5000}\right)^2 \cdot \frac{x}{2\,g_1} \cdot \cos^2 \theta$$

$$= 2\,DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \int_0^{\frac{\pi}{2}} \cos^4 \theta \cdot d\,\theta$$

$$= \frac{3}{8} \cdot \pi\,DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1}$$

In this case, a displacement in the direction of the field axis would actually cause a greater unbalanced pull than a displacement in the neutral diameter. The upper limit for B in these formulas would be B_m and we obtain pole factors of 0.25 and 0.375 respectively. The relation of these pulls changes, however, if we increase the saturation. As long as the induction is on the straight line characteristic, $B^2 \cdot \frac{dB}{dH}$ is highest for the point a in the field axis (Fig. 16); this point contributes, for a displacement in the direction of the axis, the full value of its radial pull as vertical component; for a displacement in the direction of the neutral diameter, the horizontal component of the radial pull is zero. If, however, we increase the saturation above B_m, than the point of the maximum $B^2 \cdot \frac{dB}{dH}$ will shift the farther away from the field axis the higher the saturation goes, and the horizontal component of this maximum pull will grow, while the vertical declines.

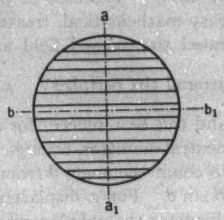

FIG. 16

As here the saturation curve comes into play, we shall abandon the analytical treatment and the assumption of a sine-shaped field and return to a field form like that represented in Fig. 13.

It is clear to us from the beginning that to obtain the highest possible unbalanced pull for a given displacement in the neutral diameter, we shall have to choose the excitation high, so as to obtain the value B_m already for a small angle θ, the cosine of which is near unity.

A four-pole machine with winding distributed evenly over the whole surface and with a zero value of field in the horizontal has a field maximum under an angle of 45 deg. to the horizontal. The maximum pole factor for such a machine was 66.5 per cent for the saturation curve of Fig. 12. By far the greatest part of the magnetic pull, caused by a displacement in horizontal direction, is contributed by the rotor zone lying between 0 deg. and 45 deg., and if we are free to choose the induction of the two-pole rotor for the zone from 0 to 45 deg. exactly equal to that of the four-pole rotor, the unbalanced pull of the two-pole machine will not greatly differ from that of the four-pole machine. In the zone from 45 to 90 deg., it is true, the induction of the four-pole machine will fall again, while that of the two-pole machine will rise from 45 to 60 deg. and remain constant from 60 to 90 deg. But the contribution for the total pull from this zone does not count for very much. The maximum value of the unbalanced pull will be reached in the two-pole machine for an appreciably higher peak saturation than in the four-pole machine.

This is born out in Fig. 17. Curve I represents the induction for one-half pole (from 0 deg. to 90 deg.). The peak induction (from 60 to 90 deg.) is very high viz. 11,100 lines per sq. cm. or 75 per cent above the critical induction. The latter one is reached already at an angle of 20 deg. Curve II represents the local radial pull per sq. cm. for a constant displacement. Both curves I and II are transferred from Fig. 12, only the horizontal scale being changed. To obtain the component of the radial pull in the neutral diameter, we multiply every ordinate of curve II with $\cos^2 \theta$ and obtain thus curve V. For instance, the ordinate of V for an abscissa 10 deg. is equal to that of curve II multiplied with $\cos^2 10$ deg. $= 0.97$. For small angles, the curve V therefore hugs closely the curve II. The greater the angle, however, the smaller the ordinates of V become compared with those of II.

The average value VI of the ordinates of V taken from 0 deg. to 90 deg. is, in this case, 31 per cent of the peak value of the pull curve II. This also would not change appreciably for an *equally* distributed winding, which would reach still higher saturation from 60 to 90 deg.

The same calculation carried through for a peak induction of 9700 gives 28 per cent as the average value.

To compare this with multipolar machines we must consider that there the average of 66 to 70 per cent which we took from the curves III and IV of Fig. 12 was to be multiplied with one quarter of the total pole-face area, while here we have to multiply with one half of the total pole-face area. To bring the result into line with the formula for multipolar machines, the value 0.31 is to be doubled to give the "pole factor". The pole factor is therefore 62 per cent which is very near the value of $^2/_3$ obtained in multipolar machines with distributed winding.

For displacements in the direction of the pole axis we have to

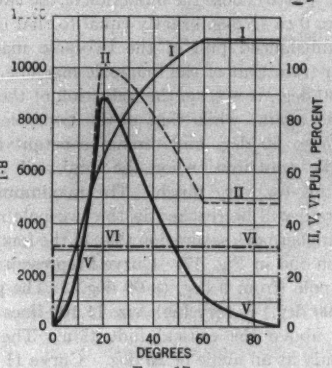

FIG. 17

multiply the ordinates of curve II by $\sin^2 \theta$ instead of $\cos^2 \theta$, and to half the values thus obtained. The parts of the ordinates between the curves V and II in Fig. 17, if halved, represent this value. The average value of the ordinates between V and II equals here just about the average value of curve V ordinates. The result is, therefore, that for the case represented by Fig. 17 the unbalanced magnetic pull for displacement in the direction of the field axis is about half of the pull for displacement in the neutral diameter.

If the middle part of each pole (60 deg. to 120 deg.) remains unslotted, this will have on the unbalanced pull for a displacement in the direction of the neutral diameter, a small effect

which may be deemed to be covered already by the small increase (from 62 per cent to $^2/_3$) which we have made in the value of the pole factor.

The ultimate result is that for multipolar salient-pole and cylindrical fields and also for two-pole cylindrical fields the maximum magnetic pull can be figured from the same formula (**6a**) *and, with approximate accuracy, with the same pole factor, viz.,* $^2/_3$.

C. Increase of Eccentric Displacement Due to Unbalanced Pull

A displacement of the rotor from its true central position causes a one-sided magnetic pull. A one-sided magnetic pull, on the other hand, causes a further displacement. There is a force working on the rotor and stator pulling them towards each other in the direction where the air gap is a minimum. This will cause not only an elastic deflection of the rotor shaft but also of the stator frame, of the bedplate and of the pedestals. If the machine is not excited, all its parts experience a certain deflection due to the weights. The elastic deflections of all parts will change if a one-sided magnetic pull occurs, whatever its direction may be. A magnetic pull on the rotor directed upwards will reduce the apparent weight of the rotor and increase the apparent weight of the stator. The transfer of this weight from the rotor bearings to the feet of the stator frame will also change the elastic deflection of the bedplate. The absolute value of the difference in deflection will be the same, whether the unbalanced pull on the rotor is directed up or downwards. But if the pull is directed sideways, the deflection of the bedplate and of the bearing pedestals will be entirely different. All parts of the machine contribute their share to the combined deflection. In many cases the contribution of pedestals and bedplate may be negligible. Rotor shaft and frame, however, must be calculated in *every* case, so as to be strong enough not to increase unduly by consequent deflection an existing initial displacement. All the deflections work in the same sense, so as to reduce the air gap in the spot where it was originally reduced and to increase it where it was originally increased.

The force F_x of the one-sided magnetic pull, is proportional to the displacement x. The elastic deflections of all the machine parts caused under influence of the force F_x are proportional to this force, therefore also proportional to the initial displacement x. The sum of these deflections constitute the "first increment"

of the original displacement. This increment in displacement will cause an increment in the one-sided magnetic pull which bears the same ratio to the original pull as the increment in displacement bears to the original displacement. The increment in magnetic pull will cause a second increment in displacement, this a second increment in pull, the latter a third increment in displacement etc. etc. As long as the proportionality between displacement and pull exists, every following increment will have the same ratio to its immediate predecessor as this to its own predecessor. In other words, the initial displacement and its increments are represented by a geometric series which we may write in this form:

$$x + q\,x + q^2\,x + q^3\,x + \ldots.$$

The sum of the series can only have a finite value, if $q\,x$, the first increment of the displacement, is smaller than the initial displacement x, or if q is smaller than unity. The sum of the geometric series of the final static displacement is represented by the formula

$$X = \frac{x}{1 - q} \tag{8a}$$

By substituting into the previous formulas X instead of x, the final static force of the magnetic pull is obtained.

Up to now we have considered static forces and displacements. If, however, a machine is *suddenly* excited, as it is generally the case with an induction motor, the shaft, frame and other parts of the machine will not only deflect so as to increase the initial displacement x to the final value X, but all the parts of the machine being elastic will overshoot the position of the new static equilibrium like a spring or a chord. The combined distance travelled by the parts of the machine from the position of the old into the position of the new equilibrium is $X - x$, and they will overshoot this position by approximately the same amount. The *momentary* displacement therefore will be

$$X_{mom.} = x + 2\,(X - x) = 2\,X - x = x \cdot \frac{1 + q}{1 - q} \tag{8b}$$

The original mechanical displacement must therefore be multiplied with the factor $\frac{1 + q}{1 - q}$ to obtain the displacement occurring at the moment of switching in.

We can now formulate the requirement, that a machine, say an induction motor, should be free from "pulling over," even if the initial displacement of the rotor center is equal to one-half of the mean air gap. In other words, the displacement at the moment of switching in, should be less than twice the initial displacement.

$\frac{1+q}{1-q}$ must be smaller than 2. This gives the result:

q smaller than $\frac{1}{3}$.

The smaller q, the safer is the machine against pulling over. If, for instance, $q = 0.2$, the displacement at the moment of switching in, will be only $\frac{1+0.2}{1-0.2} = 1.5$ of the original mechanical displacement. In other words, the machine would only pull hard over, if the original displacement were two-thirds of the original air gap.

This calculation considers only elastic deflections. It is, however, quite feasible that there is, at the moment of switching in, at first a movement of the machine parts through a space without any appreciable resistance being encountered. Suppose, for instance, that there is a clearance in the bearings and that nevertheless the air gap between stator and rotor core is smaller at the top than at the bottom. At the moment of excitation the rotor will travel freely throughout the clearance of the bearing. Only after the clearance is taken up, an opposing force will be caused through the elastic deflection of the machine parts. The rotor travelling through the clearance has obtained a kinetic force which carry it further than according to formula (**8b**).

A "momentary" appearance of the field does not only occur in induction motors but also in generators, if a circuit breaker opens after a short circuit has taken place. While the short circuit lasts, the actual value of the magnetic field is very small; at the moment of the opening of the circuit breaker, the field suddenly rises and it passes certainly also through the value of the "critical" induction.

A measure for the stiffness of the shaft is the deflection, produced by the action of gravity. If the machine has a vertical shaft, we must, for the purpose of this comparison, figure out the deflection which *would* occur, if the shaft were placed horizontally. We shall call this deflection "gravity deflection." Suppose the frame and the other parts of the machine were

infinitely stiff, so that we need not consider any deflection of these parts caused by the magnetic pull. We then could allow such an unbalanced pull as to produce in the rotor shaft a first increment in displacement equal to ⅓ of the original displacement. Let us call f the gravity deflection caused under the static influence of the rotor weight G. If now we take f as original arbitrary displacement, a first increment $f/3$, produced by the magnetic pull, would be permissible. A deflection of $f/3$, however, is produced by a force equal to $G/3$, provided the second force attacks in the same part of the shaft as the rotor weight. The latter condition would be exactly fulfilled, if we exclude from G the weight of the ends of the shaft protruding over the rotor body. But even if this is not excluded, the inaccuracy is very small. We can therefore say: The gravity deflection of the shaft must be within such limits, that the unbalanced magnetic pull produced by a displacement equal to that deflection is less than one-third of the rotor weight.

The deflections of the stator frame and the other parts may be negligible in small machines and also in turbo-generators and high-speed induction motors. In machines with large diameters, however, as low-speed induction motors and engine type generators, they cannot be neglected. There we may divide the quotient q into two parts

$$q = q_r + q_s,$$

q_r being the ratio of the rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull, and q_s being the ratio of the combined deflection of the other machine parts (principally the stator frame) to the displacement. Then the force of unbalanced pull caused by a deflection f is

$$F_f = q_r \cdot G \tag{9}$$

If we allow

$$q_r = q_s = \frac{q}{2} = \frac{1}{6}$$

we may say that *the shaft must be strong enough so that a displacement equal to the gravity deflection should not produce an unbalanced pull more than* 1/6 *of the rotor weight, provided that the combined deflection of the other parts of the machine under a certain unbalanced pull does not exceed the deflection of the rotor under the same pull.* Should the deflection of the other machine parts under the influence of a certain pull be doubly as great as that of the rotor shaft, the highest permissible value for q_r would be 1/9.

For the calculation of the deflection of the stator frame the force, of course, must not be taken as attacking concentrated in one point, but must be distributed, the parallel components attacking under an angle θ from the symmetry diameter being proportionate to $\cos^2 \theta$.

From formula (**9**) the permissible gravity deflection can be directly deduced. The machine, the characteristic of which is given in Fig. 7, has a rotor weight of approximately 9000 kg. We want to allow an unbalanced pull of not more than say $\frac{1}{6} \cdot 9000 = 1500$ kg. for a displacement equal to the gravity deflection. If now the unbalanced pull for 0.1 cm. displacement is 7100 kg., the permissible gravity deflection is $\frac{1500}{7100} \cdot 0.1 = 0.021$ cm. and the shaft must be made strong enough so that this deflection is not exceeded.

We can establish a direct formula to give us the permissible gravity deflection from the point of view of unbalanced magnetic pull without at first having recourse to the "arbitrary displacement" x.

We use formula (**6a**) for our purpose which is applicable to all machines, and substitute the gravity deflection f for the arbitrary displacement x.

$$F_f = c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{f}{g_1}$$

On the other hand is

$$F_f = q_r \cdot G$$

It follows

$$f = \frac{q_r \cdot G}{c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2} \cdot g_1 \qquad (10)$$

or

$$q_r = \frac{f}{g_1} \cdot \frac{c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2}{G} \qquad (10a)$$

In the previous example $c \cdot \pi D L$ is 40.650 and we obtain

$$f = \frac{1/6 \cdot 9000}{40.650 \cdot \left(\frac{6300}{5000}\right)^2} \cdot 0.58 \text{ cm.} = 0.021 \text{ cm.}$$

Suppose the same frame had to be used also for an induction motor with a radial air gap of 0.2 cm. In the generator the virtual air gap was 0.07 cm. larger than the real airgap. Let us assume the same for the motor, then g_1 is 0.27 cm. The critical value of B_m is likely to be reduced in an induction motor, as there, in general, a lower flux is applied and therefore the teeth section correspondingly reduced. This, as a rule, is already necessitated by the greater number of slots which are usual in induction motors. Let us assume that B_m is 5000 for the motor. The rotor core has a diameter of 385.6 cm. and a gross length of 33 cm. from which 5 ventilation ducts of together 6.5 cm. are deducted, giving a net length of 26.5 cm. The total pole surface $\pi D L$ is therefore

$$\pi \cdot 385.6 \cdot 26.5 = 32{,}100 \text{ sq. cm.}$$

The permissible gravity deflection for $B_m = 5000$ and $q_r = 1/6$, assuming the rotor weight to be again 9000 kg. is

$$f = \frac{1/6 \,.\, 9000}{\frac{2}{3} \cdot 32{,}100 \cdot \left(\frac{5000}{5000}\right)^2} \cdot 0.27 \text{ cm.} = 0.019 \text{ cm.}$$

In the assumed case the permissible gravity deflection is nearly the same as the one for the generator, but everything depends on the sections of yoke, rotor and stator teeth, for they determine the value at which the saturation makes itself felt. If the critical induction of the motor were 6000 instead of 5000, the permissible deflection would be reduced to 0.013 cm., and this would also require a stiffer stator frame to comply with the condition that the deflection of the other machine parts, caused by the unbalanced pull, should not exceed, the rotor deflection caused by the same pull. If the stator cannot be made stiff enough, q_r would have to be reduced, and the permissible deflection would be less than 0.013 cm. Should it be impossible to provide such stiffness in the shaft, then the machine should always be provided with a *winding with equalizing connections*.

We see that in all machines (apart from windings with equalizing qualities) the consideration of the unbalanced magnetic pull fixes a limit to the permissible deflection of the shaft under the influence of the rotor weight, just as in high-speed machines the consideration of the critical speed fixes such a limit.

D. Inequalities in Exciting Coils

If, *in a multipolar machine*, one pole is excited with a greater or smaller number of ampere turns than the others, the induction in the pole in question and in the adjoining parts of the two neighbor poles will be higher or lower than in the rest of the machine, and an unbalanced pull is set up. The induction is, in a working machine, produced by the action of the primary exciting coil and the reaction of the armature coil combined. Whether the ampere turns of one of the primary coils are different from those of the others and all the armature coils give the same number of ampere turns, or whether the primary coils work identically amongst themselves and one of the armature coils different from the others, the effect will be an unbalanced pull which is stationary, if the disturbing part is stationary, and rotating, if the disturbing part is rotating. Specially, if a coil of the rotating part is accidentally short-circuited, heavy vibrations will be set up. A disturbance in the balance of the machine which only occurs when the machine is excited, accompanied by the necessity of an increase in exciting current in order to obtain a given voltage, is an indication of a partial or complete short circuit of a rotor coil.

In *two-pole turbo-generators* with cylindrical field and distributed winding, the flux density is highest in the middle part of the pole and is gradually reduced to zero, when approaching a line at right angles to the pole axis. As long as this distribution is the same in two opposite poles and the rotor is in a central position, no unbalanced pull occurs. If, however, one of the coils of one pole is short-circuited, the flux distribution in one pole will be different from that of the other pole, and an unbalanced pull will be set up, rotating with the rotor and producing vibrations of the stator.

E. Equalizing Connections

In Fig. 18 the winding of an a-c. machine is indicated, consisting of 8 coils, the four coils of the upper half being connected in series, as are also the four coils of the lower half. The two paths are connected in parallel. The voltage and periodicity impressed on the two parts are alike, and as both parts consist of the same number of identically situated coils, the total flux passing the upper half will be nearly identical with the total flux in the lower half. Any dissymetry would be practically wiped out by a slight increase of magnetizing current in the one and corresponding reduction of the magnetizing current in the other.

An arrangement like this will therefore prove effective against the consequences of a vertical displacement of the rotor from its true central position. It is, however, entirely ineffective with regard to horizontal displacement. If the rotor is displaced to the left from the true central position, the coil 1_a through which the same magnetizing current is flowing as through the coil 4_a, will cause a stronger magnetic field than the latter, and so will the coil 4_b create a stronger field than the coil 1_b. The result therefore will be an unbalanced magnetic pull tending to create an increment of the displacement which is directed to the left of the rotor.

In a two-phase machine it is possible to arrange the windings of one phase with a horizontal symmetry line, the windings of the other phase with a vertical symmetry line, and so to provide to a certain extent for two components of the magnetic pull.

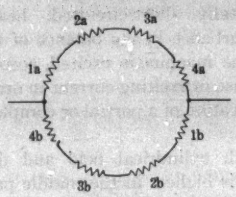

FIG. 18

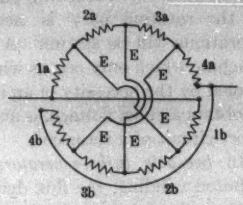

FIG. 19

In a three-phase machine the symmetry lines of the windings can be arranged mechanically under a 60-deg. angle.

A far more effective arrangement, however, is obtained, if equipotential connections *E E E* are added to the windings of every phase, strictly equalizing diametrically opposite coils, as shown in Fig. 19. The coil 1_a is equalized against the diametrically opposite coil 1_b. On both coils the identical voltage is impressed, the flux and the magnetic pull on both parts will therefore be very nearly identical. This is true of all the coils around the whole circumference and therefore the unbalanced pull will amount to a negligible figure.

The necessary requirement for magnetic equalization is of course, that the coils which are equalized against each other, contain the same number of identically situated turns. If, however, a turn should accidentally become short-circuited or open-circuited, the magnetic balance would of course immediately be upset. Equalizing arrangements in the windings therefore do

not free the designer from the duty to design the mechanical parts strong enough so as to deal, in case of accidents, with the force calculated in formula (1), but for ordinary running conditions the unbalanced pull need not be considered.

In multipolar *continuous current machines* with multiple winding but without equipotential cross connections and with as many brush arms as poles, an incomplete magnetic equalization between pole pairs is produced by currents flowing between the brushes of the same polarity. The part of the armature winding moving in a stronger field generates more current, the reaction of which weakens the field. As, however, large continuous currents are required to produce any marked weakening effect by armature reaction, the commutation becomes, as is well known, very seriously impaired, if there are any sensible magnetic inequalities. The tendency to equalization of pole strength refers here to all north poles and equally to all south poles round the machine.

If equipotential cross connections are applied, equally situated armature coils are subjected to the same *alternating* voltage, just as the coils of Fig. 19.

The current flowing through the equalizing or equipotential connections as "wattless" current, is as a rule only small, as comparatively few ampere turns in the armature are sufficient to correct the difference in field strength caused by small differences in the air gap. If, however, by accident an exciting coil is short-circuited, a very large current will flow into the part of the armature winding influenced by the pole in question, and therefore it is not wise to stint in the section of the equalizing connections. Cases are known of four-pole constant-current turbo-generators in which the equalizing connections of the armature have been burnt out due to accidental occasional short circuit of a series field coil.

Equalizing connections between coils excited with continuous current are, of course, useless for our purpose, as the current distribution in such coils is not influenced by the field strength, but solely by the ohmic resistance of the parallel paths. In a machine at rest, excited with direct current, no equalization of flux or pull takes place.

F. Effect of Unbalanced Pull on Rotating Machine

The effect of the unbalanced magnetic pull due to eccentricity may be of a twofold character. If the outer rotor surface and

the inner stator surface are truly cylindrical but not concentric, while the rotor taken by itself is running true, the forces exercised by the unbalanced magnetic pull have always the same direction in space independent of the rotation and are in the case of multipolar machines constant, and only in the case of two-pole rotating fields variable.

If, however, the rotor itself is untrue, no matter whether the field or the armature is rotating, then the place of the smallest air gap is rotating round with the rotor and therefore the force of the unbalanced pull rotating round. As far as the rotor is concerned, the effect will be the same, as if its weight were not balanced mechanically, only with the difference that the unbalance is dependent upon the excitation of the machine. The rotating force acts, however, also on the stator and will set up in low-speed machines visible "breathing"; in high-speed machines sensible vibrations of the frame, the frequency of vibrations being equal to the number of revolutions. We shall not go, in this paper, into the question what "phase difference" exists between the passing of the eccentric part of the rotor and the radial oscillations of a point of the stator, but shall assume in the following section that the stationary parts of the machine are infinitely stiff, so that the above question is eliminated from the following investigation.

G. Influence of Unbalanced Pull on Critical Speed

If the center of gravity of a rotor does not coincide with the axis of rotation, but has a distance x from the same, a centrifugal force will be developed which is proportionate to the displacement x and to the square of the number of revolutions per minute. At a certain speed the centrifugal force will be so great that it would cause (in the absence of friction etc.) a deflection of the shaft equal to the original displacement. The displacement being doubled, the centrifugal force would be doubled causing a double increment in deflection etc. At this speed therefore, the so-called "critical" speed, an original displacement, if not infinitesimal, would cause an infinite final displacement, if there were no resistance to these movements. Although in reality the resistance will keep the displacement down to finite values, the great increase of vibrations at the critical speed is highly objectionable and there is always careful calculation in the design of high-speed machines to make sure that the running speed is well above or well below the critical speed.

In an electric machine, the unbalanced pull due to the rotor displacement will work in the same direction as the centrifugal force and is itself proportionate to the displacement, but independent of the speed. If we call x the displacement in centimeters, the displacement measured in meters will be $\frac{x}{100}$. Let G be the rotor weight in kilograms, then the mass of the rotor in terrestrial units is $\frac{G}{9.8}$, the acceleration of gravity being 9.8 m. per second per second. Let n be the number of revolutions per minute. Then the centrifugal force is

$$4 \cdot \frac{G}{9.8} \cdot \frac{x}{100} \cdot \pi^2 \cdot \left(\frac{n}{60}\right)^2 = x \cdot G \cdot \left(\frac{n}{300}\right)^2$$

The unbalanced magnetic pull for a displacement equal to the gravity deflection f is $q_r \cdot G$ and for a displacement x is

$$\frac{x}{f} \cdot q_r \cdot G$$

Assuming that the stationary machine parts are infinitely stiff (so that $q_r = q$), the same formulas hold good for the rotating machine.

The sum of centrifugal force and unbalanced pull is

$$x \cdot G \cdot \left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q \cdot G$$

The elastic gravity deflection, caused by the weight G is f, the elastic deflection caused by one kilogram is f/G. To obtain the elastic deflection caused by the above sum of forces, we have to multiply by f/G.

$$\frac{f}{G} \cdot \left[x\,G\left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q\,G\right] = x \cdot \left[f \cdot \left(\frac{n}{300}\right)^2 + q\right]$$

For the critical speed this deflection will be equal to the original displacement x, the expression in brackets therefore will be unity for the critical value of n

$$f \cdot \left(\frac{n_{crit.}}{300}\right)^2 + q = 1$$

$$n_{crit.} = \sqrt{1 - q} \cdot \frac{300}{\sqrt{f}} \tag{11}$$

For the unexcited machine, q is zero and the critical speed is $\frac{300}{\sqrt{f}}$, (f given in cm.), which formula, of course, is well known. The critical speed is reduced, if the machine becomes excited and will be the lowest for that excitation which gives the highest value of the unbalanced magnetic pull. If q is equal to 1/6, the critical speed will be

$$\sqrt{\frac{5}{6}} \cdot \frac{300}{\sqrt{f}} = 0.912 \cdot \frac{300}{\sqrt{f}}$$

If q is equal 1/3, the critical speed will be

$$\sqrt{\frac{2}{3}} \cdot \frac{300}{\sqrt{f}} = 0.817 \cdot \frac{300}{\sqrt{f}}$$

An electric machine without equalizing connections has not *one* definite critical speed, but *a range* of critical speeds according to its excitation. In two-pole turbo-generators it must also be considered that the unbalanced pull in the diameter of the field axis is different from that at right angles to the field axis, and therefore such a machine will even for a definite excitation have different critical speeds in respect to these two diameters, similar to a rotating shaft with rectangular section.

To illustrate the numerical influence of the unbalanced pull on the critical speed, we may take a rotor of 60-cm. diameter and 100-cm. rough length of the iron core. After deduction of air-ducts and slot openings, 80 per cent of the cylindrical surface, that is $0.8 \cdot \pi \cdot 60 \cdot 100 = 15{,}100$ sq. cm. may be considered as effective. We will assume that the rotor belongs to a two-pole machine with distributed field winding, running at 3000 rev. per min., that the actual radial air gap is 1.5 cm., the virtual air gap 1.6 cm., and will consider as alternatives first, a mechanical critical speed of 3600, then of 1800. The weight of the rotor may be 3000 kg. We will further assume that the critical induction for which the unbalanced local pull is a maximum, is 6000 lines per sq. cm.

As the mechanical critical speed is given, we can figure directly the gravity deflection of the rotor shaft

$$f = \left(\frac{300}{n_{crit.}}\right)^2 = \left(\frac{300}{3600}\right)^2 = 0.0070 \text{ cm. and } \left(\frac{300}{1800}\right)^2$$
$$= 0.0278 \text{ cm. respectively}$$

We can figure $q = q_r$ directly from formula (**10a**). The pole factor c we will take as $^2/_3$.

$$q = q_r = \frac{f}{g_1} \cdot \frac{c\,\pi\,D\,L \cdot \left(\frac{B_m}{5000}\right)^2}{G} = \frac{f}{1.6} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1.44}{3000}$$

$$= 3.02 \cdot f$$

that is, 0.021 and 0.083 respectively.

$\sqrt{1-q}$ is $\sqrt{0.979} = 0.989$ and $\sqrt{0.917} = 0.956$ respectively. The influence of the magnetic pull on the critical speed is here small, it lowers the cricical speed only 1.1 per cent in one case, 4.4 per cent in the second case.

If the rotor belonged to a four-pole machine with half the virtual air gap, and a mechanical critical speed of 1800, q would be 0.166,

$$\sqrt{1-q} = \sqrt{0.834} = 0.913,$$

which would reduce the critical speed to 0.913 . 1800 = 1640 rev. per min. a value which is already uncomfortably near the running speed of 1500.

In the case of an induction motor with wound rotor and with the same core dimensions, the same rotor weight, a virtual airgap of 0.3 cm. (mechanical 0.2 cm.), and a critical induction of 5000 lines per sq. cm. and a critical speed of 3600 and alternatively 1800, q would be

$$q = \frac{f}{0.3} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1}{3000} = 11.2 \,.\, f$$ that is 0.078 and 0.31 respectively. In the latter case, the critical speed would be reduced by the magnetic pull to $\sqrt{0.69} \cdot 1800 = 1490$ rev. per min. This could not be accepted for a machine running at 1500 rev. per min. and it would be necessary to stiffen the shaft.

This example has been simplified by assuming the same rotor weight, whether the rotor is destined for 1500 or 3000 rev. per min., for a two- or four-pole machine, generator or motor. In reality, for an exact calculation, differences in weight would have to be considered, also certain minor differences in effective surface, critical induction, pole factor etc., etc.

Conclusions

The main conclusions arrived at in this paper are as follows:

1. The mechanical parts of a multipolar machine, whether its winding is provided with equalizing connections or not, should

be designed strong enough to withstand, as an emergency condition, without overstraining, the magnetic pull produced, if only one-half of the machine is magnetized up to full saturation of the iron, while the other half is unexcited. In machines with small air gap the rotor core may pull hard over towards the stator core, under these conditions.

2. Under normal working conditions, a reduction x of one air gap and an increase x of the diametrically opposite air gap causes in multipolar machines locally an unbalanced magnetic pull per square centimeter, given by the formula

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x \cdot d\,B}{d\,H} \text{ kilograms} \tag{3}$$

This pull has a maximum value for a comparatively *low* state of saturation. For higher saturation the unbalanced pull falls off. If the magnetic characteristic of the machine is represented by an initial straight part of appreciable length, followed by a distinct "knee", the "critical induction" is in the very beginning of the knee. For an approximate determination of the maximum pull take the induction B_m of that point of the magnetization curve in which the gradient is 5/6 of that of the straight line part and consider B_m as the end point of the straight line characteristic. The maximum unbalanced pull in kilograms per square centimeter is

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

g_1 being the "virtual" air gap of the machine.

(Virtual air gap, an air gap which would take for an induction within the straight line characteristic as many ampere turns as the real air gap and iron path of half the magnetic circuit together.)

3. In multipolar machines, the total unbalanced pull, for a given displacement x of rotor and stator centers, is independent of the rotor position. Its maximum value in kilograms is

$$F_x = c \cdot \pi\, D\, L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \tag{6a}$$

c, the "pole factor", being approximately $^2/_3$, both for machines with salient poles and for those with cylindrical fields and distributed windings. In machines with salient poles and constant air gap the maximum unbalanced pull is reached at a low saturation and falls off distinctly at higher saturations.

In machines with distributed field winding and also in salient-pole machines with graded air gap, the maximum pull is reached for higher saturations and changes less with changes in saturation.

In machines with cylindrical fields which are not slotted evenly around the periphery, and machines with salient poles and graded air gap, the saturation curve from which B_m and g_1 are taken, is a mean of the component saturation curves.

4. In bipolar machines, the unbalanced pull has different values for displacement in the direction of the field axis and at right angles thereto. The highest value for a cylindrical field with distributed winding is reached for high peak saturation, and can be calculated from formula (**6a**) with a pole factor of approximately $^2/_3$. Only the saturation curve of the *slotted* part should be taken to determine B_m and g_1, in bipolar machines which contain a non-slotted middle part of each pole. For salient-pole machines with 120 deg. pole arc, the pole factor is about 0.4. The saturation curve, from which B_m and g_1 are taken, should exclude in bipolar machines the m.m.f. required for the bodies of stator and rotor cores.

5. Assuming that under the influence of unbalanced magnetic pull the air gap experiences equal changes through the deflection of the rotor shaft and through the combined deflection of all other machine parts, the magnetic pull caused by a displacement equal to the "gravity deflection" of the shaft must not exceed 1/6 of the rotor weight, to ensure that the displacement, at the moment of sudden excitation, does not exceed twice the value of the original mechanical displacement.

If the other parts of the machine were infinitely stiff, the magnetic pull caused by a displacement equal to the gravity deflection could be allowed to approach ⅓ of the rotor weight.

("Gravity deflection" is the static deflection of the *horizontally situated* shaft under the influence of the rotor weight.)

6. The critical speed is lowered by the unbalanced magnetic pull and varies with changes in excitation. For the greatest permissible value of unbalanced pull, mentioned in the preceding rule, the reduction of critical speed amounts to 18 per cent. In machines, the winding of which has full equalizing qualities, no reduction of the critical speed takes place.

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ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

BY FRANK G. BAUM

ABSTRACT OF PAPER

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

IT IS, of course, well known that steam power is usually less expensive for low-load factors than hydroelectric power, and the latter becomes economical only when the load factor is favorable. To determine the economical division between the two there are usually given curves of cost varying with load factor. Such curves show that at certain load factors the cost of steam power exceeds the cost of water power, but the actual yearly cost of power for any assumed proportion between water power and steam power must be calculated for each case. This becomes laborious.

The results can, however, be shown in a much more illuminating way if presented as shown in Fig. 1. In this figure, abscissas from left to right (from O to O^1) show percentage of total load carried by water power and from right to left (from O^1 to O) the abscissas show the percentage of total load carried by steam power. The sum of the steam power and water power must of course equal 100 per cent for every condition, hence the sum of the two abscissas is 100 per cent at any point.

If now we take the yearly cost per kilowatt of hydroelectric power as $O^1 h$, taken in the figure at \$22 per kilowatt-year, and draw the straight line $O h$, this line will represent by any ordinate the yearly charge per kilowatt against the water power for any proportion between steam and water power. For it is clear that if we have one-half water power then the yearly charge per kilowatt against the entire 100 per cent load is \$11.

(In comparing the cost of power we must of course include total cost of delivery to center of load.)

Similarly, if we take the yearly fixed cost per kilowatt of steam power as $O\ s$, taken in the figure as $11 per kw-year, and draw the line $O^1\ s$, this will represent by any ordinate the yearly charge per kilowatt against the steam power for any proportion between steam and water power.

The straight line $s\ h$ then represents the total fixed charge against the steam and hydroelectric power for any proportion of steam and water power. ($O\ h$ and $O^1\ s$ being straight lines and $s\ h$ being derived by adding the ordinates, gives another sraight line.)

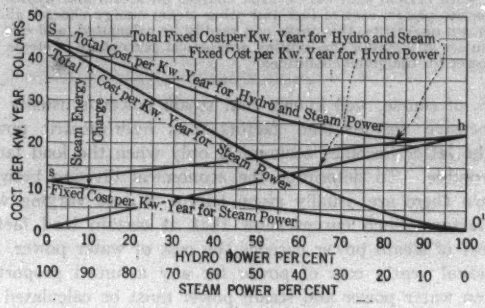

FIG. 1—PROPORTION OF WATER TO STEAM POWER

Water power cost per kw-yr	$22.00
Steam, fixed cost per kw-yr	11.00
Steam energy charge per kw-yr, and 100% load factor	44.00

For example, let total load equal 100,000 kilowatt, divided 70 per cent hydroelectric and 30 per cent steam; then the yearly charge against the water power and steam will be

Water power, fixed charge	70,000 kw. × $22	$1,540,000
Steam power, fixed charge	30,000 kw. × $11	330,000
Total power, fixed charge		$1,870,000

or $18.70 per kw-year, as shown by the ordinate of the line $s\ h$ at 70 per cent water power, 30 per cent steam power. All water power fixed charge would cost $2,200,000 and all steam power fixed charge $1,100,000 per year. All steam power costs $4,400,000.

For any other assumption of cost per year of water power or steam power, it is only necessary to determine the yearly fixed charge against steam power ($O\ s$) and water power ($O\ h$) and draw the line $s\ h$, and we have immediately the total fixed charge for any proportion of water power and steam power. This very much simplifies the problem and visualizes the results.

To determine the total charge per kw-year against the combined steam and water power, it is necessary to add the kw-hr.

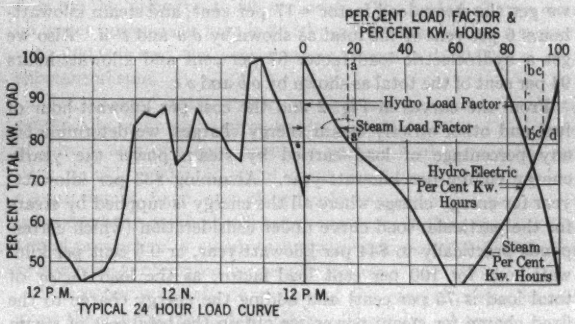

FIG. 2—LOAD FACTORS AND KW-HR. FOR STEAM AND WATER

—EXAMPLE—

For 20% peakload carried by steam and 80% by water the curves show

Steam load factor	= 17%	= $o\ a$
Water load factor	= 87%	= $o\ b$
Water kw-hr.	= 94%	= $o\ c$
Steam kw-hr.	= 6%	= $c\ d$

NOTE—Steam carries everything above 80% line in example, and the curves will show per cent load factor and per cent kw-hours for any other per cent load taken.

charge against the steam power. (It is of course assumed that all charges against the water power are fixed charges.)

To determine the yearly charge against steam power for any load factor we must start with the load curve of the power system. The load curve assumed is that shown on the left of Fig. 2. The ordinates, it will be noted, are plotted in percentage of the load, the peak load being 100 per cent.

Now we must determine the kw-hours carried by steam for any percentage of the total load carried by steam, it being assumed of course that the steam power takes the load off the

top of the curve. For this purpose we determine the curve of kilowatt-hours to be carried by steam power when 10 per cent, 20 per cent, etc. of the top of the load curve is carried by steam. To do this we take the area of the load curve above 90 per cent for example, and determine what percentage this is of the entire area of the load curve. Similarly for areas above 80 per cent, 70 per cent, etc. The results are shown in the curves to the right of the load curve.

For example, if all load over 80 per cent is carried on steam, we get the steam load factor = 17 per cent, and steam kilowatt-hours 6 per cent of the total as shown by $o\ a$ and $c^1\ d$. Also we get a hydroelectric load factor 87 per cent and kilowatt-hours 94 per cent of the total as shown by $o\ b$ and $o\ c$.

From the curves in Fig. 2 and the cost per kilowatt-hour of fuel and other strictly steam energy charges, we determine for any percentage of load carried by steam power the yearly energy charge per kilowatt-year. Assuming $33 per kilowatt-year for energy charge where all the energy is supplied by steam for the particular load curve under consideration (which corresponds practically to $44 per kilowatt-year, or 0.5 cent per kilowatt-hour for 100 per cent load factor, as the load factor of total load is 75 per cent) and adding the energy charge to the fixed charge for steam power, we obtain the total cost of steam power as shown by the curve "cost steam power" $O^1\ S$ in Fig. 1.

To obtain now the total cost of all power for any proportion of steam power to water power, we add the ordinates of $o\ h$ the "fixed cost of hydroelectric power" to the ordinates $o^1\ S$ the "Total cost of steam power" and obtain the curve $h\ S$ the "total cost per kilowatt-year hydroelectric and steam power." This curve starts at $22 per kilowatt-year if all power is water power. By adding some steam power to take off the peaks, we see there is a slight decrease in the yearly charge until the steam power carries about 15 per cent of load. At 20 per cent of load the cost again comes to about $22 per year, and then a gradual increase in power cost results.

At 30 per cent load carried by steam for this particular load curve there is little difference in the yearly charge per kilowatt and we would not for this case be warranted in installing less than 30 per cent steam; for naturally, unless very material savings result, the decision will always be to install steam power because of the smaller capital cost.

At 50 per cent of total load taken by steam installation and 50 per cent by water power, we have the yearly charge as follows:

50 per cent × \$22 = \$11.00..yearly fixed charge water power
50 per cent × \$11 = 5.50..yearly fixed charge steam power
Total..... \$16.50..yearly fixed charge total power

At 50 per cent load factor practically 33⅓ per cent of the kilowatt-hours are carried by steam and this adds the energy charge of \$33 × 33⅓ per cent or \$11. Therefore the total yearly charge is \$16.50 + \$11 = \$27.50, as shown by the curve *h S* by the ordinate at 50 per cent. The added charge over all water power costs here is \$5.50 per kw. year, but on a system with 100,000 kw., the yearly excess charge is \$550,000, a very substantial sum.

The curve of total yearly power costs per kilowatt shows graphically what we want to know, and after we have the curve of energy cost of steam power at various load factors, we can very quickly make up total costs per kilowatt-year for any assumption of fixed cost of hydroelectric and steam power. It is believed this method will assist engineers in their work. The actual proportion of steam to hydroelectric power will of course be somewhat influenced by service insurance conditions.

This paper is merely an outline of the general principles and gives the general method to follow. There are, of course, many details and different conditions in different sections of the country. It is hoped that the method presented will appeal to engineers and managers.

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APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES

BY WALDO V. LYON

ABSTRACT OF PAPER

It is shown that the flux distribution in the air gap of a synchronous machine consists of a series of component distributions that are simple harmonic wave trains, either stationary or moving at constant velocities. Methods are suggested for determining the effects of slots and the saturation of the magnetic circuit on the magnitudes of these component distributions. Expressions for the voltage generated and the power developed thereby are given. The theory is applied to the operation of a three-phase synchronous machine under different conditions of load, both qualitatively and quantitatively. A table of comparative calculated field currents is appended.

A COMPLETE theory of the performance of alternating-current machinery must recognize the presence of harmonics in the current and voltage and in the distribution of the air gap flux. The principle of the following analysis is well known but the author believes it has never been so fully developed. It seems best to show how the method may be applied in a variety of cases rather than to carry any one case into all of its manifold refinements. Much of the analysis is qualitative rather than quantitative. It is useful to know, however, what harmonics may be present even when their magnitudes cannot be calculated. Where it is possible to determine certain coefficients with sufficient accuracy the method should prove of considerable practical importance. The fundamental principle upon which the analysis is based is that of the superposition of magnetic fields. For example, if a constant magnetizing force produces at any point a certain magnetic flux density when it alone is acting, and another constant magnetizing force produces at the same point another flux density when it alone is acting, then when both magnetizing forces act together the resultant flux density at the point in question is the vector sum of these two flux densities, provided the permeability at every point in the region concerned is the same in each of the three cases. Thus the component flux densities should be calculated for a condition of the magnetic region which is assumed to be identical with that existing when the components act simultaneously. When the magnetic forces

are variable the theorem does not hold if eddy currents are produced. If the resultant magnetizing force is constant, however, the eddy currents that may be produced by the component magnetizing forces should be neglected. With the laminated magnetic masses that are usually employed when the resultant flux density is variable, the effect of the eddy currents can often be neglected without undue error. Such is the case at 25 or 60 cycles with laminations no thicker than are ordinarily used in transformers. At high frequencies or with laminations of two or three times this thickness, however, the effect of the eddy currents may be comparatively great. This would occur with the practically solid rotors of large turbo-alternators.

The procedure followed in this analysis is to decompose the actual flux distribution in the air gap into a series of simple sinusoidal distributions and then to determine how each of these is effective in the generation of voltage and in the development

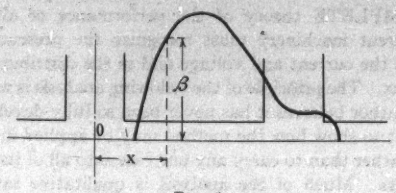

FIG. 1

of power. At any instant the flux distribution, or flux density, in the air gap of an alternating current machine may be represented by a curve such as shown in Fig. 1. The abscissas, x, of this curve are measured in electrical radians from some point fixed of reference, O, in the airgap. In the case of a synchronous machine it is convenient to choose this point as half way between the centers of adjacent field poles. It is also simpler to assume that this point of reference is stationary and that the armature is rotating. This will be done throughout the analysis. The ordinates represent the normal flux density in the air gap at the surface of the armature—or, at the surface of the field poles if this is desired. A distribution of this sort may be of constant magnitude and wave shape, or both may vary periodically, and in either case it may be fixed or moving at a constant or a variable angular velocity. In any case the most complex distribution can be decomposed into a series of simple sinusoidal dis-

tributions of different wave lengths and amplitudes all of which move at a constant but not the same velocity. In fact, some move in one direction through the air gap and some in the opposite direction.

As an illustration of this process of decomposition, consider the case of a salient-pole alternator in which but one armature coil per pole is carrying current. At any instant the flux distribution per ampere (see Fig. 1 for example) can always be represented by a Fourier series consisting of both sine and cosine terms. As the coil moves through the air gap the shape of this distribution will vary periodically due to the non-uniformity in the reluctance of the magnetic circuit. If, therefore, the coefficient of each term of the distribution series is plotted against time its value may be represented by another Fourier series. That is, the air gap flux distribution per ampere may be represented by a series of products of sine and cosine terms. Each of these products may be broken up into a sum or difference of sine or cosine terms of constant amplitude, wave length and angular velocity. If instead of there being one ampere in the coil the current in it should vary periodically, each of these sine or cosine terms must be multiplied by still another Fourier series, representing the current. The resulting products of sine and cosine terms would again be broken up into a series of sine and cosine terms which represent simple harmonic wave trains that are either stationary or moving at constant angular velocities.

General Expressions for Voltage and Power

Let the equation for one of these component wave trains be

$$\beta = B_q \cos (q\,x - j\,\omega\,t + \alpha_{qj}) \tag{1}$$

This represents a sinusoidal distribution of maximum value B_q and of wave length $2\,\pi/q$, such as might be produced by $q \cdot P$* poles moving through the air gap at an electrical angular velocity of $j\,\omega/q$ in the same direction as does the armature. See Fig. 2. If the sign before $j\,\omega\,t$ is positive the direction of rotation is opposite to that of the armature. If the coefficient j is zero the distribution is stationary with respect to the field poles. The angle α_{qj} determines the phase of the distribution.

When $\omega\,t$ equals zero or any multiple of π the value of the flux

*P represents the actual number of field poles. q is throughout the harmonic order of the flux distribution and is usually 1, 3, 5, 7, etc. j is usually 0, 2, 4, 6, etc.

density at the reference point, O, due to this distribution alone is $\pm B_q \cos \alpha_{qj}$.

The frequency of the voltage generated in the field winding by this distribution is proportional to the equivalent number of poles, $q \cdot P$, and the relative velocity, $j\,\omega/q$. The frequency is jf, where f is the fundamental frequency. If the field circuit is closed a current of this frequency will be produced in it and a torque will be developed tending to move the field in the direction in which the distribution is moving. This alternating current in the field winding will also produce a harmonic variation in the field flux which will in turn cause a higher and a lower harmonic in the armature electromotive force.

The rotating flux distribution will also directly produce an electromotive force in the armature winding, the frequency of which is proportional to the equivalent number of poles, $q \cdot P$,

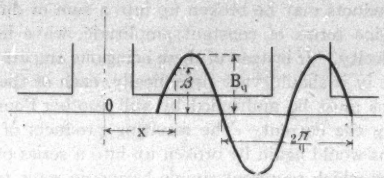

FIG. 2

and the relative angular velocity, $(\omega - j\,\omega/q)$. The frequency is $(q - j)\,f$. If there exists a current of this frequency in the armature winding, power will be developed if the phase relation of the current and the generated voltage is favorable. The equation for the voltage produced in the armature winding by this flux distribution of the qth order is

$$e_{(q-j)} = \sqrt{2}\,E_{q-j}\,[\cos\,(q - j)\;\omega\,t + \alpha_{qj}]^* \qquad (2)$$

The average power developed in a single-phase winding by

*The effective value of this voltage is

$$E_{q-j} = \frac{K_q}{K_1}\;\frac{B_q}{B_1}\;\frac{q-j}{q}\;E_1$$

where E_1 is the effective voltage due to the fundamental flux distribution, whose maximum value is B_1, and K_q and K_1 are the voltage reduction factors for the qth harmonic and the fundamental flux.

this electromotive force and a harmonic current of the s order whose equation is $i_s = \sqrt{2}\, I_s \sin(s\,\omega\, t + \theta_s)$† is

$$P = \pm E_{q-j} \cdot I_s \sin(\alpha_{qj} \pm \theta_s)‡ \tag{3}$$

The average power is zero, however, unless $(q - j \pm s)$ is zero, that is to say, unless the voltage and current have the same frequency. When the average power is positive electrical power is developed, but when it is negative mechanical power is developed.

In a balanced three-phase alternator, power is developed only when $q \pm s = j = 0, 6, 12$ etc. if there are no even harmonics present or 0, 3, 6 etc. if there even harmonics in the current. The power is

$$P = \pm 3\, E_{q-j} \cdot I_s \sin(\alpha_{qj} \pm \theta_s) \tag{4}$$

Electrical power is developed when $(\theta_s - \alpha_{qj})$ is an angle of lead, and mechanical power is developed when it is an angle of lag. For this reason it may happen that though power is developed in each of the phases of a three-phase alternator, the net power developed is zero. In this case at least one of the phases would develop electrical power while at least one of the other two would develop mechanical power. That is, the windings of a three-phase alternator may act as a self-contained motor-generator set.

Reduction Factors for Flux. Most frequently the flux in the air gap is due to the action of several magnetizing coils per pole. The resultant flux distribution is the sum of the distributions due to the individual coils acting separately. If these component distributions are decomposed into their elementary sinusoidal parts, as can always be done, those of the same wave length, which are stationary or which travel in the same direction at the same velocity can be combined according to the principles of vector addition. This process of addition is simplified if the components are of the same amplitude and are equally displaced from one another in space-phase position. In this case the method of addition is the same as that for a number of equal voltages of the same frequency differing from each other in time phase by the same amount. With n such distributions of wave length $2\,\pi/q$ differing from each other in position by an angle γ on the fundamental scale, in which the pole pitch equals

†When the conductors of a phase belt are at the zero point midway between the poles, the s harmonic component of the current in that belt has a value $\sqrt{2}\, I_s \sin\ \theta_s$

‡In this equation the signs are either both plus or both minus.

π radians, the sum is what it would be if they were coincident multiplied by

$$K_b = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}}$$

This is the belt reduction factor.

If it is further desired to add two such distributions which are equal in magnitude but differ in phase position by an angle λ measured on the fundamental scale, their sum is what it would be if they were coincident multiplied by $K_p = \cos \frac{q\,\lambda}{2}$. The angle λ is some multiple of γ. This is the pitch reduction factor.

The total reduction factor for the qth harmonic flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2}$$

This holds true only when the flux per ampere is independent of the position of the coil, that is, when the air gap is uniform.

Reduction Factors for Voltage. Having calculated the harmonic components of the resultant flux distribution in the air gap, the electromotive force generated in either the stator or rotor winding that links this flux can be found. The belt reduction factor for voltage depends upon the space displacement of the coils and upon the order of the harmonic in the flux distribution. It does not depend upon the velocity of the distribution. The frequency of the voltage produced in the winding, however, depends upon the order of the harmonic in the distribution and upon its velocity with respect to the winding. Thus neither the belt nor the pitch reduction factor for voltage depends upon the frequency of the voltage. If there are n equal coils per pole displaced from one another by an angle γ and having a pitch deficiency of λ, the total reduction factor for voltage for the qth harmonic in the flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cos \frac{q\,\lambda}{2}$$

Both γ and λ are measured on fundamental scale.

Flux vs. Voltage Reduction Factor. The angles γ and λ in the voltage reduction factors are always equal to the angles between the coils in which the voltage is produced. It is only with a uniform air gap, however, that the angles γ and λ in the flux reduction factor are equal to the angles between the coils which are producing the flux. If a winding is designed to eliminate a given harmonic from the flux distribution that it produces, a harmonic flux distribution of the same order will generate no voltage in the winding no matter what its relative velocity may be. There may be produced, however, an electromotive force of this order of harmonic frequency by another component in the distribution which is moving at the proper speed with respect to the winding.

Distributing a winding is thus no sure means of reducing or of eliminating a given harmonic in the voltage. For example, a fifth harmonic in the flux distribution moving at fundamental velocity will produce a fifth harmonic in the voltage, as will also a third harmonic in the flux distribution moving at five-thirds of the fundamental velocity. A winding designed to eliminate the harmonic voltage due to the first of these distributions will fail to eliminate that due to the second. All that can be accomplished by distributing the winding is in the reduction or elimination of the harmonics caused by given harmonics in the flux distribution. Another method of reducing the harmonics in the voltage would be to reduce the troublesome harmonics in the flux distribution. If the harmonic flux is in motion with respect to the field, this can be accomplished by means of a short-circuited winding set in the field poles. A winding of this sort is most effective when it is of low resistance and is brought into close proximity to the field winding it protects. The total variable flux that links this amortisseur or damper is just sufficient to account for the resistance drop in it.

In some cases, particularly when there is not an integral number of slots per pole, the reduction factors for flux and voltage must be calculated for each individual case, as no general formula is applicable.

Control of Flux Distribution. The distribution of flux in the air gap of an alternator may be controlled to a certain extent in either of two ways,—by shaping the pole surface if salient poles are used, or by properly distributing the magnetizing winding if the air gap is uniform. The first method will not be discussed.

The first and simplest case assumes that the air gap is uniform,

as it approximately would be in a turbo-alternator. The direct effect of the slots in both the field and armature in producing harmonics is neglected. This effect can be largely eliminated by closing the slots with magnetic wedges and by choosing the proper relative numbers of stator and rotor slots. As is customary the reluctance of the field and armature cores is neglected. An approximate method that corrects for this omission is given later.

A single field coil with a pitch equal to the pole pitch will then produce a rectangular flux distribution in the air gap which may be represented by the Fourier series

$$\beta = \frac{4}{\pi} B_0 \sum \frac{1}{q} \sin q\, x \tag{5}$$

q is the order of the harmonic in the distribution. β is the flux density in the gap at an angular distance x radians from one side

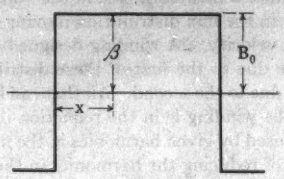

FIG. 3

of the coil, and B_0 is the average flux density in the gap. See Fig. 3. The relative magnitudes of the harmonics are: 1.00: 0.33 : 0.20 : 0.14., etc.

If the field winding is distributed in more than one slot per pole the relative magnitudes of the harmonics are reduced. The amount of this reduction depends upon the method of distributing the field winding. With n coils per pair of poles, each of unity pitch* and having the same number of turns, and equally displaced by an angle γ, the flux density at an angular distance x from the middle of a belt is

$$\beta = \frac{4\, B_0}{\pi} \sum \frac{\sin q \frac{n}{2} \gamma}{n \sin q \frac{\gamma}{2}} \frac{1}{q} \sin q\, x \tag{6}$$

*As a matter of fact the field coils are arranged concentrically but the magnetic effect is the same as if the end connections were made to give unity pitch. See Fig. 4.

If the winding were concentrated the average flux density would be B_0. The relative magnitudes of the harmonics depend upon the number of magnetizing coils, n, and the angle, γ, between them. If, for example, there are six coils per pair of poles displaced by 20 degrees, the relative magnitudes of the harmonics are 0.83 : 0.0 : 0.038 : 0.022. The third harmonic in the distribution vanishes and the phase of the fifth harmonic is reversed. The qth harmonic or any odd multiple of it will vanish if $\frac{q\,n\,\gamma}{2} = \pi$ or any multiple of π. Thus, by properly choosing the breadth of the winding, any harmonic in the flux distribution may be theoretically eliminated, although practical winding conditions will limit this possibility. Since the harmonics are much less prominent than with a concentrated winding the distribution is more nearly sinusoidal. The fundamental flux is now 83 per cent of what it would be if the winding were concentrated. This might be called the efficiency of the winding.

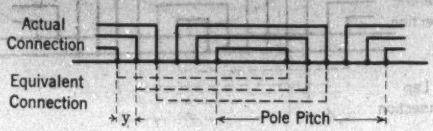

FIG. 4

If the field winding is equivalent to two similar distributed windings displaced by an angle λ as shown in Fig. 5, the relative magnitudes of the harmonics are still further reduced. By properly choosing the pitch deficiency angle, λ, any harmonic in the distribution may be theoretically eliminated. The angle, λ, is always some multiple of the angle, γ, between the slots. The distribution is represented by

$$\beta = \frac{4}{\pi} B_0 \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2} \cdot \frac{1}{q} \sin q\,x \qquad (7)$$

The average flux density for a concentrated winding having the same number of turns is B_0. Fig. 5 shows two windings, each like that shown in Fig. 4, which are displaced by two slots or 40 degrees. The relative magnitudes of the harmonics in this case are:

0.78 : 0.0 : 0.0066 : 0.017

The effect of the pitch factor is to reverse both the fifth and seventh harmonics so that the fifth now appears in phase with, and the seventh in opposition to the fundamental. The harmonics are very small. The efficiency of this winding is 78 per cent.

By properly choosing the breadth of one belt or layer, $n\,\gamma$, and the pitch deficiency, λ, any two harmonics may be eliminated theoretically. There is a practical limitation to this, however, fixed by the minimum allowable angle between the slots. If the angle between the slots is $\pi/15$, and the breadth of one layer of the winding is 10 slots and the pitch deficiency is three slots, the third harmonic will be eliminated by the distribution, and the fifth harmonic by the pitch, of the winding. In this case the seventh harmonic is 1.4 per cent of the fundamental. The efficiency of this winding is 0.79.

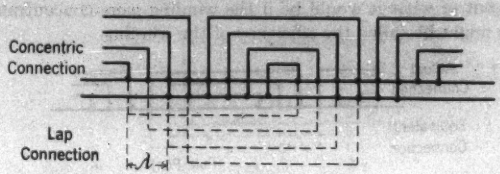

FIG. 5

Correction for Saturation. Thus far the reluctance of the field and armature cores has been neglected. The following method of correction is an approximate one designed to compensate for this omission. Both the field and armature are assumed to be uniformly slotted. The flux distribution in the air gap due to a single magnetizing coil instead of being rectangular, is less at the center than at the conductors, as shown in Fig. 6* The simplest equation that can be given such a curve between $x = 0$ and $x = \pi$ is

$$\beta = B_0 \left(1 + \frac{b_1}{B_0} \cos 2\,x\right)$$

*Because some parts of the magnetic circuit are more highly saturated than others the component distributions due to the individual coils will not be alike. This effect, however, is disregarded, although it may be of prime importance in determining the magnitudes of the harmonics. Other assumptions as to the shapes of the component distributions might be made which would increase the precision of the results.

B_0 is the average flux density in the gap and $(B_0 + b_1)$ and $(B_0 - b_1)$ are the maximum and minimum flux densities respectively. The ratio of b_1 to B_0 is approximately equal to the ratio of the ampere turns required for the field and armature cores to those required for the air gap and the teeth.

The Fourier series that represents this distribution is

$$\beta = \frac{4\,B_0}{\pi} \sum \left(\frac{1}{q} + \frac{b_1}{B_0} \frac{q}{q^2 - 4} \right) \sin q\,x \qquad (8)$$

If the ratio $\frac{b_1}{B_0}$ is 10 per cent, the relative magnitudes of the harmonics produced by a single magnetizing coil are

$$1.0 : 0.406 : 0.232 : 0.163$$

instead of

$$1.0 : 0.333 : 0.200 : 0.143$$

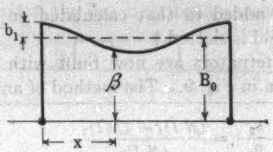

FIG. 6

as they are on the assumption that the reluctance of the field and armature cores is negligible.

The corrected series for a distributed fractional pitch winding is

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2} \left(\frac{1}{q} + \frac{b_1}{B} \frac{q}{q^2 - 4} \right) \sin q\,x \qquad (9)$$

At higher flux densities the ratio $\frac{b_1}{B_0}$ is greater and the harmonics become more prominent. The general effect of this is to make the flux distribution in the air gap flatter than it is at lower saturations.

Correction for Unslotted Portion of Field Surface. Usually the field winding is distributed over only a portion of the polar pitch as shown in Fig. 7. In this case the flux density between the points a and b is greater than it would be if the field were slotted in this region. The full ampere turns of one pole are acting to produce the flux density here and the increase in the density between a and b over what it would be provided the entire field were slotted may be represented roughly by the curve shown in Fig. 8. The length of the unslotted portion is η. The Fourier series that represents this curve is:

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{b_2}{B_0} \sin \frac{q\,\pi}{2} \cdot \sin \frac{q\,\eta}{2} \cdot \frac{1}{q} \sin q\,x\,^* \qquad (10)$$

B_0 is the average flux density with a uniformly slotted field produced by a concentrated field winding having the given number of turns per pole. This series represents the flux density which should be added to that calculated on the assumption that the entire field is slotted.†

Large turbo-alternators are now built with a field winding arranged as shown in Fig. 9. The method of analysis, including

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* $$\frac{b_2}{B_0} = \frac{(N\,I)_1 - (N\,I)_2}{(N\,I)_2}$$

where: $(NI)_1$ = ampere turns required to produce B_0 with a uniformly slotted field.

$(NI)_2$ = ampere turns required to produce B_0 with an unslotted field.

†The magnitude of the correction is shown by the following example in which $\frac{b_2}{B_0}$ is taken equal to 0.20 and η equal to $\frac{\pi}{3}$.

$$\beta = \frac{4\,B_0}{\pi}\,(0.10 \sin x - 0.067 \sin 3\,x + 0.020 \sin 5\,x + 0.014 \sin 7\,x + \ldots)$$

The corrected distribution for the winding shown in Fig. 4 if $\frac{b_1}{B_0} = 0.10$ and $\frac{b_2}{B_0} = 0.20$ is:

$$\beta = \frac{4\,B_0}{\pi}\,(0.91 \sin x - 0.067 \sin 3\,x - 0.023 \sin 5\,x + 0.039 \sin 7\,x + \ldots)$$

The third harmonic which had been cut out by the breadth factor is now 7.4 per cent. of the fundamental.

The corrected efficiency of the winding is 85 per cent.

The uncorrected efficiency as previously calculated was 83 per cent.

the corrections for the effects of saturation and of the unslotted portions of the field, is similar to that already outlined.

The magnitude of any harmonic in the flux distribution depends not only upon the pitch and breadth reduction factors but also upon the degree of saturation and the breadth of the unslotted portion of the field. Harmonics that have been

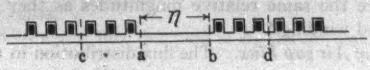

FIG. 7

entirely eliminated by the first two factors will still appear on account of the last. It is only with a uniform air gap that harmonics can be surely eliminated by the distribution of the field winding. Field slots that are closed by magnetic wedges tend to produce this condition. In general the effect of increased saturation is to flatten the flux distribution curve.

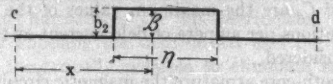

FIG. 8

Flux Distribution with Special Winding. It has been shown that if the equivalent field winding consists of equally spaced coils having the *same* number of turns, *any two* harmonics or their odd multiples can be eliminated from the flux distribution by properly choosing the breadth of the winding and the pitch of the coils. Thus if a winding is chosen that eliminates the third

FIG. 9

and fifth harmonics, it will also eliminate the ninth, fifteenth, twenty-first, twenty-fifth, etc. On the other hand, if the field winding consists of coils having *unequal* numbers of turns more can be accomplished in the elimination of the harmonics. Only the case in which the field is uniformly slotted will be considered. the second correction does not then have to be applied. The illustration here given is for a field winding consisting of four

coils per pole contained in slots displaced by 20 degrees as shown in Fig. 16. See Appendix I. The central slot is not occupied. If the relative numbers of turns in the coils 1 to four are respectively 1.0, 0.88, 0.65 and 0.35, all of the harmonics in the flux distribution will be eliminated up to and including the fifteenth. The first harmonics that do occur, viz. the seventeenth and nineteenth, have the same relative magnitudes as they do with a concentrated winding.*

Analysis of Air gap Flux. The flux distribution in the air gap of a synchronous machine can be modified to a considerable extent by shaping the pole surfaces, or, as has been shown with non-salient poles, by properly distributing the field winding. In either case the value of the flux density in the air gap may be represented by a Fourier series which may be written:

$$\beta = i_f (\Sigma A_q \sin q x + \Sigma C_q \cos q x) \qquad (11)$$

x is the electrical angle measured from a point midway between the poles, i_f is the value of the exciting current, and the coefficients, A_q and C_q are the maximum values of the various harmonic distributions per ampere of field current at the particular saturation considered.

With a smooth-core armature the magnetic circuit is constructed so that the flux distribution is at all times symmetrical about the center of the field poles. This disregards the effect of the cross magnetizing action of the armature currents which increases the saturation of one-half of the magnetic circuit above that of the other half. With a symmetrical flux distribution the C coefficients are all zero. When the armature is slotted, as it almost invariably is, the magnetic circuit is symmetrical only when a slot or a tooth is exactly at the center of a field pole. The C coefficients are no longer zero but vary periodically with the time.

There are three cases to consider. The first is for a smooth armature and a constant exciting current. The flux distribution is then independent of the position of the armature and i_f and the coefficients A_q are all constant. The second case is for a smooth core armature and a periodically varying field current. i_f is then a periodic function of the time but the coefficients A_q are still assumed to be constant. The third case is for a slotted armature core but with a constant field current. Here i_f is constant but the coefficients A_q and C_q are periodic functions of the time.

*See appendix I.

The armature winding in which the electromotive force is generated is distributed in n slots per pole per phase, displaced by an angle γ and with a pitch deficiency, λ.

I. *Constant Flux.* In the first case, which is the simplest, the electromotive force generated in any armature conductor has the same wave form as the flux distribution. This is not true of the other cases.

If the fundamental flux per pole is ϕ_1 the qth harmonic flux per pole is

$$\phi_q = \frac{1}{q} \frac{A_q}{A_1} \phi_1$$

The voltage produced in the armature winding due to this harmonic flux is

$$E_q = 4.44\, K_q\, N\, q\, f\, \phi_q\, 10^{-8}$$

N is the number of turns in one phase of the armature winding and f is the fundamental frequency. K_q is the total reduction factor for the qth harmonic in the flux distribution.

By properly choosing the distribution and the pitch of the winding it is theoretically possible to eliminate any two harmonics from the electromotive force. A three-phase winding arranged in 15 slots per pole, having a phase belt of 10 slots and a pitch deficiency of three slots will entirely eliminate the third and fifth harmonics. In a well distributed three-phase winding, a pitch of five-sixths reduces the fifth and seventh harmonics to about four or five per cent of their full value. Special windings can be designed to eliminate several harmonics. For example, a single-phase winding such as previously described in connection with the flux distribution will eliminate all of the harmonics up to and including the fifteenth.

Power will be developed in the armature only when there is a harmonic in the current of the qth order and then only if the time-phase displacement is not $\pi/2$.

Since the flux is of constant magnitude it produces no voltage in the field winding and therefore no reaction with it.

II *Variable Field Current.* In the second case it is simplest to assume, though incorrectly, that the distribution of the flux in the air gap has a fixed wave form. If the variations in the field current are not large the approximate effect of a periodic field current may be represented by

$$i_f = I_0 + k\, \Sigma\, I_s \sin\,(s\, \omega_1\, t - \theta_s)$$

The frequency of the fundamental variation in this current is $f_1 = \frac{\omega_1}{2\pi}$. It may or may not be equal to the frequency of the fundamental in the armature electromotive force. The constant term I_0 produces the harmonics that were calculated in the first case. Any harmonic in the field current produces a flux distribution in the air gap which is equivalent to two distributions of the same magnitude and wave length, which rotate in opposite directions with respect to the field poles. The frequencies of the voltages produced in the armature by these rotating fluxes are $(q f \pm s f_1)$. Thus in a 60-cycle alternator, a 25-cycle current in the field winding produces voltages having frequencies of 35 cycles and of 85 cycles, due to the variation in the fundamental component of the flux distribution.

If the flux distribution contains a third harmonic, the 25-cycle field current produces voltages in the armature winding with frequencies of 155 cycles and of 205 cycles. This method of resolving a pulsating field into oppositely rotating fields was used by Ferraris in analyzing the operation of the single-phase induction motor.

III *Effect of Slots.* In the third and last case to be considered the field excitation is constant but the *permeance* of the magnetic circuit varies periodically on account of the slots in the surfaces of the field and armature. The wave form of flux distribution in the air gap varies cyclically while the armature is advancing the distance of the armature tooth pitch.* The periodic variation in the shape of this flux distribution produces a corresponding variation in the coefficients of its harmonic components. Each of these coefficients may then be represented by a Fourier series.

If n represents the number of slots per pole the series which represent any of the coefficients such as A_q and C_q are

$$A_q = \Sigma_p a_q \cos 2\,n\,p\,\omega\,t$$

and

$$C_q = \Sigma_p c_q \sin 2\,n\,p\,\omega\,t$$

The first is a cosine and the second is a sine series for the same reasons as outlined in the analysis of the armature reaction.

*Time is measured from the moment that an armature slot is directly opposite the zero point, that is, midway between the field poles.

(See Appendix II). The general expression for the flux distribution in the air gap reduces to

$$\beta = \frac{I_0}{2} \{ \Sigma ({}_pa_q + {}_pc_q) \sin (q\,x + 2\,n\,p\,\omega\,t) + \Sigma ({}_pa_q - {}_pc_q) \sin (q\,x - 2\,n\,p\,\omega\,t) \}\dagger \qquad (12)$$

${}_pa_q$ is the coefficient of the pth harmonic term in the Fourier series which represents the variation in the coefficient of the q_{th} harmonic sine term, A_q, in the flux distribution, and ${}_pc_q$ is the coefficient of the pth harmonic term in the Fourier series which represents the variation in the coefficient of the qth harmonic cosine term, C_q, in the flux distribution.

The flux distribution in the air gap is thus resolved into two components one of which (for $p = 0$) is stationary with respect to the field poles and the other of which (for $p = 1, 2, 3$ etc.) is rotating. This second component can be further resolved into two wave trains which rotate in opposite directions through the air gap. The effect of the stationary component has already been analyzed. The harmonics existing in it may be due to the shape of the pole surface or, in the case of non-salient poles, to the distribution of the field winding or to the field slots. The rotating fluxes are due wholly to the presence of the armature slots although their magnitude is influenced by the shape of the field poles or the arrangement of the field winding. The fundamental slot harmonics, *i. e.*, $p = 1$, are probably the most important.

The periodic variations in the A coefficients indicate corresponding variations in the flux per pole, while the periodic variations in the C coefficients indicate corresponding distortions of the flux distributions. The latter are equivalent to an oscillation of the flux across the pole face.

The coefficients ${}_pa_q$ and ${}_pc_q$ can be determined by analyzing the flux distribution for successive positions of the armature with respect to the field poles. If the values of the coefficients of any particular harmonic are then plotted against time, this curve can be analyzed for the coefficients of its harmonic components, *i. e.*, ${}_0a_q$, ${}_1a_q$, ${}_2a_q$, etc.

The order of the harmonic in the generated voltage depends upon the order of the harmonic in the flux distribution which is affected by the slot pulsations, and the number of slots per pole. It is $(q \pm 2\,n\,p)$. The most important slot harmonics are due to the variation in the fundamental flux and are of the orders $(2\,n \pm 1)$.

---

†I_0 is the field current.

Distributing an armature winding has no effect in eliminating the harmonics caused by the pulsations due to the slots except in so far as it modifies the reduction factor for any particular harmonic in the flux distribution.

Harmonics due to the armature slots may be very prominent in the flux distribution and yet generate little or no voltage in the armature winding. This is because these harmonics move synchronously with the armature and will thus be effective in producing voltage only when their magnitudes vary so that they produce a net variation of flux through the armature coils.

In Fig. 10 assume that the flux distribution in the air gap is represented by the expression

$$\beta = B_0 \sin x \left[1 - a \cos 2n(x - \omega t)\right]$$

where n is the number of slots per pole. The value of a determines the magnitude of the slot harmonics. This flux distribution may be thought of as the result of a sinusoidal distribution of magnetomotive force acting on a magnetic circuit whose permeance varies with the time as shown by the bracketed term. The flux linking a coil of unity pitch is

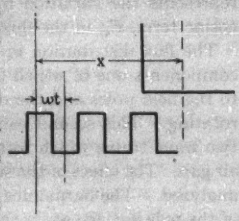

FIG. 10

$$\phi = 2B_0\left(1 + \frac{a}{4n^2 - 1}\right)\cos \omega t\,^*$$

Thus, in this case the flux through the coil varies sinusoidally and there are no higher harmonics in the voltage, although the armature slots are the cause of large harmonics—depending upon the value of a—in the air-gap flux distribution. These harmonics are of the orders $(2n + 1)$ and $(2n - 1)$, but their amplitudes vary with a frequency of $2n$ times the fundamental frequency. The net effect of this is to produce a fundamental voltage only. Further, the flux from one pole is

$$\phi_0 = 2B_0\left(1 + \frac{a}{4n^2 - 1}\cos 2n\omega t\right)^\dagger$$

*This assumes that the center line of the slot marks the division between the flux which enters the tooth at the right and the one at the left. This is of course not strictly true.

$$^\dagger\phi_0 = \int_0^\pi \beta\, dx \text{ (assumed)}$$

The flux is greatest when a slot is midway between the poles, and least when a tooth is midway between them. The slots may thus cause a periodic variation in the total flux per pole and yet there may be no corresponding harmonics in the generated voltage.

Armature Reaction. The flux produced by the armature current will be divided into two distinct parts; that which exists in the air gap and that which does not. The magnitude and distribution of the first varies with the excitation and power factor, but the second portion is very nearly independent of each of these factors, and for the sake of simplicity it is assumed to vary only with the armature current.

The effect of the armature current of an alternator in modifying the air gap flux generally influences the operation of the machine more than anything else. The method here used to analyze this effect is to first obtain an expression which shows how the flux density in the air gap, due to a single armature coil of unity pitch, varies when the coil is placed in *any* position relative to the field poles. The resultant effect due to a single-phase winding is the sum of the separate effects due to the individual coils of the winding. An extension of this method of combination will give the resultant reaction for a three-phase alternator.

In Fig. 1, β is the normal component of the flux density in the air gap due to a single coil in which there is a current of i amperes. The equation of this distribution may be written in the form of a Fourier series.

$$\beta = \frac{N_c}{N_f} i \left\{ \Sigma A_q \sin q (x - \omega t) + \Sigma C_q \cos q (x - \omega t) \right\} \qquad (13)$$

N_c and N_f are respectively the turns per armature coil and the turns per field pole. A_q and C_q are respectively the instantaneous maximum values of the qth harmonic sine and cosine flux distributions for an armature current equivalent to one ampere in the field circuit.

As the coil moves through the air gap, *i. e.*, as ωt varies, the coefficients A_q and C_q of the series will vary periodically at double frequency. If the magnetic circuit is assumed to be symmetrical about the center line of the poles the series for A_q will consist of cosine terms only and the series for C_q will consist of sine terms only. The proof of this is given in Appendix II.

The series for A_q and C_q may be written

$$A_q = {}_0a_q + \Sigma\, {}_pa_q \cos 2\, p\, \omega\, t$$
$$C_q = \Sigma\, {}_pc_q \sin 2\, p\, \omega\, t$$

where p may equal 1, 2, 3, 4, etc.

If these values are substituted in equation (**13**) and the products of sine and cosine terms are expanded, the result gives

$$\beta = \frac{N_c}{N_f}\, i \left\{ \sum {}_0a_q \sin (q\, x - q\, \omega\, t) + \right.$$
$$\frac{1}{2} \sum \sum ({}_pa_q + {}_pc_q) \sin (q\, x - (q - 2\, p)\, \omega\, t) +$$
$$\left. \frac{1}{2} \sum \sum ({}_pa_q - {}_pc_q) \sin\, (q\, x - (q + 2\, p)\, \omega\, t) \right\} \quad (\mathbf{14})$$

This represents the flux distribution in the air gap due to a single armature coil carrying a current of i amperes. The angular displacement of the coil from the center of the pole is $\omega\, t$. ${}_0a_q$ is the average value of the amplitude of the qth harmonic in the flux distribution. ${}_1a_q$ and ${}_1c_q$ are respectively the fundamental amplitudes of the variation in the maximum values of the qth harmonic sine and cosine distributions.

With a distributed winding the values of $\omega\, t$ are different for the individual coils and the total reduction factors are

$$K_{q \pm 2\, p} = \frac{\sin \dfrac{(q \pm 2\, p)\, n\, \gamma}{2}}{n \sin \dfrac{(q \pm 2\, p)\, \gamma}{2}} \cdot \cos \frac{(q \pm 2\, p)\, \lambda}{2}$$

If the expression for the current is $i = \Sigma\, \sqrt{2}\, I_s \sin (s\, \omega\, t + \theta_s)$ the general expression for the flux distribution, due to the armature current alone, in the air gap of a single-phase alternator reduces to

$$\beta = \frac{1}{\sqrt{2}} \frac{N_p}{N_f} \left\{ \sum \sum K_q \cdot {}_0a_q \right.$$
$$\cdot\, I_s \left[\cos (q\, x - (q + s)\, \omega\, t - \theta_s) \right.$$
$$\left. - \cos (q\, x - (q - s)\, \omega\, t + \theta_s) \right]$$

$$+ \sum \sum \sum K_{q-2p} \cdot \frac{({}_pa_q + {}_pc_q)}{2}$$
$$\cdot I_s \left[\cos (q\,x - (q - 2\,p + s)\,\omega\,t - \theta_s) \right.$$
$$\left. - \cos (q\,x - (q - 2\,p - s)\,\omega\,t + \theta_s) \right]$$

$$+ \sum \sum \sum K_{q+2p} \cdot \frac{({}_pa_q - {}_pc_q)}{2}$$
$$\cdot I_s \left[\cos (q\,x - (q + 2\,p + s)\,\omega\,t - \theta_s) \right.$$
$$\left. - \cos (q\,x - (q + 2\,p - s)\,\omega\,t + \theta_s) \right] \Big\} \; *$$

(15)

Three-Phase Armature Reaction. In a three-phase alternator the resultant flux distribution due to the armature current alone is the combination of three such distributions. With a balanced load the currents are equal in magnitude and differ in phase by 120 degrees. The three armature windings also differ in their relative positions by 120 degrees. The sum of the three single-phase distributions will then be zero except when the coefficients of $\omega\,t$ in equation (15) are zero, three, or some multiple of three. In the coefficient of $\omega\,t$, q is always odd; $2\,p$ is always even and ordinarily the current contains no even harmonics and s is therefore odd. $(q \pm 2\,p \pm s)$ is thus either zero or even. If, however, the current contains even harmonics, this coefficient may be odd.

The general expression for the armature reaction flux in a three-phase alternator is given in equation (15). If the current contains no even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, six, or some multiple of six. If the armature current contains none but even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, three or some odd multiple of three.

In a turbo-alternator with a uniform air gap, the armature flux is independent of the position of the armature coil with respect to the field poles. In this case all of the coefficients except ${}_0a_q$ vanish, *i. e.*, p has no value but zero, and the complete expression for the armature reaction is much simplified, consisting as it does of only the first two terms in equation (15).

---

* N_p and N_F are the armature and field turns per pole.

For the method of calculating the coefficients ${}_pa_q$ and ${}_pc_q$ and their values see Appendix III.

The only assumption made in deriving this expression for the armature reaction flux is in regard to the symmetry of the magnetic circuit. But for this it is perfectly general.

Armature Flux, Voltage and Power. In an alternator with a uniform air gap the armature reaction is stationary with respect to the field poles only when the harmonic orders of the component flux distribution and the component armature current are the same. With a salient-pole alternator, however, the fundamental current may produce a stationary third harmonic in the armature flux, or a third harmonic component in the armature current may produce a stationary fundamental component in the armature flux distribution. With a salient-pole alternator there is likely to be a considerable third harmonic component in the armature reaction which is stationary with respect to the field poles and is due solely to the fundamental part of the armature current.* If the armature is delta-connected, a third harmonic current will exist in the windings which may of itself produce a stationary fundamental component in the flux distribution.† The latter would affect both the fundamental voltage and the power developed.

With a uniform air gap, in which case $p = o$, no average power is developed. That is, no power is developed in an alternator having a uniform air gap if the field excitation is zero. With a salient-pole alternator, however, in which case p is an integer, power may be developed due to the armature reaction flux alone, or, in other words, when there is no field excitation. This power is proportional to the sine of twice the lead angle of the fundamental armature current. The power is greatest with a phase angle of 45 degrees. Thus an unexcited synchronous machine may develop considerable power as a generator if the current leads, and considerable power as a motor if the current lags. These are well known facts. Furthermore a current of one harmonic order may produce a flux that reacts with a current of a different harmonic order and produces power.

When a synchronous motor is being brought up to speed by hysteresis and eddy current action, the fundamental current is relatively a fifth harmonic at one-fifth of synchronous speed, a third harmonic at one-third of synchronous speed and a second harmonic at one-half of synchronous speed. If one of these har-

*See fourth term, equation (15) $q = 3$, $p = 1$, $s = 1$.

†See sixth term equation (15) $q = 1$, $p = 1$, $s = 3$.

monics produces a torque greater than that due to hysteresis and eddy currents, the motor will tend to lock in at that speed and not come up to synchronism. Whether it does or does not will depend upon whether the momentum of the rotating part is sufficient to carry it into a phase position that is unfavorable for the development of motor torque due to the reaction of the air gap flux and the current. The hysteresis and eddy current torque is greater at one-fifth speed and the synchronous torque developed by the current is smaller than at either one-third or one-half speed. The tendency of the motor to lock in at one-half speed is greater than at either one-third or one-fifth speed. It depends upon the design and principally upon the ratio of pole arc to pole pitch.

Harmonics in Transient Short-Circuit Condition. The transient short circuit in an alternator gives a good illustration of the effect of even harmonics in the armature current. We will consider only the case of a three-phase turbo-alternator which has its field and armature windings so distributed that the harmonic components in the flux distributions they produce are negligible compared with the fundamental components.

The general equation for this current is

$$i = \Sigma A e^{-\alpha t} + \Sigma B e^{-\beta t} \sin s \omega t + \Sigma C e^{-\gamma t} \cos s \omega t \qquad (16)$$

As far as the writer is aware the constants in this equation have never been evaluated.

Due to the first term in this expression there is a decaying flux distribution which rotates at synchronous speed with respect to the field poles. There will thus be produced a current of fundamental frequency in the field winding. The periodic flux produced by this current will generate a voltage of double frequency in the armature winding, causing a second harmonic current, which will react on the field circuit and produce a third harmonic current in it. This third harmonic variation of the field flux will produce a second and fourth harmonic in the armature winding. The effect of a second harmonic has already been accounted for. The fourth harmonic in the armature current will produce only a third harmonic in the field winding, the effect of which has already been considered. The fifth harmonic which would have been produced in a single-phase alternator is not present, since the component fluxes which would produce it sum up to zero in the case of a three-phase alternator.* There will thus be a fundamental and a third har-

*First term, equation (15) $q = 1$, $s = 4$.

monic in the field current, and a fundamental, a second and a fourth harmonic in the armature current, but none higher. The magnitude of these harmonic components will be influenced to a considerable extent by the eddy currents produced in the practically solid rotor.

Calculation of Field Current. One of the most interesting applications of this theory, is to the problem of calculating the field current of an alternator for a given load condition. A three-phase, Y-connected generator, with neutral free, is chosen so that there can be no troublesome third harmonics in the armature current. There is generally ittle error in neglecting the effect on the terminal voltage of the fifth or higher harmonics in the armature flux if the magnetic circuit is designed so that there is no prominent fifth harmonic in the flux distribution due to the

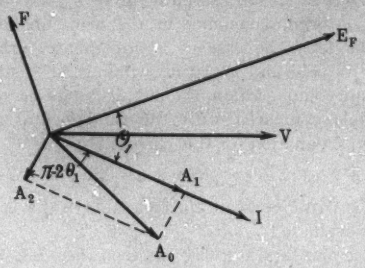

FIG. 11

field. If it be assumed that the armature currents are sinusoidal and balanced, that the harmonics in the armature flux above the third are negligible and that the harmonic variations in these components above the second are also negligible,* the expression for the stationary armature reaction flux distribution reduces to

$$\beta = \frac{1}{\sqrt{2}} \frac{N_p}{N_s} K_1 I_1 \left\{ -{}_0a_1 \cos(x + \theta_1) \right.$$

$$+ \frac{{}_1a_1 + {}_1c_1}{2} \cos(x - \theta_1) - \frac{{}_1a_3 + {}_1c_3}{2} \cos(3x + \theta_1)$$

$$\left. + \frac{{}_2a_3 + {}_2c_3}{2} \cos(3x - \theta_1) \right\} \qquad (17)$$

*In equation (15) $s = 1$, $q = 1$ and 3 and $p = 1$ and 2 only.

The first of these terms is that ordinarily given for the armature reaction and it is the only one present when the air gap is uniform. The other terms are due to the non-uniformity of the magnetic circuit for different positions of the armature coils on account of the spaces between the salient poles. The third and fourth terms are due to the presence of variable third harmonics in the flux distributions produced by the armature coils. They have no direct effect on the fundamental armature voltage but they do alter the saturation of the main magnetic circuit and may be important on this account. If they increase the flux from each pole it requires more field current to give a specified armature electromotive force than is shown by the no-load saturation curve.†

The vector diagram showing the phase relations of the fundamental armature reaction, the armature current and the air gap flux is given in Fig. 11. F and E_F are the impressed field and the voltage it would produce on open circuit. V is the terminal voltage. The armature current, I, lags behind the excitation voltage E_F by an angle θ_1. A_1 and A_2 are the first and second terms of the resultant armature reaction A_0. From equation (**17**) it is seen that

$$A_1 = \frac{1}{\sqrt{2}} \cdot \frac{N_F}{N_F} \cdot K_1 \cdot I_1 \, ({}_0a_1)^*$$

and

$$A_2 = \frac{1}{\sqrt{2}} \cdot \frac{N_F}{N_F} K_1 \cdot I_1 \cdot \frac{{}_1a_1 + {}_1c_1}{2}^*$$

A_1 is in phase with the current while A_2 lags behind the current by $(\pi - 2\,\theta_1)$ if $({}_1a_1 + {}_1c_1)$ is positive, as it usually is. Thus the resultant armature reaction lies more nearly in line with the impressed field than if the air gap were uniform. This condition occurs for two reasons; first, because an armature coil produces more flux when opposite a pole than when half way between poles $({}_1a_1 > 0)$, and second because the axis of the armature flux tends to conform with the axis of the field poles $({}_1c_1 > 0)$. At unity power factor, A_1 and A_2 are in opposition, while at zero power factor they are in conjunction. Thus the magnitude of the armature reaction flux is much greater at zero power factor than at unity power factor.

†See *Effect of Third Harmonic on Saturation.*

*Values of the coefficients ${}_0a_1$, ${}_1a_1$ and ${}_1c_1$ etc. are given in appendix III.

Fig. 12 shows the vector diagram which may be used in calculating the regulation of an alternator. The power factor (external) at which the alternator is operating is cos θ.* The sum of the terminal voltage, V, the resistance drop, $I\,r_a$†, and the leakage reactance drop, $I\,x_a$†, gives the voltage E_a, which is produced by the resultant fundamental flux in the air gap. This flux is due to the effect of the field and armature currents acting in conjunction on the magnetic circuit. If the permeability of all parts of the magnetic circuit is now assumed to be constant, the air gap flux which produces E_a may be replaced by the component fluxes which would be produced in the air gap by the independent action of the field and armature currents. With such an assumption the fluxes and the equivalent field currents

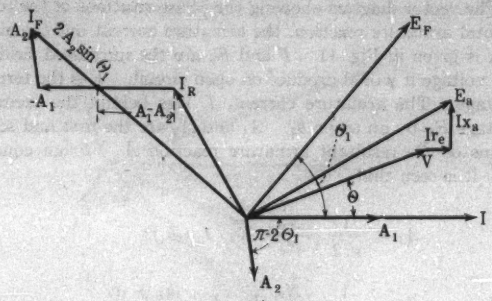

FIG. 12

producing them are proportional. I_R is the field current corresponding to E_a, obtained from the open-circuit saturation curve. A_1 and A_2 are the equivalent field currents corresponding to the two parts of the armature reaction. I_F is the actual field current. It is readily proved by geometry that the vector drawn from the terminus of I_R to the intersection of A_1 and I_F is equal to $(A_1 - A_2)$ and that the distance from this intersection to the terminus of I_F is $2\,A_2 \sin \theta_1$. Thus the phase position of I_F is determined by the armature reaction at unity power factor.

For the purpose of calculation it is more convenient to rotate I_R and I_F through 90 degrees clockwise so that the vector diagram appears as in Fig. 13.

*An angle of lead is positive and an angle of lag is negative.

†r_a and x_a are the equivalent single-phase values.

The armature leakage reactance may be determined from the open- and short-circuit characteristics. At short circuit the internal power-factor angle, θ_1, is essentially 90 degrees. Calculate the field current equivalent to the total armature reaction at zero power factor, viz. $A_1 + A_2$. Observe the voltage E_{AO} that would be produced on open circuit by this field current. Referring to Fig. 14 the leakage reactance is shown to be

$$x_a = \frac{E_F - E_{AO}}{I_a}$$

There are two minor corrections which will increase the precission of the results. They are the effect of the third harmonic and the increased pole leakage.

Effect of Third Harmonic on Saturation. The flux in the air gap

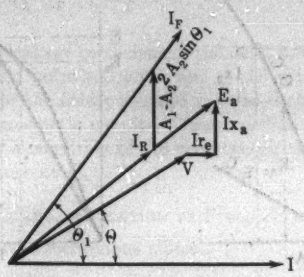

FIG. 13

and in the main magnetic circuit is produced by the magnetizing action of the field current and the armature current. Part of this flux is sinusoidally distributed and produces a measurable voltage across the terminals of the armature, while a small part of it is a third harmonic distribution and produces no voltage across the terminals. The saturation of the magnetic circuit, however, is determined by the entire flux. The armature voltage, E_a, (See Fig. 13) is due to the combined action of the field current and the sinusoidal portion of the armature reaction. The third harmonic portion of the armature reaction increases the flux per pole, when the current is a lagging one, and therefore more net field current is required to produce E_a under load conditions than at no-load. The higher harmonics in the armature flux may also increase the saturation, but their effect is

usually small. The magnetizing action of the third harmonic component, in equivalent field current, is somewhat less than

$$A_{03} = -\frac{1}{3}\,\frac{1}{\sqrt{2}}\,K_3\,\frac{N_P}{N_F}\,I_1\left(\frac{{}_1a_3 + {}_1c_3}{2} + \frac{{}_2a_3 + {}_2c_3}{2}\right)\sin\theta_1{}^*$$

In Fig. 15 E_a is the armature voltage which would be produced by I_R on open circuit. Under load, because of the increased saturation due to the third harmonic, it requires I'_R amperes to produce this same voltage. The entire magnetizing action for this condition is equivalent to a field current of $(I'_R + A_{03})$ amperes. On open circuit this would produce a voltage E. Perhaps the simplest method of determining the field current,

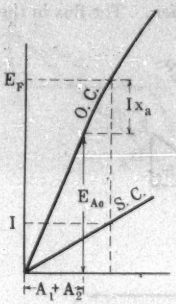

FIG. 14

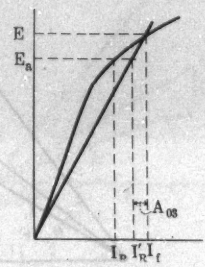

FIG. 15

I'_R, necessary to give the required armature voltage is to assume two values of E somewhat above the actual armature voltage, E_a, to calculate the corresponding values of E_a and then to interpolate between them. The values of I'_R and E_a corresponding to an open-circuit vo tage E are

$$I'_R = I_f - A_{03} \text{ and } E_a = \frac{I'_R}{I_f}\,E$$

It is suggested that values of I_f be chosen equa to $I_R + A_{03}$ and $I_R + 3\,A_{03}$. This correction is quite small unless the saturation is high and the power factor is low. Except in a few cases to which attention is called no account of this effect is taken in the calculations that follow.

*For a lagging current sin θ_1 is negative.

Increased Field Leakage. On open circuit the field current produces not only the flux in the airgap but also the leakage flux between the poles. Under load conditions the field current must usually be increased if the flux is to be maintained constant, on account of the demagnetizing effect of the armature reaction. Since the pole leakage is nearly proportional to the field current, it will usually be greater under load conditions than at no-load for the same armature voltage. The increased leakage requires additional field current, which is proportional to the difference between the calculated value of the field current, I_F, and that required to produce the armature voltage, namely I'_R. An approximate value of this added field current is

$$\left(\frac{v-1}{v}\, u\, y\right) \cdot (I_F - I'_R)$$

in which v is the leakage coefficient of the field, u, the saturation factor for the field poles and yoke, and y, the ratio of ampere turns required for the field poles and yoke to those required for the rest of the magnetic circuit. These coefficients should all be calculated for a condition of the magnetic circuit corresponding to the field current I'_R.

Numerical Calculation of Field Current

1080-h p., 3-phase synchronous motor.

Volts	6600
Poles	6
Slots	108
Series conductors per slot	14
Coil pitch	0.83
Turns per field spool	462
Pole arc: pole pitch	0.642
Field Leakage coefficient	1.083
Ratio $\frac{\text{amp. turns, poles and yokes}}{\text{amp. turns airgap, teeth and arm. core}}$	0.026
Saturation factor—field poles and yokes ($E_a = 6025$)	5.6
${}_0a_1$	0.803
$\frac{{}_1a_1 + {}_1c_1}{2}$	0.269

Calculation of field current when 98.5 amperes is delivered at a power factor of 0.42, leading

$$0.707\,K_1 \frac{N_p}{N_s} I_a = 0.707 \times 0.923 \times \frac{18 \times 7}{462} \times 93.1$$

$$= 16.57 \qquad \text{for } I_a = 93.1$$

$$= 17.52 \qquad \text{for } I_a = 98.1$$

Armature reaction at zero power factor

$$A_0 = 1.072 \times 16.57$$

$$= 17.76 \text{ equivalent field amperes.}$$

Field current on short circuit for armature current of 93.1 amperes is 19.2 amperes. Armature voltage on open circuit for field current of 50 amperes is 2250 volts.

$$I\,x_a = \frac{19.2 - 17.76}{50} \times 2250$$

$$= 648 \text{ volts for armature current of } 93.1 \text{ amperes}$$

$$= 686 \quad \text{" " " " " } 98.5 \text{ "}$$

$$V = 6600 \cos\theta - j\,6600 \sin\theta \qquad \cos\theta = 0.42$$

$$= 2772 - j\,5990$$

$$I\,z_a = 92 + j\,686 \qquad I\,r = 92. \text{ volts}$$

$$E_a = 2864 - j\,5304 \qquad \text{(between terminals)}$$

$$= 6025 \text{ volts}$$

Corresponding field current from saturation curve.

$$I_R = 14.2 \text{ amperes}$$

$$= \frac{2864}{6025} \times 14.2 - j\,\frac{5304}{6025} \times 14.2$$

$$= 6.75 - j\,12.49$$

Armature reaction at unity power factor for armature current of 98.5 amperes.

$$A_1 - A_2 = 0.535 \times 17.52$$

$$= 9.38$$

$$I_R + j\,(A_1 - A_2) = 6.75 - j\,3.11$$

$$= 7.43$$

$$2\,A_2 \sin\theta_1 = 0.537 \times 17.52 \times \frac{3.11}{7.43}$$

$$= 3.94$$

$$I_F = 7.43 - 3.94 \qquad \theta_1 \text{ is an angle of lead}$$

$$\frac{v-1}{v}\,u\,y = \frac{8.3}{108.3} \times 0.026 \times 5.6$$

$$= 0.011$$

$$I'_v - I_v = 0.011\,(3.49 - 14.2)$$
$$I'_v = 3.37$$

Measured value of field current = 2.0

It is difficult to predetermine accurately the field current of a synchronous generator for a given load condition, particularly when it is delivering a leading current. If the so-called magnetomotive force method or the method proposed by the A. I. E. E. is taken as the standard of simplicity, then it can be said that there is no simple method of calculating the field current that will give reliable results for every load condition. The effect of the armature current is much too complicated to be calculated so easily. The general method, in which the leakage reactance and armature reaction are determined by the Potier triangle, is perhaps as good a compromise as any between accuracy and simplicity.

TABLE OF COMPARATIVE FIELD CURRENTS.

Output	Voltage	P.F.[1]	FIELD CURRENT				
			Gen. method	M. M. F.	A.I.E.E.	Proposed	Observed
500 kv-a.	240	−0.0	.....	121.5	.....	134	135.5
1,500 kv-a.	2,300	−0.0	.....	120.5	.....	135.7	130.
2,000 kv-a.	2,300	−0.0	.....	199.	.....	214.	227.
2,000[2] kv-a.	2,100	−0.0	.....	52.5	.....	60.5	64.5
2,000[2] kv-a.	2,100	+0.0	−12.4	−5.6	.....	−8.7	−10.0
2,500 kv-a.	2,300	−0.0	.....	257.	.....	310 327[4]	330.
2,500 kv-a.	2,300	+0.0	−12.	10.[5] −42.[6]	67	−6	−4.8
5,000 kv-a.	2,300	−0.0	.....	235	.....	258	251.
5,000 kv-a.	6,600	−0.0	.....	221	.....	246[4,7]	250.
5,000 kv-a.	6,600	1.0	173	171	165	170	169.
83.5 amp.[3]	11,150	1.0	40.0	39.5	36	41.1[4,7]	42.8
68.0 amp.[3]	11,550	+0.784	53.	48.1	56	58.2	59.
12,000 kv-a.	6,600	−0.0	.....	372	.....	406	410.
93.1 amp.	6,600	−0.0	.....	35.2	.....	36.8	36.9
110 amp.	6,600	+0.72	16.6	16.5	16.7	13.	12.0
98.5 amp.	6,600	+0.42	9.2	9.0	16.1	3.37	2.0

1. + sign indicates a leading current; − sign, a lagging current.
2. Two-phase.
3. As a motor.
4. Corrected for effect of third harmonic.
5. Armature m.m.f. from short-circuit data.
6. Armature m.m.f. from zero power factor data at rated voltage.
7. Not corrected for increased field leakage.

The author wishes to aknowledge his indebtedness to the General Electric Company and Westinghouse Electric and Manufacturing Company for the data from which these results were calculated.

APPENDIX I

Refer to Fig. 16. N_1, N_2, N_3 and N_4 indicate the numbers of turns in the coils. Let B_0 be the average flux density per ampere turn when one coil alone is acting. The flux density per ampere at a distance x, measured from the center of a belt of conductors, reduces to

$$\beta = \frac{8\,B_0}{\pi} \sum \left(N_1 \cos \frac{q\,\gamma}{2} + N_2 \cos \frac{3\,q\,\gamma}{2} + N_3 \cos \frac{5\,q\,\gamma}{2} + N_4 \cos \frac{7\,q\,\gamma}{2} \right) \frac{1}{q} \sin q\,x$$

If any harmonic is to be eliminated its coefficient in this series must be zero. Since there are three independent variables in each coefficient, viz. $\frac{N_2}{N_1}$, $\frac{N_3}{N_1}$, $\frac{N_4}{N_1}$, at least three harmonics and their odd multiples may be eliminated. Choose these

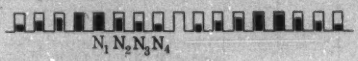

FIG. 16

variables so that the third, fifth and seventh harmonics will vanish. Since there are 9 slots per pole the angle $\gamma/2$ equals 10 degrees. The equations are

$$N_1 \cos 30 + N_2 \cos\ 90 + N_3 \cos 150 + N_4 \cos 210 = 0$$
$$N_1 \cos 50 + N_2 \cos 150 + N_3 \cos 250 + N_4 \cos 350 = 0$$
$$N_1 \cos 70 + N_2 \cos 210 + N_3 \cos 350 + N_4 \cos 490 = 0$$

The ninth harmonic vanishes for any number of turns in the coils. The equation for the eleventh harmonic coefficient is identical with that for the seventh except that the signs are changed. It therefore vanishes. The same is true for the thirteenth and fifteenth harmonics. Thus the first harmonic that appears is the seventeenth. The other higher harmonics that may be present are the nineteenth, twenty-third, twenty-ninth, etc. The first harmonics that appear, viz. the seventeenth and the ninteenth are of the same order as would be produced by the rotor slots. They could be reduced to small proportions by closing the slots with magnetic wedges. The solution of the above equations for the relative numbers of turns in the field coils gives

$$\frac{N_2}{N_1} = 0.88$$

$$\frac{N_3}{N_1} = 0.65$$

$$\frac{N_4}{N_1} = 0.35$$

The efficiency of the winding is 80 per cent.

The reduction factor for the seventeenth harmonic is the same as that for the fundamental but the harmonic is reversed in phase. The same is true of the nineteenth harmonic. Each of these harmonics thus occurs in the same relative magnitude as it would in the distribution due to a concentrated winding. Making the correction for saturation, neither alters the efficiency

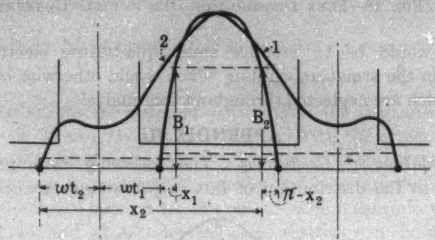

FIG. 17

of the winding nor the reduction factors for the seventeenth and nineteenth harmonics, but does alter their relative magnitudes. If $\frac{b_1}{B_0}$ is 0.10, they are both about thirteen per cent greater than at low saturation.

APPENDIX II

In Fig. 17, $B_1 = B_2$ if $x_1 = \pi - x_2$ and $\omega t_1 = -\omega t_2$ and the magnetic circuit is symmetrical about its center line.

$$B_1 = \Sigma A_q' \sin q x_1 + \Sigma C_q' \cos q x_1$$

$$B_2 = \Sigma A_q'' \sin q x_2 + \Sigma C_q'' \cos q x_2$$

But since $x_1 = \pi - x_2$, and q is always odd

$$B_2 = \Sigma A_q'' \sin q x_1 - \Sigma C_q'' \cos q x_1$$

From this it follows that $A_q' = A_q''$ and $C_q' = -C_q''$.

But A_q and C_q are periodic functions of $2\omega t$ and since $\omega t_1 = -\omega t_2$, A_q must be a cosine series and C_q a sine series.

If one tip of the field pole is more highly saturated than the other, as it usually would be, the magnetic circuit will not be quite symmetrical about its center line. This distortion is probably never very great since most of the reluctance is in the air gap. Its effect would be to introduce sine terms into the series for A_q and cosine terms into the series for C_q. The final

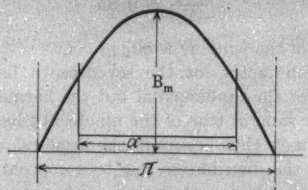

FIG. 18—FLUX DISTRIBUTION DUE TO FIELD CURRENT

result would be to produce small quadrature electromotive forces in the armature winding which would otherwise be absent and which are neglected throughout this analysis.

APPENDIX III

Calculation of Coefficients. The following assumptions in regard to the distribution of flux in the air gap are considered

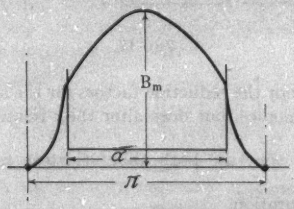

FIG. 19—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. ZERO DEGREES.

ideal. They are chosen to represent the actual conditions as nearly as possible and still allow relatively simple calculations to be made. With these assumptions, however, it requires about fifteen pages of close calculation to obtain the desired coefficients. In practise, it may happen that the departure of the actual distributions from these ideal forms may be so great as to warrant new calculations that will more nearly fit the

special case. The flux distributions can also be determined experimentally and the coefficients deduced therefrom.

With one ampere-turn acting per field pole the flux distribution in the air gap is assumed to be sinusoidal, with a maximum value of B_m. See Fig. 18. For the same saturation of the main magnetic circuit with one armature turn carrying one ampere, the

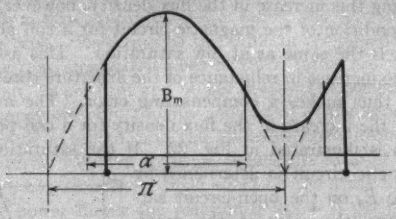

FIG. 20—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 45 DEGREES

flux distributions for different positions of the turn are shown in Figs. 19, 20 and 21. At zero degrees, with the armature coil directly opposite a field pole, the distribution is an inverted sine curve between the poles and coincides with the distribution due to a field ampere-turn under the poles. The maximum value is B_m. At 45 and at 90 degrees, the distribution is also an

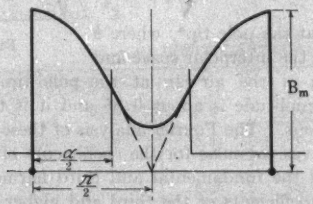

FIG. 21—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 90 DEGREES.

inverted sine curve between the poles and coincides with the distribution due to a single field ampere-turn under the poles. Taking the maximum value in each of these four cases as B_m assumes that the reluctance of the armature core, pole and yoke is negligible compared with that of the air gap and teeth. This is nearly so at low saturation. For operating conditions the

saturation is higher and the flux densities due to an armature coil will be relatively greater for those positions that give flux paths that do not include the field poles, yokes and core. In order to make the calculations as simple as possible it is assumed that this applies only to the distribution at 90 degrees. It should also apply to the distribution at 45 degrees. The method of estimating this increase in the flux density, however, assumes that the condition of the magnetic circuit for a coil position of 90 degrees is the same as at low saturation. This assumption neglects the increase in reluctance of the armature core and pole faces and thus makes a compensating error. The method of estimating the increase in the flux density for a coil position of 90 degrees is illustrated in Fig. 22. If the saturation of the main magnetic circuit is determined by the voltage E_a on the open-circuit saturation curve, the value of the maximum flux density should be taken as $\frac{E'}{E_m} B_m$ for the distribution at 90 degrees.

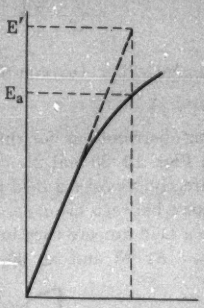

FIG. 22

In the two latter positions of the armature coil the amplitude of the inverted sine curve is taken as $1 - \frac{2}{\sqrt{4 + \frac{b^2}{g^2}}}$ of the density at the pole tip*, where b is the width of the interpolar space and g is the length of the air gap at the pole tip. An average value of this amplitude is about 0.85 and it is taken as such in the calculations. The Fourier analysis of these assumed flux distributions gives formulas for the coefficients of the sine and cosine terms of the fundamental and third harmonic distributions. If the coefficients of the third and higher harmonics in the *variations* of these coefficients are assumed to be negligible, the approximate values of ${}_0a_1$, ${}_1a_1$, ${}_1c_1$, ${}_1a_3$, ${}_2a_3$, ${}_1c_3$ and ${}_2c_3$ can be calculated. At the end of this appendix these coefficients are tabulated for a ratio of pole arc to pole pitch of 0.70 for different saturations. For ratios of pole arc to pole pitch between 0.65 and 0.75 the armature flux is nearly constant.

*This formula is due to Mr. F. W. Carter, who, however, uses it in connection with the problem of determining the flux distribution opposite an armature slot placed in an otherwise uniform air gap.

Different assumptions in regard to the shape of the flux distributions for different positions of the armature coils would give different values for these coefficients. Considerable research may be necessary before reliable representative values can be chosen. Oscillographic measurements made at low saturation on a 7.5-kw. generator give a ratio of third harmonic to fundamental component at unity power factor of 52 per cent, and at zero power factor of 18 per cent. The ratio between the fundamental fluxes for unity power factor and zero power factor is 45 per cent. Referring to the table of coefficients it will be seen that the calculated values of these ratios are respectively 64 per cent, 22 per cent and 50 per cent. The coefficients given in this paper are only roughly indicative of the general results that may be expected, and the following conclusions are liable to some error. Within its usual values the ratio of pole arc to pole pitch has apparently little effect on the armature reaction. At unity power factor the reaction is about 3 per cent greater for a ratio of 0.75 than for one of 0.65. For a ratio of the length of the air gap at the pole tip to the length of the interpolar space of 0.05 the reaction at unity power factor is about 2.5 per cent less than for a ratio of 0.10. Saturation has no effect on the armature reaction at zero power factor but at unity power factor the armature reaction increases with the saturation. At higher saturations the third harmonic in the armature reaction is more prominent than at lower saturations at unity power factor, but at zero power factor the third harmonic component is practically constant.

TABLE OF COEFFICIENTS.*

Saturation	1.0	1.11	1.2	1.3
s_{01}	0.80	0.82	0.83	0.85
$\frac{s_{01} + s_{21}}{2}$	0.27	0.26	0.24	0.23
$\frac{s_{03} + s_{23}}{2}$	−0.29	−0.30	−0.32	−0.33
$\frac{s_{05} + s_{25}}{2}$	0.05	0.07	0.09	0.10

*To obtain the best results the coefficients should be calculated for the proper ratio of pole arc to pole pitch and for air gap to interpolar space.

DISCUSSION ON "APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES" (LYON). (PRESENTED BY PUBLICATION.)

N. S. Diamant: I would like to call attention to a few points in connection with this paper. As the author states very clearly, the principle of the analysis given in the paper is well known and all that the author claims and expects to do is to develop it more fully and put it in a form more usable for the average engineer. It is difficult to see whether the author has succeeded in this for the following reasons.

In a paper of this nature it is well, if not necessary, to connect up one's work with that of previous investigators by giving references at the proper places. This, however, the author has not done. Therefore, it may be of interest to state that a very complete resumé of generation of voltage in dynamo electric machines was given by S. P. Smith and R. H. Boulding, *Journal I. E. E.*, 1915, Vol. 53. In connection with the general method of analysis referred to by the author in the first three paragraphs of the paper, the following references might prove useful to the reader who wants to go further into the subject: *Journal de Physique* 1897, 3 series, T. VI. p. 341 and 483. A. Blondel, *L'Eclairage Electrique*, 1895, T. IV, pp. 241, 308 et 358. T. V. p. 268. See also P. Janet, *Lecons d'Electrotechnique generale*, T. III, chp. V. Alex Russell, *Altern. Currents*, Vol. II, Chp. XIV.

The above remarks are not made in order to detract from the value of the paper, but because during the last few years, in connection with the study of similar subjects, I felt the need of a paper or book summarizing in a general and clear way the best results of work that had been done along this line of the application of harmonic analysis to the theory of synchronous machines. Thus, in a paper* presented at the Atlantic City Convention on "Sustained Short-Circuit Phenomena and Flux Distribution of Salient-Pole Alternators," I was compelled to include some introductory material bearing on the flux distribution and flux pulsation and oscillation. See Section 3 and Supplementary Notes at the end of the paper referred to. It is rather disappointing to find that the author has not quite filled this need of a general critical summary of the subject.

I think it would be well in a paper of this nature to make perfectly clear when one refers to time harmonics called multiple fields. For example, referring to page 1184 and following the author's illustration it can be shown that an ordinary armature winding with N conductors per slot and one slot per pole per phase, supplied with a sinusoidal, in time, current, will be exactly equivalent to: (1) flux, sinusoidally distributed, in space, having a pitch of τ cm. or inches. (2) a flux sinusoidally distributed having a pitch of $\tau/3$ and 1/3 as many turns as the

*TRANS., A. I. E. E., 1918, Vol. XXXVII, p.

first, etc. In general, in case of a polyphase alternator with a sine-shaped armature current with regard to time, and any ordinary type of winding, non-sinusoidally distributed in space, the resultant m.m.f. can be considered as consisting of series of m.m.f. curves, all of fundamental frequency, in time, and having a pole pitch τ/k and traveling at a speed of k times the speed of the fundamental wave. Thus, it is extremely important and makes it very easy reading if the author in every case would make it perfectly clear whether he refers to time or space harmonics.

Mathematically speaking, it is also very well to make clear whether the author refers to traveling waves or stationary waves.

In connection with the effect of slots given on page 1492 it would have been well if the author had explained more in detail the origin and production of the tooth harmonics, particularly as there seems to be considerable difference of opinion in regard to this as referred to in my paper on 1493. The author states that "the periodic variations in the shape of this flux distribution produce a corresponding variation in the coefficients of its harmonic components. Each of these coefficients may then be represented by a Fourier series." On the next page after some mathematical transformations he is contented to state further "that the periodic variations in the A coefficients indicate corresponding variations in the flux per pole, while the periodic variations in the C coefficients indicate corresponding distortions of the flux distribution. The latter are equivalent to an oscillation of the flux across the pole face."

These statements I am afraid are at their best too abstract and mathematical, and personally I would greatly appreciate having a little more detailed interpretation and explanation of the underlined phenomena. I would be glad for example to have the author explain how he starts with periodic variations in the flux and after some mathematical transformations he gets a variation plus a to-and-fro oscillation of the flux across the pole face. The coefficients A and C may indicate this but it would help very much if he will explain to us the physical mechanism involved.

With reference to the harmonics in sudden short-circuit conditions I would appreciate having an explanation of the derivation of equation (**16**) and just what is meant by the remark that "so far as he is aware the constants in this equation have never been evaluated." This statement may be true but for the sake of the reader who wants usable data on the subject as to theory I may refer him to Boucherot and Berg, and as to experimental data and methods of calculation to papers published during the last few years in the TRANSACTIONS A. I. E. E., *Electrical World*, *General Electrical Review*, and *Electric Journal*.

W. V. Lyon: The author agrees with Mr. Diamant that a comprehensive bibliography would be useful to the student of this subject.

The fundamental equation (1) on page 1479 represents a sinusoidal distribution of flux in the air gap that may be stationary or traveling with respect to the field poles, depending upon whether j is or is not zero. For any fixed value of time the equation shows that the flux density varies sinusoidally as we progress around the gap. The wave length is $1/q$ of the wave length of the fundamental distribution, *i. e.* $1/q$ of twice the polar pitch. The equation also shows that the density at any fixed point in the gap varies sinusoidally with the time. Should we progress around the gap at such a rate that $q\,x$ is always proportional to the time and equal to $j\,\omega\,t$ the density at this traveling point is constant. In this way the velocity of the distribution is determined. The velocity, x/t, thus equals $\frac{j\,\omega}{q}$ as stated on page 1479. When j is zero the distribution is stationary with respect to the field poles. Otherwise it is moving in the same direction as the armature if j is a positive quantity and in the opposite direction if j is a negative quantity. As stated, the poles are assumed stationary. Relative motion is of course the only thing that counts. The advantage of this form of expression for the flux density is that it at one and the same time indicates the shape of the distribution and its relative velocity with respect to the field poles.

Unfortunately the question of the harmonics in the e. m. f. due to the action of the slots cannot be handled with mathematical exactitude due to the fact that as the slots come under the pole tip the division of the flux between that which enters the tooth at the right and that which enters the tooth at the left is not at the center line of the slot. Thus it is not possible to calculate the instantaneous flux linkages with the armature coil lying in this and its corresponding slot by integrating the flux density function between limits, since the limits are not known. They might be determined experimentally by exploring the field within a slot when it occupies different positions with respect to the pole. In the absence of such data the author integrated from the center of the slot, but noted the assumption, page 1494. Referring to equation (11) on page 1490 it is seen that the flux per pole due to the field current alone is approximately equal to the integral of this expression from 0 to π. This is not of course strictly true since the density may not be zero at these two limits. The integral of the cosine terms is zero between these limits, *i. e.* the cosine terms contribute nothing to the polar flux. They do however alter its distribution, and, if variable, account for oscillations of the flux across the polar surface. On the other hand if the A coefficients vary the entire polar flux pulsates in magnitude. These oscillations and pulsations are due to the action of the slots as is well known. They are not always the cause of harmonics in the armature e. m. f. And although the question has been the subject of considerable investigation, the

author has seen diametrically opposed conclusions in regard to the cause and suppression of these harmonics. Some combinations of pulsations and oscillations may produce no net effect in the winding as shown on page 1494. A simple method of reducing these variations in flux density is to make the ratio of slot opening to air gap as small as possible.

The author believes that the statement in regard to the transient short-circuit current is correct. While much work has been done in this important subject no solution has been given that is based on as rigorous theory as the question merits. The general form of equation (**16**) is of course determined by a consideration of the result produced by a disturbance in two inductively coupled circuits like the field and armature windings of an alternator. If further proof is needed it is supported by oscillographic records such as Mr. Diamant has taken.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., September 16, 1918.

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ELECTRIC WELDING—A NEW INDUSTRY

BY H. A. HORNOR

ABSTRACT OF PAPER

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the shipbuilding industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

INTRODUCTION

ABOUT a year ago the Chairman of the Standards Committee of the Institute was requested to investigate and standardize spot welders and the apparatus connected with them. It occurred to the members of this committee that electric welding could perform an important function in increasing the progress of steel ship construction. The work which was started by the Standards Committee was then transferred to the General Engineering Committee of the Council of National Defense. Last winter the Council of National Defense abolished all advisory committees but at this time the Emergency Fleet Corporation of the U. S. Shipping Board had become so much interested in the subject that they decided to adopt the Committee. The Committee is composed of representatives covering broadly the whole field of welding activities in this country and, although electric welding has been the subject of all the investigations up to the present time, it is now proposed to include gas welding with representatives from all the gas welding associations and companies connected with this industry.

The two main processes of electric welding, namely, arc welding and spot welding, were found by this committee applied in the first case to repairs and in the second case to certain factory quantity production jobs. The work done was in the case of spot welding only on light material, and in neither case very extensive. The processes to be successful in their application

to the construction of merchant vessels would have to show reliability in the joining of steel plates from a half-inch to one inch in thickness. To this and kindred problems the committee immediately turned its attention.

The work had all been done in the field where it had been applied by practical men. It was first necessary to formulate the proper nomenclature and symbols. This was thoroughly investigated and a very comprehensive set of symbols has been approved by the committee and is in daily use by those now actively engaged in this new application. The approved nomenclature introduces the subject to the designing and calculating engineer and gives him the instrument by means of which he is able to place his thoughts rapidly and conveniently on drawings.

The manufacturers of apparatus joined the practical man in the study of the problems of electric welding. Apparatus and so-called processes introduced various types of machines suitable for the conversion of electrical supply to the proper values of current and voltage needed at the arc or at the spot. The manufacturer in his eagerness to meet the problem naturally encountered many difficulties. These difficulties increased until a point was reached as referred to above where he demanded some standards upon which his apparatus could clearly be rated. Therefore, the manufacturer was only too pleased to co-operate with the Welding Committee and is today conscientiously aiding in straightening out the difficulties in which he was involved prior to last year.

Arc welding in this country has largely been done in the railroad repair shops. It was discovered that the process was much cheaper and could be performed more rapidly than by any of the gas welding methods. It also could be applied without preheating and in many cases without the expense of disassembling complicated pieces of machinery. Spot welding besides being used in many different industries was sought for by the railroad man and there has been built a gondola car which has seen some seven or eight years of service. It is interesting to note here the difference in practise between Great Britain and the United States. The former knowing little or nothing about spot welding had the practise and application of arc welding very well under way; the latter exactly the reverse.

Apparently the attempts to train operators were rather crude and it was early observed that the reliability of the electric weld depended substantially upon the skill of the welder. The manu-

facturers of apparatus and the superintendents in railway shops had struggled with the problem of training operators but intensive study had not been given the subject so that there existed in this respect a great deal of groping in the dark.

Present Status of Electric Welding

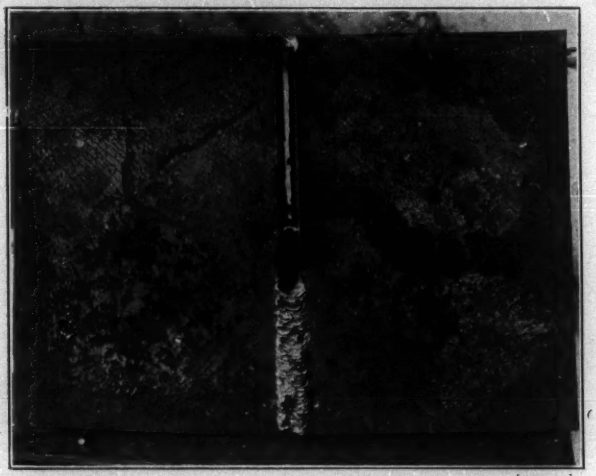
Investigations were immediately undertaken to answer the question whether spot welding could be successfully accomplished using one-inch thick steel plates. An experimental apparatus of large size was erected and put into operation, the results showing that no difficulty was encountered with half-inch and three-quarter inch plates. The same remark applies to one-inch steel plates. In fact, this experimental machine was successful in welding three thicknesses of one-inch plate, a condition which far exceeds the requirements of merchant ship construction. This operation has its historical significance in that this was the first time that any spot welding of this magnitude had been performed. The successful outcome of these experiments has led to the design and construction of large spot welders to be used in the fabrication of ship sections. The practical application of a large five-foot gap spot welder will be made at a demonstration of a forty-foot section of a standard 9600-ton ship to be built at the plant of the Federal Shipbuilding Company, Kearney, New Jersey. This is the largest portable spot welder ever built. It will prove two points in ship construction by the electric method, namely, the clamping of the ship's structural parts for assembly thereby reducing the time in working the material as well as for the erection of the ship material; and secondly, by the speed of spot welding it will prove the decrease in time for joining the material together. The consensus of opinion is that the large stationery spot welder of five- or six-foot gap will undoubtedly play an important part in increasing the speed of fabricating sections of standard steel vessels. Further investigations are being made and designs are being worked out for special spot welders for use in the construction of bulkheads. The designs proposed are chiefly for shop processes, but it can be asserted that such apparatus will be of undoubted value in the saving of time and man power.

Arc welding had been tried in a great variety of work but there was no conclusive evidence that it could be developed to the stage of joining ship plates with the certainty of full strength. The first stage of this investigational work is now

almost completed. Sample welds of half inch ship structural steel were taken by a special sub-committee to fourteen or fifteen different places where electric welding was being performed. This sub-committee saw the welding done, noted the conditions of current, voltage, electrode, operator, etc., and then prepared the welded samples for tests. The samples were forwarded to the Bureau of Standards in Washington so that the tests should be conducted by parties absolutely disinterested and without knowledge of how the samples were obtained. The results of these tests showed a remarkable similarity especially when it is realized that they were made by several firms with different electrode materials and under varying conditions of the electrical circuit. Practically all of the welds pulled at over 50,000 pounds per square inch and several over 60,000 pounds the average being about 58,000. On the bending test one of the samples was bent to an angle of 78 degrees before a crack started and final failure reached 80 degrees. In another case the sample was bent to 65 degrees before the crack started and final failure did not occur until 86 degrees. The point of importance here is that all the welds showed a reliability and satisfactoriness which makes conclusive the opinion that electric arc welding is applicable for the joining of steel where the structure is submitted to live loads, bending strains, static pressure, or the like. The Sub-committee on Research is pursuing this subject and practical samples are being prepared for similar tests using three-quarter and one-inch stock material. The results of these tests will be available as soon as the reports are presented and approved by the Welding Committee. The Research Committee is also preparing various types of joints in heavy plating. These will be submitted to all the regulation tests and in addition to shock and fatigue tests and tests to destruction.

To give a further indication of the large size practical tests which are being carried on at the present time it may be stated that three 12-foot cube electrically welded tanks are now being constructed. These tanks are built in such a way that from twelve to fifteen different designs of joints are used in their construction. After these tanks are built they will be subjected to a static strain and the deflection of the seams will be directly measured. Afterwards they will be tested by external shock and crushed to destruction. Portions of the joints will be cut, sent to the Bureau of Standards, and again tested for the sake

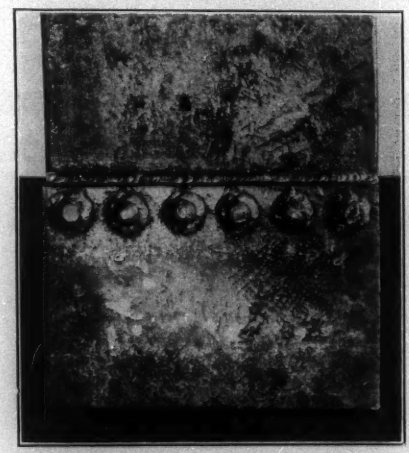
SAMPLE SHOWING INDIFFERENT AND GOOD BUILT UP WELDS



[HORNOR]

PARTIALLY COMPLETED ARC WELDED SEAM—NOTICE TACK WELDS TO HOLD PLATES TOGETHER

ARC WELDING IN THREE DIFFERENT POSITIONS—FLAT—VERTICAL AND OVERHEAD



[HORNOR]

ARC WELDING AROUND RIVET HEAD TO PREVENT LEAKAGE

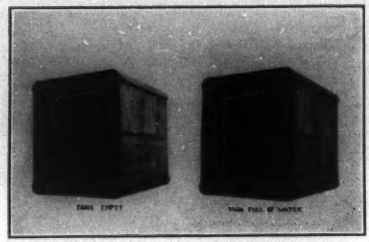
ARC WELDING BOILER FLUES

SAVING PISTON BY BUILDING UP WITH ELECTRIC ARC WELD

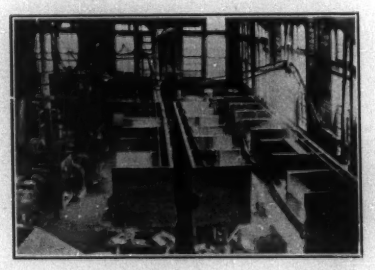
[HORNOR]

SECTION OF A SMOKE STACK—SEAMS WELDED BY ELECTRIC ARC WELDING PROCESS—METAL 1/4 IN. BOILER PLATE

BACK TUBE SHEET WELDED TO HEAD



TWELVE FOOT CUBE TANK ALL SEAMS ELECTRICALLY WELDED FOR EXPERIMENTAL PURPOSES—BUILT AT PITTSFIELD, MASS.



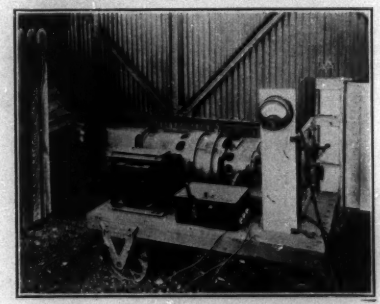
[HORNOR]

SCHOOL FOR ELECTRIC ARC WELDERS ESTABLISHED BY THE EMERGENCY FLEET CORPORATION AT THE PLANT OF THE GENERAL ELECTRIC COMPANY

FRONT VIEW WELDING BOOTH—G. E. WELDING SCHOOL

THIRTEEN FOOT MAST ELECTRICALLY WELDED

ONE-MAN A. C. TRANSFORMER OUTFIT FOR ARC WELDING

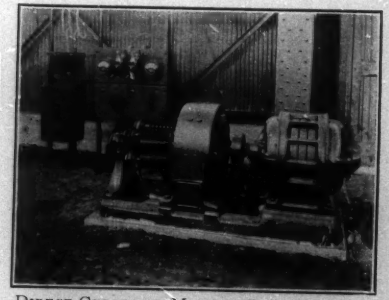


ONE-MAN PORTABLE D-C. ARC WELDING OUTFIT [HORNOR]

ELCTRICALLY WELDED SAMPLES—FAILURES IN TENSION OCCURRING AWAY FROM WELD—STEEL PLATES 3/4 IN. BY 7 IN. AND 1/2 IN. BY 6 IN.

MACHINE FOR EXPERIMENTAL WELDING FOR HEAVY STEEL BY ELECTRICITY—CURRENT CAPACITY 100,000 AMPERES— PRESSURE CAPACITY 36 TONS

ELECTRICALLY WELDED STEEL PLATES 1/2 INCH AND 1 INCH THICK AFTER BEING TESTED IN TENSION—SHOWING EFFECT OF COMPOUND STRESSES DUE TO EFFECT IN LINE OF TENSION CAUSED BY THE LAPPING OF THE POLES



DIRECT-CURRENT MOTOR GENERATOR SET FOR SUPPLYING 8 ARC WELDERS

of accumulating precise data. In this connection there is being built at the Norfolk Navy Yard a battle-towing target. The keel of the target 110 ft. long will be entirely electrically welded and the results of this practical demonstration will be carefully recorded after it has been put in regular service.

It is to be expected that the manufacturers of apparatus being keenly observant of the increased interest in electric welding as well as in the future, which is probably now unquestioned, would be active in their desire not only to improve their present facilities and their design of apparatus but also to proceed themselves to follow the trend of the investigations made by the Welding Committee. The consequence of this has been a large increase in output of apparatus and it may be unhesitatingly stated that there are no difficulties in the way of obtaining all the electrical welding apparatus that is needed. One interesting point is that certain manufacturers who were decidedly of the opinion that direct current was the only proper current to use for arc welding have within a very recent period changed their point of view and are willing to admit that alternating current may have certain advantages in the development of this art.

The electric arc requires a reduced voltage and this is difficult to attain with direct current without relatively expensive machines or a useless expenditure of energy. The practise in this country in manufacturing establishments of any size has been toward an increase in the supply voltage so that very few large manufacturing plants use less than 220 volts direct current. With this voltage the only economical method of transformation is in the use of a motor-generator set. The efficiency in this case is in the neighborhood of 50 to 60 per cent. It is possible to use a supply voltage of 110 volts with a variable resistance which cuts down the voltage to the arc volts. This gives a very poor efficiency. In the case of alternating current the supply voltage can be reduced by a transformer which will supply as in the case of direct current a sufficient voltage for striking the arc and a satisfactory reduction when the arc has been struck. On the other hand, if a low voltage alternating current is provided a simple reactance may be introduced which has some of the same wasteful characteristics of the resistance used with the direct current. The average apparatus will permit of electric arc welding consuming about six to eight kilowatts per welder but if low voltage is provided there are

certain outfits which will reduce the consumption as low as three and one-half kilowatts per welder, or even less.

Without entering into an elaborate analysis of the relative costs of electric welding, it may be broadly stated that there is hardly any question that the electric process is cheaper than any other. The same may be said as regards speed and also reduction of man power. In a recent discussion of this subject President Adams stated that at one of the Eastern shipyards the total number of parts on the welding program of the standard riveted ships now building at that yard amounted to 225,000. The labor cost for riveting these pieces is about 245,000 dollars and for welding about 99,000 dollars making a saving of 146,000 dollars. But this is only a drop in the bucket when compared to what might be profitably done in this line. He stated further that in certain particular instances the saving is as great as 90 per cent.

One of the interesting questions discussed with some fervor by the members of the Welding Committee is the advantages of the bare and covered electrode. Regarding this discussion no definite facts can be stated. In England the practise has been to use the covered electrode which protects the welding arc from contact with the air thus guarding against too great a formation of oxide. The practise in the United States up to the present time has been largely bare wire. Recently, American investigators have discovered the important fact that there are advantages in the covered electrode and many experiments are now being made, some with results. It is important to observe that in the above mentioned tests of welds, the best one of these samples was made with a coated (not an asbestos covered) electrode using alternating current. The point in this case seems to rest upon the question of the ductility of the weld and it would seem that the bare electrode does not make as ductile a weld or at least one as easily bent as the coated or covered electrode. The question of the ductility of the weld is one of much importance in the application to ship construction and will doubtless be of importance in other allied industries. It is, therefore, a question of serious importance and constitutes an important part of the work of the Subcommittee on Research.

No matter what the type of electrode is nor its composition, no matter what kind of shank material is to be welded, no matter what kind of apparatus is employed, the reliability of

the weld rests mainly upon the man who makes it. This man if he has been properly trained and is skilled in the art knows instantly whether he is making a weld or not. He becomes after much practise able to judge fairly well upon looking on a finished weld whether it is a good weld or not. The work of training electric welding operators early became a part of the functions of the Education and Training Section of the Emergency Fleet Corporation. The men connected with this work are members of the Welding Committee. Schools for the training of operators as well as for the conversion of operators into instructors, are established in many parts of the country. The objects held in view by the training department are first to give the man intensive practise work so that he becomes a good craftsman. The methods are simple to start with, as the exercise of the right arm muscles must become flexible enough to permit the operator to give the required movement to the electrode. By a graduated series of exercises this is accomplished in about eight weeks. The man is allowed to do production jobs in the shop which gives him confidence through responsibility. It becomes desirable at this time to give the man some outside work on ships and where this is practicable, it is done. The man is then turned over to an instructor who gives him an intensive course in pedagogics lasting from five to six weeks. At first sight it would not seem necessary to so instruct a man but it is not generally understood that teaching after all is itself a trade. The experience with the men in this respect is most interesting. In nearly every case the man has resented this course at the start but at the end has turned completely around and in many cases has desired an even more extensive training. What is really accomplished is to give the man the necessary confidence to impart the knowledge that he has gained to another green man. The men under training are taken from the various industries especially the shipbuilding industry and after they have finished their instructor training course are returned to their employer to carry on the instruction, in their own plant. The men who go through this training as provided by the Emergency Fleet Corporation are certificated when they have shown themselves to be entirely proficient. It is not possible nor expedient for the Emergency Fleet Corporation to require the certification of all electric welders. It is the consensus of opinion that all industries doing serious work with the electric arc should use men who are certified as to their ability in the art

of electric welding. The main reason for this opinion is that the operator must be a conscientious workman or the weld will not be of perfect quality.

This brings forward another problem upon which a great deal of experimental work has been and probably will continue to be done, namely, a practical and scientific method of testing a welded joint after it has been made. There have been a number of suggestions made for the solution of this problem. They are briefly, as follows:

(a) Mechanical. By hammering the weld or by chipping at frequent intervals.

(b) Electric. By means of resistance or voltage drop.

(c) Magnetic. By means of the permeometer or the change of conditions of the magnetic circuit.

(d) X-ray. By means of an exposure on an X-ray plate.

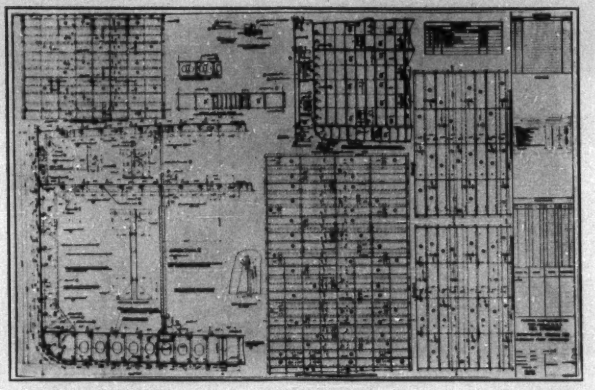
At the present time none of these suggested methods have been productive of conclusive results and recourse must be had to the purely mechanical methods of striking heavy blows on, or adjacent to, the weld or by using a chipping hammer and making intermittent examinations. It would seem by far the best procedure to make the inspector proficient in the art so that he may closely observe the welders while at work. This may be accomplished by a two or three weeks attendance of inspectors at any one of the electric welding training centers.

Methods of Electric Welding

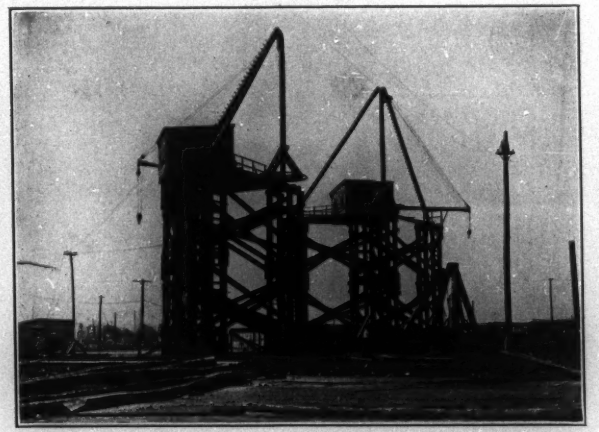
There are many methods and processes of electric welding but the two main ones that interest the committee at the present time and alone have been mentioned so far are the spot welding and arc welding. It may be a surprise to some of the old time welders to consider electric welding as a new industry. In substantiation of this statement it may be well to describe briefly what is meant by electric welding as it is practised today.

Spot welding is not much different in the methods of procedure or in design of apparatus as when it was first introduced. Copper electrodes, water-cooled in the heaviest machines, are placed on opposite sides of the material to be welded together. The joint is a lap joint. Machines are now so designed that two spot welds may be made at one time. The routine of the operation is as follows:

The electrodes are brought into contact with the materials to be joined, current is supplied sufficient to give the required heat, pressure



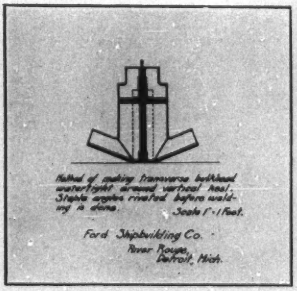
STRUCTURAL DRAWINGS OF EXPERIMENTAL SECTION OF STANDARD SHIP BEING BUILT AT KEARNEY, NEW JERSEY, UNDER THE DIRECTION OF MR. A. J. MASON



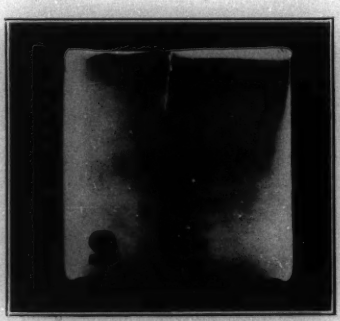
[HORNOR]

ARRANGEMENTS AT KEARNEY, NEW JERSEY, FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP

ARRANGEMENTS AT KEARNEY, N. J., FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP—NEAR VIEW SHOWING KEEL BLOCKS



METHOD OF MAKING TRANSVERSE BULKHEAD WATERTIGHT AROUND VERTICAL KEEL—STAPLE ANGLES RIVETED BEFORE WELDING IS DONE



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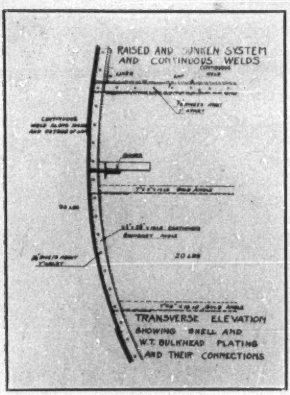
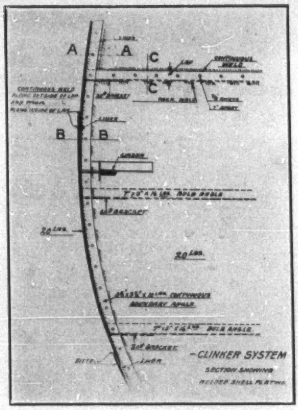
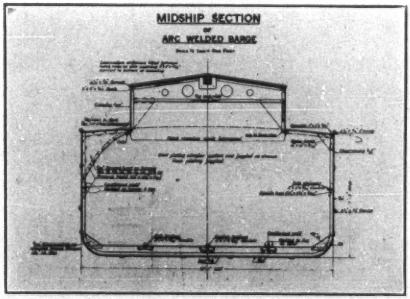
X-RAY PHOTOGRAPH OF WELD



RAISED AND SUNKEN SYSTEM AND CONTINUOUS WELDS

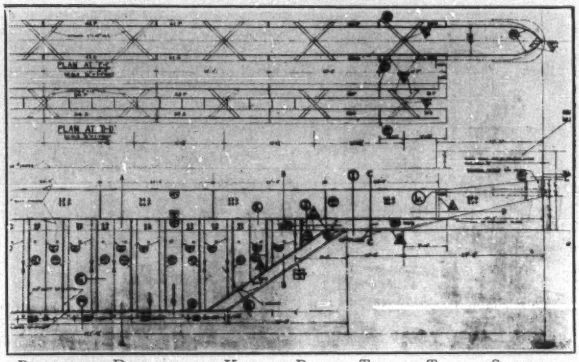


CLINKER SYSTEM—SECTION SHOWING WELDED SHELL PLATING

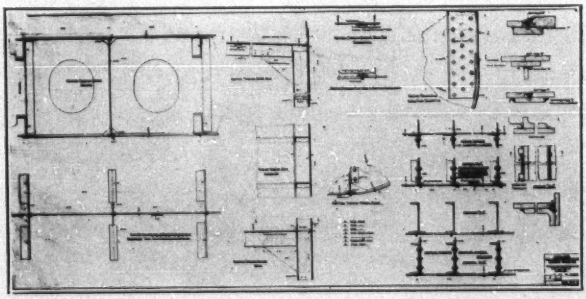


MIDSHIP SECTION OF ARC WELDED BARGE [HORNOR]

TOTALLY ELECTRICALLY WELDED BARGE BUILT IN ENGLAND



PORTION OF DRAWING FOR KEEL OF BATTLE TOWING TARGET SHOWING THE USE OF APPROVED SYMBOLS AND THE SIMPLICITY OF THE DRAWING



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DETAILED DESIGN FOR PROPOSED ELECTRICALLY WELDED SHIP

is then applied, the current is removed, and the pressure is removed, the weld is then complete.

The operator has a perfect indication of making a good spot weld by the use of a button placed under the electrode observing which he knows exactly the proper timing of the operation. There is therefore no question as to a good, bad, indifferent, spot weld. Automatic spot welders have been designed and built but it is the general opinion that they add complication to a process which in itself is very simple.

The process of arc welding is as follows:

One side of the electric circuit is connected to the material to be welded, the shank material is usually prepared by beveling the edge of the pieces to be welded together. The other side of the electric circuit is connected to the electrode. The operator is provided with a holder which carries the electrode. By touching the electrode to the shank material the arc is drawn. The skilled operator now moves the electrode from side to side of the groove giving a semi-circular motion while at the same time moving the electrode along the groove.

It is important that the arc "bite" into the shank metal creating a perfect fusion along the edges and the movement of the electrode is necessary for the removal of any mechanical impurities that may be deposited. In the coated electrode it is further necessary that the slag which forms for the protection of the pure metal be worked up to the surface and it is extremely important in the event of a second or third layer that the slag or impurities be carefully scraped away before the virgin metal is again laid on.

The operator in arc welding is protected with either a hand screen or a covering for his face with special glass through which to observe his work. The electric arc emits dangerous invisible rays in both the upper and lower spectrum scale and it is quite evident that both the infra-red and ultra-violet are dangerous in their effect, the former is pathological the latter actinic. The operator further uses gloves for his hands and for the very difficult work of overhead welding it is necessary for him to use a helmet which partly covers his breast.

Developments

The tendency of developments in spot welding has already been slightly touched upon. In their nature as applicable to shipbuilding the advancement will naturally have to proceed toward means for accomplishing spot welding in very cramped locations. This makes an exceedingly difficult problem as the

power requirements are such as to preclude any very small device. In riveting one-half of the apparatus is on one side of the work and the other half on the opposite side and it is difficult to conceive of any method of spot welding that will admit of such an arrangement. In shipbuilding it is quite probable that designs may be made that will permit of a large or at least increased amount of spot welding in the actual construction of the vessel. Certainly, present designs of riveted ships will not allow of this to any great extent. As already stated, spot welding can now take its place in the fabricating shops and it is to be expected that within a few months spot welding will begin to supplant riveting in this field. The only drawback to this will be the sufficient production of spot welding apparatus.

The tendency of development in arc welding is toward the automatic machine to obviate the responsibility that has to be placed upon the skilled operator. Intensive work has been done within the last few months in the line of automatic arc welding machines and at the present time sample tests of welds made by such apparatus have been sent to the Bureau of Standards. These machines will occupy a very important position in repetition work. They will not immediately supersede the skilled operator in repair work, or in special jobs but it may be expected that the development of such machines will bring about apparatus which can be man-handled and will eventually take the place of most of the hand work as it is now known.

Of the scientific advancement in the art of electric welding there is so much to be treated that only a general outline can be considered at this time. The research work has only just begun. Practise has preceded the scientific investigation. The field, therefore, is full of most interesting problems. Those who have been following the development of the past six months are deeply interested to know the fundamental reasons. The investigational questions may be grouped into three main divisions:

1. Metallurgical.
2. Physical.
3. Electrical.

The metallurgist has yet to tell us what the conditions of the metals are after the electrode material has fused with the parent metal, and to determine what the proper conditions must be to produce a good weld. This problem has in it a great many variables. The physicist must explain the atomic or

electronic conditions which permit of the combinations at the high temperatures involved and must explain the phenomenon of overhead welding. The electrical investigator must determine all the various phenomena connected with the preferences between and the advantages of the use of different forms of electrical energy and the varying characteristics of the electric circuit in producing different types of welds.

Conclusion

From the preceding remarks it must be conceded that the Welding Committee of the Emergency Fleet Corporation has already crystalized the problems connected with this art. The working functions of this Committee have been laid down upon the broadest possible lines. Liberal opportunity has been given every one to state in detail his opinion and to express the reasons for his preference on every point connected with this subject. The Committee goes even further than this. It furnishes those interested with every new idea that is brought to bear upon the subject after sifting from the suggestions any question of doubt or misstatement of fact. All suggestions of improvement or problems of special application are gladly taken in hand, thoroughly investigated, and reports made. It will welcome any comments that those connected with the industries may desire to lay before it. The personnel is at the present time such that it can devote not one but many minds to the solution of any specific problem that is laid before it.

The Committee early discovered that the literature of electric welding was very much clouded by misstatement of fact or half-baked theory and much of the time of the Committee has been taken up in disproving such statements. In order to spread the results of this work to all quarters a handbook is now being prepared which will contain only definite facts and results of investigations as are approved by the whole Committee. This handbook will be made available to all those who desire to acquaint themselves with the proper means of accomplishing good and reliable electric welding.

Presented at the 343d Meeting of the American Institute of Electrical Engineers, New York, November 8, 1918.

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ELECTRIC MOTORS IN THE CEMENT INDUSTRY

BY R. B. WILLIAMSON

ABSTRACT OF PAPER

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

PRACTICALLY all modern mills manufacturing Portland cement are driven by electric motors, and since the process is mainly one of crushing and grinding, a large amount of power is required.

Before considering the various motor applications, it may be advisable to give a brief outline of the process of manufacturing Portland cement, which constitutes about 99 per cent of the cement of all kinds manufactured in the United States.

GENERAL DESCRIPTION OF PROCESS

Portland cement is generally formed by an artificial combination of calcareous and argillaceous materials. These materials are mixed either before grinding or during the process of grinding. After being ground so that from 80 to 90 per cent of the material will pass through a screen having 200 meshes per lineal inch, it is burned to incipient fusion in a rotary kiln, after which the resulting clinker is again ground so that at least 78 per cent will pass the 200-mesh screen.

The Portland cement clinker, after it leaves the rotary kiln, requires no further treatment than grinding, excepting the addition of about 2 per cent of gypsum to retard the setting. In some parts of the United States natural cement rocks are found which contain nearly the proper proportions of materials to produce Portland cement; but, even in these localities it is generally necessary to add either limestone or shale in order

to get the proper mixture. Certain deposits of cement rock so nearly approach the proper proportions of materials, that at one time limestone must be added and at another time shale.

In the United States, limestone furnishes the largest supply of calcareous material for Portland cement. Other sources of supply for this class of material, in addition to cement rock mentioned above, are marl, chalk and alkali waste. Shale and clay are most commonly used for obtaining the argillaceous material for Portland cement. Cement rock mentioned above, as well as blast furnace slag, are also used as a source of argilla-

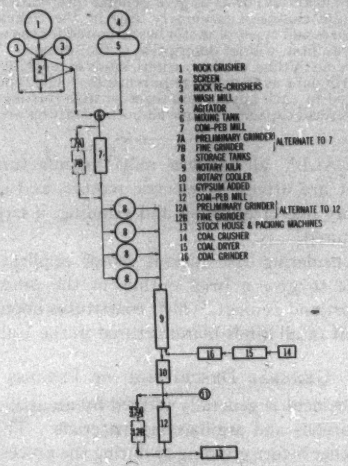

FIG. 1

ceous material. Blast furnace slag, like cement rock, contains a considerable proportion of lime and has the further advantage that this lime is found as calcium oxide instead of the carbonate; therefore, the heat necessary to dissociate the carbon dioxide gas from this part of the lime is not required.

The preliminary crushing of limestone, cement rocks and shale is ordinarily done in the crushers of the gyratory, jaw or roll type. Materials may be ground either in the wet or dry state. Fig. 1 shows a typical flow sheet for a wet mill and Fig. 2 for a dry grinding mill. In a dry grinding plant, the two classes

of materials are preferably dried separately and then weighed, after which they are ground. In a wet grinding plant, the materials are generally measured or weighed in their natural state as excavated, and then ground. Where shale or cement rock is used as the argillaceous material, it must be crushed in rock crushers. In the past, it has been the practise in certain wet grinding cement plants to dry the clay in order better to store and handle it, and also to eliminate gravel and other foreign material. The better practise, however, is to dump the clay in its natural state into a wash mill where sufficient water is

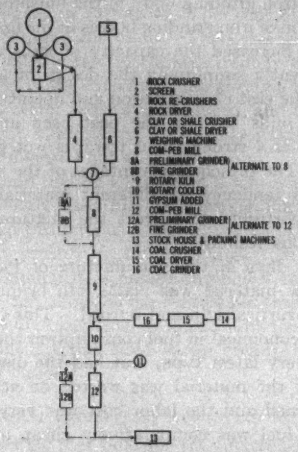

FIG. 2

mixed with it to allow the heavy pieces of gravel to settle out, and also to form a slurry which can easily be stored in tanks. The flow sheet, Fig. 1, for a wet grinding cement plant shows a wash mill installed for handling the argillaceous material. Wash mills are used only for clays, marls and similar materials, which, by mixing with water, can be kept in suspension. If shale were used as argillaceous material, a crusher of some type would have to be provided instead of a wash mill.

The same general type of grinding machinery can be used for either wet or dry grinding. In the past, it has been customary to use a preliminary and a finishing grinder; but, the latest

development in cement practise is to use a combination mill which combines the preliminary and finishing grinder in one machine, eliminating considerable expense in cost of installation and reducing materially the cost of operation. The combination mill has its greatest advantage in eliminating one set of storage bins and a great deal of elevating and conveying machinery. Combination mills consisting of a preliminary compartment charged with steel grinding balls and a finishing compartment charged with flint pebbles were tried many years ago, but mills of this type have never been successful until the introduction of the iron grinding body in the finishing compartment. The introduction of iron grinding bodies in the finishing compartment has also increased the capacity of mills of a given size, resulting in greatly simplifying the design of a cement plant, also reducing the cost of installation and operation.

While most mills have in the past been arranged for dry grinding, there is a strong trend towards wet grinding in new installations. The principal advantages of the wet process are, cleanliness, less tendency for different materials to segregate, and better opportunity to correct the mixture to obtain the exact proportions desired.

In the early days of the manufacture of Portland cement, the ground raw material was made into briquettes and then charged into a vertical kiln and burned. This process of burning was more economical in fuel consumption than rotary kilns, except of the very latest type; but had the disadvantage that a great deal of the material was wasted on account of being improperly burned and the labor cost was very high. In the United States, fuel was comparatively cheap and the cost of labor high, and, as might be expected, the rotary kiln is an American invention. The improvements made in the rotary kiln have resulted in such saving in the cost of burning cement that it is now generally used throughout the world.

The clinker leaves the rotary kiln at a high temperature, and before grinding it must first be cooled. Most modern cement plants use a rotary cooler after the kiln. The cooler is preferably placed below the kiln in order to conserve the sensible heat of the clinker, the air for cooling the clinker passing up into the kiln to support combustion. In addition, however, to cooling the clinker, it is desirable to age it before grinding, as aging not only improves the quality of the cement, but also increases the "grindability" of the clinker. The aging of cement

can either take place before grinding or after grinding; but, the cement can be more cheaply stored in the condition of unground clinker than after it is ground.

The grinding of the clinker into finished cement requires more power than any other step in the process, as the clinker ordinarily is much harder to grind than the raw materials. It is common practise in American cement plants to grind the raw material and the clinker to about the same fineness. There is at the present time, a tendency to demand more finely ground cement, specifications for Portland cement in the United States having been changed in 1916 from 75 per cent through 200 mesh to 78 per cent through 200 mesh. There is a greater tendency, however, to grind the raw material extremely fine. There are three advantages in the extremely fine grinding of raw material; fuel is saved, a better cement is produced, and the clinker from finely ground raw material is more easily ground than clinker from the same material not so finely ground.

Natural gas and fuel oil are used in a few cement plants, but the most commonly used fuel for burning Portland cement is finely pulverized bituminous coal. Bituminous coal is crushed in a preliminary crusher, or the screenings from the coarse coal are used, after which it is passed through a rotary dryer and then ground so that about 85 per cent passes through a 200 mesh sieve. Finely pulverized coal is blown into the rotary kiln by an air blast; the pressure used varies greatly but is generally from two to six ounces.

General Conditions Affecting Motors

The same reasons that hold for the preference of a-c. to d-c. motors in general industrial work make the a-c. motor more desirable for cement mill operation. There are two main reasons for this. First, the relatively higher alternating voltage that can be used, *i. e.* 440 or 550 volts or even 2200 volts for some of the motors in large mills; and second, the greater mechanical simplicity of the a-c. motor, particularly that of the squirrel cage type. The great majority of cement plants use alternating current, though there are a number of mills equipped with d-c. motors which have given excellent service. There is nothing inherently injurious about the action of cement dust on the commutators of d-c. machines, and they are successfully employed in some plants, but, all in all, the a-c. motor is preferred and used. The following features should be borne in mind in selecting motors for a cement mill.

Service. Continuous. Shut down for repairs only.

Starting Conditions. Severe on many applications and making phase-wound-rotor motors preferable or even essential.

Rating. May be affected by heavy accumulations of dust on the motor windings, thus interfering with ventilation. Machines should be frequently blown out and cleaned off.

Bearings. Should also be made dust proof by the use of felt and steel washers, as cement has a highly abrasive action and greatly increases bearing wear when present in the bearings.

Outboard Bearings. It is good practise to supply outboard bearings for belted motors of 75 h. p. and larger. Pulleys larger than usual in both diameter and length are often specified since the cement dust causes more than normal belt slippage and makes necessary tighter belts and increased strain on bearings.

Drive. Motors should not be direct geared to crushing machinery on account of excessive and severe vibration. Low-speed motors direct-connected through flexible couplings, or belted, are the proper application. So far as possible, motors and belts should be placed in a separate room for protection from dust and to prevent their agitating the dust in the room where the mills are located.

Type of Motors. Alternating-current preferred; squirrel-cage where starting conditions are fairly easy, and wound-rotor motors where starting is severe. Low- or moderate-speed motors should be used. On account of the unfavorable belt conditions large speed reduction ratios should be avoided. Moreover, low- or moderate-speed motors are more substantial mechanically than high-speed and give less bearing trouble.

Collector rings are not injured by cement dust, which, when dry, does not show the abrasive action that it does in the bearings. It is advisable, however, to protect collectors so that dust cannot settle on them in large quantities.

While the induction motor has been used almost exclusively for cement plant operation, there are indications that synchronous motors will be adapted to this work more in the future. This is specially so in the case of low-speed motors for geared tube mills as mentioned later.

Frequency. Frequencies of 60 cycles and 25 cycles are in common use. Most of the large plants that are operated in connection with steel mills have 25-cycle equipment since they usually get their power from the same source as the steel mill, and 25 cycles has been generally used for steel mill operation.

A few plants are in operation at frequencies other than 25 or 60 cycles but these are on the whole exceptional.

Voltage. 440 volts preferred to 220 on account of copper distribution; 550 volts is satisfactory in a dry process plant. In some large mills 2200 volts has been used for the large motors but this practise is not common.

Load Factor. May be as high as 95 per cent since the machines run at all times under constant full load. The load factor of the average complete cement plant runs from 65 to 75 per cent taken through the entire year and is probably the highest of any industry.

The various kinds of machines that may be employed for the different processes in a cement mill may be grouped as follows:

1. Elevator and conveying machinery for raw material.
2. Crushers.
   Gyratory Jaw Roll.
3. Intermediate crushers
   Roll Hammer mills.
4. Rotary Dryers.
5. Preliminary Grinding Machinery.
   Ball mills Rolls
   Ring-roll mills Huntington mills
   Hammer mills Griffin mills.
   Fuller mills
   Raymond mills.
6. Finish Grinding Machinery.
   Tube mills
   Combination mills.
7. Conveying Machinery for Finished Product.
8. Rotary kilns. Coal feeding machinery.
9. Cement Grinding Machinery.
   (a) Preliminary grinding
   Ball mills
   Rolls
   Griffin mills
   Ring roll mills.
   (b) Finish grinding
   Tube mills
   Ball peb mills
   Combination mills.
10. Coal Grinding and Drying Machinery. Same class of machinery as used for raw material.

ELEVATORS AND CONVEYERS

Every cement plant requires a large number of elevators. and conveyers for handling the material between the various

steps in the process. One modern 3000-barrel plant, as an example, has 14 elevators, 2 belt conveyers and 8 screw conveyers in its equipment. It is very important that these auxiliaries be kept operating, as an accident to a conveyer or elevator may shut down an entire department of the plant until repairs can be made.

Various methods of driving this class of machinery have been installed in different mills. The chief difficulties are that the driving shafts run at extremely low speeds and that elevators must be driven from the top. The first necessitates a number of countershafts and belts or gears to give the necessary speed reduction and the second means that the driving motor, if individual drive is used, must often be located in an inaccessible place and in many cases be subjected to considerable vibration due to weak foundations. A very good method of drive which is sometimes applicable, especially in the case of crushers or group drive of grinding machinery, is to belt the elevators and conveyers from a pulley on some part of the mill shafting or from a small pulley on the front end of the motor.

Another method adopted in some plants is to group several elevators and conveyers on one motor, driving through countershafts and belts or chains.

The third possibility is to put an individual motor on each conveyer and elevator. In applying individual motors to this class of work, however, there are certain points which should be borne in mind if trouble is to be avoided. The machinery is rather roughly built, especially bucket elevators and bucket and drag type conveyers. These usually consist of two parallel chains driven by rough cast or forged sprockets, and carrying the buckets between them. It sometimes happens that the chains will climb up on the teeth of the sprockets and throw heavy overload on the driving motor which may be enough to stall it. Screw conveyers occasionally wear their bearings down so far that the screw may rub against the trough for most of its length with consequent overload. These possibilities make it advisable, in the case of individual drive, always to install motors from 25 to 50 per cent larger than necessary to meet the average power demand. This is not true when group drive is used as it is very unlikely that more than one unit of the group would be subjected to overload at the same time and this would probably not affect the operation of the motor to any extent. At the same time, group drive has the

disadvantage or requiring more belts and countershafts and of the probability of crippling an entire department if an accident happens to any unit or any part of the transmission.

The lengths and types of elevators and conveyers differ so much with different plants that it is impossible to give a table which will be of much assistance in estimating the power required, but the following test results will probably be of some service in such work.

Screw conveyer on dry clay.	
Diameter	16 in.
Pitch	16 in.
Length	120 ft.
Speed of screw	65 rev. per min.
Horse power	5.5
Screw conveyer on pulverized coal.	
Diameter	16 in.
Pitch	16 in.
Length	162 ft.
Speed of screw	55 rev. per min.
Horse power	3.7
Belt conveyer on finished cement.	
Width	20 in.
Length between centers	230 ft.
Height material is lifted	40 ft.
Speed	200 ft. per min.
Horse power	10–14
Belt conveyer on crushed stone.	
Width	20 in.
Length between centers	135 ft.
Height material is lifted	22 ft.
Speed	211 ft. per min.
Horse power about	5
Elevator on crushed stone.	
Buckets	18 x 10 x 16 in.
Height material is elevated	52 ft.
Speed	83 ft. per min.
Horse power	3 to 5.5

Elevator same as preceding except elevating to 60 feet instead of 52 feet. Horse power 2 to 3 (probably due to better lubrication.)

Elevator on crushed stone.	
Buckets	48 x 8 x 12 in.
Height material is raised	62 ft.
Speed	95 ft. per min.
Horse power	8 to 10 (occasional peaks above 15)

The starting torque of elevators and conveyers is likely to be above normal running torque, but standard squirrel-cage

motors, if large enough to take care of the peak loads as mentioned above, should have no difficulty in starting them.

CRUSHERS

All dry process cement mills require extensive crushing plants for their limestone cement rock. Three types of crusher are in common use for this work. Gyratory crushers are used more than any other, although those of the jaw type, or crushing rolls, are preferred in some cases.

Gyratory Crusher. The gyratory crusher has a vertical shaft with a bearing at the top and bottom. The shaft is driven by a bevel gear and pinion at the lower end. The shaft is eccentric with respect to the bottom gear and journal so that the shaft gyrates as the gear revolves, the amount of side movement being a maximum at the bottom and a minimum at the top. Attached to this shaft is a conical crushing head of chilled iron or special steel, sometimes smooth and sometimes corrugated. The shell or mouth of the crusher surrounds the crushing head and is lined with special iron or steel plates. The shell is conical but inverted so that the distance between its surface and the crushing head is a maximum at the top and a minimum at the bottom. As the shaft rotates, its gyratory motion causes the head to alternately approach and recede from the shell so that the large pieces of rock which are fed in at the top are gradually broken up and fall lower and lower in the shell until the pieces are small enough to fall out through the space between the bottom of the head and shell on to a sloping diaphragm between the bottom of the shell and the driving gear, which directs the crushed pieces into the discharge spout.

In all cement plant crushing, the problem is to reduce the rock from the size as it comes from the quarry down to a size small enough to pass an inch or a ¾ inch ring. It has been found that it is better to divide this range into two steps instead of making the entire reduction in one machine. Therefore, in most crushing plants, the rock first passes through a large crusher which reduces it to three- or four-inch cubes, after which it is reduced to the required size (usually ¾ to 1 inch) in smaller crushers. Occasionally, revolving screens are installed to grade the material as it comes from the crusher, all that which will not pass through the screen being returned for further reduction.

There is a considerable difference of opinion regarding the

size of the first crusher. The rock as it comes from the quarry is made up of pieces of various sizes ranging from small chips up to pieces three or four feet in diameter, the only limit being what can be handled by a steam shovel. The great majority of the pieces, however, are under 20 inches. Some cement manufacturers install for the first crusher a machine large enough to take the largest pieces, while others use a crusher large enough for the average size and break up the extremely large pieces with dynamite before taking them to the crusher house. The advantage of the first method is that it eliminates this dynamiting, thus saving labor and expense in the quarry. Its disadvantage is that a crusher large enough to take care of the few exceptionally large pieces will probably have a much greater capacity than necessary so that it will be idle a great deal of the time, and will represent considerably more investment than a smaller machine, as well as a greater consumption of power.

The size of the crushers is designated by an arbitrary number. Practically the same system is used by the different manufacturers, so that a No. 8 crusher, for instance, is about the same size capacity, etc., whether one make or another; Table 1 is therefore approximately correct for all makes. As a No. 3 crusher is the smallest which is likely to be used in any cement plant, data are omitted on number 0, 1 and 2.

TABLE 1.
Power Requirements of Gyratory Crushers.

Makers No.	Size of opening on each side of top bearing arms	Size cube which can be crushed	Horse power
3	7 × 28	7	10– 20
4	8 × 34	8	15– 25
5	10 × 40	10	20– 35
6	12 × 44	12	25– 40
7–½	15 × 55	15	50– 70
8	18 × 68	18	65–100
9	21 × 76	21	100–140
10	24 × 99	24	125–175
18		36	150–200
21		42	150–200
24		48	175–225
27		54	200–250

The horse power figures given in Table 1 must be used with discretion because there are a number of factors which affect the power required by crushers. Chief among these are the hardness of the material to be crushed and the method of feeding. On soft rock or shale the power will be lower than when crushing very hard stone, the difference being about as indicated by the

two limiting powers in the above table. When crushers are fed intermittently as for example the very large crushers which take the material when it first comes from the quarry and which are of such great capacity that they can run a small carload of rock through in less than a minute, after which they may run light for several minutes, the driving motor can be somewhat smaller than for crushers of the same size fed from bins where the power is more nearly continuous. It is the usual experience that crushers are "over motored", and the horse power data given in the above table are more likely to be high than low.

Belt drive is used almost exclusively, an incidental advantage being that the belts afford some protection against damage in

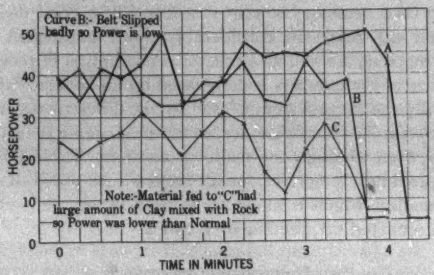

FIG. 3—TESTS ON THREE NO. 5 CRUSHERS—EACH DRIVEN BY 25 HORSEPOWER 600 REV. PER MIN.

A, CRUSHER WITH CORRUGATED HEAD—1½ IN. PRODUCT
B, " " " " —1¼ IN. "
C, " " SMOOTH " —1 IN. "

case pieces of iron etc., get into the rock accidentally. Installations seem to be about evenly divided between those using individual motors and those in which the entire crushing plant including possibly three or four crushers with their elevators and conveyers are driven in a group from one large motor.

The starting torque required by a gyratory crusher is small unless it is accidentally stopped with material in it, in which case, it is usually necessary to dig it out before it can be started.

Typical load curves on crushers are shown in Figs. 3 and 4. The former illustrates the difference that may exist between the power required by crushers of the same size due to the quality of the material.

Jaw Crushers

In jaw crushers of the Blake type the crushing takes place between the two jaws set at an acute vertical angle, one fixed and one movable; the feed opening at the top being greater than the discharge at the bottom, gravity brings the material into the crushing zone and discharges it when crushed. The movable jaw is suspended from the top of the crusher frame and is given a reciprocating motion by means of an eccentrically operated vertical pitman and a pair of toggle plates.

The most favorable condition for jaw crushers is obtained when the ratio of size of feed to product does not exceed 6 to 1. When this ratio is exceeded, the efficiency of the crusher is greatly decreased, and an increase in the power requirement will be noted. With a ratio of 6 to 1, the power requirement remains practically constant whether the reduction is from 18 in. to 3 in.,

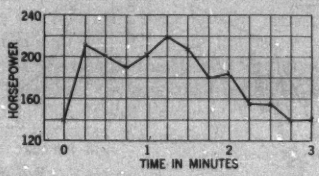

Fig. 4—Power Required to Drive Mammoth Crushers while Crushing One Carload of Rock—Feed Average Size from Quarry—Reduces to 4 in.-5 in. Size

or from 6 in. to 1 in. The amount of material crushed, however, will be in proportion to the width of the discharge opening, *i. e.*, when reducing from 18 in. to 3 in., the crusher will handle approximately three times as much material as when reducing from 6 in. to 1 in., all other things such as size of feed openings, speed, etc. remaining constant. Any increase or decrease within certain limits in the speed of the crusher will increase or decrease the volume of material handled and the power required.

The power required to operate any given size of jaw crusher, depends on the power required to operate the crusher when running idle and the power required to do the work of crushing the material.

Consider these in the order as given above: The power required to operate a crusher when it is running idle is due principally to friction, and this will vary considerably for different

sizes, speeds and makes of machines. When figuring the power requirements of a jaw crusher, it would be preferable to obtain the friction of the crusher direct from the manufacturer.

For the Blake crusher, there is one theoretically correct speed based on the law of gravity. The time period of the stroke of the movable jaw must not exceed the rate of movement of the material due to gravity. Therefore, there is one theoretical speed for all sizes of crushers, but it is obvious that there is a mechanical limitation when a certain size of crusher is reached that will necessitate the use of a lower speed.

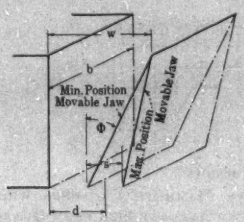

FIG. 5

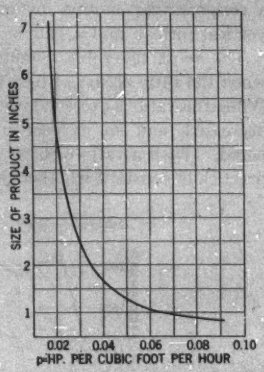

FIG. 6

This curve is drawn on the assumption that the size of the product will never be less than 1 inch and that the reduction will never be more than 6 to 1.

An approximation of the power required to operate the crusher when it is doing no work is

$$p_m = 0.016 \times b \times w$$

where

p_m = horse power

b = breadth of opening of jaws in inches. See Fig. 5

w = width of opening of jaws in inches. See Fig. 5

Power required to crush the material.

The amount of material M passed by the crusher is*

$$M = \frac{sb\,(2d + s)\,R}{57.6 \tan. \phi}$$

Where M = cubic feet of material per hour.

s = stroke in inches measured at the throat of crusher. See Fig. 5

*Wiard, "Theory and Practise of Ore Dressing."

d = average distance in inches between the fixed and movable jaws and the minimum and maximum position of the swing jaw as measured at the throat of the crusher. See Fig. 5.

b = breadth in inches at opening of jaws. See Fig. 5.

R = revolutions per minute.

The angle ϕ varies for different makes of crushers from 20 deg. to 26 deg.

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order to obtain the actual capacity, the theoretical capacity should be divided by 1.6. This is on account of irregularity in feed, hardness of material, size of material, etc.

In the smaller sizes of crushers, where the breadth b is small and the stroke s is large, comparatively, it is found that the theoretical capacity M is as much as 3¼ times the actual capacity. This is due to the fact that these smaller sizes of crushers are very inefficient when compared to the larger sizes; this is found for example to be the case of the 10 in. x 7 in. machine as shown in Table 2. Therefore, the empirical formulas as herein given will not apply to these small crushers.

Power required to crush the material (dry limestone)

$$p_c = p\,M$$

where p_c = horse power.

p = take from curve—Fig. 6.

It is assumed that the size of the product will never be less than one inch and that the reduction will never be more than 6 to 1.

The total power required to operate the crusher when doing work

$$P = p_m + p_c$$

Where P = horse power.

The above formulas have reference only to the crushing of dry limestone. The weight of one cubic foot of broken dry limestone is approximately 100 lb.

As the starting conditions are usually severe, the slip ring type of induction motor is invariably recommended to drive jaw crushers.

Fairmount Crusher

Many of the more recent crushing plants for cement works use the Fairmount type crusher. This machine consists of a single roll working against an anvil plate. The roll is provided

TABLE 2.
Power Requirements of Blake Crushers.
Dry Limestone.

Size or jaw opening $b \times w$	Capacities in tons per hour											Approx. horse power req'd
	Size of product											
	¾ in.	1 in.	1½ in.	2 in.	2½ in.	3 in.	4 in.	5 in.	ϕ	s	R	
10 × 7 in.	1.5	2.5	4	5					26°	5/8 in.	275	7
15 × 9 "		6	8	10	12				26°	7/16 "	275	10
20 × 10 "			10	15	17.5	20.			26°	1/2 "	275	15
24 × 12 "				20	25	30.	35		26°	9/16 "	275	25
30 × 18 "					20	37.5	45	50	26°	9/16 "	275	40

with knobs from two in. to three in. in height for crushing against the anvil plate and with sledging knobs from three in. to four in. in height for breaking the large pieces of stone on top of the roll, which are too large to be directly gripped between the roll and the anvil plate. These machines have a very large capacity when operating continuously, but are not objectionable for small capacity on account of their very low friction load when running idle.

The Fairmount type crusher is built in sizes 24 × 36 in., 24 × 48 in. and 24 × 60 in., also 36 × 60 in. and 60 × 84 in., in each case the size being referred to diameter and face of the roll. The size which has so far been used in cement plants is 36 in. diameter by 60 in. face. This size of machine ordinarily requires a 150-h. p. motor to drive it and it has a capacity of from 200 to 500 tons per hour, depending upon the character of the material to be crushed. One of the particular advantages of the Fairmount type crusher is that it makes a very large reduction in one operation so that the number of secondary crushers is reduced and the product from the primary crusher being smaller can be much more easily handled into and out of storage bins.

The starting torque required for the Fairmount crusher when idle, is very low so that squirrel-cage motors are often used. While it is not possible to start the crusher full of stone even with slip-ring type induction motor, the latter will give sufficient torque to start up the crusher without entirely cleaning it out, if it should happen to stop under load. Slip-ring type motors are therefore, recommended for this class of work.

Crushing Rolls

The most favorable condition for crushing rolls is obtained when the ratio of size of feed to product does not exceed 4 to 1. When this ratio is exceeded, the efficiency of the roll is greatly decreased and an increase in the power required will be noted. With the ratio of 4 to 1, the power requirement remains practically constant whether the reduction is from 4 in. to 1 in. or from 1 in. to ¼ in. The amount of material crushed, however, will be in proportion to the size, *i. e.*, when reducing from 4 in. to 1 in. the rolls will handle four times as much material as when reducing from 1 in. to ¼ in; all other things such as size of rolls, speed, etc., remaining constant. Any increase or decrease in the speed of the rolls will increase or decrease the quantity handled and the power required.

Due to irregularity of feed, variation in size and hardness of material, etc., it has been found that the power required is not steady but fluctuates as shown by Fig. 7 which is a power diagram of a 42 × 16-in. roll operating at 60 rev. per min. and reducing dry iron ore from a maximum of two inches to an average of one-half inch, the average power being 13.18 h. p.

Fig. 8 shows the relation existing between horse power per cubic foot per hour and different ratios of reduction assuming maximum size of feed material and average size of product, all friction of the rolls being neglected. This curve has been plotted as an average of points obtained from actual tests. It is to be noted, however, that while dry material has been rolled, most of the points indicate power readings taken while crushing iron ores, and that only one point refers to the rolling of dry

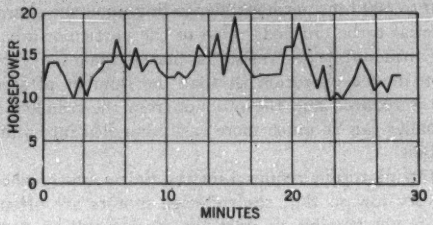

FIG. 7

limestone such as is used in the cement industry. As the iron ore is a harder substance than the limestone, it is reasonable to assume that the curve as drawn shows values slightly higher than would be obtained had more data been available on dry limestone.

The friction of crushing rolls, that is, the power required to drive the rolls at their normal speed and without load, varies considerably for different sizes, speeds and makes of machines, but an approximate average is obtained from the following:

$$P_1 = 0.0835\,(D + W)$$

where p_1 = horse power
D = diameter of rolls in inches
W = face of rolls in inches.

When figuring the power requirements of crushing rolls it is preferable to obtain the friction of the rolls direct from the manufacturers.

The amount of material passed by the rolls is

$$M = \frac{d \times W \times S}{144} \times 60$$

where M = cubic feet of material per hour
d = distance in inches from edge to edge of roll (See Fig. 9)
W = face of rolls in inches
S = peripheral speed of rolls in feet per minute.

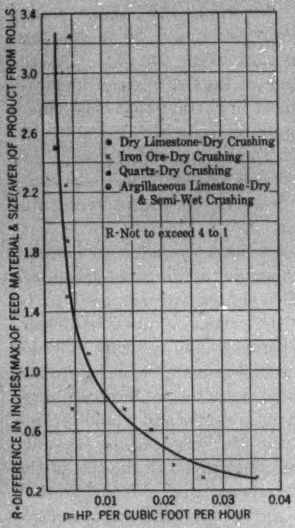

FIG. 8

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order, however, to obtain the actual capacity, the theoretical capacity should be divided by 4. This is on account of irregularity in feed, etc.

The total power required to drive rolls for any different rolling condition is

$$P = (p \times M) + p_1$$

where P = total horse power
p = horse power per cubic ft. per hr. (Fig. 8)

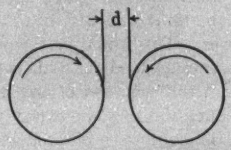

FIG. 9

As the starting conditions are not severe, the squirrel-cage induction motor makes a suitable machine for driving the rolls, provided the conditions are such that this type of motor is connected to a source of power where line disturbances are not objectionable. If line disturbances are objectionable, the slip-ring type of induction motor should be used. When shutting down the mill, the usual procedure is to first cut off the feed to the rolls, allowing same to empty themselves, before closing down the motor. No difficulty, however, is likely to be exper-

ienced with the squirrel-cage type of motor, if called upon to start the rolls when full of material and with the feed open, as even under these conditions the starting requirement is not severe.

Intermediate Crushers

Hammer Mills. These machines are made in several styles but all make use of the same principle, namely, the use of hammers revolving at high speed and abrading the material against a slotted iron casing. Unlike the crushing rolls, where a reduction not exceeding 4 to 1 is most favorable, the hammer mill is used for reducing material of from 2½ to 1 in. to 20 mesh or less, the maximum reduction in this case being 76 to 1. As the power required to drive this type of mill varies considerably, due to irregularity in feed, it is most important that this remain steady and therefore a feed regulator is highly desirable. In addition to this, any variation in the size and hardness of material, provided the feed remains fairly constant, will also cause power fluctuations which will however be within reasonable limits.

There is very little general power data available on hammer mills other than for a specific manufacturer's type, trade size and specific speed or speeds. The latter are supposed to be the most efficient speeds at which the particular mill should operate. A general rule gained by experience indicates that in order to crush one ton of dry limestone in one hour, reducing from a maximum size of 2½ in. to 20 mesh, requires six to seven horse power. This would be the output of a motor and would include the friction load, in addition to the crushing load, of the mill. The actual capacity of any mill is proportional to the size of the product and to some extent to the character of the feed so that for different sizes of product on any given mill, the power requirements will be different.

Sturtevant Hammer Mills are of two types known respectively as hinged hammer and hammer bar mills. The hinged hammer mills are of the general type noted above under this class. The bar hammer mill differs in that there is only one set of hammers instead of six or more and the hammers themselves have an axial length the full width of the rotating spider which carries them. Both mills take material in pieces from 5 to 6 in. wide. The hinged hammer mill reduces from one inch to 20 mesh and finer. It is built in three sizes and operates at a speed of 1000 to 1500 rev. per min. The hammer bar mill is built in two

sizes and reduces from ½ in. to 20 mesh. A belted wound-rotor motor is the preferred application. Squirrel-cage motors have, however, proved satisfactory, particularly if used when line disturbances at starting are not objectionable. In any case, if a mill is shut down it will have to be clear of material before started again.

Hammer bar mills are made in two sizes, No. 0 and No. 1. The No. 0 mill will handle from 1½ to 4 tons per hour, depending on material, and requires from 8 to 10 h. p. The No. 1 handles from 4 to 12 tons per hour and takes 20 to 25 h. p.

Hinged hammer mills are made in three sizes 30 by 12 in., 30 by 18 in. and 30 x 24 in. The power varies widely with the material. The 30 x 12-in. machine, for example, might take a maximum of 30 h. p. or a minimum of 10 h. p. The wider machines take increased power in proportion to the width.

TABLE 3.
Power Requirements of Hammer Mills.
(Allis-Chalmers)

Trade size	Rev. per min.	Hammer circle	Face	H. p.
No. 1	1800	24 in.	9 in.	15
" 2	1200	30 "	12 "	30 to 40
" 4	1200	42 "	24 "	75 to 90

The above requirements are based on crushing dry limestone, reducing from a maximum size of 2½ in. to 20 mesh with a constant feed of material best suited to the particular mill.

Williams' Jumbo Crusher. The machine of this make used for intermediate crushing is usually the No. 6 size. It takes stone three inches in size and under and reduces it to ¼ inch and finer. The capacity is 60 to 65 tons per hour, and the motor required is 150 h. p. The mill operates at from 720 to 750 rev. per min. It prepares material for some one of the various finishing mills.

Vulcanite Crushers. (Williams). The Vulcanite crushers are also swing hammer mills. They are built in eight sizes having capacities from two tons per hour at ¼-inch size up to 50 tons per hour at ½-inch size. The standard unit generally employed for cement work is No. 3. This takes material up to three inches and reduces it to ¼-inch for feeding ring roll mills. It handles 15 to 18 tons per hour, operates at a speed of 1000 rev. per min. and requires 50 h. p.

Universal Grinder (Williams). This is termed a tube mill feeder as it will take limestone 2½ inches and under after passing through the dryers and reduce it to a product, 95 per cent of which will pass a 20 mesh sieve. These are built in ten sizes, the two most commonly used in cement plants being the No. 3 and the No. 9. The No. 3 handles from 8 to 10 tons per hour under the above conditions, requires from 50 to 60 h. p., and should operate at a speed of 1100 rev. per min. The No. 9 size, used in plants requiring very large units, handles from 25 to 30 tons per hour, requires 175 to 200 h. p., and operates at a speed of 720 rev. per min. Wound rotor induction motors are the best application on all three mills. The preferred method of drive is by motor direct connected through a flexible coupling.

Sturtevant Rotary Crushers. This type of rotary crusher operates on the principle of the old fashioned coffee mill. It

TABLE 4.
Power Requirements of Rotary Crushers.

Machine size	Approx. capacity in tons per hour	Rev. per min.	Approx. h. p.
00	1 to 1½	300	1 to 2
0	½ to 2	250	3 to 4
1	1 to 6	300	6 to 10
1½	4 to 10	200	15
2	8 to 15	250	15 to 20

is shaped like an hour-glass or double cone, one cone mounted and reversed on the other. The coarse material is introduced at the top and is acted upon by a rotating cylindrical member which exerts a continual nipping action on the rock since the diameter of the stationary cone decreases as the material falls toward the center. Below the center the material is ground or shredded by the usual mill stone action. The lower portion of the rotating member is cone-shaped and so arranged as to leave a tapering annular space decreasing in size toward the bottom. As the material proceeds by gravity toward the bottom, it is ground and reduced in the annular space by the action of the rotating cone against the stationary one. Both these members are faced with hardened steel burrs or grinding members. These crushers are intended for moderately hard or soft rocks but are not used for hard cutting substances. They crush large rocks

to approximately ⅛ inch without screens in the smallest sizes and as coarse as 1 inch in other sizes. Their sizes, outputs and power requirements are as shown in Table 4.

A wound-rotor induction motor is the preferred application for these crushers but a squirrel-cage motor can handle them if designed with considerable margin in starting torque.

Rotary Dryers

Dryers are used to dry the crushed limestone which contains a considerable amount of moisture that must be driven out before the grinding operation can be performed. They are also used for the same reason, for drying the clay and coal. The dryer consists of a large revolving cylinder which is sufficiently inclined to roll the contents through by gravity. It is heated either by oil, natural gas, or by the waste gases taken from the kilns.

Auxiliary machinery necessary for the dryer, are fans and conveyers, and usually this auxiliary machinery is considered as integral parts of the dryer, and is driven by the same motor. For a fixed rate of feed the load is constant and as a rule the dryers are operated at a given predetermined speed; therefore, squirrel-cage motors are satisfactory for this kind of service. Squirrel-cage motors with 125 per cent to 175 per cent starting torque, are suitable.

Approximate horse power requirements are as follows:

Size of dryer	Horse Power	Recommended speed of motor.
6 ft. x 60 ft.	12	700 to 1200
6½ ft. x 60 ft.	15	700 to 1200

On some particular installations, the following was found. A dryer 6 ft. in diameter 45 ft. length, 9 rev. per min., connected to a 24-in. x 50-ft. horizontal pan conveyer and to a 14 in. x 35-ft. bucket elevator, required a 30 h. p. motor. A 7-ft. x 60-ft. dryer for rock crusher, connected to a 20 in. x 45-ft. inclined conveyer, and 16 in. x 40-ft. elevator, required 40 h. p.

Ordinarily, dryers of 5-ft. diameter and 50- to 60-ft. length, revolving at 3 to 5 rev. per min., were found to require between 10 and 20 h. p.

PRELIMINARY GRINDING MACHINERY

Ball Mills. The ball mill consists principally of a revolving horizontal cylinder lined with overlapping iron plates and containing a charge of steel balls. For the first charge, balls of

3½, 4 and 5 inches diameter are supplied, but under running conditions only the largest size are added from time to time to maintain the charge. These mills will take material up to 6 inches in size. The material as ground falls through perforations in the iron plates to the cylindrical screen or hopper which surrounds the mill, and the oversize rock falls back through the same perforations to the grinding plates. The fine product is removed by a conveyer below the machine and is generally of such fineness that all will pass a 20 to 30 mesh. When pulverizing to pass all through a 20 mesh, from 30 to 40 per cent will pass a 100 mesh sieve.

Ball mills are low-speed machines generally running from 20 to 25 rev. per min. The charge of balls revolving at this speed causes very severe vibration in the gearing, and motors geared

TABLE 5.
Power Requirements of Ball Mills.

Make	Weight of charge lb.	Capacity on cement clinker bbl. per hr.	H. p. req'd.
Gates No. 7	3,000	12–16	30– 40
" " 8	4,500	18–24	40– 50
Smidth (Kominuter) No. 53–½	3,000	.....	30
" " " 66	6,600	.....	75
" " " 88	....	.....	100
Krupp No. 7	3,520	12–16	30– 35
" " 8	4,400	16–22	35– 45
Chalmers & Williams	10,000	60–70	100–125

to their countershaft have proved unsatisfactory. Belted motors or low-speed motors for direct connection to the countershaft through flexible couplings give entire satisfaction. When starting, the charge of balls and stone must be rotated through nearly 90 deg. before it begins to roll, and the motor torque required at starting varies from 1¾ to 2 times full-load torque. Squirrel-cage a-c. motors specially designed for this high starting torque have been successfully applied but they have a lower efficiency than wound-rotor motors and make a large current demand at starting. The same starting conditions are found in tube mills and as the size of these has steadily increased of late years, and as the power required has also increased with the use of metallic grinding bodies in the mills, there has been a gradual change to the use of wound-rotor motors in place of the

high-torque squirrel-cage motor. Many of the latest mills are using wound-rotor motors for these grinders of large capacity and the saving in power due to increased efficiency, together with better performance at starting, warrants their use in preference to the squirrel-cage type. Table 5 indicates the power requirement for a few typical ball mills.

Hammer Mills. These mills which have been described above are used for preliminary grinding as well as for crushing.

Ring-Roll Mills

There are a number of different mills that may be classed generally as ring-roll. In these, the material is crushed by means of a ring or large balls rolling inside and against the ring or die. In some types this ring rolls also. There is enough difference in mechanical construction to require a brief explanation of each mill. Under this general type are classified the Kent mill, Griffin mill, Fuller mill, Bonnot mill, Raymond mill, Sturtevant Ring-Roll mill, Huntington mill and others.

These different types of mills are used in many combinations depending on the material and the locality in which they are employed. An analysis of about fifteen mills shows that ball mills are widely used as intermediate grinders taking material from the crushers on the raw side and the kilns on the clinker side and reducing it to a suitable fineness for feeding tube mills. In another combination the ball mills are replaced by ring-roll mills. Another combination uses hammer-mills to reduce the material to ¼ inch and smaller, after which it is crushed in ring-roll mills. In other plants the entire operation of grinding the clinker in the finishing mill is performed by ring-roll mills.

Fuller Mill. Three sizes of this mill are manufactured—33-inch, 42-inch and 57-inch. The smallest mill is best adapted for use on coal or similar materials while the two larger machines are adapted either to coal or harder material such as limestone, rock or cement clinker. This mill accomplishes both intermediate and fine grinding in one operation. It takes material up to ¾ inch in size and reduces it to a fineness of 85 per cent through a 200 mesh sieve. The mill is vertical in construction and the grinding is done by four chilled unattached balls rolling against a circular horizontal die. The balls are propelled by four equi-distant horizontal arms or pushers radiating from the vertical shaft. The shaft is driven at such a speed as to cause the balls to exert a pressure against the die of approximately

1600 pounds. The material is fed in at the top of the mill by a screw and enters the pulverizing zone. By means of fans and screws the material is removed as soon as it has reached the proper degree of fineness. The driving pulley is mounted at the bottom. Squirrel-cage induction motors are used very satisfactorily to drive these mills through belts running in a vertical plane. Special vertical motors have been developed for this application.

The output of these mills and the power required to operate them depends upon the weight and hardness of the material being pulverized and the fineness of finished product desired. On the 33-in. mill the output will vary from two tons per hour when grinding hard material, to four tons per hour when grinding soft material. The power will approximate 45 h. p. for hard material and 25 h. p. for soft material. The above figures are based on grinding to 100 mesh. On the 42-inch mill the output will vary from 4 to 10 tons per hour from hard to soft material, and the horse power will vary from 75 to 45 when grinding to 100 mesh.

The 57-inch mill when grinding raw cement material consisting of limestone and shale or clay or cement rock has a capacity of from 9 to 12 tons per hour and requires from 110 to 125 h. p. The same mill when pulverizing cement clinkers has a capacity of from 500 to 600 barrels per day and requires from 135 to 150 h. p. The performance of the 57-inch mill is based on grinding to 85 per cent through a 200 mesh sieve.

It should be borne in mind that the power required for the different machines as given in this paper represent average performances. It is possible under certain conditions to crowd all these machines thereby increasing their output and also increasing the power consumed so that specific instances are no doubt known to all engineers interested where the figures are somewhat higher than here given. These are matters that take care of themselves in practise.

Kent Mill. The Kent mill consists principally of a revolving ring and three rolls pressing against its inner face. The rolls are convex and the ring is concave and tracks on the rolls. Springs support the rolls yieldingly and the rolls support the ring so that the four crushing parts are free to move. The material falls from the inlets onto the inner face of the ring, and centrifugal force holds it there in a layer an inch deep as it passes under the rolls. The latter are pressed by the springs

outwardly against the rock or the ring with a pressure adjustable to 20,000 pounds, the adjustment being accomplished by means of the screws against the springs. As the rolls pass over the rock they crush it against the ring, while the crushed rock falls off each side of the ring into the casing and then into the discharge. It is claimed that 90 per cent of the rock is abraded on itself in crushing so that the wear on the parts of the machine is materially decreased.

The torque required to start this mill does not exceed full-load torque, since the moving parts are comparatively light and there is only a small amount of rock in the machine at a time. The pull-out torque of the motor supplied should be comparatively high as large pieces of rock passing between the rolls and ring will cause severe momentary overloads.

In order to obtain a fine and uniform product, the ground material is usually elevated from the discharge to a screen or separator above the mill and the oversize is returned to the feed. The elevator and separator will add from 7½ to 10 h. p. to the figures given below for the mill alone. The feed should preferably pass through a one-inch ring but anything up to two inches can be handled. The finished product will all pass a 20 mesh.

These mills are made in three sizes known as the Kent mill, the "Maxecon" and the "Big Maxecon" mill. Their capacity depends on the hardness of the material and the fineness to which it is ground. Round approximations might be 25 barrels per hour for the Kent, 35 for Maxecon and 50 for Big Maxecon. These mills operate at 200 rev. per min. and the usual sizes of motors are 30, 40 and 50 h. p. for the Kent, Maxecon and Big Maxecon respectively. When grinding clinker one h. p. per barrel is the average power consumption to pass 20 mesh; also the product will approximate 50 per cent through 100 mesh and 35 per cent through 200 mesh.

Raw rock varies so in hardness that capacities range from 7 to 16 tons per hour, and power estimates from five h. p. per ton for the hardest to three h. p. for the softest.

Sturtevant Ring-Roll Mill. This ring-roll pulverizer consists of a steel anvil ring or die secured in a head supported and revolved by a horizontal shaft. Against the inner face of this ring the three hammer rolls are equally and elastically pressed with great force and revolved by the ring. Substances to be ground are fed to the anvil face of the rotating ring and held thereto by centrifugal force and are crushed by being drawn

under the rolls, as the ring revolves. The face of the anvil ring is concave and the rolls convex, thereby crowding the material as it is finished to the outside edges of the ring. These mills may be used with elevator and separator, thus securing very uniform product. They take material up to 1½-inch size and reduce it to a fineness varying from ⅛ inch to 100 mesh or finer. They are usually employed as a preliminary pulverizer rather than a finishing mill for cement work. As is the case in mills of this type, the operation of crushing is largely performed by abrading the rock on itself so that the wear on the mill parts is reduced. These mills are made in five sizes; three sizes of single mills and two of duplex mills which are a double unit. Table 6 gives the sizes, horse power and speed. Regarding the output, the manufacturers furnish this on request, since it varies widely with the nature and conditions of the material handled. As an illustration the following figures are cited for the No. 2 duplex mill. Grinding cement clinker to 20 mesh, 80 to 100 barrels per hour, and to 80 mesh, 30 to 40 barrels per hour. Grinding limestone to 20 mesh, 16 to 20 tons per hour; and to 80 mesh, 8 to 14 tons per hour.

TABLE 6.
Power Requirements—Sturtevant Ring-roll Mills.

Size	Pulley speed rev. per min.	Ring speed rev. per min.	Approx. h. p.
0	125	125	8 to 12
1	320	80	15 to 20
2	300	63	35 to 45
No. 1 Duplex	375	80	30 to 40
No. 2 Duplex	325	64	70 to 90

The elevator and separator if used add about 8 h. p. per single mill. A squirrel-cage induction motor is the preferred type for these mills.

Huntington Mill. This ring-roll mill is a vertical shaft machine with a horizontal spider from the rim of which are suspended four spindles. These spindles carry at their lower end a heavy horizontal crushing roll. The rotation of the main shaft causes these crushing spindles to swing out by centrifugal action and roll against a ring die. The material is fed against this die and crushed by the rolls.

These mills are made in four sizes, but the largest size is suited only for wet grinding. The other three sizes are 3½ ft.,

5 ft., and 6 ft. and require 5, 8, and 10 h. p. respectively. This mill is a preliminary pulverizer and not a finishing mill. A belted squirrel-cage motor is a satisfactory application.

Bonnot Mill. This ring-roll type of mill employs air separation of the material. The driving shaft is horizontal and the grinding ring or die is held in a vertical plane in the main frame. The rolls are carried by a head or driver mounted on the main shaft and are free to be thrown outward by centrifugal force, thus furnishing the crushing pressure. The material thrown upward into the screening chamber by the motion of the grinding parts is deflected against the screen by the blades of a fan. The coarse material strikes a baffle and falls fack into the grinding chamber while the fine material passes through a screen and drops through passages into the conveyer. The 36-inch size is used for cement; it operates at 180 rev. per min. and requires 75 h. p. It takes pieces up to two-inch size and when working on clinker will handle 13 to 16 barrels per hour of which 96 per cent will pass 100 mesh and 77 per cent 200 mesh. It will be seen from this that it is a finishing mill as well as a preliminary grinder. When grinding raw material and fed through ⅝-in. grate it will handle 4 to 5 tons per hour, 97 per cent through 100 mesh. A squirrel-cage a-c. motor is the preferred application.

Griffin Mill. This mill employs in its construction the principle of a roll running against a ring or die. Power is received by a pulley running horizontally, from which, by means of a universal joint, a shaft is suspended in pendulum fashion. To the lower extremity of this shaft is rigidly secured the crushing roll which is thus free to swing in any direction within the case. The base or pan contains the ring against which the roll works and upon the inner surface of which the pulverizing is done. On the bottom of the roll are shoes for stirring up the material and throwing it against the ring, and above the roll are attached blowers for blowing the fine material through the screen which surrounds the base. When at rest the roll hangs vertically beneath the driving shaft, but when revolved it flies out to the die ring and travels around in a direction opposite to that of the driving pulley. There is a pressure of approximately 6000 lb. due to centrifugal force brought to bear on the material being pulverized between the roll and the die. These mills are manufactured in three forms known as the "Bradley Hercules," the "Bradley 3-Roll" and the "Giant Griffin" mill. The Bradley Hercules takes material two inches

and under and reduces it to a fineness of 53 per cent through a 100 mesh screen at the rate of from 40 to 50 tons per hour, using about 250 h. p. in the operation. When used as a preliminary grinder of cement clinker it has an output of from 150 to 175 barrels per hour and requires 250 h. p. to drive.

The Giant Griffin mill has an output when grinding clinker of from 25 to 30 barrels per hour to a fineness of 60 per cent through a 100 mesh screen. It takes material ¾ inch and under for feed and when operating to full capacity requires about 80 h. p. This mill is a very efficient preliminary machine on account of the high percentage of fineness and has sometimes been used as a finishing mill.

The 3-roll mill in theory is similar to the Griffin, but employs three rolls instead of one. A squirrel-cage a-c. motor is the preferred application for these mills except the Hercules which is better equipped with a phase-wound-rotor motor on account of the starting conditions on so large a unit.

Raymond Mill. This mill performs the operations of preliminary grinding and finishing in one process. The grinding ring is mounted in a horizontal plane and the grinding is performed on its inner surface by means of rolls suspended from a rotating driving head or spider. There are two, three, four, or five of these rollers, depending on the size of the mill. A prominent feature of this mill is the system of air separation employed. When handling raw material the mill will handle about one ton per hour per roll, *e. g.*, the 4-roll and 5-roll mills will handle four and five tons per hour respectively from a size of ¼ inch to 1 ½ inches to a fineness of 95 per cent to 98 per cent through a 100 mesh sieve. There are six sizes in all called two-roller mill, three-roller mill, four-roller mill (low side or high side), and five-roller mill (low side or high side). The power consumption is given as follows:

2 rolls, 45 to 50 h. p.	3 rolls, 55 to 60 h. p.
4 " 75 to 80 h. p.	5 " 85 to 90 h. p.

The driving motor is belted. A squirrel-cage induction motor with good starting characteristics is the preferred application.

Emerick Ball Pulverizer. This is a vertical mill and the crushing is done by four or five 12-inch or 14-inch steel balls which are unattached and roll in and against a grinding race. These balls are driven by a head or spider which keeps them properly separated. The centrifugal force set up in the balls by the rotation of the driving head causes them to exert great pressure

against the grinding race, thereby crushing and abrading the material in the mill. The mill is arranged with five 12-inch, four 14-inch, or five 14-inch balls. This mill may be used either as a preliminary grinder or as a finishing mill. When used as a finishing mill it is equipped with an air separator and will reduce from ½ inch and under to 95 per cent to 98 per cent through 100 mesh and 80 per cent to 90 per cent through 200 mesh. On raw material or coal it will handle six to eight tons per hour and clinker 12 to 20 barrels of finished cement per hour. The mill operates at 90 to 130 rev. per min. and requires 35 to 60 h. p. A belted squirrel-cage motor is the proper application either vertical or with a quarter-turn belt.

Symons Disk Crusher. This crusher is of a special type in which the crushing is done by two dish-shaped disks mounted on horizontal shafts which are separate but one of which gyrates slightly with respect to the other. The two disks are set with their concave sides facing each other and are rotated in the same direction at the same speed. This results in the hollow between the two disks having a wider opening between their edges at one part of a revolution than on another. This progressive pinching action admits a piece of coarse material at one side and as the disks rotate they approach one another thus crushing the material which is then thrown out through the opening between the edges of the disks if it is reduced to the proper fineness; if not, the process is repeated.

A wide range of adjustment is provided by changing the distance between the crushing disks. These crushers are made in four sizes as given in Table 7. A squirrel-cage motor with a maximum starting torque of 1 ½ to 1 ¾ full-load torque is satisfactory on this application.

TABLE 7

Power Requirements of Disk Crushers.

Size of crusher	18 inch	24 inch	36 inch	48 inch
Size of material fed	1¼ "	2½ "	3½ "	6½ "
Size to which reduced	⅜ "	½ "	¼ "	1 "
Tons per hour	5 to 8	12 to 15	25 to 30	45 to 60
H. p. required	12 to 18	18 to 25	30 to 40	50 to 65

Such material is too coarse to finish in tube mills, but could be handled by some of the types of ring roll finishing mills described in this paper.

FINISHING GRINDING MACHINERY

Tube Mills

For the fine grinding of limestone or clinker, the tube mill is used extensively, both wet and dry. The usual form is that of a rotating cylinder and the grinding is done by attrition or by the rubbing together of the grinding medium and the material to be ground. The mill is generally filled to about the center line, and the grinding medium, or charge, as it is usually called, consists of either flint pebbles, cast iron balls, or forged steel balls, etc., depending on the design of the mill and the compara-

TABLE 8

PLAIN TUBE MILLS WITH FLINT PEBBLES.

Operating in Portland Cement Plants with 20 Mesh Feed and Grinding, to 95% through 100 mesh and 80% through 200 mesh.

Size of Mill	Capacity on raw material		Capacity on clinker	Horse power		R.P.M. mill	Initial charge pebbles
	2000-lb. tons per 24 hr.	666-lb. bbl. per 24 hr.	380-lb. bbl. per 24 hr.	to start	to run		
3 ft. 6in. × 20 ft.	50	150	120 to 144	60	35 to 40	40	9,800
4 ft. 0 in. × 20 ft.	64	192	192	70	45 to 48	35	11,600
4 ft. 6 in. × 22 ft.	80	240	240	85	55 to 60	31	14,500
5 ft. 0 in × 22 ft.	117	350	350	120	75 to 80	28	20,000
5 ft. 6 in. × 22 ft.	157	470	480	150	95 to 100	26	24,000
6 ft. 0 in × 22 ft.	192	575	575	185	115 to 125	24	29,000
7 ft. 0 in. × 22 ft.	280	840	815	250	160 to 175	20	39,000

The above figures are based on the charge and material level being carried at the center line of the mill.

tive costs of the various grinding mediums. There are several types of tube mills in commercial use, designated as follows:

1. The plain tube mill using flint pebbles as a grinding medium.

2. The plain tube mill with ball-peb compartment. This type of mill has a partition located near the discharge end. The large compartment on the feed end is charged with flint pebbles, while the small compartment on the discharge end is charged with cast iron or forged steel balls.

3. The ball-peb mill using cast iron or forged steel balls as a grinding medium.

4. The combination or compeb mill with a partition located near the feed end. The small compartment of the feed end is charged with large cast iron or forged steel balls while the large compartment on the discharge end is charged with small cast iron or forged steel balls.

The power requirements of these different types of mills vary not only with the size of the mill, but also with the nature and amount of the charge used. Once the mill is started, the load is practically constant and only slight pulsations in power can be noticed. The power required to drive these mills has been determined from actual tests and refers to dry grinding conditions only. For wet grinding the power required is approximately 30 per cent less than for dry grinding, other conditions remaining the same. The power requirements for the four types of mills, as noted above, are as follows:

1. The plain tube mill using flint pebbles—see Table 8.
2. The plain tube mill with ball-peb compartment; add 20 per cent to the power requirements of the plain tube mill using flint pebbles.
3. The ball-peb mill; see Table 9.
4. The combination or compeb mill; see Table 9.

The most economical speed for any mill should be such that the bulk of the charge is carried to the highest possible point in the mill before falling and this without any of the charge or material being carried around by centrifugal force. This is approximately

$$s = \frac{195}{\sqrt{d}}$$

where s = revolutions per minute.
d = diameter of mill in inches, including the lining.

On this basis of speed and for the dry grinding of limestone and clinker, the power required to drive any of these various types of mills may be approximated by the following:

$$P = \left\{ [0.0034 \ (W + W_m)] \frac{R}{r} + 0.00015\, w \right\} \times \frac{S}{s}$$

where P = Horse power.
W = Weight of charge of pebbles or balls in pounds.
W_m = Weight of the material being ground in mill.
w = Weight of revolving elements of the mill in pounds.

TABLE 9.

BALL-PEB AND COMPEB MILLS.

Operating in Portland Cement Plants with 20 mesh feed and grinding to 95 per per cent through 100 mesh and 80 per cent through 200 mesh.

Dia.	r. p. m.	Periph speed	Length of Mill in feet																	
			5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Horse power to run mill																	
5 ft.	24	362	44	50	58	66	74	82	90	98	106	115	123	131	139	147	155	164	172	180
	25	378	46	52	60	68	77	85	94	102	110	119	127	136	144	153	161	170	178	187
	26	392	48	54	63	71	80	89	98	106	115	124	133	142	151	160	168	177	186	195
	27	407	50	56	65	74	83	92	101	110	119	128	137	147	156	165	174	183	192	202
	28	422	52	59	67	77	86	96	105	115	124	134	143	153	162	172	181	191	200	210
5½ ft.	22	368	54	61	71	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220
	23	386	56	64	74	84	94	105	115	125	135	146	156	167	177	187	198	209	219	230
	24	402	58	67	77	88	98	109	120	131	142	153	164	175	185	196	207	218	229	240
	25	418	60	69	80	91	102	114	125	136	147	159	170	182	193	204	215	227	238	250
	26	436	62	72	83	95	107	118	130	142	153	165	177	189	201	213	224	236	248	260
6 ft.	20	367	63	73	84	96	108	119	132	144	155	167	179	191	203	215	227	239	251	263
	21	385	66	76	88	101	113	125	138	150	163	176	188	201	213	226	238	251	263	276
	22	403	69	79	92	105	118	131	145	157	170	184	197	210	223	236	249	262	275	289
	23	422	72	83	96	110	124	138	152	165	178	192	206	220	234	248	261	275	289	303
	24	440	75	86	100	115	129	143	158	172	186	201	215	230	244	258	272	287	301	316
7 ft.	17	366	81	97	113	129	145	161	178	193	209	226	242	258	274	290	306	323	339	355
	18	387	86	102	119	132	154	170	188	204	221	238	250	263	285	307	324	341	358	375
	19	408	91	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396
	20	428	96	114	133	152	171	190	208	227	246	265	284	303	322	341	360	379	398	417
	21	452	101	119	139	159	179	199	218	238	258	278	298	318	338	358	378	399	418	438
8 ft.	15	369	103	129	150	172	193	205	236	257	278	300	321	343	364	386	407	429	450	472
	16	394	115	137	160	183	206	228	252	274	297	320	343	366	389	412	434	457	480	503
	17	418	122	146	170	195	219	243	268	292	316	340	364	389	413	438	462	487	511	535
	18	443	130	155	180	206	232	257	284	309	334	360	386	412	437	463	489	515	541	567

The above figures are based on the following:

5 ft. 0 in. dia. mill—charge and material level within 6 in. of center line of mill—1800 lb. initial charge per ft. length of mill
5 ft. 6 in. " " " " " " " 7 in " " " " " 2200 " " " " " " "
6 0 " " " " " " " 8 " " " " " " 2600 " " " " " " "
7 0 " " " " " " " 10 " " " " " " 3600 " " " " " " "
8 0 " " " " " " " 11 " " " " " " 4700 " " " " " " "

s = Theoretically correct speed in rev. per min.
S = Actual speed of mill in rev. per min.
R = Distance in feet from center of mill to center of gravity of mass $W + W_m$.
r = Distance in feet from center of mill to center of gravity of mass $W + W_m$. When mass comes to center line of mill. $r = 0.0177\,d$.
When a metallic grinding medium is used $W_m = 0.304\,W$.

This is based on one cubic foot of charge and material weighing 314 pounds and consisting of 280 pounds of charge and 34 pounds of material. When a flint pebble medium is used $W_m = 0.121\,W$. This is based on one cubic foot of charge and material weighing 150 pounds and consisting of 115 pounds of charge and 35 pounds of material. In the above equation S should not vary more than 15 per cent above or below s.

For operating ball mills both squirrel-cage and wound-rotor type motors are used. The tendency is towards the larger sizes of mills with metallic grinding bodies thus requiring much larger motors than was common a few years ago. These mills are rather difficult to start since in order to get them in motion the charge has to be lifted, and in the case of large squirrel-cage motors this causes a heavy current demand at low power factor even when special high-torque motors are provided. Also in the larger motors the higher efficiency of the wound-rotor type is an item to be considered. Hence for the large mills wound-rotor motors are to be preferred.

The tendency is also towards lower speed motors geared to the mills in place of the higher speed belt driven type previously used. In case of geared drive a flexible coupling should be provided on the motor as there is always considerable vibration present. The low-speed motor has the disadvantage of higher cost and lower power factor, but the saving in space, belting, etc. offsets these disadvantages to a large extent.

There is a trend in some of the latest installations towards the use of synchronous motors for these mills especially where low-speed motors are used. In such applications it is necessary to provide a clutch so that the motor can be started light, but the well known advantages of the synchronous motor are such that it is probable their use for grinding in cement plants will become much more common in the future than it has been heretofore.

Rotary Kilns

For supplying rotary kilns powdered fuel is required in most cases and the crushing plant for this uses the same class of crushing and grinding machinery as has already been described for handling limestone and cement.

The kiln itself is a long cylinder lined with fire brick, revolving on an inclined axis and fired from the lower end. The material is introduced at the upper end and works its way through the kiln in from one to two hours. A speed variation is necessary on the kiln. Therefore either a squirrel-cage motor with speed change gearing or a wound-rotor motor with resistance controller is required; the latter is preferred. The controller should be such as to permit 50 per cent speed reduction on full-load torque. Since the kiln radiates a large amount of heat it is preferable to place the motor below the floor or to install it in

TABLE 10

Power Requirements of Rotary Kilns.

Size of kiln	Rev. per min.	H. p.
6 × 60 ft.	1 to 3	5 to 7
7 × 80 "	1 to 2	8 to 12
7½ × 100 "	¾ to 1–½	12 to 18
8 × 120 "	½ to 1	15 to 20
9 × 150 "	3/8 to ¾	22 to 30
10 × 170 "	¼ to ½	30 to 40

such manner as to protect it as far as possible from the heat radiation of the kiln. For the same reason motors selected for this work must be liberally rated and have low temperature rise.

Kilns vary in size from 6 ft. 6 in. diameter by 100 ft. length to 10 ft. 6 in. by 200 ft. and revolve at speeds ranging from 3 to ½ rev. per min. They require from 10 to 30 h. p. The load is very steady and the horse power requirements are nearly proportional to the speed of the kiln, to the square of the diameter, and to the length. Approximately 125 to 175 per cent starting torque is required.

Table 10 gives approximate power requirements of rotary kilns. The power requirement varies with the speed as shown in the last two columns and the rating of the motor, on account of the severe operating conditions, should be twice the lower value there given. In selecting the resistance for controlling

the speed, care must be taken to have it proportioned so as to be capable of reducing the speed to one-half when the load is that actually required to drive the kiln at half speed.

After the material has passed through the kilns it is ground in the finishing mills. For this work the same class of grinding and conveying machinery is employed as that previously described.

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DISCUSSION ON "THE APPLICATION OF ELECTRIC DRIVE TO THE CEMENT INDUSTRY" (WILLIAMSON, KELSEY, DUDLEY, AND WEICHSEL). NEW YORK, N. Y., NOVEMBER 8, 1918.

A. M. Dudley: We are really presenting to you two things to-night, and as regards the second we would like to have you feel that you are present at the birth of an idea, if you feel that there is an idea involved.

As regards the paper itself, two years ago Capt. Martindale was appointed Chairman of the Committee on Industrial and Domestic Power. Capt. Martindale, with others on the Committee, felt that the technical committees of the Institute were not entirely discharging their duty.

This Committee had a feeling that the technical committees of the Institute should do active and constructive work every year of their existence, and that succeeding committees should build on the work of the committees that had gone before, so that the knowledge thus accumulated, built up from year to year, would be available to the membership of the Institute at large and become almost a compendium of the progress of the art in that particular line. That idea, if carried out, would not be difficult of application, and it would provide the committees with a working program, which is a very great thing, it having been the case in most instances that the program of the work of the committees depended on the personality of the man in charge of the committee.

With that in view Capt. Martindale drew up a schedule, starting with certain machine tools to which he proposed to apply the proper motor for operating, both alternating current and direct current, and giving its characteristics, its methods of control, its capacity, and all other pertinent facts.

After some discussion the Committee decided such a plan was not sufficiently comprehensive and it was decided to study the entire subject, first from the viewpoint of the industry. It will occur to you that to the man who has made a study of this, to the man who is educated to the application of industrial motors—that the electrical study should be from the standpoint of the characteristics, particularly the speed-torque characteristics of the motor, because the number of different speed-torque characteristics is limited, and these speed-torque characteristics must be applied to all work.

That is true of the specialist, but to the man who is interested in the industry and who is operating the motors, the natural view point is from the standpoint of the industry, so no matter how it is studied, to become available to the membership at large, it must be primarily presented through the industry.

With that in view the committee selected three industries, having in mind one industry of very wide scope, a second industry of somewhat more limited scope, and a third industry, which was very highly specialized. With that classification in view, the machine tool industry was selected as being a very

widespread industry, with many ramifications and touching a great many other industries; the manufacture of cement was selected as a more limited industry and passenger elevators as a highly specialized industry.

That brought the work up to the end of the first year. The next year, last year, the entire committee was reappointed because it had shown that it had an idea. We said we had not had time to carry it out. The Board of Directors said—"All right, you can have another year to show what you can do," and the entire committee was reappointed. Unfortunately, for the work of the committee, Capt. Martindale went into the service and we did not do much work last year. But by working along we came through with a paper and it was presented to you to-night.

We would like you to think of this not as a finished presentation. Of necessity all papers prepared by the designing or application engineer, rather than the operating man, have a sort of academic flavor, that is, they look as though the man who is actually up against the job had not prepared the paper, and for that reason perhaps in some details the paper is lacking or not as full as it should be, and it is the hope of the committee that you will criticize the paper fully and freely, and discuss it, and make additions to it, or subtractions from it. The results will be put in form to be available to the entire membership as virtually a study of this industry up to this time, and succeeding committees would find it part of their duty to see that the data are kept up to date, because the industries change rapidly, and there have been great changes in the cement industry in the last five years.

We would like you to consider whether the committee has chosen wisely in making this special study, and whether the idea as a whole is a good one. We feel that eventually this work will be developed to such an extent that, through the PROCEEDINGS or otherwise, the membership at large will be interested in the collection of the data, and as the Committee intends to make a study of certain industries each year, it is hoped that the membership will respond freely and cooperate, and that all who are engaged in the particular industry treated will prepare such information, or furnish such notes as they may have, and send them in to the man compiling the particular data.

As this work grows it may assume the shape of a loose-leaf pamphlet, or a card-index, or something of that nature, so that when you are discussing the paper to-night, we wish it to be discussed, just as the presentation is, as a treatise on the use of electric motors in the manufacture of cement, and we would also like in addition, to have some discussion on the work which the Industrial Power Committee hopes to do, and wishes to do, whether it will be of a constructive nature and valuable to the Institute at large, and whether the Institute can be of service in this particular manner.

Arthur Simon (read by W. C. Kalb): Mr. Williamson's paper covers very thoroughly one part of the work which the Industrial and Domestic Power Committee endeavored to carry out during the past few years, and it is a clear indication how tremendous the amount of work must be if it is intended to cover the application of electric motors in all industries in a like manner.

The writer has thought for a long time of ways and means by which the subject could be most satisfactorily and expeditiously handled and the thought occurred to him that the different industries might be treated in the form of monographs, each one covering a particular industry, or even only a section of that industry. Mr. Williamson's paper could well be made the first one of these monographs as it covers the particular subject under discussion in a very thorough manner.

My reason for suggesting monographs is that each one could be edited by an engineer who is thoroughly familiar with the industry in question and who could get the cooperation of other men equally well qualified. In this manner it would be possible to distribute the work over a larger number of contributions and by publishing the different sections in book form and separately, a market would be created for them outside of the membership of the Institute, sufficient to cover the necessary expenses, which I realize would be rather high.

On the other hand, if the material is gathered and published in the manner indicated, it would be possible to very liberally illustrate the different booklets. Photographs and cuts could be procured from the various manufacturers interested in the particular lines under discussion. Liberal illustrations would be desirable because they would enable the specialist, who has for instance only had experience with the design of electric motors and not with their application, to familiarize himself more with the installation and operating conditions. This as I understand was one of the primary reasons for the work, undertaken originally by the Industrial and Domestic Power Committee.

The idea of publishing monographs to cover the entire field in the manner proposed is, of course, not new. It has been and is being done in other lines of science and engineering, I call attention to certain lines of publications in physics and electrical and mechanical engineering abroad.

Mr. Williamson entitles his paper "Electric Motors in the Cement Industry." I believe that it would have been desirable to change this title to "Electric Motor Applications" and include in the discussion the various forms of controlling devices which are most suitable for the various motor drives.

The tendency in the past in the cement industry has been to put too much emphasis on the desirability of the squirrel-cage motor on account of its simplicity and ruggedness. I am glad to note that Mr. Williamson considers the slip-ring motors, particularly in the larger sizes, equally, or more satisfactory than the squirrel-cage motor. If the control is also taken into con-

sideration, it will probably be found that the slip-ring motor is the preferable type in the majority of cases.

The tendency in the cement as well as in other industries today is to use automatic controlling devices on account of the lack of skilled labor which can be introduced with the operation of hand controllers, and it is no doubt true that the controllers for slip-ring motors in general give better all-around results than squirrel-cage motors in combination with controllers for their operation. Furthermore, by the use of the slip-ring motor the starting currents can be greatly reduced and the risk of burning out the motor due to its being stalled at starting, is greatly reduced.

The writer realizes that this is at the present time to a large extent a matter of opinion and it would be desirable to get the benefit of the experience of operating men who had to deal with the two types of motors and also to get some recording instrument curves, showing the starting conditions for squirrel-cage and slip-ring type motors on various drives.

W. C. Kalb: Mr. Williamson, in the early part of his paper, points out that the tendency in the cement industry has been to over-motor rather than to under-motor the drive on the most of this equipment.

It seems to me in some cases while that may mean that the motor is operating at a lower degree of efficiency than where it is working up to its full rated load, there is still an economical advantage, particularly in types of machinery such as some of the crushers he mentions, where, if the machine is stalled, it is necessary to tear out all the charge before it can be started again. The saving over the loss of time and the labor required to take care of the stalling, with the consequent interruption of production, if a motor was working so near its capacity that such stalling occurred with any frequency, would rapidly pay for the installation of a motor of greater capacity.

On the question of brushes, which is a field in which I am particularly interested, we do not find such an extensive application of brushes in the cement industry as in many others, owing to the predominance of alternating current. However direct current, as has been stated in Mr. Williamson's paper, is used to some extent and with considerable success.

The application of brushes to this industry is very much the same as any other, with perhaps the exception that self-lubricated brushes, so called, or those which are impregnated with the lubricating material are not at all desirable in this industry. The presence of the abrasive material in the form of dust which can be accumulated by any lubricating material on the surface of the brush will cause an unnecessary amount of wear on commutators. Consequently this type of brush should be avoided in the cement industry or in any others where abrasive dust may be present.

As to under-cutting of commutators, in the presence of so

much dust of this kind there is not the need for it in all cases that there is in other installations. We see frequent cases where a motor requires the under-cutting of the mica, provided a non-abrasive brush is used, because after a considerable period of operation, perhaps months, the mica will gradually get high. Cases of that kind would not require under-cutting in a cement mill, the abrasion which occurs from the dust present would be sufficient to permit operation with flush mica, and non-abrasive brushes. There will be conditions, however, where undercutting would be advisable, unless abrasive brushes are used. In such cases the undercutting should not be deep—1/32 in. as a maximum will give successful operation, and it will not be deep enough to afford a pocket for the collection of dust, so that it will offer any serious trouble, if the machine is blown out with any reasonable frequency.

Probably more brushes are used on the slip-ring types of motors in cement mills than are used altogether on direct-current motors, due to the predominance of the alternating-current motor.

I would like to put in a word here in urging the greater use of the carbon and graphite brushes as against the metal-graphite composition brush. This, of course, means cutting down the current density in the brush to come within the range of the carbon-graphite brush, but that is not a serious problem as there is always plenty of room for the accommodation of holders.

An objection frequently offered by designing engineers to the use of the carbon brush on the slip-ring type of motor is that it increases the slippage. Again, for this industry, I do not think that would be a serious objection.

My reason for urging the advantage of the carbon brush, however is the tendency for all composition brushes, once abrasion has started, to of themselves continue that abrasion. Once break up the polish, the smooth surface of the brush face and the ring, and it is very slow in forming again. This rough contact affords considerable abrasion, causing short brush life and rapid ring wear. The carbon brush is not open to this objection.

In regard to the types of carbon brush having the necessarily high carrying capacity, there are practically but two from which to choose—these are the all-graphite, which is made as a molded brush, from pure natural or artificial graphite, and the electrographitic brush. I have a personal preference for the electrographitic brush, due to its greater hardness, greater mechanical strength and ability to stand up under severe mechanical conditions. It can be operated up to a current density of 60 amperes per square inch without difficulty, and where conditions are right a considerably higher current density is permissible, so it will take care of any condition to which a graphite brush can be applied.

Linn Bradley: The portion of cement plant equipment in which I have been interested is the electrical apparatus employed for the collection of dust from the gases that come from the

rotary cement kilns. This phase of the subject under discussion may be of interest to members of the Institute, although it is remotely related to the other phases which have been so thoroughly discussed this evening.

The electric circuit that I propose to discuss, consists in part of a source of alternating current, a mechanical rectifier (synchronously driven) which converts the alternating current into a pulsating direct current, and a load which operates as a leaky condenser. This load is what causes so much concern to the operator. It behaves erratically at times and its peculiar operating characteristics are such that the circuit has been aptly termed a "wild indian."

The problems encountered are numerous and difficult and the field is sufficiently developed in cement plants to warrant engineers familiarizing themselves with the technique involved. First, let me say that, instead of following the usual practise, by trying to avoid the formation of arcs, the electrical precipitation engineer is dealing with arcs in the high-tension circuit (at rectifier) as an integral part thereof. It is unnecessary to state that this introduces a number of interesting, even if vexatious problems.

Low-tension alternating current of 25 to 60 cycles, single-phase, is raised to 30,000 or 100,000 volts by means of a static transformer, and this is converted into pulsating direct current by means of the synchronously driven mechanical rectifier previously referred to. This rectifier in its simpler form comprises a disk, about two feet in diameter, made from a good insulating material, such as Micarta, fibre, Dielecto, etc. mounted upon a shaft driven by a small synchronous motor or by a direct connection to the shaft of the low-tension alternator delivering the low-tension alternating current to the step-up transformer. It is thus apparent that the synchronous motor is merely a convenient substitute for the mechanical coupling when the alternator is situated at some distance from the rectifier, *i. e.*, when the alternator is in a central power house and is used for supplying current to other pieces of apparauts. It is desirable to keep this point clearly in mind, that the two methods of driving the rectifier are almost identical, so far as the electric circuit in question is concerned.

Near the periphery of the revolving disk, but separated from it by a small air gap, four shoes or elongated conductors are placed, one for each quadrant. (See Fig. 1) The disk carries four "contactors," these so-called contacts being made through the air gaps by means of arcs. These revolving points might preferably be called "arcing-points" or "arcing-contactors." The general arrangement of the disk and shoes and a wiring diagram are also indicated in Fig. 2. It is not feasible at these high peripheral speeds, to have the shoes and contactors in actual contact,—we wish it were. Two wires connect the transformer high-tension terminals to two shoes spaced apart

horizontally, one wire connects the bottom shoe to the ground and another connects the upper shoe to the "discharge electrodes" of the precipitator or leaky condenser. In the sketch the "discharge electrode" is represented as an iron wire (No. 14) placed along the axis of a "collecting electrode", in this case a round iron pipe about 12 in. (30.4 cm.) diameter by 15 ft. long (4.5 m.) These two types of electrodes are separated by suitable insulators, the current flowing from one to the other through the dust-laden gases. There is a corona discharge taking place continuously when the apparatus is in operation. The corona is readily distinguished as negative or positive by visual observation. In practise the negative is used as it permits a higher potential with better operating results. The positive is more brilliant but it is much less effective for precipitation purposes.

The general character of the potential difference between the discharge and collecting electrodes, on the precipitator, or direct-current side of the rectifier, is indicated in Fig. 3. Fig. 4 shows the general character of the potential difference between

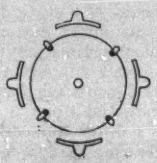

FIG. 1

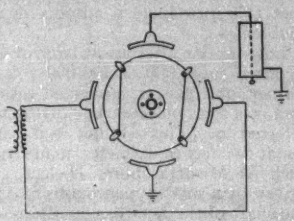

FIG. 2

the two high-tension terminals of the transformer on the a-c. side of the rectifier. *A* represents good conditions and *B* showing poor conditions. An examination of these sketches will point out some of the problems which must be kept in mind in designing, as well as in operating apparatus in conjunction with this process. It is evident that the potential difference does not fall to zero, between the precipitator electrodes, unless a spark or an arc occurs, in the interval between the so-called contact periods at the rectifier, owing to the fact that the rate of leakage decreases rapidly with a reduction in potential, and when the critical corona potential is passed the rate of leakage is too small to permit the condenser to become discharged in the short time which elapses before the transformer is again feeding electric energy into the precipitator (*i. e.*, the leaky condenser). This interval obviously is less with 60-cycle power than with 25-cycle. When the "contact" is renewed, current flows into the precipitator so long as the potential between the transformer terminals is greater than the potential between the electrodes of the precipi-

tator, allowance being made for the resistance of the circuit including the arcs at the rectifier. When the transformer potential decreases to a point just below that of the condenser or precipitator, the contacting period at the rectifier should cease. However, the current and potential are not in phase, and this adds some complications. It would be ideal to end the contact period when the potential difference of the transformer terminals and the precipitator electrodes were equal and the current flow had become zero. Since the current and potential are not in phase, this condition is not obtainable in present practise.

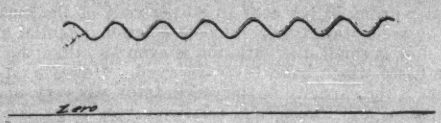

FIG. 3—VOLTAGE WAVE (FLUCTUATION APPROXIMATELY 20 PER CENT OF MAXIMUM

Current flows into the precipitator for two distinct purposes: First, to charge the condenser and raise its potential; second, to cause ionization of the gas between the electrodes and an electric current to flow between them. These two elements of the current may be divided into one which is purely a capacity current and the other which is purely a load current, the one being out of phase with the potential and the other being in phase with the potential which produces it. These vary relatively when the potential and other conditions change. Until sufficient electric energy has been delivered to the precipitator to raise the condenser to a point where the potential difference between the

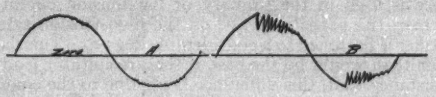

FIG. 4

electrodes is sufficient to produce a corona discharge, the current is almost entirely a capacity current (resistance of the circuit is small). When the critical corona potential has been passed, current is required to increase the capacity charge and raise the potential between the electrodes and also to provide the current which flows through the gas between the electrodes. Since the rate of current flow due to the corona discharge, increases as the square of the increase in potential difference, there will be a change in the ratio of capacity current to load current, and this change of ratio will be taking place throughout the entire period of current flow from the transformer. However, the change in

ratio will not be affected by increasing the length of the pipe, or what is equivalent by adding more pipes of the same kind, provided the wave form, etc. at the transformer remain fixed. The only point I desire to bring out right here is that the amount of lead of the precipitator current over the potential will be practically the same for a precipitator of a few pipes as for one of several pipes. Oscillograms show very plainly the leading characteristic. Oscillograms also show that when a precipitator is operated at near the maximum potential, without spark over, the voltage between the electrodes varies so that the minimum potential is from 70 per cent to 80 per cent of the maximum. When the precipitator is operated at a lower potential and the current flow is small, the variation is even less than the above, in some cases which have been observed. However, it would seem that if the capacity of the precipitator was very small and the alternator and transformer relatively very large, the potential difference between the electrodes of the precipitator would vary with the transformer potential during the time the rectifier was in contact, and that therefore the degree of fluctuation would depend directly upon the number of degrees of the wave which was being rectified. On the other hand, if the alternator and transformer were of small capacity and the precipitator was of relatively large capacity or capacitance, there is a possibility that enough energy might not be delivered to the precipitator during contact periods to materially raise the potential. It has been observed that when the precipitator was large, taking a relatively large load current, and the alternator and transformer were small, the degree of fluctuation of the precipitator potential was quite small, the extent depending considerably upon the number of degrees of the wave being rectified. This condition produces a distortion of the wave form, and, when a heavy resistance is used in the primary or low-tension circuit of the transformer, the potential wave on the high-tension side of the transformer is flattened to a considerable degree. This flattening is greater than would be the case if the precipitator load were a pure resistance load, this difference being due to the character of the corona load.

In practise it is usual to have the rectifier contact period cover about 90 to 120 electrical degrees, the lower portions of the wave being of no value in charging the condenser or providing for the corona load. The potential maintained between the precipitator electrodes does not fall to zero, whereas the potential on the transformer passes through zero during each cycle. The current into the precipitator, on the d-c. side of the rectifier, however, does start at zero, rises to a maximum and again goes to zero. The shape of this current wave depends upon the extent to which the condenser is charged, the amount of load current, the ratio of actual potential to possible maximum potential, the ratio of capacity current to load current, the number of degrees rectified, the portion of the wave used, and to other but more minor factors.

If an arc occurs in the precipitator, the potential between the electrodes falls nearly to zero, and during the next rectifier contact the current flow is heavier than normal and the precipitator becomes charged nearly to its normal, and is again normal in one or two contacting periods. With a single-phase a-c. source, such as described, the arc is broken between contacting periods. With a source of true direct current, the arc continues until broken by special means or until the arc extinguishes itself by becoming elongated to the breaking point. Three-phase alternating current acts in a manner similar to true direct current in this connection. For this reason, single-phase alternating current as a source of the pulsating direct current possesses some practical advantages over non-pulsating direct current.

A study of the arcs which occur at the contacting-points of the rectifier reveals some interesting phenomena. With a rectifier built as described, the four revolving arms and points act as a fan which produces heavy windage in the space between the shoes and the revolving points. This windage tends to blow out the arc and thus produces a very unsteady condition. The more unsteady the conditions the more pronounced are the surges. It has been found that a solid disk produces less windage than a disk built up from arms which project out from the hub, and that smoother operation is obtained. Better results would be obtained if actual contact were possible. At the shoes an arc is established as soon as the point approaches the shoe, and then this arc is drawn out until the resistance is so great that the arc breaks and another arc forms. It is possible that the new arc forms before the old one is broken, but this is not easily proven. Visually these arcs appear to overlap a fraction of an inch. Generally three distinct arcs are observable with the solid disk, each arc being increased in length and decreased in cross section as it is drawn out, indicating an increase in resistance and a decrease in rate of current flow. This condition readily promotes surging. It is difficult, indeed, to get oscillograms which show clearly all of these phenomena, but the phenomena are clearly shown in these arcs.

Furthermore, one can obtain a good view of the so-called static sparks which are shown as little white streamers within the body of the arcs. These are quite close together, thus indicating in some measure, their frequency. Considerable resistance in the high-tension circuit partially clears the arc and eliminates some of these streamers.

It is believed that these white streamers are an indication of the source of the trouble experienced with transformer coils. The end coils of the transformers readily fail unless they are provided with excessive insulation and are otherwise protected against excessive potential strains. The amount of potential difference which can be set up between adjacent turns of a coil depends upon the rate of change, and since the rate of change is very high with these oscillations, a very high potential gradient

is readily established when the coil is placed around an iron core. The current flowing in the high-tension circuit consists of the regular charging and load current wave, upon which are superimposed these numerous oscillating current waves. By placing a choke coil on the terminal of each transformer, the load and charging current readily pass through the choke coil while the oscillations are reflected thereby, due to their much higher frequency. They are caused to pass through resistance and thus dissipate themselves in heat. In this way the transformer coils are protected against break-down. A further step is to take advantage of the difference in rate of change, or steepness of wave front, allowing the load and charging current to flow through the choke coil and compelling the oscillations, having higher frequency or steeper wave front, to shunt the coil and pass through a very high but relatively non-inductive resistance, such as a carborundum resistance rod. By utilizing these methods, transformers now operate satisfactorily without frequent break-down, and smooth and fairly steady operation is obtained in this special field of application.

Recently a considerable amount of discussion has taken place and some articles have appeared in the literature relative to the virtues of synchronous motor drive as compared with connecting the mechanical rectifier direct to the shaft of a motor-generator set. From a study of the above, it seems evident that there is no fundamental difference in the two methods, as the synchronous motor merely acts as a substitute for the flexible mechanical coupling used in the other case. The hunting of the synchronous motor or failure to keep in step are no problems in practise, when the power line has satisfactory characteristics. When the line is unsuitable, it is obvious that a motor-generator set is preferable. The other troubles or problems discussed above are much more important and fundamental and point the path for those responsible for designing the electrical equipment, and should be of considerable assistance to those who are charged with the installation and operation of this type of equipment. Designers should remember that the load is due to a corona discharge and that for a given increase in potential the current resulting is greater than if the load were an ordinary resistance with no temperature coefficient. Again, the rectifier is in contact only during a portion of the wave, the current flow taking place during this interval which may be relatively very short. It is not safe to rely entirely upon meter readings placed in the low-tension circuit, as they fail to record the rate of change or rate of current flow. The actual voltage on the high-tension terminals of the transformer can not be correctly read from the low-tension voltmeter reading and the transformer ratio. The error is the greater when the number of degrees rectified is the smaller. For instance, if there was only a point to point contact at the rectifier, the low-tension voltmeter might read say 200, and one might assume that the actual potential delivered to the pre-

cipitator electrodes was (200 : 100,000 volts transformer ratio), whereas it might be that during the interval the rectifier was in contact, the transformer potential may have been nearly zero due to the very high rate of current flow during the short interval, which would be the case if there was an arc-over in the precipitator. The peculiar power factor of this circuit may be understood by a consideration of this point. Another point of interest is the flattening effect upon the low-tension and high-tension waves of the transformers when a high non-inductive resistance is placed in the low-tension side of the transformer. This flattening effect is very pronounced when a small alternator is used and the corona current is quite large, and exceeds that which would result if an ordinary resistance load were used. As the voltage increases, the current flow increases rapidly, produces a greater potential drop across the resistance and this in turn makes the potential across the low-tension terminals of the transformer that much lower. The larger the resistance the greater the flattening effect upon the wave shape. The rate of current flow during the time the rectifier is in contact should determine the size of the alternator. The maximum rate of current flow from an alternator for this service is greater than for a lighting load where the entire wave is used and the load is a resistance load. Small alternators should be designed with this in mind. Poor wave shapes from small alternators are due in part to this feature. On the other hand, if a large alternator is used, and a small amount of current is taken from it, the effect upon the wave shape is going to be negligible as compared to the same amount of current taken from a small alternator. This probably accounts for the reason why the synchronous motor operation, taking the power for the precipitator from a large alternator, gives better operating conditions and better wave shape, and enables one to operate with higher potential on the precipitator, than when an undersized or improperly designed small alternator is used. Of course, a power line must not be used if it is subjeot to excessive voltage variations, since good voltage regulation cr control must be had for the precipitator circuit.

One of the principal uses of this process in a cement plant is to recover potash as a by-product from the gases coming from the rotary kilns. Considerable progress is being made in this field, and it is quite probable that the process will be universally adopted in cement plants in time. Therefore engineers should consider some of these special uses and applications and become familiar with the technique and the problems arising from such operation. With this process in cement plants, a considerable tonnage of potash can be recovered as a by-product while cleaning the gases and overcoming a nuisance in the plant and immediate neighborhood, and it is of special interest, that this potash can be recovered at a cost which will permit of competition with the potash which can be supplied by Germany. The experts of the U. S. Bureau of Soils and of the U. S. Bureau of Mines estimate

that the amount of potash which can thus be recovered from cement plants per year will be from 50,000 to 100,000 tons.

W. L. Merrill: It has been my experience that in the application of electric drive to various industries that there is a pronounced tendency to complicate, rather than simplify control apparatus. Generally speaking, control apparatus for industrial motors should be as simple as possible to accomplish the desired results and of course it is an obvious fact that the more complicated the duty cycle of the motor is, the more complicated will be the control. We have at the one extreme, the reversing mill motor controls and the reversing planer controls, and at the other extreme we have the squirrel-cage motor on all day service and a simple line switch, or a simple type of starting compensator. Between those two extremes of magnetic control and simple switch control we have the various types of drum, automatic and semi-automatic, and magnetic push button, etc., etc.

As stated in the paper, the cement mill load is long and continuous and fairly uniform, with exceptions of one or two types of machines. Therefore, following the above reasoning it is evident that the control for cement mills should be very simple and most of the successful installations which have been made have followed this policy. This reduces itself to a simple rugged drum-type of controller for collector ring motors, either with primary contacts and some auxiliary overload protection, or simply secondary contacts and primary oil switch with overload and possibly low voltage protection. For the squirrel-cage type of motors, the simple type of hand operated compensator is all that is required and there is at least one successful installation with motors running up to 250 h.p. in which the only control installed is a knife switch; motors being thrown directly on the line.

We must consider the class of operators with which we have to deal in the cement mill and in so doing remember that ruggedness and simple control will give the best results. There have been installations in which very complicated systems of control have been established in order to bring the control of the various motors situated throughout the different parts of the plant to one central station. It is needless to record that the advantages given by this (if there are any) are very much outweighed by first cost, added equipment, many additional lines and circuits not only to keep in repair, but to add to trouble.

There is always a happy medium between simplicity and complication. Many engineers apparently foresee supposed conditions arising which make it necessary to install protective features and the like, so that if the expected happens much danger is not the result. This is often carried to extremes so that the additional protective apparatus and apparatus for the convenience of operation out weighs the original fundamental equipment. The electricians and trouble men devote more of their time to keeping these auxiliaries in working condition than is necessary for the fundamental equipments.

In many industries conditions will warrant magnetic control instead of drum control and of course there is no particular reason why the master controller and the magnetic control panel would not start a collector ring type of motor on a rockcrusher, but it is perfectly possible to do this with a drum controller, and a magnetic control under the atmospheric conditions of a cement mill should be so thoroughly boxed in and protected that the expense of the equipment is not only high but that any claim of advantage over the drum type would be more than outweighed by the complication and cost and probably maintenance, since remote control of cement mill machinery is in no way essential, as the man may as well start and stop his motor by drum controller as by pushing a button.

In conclusion, I wish again to call attention to the advisability of using the simplest and strongest types of control for use in the cement mill industry.

H. D. James: I wish to add a memorandum to Mr. Williamson's paper suggesting suitable control apparatus for use with the principal motors recommended, which are the squirrel-cage induction motor and the wound-secondary induction motor. I have had no experience in the application of control to direct-current motors or synchronous motors for use in cement mills, and must leave comments on this to someone else.

General Features. All control equipment, with a very few exceptions, is for constant speed non-reversing motors. Most of these motors are of the squirrel-cage induction type, except a few applications requiring speed adjustment, or where the wound secondary induction motor is necessary in order to develop the required torque at starting with a reasonable demand on the power circuit. In many cases, the standard squirrel-cage induction motor does not have sufficient starting torque, so that special motors must be designed and the general purpose motor cannot be used, even when installed in a separate room and protected from dust. The use of a transformer type starter for these motors is entirely feasible if the motor will develop the requisite torque at 80 per cent of line voltage or less. Where 90 per cent of line voltage is required to start the load, there is practically no saving in using a current limiting device, and the motor should be thrown directly on the line. This represents very severe starting conditions and may require special bracing of the motor windings to withstand the heavy starting current. The switching mechanism in such cases must be very strong and capable of handling the locked current of the motor without undue wear.

Control apparatus for the wound secondary motor need not differ from standard if it is installed in a separate room and protected from dust.

The atmosphere in these plants is very dusty and some precaution must be taken to protect the electrical apparatus from cement dust and coal dust where powdered coal is used as fuel.

This protection can either take the form of a cabinet or dust-proof enclosure around each piece of control apparatus, or the motors and controllers may be located in an adjacent room from which the dust is excluded. This latter arran gement is preferable as standard equipment can frequently be used and the maintenance charges will be reduced.

All wiring should be in conduit and all controller frames or enclosures grounded. Special care should be taken to ground the framework of push-button stations, master switches, or manual controllers, particularly in the presence of coal dust, in order to protect the operator from accidental shock.

The National Electric Safety Code requires all live parts of control apparatus to be protected from contact by unauthorized persons. This requirement can be met by one of the following arrangements:

(a) Enclosing cases.
(b) Mounting control equipment at least eight feet above the floor.
(c) Placing the control equipment in a separate room not accessible to an unauthorized person.

Usually the first or third arrangement is followed in cement mills.

Mr. Williamson has brought out very clearly the severe starting conditions required for many of the machines. This should not be overlooked in specifying control equipment. Standard general purpose controllers have been classified by manufacturers with reference to the time of starting as "light" and "heavy" duty. The light duty starting should not exceed 15 seconds once every four minutes. Heavy starting should not exceed a 30-second period every four minutes. The heavy starting class will be required for many of the machines in cement mills.

The following features should be embodied in control equipment:

Manual Starters for Squirrel-Cage Induction Motors. Starters for all except the smaller motors should be provided with some form of current limiting device. The preferable form of current limiting device for the larger motors is of the transformer type, known to the trade as auto-starters or compensators. These starters should be equipped with time-element overload relays affected by two legs of the circuit and by low-voltage protection. The handle should be arranged so that it must be held in the starting position. A latch should prevent the accidental throwing of the handle into the running position before it is first thrown in the starting position. Provision should be made for locking the handle in the off position to prevent accidental starting of the motor if work is being done on the machinery. Where the switching member is oil immersed, it is particularly necessary to see that dust is excluded from the case.

Manual Starters and Controllers for Wound-Secondary Motors. These controllers are usually of the drum type, as this design is

enclosed and usually very compact. All manual starters and controllers should be provided with a protective panel equipped with a time-limit overload relay, affected by two legs of the circuit and low-voltage protection. The panel should have a magnetic contactor opening at least two legs of the circuit. In addition, a disconnecting switch with a safety lock should be provided. This disconnecting switch may be mounted on the protective panel or separately, depending upon the requirements.

Automatic Starters and Controllers for Wound Secondary Motors. Automatic controllers are preferable for the large size motors, which are generally of the wound-secondary type. These controllers provide automatic acceleration and should be equipped with time-element over-load protection affected by two legs of the circuit, and should be connected for low-voltage protection. A disconnecting switch with a safety lock should be provided. Usually this switch is mounted on the panel with the contactors. Where speed regulation is not required, the master switch can consist of a start and stop push button. Several of these push-button stations may be located around a machine when necessary. Where speed control is required, a drum type of master switch should be used. The starting resistor for an automatic panel should give approximately 150 per cent of full-load current on the first notch. Where speed regulation is required, a resistor of continuous capacity should be used and the starting current may be reduced to 70 per cent of full-load current if the application requires this.

I heartily endorse Mr. Merrill's statement that the control should be as simple as possible. If the motor and line conditions will permit the squirrel-cage motor to be used and thrown directly on the line, that is the preferable arrangement, but the engineer laying out the plant will have to determine the type of motor his line and load will require.

After you determine the motor, the question remains, if it is the slip-ring type, whether you shall use remote control with the larger motors. The principal advantage in remote control is the question of safety. We are giving more and more attention to the question of safety, and it is possible to remove the main switching equipment from the neighborhood of the operator and leave only a small power circuit of the push-button type, and this is in the direction of safety. It adds to the matter of complication, unquestionably, but that must be adjusted and balanced in each particular installation.

It is difficult to lay down a rule to cover every case, and my suggestions are to be taken merely as suggestions along the lines we are discussing.

Francis J. Burd (read by R. H. Goodwillie): I am impressed with the importance of Mr. Williamson's paper and the attempt which is made to put into tangible form, a record concerning the application of motors to the cement industry which is bound to be of mutual benefit to the electrical and cement manufacturers. It should tend to bring the two industries closer together.

It has been the writer's experience in selling electrical equipment that it is generally necessary to thoroughly understand the customer's problems as it relates to the electrical equipment involved before confidence is established to the degree of purchasing the equipment. I feel that papers such as presented by Mr. Williamson will accomplish considerable in bringing about more intimate relations between the electrical manufacturers and more especially the Institute as a whole to many other industries beside the cement industry.

I don't believe that it can be said, that the Institute has heretofore encouraged to any great extent measures to promote closer relations between the electrical and other industries. I believe the tendency has been to drift along with the problems presented by its own members.

Papers such as this one on the cement industry would certainly go a long way toward promoting the relations referred to and I would recommend that the Institute encourage and give publicity to papers of this sort. Such encouragement and support of the Institute as a whole would certainly be of material benefit to all concerned.

It would seem that every member of the Institute could do something in support of this kind of work in the spirit of supporting the Institute as a whole. If the members will only look upon the Institute as an institution of their own and give it their full support, I believe the Institute could easily fulfill its share of the work.

Not very long ago, the Institute members were asked on a questionnaire form to register their training, experience and vocations. It was understood that this data was for the Institute records. On the assumption that the membership responded satisfactorily, those records should form a good basis on which to segregate the membership according to industries.

With committees appointed by the proper authorities to collect data as it relates in an electrical way to certain industries, it would seem that members to obtain the desired information could easily be selected from the questionnaire records by the committee.

I believe this method offers to each member an excellent opportunity to assist the Institute in the manner in which he is best qualified.

I would not recommend that any special file be kept of the collected data, at Institute headquarters. I believe the proceedings would provide ample means for a permanent and continuous record. However, I would recommend that the Institute allow publicity of papers in such a manner that they would appear before the industry concerned.

Practically every industry in the country has one or more associated trade journals. These journals offer an excellent opportunity for the Institute to get before a given industry interesting articles concerning the application of electricity to

their processes. It would seem that this is where the Institute could promote closer relations between the electrical and other industries.

E. Friedlaender (read by R. H. Goodwillie): The committee on "Industrial and Domestic Power" is deserving of a great deal of credit for the work it has done in compiling these data. I think the Institute should encourage in every way the presentation of such practical descriptive papers as this one. A great number of our members are practical operating men, who are greatly in need of this kind of data.

A great amount of descriptive matter is gotten out by the manufacturers in regard to the construction of their apparatus and machinery, but very little instruction is available for the customer, for the proper selection, use and installation of the apparatus.

Close co-operation between manufacturers, designers and users is absolutely necessary. The best motor and controller will fail to do the work satisfactorily if it has not been properly installed. Responsibility of the manufacturer of electrical apparatus does not end with the designing and furnishing of same, but should include the additional service of selecting and installing apparatus best suited for the work.

Committees on Power Stations, Transmission and Distribution, Industrial and Domestic Power, Iron and Steel Industry, etc., etc., should compile practical data and present them to the Institute meetings for discussion.

During these trying times, large production rather than new development has been the aim of the electrical industry, in order to help the government to prosecute the war. Wages have about reached their maximum, at the same time efficiency of man power is at its minimum. It is therefore only natural that in the near future great amounts of money will be expended to replace, wherever possible, man power by machinery, and skilled work will be performed by unskilled labor at lower cost with increased production.

The attitude of the public is very favorable towards the use of electric motors. The advantages of motor drives are well known, such as saving in floor space, convenient arrangement and spacing of machinery, safer and cleaner factories, better control and efficiency, greater overload capacity, ability to shut down each individual machine without affecting any other, etc.

Where the output depended on the endurance and physical strength of the operator, controlling machinery manually, the electric motor and automatic controller has greatly increased production. Unexperienced and ignorant operators manipulate large, ponderous machines without the least effort and cannot do any damage to same until control fails. The employment of women in place of men in the iron and steel industry has been possible in many places where machinery is electrically driven.

Naturally electrical installations of such nature are compli-

cated and require experts to look after them. Neither motor nor controller will give sufficient warning of trouble until too late, and regular supervision and inspection is absolutely necessary, especially where continuous operation must be maintained.

In motor applications in general, a number of points must be carefully considered such as: Description of machine and drive; load conditions; location; speed control; voltage and kind of current.

Where a large amount of dust and abrasive material is liable to get in electrical machinery, as in steel mills and cement plants, bearings should be dustproof, all collectors and commutators should be protected, all controllers—especially relays—should be covered. Motors should preferably run at low speeds and be installed in separate, clean, dry and cool rooms, wherever possible, and connected to rough-running machines by means of flexible couplings.

Not over 220-volt current should be used where men are liable to come in contact with same. If higher voltages are to be used electrical machinery must be isolated.

The cement industry is getting its power mostly from steel plants and blast furnaces from valuable by-products and waste gases. Naturally, alternating current is used exclusively on account of transmitting large amounts of power over considerable distances.

Although direct-current motors have ideal speed control and torque characteristics, alternating-current motors are preferable for the cement industry for the following reasons: No commutator troubles, especially on account of great amount of dust; constant speed of motor; stand greater abuse; no transformation losses from alternating current to direct current etc.

Where motors are often started and stopped, especially under load, the proper selection of motor and controller is very important, especially when current is furnished by central stations. Voltage disturbances in line through high current peaks, while starting, must be avoided; also low power factors. In many cases phase-wound motors must be used in place of the simpler squirrel-cage motors. The use of synchronous motors, especially of large size, where starting conditions are not too severe, is advisable for correcting power factor and regulating voltage, especially now when copper is high in price.

Auto-transformers and starting switches must be very strong as the service in a cement plant during starting period is often very severe.

The location of transformers is important. It is often advisable to locate individual transformers as near to the motors as possible in each building, instead of installing a central transformer station and long secondary transmission lines.

W. T. Snyder (read by R. H. Goodwillie): The author makes a reference to out-board bearings; it is considered by many in the steel industry that out-board bearings on motor sizes lower than 75 horse power is good practise, on many applications.

In the steel industry there are many applications, similar to cement-mill drives to which the motor is successfully geared and it has been proven that applications formerly considered suitable only to direct-connected low-speed motors can be successfully driven by high-speed motors rigidly coupled to cut gears at a low initial cost compared to the low-speed motor; and also compare favorably in maintenance cost; and at much lower maintenance and more satisfactory operation than belt drive. The motor and gear-box should be on a common bed-plate or at least on a common foundation.

In the steel industry 220 volts is considered preferable to 440 volts from a safety standpoint. The saving in copper by use of the higher voltage is often over-balanced by the additional safe-guards required. Practically the same safety precautions should be taken against shock with 440 volts as with 2200 volts or higher.

L. E. Underwood: Mr. Williamson says: "Bearings should also be made dust proof by the use of felt and steel washers, as cement has a highly abrasive action and greatly increases bearing wear when present in the bearings." That is undoubtedly true, but there is one application of dust-proofing which has not been mentioned. If in the lip of the housing, a very small clearance, perhaps 0.005 of an inch is obtained, or some amount like that, and circular grooves are put in this lip and filled with a heavy grease, an essentially dust-proof bearing can be made. The grease rubbing on the shaft catches the first particles of dust that come through, and they collect and form their own seal. In addition to that it is very essential that the oil-well opening be permanently covered, that is to say, the cover should be locked in place, so as to make it very difficult to remove it. I think these two features make as nearly a dust-proof bearing as anything I have seen in my experience.

Selby Haar: About nine years ago I had occasion to design some motors, I think, for operating crushers in a cement plant. They were of 150 h. p. capacity at 165 revolutions. They were a-c. motors and it was a rather unusual design of motor. It happened that there were 32 or 33 of them, all alike, which was still more unusual, and on account of the fact that there were so many of them I investigated the results of the tests, and was pleased to find that these motors agreed within two or three per cent of each other in their characteristics.

Since the committee has asked for some suggestions in regard to the construction of this paper, I will say it might possibly be a help to the people who read this paper if it were known that the data on which the paper is based, and which Mr. Dudley, I believe, said were accumulated previous to preparing the paper, are all deposited in the library of the Institute or in the general library of the United Engineering Societies and are available to people who wish to study the subject. I had intended to suggest that the Committee prepare a selected bibliography of the

important articles on the subject, but if I am correct in assuming that the underlying facts for this paper are available in the library that would make it unnecessary for the Committee to go to the labor of preparing a bibliography.

J. N. Mahoney: In connection with control problems comes the matter of the degree and character of overload protection. I have in mind the case of a cement mill where the problem of overload protection is a very important one. The power supply to the cement mill electrical apparatus comes through a bank of 4500 kv-a. of transformers at 2200 volts, backed by a very large high-voltage system. It is evident the short-circuit energy which can reach the motor and control apparatus is very great. While this problem is not beyond the application of reasonable apparatus arrangement and cost, I want to call to your attention here that this phase of the situation is usually not well considered in making an electrical apparatus application.

If overload or short-circuit protection is provided at every motor or branch circuit the cost will be large if the power supply units are not properly selected. On the other hand if overload protection is provided only at a few main supply points the cost of shutting down a large portion of the service may be large.

It is well to provide a power limiting arrangement in a large capacity installation, rather than use unusually capable control apparatus to handle the short-circuit energy that may reach the control equipment and probably the motors without such consideration.

W. L. Merrill: In connection with this last point we have had brought to our attention, I believe, the fact that the proper place for control apparatus under these conditions is where the lines go into the cement mill and not at each individual motor.

In any of these industries, particularly in the case of the cement industry, we must bear in mind that what the mill is put up for is to make cement. No motors should go into a cement mill that are not capable of standing up and doing the work. We should not have to surround them with a lot of control apparatus which we have to put off in a separate room in order to keep the operation safe.

If we go to that extreme we will have a high grade power station with a lot of expensive engineers and electricians to keep it running, and not have our minds fixed on the making of cement. That is what the mill is for, and the simpler and stronger the pieces of apparatus we can put in such a mill, the better the output and the less the cost.

R. J. Dearborn: As I glanced over Mr. Williamson's paper before coming here I noticed on the first page a prominent heading, "General Description of Process". I made up my mind that it would be an interesting paper aside from its technical value, and so I came to the meeting.

In listening to the verbal presentation by Mr. Williamson I was disappointed to find that he skipped that part of the paper

entirely, and I would like to suggest as a matter of human interest that the speakers in the presentation of papers of this kind give a brief abstract of the general description of the process and try to visualize it to the mind of the listeners in such a way that all may have a good general idea of what the process is, after they leave the meeting, even if little was known about it in advance.

Will it not promote the general interest and increase the attendance of the meetings if the committee on papers assists the writer in the preparation of a digest for verbal presentation, which particularly has the listener in mind. The papers are excellent but the presentation often leaves something to be desired.

James Dixon (abstracted by A. M. Dudley): Direct-current motors of the open type installed in locations where they are exposed to cement dust are noteworthy for the general good condition of the commutator surface. After a comparatively short time in service the commutator face acquires a fine polish and the later wear on commutator and brushes is abnormally slow. In one case emergency repairs were made on a grounded commutator. No facilities for turning it true were available, it was filed to approximate roundness with no sharp irregularities and put in service, the load being about 10 per cent above the motor rating. The sparking which existed at first disappeared after two days and in a week this commutator was as good as any in the plant. The eccentricity of course, still remained but was taken care of by the brush gear.

This tendency of commutators to acquire unusually good surface when exposed to non-abrasive dust has also been noted by the writer in a candy factory and in a chocolate factory. In both of these the motors were exposed to finely pulverized sugar.

As might be expected, long life of commutator and brushes is a result of the highly polished contact face.

A. L. Hadley (abstracted by A. M. Dudley): There is one question which might be brought up for discussion, or for information as to the usual or best practise, and that is relative to the use of commutators with undercut mica for direct-current machines. The author of the paper states: "There is nothing inherently injurious about the action of cement dust on the commutators of d-c. machines, and they are successfully employed in some plants, but, all in all, the a-c. motor is preferred and used."

No doubt the a-c. motor for most new installations is being employed. The fact that cement dust on a commutator does not make it inoperative, would lead to the conclusion that the cement dust is sufficiently abrasive to keep the mica flush. It will be of interest to know whether cement mills have used commutators with undercut mica with any better success than those using commutators with flush mica.

C. H. Sonntag (abstracted by A. M. Dudley): At the time that electric motors were first introduced in cement plants the

average mill consisted, at least in its grinding departments, of long lines of shafting carrying many friction clutches, from which the grinding machines were driven by belts. In the cheaper installations the clutches were mounted directly on the line-shaft, and in the better class mills the clutches were of the quill type. In either case, trouble with any clutch meant a shut down of the whole line-shaft until the difficulty could be remedied. The line-shafts were driven by Corliss engines through belts or ropes, or sometimes by direct coupling. This required that the power house be built as an integral part of the mill proper, with the attendant exposure of high-grade machinery to the ever-present dust.

Electric drive was hailed as the best remedy for these conditions. The power house could be removed to a convenient distance, where it could be kept clean, and where it really became a factory in itself, manufacturing kilowatt hours. This put the generation, measurement and use of power on a scientific basis that was practically impossible in the old type of mill. It also permitted the use of turbo-generators, with their attendant economies of cost, space, and steam consumption, a thing out of the question with the old line-shaft arrangement, unless large motors were installed to drive these shafts. This was done in only a few cases, as the advantages of the individual motor drive were too obvious to be overlooked.

Mr. Williamson has given a very good summary of the requirements of motors for this class of service. There are, however, a few points that should be emphasized in selecting additional equipment.

1. *Type.* The a-c. motor is of course the only choice when turbo-generators are used in the power house. They are more desirable in any event, because they are practically fool-proof.

2. *Speed.* To the average man who is putting up the money to build a plant, the cheapest motor is apt to look the best, when the choice lies between the products of reputable builders. A high-speed motor is cheaper than a low-speed one. In the stress of competition that attends the sale of a fair sized list of motors, a salesman of narrow vision who is not familiar with the conditions in a cement plant is apt to recommend to the powers that be, a selection of higher speed motors, because he can thereby underbid his competitors. If this comes to the attention of the financial man of the company, pressure from above may be brought to bear on the designing engineer which it may be very hard to resist. Especially is this so where a plant is being built by a promotor who expects to take his profit and depart for other fields, leaving the enterprise in the hands of some one else. The selection of high-speed motors may handicap the plant throughout the life of the motors. There is a very definite reason for this. The use of high-speed motors involves the use of small pulleys to keep the belt speed within bounds. In the dusty atmosphere of a cement plant the small pulley is a constant

source of trouble. The belt must be kept tight to do its work, and probably doped with sticky belt dressing. Even if made of endless leather its life will be shorter than it should be, and leather is expensive these days. If a textile belt is used, the repeated bending of the splice over the small pulley will soon break the belt at the splice. It will not take very many shutdowns because of belt repairs to pay the difference in price between the high-speed motor and one suited to the work. The writer has gone into this point at length, for the foregoing is the result of actual experience. For general belted service a motor with a speed higher than 900 rev. per min. should not be used, and 720 rev. per min. is better. This applies only to small motors up to say 25 h. p. For larger motors 600 rev. per min. should be the upper limit of speed.

3. *Special Types.* The various sizes of Griffin, Fuller and Raymond mills have pulleys with vertical shafts, and to drive these, as has been mentioned in the original paper, at least three makers have developed vertical motors adapted for belt drive. The design of these motors is entirely satisfactory, and the only object in speaking of them again is to call attention to the rather unfavorable conditions under which they operate, due to the low belt speed necessary.

A 30-in. (76.2 cm.) Griffin mill requires 30 h. p. to operate it, which must be transmitted through a 10-in. (25.4 cm.) belt running at only 1650 ft. (502.9 m.) per min. This means that a 500 rev. per min motor may have only a 12½-in. (31.7 cm.) pulley. A 42-in. (106.6 cm.) Fuller mill requires 75 h. p., and if fitted with a 54-in (137.1 cm.) pulley has a belt speed of 2500 ft. (762 m.) per min., calling for a 19-in. (48.2 cm.) pulley on a 500 rev. per min. motor. These small pulleys mean that the belts must be kept tight, both as to initial tension to prevent slipping and as to working tension to transmit the necessary power. There is some justification for this, in that these machines were originally designed for operation from low-speed line-shafts through quarter-turn drives, so that larger driving pulleys could be used. If there were an insistent demand for it, these machines could be re-designed with larger pulleys, so that a more favorable belt speed could be had.

4. *Ease of Cleaning.* The dust in the air of a cement plant has a tendency to collect in every protected nook and cranny in a motor, and particularly around the ends of the coils. Most of the older types of motors have the end bells of quite open design, so that the natural fanning action of the machine, assisted by the occasional use of compressed air will keep them in fair condition. But there is one late model in which the outer parts of the end bells are solid, and dust accumulations in this type of motor will give serious trouble if not watched. In the plant with which the writer is connected we have had to cut holes in the lower parts of the ends of this type so that the air circulation may carry the dust out.

5. *Dust-proof Control.* The well-known types of auto-transformer starting devices are sufficiently dust-proof as to give no trouble on that score, but where the size or location of the motor calls for remote control through magnetically operated contactors, these may give considerable annoyance by the interference of dust with their operation. Again, in older installations, cartridge fuses on open panels are relied on for overload protection. Dust falling on the upper end of the fuse will work its way between the clips and ferrule, and finally make the contact so poor that heating and destruction of the entire fuse and cartridge occurs when there is no overload. The best way out of this that the writer has seen is the use of inverse-time-limit overload relays which open the circuit of the low-voltage release coil. These may be had in perfectly dust-proof cases, and they make a very satisfactory installation on which there is no up-keep, as there are on fuses.

6. *Motors and Silent Chains.* The use of silent chain is being tried out in two or three plants in connection with kiln, ball and tube mill drives as a substitute for belts. Their makers prefer not to operate them at more than about 1400 ft. (426.7 m.) per min., which calls for a high working tension in the chain to transmit the power. This is of interest to the man installing them in connection with motors because provision must be made for safely caring for the chain tension. The bearings of smaller motors seem to be able to handle this without trouble. We have two 30-h. p., 850-rev. per min. two-bearing motors driving rotary kilns through silent chains, and their operation is perfectly satisfactory, and far superior to that of the belt drives that they displaced. We also have a 375-h.p. silent chain drive on a large tube mill loaded with iron balls. The driving shaft of this chain is mounted in bearings of its own, the motor being direct-connected to the end of this shaft through a flexible coupling. The very evident high tension in this chain when in operation has justified us in arranging to relieve the motor of this strain.

7. *Synchronous Motors.* The outstanding feature of a cement mill load from the power house stand-point is its low power factor, due to the use of induction motors. It is never over 80 per cent, and is usually nearer 70 per cent. We all know the effect of this on the capacity of the generators. The obvious remedy is the use of synchronous motors. They are already in use to a certain extent for driving air compressors, where they can be started light. Their inability to start under a heavy load is all that prevents their use for the larger sizes of crushers, and for ball and tube mills.

The average cement mill crushing and screening plant is usually rather compactly built, giving a good opportunity for the use of group drive with only a few motors, of which one, at least, could be of fair size. If the crushers are always started empty, which is usually the case, this motor could be of the syn-

chronous type, though the writer knows of no case where a synchronous motor is so used.

As stated in the paper, the use of this kind of motor for ball and tube mills would make imperative the use of a friction clutch to start the mill after the motor is up to speed, for these mills always start with a heavy over-load. Cement men were so glad to get rid of friction clutches when the old line-shafts were taken out that they are chary about taking them up again. If there is a ball or tube mill installation on this plan, a discussion of its operation and the cost of up-keep of its clutches would make interesting reading.

D. B. Rushmore (abstracted by A. M. Dudley): The rapid strides in the use of cement for building and construction during the past few years naturally creates an interest in the industry itself, the raw materials, process of manufacture and its magnitude, and before discussing the application of motors to the industry it might not be amiss to refer to the Census of Manufacturers issued by the Bureau of Commerce.

The government statistics report a total number of plants of 133 in 1914 employing 27916 wage earners and give the following data:

Capital	$243,485,000.00
Wages	18,192,000.00
Cost of materials	51,987,000.00
Value of product	101,756,000.00
Value added by manufacture	49,769,000.00

These statistics, however, include other kinds of cement than Portland cement as is indicated by the detailed output for 1914

Puzzolan	68,311. bbls.	$63,358.00
Natural	751,285. "	351,370.00
Portland	86,437,956. "	80,118,475.00
		$80,533,203.00
		21,222,797.00
		$101,756,000.00

The more important manufactured cements in this country are the Natural and the Portland, but the latter is by far the most important from the standpoint of quality and also to the electrical manufacturer because it requires considerably more power per barrel to produce than the Natural cement. The annual production of Natural cement has been steadily decreasing during recent years while the production of Portland cement has increased very rapidly. Owing to this fact it is not necessary to consider the manufacture of Natural cement.

A large number of the cement mills are already electrically operated as is indicated by the power statistics.

Primary Horse Power—1914	
Total	490,402.
Owned	
Steam engines and turbines	291,321.
Water wheels and motors	6,346.
Internal combustion engines	28,366.

Rented.	
Electric	164,369.
Electrical horse power	
Owned and rented	336,516.
Fuel Consumption—1914	
Anthracite (long ton)	226,474.
Bituminous (short tons)	6,731,438.
Coke (short tons)	20,072.
Oil including gasoline bbls.	2,502,015.
Gas (cu. ft.)	5,525,894.

Portland cement is of a somewhat uncertain chemical constitution but consists primarily of tri calcium silicate ($3\ CaOSiO_2$) and tri calcium aluminate ($3\ CaOA1_2O_3$) with traces of iron, magnesium and other materials which seems to have small effect on the quality of the cement.

There are two classes of raw materials used in the manufacture, one being rich in calcium in the form of carbonate, such as limestone, marl and chalk and the other containing a large proportion of silica and alumina as clay, shale, slag and natural cement rock, although the latter is sometimes found so rich in calcium that it may be placed in the former class, as clay must be added to bring the silica and alumina up to the proper porportions.

There are two processes used in the manufacture of cement, *i. e.*, the wet process and the dry process. Those plants using the wet process are found principally in Michigan and employ marl (calcium carbonate) and clay. The raw materials are mixed and pulverized while in a semi fluid state, the wet mixture containing about 60 per cent of water which makes it sufficiently fluid to be handled by pumps. It is stored in tanks equipped with agitators to prevent settling and after being properly pulverized and mixed by means of tube or ball mills is pumped into the kilns for burning and from this point on the process is similar to the dry process.

The dry process which is employed by the bulk of the cement plants consists principally of (1) Crushing, (2) Grinding and mixing of raw material, (3) Burning, (4) Grinding the clinker to finished cement. There are various types of machines used in the process of manufacture.

Crushing
- Gyratory (from quarry size to 3 in or 4 in.)
- Gyratory (from 3 in. to 4 in. size to ¾ in or 1 in.)

Elevators.
Storage Bins.
Bins.
Preliminary Grinding
- Hammer Mills
- Kent Mills
- Rolls
- Ball Mills
- Kominuters

Dryers.
Mixing Bins.
Coal Storage.
Crusher
Dryer
Pulverizer Bins.
Grinding
- Fuller Mills
- Griffin Mills
- Raymond Mills

Elevator
Screw Conveyor
Pulverized Coal Bins.

Final Grinding-Raw
- Fuller Mills
- Griffin Mills
- Raymond Mills
- Tube Mills

Kiln Bins.
Kilns

Coolers.
Clinker Storage.
Gypsum Storage
Pulverizer Bins.
Final Grinding—Finished.
- Fuller Mills
- Griffin Mills
- Tube Mills

Cement Storage
Packing Bins
Packers
Shippers

Both direct and alternating current have been used in the electrification of cement plants and all of the apparatus has proven commercially satisfactory so that it is impossible to say that any one frequency or voltage must be used. However, alternating current is certainly preferable to direct current and 25-cycle equipment is preferable to 60-cycle unless the plant intends to purchase power or wishes to tie in with some existing 60-cycle system, or in cases where the lower cost and better economy of 60-cycle turbines is a determining factor.

The advantages of the 25-cycle equipment are due to the fact that many of the machines run at comparatively low speed and require high starting torque. Furthermore the continuous service in an extremely dusty atmosphere make low-speed motors more desirable from the mechanical standpoint.

Low frequency is best adapted for low-speed motors with reasonably high starting torque and good power factor so that from the motor standpoint 25 cycles is better than 60 cycles. The extreme case of this is shown in the low-speed tube mill drives, where the motor is direct connected to the counter shaft, which would be impracticable with anything higher than 25 or 30 cycles.

Direct-current apparatus operates under the trying conditions of dust much better than would be supposed, but as there is bound to be considerable wear on the brushes and commutators, and as the first cost is higher than that of the a-c. equipment, it is seldom used. Where d-c. motors are used it is customary to enclose them in housings.

The effect of cement dust on bearings is a feature that must be taken care of and it has always been the practise of some manufacturers to furnish dust-proof bearings, but the best insurance

against bearing wear is to install low-speed motors and avoid locating them in the dustiest places. The effect of cement dust on motor windings has never been found to be harmful and standard insulation is always used. The dust does collect on the coils and in the air ducts, however, and may interfere seriously with the ventilation if it is not blown off.

In order to eliminate as much as possible the complications in both motors and control and reduce the wearing parts to a minimum it is desirable to use squirrel-cage motors and the tendency has always been to install these in cement plants up to a size somewhat larger than would ordinarily be used in other industries. There are a few machines on which slip-ring motors must be used, such as the kilns on which a variable speed is sometimes required, hammer mills which have a very heavy revolving element and the possibility of being shut down when full, and the larger tube mills; in such cases the motors should be furnished with enclosed collectors and the drum controllers made as nearly dust proof as possible.

The question of the proper voltage to be used in new installations is one that deserves attention on account of the large amount of power involved. Cement plants usually cover considerable ground, and it is often desirable to locate the power house apart from the main plant, so that the feeders may be as long as 1000 ft. In the interest of economy in transmission copper, it has been our practise to recommend 550-volt motors although some of the largest plants, use 440 volts. There are also several 2200-volt plants which have been in successful operation for several years using this voltage for all of the larger motors and there is no objection to this where it is found to be the most economical. It is often questionable however whether the saving in transmission copper over a 550-volt installation is not more than offset by the increased cost of the necessary conduit wiring and the higher price of 2200-volt motors.

The load on the generating station of a cement plant is exceptionally steady as compared with some other industries and there are no special problems in this connection. The principal requirement is that the source of power should be reliable and adapted to continuous operation 24 hours a day, seven days a week. It is customary in new plants to install either two or three generating units, the total capacity being somewhat greater than the average demand of the mill. This gives such a flexible arrangement that interruptions due to generating troubles are very infrequent.

The distribution of load between the different departments of a cement plant is interesting, and records of it may lead to economies in production. However, readings based on a single day are apt to be misleading on account of storage of material in the bins. That is, for a given day the crushing department may crush more or less than the raw grinding department uses, while the latter may grind more or less than the finishing depart-

ment, etc. The following table gives results in round numbers obtained over an entire year in a typical 2000-barrel dry-process plant using limestone and shale:

Department	Total kw. hr.	Per cent
Crushing	1,800,000	14.7
Raw Grinding	3,400,000	27.7
Coal grinding and kilns	900,000	7.3
Finishing	5,800,000	47.3
Lighting, machine shop and miscellaneous	371,000	3.0
	12,271,000	100.0

We are inclined to believe that in the usual plant there is no such great difference between the power used in the raw and finishing grinding departments as is shown in this table. A 3000-barrel plant, also using limestone and shale showed about the following divisions:

Crushing	15 per cent
Raw grinding	30 " "
Coal grinding and kilns	10 " "
Finishing	40 " "
Lighting and miscellaneous	5 " "

In estimating on cement plants it has been customary to assume that the power required will be about a horse power per barrel per day, *i. e.*, that, for example, a 2000-barrel plant will consume about 2000 h. p. That is fairly accurate, but in actual plants the horse power installed per barrel rated output varies considerably, because the ratings are not always an actual measure of the plant capacity, and because the tendency is always to put in somewhat larger motors than absolutely necessary. The average seems to be about 1¼ h. p. nameplate rating of the motors per barrel per day capacity of the plant.

T. E. Simpers (abstracted by A. M. Dudley): Mr. Williamson mentions certain advantages for the wet process as compared with the dry process. One of these is cleanliness. It is not mentioned that the finishing mill of a wet process plant uses the same operations as that of a dry-process plant, so that this part of the mill is just as dirty in plants using either process. Furthermore, the finishing mill is in most plants by far the dirtiest part of the plant. Also one of the necessities in a wet process plant is a supply of cheap clean water. A number of plants are located so that this is not available.

I would consider the value of the load factor as mentioned by Mr. Williamson, as being too optimistic. It is a fact that all of the apparatus in cement plants is in operation practically all the time, yet considering shut-downs throughout a year on account of operating and labor conditions, shut-downs for repairs, and the usual underloading of motors on account of the severe service, it will be found that the load factors will be less than those values mentioned by Mr. Williamson. A review of

complete tests on nine cement plants showed the highest average load factor obtainedto be 66 per cent. A number of these plants operated at values between 55 and 65 per cent, while almost an equal number operated as low as 30 per cent. There is no question however, but that with good engineering judgment and neglecting extensive shut-downs on account of weather or labor conditions that the figure of 65 per cent could be made an average load factor. This is of course, rather higher than obtained in plants of other industries.

Mr. Williamson states that the starting torque required by a gyratory crusher is small, unless it is accidentally stopped with material in it, in which case it is usually necessary to dig out the material before machine can be started. Considering crushers of the smaller capacity, it has been our custom to apply either wound-secondary motors or squirrel-cage motors with starting torque of at least two times full-load torque, which motors are capable of starting the crushers under practically all circumstances, in order to take care of the shut-downs when the material is not run out of the crusher.

Under the sub-subject "Griffin Mill" it is not brought out by Mr. Williamson that this mill requires a motor with a veritcal shaft. Furthermore, as far as the writer knows, a wound-secondary motor has never been applied to this mill, except where it has been driven on a horizontal shaft with a quarter turned belt, and this method of application has practically been discarded. A squirrel-cage motor with a vertical shaft for belt drive is the usual method of connecting the motor application to this mill.

Mr. Williamson mentions the fact that the low-speed motor has the disadvantage of lower torque and lower power factor, but the saving in space, belting, etc. offsets these disadvantages to a large extent. I would call attention to the fact that the maintenance of auxiliary apparatus is an important item in connection with such drives. The Universal Portland Cement Co. use 18-pole 25-cycle squirrel-cage motors direct connected to the counter-shaft of their tube mills. These motors are naturally much more expensive than the belted motors, but they figure that they represent an eventual saving on account of the fact that by using the low-speed motors, belting or a gear reduction may be omitted, and while the first cost of the apparatus is high, the maintenance of same is low as compared with plants using higher speed motors.

Mr. Williamson mentions the use of synchronous motors in connection with tube mill drive. Naturally the synchronous motor, considering running characteristics, represents an ideal installation in that low-speed motors can be obtained with high efficiency, and with capability of correcting power factor of the remainder of the plant. Since the synchronous motor cannot start the mill, it is necessary to use a clutch in connection with same. It has been the writer's recent

experience in connection with somewhat similar applications that the clutch is a source of annoyance on account of excessive maintenance, and furthermore, it is very expensive for the size of application under consideration. Considering such an installation, it would be of interest to know if there have been installations made of motors starting light, and with a magnetic clutch used for connecting the load on the motor, to make a comparison of such installation with installations where no clutch was used.

Furthermore, in respect to the synchronous motors, it would be of interest to know if such installations have been made. It is recognized that the synchronous motor has no slip whatever. Therefore, when a heavy load is thrown on the motor by means of a clutch, either the clutch must slip, thereby causing excessive wear to the clutch, or if it does not slip a very considerable shock will be experienced by the entire driving system.

H. Weichsel (abstracted by A. M. Dudley): Mr. Williamson's paper gives very valuable information regarding the requirements placed on electrical motors when installed in cement mills.

In order to select the most suitable motor for any kind of service, it is absolutely essential to consider the following three factors:

First: The characteristic of the machine to be driven.

Second: Kind of service to which motor and driven machine is going to be subjected.

Third: The kind of supply circuit available.

Mr. Williamson has, in an admirable manner, presented to us the most prominent factors entering in the three items listed above.

From the characteristics of the machine to be driven, it is possible to determine which type of motor should be used, such as for instance, squirrel-cage motor, slip-ring motor, adjustable speed motor.

The kind of service influences only a little the type of motor, but has a large bearing on the most suitable design of the motor, so as to give satisfactory life and service.

Mr. Williamson gives in his paper means for determining the approximate horsepower requirements for the different classes of machines, and also informs us regarding the necessary starting torque. It is quite frequently the starting torque required, which is one of the leading factors in selecting a suitable motor. If no speed adjustment is required, quite frequently either a squirrel-cage motor or a wound-rotor motor can be used for the running period of the machine, but the necessary starting torque might in some cases absolutely prohibit the use of squirrel-cage rotors.

The third factor mentioned above, which is the frequency of the supply circuit, has a great bearing on the decision if a squirrel-cage or a wound-rotor type motor should be used. We

assume here that polyphase induction motors are under consideration. Mr. Williamson has pointed out that most of the cement mill machines operate at rather low speed. In order to avoid a large ratio between the pulley of the motor and the pulley of the driven machine, it is therefore essential to make use of low-speed motors. For a given motor speed, however, the 60-cycle motor requires more poles than a 25-cycle motor, and standard squirrel-cage motors designed for 60 cycles have, on account of the higher number of poles the higher frequency, a lower starting torque than 25-cycle motors of equal speed. It might therefore readily occur that, for driving a given machine at 25 cycles, a squirrel-cage motor will have ample starting torque for starting the machine, while a 60-cycle motor, of approximately the same speed, will be unable to perform the starting operation satisfactorily.

If a 25-cycle supply circuit is used, it will make it therefore possible to use a larger number of squirrel-cage motors than if a 60-cycle supply circuit is used.

From the operator's point of view, the squirrel-cage motor, no doubt, has many advantages, on account of its strength and simplicity. The heavy starting current of squirrel-cage motors is practically of no objection in case of cement mills, because an installation of this kind consists invariably of a great number of machines, and as never more than a few squirrel-cage motors will be started simultaneously, it follows that the heavy starting current of squirrel-cage motors cannot influence the power plant to any extent.

If, therefore, the engineer designing the cement mill is free in selecting the frequency of the supply, it seems advisable to use 25 cycles, as this will allow him to make the fullest use of the advantages of squirrel-cage motors. Besides the advantages previously mentioned, it should be considered that the power factor of 25-cycle motors is considerably higher than the power factor of 60-cycle motors with equal speed and horse power capacities. Furthermore, the power factor of squirrel-cage motors is usually slightly superior to the power factor of motors with a wound rotor. Besides this, we must consider that the service conditions in cement mills, as a rule, are extremely severe, partly on account of the low class of labor available, and partly on account of the unavoidable dust. The dust will soon settle inside of the rotor winding of a wound rotor, and is therefore liable to increase the heating of the machine. The squirrel-cage motor, on the other hand, offers little or no opportunity for the dust to settle in the rotor winding, and an increase in temperature rise of a squirrel-cage rotor, has, no influence on the life of the machine. Due to the inferior class of labor available, it will frequently be found that the motors are subject to all kinds of abuses, and not unfrequently, foreign matter, such as small stones, enter the rotor. Such foreign matter is naturally more liable to cause trouble in a wound-rotor than in a squirrel-cage rotor.

The question of dust and foreign matter in the motor, demands further as large an air gap as is consistent with the design of the motor. A large air gap can readily be used on 25-cycle motors without impairing the power factor materially. This, however, is not the case on 60-cycle motors of equal speed.

The bearing construction is another feature which demands careful consideration on account of the dusty atmosphere always present in cement mills. Only high grade babbit should be used, and the bearing housings should be made as dust-proof as possible. Even with the best babbit and dust-proof bearings, it will be necessary to reline the bearings occasionally, and for this reason, it is advisable on larger motors to use split bearings, as these allow removal of the bearing shell without removing the end-plates or rotor.

In case many squirrel-cage motors are used, it is frequently advisable to do away with auto starters, and to install a separate starting circuit, which is fed either from a separate set of transformers, or from taps on the main bank of transformers. In this case, the only starting apparatus necessary is a 3-pole double-throw switch, which preferably should be of the oil immersed type.

Many of the cement mill motors are liable to sudden overloads, due to choking of the cement mill machinery. These overloads might be temporary and light, or might be so severe as to stall the motor, or cause the belt to slip. To avoid disastrous results from these unavoidable overload conditions, it is advisable to protect all motors with an overload relay of the inverse-time-element type. Furthermore, it is advisable to install in each motor circuit an ampere meter, which is so located as to be easily observed by the operator.

The information given in Mr. Williamson's paper is of such an important and far reaching nature, that it seems that this information should be made accessible to all parties interested in the subject. This would help the electrical engineer, the cement industry, and the economic life of the nation as a whole.

Mr. Williamson's paper has been presented under the auspices of the Industrial and Domestic Power Committee. No doubt, many other papers will in future be presented by this committee, all of which will be of interest to many people outside of the electrical profession. Papers of this kind will enable the superintendent or engineer of a given industry to familiarize himself with the fundamental principles and characteristics of the various types of motors, and the actual experiences which the electrical engineers had with certain types of motors in certain industries. On the other hand, the discussions will be partly carried on by engineers who are in daily contact with the different industries for which the motors are used. Mr. Williamson's paper, for instance, has influenced several prominent men of the cement industry to contribute valuable information in their discussions.

Let us hope that Mr. Williamson's paper has been the signal for the free cooperation of all parties interested in the progress of the electric drive.

In order to give everybody interested in this subject the fullest opportunity to benefit by such papers as have been presented today, and which, in all probability, will be presented in the near future by the Industrial and Domestic Power Committee, I propose that the Institute devise some means to make such information as available as possible. This, for instance, could be accomplished, by printing these papers in pamphlet form, which the general public could buy for a small amount of money. Furthermore, a card index would be of great value, if it could be arranged according to industries, using one card for each type of machine to be considered. This card could give the fundamental characteristics of the machine to be driven, the most suitable kind of motor, and with possible references to articles published on the electric drive of such machine. If such records could be kept at the headquarters of each institute section, they would probably be of great value.

Naturally, records and investigations of this nature can never be considered as completed, due to the daily progress made. For this reason, it seems that these records should be kept up to date. The Industrial and Domestic Power Committee could probably make arrangements to keep these records up to date, and collect permanently new data, and further, keep up records of all publications on this subject made in home journals, as well as in foreign magazines. If engineers all over the country could be induced to forward to the Industrial and Domestic Power Committee experiences and data on the subject of electric drive of certain industries, it would undoubtedly help to make the work of this committee of the greatest value to all industries interested, and to the economic life of the nation as a whole.

S. H. Harrison (abstracted by A. M. Dudley): I was interested in the formulas given in the paper in connection with the output of crushers, rolls, etc., and thought that perhaps in connection with the subject of rotary kilns, a formula expressing the relationship between the capacity and the diameter and length of the kiln may be of interest to you.

The only formula expressing the above relationship which I have seen heretofore was one suggested by Prof. R. C. Carpenter (See page 73—Powdered Coal as Fuel, by C. F. Herington—D. Van Nostrand Co., N. Y.) Prof. Carpenter gives the following formula for kilns operating on dry material.

$$C = \frac{D^2 L}{24}$$

where C = capacity in 24 hours in barrels of 380 lb.
D = outside diameter of kiln in feet.
L = length of kiln in feet.

From the structure of this formula it is evident that the capacity is assumed to be directly proportional to the volume of the kiln.

From data which I have collected concerning the output of a large number of kilns it appears that the output is not proportional to the volume of the kiln, but to the heating surface or very nearly so. Assuming then that the capacity of the kiln is directly proportional to its diameter and also to some function of its length, the following formula was assumed,

$$C = K\,D\,L^n$$

where K = a constant
D = diameter in feet
L = length in feet
n = constant
C = capacity in barrels of 380 lb. per 24 hours.

From the data which I had covering a large number of kilns in actual operation, fired with powdered coal and calcining dry raw material, I found the following relationship to hold true.

$$C = 0.95\,D\,L^{0.9}$$

In plotting the value $0.95\,L^{0.9}$ against L as abscissas the resulting curve was almost a straight line, so much so that for practical purposes the formula

$$C = 0.6\,D\,L$$

can be used giving results which are a trifle lower for values of L under 100 ft. (30.4 m.) and at trifle higher for values above this number than that given by the exponential formula. In view of the fact that different cement companies obtain different results from kilns of the same size the formula

$$C = 6/10\,D\,L$$

may be depended upon to give results which very closely agree with the outputs obtained in practise.

The table below may be of interest.

Size of kiln	Formula	Formula	Results obtained in practise
	$C = 1/24\,D^2\,L$	$C = 6/10\,D\,L$	
5 ft. x 60 ft.	62.5 bbls.	180	160 to 200
8 ft. x 125 ft.	333 "	600	550 to 650
10 ft. x 200 ft.	833 "	1200	1000 to 1300

The formulas

$$C = 0.95\ \text{D. L.}^{0.9}, \text{ or } C = 0.6\ \text{D. L.}$$

are to be used for kilns of commercial sizes; they do not hold for kilns of unusual lengths or diameters, for instance, 3 ft. in dia. x 100 ft. long. In other words the formula was derived from actual kilns operating under practical conditions.

W. I. Slichter: I would like to say a word of appreciation and approval of the policy which this committee has inaugurated in this meeting, the policy of making an organized survey and study of the various industries included in the scope of its activities. These matters have been discussed at their committee meetings and the conclusions arrived at have been presented to the Institute in the form before us.

This policy might well be followed by many other Institute committees. The committee by arriving at an agreement on a definite statement of the facts of the problem have no difficulty in agreeing upon a solution. The electrical engineers then present a united front in attacking the problem of the electrification of a given industry.

A noticeable feature of the papers this evening is the absence of any plea in justification of the use of electric motors in these industries. Only a few years ago any paper of this character would have given considerable prominence to arguments showing the advantage of electric as compared to other systems of drive but this is no longer disputed. In fact the flexibility of the electric drive is what made it possible to develop the capacity of our factories so rapidly to meet war conditions.

It is stated on reliable information that the applications of electric motors to manufacturing purposes and particularly to factories using units smaller than 50 h. p. has been so standardized that it is only necessary to know the process of manufacture and the type of building in order to place an order for the whole equipment of motors, starters, switches and wiring. This is particularly valuable because when the present war activities are over and the industries resume their peacetime processes these factories built to make war necessities may be easily converted to some reconstruction work.

G. H. Rowe (communicated after adjournment): Mr. Williamson mentions that the use of synchronous motors might work out advantageously for driving certain machines in cement mills.

While the induction motor is undoubtedly best adapted in the majority of applications, it would seem that synchronous motors would find an ideal application for machines running practically continuously and requiring only moderate starting torques. Besides rugged mechanical design, large gap, high efficiency, freedom from dust troubles, they have the added advantage of power factor correction. The saving in line losses during a year due to power factor correction alone would, in the majority of cases, be a very considerable item in plants using so many relatively small induction motors operating at 220 and 440 volts.

Is there any really good reason why induction motors and synchronous motors are not used in combination, even though it becomes necessary to start the latter with a clutch at rather infrequent intervals.

A. L. Hadley (communicated after adjournment): The latest designs of synchronous motors such as used for driving air compressors, etc., are built with starting torque of 30 to 40 per cent on compensator tap of 50 per cent without abnormal current taken from the line. The question of the use of synchronous motors for cement machinery wherever the load is constant and continuous, can well be considered both for low and moderate speeds. I would suggest that this matter be brought up for discussion. A statement of the approximate

starting torque in per cent of normal full-load torque needed for each kind of cement machine would be a good indication as to whether a synchronous motor might be used. Doubtless some applications could be made without the use even of a clutch.

R. B. Williamson: The question of squirrel-cage and slip-ring motors has been brought up. I think there is no doubt that in the past there has been more or less prejudice against slip-ring motors in cement mills. Squirrel-cage motors were sometimes forced to do what they were not designed to, and could not do, to the best advantage. The prejudice against the wound-rotor motor was due to the fact that they had collector rings, and cement mill operators were afraid there would be more or less trouble with the brushes. In the later plants slip-ring motors are being used more, and the fact that mills have become so much larger, and must use larger motors, has forced this to a large extent.

There was one point regarding which I hoped to hear more discussion, and that is the possibility of using synchronous motors, to a greater extent in this work. For example, in a ball mill or tube mill, the load is very steady after the mill is once started; the amount of power required is large, and if synchronous motor can be used for that work it would be quite an advantage. The tendency in the newer mills is to cut out belting and use lower speed motors, some of the motors being as large as 400 or 500 h. p.

Most engineers agree that in the future there is going to be a great deal of development in the way of supplying power from large central stations over wide districts. For a number of reasons, not necessary to go into at this time most of this power will be generated at 60 cycles. The 60-cycle frequency, combined with the low-speed induction motor, makes a bad combination from the point of view of power factor, especially with motors having a liberal air gap, such as is necessary for cement mill work.

Synchronous motors on ball mills and tube mills would be a great aid in keeping up the power factor on the plant as a whole, and their successful use hinges very largely on the operation of the clutch between the motor and the mill. There is always more or less prejudice against the use of clutches, but there are some successful installations of this kind in foreign mills and some installations in this country at the present time. These use a magnetic clutch, and it is possible to arrange a magnetic clutch, with an automatic resistance which will gradually increase the exciting current on the clutch and thus giving a gradual gripping effect so as to start the mills. The synchronous motor in addition to its high power factor has a large air gap; it is very simple mechanically, and aside from the clutch and the added complication of providing exciting current for it, it seems to me there is no reason why it may not be adapted to this work in the future.

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ELECTRIC POWER GENERATION IN ONTARIO ON SYSTEMS OF HYDROELECTRIC POWER COMMISSION

BY ARTHUR H. HULL

THE Province of Ontario is the largest and most populous of the Provinces in the Dominion of Canada, having an area of 407,262 square miles and a population (census 1911) of 2,523,274. Ontario's population is about 35 per cent of the total of Canada's population, and its area is 10.9 per cent of the total area of the Dominion. Ten per cent of its area is water.

Ontario has developed into the greatest manufacturing Province in Canada and as there are no coal deposits in the Province, its abundant water power resources are being more and more utilized to furnish the power required for its varied and increasing industrial development. Other fuels, such as natural gas and petroleum are found in the southern part of the Province, gas along the north shore of Lake Erie and petroleum near Sarnia. The natural gas supply, however, is insufficient for present purposes and the quantity of petroleum produced does not begin to meet the needs of the Province. The demand for electric power has increased during the past ten years to such an extent, that, with the war industries in operation, it became necessary to impose restrictions on private and municipal consumers in order to provide the power required by the war industries. The great saving in coal effected by the use of electric power in Ontario's industries is one of the most important aspects of the electric power situation, and further great savings can be, and will be before long, brought about by the electrification of the steam railroads.

Mr. H. G. Acres* in a monograph on Water Powers of the Province of Ontario, written in 1915, gives the following sum-

*Hydraulic Engineer, Hydroelectric Power Commission.

mation of water power capable of development, and of water power already developed in the Province.

Division	Potentiality	Developed.
Ottawa River & Tributaries	688,000 h.p.	71,000 h.p.
Great Lakes Tributaries	446,000 "	137,000 "
Hudson Bay Slope	250,000 "	22,000 "
James Bay Slope	1,500,000 "	70,000 "
International Boundary Rivers	2,045,000 "	462,000 "
	4,929,000 h.p.	702,000 h.p.

Of the above total for power developed, about 69,000 horse power is used in pulp and paper manufacture, about 59,000 horse power is used as hydraulic power directly applied, and the balance 574,000 horse power is converted into electric energy for light and power.

This paper will deal only with the generation of electric power by the Hydroelectric Power Commission of Ontario which is now the largest producer and distributer of electric energy in the Province.

The Hydroelectric Power Commission of Ontario, which will be referred to hereafter as the Commission, was formed by the Ontario Government in 1906, and first furnished power over its 110,000-volt lines from Niagara Falls in October 1910, the power being purchased from the Ontario Power Company of Niagara Falls, Ontario. It was not until August 1st, 1917, that the Commission obtained direct control of the generation of power at Niagara Falls, through the Ontario Power Company, but in other parts of the Province, the Commission had started to develop its own power by constructing a hydroelectric plant at Eugenia Falls on the Beaver River near Flesherton, which was put into operation in November 1915, and one at Wasdell's Falls on the Severn River which was started in service in October 1914. In 1914 the Commission acquired by purchase the Simcoe Railway and Power Company, whose generating station at Big Chute on the Severn River had previously supplied power under contract to the Commission's Severn System, supplying the district around the southern part of Georgian Bay. In March 1916 the Government of Ontario purchased the entire holdings of the Electric Power Company which, through subsidiary companies, was generating power on the South River near Nipissing, on the Otanabee River

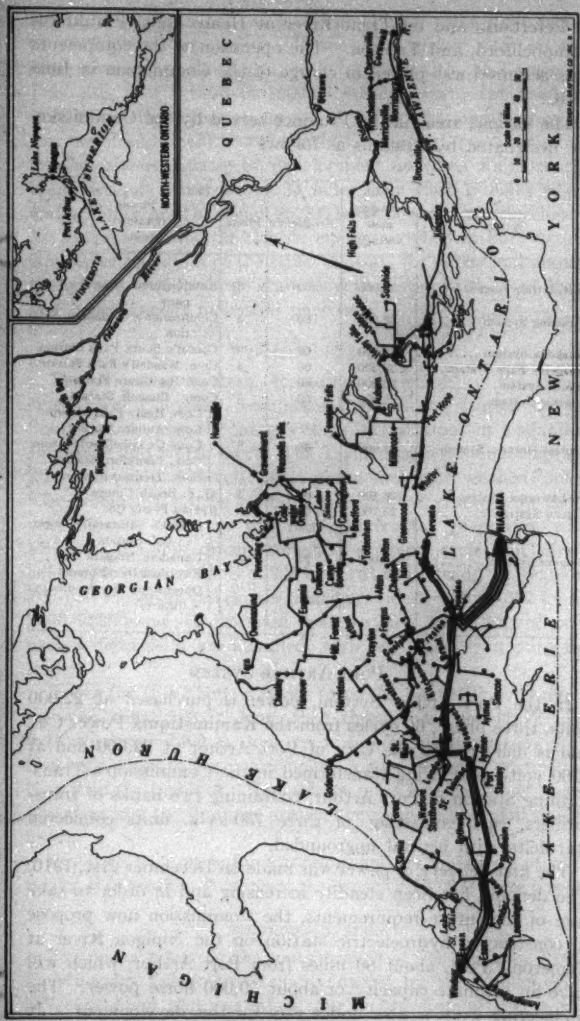

FIG. 1—ELECTRIC GENERATING AND TRANSMISSION SYSTEMS OF THE HYDROELECTRIC POWER COMMISSION OF ONTARIO

at Peterboro, and on Trent River at Healy Falls, Frankford, Campbellford, and Trenton. The operation of the equipments thus acquired was placed in charge of the Commission in June 1916.

The various areas in the Province served by the Commission are designated by Systems as follows

System	Main transmission voltage	Frequency	Phase	Power obtained from
Port Arthur System	22,000	60	3	Kaministiquia Power Company
Nipissing System	22,000	60	3	Commission's Nipissing Station
Muskoka System	22,000	60	3	Comm'n South Falls Station
Wasdell's Falls System	22,000	60	3	Com. Wasdell's Falls Station
Severn System	22,000	60	3	Com. Big Chute Station
Eugenia System	22,000	60	3	Com. Eugenia Station
Central Ontario System	44,000	60	3	Com. Healy Falls Station Com. Auburn Station Com. Campbellford Station Com. Frankford Station Com. Trenton Station
St. Lawrence System	26,400	60	3	M. F. Beach Company
Rideau System	25,700	60	3	Rideau Power Co.
Niagara System	110,000	25	3	Comm's Ontario Power Co's Station Canadian Niagara Co. Electrical Development Co. Queenston Development (future)

Port Arthur System

In the Port Arthur System, power is purchased at 22,000 volts, three phase, 60 cycles from the Kaministiquia Power Co., and is delivered to the City of Port Arthur at 22,000 and at 2200 volts, part being transformed in the Commission's Transforming Station in Port Arthur, containing two banks of transformers, each consisting of three 750-kv-a. units connected star-delta with neutral ungrounded.

The first delivery of power was made on December 21st, 1910. The demand has been steadily increasing and in order to take care of the future requirements, the Commission now propose to construct a hydroelectric station on the Nipigon River at Cameron's Pool, about 80 miles from Port Arthur, which will have an ultimate capacity of about 50,000 horse power. The engineering work is now under way for this development. It

is proposed to transmit the power at 110,000 volts, three phase, 60 cycles, to Port Arthur. Three other power sites on this same river, when developed, will, together with the Cameron's Pool site, give a total of 150,000 horse power. The needs of the Port Arthur district should, therefore, be amply provided for, but if additional power should be required for this district, another site is capable of development at Silver Falls on the Kaministiquia River about 25 miles from Fort William where with a 350-ft. head a total of 25,000 horse power can be developed. At the present time the electric energy supplied at Port Arthur is used for operation of the street railway, for public and domestic lighting, and for large grain elevators, ship yards, coal docks and miscellaneous industrial purposes.

Nipissing System

The Nipissing System, formerly controlled by the Nipissing Power Company, comprises a generating station on the South River near Nipissing Village, sub-stations in Callander, Powassan and North Bay. The Transmission system comprises 26.5 miles of 22,000-volt, three-phase circuit on wooden poles. The generating station contains two 450-kw. three-phase 2200-volt 60-cycle 450-rev. per min., horizontal generators with 12.5-kw. 125-volt direct-connected exciters, each unit direct coupled to a water wheel of 925 b.h.p. at 86 ft. head. A 37.5-kw. 125-volt motor-driven exciter is also provided. Three 300-kv-a., 2200/22000-volt single-phase water-cooled transformers connected delta-delta are installed, with one 22,000-volt outgoing feeder.

Water for this plant is taken from the South River through an open canal 900 ft. long, then through a wood stave pipe 6 ft. diameter 2300 ft. long to a differential surge tank 72.5 feet high, close to the power house, a steel penstock connecting thence to the turbines. The storage pond has an area of about 100 acres and the drainage area of the river is about 350 square miles.

Provision was made for extension, and by additional storage works, it is possible to increase the capacity to 2500 horse power.

Muskoka System

The power house on the south branch of the Muskoka River at Muskoka Village was formerly owned by the Municipality of Gravenhurst and was taken over by the Commission in the

latter part of 1915, and has been remodelled and enlarged, and now supplies power over a single circuit 22,000-volt, three-phase, 60-cycle line about 26 miles long to Huntsville, and at 6600 volts three phase, 60 cycles, to the Municipality of Gravenhurst over the Municipality's line.

The equipment in this station now comprises one 450-kv-a., three-phase, 6600-volt, 720-rev. per min., 60-cycle generator, direct connected to a 500 b.h.p. turbine; one 750-kv-a. similar 720-rev. per min. generator, direct connected to a 1000 b.h.p. turbine; one bank of three 400-kv-a., 6600/22,000-volt transformers connected delta-delta; double low-tension bus, and single high-tension bus. Four outgoing 6600-volt feeders are

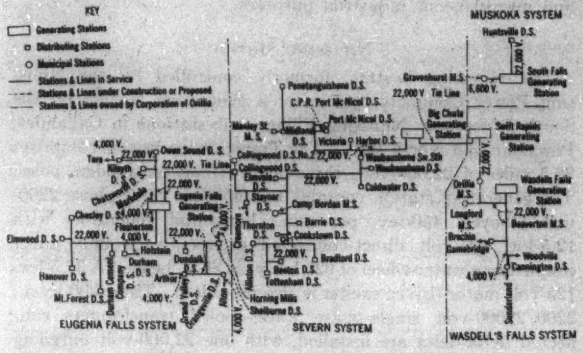

FIG. 2—DIAGRAM OF EUGENIA FALLS, SEVERN, WASDELL'S FALLS, AND MUSKOKA SYSTEMS—60 CYCLES

provided, two feeding to Gravenhurst and two for future use. One 22,000-volt feeder leaves the station to supply Huntsville. Provision is made in the existing building for a second high-tension feeder.

When this station was taken over by the Commission, only one 450-kv-a. generator was installed. The station was remodelled and enlarged for the larger unit mentioned above, and provision was made for the extension of the building at a future date and for the replacing of existing units so that, when future load conditions require it, there will ultimately be three 1500-kv-a. generators with a turbine capacity of 6000 horse power installed.

The present excitation at this plant is provided by a turbine-driven exciter for the 450-kv-a. unit, a motor-driven exciter for the 750-kv-a. unit, which also has a belted exciter for emergency use, which is so arranged that it may excite either generator.

The hydraulic head at this plant is 102 ft. The water is conducted through one steel and one wood stave pipe, each 946 ft. long to the turbines.

Wasdell's Falls System

This system comprises a hydraulic generating station on the Severn River at Wasdells' Falls near Severn Bridge, and substations at Beaverton and Cannington. Power is transmitted to Beaverton and Cannington over a single-circuit steel-conductor line at 22,000 volts, three phase 60 cycles, and over a single-circuit aluminum tie line at the same voltage to the Commission's Severn System, connecting at Longford with the Town of Orillia 22,000-volt lines from Longford via Orillia to the Big Chute generating station.

The generating station which was constructed by the Commission in 1914 is interesting on account of the low hydraulic head. The normal head is 12 ft., but variations from 9 to 15 ft. occur. The equipment consists of two 400-kv-a., 90-rev. per min., vertical type 60-cycles, three-phase, 2300-volt generators connected through flexible couplings to turbines rated at 600 horse power at 100 per cent gate opening with 12-ft. head. Two exciters, one 20-kw. turbine-driven, and one 30-kw. motor-driven are provided. Two banks of transformers, each consisting of three 2200/22,000-volt 150-kv-a. units connected delta-delta are installed. Two outgoing 22,000-volt lines were provided.

There are in this system, 37 miles of 22,000-volt circuit and 23 miles of 4000-volt circuits from the two sub-stations.

Severn System

This system comprises a hydraulic generating station on Severn River at Big Chute with substations as shown in the diagram.

The generating station as originally built in 1909 by the Simcoe Railway and Power Company contained three 900-kv-a., three-phase, 2200-volt, 60-cycle, 300-rev. per min., horizontal-shaft generators direct connected to turbines each

rated at 1300 horse power under 56-ft. head; two 100-kw. 580-rev. per min., 125-volt exciters direct connected to 200-h.p. turbines; two transformer banks each consisting of three 600-kv-a., 2200/25,000-volt water-cooled transformers connected delta-delta, with two outgoing 25,000-volt lines. An extension to the building has been made by the Commission during the past year, and there is now being installed one 1600-kv-a., three-phase, 60-cycle, 2200-volt 300-rev. per min. horizontal generator direct-connected to a 2300-h. p. turbine. The switching equipment is being rearranged to provide a double high-tension bus for greater flexibility in operation. Space is left for a third bank of transformers, and for two future 22,000-volt line equipments. A double low-tension bus is installed. All power is transmitted at 22,000 volts, three phase, over four lines.

An interesting feature of the transmisssion lines of this system is the long spans across Matchedash Bay at Waubaushene, one being 1135 ft. long and one 858 ft. long. The west shore tower is 175 ft. high, the middle and east shore towers are 88 ft. high. No. 00 B. & S. 19-strand copper conductors are used on these spans.

The water is taken from the river through a canal 500 ft. long to the head works and then through two steel penstocks 170 ft. long to the turbines.

Eugenia System

This system obtains power from a hydraulic generating station constructed by the Commission at Eugenia on the Beaver River which was placed in service in November 1915. Extensions are now almost completed to provide double the output. This development is one of the most interesting in Ontario, and has the distinction of having with one exception, the highest hydraulic head of any plant in Canada, being 552 ft. gross. It is also one of the highest heads in the world using reaction wheels.

The first installation consisted of two 2250-h.p. turbines each direct connected to 1410-kv-a., three-phase, 4000-volt, 60-cycle 900-rev. per min., horizontal-shaft generators having neutral grounded without resistance; one bank of three 900-kv-a., 4000/25,000-volt transformers connected delta-delta, single high-tension and low-tension busses, two outgoing 25,000-volt feeders and two 4000-volt feeders, one each to Markdale and Flesherton. The extensions cover enlargement of the

building to accommodate two additional generating units, one of which is now being installed, and for double high-tension and low-tension busses, and for six 25,000-volt feeders. The new unit consists of a 2810-kv-a., three-phase, 60-cycle, 720-rev. per min. horizontal maximum-rated generator direct connected to a 4000-h. p. turbine. Each generator has a direct-connected 125-volt exciter of sufficient capacity to excite two generators.

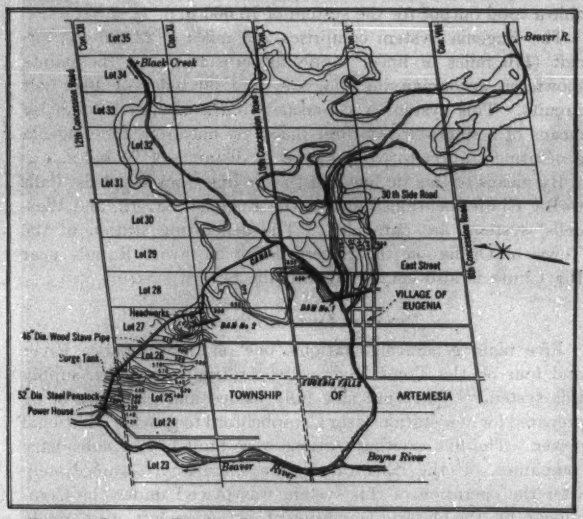

FIG. 4—EUGENIA FALLS—GENERAL PLAN OF POWER DEVELOPMENT ON BEAVER RIVER

The success of the development depended upon the storage of the water of the Beaver River and for this purpose two large storage dams were constructed as shown on accompanying plan. From the head works, a wood stave pipe 46 in. in diameter conducts the water 3400 ft. to the Johnson differential surge tank which is 105 ft. high. From this tank a 52-in. diameter steel pipe is carried 1550 ft. to the power house, which is a brick building 69 ft. wide, 112 ft. long and 34 ft. high above the generator room floor. Actual tests made at this station

after the first installation was completed in 1915 gave an overall full-load efficiency of 80 per cent which shows how carefully the design was worked out.

Additional power for the Eugenia System will be obtained, when required, by the construction of further water storage systems and of a second pipe line at the Eugenia Falls development and the installation of a fourth unit, also by the construction of a station on the Saugeen River near Lake Huron which operating in parallel with the Eugenia Station will make available a total output for the system of 15,000 h. p.

The Eugenia System comprises 245 miles of 22,000-volt circuit (176 miles of lines) supplying substations at the points shown on accompanying diagram, and 50 miles of 4000-volt circuits. This system is connected to the Severn System by means of a single-circuit three-phase tie line from the Eugenia generating station to Collingwood, a distance of 24 miles.

By means of this tie line and the tie line from Wasdells' Falls to Big Chute mentioned above, the Eugenia, Severn, and Wasdells' systems are paralleled. The generating station of the Town of Orillia on the Severn River at Swift Rapids near Big Chute is also connected into the Severn system.

Central Ontario System

Five main generating stations, one on the Otonabee River and four on the Trent River, all operating in parallel, supply this system. These are now fully loaded and plans are being prepared for new stations near Campbellford to provide additional power. The five exciting stations were built by the subsidiary companies of the Electric Power Company. Immediately after the operation of this system was placed under the Commission by the Ontario Government, arrangements were made to install a new unit in the Healy Falls generating station and work on the same is now nearing completion.

The following table gives the data relative to the existing developments, all generators being three-phase, 60 cycles, with 25 per cent overload guarantees:

In addition to the above stations, a 1000-kv-a., three-phase, 60-cycle, 120-rev. per min. horizontal generator owned by the Town of Campbellford in its generating station, a short distance north of the Campbellford station, delivers its output to the 2400-volt bus in this station. Also at Fenelon Falls, a small generating station is operated, containing two 400-kw., 600-volt,

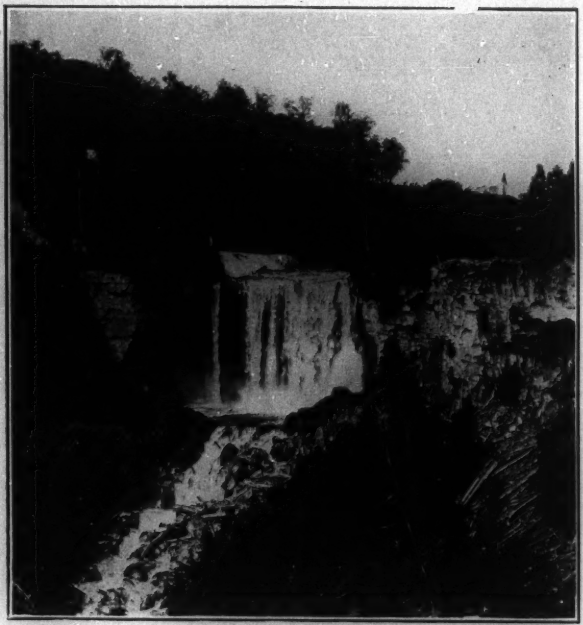

FIG. 3—EUGENIA FALLS [HULL]

FIG. 7—VIEW OF NIAGARA TRANSFORMER STATION LOOKING SOUTHEAST

FIG. 8—VIEW OF NIAGARA TRANSFORMER STATION LOOKING SOUTHWEST

FIG. 11—ARMORED CABLE INSTALLATION [HULL]

Location	River	Gross head feet	Units				Speed rev. per min.	Type	Generator voltage.
			Number		Rated capacity.				
			Present	Future	Generator kv-a.	Turbine h.p.			
Trenton	Trent	20	4	..	937.5	1400	120	Vertical	6600
Frankford	Trent	18	4	..	812.5	1200	112.5	Vertical	6600
Campbellford	Trent	23	5	..	750	1100	150	Vertical	2400
Healy Falls	Trent	76	3*	1	3750	5600	240	Horizontal	6600
Auburn	Otonabee	18	3	1	625	950	150	Horizontal	2–6600 1–2400

*One unit now being installed.

three-phase 60-cycle, 200 rev. per min. generators connected to two 700-h. p. turbines. These feed into the 44,000-volt net work of the Central Ontario system at Lindsay the voltage being stepped up in the generating station through two banks of transformers to 11,000 volts for transmission to Lindsay.

All these stations are on the route of the Trent Valley Canal, the dams having been constructed by the Government of the Dominion of Canada.

The Trenton station, known as Sidney Dam 2 development, being at Dam 2, section 1 of Trent Valley Canal, about one mile north of the Town of Trenton, was placed in operation by the Sidney Electric Power Co. in August 1911. The four generators feed into a single sectionalized bus, and two 6600-volt feeders conduct the current to a transforming station across the road, which station also receives the output of the Frankford generating station at 6600 volts. Two 75-kw. 125-volt exciter units are installed, one being a vertical turbine-driven unit and the other a motor-driven unit. Each of these exciters is capable of exciting all generators at 25 per cent overload with 65 per cent power factor current lagging.

The Frankford station, known as Sidney Dam 5 development, is located at Dam 5 on Section 1 of Trent Canal, about four miles north of Dam 2. This station was constructed by the Sidney Electric Power Company and placed in operation a short time after the Trenton station. It has a single unsectionalized 6600-volt bus and its entire output is fed over two 6600-volt circuits to the transforming station at Dam 2.

The Trenton transforming station contains three 3000-kv-a., 6600/44,000-volt, three-phase, 60-cycle, shell-type transformers connected delta-star with ungrounded neutral, double high-tension and low-tension buses, switching equipment for three 44,000-volt lines and for six 6600-volt lines.

The Campbellford station, known as the Stephen's Dam station, was first placed in operation in 1909 and is near Dam 1, Section 5 of Trent Valley Canal. This station contains four 1125-kv-a., three-phase, core-type, 2400/44,000-volt transformers and two outgoing 44,000-volt line equipments. Excitation for the generators is obtained from two 60-kw. 125-volt exciters, one turbine-driven and one motor-driven, with two 17½-kw. belted exciters for emergency use. The station is situated some distance below the dam, the water being conducted to it through an open head race.

The station at Healy Falls is situated about seven miles above Campbellford. Water is conducted through steel penstocks from the head gates to the turbines. A long tail race excavation through rock was necessary and the enlargement of this tail race, necessitated by the installation of the third generator, is now being completed. This excavation work has been slow and difficult on account of the nature of the rock.

There are three 3750-kv-a., three-phase, shell-type, 6600/44,000-volt transformers installed, with very flexible switching equipment consisting of double 6600-volt buses, two 44,000-volt outgoing lines, a sectionalized 44,000-volt single transfer bus, with a transformer and an outgoing line forming a unit which may be operated independent of the bus. Provision is made for a fourth generator and transformer and for two additional high-tension line equipments. Excitation is provided by two 160-kw. 125-volt exciters, one being turbine-driven and one motor-driven.

The Auburn generating station is located in the north of Peterboro, on the Otonabee River 1200 ft. below the Auburn Dam. Water is taken to the turbines through an open head race 1200 ft. long paralleling the river. This station was built by the Auburn Power Co. and placed in operation in 1912 and no extensions have been made. The turbines are of the four runner type in open wheel pits 40 ft. long, 16 ft. wide and 14 ft. deep. A travelling steel gate 15 ft. 3⅜ in. by 14 ft. 3 in. is provided with motor-driven lowering and hoisting mechanism. The gate is designed to be lowered against the running water in 2½ minutes. Two 90-kw., 125-volt exciters, one being turbine driven and one motor-driven, furnish the excitation of this plant. One generator is wound for 2400 volts and feeds a section of the bus from which 2400-volt feeders may be taken. This section of bus is connected to the 6600-volt section of the bus through a bank of three 200-kv-a., 2400/6600-volt transformers. Feeders at 6600 volts, three-phase, connect this station to the transformer station adjacent which contains two 1875-kv-a., three-phase, core-type 6600/44,000-volt transformers and one outgoing 44,000-volt line connecting into the 44,000-volt network of the Central Ontario System. Some power is also delivered to Peterboro at 6600 volts from the generating station.

The accompanying map, Fig. 1, and the diagram, Fig. 5, show the location of the generating stations of the Central

Ontario System, and the extent of the transmission net work with substations. Other power sites along the Trent Valley Canal together with those described briefly above will, when developed, provide about 60,000 electrical h. p. with a maximum capacity of 75,000 electrical h.p. The present power output of the system is used for lighting, street railway and manufacturing purposes, a considerable quantity being required at Campbellford for a pulp and paper mill, and near Belleville for cement mills. All transmission lines are constructed on wood poles. The total mileage of 44,000-volt circuits is 372, with 15 miles of 11,000-volt circuit, 16.4 miles of 6600-volt circuits and 52 miles of 4000-volt circuits.

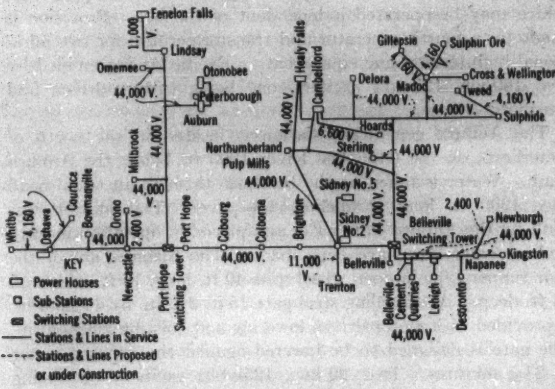

FIG. 5—CENTRAL ONTARIO SYSTEM

ST. LAWRENCE SYSTEM

This system at the present time has no generating station owned by the Commission. Power is secured by contract from the hydraulic station of the M. F. Beach Company at Iroquois, but the amount obtainable proved inadequate and was supplemented by power obtained from the steam generating station of the town of Brockville. As additional power is needed arrangements are now being made to obtain an adequate supply from another source, near Cornwall. To take this power the Commission is now constructing a transforming station near Cornwall which will contain one bank of three 1250-kv-a., single-phase, 63,500/26,400-volt, 60-cycle transformers connected

star-delta and switching equipment for two incoming 110,000-volt circuits and for two outgoing 26,400-volt circuits.

This system now supplies power to Brockville, Prescott, Morrisburg, Winchester and Chesterville. Fig. 6 shows this system together with the Rideau system.

Rideau System

A new net work is being developed called the Rideau system and covering a district in the neighborhood of the Rideau River. Plans are now being prepared for a hydraulic generating station at High Falls on the Mississippi River near Clarendon, a point about 50 miles northerly from Kingston, Ontario. The

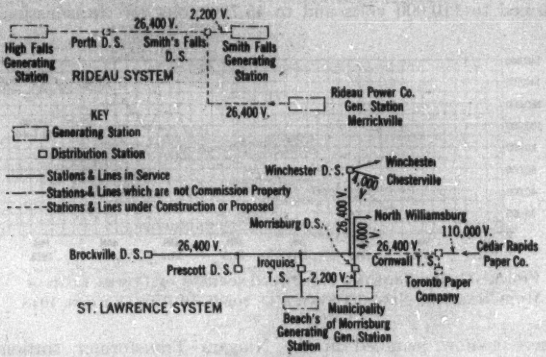

Fig. 6—St. Lawrence and Rideau Systems—60 Cycles

installation at this point will consist of four 350-kw., 2200-volt, three-phase, 60-cycle, 300-rev. per min. horizontal generators connected to two turbines, one generator being at each end of turbine; and one 875-kv-a. 80 per cent power factor, three-phase, 60-cycle, 2200-volt, 300-rev. per min. generator direct-connected to its turbine with necessary switching and transformer equipment to transmit the full 2100-kw. output, at 26,400 volts to the Rideau system.

A portion of this system is now in operation, power being obtained at 26,400 volts, three-phase, 60-cycles, under contract from the Rideau Power Co. at Merrickville, and transmitted to Smith Falls where at the substation the municipality's own

generating stations are paralleled with this system on the low-tension bus.

This system may ultimately be connected with the St. Lawrence System.

Niagara System

The Niagara System comprises all lines and substations that receive power from Niagara Falls, and covers the entire district from Niagara River to the Detroit River and River St. Clair and north to Goderich and Harriston and east to Toronto. Power is received at a transformer station at Niagara Falls from the Ontario Power Co. (now controlled and operated by the Commission) and from the Canadian Niagara Power Company, at 12,000 volts, three-phase, 25 cycles and is transformed to 110,000 volts and to 45,700 volts for transmission.

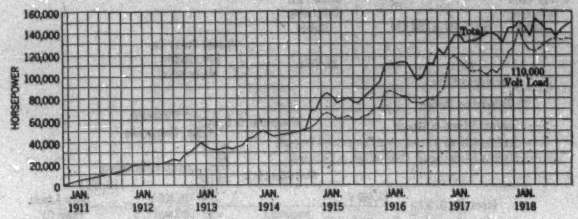

Fig. 9—Curve Showing Monthly Increase of Power Load of Municipalities Niagara System October 1910 to October 1918

There is now installed in the Niagara Transformer station 132,000 kv-a. of 110,000-volt transformers in eight banks and 42,000 kv-a. of 45,700-volt transformers in four banks, not including spare units. Four banks of 110,000-volt units are composed of twelve 7500-kv-a., single-phase, shell-type 12,000/63,500-volt transformers, all other transformers being of 3500-kv-a. rating. This station, in point of transformer capacity, is the largest in the world, having a total capacity of 174,000-kv-a, with 7000 kv-a. in spare units additional. Figs. 7 and 8 show two views of this Niagara transformer station.

Four outgoing 110,000-volt circuits feed to Dundas transformer and switching station and from there power is distributed to thirteen 110,000-volt transformer stations. Four 45,700-volt lines feed to Welland to the substations of the Electric Steel & Metals Company, the Union Carbide Co. and

the city of Welland. From this latter point a single-circuit, 45,700-volt line runs to Dunnville.

This system was placed in operation in October 1910. The curve in Fig. 9 shows the monthly increase in power load of Municipalities on this System from October 1910 to October 1918. This gives some idea of the growth of the system in this period but the latter portion of the curve cannot be regarded as normal owing to the increase in power required for munitions

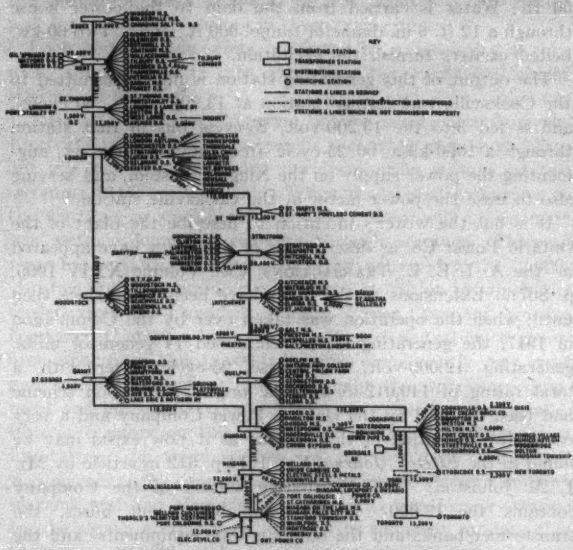

FIG. 10—DIAGRAM OF STATIONS—NIAGARA SYSTEM

manufacture and the consequent restrictions on other uses to cut their requirements to a minimum.

The transmission net work, see Fig. 1 and Fig. 10, on this system contains 760 miles of 110,000-volt circuits; 65 miles of 45,700-volt circuits; 529 miles of 26,400-volt circuits; 489 miles of 13,200-volt circuits; 27 miles of 6600-volt circuits; 180 miles of 4000-volt circuits; and 20 miles of 2200-volt circuits. The 110,000-volt and 45,700-volt circuits are carried on steel towers (with exception of one 45,700-volt line from Welland to Dunnville), while the

other circuits mentioned above are standard wood pole line construction.

The Commission early in 1917 purchased the Erindale Power Company which had a hydraulic generating station on the Credit River at Erindale, 14 miles west of Toronto, containing two 600-kw. three-phase, 60-cycle, 13,200-volt, 200-rev. per min. generators, each direct-connected to a 1000-h. p. turbine designed for a 60-ft. head. The present operating head is about 50 ft. Water is carried from the dam to the power house through a 12 ft. 6 in. diameter tunnel 900 ft. long. Two 60-kw. belted exciters furnish the excitation.

The output of this generating station is now transmitted to the Cooksville transformer station at 13,200 volts, 60 cycles, and is fed into the 13,200-volt, 25-cycle bus in that station through a 1000-kv-a. 60/25-cycle frequency changer set, augmenting the power supply on the Niagara system, and serving also to raise the power factor at the Cooksville station.

It is not the writer's intention to describe the plant of the Ontario Power Co. as descriptions of this plant have appeared in the A. I. E. E. TRANSACTIONS (See Volume XXIV 1905, p. 807). Extensions to the plant have been made since then until, when the operation was taken over by the Commission in 1917, the generating station contained 14 generator units, generating 12,000-volt, three-phase, 25-cycle power with a total rating of 149,012-kv-a. The original excitation scheme had been changed by the Ontario Power Company and a very complete description of this scheme as it now exists may be found in the *Electric Journal* Vol. 1914, p. 612 in article by Mr. J. A. Johnston. The distributing station of the Company contains the 12,000- 30,000- and 60,000-volt buses, the transformer banks and the outgoing line equipments and the control room for the plant. The switching scheme as originally laid out has been considerably re-arranged the buses being sectionalized and connected through reactors, these changes being due to the great amount of power generated and to the large proportion leaving the station at generator voltage.

The Commission, on behalf of the company, is now making extensions to the generating station. The building is being extended north about 90 ft. and a third pipe line of temporary nature consisting of wood stave pipe is being constructed. Two additional generating units are being installed each rated at 15,000 kv-a. maximum rating at 75 per cent power factor.

These new generators are the same speed as the previous units namely 187.5 rev. per min. and the frames are the same size as the 8775-kv-a. units most recently installed. They have however a higher temperature guarantee and are so designed that they may be operated as synchronous condensers at zero power factor if it ever becomes necessary to remove them from this station. These generators were made in Canada, under war conditions, and all parts, except the laminated steel and the insulation, were produced in this country. This speaks well for Canadian industry in these trying times.

Each of these new units is excited by a 125-kw. 250-volt, motor-driven exciter, thus extending the excitation scheme referred to above.

The Ontario Power Company obtains some power from the Electrical Development Company's generating station at Niagara, this being brought into the distributing station on two incoming 12,000-volt feeders. When the present extensions to the distributing station are completed, there will be six main bus sections connected together through reactors. Three of these bus sections will supply the power delivered to the 12,000-volt bus in the Commission's transformer station to which the feeders from the Canadian Niagara Power Company are also connected through a bus reactor. The concentration of generator capacity on this 12,000-volt bus is consequently very great and has necessitated material changes in the switching equipment and bus construction. The studies covering the installation of reactors on this 12,000-volt system have been most interesting. As these studies are still not fully completed, no diagram of the connections for the Ontario Power Company stations and the Commission's Niagara transformer station can be shown at this time. It will be sufficient to state that there is no 12,000-volt bus in the generating station, and that each generator feeds through an automatic oil circuit breaker in the generating station to its individual cables in the cable tunnels up the hill, thence to its group of circuit breakers in the distributing station where switching arrangements are such that each generator and each feeder may be connected to either of two bus sections. In the Commission's Niagara transformer station there is a single sectionalized 12,000-volt transfer bus system so arranged that feeders are connected through an auxiliary bus to the main bus or direct to a transformer bank.

It is of interest to note that the cables for the two new 15,000-

kv-a. generators were purchased after laboratory tests were made on manufacturers' samples to determine the dielectric losses and tendency of compound to flow, and guarantees on dielectric losses were obtained from the contractor.

Another interesting point in connection with the feeders entering the Commission's transformer station is that eight of these are three-conductor, armored cables laid directly in the earth without ducts. These armored cables all have sector-shaped conductors, with 8/32-in. x 8/32-in. paper insulation, a lead sheath and a double steel tape armor with jute bedding and jute covering. They are placed three feet below the surface and two and three feet apart center to center on private right of way. Where these buried cables cross ducts containing other cables, arrangements are provided to moisten the surrounding earth either by sprays above the surface or by porous tile in the ground. Figs. 11 and 12 show views of this armored cable installation, Fig. 10 showing the ventilating pipes at street crossing, the cables being in 6-inch cast iron pipe across the street.

The Queenston Development

When the Commission in March 1908 contracted with the Ontario Power Company for 100,000 horse power, it was thought by many that such amount of power would meet the requirement for many years. This supply was however exhausted in 1915 that is, in 5 years from date of first delivery of power, and the additional power secured by arrangements with the Canadian Niagara Power Company has proved insufficient to meet the demands. It became necessary therefore to look to a new development to secure a further supply and the final decision was to proceed with a development called the Queenston Development which is authorized by an Act passed by Ontario Government in April 1917 called The Ontario Niagara Development Act.

Between Lakes Erie and Ontario there is a difference in elevation of 330 feet. The greatest net head now utilized on the Canadian side at Niagara Falls is about 160 ft. Canada is entitled to divert 36,000 cu. ft. per second from the Niagara River, and of this amount, by an Ontario Government Order in Council in 1915, the Commission were allotted 6610 cu. ft. per second. The Ontario Power Company were allotted 11,180 cu. ft. per second, so that, within the control of the Commission, there is now available 17,799 cu. ft. per second.

FIG. 12—ARMORED CABLE INSTALLATION

FIG. 14—QUEENSTOWN POWER HOUSE SITE [HULL]

FIG. 15—LARGE ELECTRIC SHOVEL

FIG. 16—ELECTRIC LOCOMOTIVE AND DUMP CARS [HULL]

To obtain the greatest amount of power from the water available, after careful surveys and studies, it was decided to locate the generating station just above Queenston, and to take water from the Niagara River, through the Welland River (reversing the flow in same) and canals encircling the City of Niagara Falls to Queenston. The first canal is now under construction and the accompanying map of the Niagara District, shows its location. Fig. 13 shows a general plan of this scheme.

The total length of waterway from the Niagara River to the generating station site is about 12½ miles, 4¼ miles of this

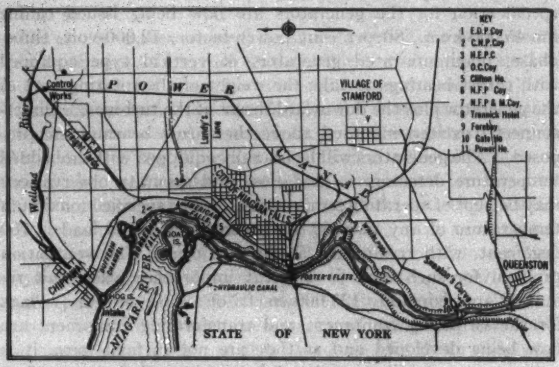

FIG. 13—GENERAL PLAN OF NIAGARA POWER DEVELOPMENT—HYDRO-ELECTRIC POWER COMMISSION

distance being in the Welland River, and the balance in an excavated canal. The net effective head will be about 305 ft. and the first canal is normally designed for 10,000 cu. ft. per second at minimum low water.

The generating station will be located about one mile up stream from Queenston, in the Gorge just at the end of the last rapids in the river. At this point the banks are steep and short penstocks only will be required. Provision is being made for extensions, and right-of-way for two additional canals has been purchased. Fig. 14 shows a view of power house site.

To show the effect of utilizing the greatest possible head of water, it may be pointed out that about 30 h. p. will be

developed for each cubic foot per second in this development, whereas about 14 h. p. is all that is obtained in existing plants at Niagara Falls.

The development now under construction is designed as regards canal, forebay, gate house substructure and power house substructure, for a capacity of 300,000 h. p. It is proposed to construct the gatehouse and power house superstructure for an initial installation of 200,000 h. p. in four units. The designs are made so that extensions of power house and gate house can be made to almost any extent. Future plans contemplate the use of 100,000-h. p. units.

The turbine speed has been fixed at 187.5 rev. per min. and specification for the generators are now being issued calling for 45,000-kv-a., 80 per cent power factor, 12,000-volt, three-phase, maximum-rated generators of vertical type equipped with thrust bearings to take the weight of the rotating part of the generator plus the downward thrust of the turbines. Direct-connected exciters mounted above the thrust bearings are proposed. The generators will be liberally equipped with embedded temperature detectors so that close and accurate observations may be kept of operating temperatures. The specified maximum temperature of any part of the generator at rated load is 100 deg. cent. with cooling air at 40 deg. cent. The ventilation problem for such units is of great importance, each unit requiring approximately 125,000 cu. ft. of air per minute. Plans for the switching equipment and transforming equipment are now being developed and as they are not in final form, it is not opportune to give any detail description. It is proposed to install double 12,000-volt sectionalized buses, banks of transformers consisting of three 15,000-kv-a., single-phase units, and to lay out the switching equipment so that one generator, one bank of transformers, and one outgoing high-tension line will be a unit. Bus tie reactors will be provided, and the short-circuit current will be limited to a value that can safely be handled on the circuits and circuit breakers.

It is proposed to install two small service generators to supply power required for the station services such as pumping, cranes, lighting, machine shop, ventilating, etc.

To carry on the construction work on this development it was decided to make as extensive use of electric power as possible. Two large electrically operated revolving shovels each fitted with an eight-cu. yd. bucket for earth excavation and of capacity

to handle a five-ton bucket in rock were provided. See Fig. 15. The larger of these two shovels has a boom 90 ft. long and a dipper stick 80 ft. long, while the smaller shovel has a boom 80 ft. long and a dipper stick 58 ft. long. Each shovel has motors of a nominal total rating of 715 h. p. on a half-hour rating. Three shovels weigh over 300 tons each and have a capacity of 5000 cu. yd. of earth per day. Five other electrically operated shovels are also provided having bucket capacities ranging from 7/8 to 4½ cu. yd.

The railway equipment of the construction work includes 150 dump cars of 20 cu. yd. capacity each, six 40-ton steam locomotives and twelve 50-ton electric locomotives, as shown in Fig. 16.

The electric power required for shovels, trains, air compressors etc., is obtained from the Ontario Power Company's station and is transmitted over two overhead feeders to the Whirlpool Substation located near the Whirlpool. A similar substation will shortly be erected near Montrose on the southerly section of the canal. The Whirlpool substation is of semi-permanent construction and contains switching equipment for the two 12,000-volt incoming lines, and for three 1500-kv-a., 12,000/4000-volt transformers, four 500-kw. 600-volt, d-c. synchronous converters with their 12,000-volt transformers. An adjoining room contains eight air compressors of 1000 cu. ft. per minute capacity each against 125 lb. pressure, belt driven from 550-volt motors, supplied through two banks of 200-kv-a., 2300/550-volt transformers. Air is distributed up and down the canal for a distance of three miles from this substation to operate rock drills, channellers and forges, etc.

Power is distributed up and down the canal by 4000-volt, 25-cycle, three-phase, four-wire, grounded-neutral circuits to which the shovels are connected through flexible armored cable. A double track railway has been built for the full length of the canal with a branch to the main dumping point at St. Davids. These railway lines are electrified, the trolley wires being offset to one side of the track so as not to interfere with the shovels.

The substation at Montrose will be identical with that at Whirlpool and it will similarly serve to supply power for the work on the upper end of the canal.

The extensive use of electric power on the construction of works of the magnitude of this development is working out well and is resulting in a great saving of coal.

The enormous amount of power to be transmitted from this Queenston development involves much engineering study to work out a solution particularly when it is borne in mind that a large amount of capital is invested in existing stations and lines operating at 110,000 volts. These studies are now under way and it is to be hoped that at some later date, there will be placed before the Institute a more complete description of this Queenston Development and its transmission lines connecting in to the Commission's Niagara System.

For those who wish further information on the subject of electric power generation in Ontario, reference may be made to the following:

1. Annual Reports of Hydro Electric Power Commission of Ontario from 1908 to date.

2. The Power Commission Act with Amendments—Ontario Government.

3. Report of Commission of Conservation, Canada, on "Water Powers of Canada" 1911, also 8th Annual Report (1917) of this Commission.

4. Report of Dominion Water Power Branch (Canada) on "Water Powers of Canada."

5. Volumes of "Canadian Engineer" Toronto.

6. Volumes of "Electrical News" Toronto.

7. Transactions of Canadian Society of Civil Engineers, (now the Engineering Institute of Canada.

DISCUSSION ON "ELECTRIC POWER DEVELOPMENT IN ONTARIO" (HULL), TORONTO, CANADA, NOVEMBER 22, 1918.

H. U. Hart: The Hydroelectric Power Commission has built up a remarkable system, having in operation 760 (1,223 km.) miles of 110,000-volt transmission lines, and the largest transformer station in the world—174,000 kv-a. installed transformer capacity. The engineers of the Commission have built many switching and transformer stations of all sizes and voltages, and from their experience have made up what they term standard stations, the drawings for the complete building including the location and dimensions of transformers and switching apparatus for various sized stations for 110,000, 46,000, 22,000, and 2200-volt substations. This standardization saves a great amount of engineering work to the manufacturer and in the erection of the station.

It is a matter for congratulation to the engineers of the Commission and to the manufacturers associated in the work to see how successfully the 110,000-volt transformers and other apparatus have performed their work.

I am very sorry Mr. Hull did not have time to tell us of the early operation of these plants, and possibly some time later he may do so. Reactance coils at that date were not developed or used to such an extent as at the present time, and due to line trouble at the start of operation which occurred frequently, trouble developed in the circuit breakers due to the carbonization of the oil from frequent openings, and also to the great amount of power in the arc under short-circuit conditions. The circuit breaker tanks were not strong enough for this abnormal service.

A quick remedy for this trouble in 1910, was to connect two 110,000-volt circuit breakers in series, making one a resistance breaker which tripped first. The 110,000-volt resistance was made up of cast iron grids mounted on pillow-type porcelains. The use of this resistance breaker alone permitted operation at that time and I believe is still giving satisfactory service.

The Queenston development, when completed, will be the largest in size of any of the hydroelectric developments, both as regards the size of units, and the total capacity of station.

W. G. Hewson: I would like to ask Mr. Hull, or any other person who can advise us, regarding the relative efficiency of the various plants in the Niagara district, for instance the DeCew plant. These are days of efficiency and you must make full use of everything you have. I think there is an advertisement running through the papers which asks the question "How many hides has a cow?" which would seem to indicate that all the hides must be used. Those of us who attend our own furnaces know that the coal mine owner is selling us not only his coal, but the rock and earth found along with it.

I noticed an article the other day in a hand book published by the Institute in 1904 describing the developments found in

existence at Niagara Falls. Apparently the plants that are now supposed to be inefficient were then regarded as highly efficient, when compared with the three former schemes that had been proposed for utilizing the power at Niagara Falls. The first scheme was a pneumatic one in which air compressors in a central power house were to be driven by turbines and the air then piped to air motors in surrounding plants. Another scheme was to run horizontal radial shafts from the generating station and connect the various plants to these shafts by means of clutches.

Arthur H. Hull: Answering Mr. Hewson's question, I will say we have no efficiency figures available for the machines in the Queenston plant. I do not think the time is opportune to make any comparison. It is hard to say whether the generator efficiency will be 95 or 96, or whether the turbine efficiency will be 90 or 85. The designers calculate on certain figures, but they may or may not get them in these large units. The units are particularly large. I believe it is the largest hydroelectric turbine that has been built for this head and the generators are the largest of the type made for the speed.

Personally, I have no figures on the efficiency of the De Cew Falls plant. If Mr. Acres were here, he could probably give some idea of it.

Thomas A. Worcester: I was in touch with the earlier developments of the hydroelectric transmission schemes, and I marvel when I look at the diagrams which Mr. Hull has shown to see how the whole undertaking has grown. It is many times larger than was anticipated when the original layout was made.

There is one thing which I think is quite noticeable, the original 110,000-volt distribution system was developed for 25 cycles. On all of the smaller systems throughout the northern part of the territory, the development seems to tend toward 60 cycles. There must be a definite reason. We possibly all know the reasons for the early development of the 110,000 volts and 25 cycles, but we may not know why the other developments were made at 60 cycles. I think it would be interesting if Mr. Hull would explain this. It is also noticeable that the new Queenston is to be made at 25 cycles. Furthermore, it is proposed to make the high-tension voltage 110,000 volts. There is no doubt a reason for this. Possibly the load can be picked up within the range of 110,000-volt distribution, but when we notice the extremely large territory to be covered, and the fact that there is a large concentration of power at the Falls, and possibly not so much available in the northern part of the province, one wonders whether 110,000 volts will be sufficient for the ultimate development of the Queenston plant.

I believe Mr. Hull mentioned 300,000 kv-a. as the immediate prospective development. No doubt, this will not be the ultimate, but perhaps it will be a million. Will 110,000 volts be suitable for distribution for 1,000,000 kv-a? I think if Mr.

Hull could answer a few of these points in detail, it would be interesting to all of us.

Arthur H. Hull: In regard to the 60-cycle frequency for the smaller system, I will point out that these systems were acquired by the Commission through purchase, and they had themselves grown from small isolated steam plants, plants where possibly 133 cycles were used, and 60 cycles was the next step.

On the Central Ontario system, I believe, the decision to use 60 cycles was made on account of the feeling at that time that 25 cycles did not give the most beneficial results for incandescent lighting.

In addition, their transmission distances are comparatively short. The Central Ontario system has power houses scattered through its system. The Wasdell's, Muskoka and Eugenia systems are in a district whose adjacent sections were served by the Severn system which operated at 60 cycles and naturally followed the frequency of the Severn system.

As to the system near North Bay, that was served by a steam plant in North Bay, operating at 60 cycles, and the generating station was built for 60 cycles, in order to be able to parallel with the steam plant, and that steam plant is now being used at North Bay as an auxiliary serving as synchronous condensers for voltage regulation or for emergency supply as conditions demand.

As regards the use of 110,000-volts on the transmission lines from Queenston you have asked a big question. It is a problem we are working on at the present time. The possibilities are that the plant will be built for about 190,000 volts, which is the *Y* voltage of 110,000 volts. The transformers may be operated at 110,000 volts delta to start with.

There will be no necessity for paralleling the low-tension side of the Queenston plant with the other generating stations at Niagara Falls, so that they could be paralleled on the high tension with the Queenston low-tension bus out of phase. The present indications are that there will be 110,000 volts for Toronto for many years to come, and there may be 190,000 volts transmission in the western districts by new lines, that is, the district extending to Windsor.

But the whole problem has not been solved yet, and we are making a lot of calculations on voltage regulation of lines, synchronous condenser application, and things of that kind, and until these are actually completed, I would not like to say it will be this or that. I do not see any great difficulty in using 190,000 volts, and the chief trouble at present is possibly in the circuit breakers. I do not think the transformer manufacturers are afraid of the proposition in any way, and I believe that the circuit breaker manufacturers are ready to take it on. The whole question works out on the basis of voltage regulation and line loss.

E. M. Ashworth: There is just one feature which I note has not been emphasized, and which is a feature of interest, par-

ticularly no doubt to our visitors from the other side, and that is, that this large, and I think successful, system has been initiated and carried out under public ownership.

H. R. Summerhayes: I would like to ask Mr. Hull what difficulties they have in the matter of regulation of voltage. We usually think of 110,000 volts as a trunk line transmission, and this seems to be a system where it is 110,000-volt distribution with power taken off at various points. Is it necessary to use synchronous condensers to get good voltage regulation? Also, is it contemplated to connect the Niagara Falls and the Toronto district with other parts of the Dominion, by trunk lines of high voltage?

Arthur H. Hull: In answer to Mr. Summerhayes, I might say that we have at the present time 8000 kv-a. in synchronous condensers operating in Toronto. There are some synchronous motors operating at Hamilton, some at London, but there are not very many in this south western section. We have a double circuit line to Windsor, and four circuits from Niagara to Dundas, and we find sometimes during periods of light load, we can cut out a section of line and get some regulation in that way. It is not possible to vary the bus voltage very much at Niagara Falls at the present time, that is, the generating bus voltage, owing to the fact that the bus sections are tied together through reactors and there are other customers of the Ontario Power Company, that have to be looked after. Some of the smaller stations have potential regulators, working on voltages of 4000 or 2300 volts, of capacity sufficient to handle the lighting load of the towns, and we have not had very much trouble with line voltage regulation. It would be better if we could vary the voltage at the generating station to take care of it.

The next step would be to put synchronous condensers at various points in the system. As a start on that, we recently installed 8000 kv-a. in Toronto, which is now being used. I do not know that I understood the second part of the question.

H. R. Summerhayes: It was whether you contemplated to connect the Niagara and Toronto districts, with 110,000-volt distribution, with the other systems in the Province.

Arthur H. Hull: No, not at the present time. These systems (*i.e.* other than the Niagara System) are now generating enough power for the present demands, and by further development in the available power sites, there will be sufficient power to take care of the demand for some years to come. Of course, it is pretty hard to say what is going to happen ten years from now. All this development which we have had here (*i.e.* in the Niagara System), has been made inside of ten years, and I would not undertake to predict so far ahead, as to what will occur within the next ten years.

F. A. Gaby: I will go further and say that the Commission already have prepared estimates on connecting the Niagara System with the St. Lawrence system, by a 220,000-volt line,

with frequency changer sets at Toronto to tie the 25-cycle Niagara system with the 60-cycle St. Lawrence system.

Arthur H. Hull: No doubt the Long Sault Rapids on the St. Lawrence will eventually be developed by somebody, and, of course, Ontario will get a lot of that power, which ought to bring about in this district, (eastern Ontario) an enormous development. There is a new system now being developed, called the Rideau System which when completed will pretty well take care of the present needs of the district near the Rideau River. I may point out, however, that there are no water powers of any magnitude in the Niagara System district outside of those at De Cew Falls and Niagara Falls. This part of the country is pretty well cleared up, and there is very little chance of any hydraulic developments other than near Niagara so that there are good prospects of a great amount of additional load in this Niagara section.

Wills MacLachlan: It might be pointed out, for the benefit of the visitors, that in the north country there are large powers at French River, and also in the mining districts of the north there are other power developments. For those that do not know the conditions, they may think that almost all electrical Ontario has been shown on the map put on the screen by Mr. Hull, and these developments all apply to the south end of the Province. There are future developments that will positively obviate the necessity of running a high-tension transmission line to carry the power in the northern part of the province.

Arthur H. Hull: The paper only covers the systems of the Hydroelectric Power Commission. At Fort Francis there is a generating station, there is a development at Sault Saint Marie, and in the Sudbury district, also in the Cobalt district, as well as in the Porcupine districts, and there are some generating stations for pulp and paper manufacture up in this far north country (District of Patricia). There are power sites scattered all through here (north of Lake Superior) and the trans-continental railways run through this district, so if plans of electrification of these railways are carried out there will be an abundance of water power available in those sections to supply them and develop the districts.

Perry A. Borden: There were some articles published some months ago in the local papers dealing with the proposed scheme to divert the Grand River from its natural course, into the Western end of Lake Ontario by way of the Dundas Valley. May I inquire if anyone can give me information as to the authenticity or practicability of this scheme?

Arthur H. Hull: The Grand River starts in Grey County and empties into Lake Erie. There is a valley near the river that runs down to the west end of Lake Ontario. The proposition would be to divert the Grand River into Lake Ontario at this point (Dundas). I cannot say personally, Mr. Borden, what there really is in that scheme. Possibly Mr. Gaby might say something about that.

F. A. Gaby: The only suggestion that has been made is the general suggestion. I do not believe any studies have ever been made on the diversion of the Grand River into the escarpment at Dundas.

The Commission in 1910 and 1911 did make numerous studies on the possibility of using the plateau above the Dundas Valley as a storage basin for an auxiliary plant to be operated in conjunction with the Hydroelectric Power Commission system, as a hydroelectric system, that is, to pump water from Lake Ontario, 300 or 400 feet up to the plateau or storage basin, and then use this during peak load periods to augment the supply of power from the Ontario Power Company, rather than buy its power on the peak load, possibly, and in this way it might be possible to finance the expenditure necessary on such an equipment. This is the method used in the case of large sources of power on low power factor loads in Switzerland and other countries, but I do not know of any studies being made for the diversion of the Grand River, and further, I think it would be very difficult, in view of the uses the Grand River is being put to at the present time.

The Commission had been making a study before the war of possibility of starting to regulate and obtain power on the Grand River. These hydrographic surveys are proceeding at present, and it is hoped several years from now, when we get sufficient information on the characteristics and features of the streams, we will be able to make a comprehensive report of the possibility of regulating the flow and generating the power on this small flow. It is a river with variable flow, very high flood periods, and the minimum flow is very small, and its power possibilities are very small.

Presented at the 344th Meeting of the American Institute of Electrical Engineers, Toronto, Can., November 22, 1918.

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ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE

BY WILLIAM G. GORDON

ABSTRACT OF PAPER

The author describes the electrification of the tunnel through Mt. Royal at Montreal which was built to give the Canadian Northern Railway entrance into the heart of the city. The tunnel emerges from the mountain several feet above the level of the city and it is proposed to extend an elevated line at the same uniform grade to connect with the proposed viaduct on the lines of the Harbor Commission. The tunnel is 3.1 miles long and the method of construction is described in detail.

The power is purchased and delivered to a substation near the west portal of the tunnel. The equipment of the substation is described. At present there are six electric locomotives in operation having a one-hour rating of 1280 h.p., and it is proposed to add multiple unit motor cars for handling the local traffic. Details of the equipment and dimensions of both the locomotives and motor cars are given.

The catenary system, which is described in detail, has a number of unusual features due to special local conditions and the extremely low temperatures which sometimes prevail in Montreal.

THE City of Montreal is divided into two principal levels; the commercial and financial quarter being on a plane only a few feet above high water, and the residential and shopping districts being at a height of about 75 ft. above the river. As the space between Mount Royal and the St. Lawrence River is limited, this district has become very congested. Business has largely forced the residence section up and down the River and around the mountain. The tunnel under Mount Royal was built with the idea of giving the Canadian Northern Railway, which property now belongs to the Dominion Government, an entrance into the heart of the City, and to render available a large area for residential purposes, only a few minutes by train from the main terminal.

The location of the present terminal is about midway between the two levels and it is proposed to extend an elevated line, at the same uniform grade, which will connect up with the proposed viaduct on the lines of the Harbor Commission, thus giving

direct access to Trans-Atlantic steamers, and all the harbor facilities.

The tunnel as built is 3.1 miles long and is the shortest line that can be devised to take advantage of the geological formation. The tunnel has a uniform grade of 0.6 per cent toward the city to insure proper drainage.

In order to meet the various physical conditions, different cross sections were used; where hard sound rock, unsound rock and soft ground were encountered respectively.

The twin section type of tunnel was adopted for

1. Economy in construction.
2. Ease and economy in ventilation.
3. Protection and safety in case of derailment or accident.

In addition to working from both ends of the tunnel a shaft was sunk one mile from the West Portal at Maplewood Avenue.

It is of interest to note that when the heading from the West Portal met that being driven from the Maplewood Avenue shaft, the lines checked within 1/16 in. on the alignment, and ¼ in. in grade, and that where the headings from the Maplewood Avenue shaft and that from Dorchester Street met under the highest point of Mount Royal, the error was ¾ in. in alignment, and ¼ in. in grade.

The method employed in driving the tunnel was to drive a bottom center heading about 8 ft. high by 12 ft. wide, as this heading could be driven ahead rapidly without much regard to the character of the ground, and from which the full size excavation could be developed at as many places, simultaneously, as desired.

Four drills were used in each heading, supported on a horizontal bar; the drills being operated by compressed air at a pressure of about 100 lb. per sq. in. The break-ups, where the upper part of the tunnel section was excavated to its full width and height, were opened at intervals of from 500 ft. to 800 ft. along the center bottom heading, the practise being to open up as many of these as necessary to keep up with the heading progress.

The compressed air used for operating the drills and other pneumatic machinery was obtained from two plants, one at each end of the tunnel, with an aggregate capacity of 11,000 cu.ft. of free air per minute, compressed to 110 lb. per sq. in.

The muck from the tunnel was handled by two 10-ton and one 8-ton trolley locomotives, and six 5-ton storage battery locomotives.

The load curve was worked up from the following data:

TRAIN WEIGHTS AND SPEEDS ASSUMED FOR MONTREAL TERMINAL ELECTRIFICATION

Class.	Trailing tons.	Speed level.	0.6 per cent up-grade.	Schedule.
Transcontinental	1130	37.0	26.5	21.2
Express and local	550	37.5	27.1	21.6
One motor coach	60	50.0	41.5	22.2
Three motor coaches	180	50.0	41.5	22.2
Three motor coaches and two trail coaches	260	47.8	34.8	21.8
Freight	1000	32.5	23.5	....

The substation is a handsome building, and will harmonize with the buildings which will be erected in the neighborhood.

Power is purchased from the Montreal, Light, Heat & Power Company at 63 cycles, 11,000 volts, three-phase. It is delivered to the substation by a lead-covered, three-conductor cable carried in a duct through the tunnel and also by an overhead line to insure continuity of service. The general arrangement and capacity of the switching equipment provides for the later addition of a steam auxiliary plant at the Back River near the Cartierville Yards for extension of the electrification of the main line to Ottawa.

There are two motor-generator sets with provision for a third, later. Each of these sets consists of a synchronous motor direct coupled to and on a common bedplate with two 750-kw., 1200-volt, d-c. generators, the set running at 600 rev. per min. The generators are connected in series giving 1500 kw. at 2400 volts per unit.

The sets have an overload capacity of 200 per cent for five minutes. The heavy overload capacity of these machines is obtained by the use of a pole-face winding. This winding of tubes and rods through holes near the pole faces is so connected as to directly oppose the armature reaction, thus insuring satisfactory operation up to the heavy overload mentioned. The pole-face windings and the series and commutating field windings are all connected on the ground side of these machines.

The shunt fields of the d-c. generators and the synchronous motor fields are arranged for 125-volt excitation. Each of the synchronous motors is started by a three-phase, 11,000-volt

compensator. This auto-transformer has one coil per phase with suitable starting taps brought out.

The three exciter sets each consist of a 50-kw., 125-volt d-c. generator driven by an induction motor. The generators are commutating pole type, flat compounded for the specified voltage and are especially adapted for exciter work and voltage regulator control. A bank of six 100-kw. single-phase transformers supply the induction motors of the exciter sets and miscellaneous station requirements.

All oil switches on the 11,000-volt circuits, except the synchronous motor magnetizing and starting switches, are enclosed in masonry cells and have two breaks per pole, each break in a separate tank. These switches have a rupturing capacity of 16,000 arc amperes at 11,000 volts. They are motor operated and will open automatically on overload, the incoming line switches excepted, either instantaneously or with a time-limit action, as desired. The incoming line switches operate automatically on the reversal of power only.

The synchronous motor starting switches are remote-control, solenoid-operated, mounted in cells, and have a rupturing capacity of 2000 arc amperes at 11,000 volts.

The main switchboard is of three-section panels of natural black slate, 90 in. high. The 2400-volt direct-current circuit breakers and lever switches are mounted on a panel back of and above the main switchboard. They are operated by insulated handles on the front of the main board so as to eliminate any possibility of the operator coming in contact with the 2400-volt circuit.

The circuit breakers are mounted between fireproof barriers and are equipped with powerful magnetic blowouts. The field switches are mounted on a base back of the panels with the operating handles on the front of the main board.

There are six locomotives in operation. Each locomotive has four axels with all the weight of the locomotive upon the eight driving wheels. The running gear consists of two four-wheel trucks, articulated with a heavy hinge. The equalization of the trucks is accomplished by a semi-elliptic leaf spring over each journal box, connected through spring hangers to the frame and to the equalizer bars. The equivalent of a three-point suspension is thus obtained through the side equalization of one of the trucks and both side and cross equalization of the other truck.

The friction draft gear is mounted in the end frame casting of

FIG. 1—APPROACH TO WEST PORTAL OF MOUNT ROYAL TUNNEL

FIG. 2—TWO-DEGREE CURVE IN THE TUNNEL [GORDON]

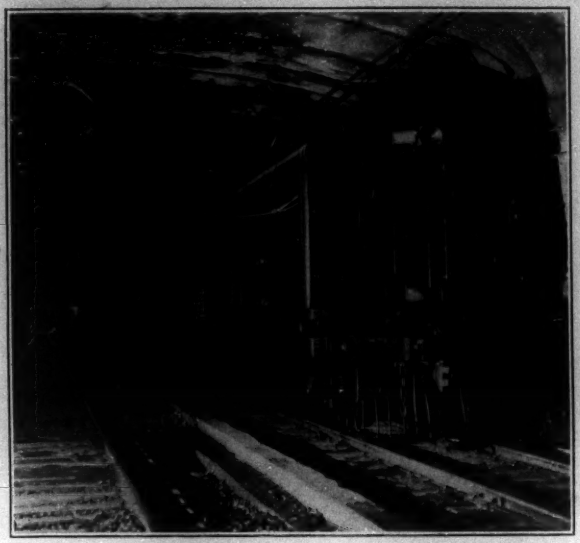

FIG. 3—LOCOMOTIVE PULLING IN MESSENGER AND TAKING CURRENT FROM OPPOSITE TRACK

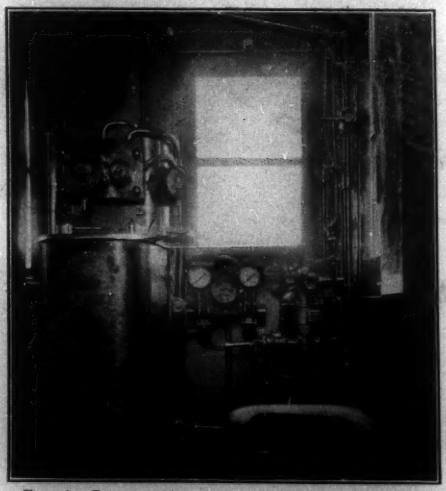

FIG. 4—CONTROL APPRARATUS IN OPERATOR'S CAB [GORDON]

FIG. 5—VIEW OF SWITCHBOARD IN SUBSTATION

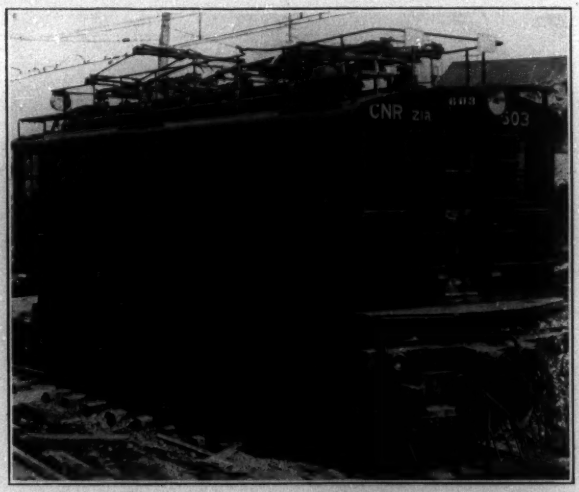

FIG. 6—LOCOMOTIVE WITH LOW CATENARY CONSTRUCTION [GORDON]

FIG. 7—CATENARY CONSTRUCTION ON TANGENTS [GORDON]

the truck. This type of construction restricts the hauling and buffing stresses to the truck side frames and articulated joint, thus relieving the cab and apparatus from the effects of severe shocks. The cab, which is of the box type, is divided into three compartments, the center compartment for the apparatus, and the two end compartments for the operator. Each operator's compartment is supplied with controller, control switches, ammeter, air brake and pantograph control, air gages, 2400-volt cab heater, bell rope, and control for the whistle and sanders, thus providing the locomotive with complete double end control. The motors are nose-supported in the usual way and geared to the axle by means of twin gears, each of four-inch face.

The motor equipment consists of four commutating pole motors wound for 1200 volts and insulated for 2400 volts, two of these motors being permanently connected in series for operating on the 2400-volt trolley circuit. The one-hour rating of each motor is 320 h.p. at 1200 volts. The motors are designed for forced ventilation which is obtained by means of a blower in the locomotive cab. Either pair of motors may be cut out by a special handle on the change-over switch. The locomotives are geared for a free running speed on tangent level track of approximately 45 mi. per hr. with ten points in series and nine points in series-parallel. The master controller used is of the non-automatic type and has two handles, one regulating the applied voltage at the motors and the other controlling the direction of rotation of the motors. The rheostats which form the external motor resistance are placed near the roof of the cab and provided with ample natural ventilation.

The master controller and contactor energizing circuits are designed for 125 volts. Each contactor is easily accessible without any disturbance to adjacent contactors. A special electropneumatic change-over switch is used for making the transition between series and series parallel connection of the pair of motors.

The 125-volt current for operating the contactors and for lighting the cab and headlights is obtained from a motor-generator set, the motor of which has two 1200-volt windings and two 1200-volt commutators in series for operation on 2400 volts. This set is mounted in the center cab and also drives the blower for providing forced ventilation to the main motors.

Fuses of the copper ribbon type placed in the fuse boxes provide protection for each individual circuit as well as the main

circuit from the trolley. These fuse boxes are all arranged to blow into a common chamber designed to take care of the arc. In addition to the fuse on the main circuit, a main switch is also provided. This is of the knife-blade type, being opened and closed by a handle in a position for easy operation in case of emergency, or when it might be necessary to open the circuit while carrying current. This main switch blows into the chamber provided for the fuses and has a powerful magnetic blowout.

The trolleys are of the slider pantograph type, pneumatically operated and mounted on insulated bases. Two pantographs are used per locomotive.

A speedometer, similar to the type largely used on automobiles, but especially designed for locomotives, is located in each operating cab. These are connected to the driving wheels of the locomotive by means of flexible shaft and gearing.

A combined straight and automatic airbrake equipment is provided on each locomotive. This equipment includes a 2400-volt, motor-driven air compressor, the set consisting of two 1200-volt motors operating in series on 2400 volts and direct-connected to an air compressor having a displacement of 100 cu. ft. of free air per minute. The approximate total weight of each locomotive is 83 tons. Some of its principal dimensions and characteristics are given in the following table:

Length inside knuckles	37 ft.	4 in.
Length over cab	31 "	0 "
Overall height, pantograph down	15 "	6 "
Height over cab	12 "	10 "
Overall width	10 "	0 "
Total wheelbase	26 "	0 "
Rigid wheelbase	8 "	8 "
Total weight, all on drivers	83 tons	
Wheel diameter	46 in.	
Tractive effort at 30 per cent tractive coefficient	49,800 lb.	
Tractive effort at one-hour rating	20,300 lb.	
Tractive effort at continuous rating	16,200 lb.	
Speed at rated amperes, one-hour rating	23.4 mi. per hr.	
Total horse power, one-hour rating	1280 h.p.	
Speed at rated amperes, continuous rating	24.6 mi. per hr.	
Total horse power, continuous rating	1090 h.p.	
Gearing 80–25	Reduction 3.2	

The multiple unit motor cars for handling local traffic are not yet in operation. The principal dimensions of these cars are given in the following table:

Length over buffers	67 ft.	5¾ in.
Length over body corner posts	57 "	6¼ "
Truck centers	42 "	9 "
Width over side sill angles	9 "	10½ "
Width over eaves	10 "	2¾ "
Height top of rail over roof	13 "	0 "
Height top of rail to underside of side sill	3 "	7½ "
Center to center of body side bearings	4 "	10 "
Center to center deck sills	5 "	6 "

The approximate weight of the car loaded and equipped is 160,000 lb. The electric hot air system of car heating is used. One complete heater is placed underneath each car and receives its energy direct from the 2400-volt supply. The heater has a capacity of approximately 25 kw. and is constructed for two heat combinations so as to provide for the changes in temperature conveniently and economically.

The complete heating equipment consists of the heating unit, blower and regulating mechanism, the controlling switch and thermostat of the regulating mechanism being arranged for operation from the 600-volt supply. Air is forced over the heating unit by means of the blower and distributed to the car through the air ducts along the sides of the car. The blower used for the circulation of the air is operated by a motor which is connected in series with the heating unit on the ground side. The capacity of the blower is approximately 1000 cu ft. of air per minute.

The motor equipment consists of four fully ventilated, 125 h.p. 1200-volt, commutating-pole motors insulated for 2400 volts. Two of these motors are permanently connected in series for 2400-volts operation. Ventilation of the motor is accomplished by drawing air into the armature at the pinion end by means of the fan on the armature shaft. The air passes longitudinally through the whole interior of the motor and is expelled through an opening in the frame at the commutator end, protected by wire mesh.

The control is of the non-automatic type for multiple-unit operation. The equipment includes a motor-generator set for furnishing 600-volt current for the control circuits, the air compressor and lighting circuits. This set consists of two 1200-volt motors, operating in series on 2400 volts, direct connected to a 600-volt generator. The master controller, contactors, switches, reversers and pantograph are essentially the same construction and appearance as those already described for

the locomotives. The controller has five steps in series and four steps in parallel. It differs from the locomotive controller in having the usual motorman's operating handle instead of a lever. This handle is provided with the so called "dead man's" feature for cutting off power and applying the air brakes in case the motorman removes his hand.

Copper ribbon fuses similar to those on the locomotive are used, and an aluminum-cell lightning arrester is installed on each car.

Special local conditions and extremely low temperatures introduced features making the design of the catenary system for this electrification somewhat out of the ordinary. The electrified track at present is about 10 miles long, and in this distance there is a passenger terminal station and coach yard in the city, a double track tunnel, double tracks in a cut with low clearances under highway bridges, a long stretch of single track, both tangent and curve, and a large freight yard with repair shops and storage tracks. The temperature in the coldest winter weather reaches 35 deg. below zero, while in the hottest summer weather it will go as high as 110 in the sun. In the early spring severe sleet storms sometimes occur.

The poles are of Eastern white cedar. The specifications for these poles and also for the creosote oil used as a preservative, were based upon those of the National Electric Light Association. Steel poles are used in the terminal yard in the city on account of their more sightly appearance. The wood poles are set seven feet in the ground and are all back-guyed. They are long enough to carry two cross-arms for feeders, signal circuit and a three-phase transmission line for supplying the shops in the Cartierville yard with electric power. On top of the poles there is a No. 000 copper ground wire which serves both as a protection against lightning for the circuits on the poles and also as a preventive against any trouble that might be caused by breakage of the rail bonds, which latter are of the welded U type.

The poles throughout the single track construction are spaced 150 ft. on tangents and 20 ft. on the two-deg. curve. On the double-track portion, where the overhead clearance is limited, the spacing is reduced to 105 ft. on tangents.

The messenger for the electrification outside the tunnel consists of a ½ in. seven-strand Siemens Martin steel cable with an ultimate strength of 11,000 lb. and an elastic limit of 6600 lb.

Two No. 0000 copper feeders are installed, on the full length of the electrification outside the tunnel and the other for about one mile west of the substation.

The messenger is anchored every half mile. This is accomplished by running the end of one half-mile length past the end of the next for a distance of one span. It is then made fast to an anchor eye on the bracket through an insulator and turnbuckle, and the same point of the bracket guyed back to the next pole, which in turn is guyed against this strain. The two messengers where they pass each other, are kept from eight to ten inches apart. By anchoring the trolley wire on the same bracket the anchorage becomes a section insulation, the air space between the messenger and trolley wire forming the insulation. Where a section insulator is not required a copper jumper is placed between the messenger and trolley wires.

For the double-track portion of the line cross-span construction, is used, the cross-span being a ⅜-in. seven-strand Siemens Martin steel cable. The messenger is fastened to this by means of a small malleable clamp. This cross-span is made up with a turnbuckle, strain insulator, and wedge grip in each end, and fastened to the poles by means of eyebolts.

In yard work spanning more than two tracks the construction is similar, but with the addition of a cross messenger of ½-in. cable above the ⅜-in. cable. This cross messenger is made fast to the poles directly, without insulators or turn-buckles, and carries the weight of the spans below through lengths of ¼-in. steel cable. These fasten to eyes in the tops of the messenger hangers and to the cross messenger by means of Crosby clips, There is a strain insulator in each of these lengths.

Pull-offs are used on curves for holding the contact wire and messenger in the correct position over the track and at intervals on long tangents for steadying the contact wire. The pull-offs are made of sherardized steel tubing bent to avoid fouling the pantograph. Each pull-off is fitted with a clamp ear at one end and an eye at the other. Adjustable links are sometimes required with the pull-offs to keep the trolley wires the right distance apart at certain points, such as where the trolley wire for a turnout approaches the main trolley wire at an angle. Each link is composed of two malleable iron brackets, with clamp ears, connected by means of a ½-in. pipe, the length of which is adjusted between the brackets and held by set screws.

Porcelain strain insulators are used in two sizes. The larger,

used with ½-in. and ⅜-inch. steel cable, withstands a wet flash-over test of 14,000 volts and has a breaking strength of 22,000 lb. The smaller, used with ⅜-in. and ¼-in. steel cable has a breaking stength of 12,000 lb.

The insulator used on the bracket construction is of the ordinary glazed porcelain, double petticoat, pin type, 4½-in. in diameter. It has a wet flash-over test of 20,000 volts. The messenger rests in the groove in the top of this insulator and is not tied except on curves.

The contact wire is of special bronze composition, size No. 0000, with a breaking stength of 65,000 lb. per sq. in. and an elastic limit of 39,000 lb. per square inch. Its section is the standard of the American Electric Railway Association for No. 0000 grooved trolley wire. The use of this wire instead of hard-drawn copper was thought advisable both because of its longer life when subjected to the wear caused by sliding pantographs, and also because it could be pulled up tighter than copper on account of its greater strength. This latter reason was considered of special importance because of the wide variation in temperature in Montreal, with the consequent great variation in the sag of ordinary copper trolley wire between winter and summer.

The trolley wire is hung straight over the center of the track as the natural side sway of the pantograph is sufficient to prevent wearing grooves in the contact strips.

The height of the trolley wire above the top of rail is ordinarily 23 ft. except along the double track construction and in the tunnel, where it is 16 ft. In this section two wires are used over each track. They hang side by side, supported from the same messenger, the hangers of one wire being staggered with those of the other. These double wires do not raise the hanger loops as high as would a single wire when a pantograph passes along, which is an obvious advantage where the head room is limited. Sparking and consequent wear both of the contact shoes and contact wires is reduced to a minimum, as there is always good contact between the slider strips and one of the contact wires.

The hangers are all of the long-loop type, having a malleable iron, single-bolt clamp ear, and a strap varying in length to suit its position in the span. All parts are sherardized. In spans of all lengths from 150 ft. to 90 ft. the hangers are spaced 15 ft. apart.

Lightning arresters of the magnetic blowout type are installed

at half-mile intervals. The arrester is placed near the top of the pole and the ground wire runs down the pole to a ¾-in. iron pipe driven about ten feet into the ground. Before driving this pipe, a 2-in. pipe was driven down about 5 ft. then withdrawn and the hole filled with rock salt. The ¾-in. pipe was driven down through the salt.

In addition to these arresters on the poles, aluminum cell arresters are installed in the substation on the positive busbars and on each feeder.

In order to string the messenger cable with the proper tension a dynamometer was used. It was therefore necessary for the foreman of the line gang to know what the tension should be at different atmospheric temperatures. The right sag at any given temperature was also of importance as a check on the tension. This information was supplied in the form of tables to which the line gang worked, the sags and tensions being given at 5 deg. intervals.

In the tunnel the overhead clearance was so limited that the catenary had to be very flat. This meant pulling the messenger up very tight for spans of reasonable length. A cable of phosphor bronze was decided upon, composed of nineteen wires, and having an overall diameter of 0.888 in. This cable has an ultimate breaking strength of 22,000 lb. and an elastic limit of 18,600 lb.

This messenger is supported every 90 ft. from the roof of the tunnel by a combination of iron yokes held in the concrete by four one-in. bolts. The cross yoke carries the messenger insulator and is supported on two insulators carried on the two end yokes, so that there are two insulators between the messenger and the ground. The insulators are of glazed porcelain and have a wet flashover test of 20,000 volts. All clamps and small parts of the messenger supports are of malleable iron sherardized. The yokes are of 2-in. by ⅝-in. and 1½-in. by ⅞-in. mild steel, painted with an asphaltum compound as a protection against rust. Two No. 0000 phosphor-bronze contact wires hang side by side from the messenger. The hangers for each contact wire are spaced 15 ft., or 7 ft. 6 in. between adjacent hangers. The hanger lengths vary from 6 in. to 13¾ in. with a 90-ft. span. The two hangers nearest the messenger support, namely those 11¼ and 13¾ in. in length, are made with two loops one sliding inside the other, where the clearance to the roof is small. The remaining hangers are similar to those used outside the tunnel, except that the loop is wider in order to take the

larger messenger. It was found that the two messenger cables and the four contact wires over the two tracks in the tunnel would give ample conductivity, so that no feeders through the tunnel were required. Both the messenger and contact wires are anchored every half mile. Two "bridles" of ½-in. steel cable are fastened to the messenger by means of six ⅞-in. Crosby clips, and the ends of these bridles fastened each way, through two cemented-type strain insulators in series, a turnbuckle and wedge grip to roof plates. The contact wire is anchored by lapping the ends for a distance of one span and then carrying each end up and slightly to one side of the center, making fast to a roof plate through two insulators, a turnbuckle and a wedge grip.

At the only curve in the tunnel, one of two deg. two pull-offs are placed in each span, over each track, one for each of the contact wires. The pull-offs are fastened to the tunnel arch through two strain insulators in series by means of an expansion bolt. The two pull-offs are placed 7 ft. 6 in. apart and this arrangement prevents hard spots and at the same time keeps the two contact wires close enough together for satisfactory operation.

DISCUSSION ON "ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE" (GORDON), TORONTO, CANADA, NOVEMBER 22, 1918.

W. G. Hewson: I would like to ask Mr. Gordon if he knows the reason for putting the substation so close to the end of the line. Undoubtedly the 2400-volt system was not adapted for pioneer work—it must have been with the idea of extending the system in the future, and it seems as if the substation is very close to the end of the line.

W. G. Gordon: I think I covered that, Mr Hewson, in my presentation of the paper. I mentioned that the general layout was with the idea of extending the electrification later on, as far as Ottawa. The substation was laid out to tie in with a turbine plant later on to be installed at Back River. The selection of the location was not just for the electrification of the present length of line.

W. G. Hewson: Is that installation going to be extended to Ottawa?

W. G. Gordon: That was the original intention.

W. G. Hewson: Why was not the substation put farther out on the line?

W. G. Gordon: It was more convenient in connection with the residential district scheme they had in mind, in opening up, to locate the substation where it is, and the few miles that would be gained in regard to the spacing to the next substation location was not worth considering at the time.

W. A. Bucke: There is one point that has nothing to do with the electrification of railways, which I would like to bring forward. I wondered how the matter was working out in actual practise. That is, the question of grounding the lightning arresters along the line, which is an important part of the system of protection. I understood Mr. Gordon to say they drove in a 2 in. pipe, filled the hole with salt, and then drove in a smaller pipe. I wondered how it had worked out in practise. It seemed to me it would not be many years before the pipe would be eaten away leaving a mass of iron oxide, etc. This would shake loose, and you would not have any ground. I would like to know how that arrangement will work out.

W. G. Gordon: The idea is to start the ground with a thoroughly saturated saline condition; but as far as it will actually work out in time, I have not seen it tried out exactly in this way. So far they have had remarkably good results. Last summer there were some very severe lightning storms, and they did not get by the arresters into the station.

W. A. Bucke: I should think it would be effective as long as it lasted.

Arthur H. Hull: That brings up a point to be emphasized—the importance of maintaining good grounds. The ordinary practise in the past seems to have been to drive in a pipe any old place, and whether they had a good ground or not seemed to be immaterial, as long as it looked like one. It seems to me that

it is very important to test the ground connections and see that the grounds are maintained.

W. G. Hewson: If it is not switching the discussion from that point, may I ask Mr. Gordon about the motor cars. I think he mentioned that they weighed 160,000 lb. or 80 tons. That is heavier than what is in ordinary use for suburban work, but I presume the operation in the tunnel has something to do with it.

Another point—he might tell us something about the air brake arrangements. I did not understand the exact location of the single compressor. I should think it would be very necessary with the locomotives on a 0.6 per cent grade, leading down into conditions like they have on that line, that the air brake should be very reliable, and I was wondering whether these compressors were put on the locomotives or on the cars, and whether they used one or two compressor units.

W. G. Gordon: They have a single air compressor on the locomotives, with a capacity of 100 cu. ft. per min., but there is nothing more to be done, as regards operating down that 0.6 per cent grade, than to see that the air is in proper operating condition. There is no question of the reliability of the operation of the air brake, if the pressure is up on the reservoirs. If a man is taking his locomotive through the tunnel by itself, he is working on straight air, and not taking any chances on the triple sticking. In taking a train through in the normal way, he operates on the triple, with the automatic braking feature. The men who operate the locomotives are all ex-steam men, familiar with air. They watch the gauges, to see that they have proper pressure, and naturally would not take any chances on starting off on the down grade without having tested the brakes. There is no chance of them getting into trouble, provided the proper supply of air is there. Regarding the weight of the motor cars, as you say, they are heavy. The standard steam coach design was taken and the equipment mounted on that, and that equipment runs into a good deal of weight, as you know. The figures I have given represent the weight loaded and completely equipped.

Harry U. Hart: I was wondering if the circuit breakers are the type that open in a few thousandths of a second. They must get some heavy short circuits, like all railways, and I would like to know if they are like any of those recently shown in the technical journals.

Another point in the paper which raised an inquiry in my mind was why a synchronous motor and two generators were run as a unit, instead of using synchronous converters.

W. G. Gordon: In regard to the breakers they have the standard type—the ones shown were the standard type of breaker—no special quick-acting feature. As a matter of fact, they have no quick-acting negative breakers installed, and up to date they have not felt the need of them. They have some pretty severe short circuits, and one or two have caused consider-

able damage to the brush holders, but not sufficient to put them out of commission, and it is exceptional to have a bad flash-over.

With 60 cycles however, the use of rotaries was out of the question as each rotary would only be good for from 600 to 700 volts.

With regard to the use of synchronous motor-generator sets as against rotaries, of course, that is a question on which where either is suitable you will find arguments on both sides. Had it been 25 cycles instead of 60 cycles by putting two in series we could have used 2400 or 3000 volts.

Arthur H. Hull: I ask Mr. Gordon if the feeders in the substation are tapped on to the trolley wires in the vicinity of the substation, or are they carried out along the lines some distance?

W. G. Gordon: They carried the feeder out a little over a quarter of a mile, but decided there was nothing gained over tapping directly in, as far as mitigating in any way the severity of flash-overs.

Arthur H. Hull: Do you think to carry the feeder out a considerable distance is impracticable?

W. G. Gordon: Against carrying out the feeder is the constant loss in the feeder with the questionable insurance you are getting, as to how far this will cut down the serious effects of the flash-overs. In other words, the saving in the seriousness of the flash-overs might not begin to meet the investment in the copper or the steady losses over that extra length of feeder.

W. H. Mulligan: I ask Mr. Gordon if the tunnel is lighted.

W. G. Gordon: They carry a lighting service all along one side of the tunnel. There are regular incandescent lights for the purpose of lighting the tunnel.

Wills MacLachlan: In connection with these high-tension generators, is there any provision for the protection of the operators in the case of a flash-over? There have been cases of men badly pitted with copper in going in front of one of the generators at the time of a flash-over.

Another point—in connection with the design of the switchboard, back and above the main switchboard, there is a platform, and running on the back of the switchboard there is a bus, and there is no protection to prevent forming a short circuit across the bus.

W. G. Gordon: There is no special provision taken for providing against the conditions you speak of, in case a man happens to be right over a commutator at the time of flash-over. It is a rare occurrence that you do actually find, in this class of work, that a man is burned by being right over the commutator, or passing the commutator, in pursuance of his work, at the time of flash-over.

On these generator sets, there is a considerable distance from the commutator to the outboard bearing, and the probabilities are that a man would not get anything beyond a scare even if there was quite a bad flash-over.

With regard to the small platform shown on the back and top of the tie rods of the main board, that is never used, except when examining these 2400-volt switches or breakers, with the current off, and when they are dead, the corresponding busses behind them are also dead.

W. M. Gifford: I would like to ask Mr. Gordon what the mileage obtained with the pantagraph is.

W. G. Gordon: I cannot speak definitely for these locomotives, but we are operating exactly the same type of pantagraph on the London & Port Stanley System. The life they obtain from the two copper wearing strips is from 10,000 to 12,000 miles. We find once wear is established, once a minute film of oil is established on the contact wire, and it does become so established, the wear on the contact wire is almost negligible. We have found that, after operating sliders on just the ordinary hard drawn copper trolley wire for a number of years.

R. R. Stevenson: In a case where a heavy electric trunk road is taking power from a central station system, I ask Mr. Gordon if there are any special features in the power supply contact which will affect the layout of the switching and transformer equipment. Were there any special provisions made in the power supply contact which would affect the overload protection which would have to be supplied?

W. G. Gordon: In practically every contract—while I am not at liberty to speak freely in regard to this one in question—they have certain provisions as regards the power factor below which they must not operate, and also the contract is made on a maximum demand basis on peaks of definite duration, but the switching in general in this case is laid out to take care of the maximum pull on the substation.

Presented at the 344th meeting of the American Institute of Electrical Engineers, Toronto, Canada, November 22, 1918.

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110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER

BY S. SVENNINGSON

ABSTRACT OF PAPER

The paper deals with some remarkable construction recently completed by the Shawinigan Water & Power Company near Three Rivers, Quebec. The St. Lawrence river is crossed by transmission line wires on a span of 4800 feet, being the longest span in the world. Due to necessities of navigation clearance, the towers are 350 feet high.

The preliminary investigation leading to the adoption of this construction is outlined, and a general description is given of the design and construction of the towers, insulators and cables. The provisions for protection from ice and the method of sag calculations are also given.

THE Shawinigan Water & Power Company's principal hydro-electric development is located at Shawinigan Falls on the St. Maurice River about 20 miles north of Three Rivers, where the St. Maurice empties into the St. Lawrence River, and about 85 miles north east of Montreal.

The Company has for a number of years been transmitting power to Towns south of the St. Lawrence River. Two lines carry the current to a switching station at Victoriaville, 35 miles south of the St. Lawrence River. At Victoriaville the lines branch, one branch 50 miles long running to Sherbrooke and supplying various towns and industries between, the other branch 40 miles in length feeding the asbestos mines and other industries in the Thetford district. The current has been transmitted at 50,000 volts from Shawinigan Falls to the St. Lawrence where the voltage was stepped down to 25,000 for transmission across the river over Submarine Cables, then stepped up again to 50,000 and transmitted at this voltage to Thetford and Sherbrooke.

The first submarine cable was installed in 1906. At this time the alternative of putting in an overhead crossing was considered, but the amount of power to be transmitted was so small that it was decided that the expense of an overhead crossing was not warranted. However, the demand for power on

the south shore steadily increased until by the beginning of 1916 five submarine cables were in operation, two three-conductor cables and three single-conductor cables, and the capacity of the transformer house, 10,000 kw. had been reached. The company was then faced with the necessity of increasing the capacity of the crossing and the question of an overhead crossing to replace the submarine cables again came up for consideration.

Submarine cables have always been a weak point in this part of the system and a source of more or less trouble and expense. The river bed is hardly suitable for a submarine crossing. It is sandy but with a generous sprinkling of rocks and boulders. The bottom slopes out very gradually from both shores to the steamboat channel, the sides of which are rather steep. Owing to this abrupt change in slope the cable in the channel is partly suspended and kept constantly swinging by the current in the river so that the armouring quickly wears through and the cable burns out at the points of suspension. The current in the channel carries the cable down stream and has even been strong enough to pull them loose from their moorings and break their connections in the cable house. In the winter the ice usually puts at least one of the cables out of commission, and it has been found necessary at times to erect temporary pole lines across the ice to maintain the service to the south shore. When, therefore, in the Fall of 1916 the demand came for more power for the south shore partly for war work and it became a question of putting in an additional submarine or an overhead crossing the Company decided in favor of the latter.

In order to increase the capacity of the submarine crossing it would have been necessary, in addition to purchasing and installing new cables, to build extensions to our cable houses and install new transformers with their necessary switches lightning arresters and other equipment. This would have involved an expenditure of at least $150,000. and at the same time the weak point in the line would not have been improved.

The overhead crossing was estimated to cost $200,000. the difference between the two being offset, in the opinion of the company, by the elimination of the weak link, by obtaining greater security from interruptions to the service and by a gain of from 2 per cent to 3 per cent in regulation by cutting out the transformers. A considerable amount of operation and maintenance expense would also be eliminated and the transformers, cables and other equipment tied up in the submarine crossing

were needed and could be used to advantage in other parts of the system.

Preliminary Investigation

The two shores of the St. Lawrence River upstream, as well as downstream of the cable houses were carefully surveyed in order to find the most advantageous point of crossing. As a result of this preliminary survey it was finally decided to investigate in detail two alternatives:

a. A three span crossing at Point-du-Lac, each span approximately 2200 ft. long.

b. A single span crossing between the cable houses 4800 ft. long.

Point-du-Lac is situated at the eastern end of Lake St. Peter about six miles up the river from the cable crossing. From a construction point of view the site at this point appeared at first to be the most favorable for an overhead crossing. The St. Lawrence River at this point is about 7000 ft. wide, but as the water is very shallow except for a distance of 2000 ft. in the center a crossing could have been built using three spans of approximately 2200 ft. each. The towers on either side of the main channel would have been about 205 ft. high, while the other two towers would have been about 110 ft. high. The crossing at this point probably would have been somewhat cheaper than the other alternative but it would have necessitated the construction of about 15 miles of double circuit high-tension pole lines in order to connect up with the main transmission lines. The cost of these connecting lines would have brought the total cost up to that of the single span scheme. A fairly strong point against the three-span crossing was the inaccessibility of the towers during certain periods in the spring and fall when the river is full of floating ice. The single span scheme was finally decided on as being the most advantageous, although it was fully realized that there were many difficult problems to solve in connection with the design and construction.

General Description

The crossing as completed consists of a central span 4801 ft. long and two anchor spans, the north shore span being 571 ft. long and the south shore span 951 ft. long.

There are two towers 350 ft. high and 60 ft. square at the base, the upstream and downstream faces tapering to a width of 14 ft. at the top. A crossarm at the top, 14 ft. wide by 100 ft. long,

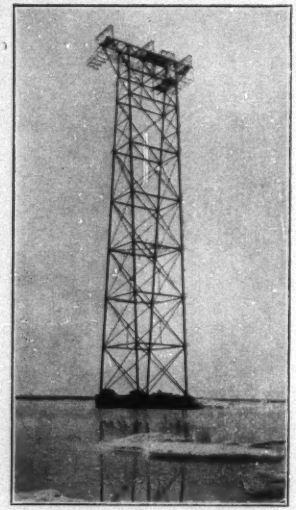
carries three double-groove sheaves 50 ft. apart over which the anchor cables pass. The tower foundation is made up of four circular reinforced concrete piers 11 ft. in diameter placed on the corners of a 60 ft. square. These piers are connected by heavily reinforced concrete beams 4 ft. wide by 8 ft. deep. The tip of the foundation is 14 ft. above the normal water level.

Three lines of 1 3/8 in. diameter steel cables 50 ft. apart span the river between the two towers. To each end of each center span cable is yoked two anchor span cables. These are carried over the tower on the main sheaves and then down to a point about 20 ft. from the anchors. At this point equalizing beams are cut into the lines and the load is transmitted from them to the anchor piers by means of short straps of 1¾ in. diameter cable. The cables are gripped at the ends by means of heavy steel bridge sockets in accordance with the usual practice for suspension bridge cables and other structures of this type.

It was originally intended to use the main cables as conductors and to insulate them from the towers by means of specially designed insulators. Unfortunately these insulators were not completed in time for erection and for the present the main cables are used as messengers from which No. 1/0 stranded copper conductors are suspended. These suspended lines are supported every 250 ft. by suspension insulators of eight units to a string.

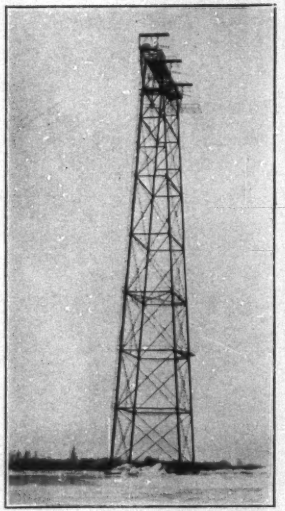
The original scheme contemplated the use of part of the submarine crossing as a standby until some future date when an additional overhead crossing identical with the present one would be installed. However, the success that has attended the operation of the steel cables as messengers has led us to the conclusion that it is possible to use the present crossing for two circuits and thus to eliminate most of the expense of an additional overhead crossing. We have at present under consideration two alternatives for the accomplishment of this object. One alternative provides for the use of the steel cables for one circuit and the suspended conductors for the other circuit. The other alternative contemplates the carrying of two conductors on each steel cable. This would be accomplished by suspending the two conductors from the ends of light triangular steel or aluminum frames attached to the steel cables at intervals of 200 to 500 ft.

The anchor piers are large mass-concrete "dead men" each anchor being designed to take the full overturning moment when submerged.



VIEW OF CROSSING LOOKING UPSTREAM

Towers are 350 ft. high and 4,801 ft. center to center.



VIEW OF NORTH SHORE TOWER

Showing ice piled up around the foundation after one of the ice shoves.

VIEW OF INSULATOR ERECTED

Parts of crates on insulators not removed—Insulator is 8 ft. in diameter, 13 ft. high, weight 6 tons—Anchor cables attached to the upper spider—Center span cable passes through center of the ring girder and is attached to middle spider.

[SVENNINGSON]

PROCEEDINGS
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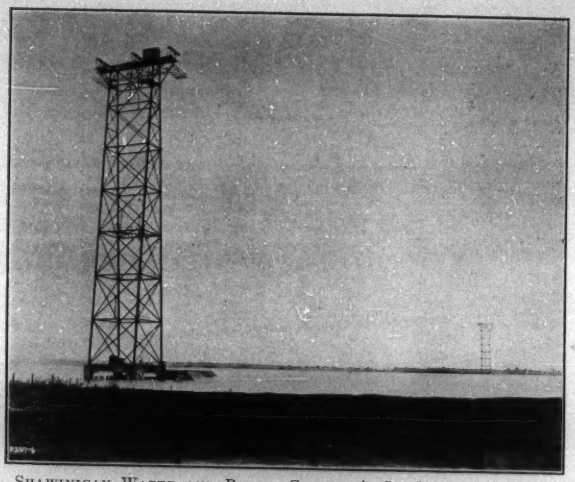
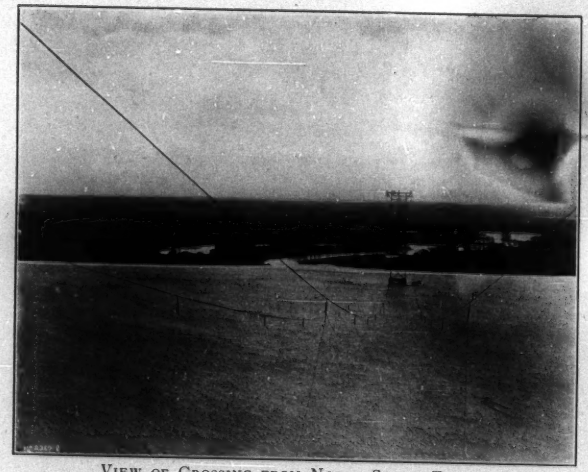
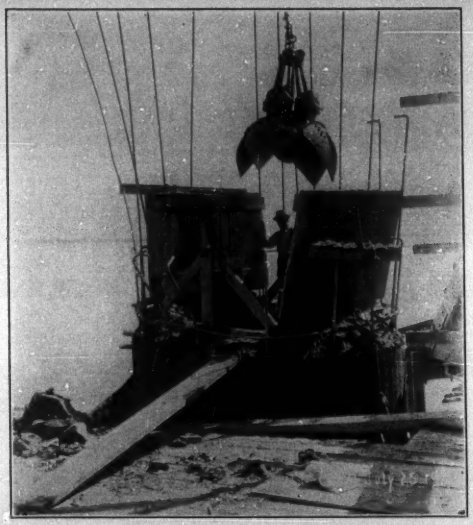


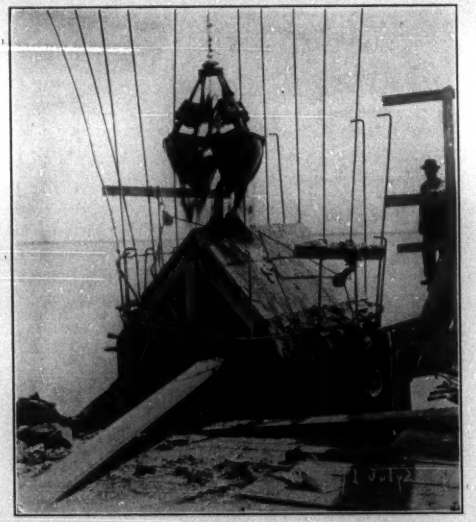
SHAWINIGAN WATER AND POWER COMPANY'S ST. LAWRENCE RIVER CROSSING. VIEW FROM NORTH SHORE.



VIEW OF CROSSING FROM NORTH SHORE TOWER



CASSION CONSTRUCTION—VIEW SHOWING METHOD OF EXCAVATION WITH ORANGE PEEL BUCKET—TRAP DOOR OPEN TO LOWER BUCKET



CAISSON CONSTRUCTION—VIEW SHOWING METHOD OF EXCAVATION WITH ORANGE PEEL BUCKET—TRAP DOOR CLOSED TO DUMP BUCKET

Foundations

During February 1917 a number of borings were taken about the site of the towers to determine the nature of the river bottom. These borings penetrated to a depth of 100 ft. and we found that the foundation on which we would have to build our towers consisted for the full depth of these borings of very fine white sand with occasional strata in which a little clay was mixed with the sand. The difficulty of obtaining a secure pile foundation in this kind of soil and the uncertainty as well as the cost of placing a mat foundation in the dry led us to adopt the form of pier foundation which we used.

The piers were constructed in the form of hollow cylinders of reinforced concrete with an outside diameter of 11 ft. and an inside diameter of 7 ft. These cylinders or caissions were poured in 6 ft. lifts, the first lift tapering on the inside towards the bottom to a diameter of 10 ft. and being shod with a 6 in. by 6 in. angle cutting edge. This lift was poured on a working platform and lowered into the water by means of four 2-in. screws. The second lift was then poured and after the concrete had set the bottom was excavated by means of an orange peel bucket rigged up on a derrick. As the caissons gradually settled successive lifts were poured until they had penetrated to a depth of about 40 ft.

Little trouble was experienced on the north side, but on the south side we encountered large numbers of boulders, some of which were so large that they could not be picked up by the bucket, so that we had to drill and shoot them. In order to do this the caissons had to be unwatered, a tedious process which delayed the work considerably. When a caisson had reached its penetration of 40 ft. a plug of rich concrete was poured in the conical section at the bottom and the inside of the caisson was then filled with mass-concrete. The four piers forming one foundation were finally connected by reinforced concrete beams.

This work was begun early in the year and we expected to have it finished by mid summer, but high water, high winds, rain and labor troubles delayed us so much that it was not completed until about the middle of September.

Tower Design and Erection

The towers were designed for the following loads:

A vertical load of 530,000 lb. made up of 350,000 lb. due to the weight of the tower itself and 180,000 lb. due to the vertical component of the tension in the cable.

A horizontal load of 42,000 lb. at the top of the tower parallel to the line due to the horizontal component of the tension in the cable.

A horizontal load of 26,000 lb. at the top of the tower normal to the line due to wind load on the line.

A wind load on the towers of 400 lb. per foot height or 140,000 lb. total.

The maximum compression on each of the front legs was estimated to be 575,000 lb. and the maximum uplift on the rear legs 233,000 lb. per leg.

The calculated deflection of the towers under maximum load is 4½ in. The wind and dead load stresses in the tower members do not exceed 20,000 lb. per square inch.

The bottom sections of the tower legs are composed of two 18-in. I beams weighing 70 lbs. per foot. These sections get lighter towards the top, the top sections being made up of two 12 in. channels weighing 20½ lbs. to the foot. The two members composing each leg are laced together with 2 in. by 2 in. angles.

Access to the top of the towers is obtained by means of Otis-Fensom Automatic Elevators, operated by electric hoists at the top and controlled by push buttons at top and bottom. The elevated baskets are composed of angle frames with plank floors 3 ft. wide by 4 ft. long. The elevator guides consist of four ⅝ in. diameter cables one at each corner of the basket, suspended from the top and weighted at the bottom to provide a constant tension of 1000 lbs. A steel ladder along one of the legs also provides access to the top and to the various levels at which insulators are strung. The ladder is enclosed from the 50 ft. level to the top with a wire cage and provided with trap doors and seats every 50 feet.

The towers were erected as follows:—

A timber erection tower 10 feet square was built up to a height of about 20 feet on piles which had been driven in the center of the working platform. A steel derrick with a 60-ft. boom was hoisted to the center of the erection tower by means of a jin-pole. The material for the towers was unloaded from the cars on the wharf at Three Rivers and transported to the site in barges. The bottom section, with its cross-bracing and girts was erected in place by means of the steel derrick. The erection tower was then raised another 50 feet, the derrick hoisted to the top as before and the next section of tower erected and so on to the top. The erection tower was guyed to the legs of the steel

tower as it went up. Rivetting gangs followed close on the heels of the erection gang. As soon as the north shore tower was completed work was begun on the south shore tower. The erection of the north shore tower was begun early in September. The winter set in rather early and working conditions were so adverse that, in spite of the fact that the Shawinigan Company paid the men a substantial bonus, the south shore tower was not fully completed until after the first cable was erected in March.

Each of the towers is grounded by means of a copper plate four feet square which is buried in the river bed and connected to one of the tower legs.

The three sheaves at the top of each tower are eight ft. in diameter and weigh about 2.5 tons each. The rims and hubs are of cast steel and were machined before the sheaves were assembled. The rims are in four sections bolted together. The spokes are built up of steel plates and angles rivetted together and bolted to the rims and hubs. The shafts are 5 in. in diameter of medium open-hearth steel. After the sheaves were assembled the hubs were shrunk on the shafts. The sheave bearings are of cast steel bushed with bronze. Lubrication is provided by means of wicks.

Insulators

The insulators which we proposed using in the steel lines were devised by our Engineering Department in conjunction with that of the Canadian Porcelain Company. They consist of a large ring-girder and two spiders.

The ring-girder is 8 ft. in diameter and made up of two 9-in. channels 12 in. apart with ⅜-in. cover plates. The spiders are built up of plates and angles with a heavy steel casting in the center. The upper spider is connected to the ring-girder by means of three 2½-in. bolts 10 ft. long, one at the end of each spider arm. The center spider is supported on the ring-girder by six porcelain insulators of eight skirts each, two insulators at the end of each spider arm. The clear distance between the spiders is about 36 in.

The porcelain insulators used are special compression insulators having a tested breaking strength of 60 tons each, this is about four times the estimated maximum load. Electrical tests showed a dry flash-over of 302,000 volts and a wet flash-over of 262,000 volts.

The completed insulator has a net weight of about six tons.

Cables

The cables are 1⅜ in. in diameter, of galvanized plough steel, made up of six strands of 19 wires each and a stranded steel core of 30 wires with a small hemp centre. Tests made at McGill University showed that the individual wires had an average yield point of 221,000 lb. per square inch, and an average breaking strength of 258,000 lb. per square inch.

The completed cable was tested, the yield point being found to be 158,500 lb. and the ultimate strength 186,400 lb. or 193,000 lb. per square inch and 227,000 lb. per square inch respectively.

The test of the completed cable indicated a modulus of elasticity of 7,250,000 lb. or 8,800,000 lb. per square inch. We are in doubt as to the correctness of our test in this regard on account of the fact that the usually accepted value for the modulus of elasticity for stranded steel cables is about 21,000,000 lb. per square inch. However, the behavior of the cable during erection bore out the results of the test.

No electrical tests were made on these cables but their sectional area would indicate a conductivity equivalent to that of a 100,000-cir. mil copper cable.

The bridge sockets used for connecting the cables were machined out of solid blocks of steel. Those attached to the center span cables are 15 in. long, taper in width from 5½ in. at the front to 8 in. at the back and are 9 in. thick, providing a grip of this length on the cable. The sockets attached to the anchor span cables are slightly smaller. A conical hole, tapering in diameter from 1 9/16 to 5 in. is bored through the center of the block from front to back and on each side of this hole a 2 5/8-in. diameter hole is provided for the connecting bolts.

The cable was passed through the tapered hole in the center of the bridge socket and broomed out on the end for a length of 15 to 18 inches. The wires were then cleaned with gasoline and held in place by means of a templet made of ⅛-in. steel plate which fitted over the back of the bridge socket. The bridge socket was then suspended with the back up and heated by gasoline torches for about half an hour when spelter was poured into the conical hole through a one-inch diameter hole in the center of the templet. After the bridge socket had cooled the ends of the wires projecting from the templet were cut and the templet was removed.

Before adopting this form of connection tests were run under our direction at McGill University to determine the depth of

socket required. We found that if the spelter was heated to just the right temperature *i. e.* just hot enough to ignite a sliver of wood thrust into it, that the full breaking strength of the wire was in the majority of cases developed in a length of six inches.

Shortly after the bridge sockets were poured we found it necessary to shorten two of the cables and the speltered end was cut off. We had one of these cones of spelter cut in the machine shop and found that the spelter adhered so firmly to the wires that the section could be machined without lifting the wires out.

Erecting Cables

Owing to a constant succession of delays that occurred in the construction of the foundations and in the erection of the towers we had to abandon our original plan of stringing the cables in the fall of 1917 before the ice formed in the river and so decided to do this part of the work after the ice had become thick enough to support the weight of the heavy reels of cable.

Throughout the heavy snows of January and February we managed by constant rolling and scraping to keep a road open between the two towers. Early in March the center span cables were laid out along this road. The anchor cables were then laid out, measured, and cut, and their bridge sockets attached.

The three lines were erected one at a time, the middle line first and then the down stream and the upstream lines in succession. The ends of the anchor cables were hoisted over the towers, the south shore cables made fast to the center span cable, drawn over the tower until the bridge sockets touched the main sheave, tied to the top of the tower and attached to the anchor pier. The north shore cables were next attached to the center cable, the suspension insulators and copper line fastened to this and the cable hoisted into place. The proper location of the suspension insulators was determined beforehand for both the steel and copper lines, due allowance being made for stretch in the cables and these distances were chained off on the cables as they lay on the ice. After the cables had been hoisted into place the end spans of the copper lines were attached to strain insulators on the towers and drawn up until the suspension insulators hung vertically.

The hoisting was done by a steam hoist braced against the center anchor pier on the north shore. Two 5/8-in. steel hoisting lines reeved through two pairs of three-sheave blocks were used to draw the end of the cable up to within 40 ft. of the anchor

pier, the final 40 feet being taken up by means of two ¾-in steel cables reeved through two pairs of six-sheave blocks.

This method of pulling up the cables appealed to us as being quite simple and easy of accomplishment. However, we found that it was a nerve racking process the first time we tried it. The cable seemed to come up easily to within a short distance of the anchor pier when the hoisting engine began to show signs of distress. When the end of the cable was within three ft. of the anchor pier the hoist suddenly coughed and quit. Those of us who had calculated the length of the cable and measured it as it lay stretched out on the ice wondered if we had slipped up on our figures or our measurements for at the calculated tension in the cable, allowing generously for friction in the blocks the hoist should have been capable of pulling the cable right home through twelve parts of line without any trouble. The riggers didn't believe that we were able to calculate the tension in the cable and were wondering if their pins and lashings would hold. A wait of ten minutes for the hoist to recuperate and for an inspection of the tackle and connections, a little sand in the clutch, a few more revolutions of the drum and the cable came in another foot. Another ten minutes for inspection and recuperation, another puff or two, a few more inches gained and finally, by dint of nursing the hoist and pulling for it on the part of all concerned the cable was hooked-up and made fast. A slight change in the tackle made it possible to pull up the other two lines without any mental effort whatever.

The copper conductor in each line is supported by 17 suspension insulators spaced about 250 ft. apart, the end insulators being about 400 ft. from the towers. The copper lines drop from the end insulators to strain insulators on the towers at the 150-ft. level, pass through the towers to the back where they are connected to another set of strain insulators. On the north shore the lines pass directly from the main tower to a transmission line tower on the shore a distance of about 600 feet. On the south shore a light structural steel truss, 50 ft. long, hung from two sets of the anchor cables, provides an intermediate point of suspension, forming two spans of 500 ft. each. Access to the insulators attached to the truss is provided by a foot bridge running up from the anchor pier and suspended from the anchor cables.

After the cables were erected we noticed an almost constant vibration in them, varying in intensity and somewhat similar to

that in a violin string, with definite nodes 12 to 15 ft. apart as nearly as could be judged. About a month after the line was put in service this vibration shook loose the bolts connecting two of the suspension insulators to the cable and they dropped and hung suspended on the copper line.

This condition brought us face to face with a problem which we had considered with misgiving and which had led us to discard the idea of suspended conductors when the crossing first came up for consideration. However, the difficulty was solved for us by two of our riggers who volunteered to go out on the steel cable, for a consideration, and to fish up and re-attach the insulators. A trolley was devised and constructed of two sheaves on which a small platform was hung and they had little difficulty in getting out to the point from which the insulators had fallen, about 1000 feet from the tower. By means of a small tackle line they hauled the insulators back into place and started back towards the tower. Unfortunately the sheaves on the trolley jammed and they could not pull themselves up. At this point one of the riggers lost his nerve and the episode might have had a tragic ending but for the fact that the other kept his head. He lowered his mate to a boat waiting in the river 250 ft. below, then lowered the trolley and finally passed the line over the steel cable, slid down to the boat and pulled the line down after him. A short time later an insulator on one of the other lines broke loose and it was similarly reconnected. This time, however, we profited by our former experience and provided a tail line by means of which the riggers were pulled back to the tower. Since then we have experienced no trouble from this source.

So far as we have been able to judge the suspension insulators had not been damaged in any way by the vibration in the line but there is a possibility that it may cause the cement in which the caps and bolts are set to disintegrate. We are at present carrying on investigations with the object of devising some means of absorbing this vibration between the steel cables and the insulators.

The cables as originally strung allowed the following clearances between the copper conductors and the average water level during the season of navigation: Downstream, 172.5 ft.; Center, 178.8 ft.; Upstream 180.6 ft.

The temperature at the time of erection was about 20 deg. fahr. As there is a change in sag of approximately one foot for

each 10 deg. change in temperature the above would correspond roughly to clearances at 110 deg. fahr. of 163.5, 169.8, and 171.6 feet respectively.

At the time these cables were erected we naturally expected the sag to increase as the cables stretched under the load until the strands were drawn tightly together. There was no data available with regard to the amount of stretch to be expected so that it was impossible to allow for this in sagging the cables. The hoist, therefore, was left in position so that we could pull up the cables when the sag became too great.

In May of this year we found that the sag in the cables had increased by from 24 to 27½ ft. and that in order to obtain the necessary clearance over the channel we would have to take up 24 ft. in the sag of the downstream cable and 13 ft. and 14 ft. in that of the center and upstream cables respectively. The amount by which a cable will stretch in taking up a given amount in the sag varies inversely as the modulus of elasticity of the cable. Owing to the low modulus which we worked out for the cable from the results of the tests made at McGill University we were in doubt as to the amount of take-up required. We found that in order to take up 24 ft. in the sag we would need to pull the cable in between 7.2 and 10.4 ft. depending on the value of this modulus. This cable was taken in about eight ft. with a consequent reduction in the sag of about 25 ft. This corresponds to the result that would be obtained if the modulus of elasticity of the cable were 17,000,000 lbs. In other words it would appear that from the time of the original sagging of the cable to the time the cable was re-sagged the modulus of elasticity had increased from 7,250,000 lb. to 17,000,000 lb. This change in modulus is no doubt due to the gradual stretching of the cable causing the wires and strands to draw more closely together under the constantly applied tension of the span.

Ice Protection

Ice conditions in the St. Lawrence River at this point are at times very troublesome and we considered it advisable to construct some kind of guard piers outside the towers to obviate the possibility of damage from this source. During the winter we deposited about 3000 tons of field stone on the river bed on each side about 75 ft. from the upstream and river faces of the towers, carrying the rock up to an elevation about three ft. above the level of the ice. The ice usually goes out about this level, but

last year conditions were exceptional, and before the ice moved it had risen above the tops of our ice-breakers and passed clear over them piling up around the tower foundations to a height of 25 or 30 feet. Fortunately no damage was done. We are at present completing the guard piers, by means of reinforced concrete cribs filled with rock and carried to about the level of the maximum recorded high water.

Sag Calculations

In our calculations for sag, tension, length of cable, etc. under various conditions, we used the parabolic formulas in preference to the hyperbolic formulas for the catenary, on account of the greater simplicity of the former. Comparison was made however between the two sets of formulas and we found, as we had expected, that at working tensions the difference was negligible. The formulas for the parabola gave us about six in. more sag and about one ft. less length of cable than the catenary formulas for the same conditions of tension and temperature.

We assumed the maximum load on the cable to be three-fourths of an inch of ice all around and 10 lb. of wind per square foot of projected area, for both the steel and copper lines, at a temperature of 0 deg. fahr. Under these conditions the calculated tension in the cable is about 106,000 lb. with a sag of 228 ft. The normal tension at summer temperatures is about 61,000 lb. with a sag of 185 ft.

Conclusion

There are still a few details to be worked out in connection with the crossing before we can consider the undertakings completed. The questions of providing two circuits on the crossing and of devising some means of absorbing the vibration are still under consideration. The problem of inspection and renewal of the suspension insulators has been only partially solved.

The crossing has been in uninterrupted service now for about nine months. It has not yet weathered a winter with its low temperatures, gales, and sleet storms, so that we still have something to learn about its action under these conditions. The allowable stresses, however, have been kept within reasonable limits and we hardly expect serious trouble from this source. The success that has attended the operation of the crossing since it was put into service gives us reason to expect a satisfactory solution to the problems still remaining.

DISCUSSION ON "110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER" (SVENNINGSON), TORONTO, CANADA, NOVEMBER 22, 1918.

C. W. Baker: There are two questions I would like to ask. First, in the crossing with the submarine cables, why, suspend them in the steamer channel, why not let them lie right on the bottom? Would that obviate the trouble you had with the swinging of the cable in the steamer channel, breaking it off at the points of suspension?

Second, in blasting out the boulders you encountered in sinking the caissons, at the south shore, how did you prevent destroying the caissons at the same time?

S. Svenningson: With regard to the submarine cables, the reason for the cables hanging loose was on account of the very steep slopes at certain points of the river and at the same time the very strong current. Of the five submarine cables we have in place, there has really been one cable which has given absolutely satisfactory service, and that happened to be a very heavy cable, three-core conductor with a very heavy armoring feature. The river bottom in the St. Lawrence changes from time to time on account of shifting of fine sand and light stones which form the river bed. We tried very hard to weight down the cables and then put weights on them, but did not succeed.

With regard to blasting of the stones in the caisson, we had to take particular care that the caissons were emptied before the shot was fired. For that reason, we had a steam pump that would pump the caissons dry, and then we went down and drilled the stone and placed the charge. In spite of that we did have an accident. One caisson cracked for a distance of five feet, but fortunately it held together and no damage was done.

Ernest V. Pannell: I do not think sitting in this room we can appreciate the stupendous character of the work described by Mr. Svenningson's paper. It would be necessary for us to take a trip to Three Rivers. If we imagine the Excelsior Life Building, 100 ft., placed on top of the Royal Bank Building, 250 ft. high, we shall have some conception of the height of the towers employed in this crossing.

I wish to ask Mr. Svenningson about a point which I do not think was covered in the paper. That is the formula he used for the compression legs of the towers. I examined a few of these formulas lately and find that the results vary as much as 100 per cent, according to the different expressions used, and it would make the paper more complete if, when he revises it, Mr. Svenningson would express the formula used for obtaining the compression in the legs, and also the value of L/r for these legs.

The point about the modulus of elasticity of the cables is very interesting. I have tested out wire myself and expected to get a modulus of nine million, and found that it came out three and one-half million; that is, the actual modulus is about one-third of the theoretical modulus, and I cannot account for it,

and I think that Mr. Svenningson may have some theory, as to whether the difference is brought about in the limited lengths of cable tested between grips, or whether there is something about a stranded cable which there is not about solid material. I do not know, but has Mr. Svenningson any theory why, when the theoretical elasticity limit of the cable is 21,000,000 it should test out at 8,000,000. That is not only true of steel cables, but it is also true of other forms of cables.

S. Svenningson: In regard to the formula for the compression of legs of the tower, it is a standard formula used by the Canadian Bridge Company. I do not remember exactly that formula. L/r was somewhere around 80.

In regard to the modulus of elasticity, I believe that our observation at Three Rivers has shown that the difference in the modulus of elasticity, the theoretical from the practical, is due to the effect of a stretching of the cable the first month, or six weeks and, as I said in the paper, that might introduce a great deal of vibration, very violent at times, which I believe was due to the center core taking relatively more load than the outside strands. In making up the cores, the center was made of a softer material than the outside. It was made of soft steel to allow for a good deal of stretching, and, as I remarked, the modulus of elasticity that we found is about 7,000,000 lb., while after the cable had been up for six weeks or two months, it had apparently increased to about 17,000,000 lb., which I believe is due to all the wires taking their proportion of the load. In other words, when you test a small piece of cable in a testing machine, possibly 25 per cent of the wires will take 75 per cent of the load.

H. B. Dwight: What precautions are taken to protect the steel cable against corrosion as the years go on? I presume the galvanizing is good for a few years, but probably after that time something will have to be done for a structure of such importance. Its life, to be worth while, must necessarily be a good many years.

S. Svenningson: When the time comes the galvanizing will flake off, and, of course, we have to paint the wire and use what we call "cable dope." We send a man out on the cables to paint them.

H. B. Dwight: How often would that have to be done?

S. Svenningson: I believe it would have to be done about once a year, and it may be, after the cable has been up ten or fifteen years, the cable will have to be replaced.

H. B. Dwight: While you are answering that question, may I ask if you have had any experience with the ground cables on your transmission lines; have you had any necessity for replacing the steel ground cables?

S. Svenningson: No, we have not.

H. C. Don Carlos: Mr. Svenningson mentions as one of the deciding features in determining the long span construction as against the short span construction, the inaccessibility of the towers in a short span construction. It seems to me that would

probably be more than offset by the inaccessibility of the insulators in the long span construction, and the suspended conductor from the messenger. I think, undoubtedly, the insulators will require more attention than the towers would, and will have to be gotten at more often. In the short span construction, it would have been a very simple matter to have provided a messenger cable for getting at the towers under all conditions.

In reading over the paper and noting the very wide difference of tension between the summer conditions and the winter conditions with the ice load and heavy wind pressure, it occurs that it might have cut down the cost of the construction considerably to have counter-weighted these cables and provided uniform tension rather than approximating a uniform sag. During the winter, when you have a heavy ice load, it would obviously have been unnecessary to provide navigation clearances. It might be, with the ice loading and the wind loading, they would not have sufficient clearance from ground for safe operation; but if it did work out that the sags were not too great, it seems to me that it would have been a much safer construction and probably much cheaper construction, to have counter-weighted the cables. I would ask if they considered that proposition at all.

Another question I would like to ask is this—I notice there is considerable difference between the clearances of the different cables, but I could not figure out just why that was, and I was wondering whether that would be accounted for by the grade in the river.

Another thing that struck me as being a rather dangerous proposition in this construction is the possibility of damage to the messenger cable through an insulator breakdown or flashover, and the arcing to the grounded messenger cable might pit the messenger cable, and if anything should happen to the protective relays to prevent the line dropping out quickly, it might even seriously damage the cable and weaken it by the arc hanging on. I believe the Shawinigan Company operated with a dead ground neutral, and it probably would not be possible to hang on to an arcing ground for any length of time, but it struck me there was that possibility of an arcing ground damaging the cable and burning it to the point of failure.

What is the horizontal displacement of the cable under the maximum wind conditions that they have experienced. Have any observations been made along that line?

Also, what is the length of the next longest span? I believe up to the time of this construction the longest span in existence was across San Francisco Bay, but I do not recall what the length of that span is, and probably the author of the paper will tell us.

S. Svenningson: As regards the inaccessibility of the towers at Point du Lac of course, it should be borne in mind to start with that we did not intend to have any suspension insulators on the main cable. That is one reason. To get out on the main cable, we use the present crossing cables, and in that case the insulator which we have to replace will would only be of light

weight, while if one of the heaviest pieces, in a series of insulators, on the Point-du-Lac scheme, should break down, it would be hard to replace them.

H. C. Don Carlos: I suggest that in a case of the 2000-ft. span which you had for the short span construction, you would have only used standard insulators?

S. Svenningson: We would have to use standard insulators in parallel.

H. C. Don Carlos: Yes.

S. Svenningson: Assuming you adopt a similar type, the same as you start with on the cable line, if a series of insulators, or one insulator breaks, the whole thing falls down practically, and that means that you must pull back the cable.

In regard to counter-weighting, we considered that, but, of course, the heavy windstorms and the sleet will occur at times of the year when the river is still open to navigation,—for instance, we had at the beginning of this week a very heavy sleet storm and a great rise in the tide, so that the water in Quebec rose about fifteen feet. Under such conditions, if we had employed counter-weighting, and a steamer should come up the channel, something might happen.

Another thing is that on account of that slight angle where the cable leaves the tower, the horizontal pull on the tower is slight, due to the difference in tension. The maximum tension on the tower from each cable is only 14,000 lb., and the weight of the tower would be practically the same, due to its height and the wind pressure which is exerted on the tower.

You asked a question about the displacement of the cable. Do you mean the horizontal deflection?

H. C. Don Carlos: Yes.

S. Svenningson: No, we have not seen much of that.

Another question was the difference in sag of different cables. There is a difference of sag of about 10 ft.—I think that was the maximum—and that was undoubtedly due to the difference in the stretching of the cable during erection. The cables were measured, and they were exactly of the same length, but during erection one cable was stretched, probably, a little more than the other.

With regard to the arcing over of the insulators, the line is now operating at 50,000 volts, and we have eight skirt insulators in use, so that the possibility of arcing over is rather remote.

In regard to the next longest span, as Mr. Don Carlos mentions, that is the span over San Francisco Bay, which is 4400 ft. in length. I believe the United States Aluminum Company is erecting a span in the South somewhere, which is over 5000 ft. in length. That is a span between two mountains and the cable only consists of a 7-in. cable.

H. C. Don Carlos: Steel core aluminum?

S. Svenningson: I do not know.

T. A. Worcester: May I ask Mr. Svenningson if it is contemplated to abandon the use of the special insulator and

continue to use the suspension insulator with copper conductors?

S. Svenningson: That has not finally been decided. We have the two schemes. One scheme is to put in the heavy insulator, which means we have to take down the lines, and that will have to be done in the winter time when we can build a pole line across the ice. With the other scheme we also find it necessary to have a pole line, but that can be coincident with taking down the wires. That has not been finally decided on.

F. J. Wyman: I think it occurs, perhaps quite frequently, where we have a combination of submarine or underground cable and overhead cable, and apparatus at each end, that the combination of inductance and capacity causes more or less disturbance in the lines in the nature of surges and static disturbances, consequently I would like to ask Mr. Svenningson if, in changing this line over, there has been any diminution of any kind in the disturbances, that is, if they were noticable before to any extent, and whether, since the change over, there has been any diminution. Were there any oscillograms taken before the change, and has there been any since?

S. Svenningson: We have two transmission lines leading down to the St. Lawrence River, and two transmission lines beginning from the South Shore down to the Sherbrooke district. As soon as the overhead crossing was built one line, the Thetford line, went over the overhead crossing, and the submarine crossing took care of the Sherbrooke line; but, so far as I know, they were not operated in parallel.

No tests have been made of the crossing, and so far as disturbances are concerned, I really do not know that I have noticed any difference.

T. A. Worcester: Another question—in the original plan, when it was contemplated to use the special strain insulator, was the problem of sleet considered, and were any arrangements made for melting the sleet from the conductors by heating them, either by the normal current, or by special short-circuit arrangements at times of light load? Would such an arrangement materially reduce the size of conductors which were required?

S. Svenningson: No, that is not considered, because very often we have disturbances at a time when we have no current to melt the sleet.

W. P. Dobson: I was interested in the two types of formulas for calculating spans, especially in connection with shorter spans, and I think this work Mr. Svenningson has done shows that the simpler formulas and simple methods of calculation are quite suitable for spans of practically any commercial limits.

I ask Mr. Svenningson what the stress in the porcelain insulators at the ends of the span was.

S. Svenningson: The stress per square inch?

W. P. Dobson: Yes.

S. Svenningson: The total area was 300 sq. in. and the maximum tension is 106,000 lb. That would be about 300 lb. per sq. in.

Presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., December 13, 1918.

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THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL

BY J. B. CRANE

ABSTRACT OF PAPER

This paper gives figures as to the power cost and current consumption of anthracite mines and the reasons for these being in excess of the requirements for bituminous mines.

Estimates are also given as to the additional coal that will be released by the electrification of the anthracite mines.

Illustrations are included showing representative installations of electric drive.

THE production of anthracite coal in the United States for the past five years was as follows:

Years	Total Gross Tons
1913	81,809,782
1914	81,580,479
1915	79,803,374
1916	78,406,387
1917	89,720,982

The undeveloped beds are to such a large extent owned or controlled by the large producers that no extensive opening of new mines has taken place in recent years nor is to be expected in the near future.

In the bituminous fields there are many independent holdings, and when the coal business was flourishing many new companies were formed and opened up new mines, using electric drive, mainly on account of lower first cost for installation of machinery. The resulting economies were such as to force the former steam-operated mines to electrify even where it was necessary to throw away expensive steam equipment.

Coal producers are conservative and not having the many examples of successfully operated electrified mines before them, the anthracite operators have been slower in seeing the benefits to be derived and the economies to be effected by throwing out their steam-operated and replacing it with electrically-operated

equipment. The margin of profit, due to the better organization of the producers, was also higher and the necessity of saving in the cost of production had not been brought home to them with the same force. For these reasons the possibilities of making a greater progress, in the decrease in amount of coal burned for power, in increased production of coal, in decrease in amount of labor and in the general living conditions for the employees, exists to a greater extent in the anthracite than in the bituminous coal regions.

The anthracite operators, are at the present time keenly alive to this fact and are making every effort to take advantage of the improvements to be made by electrification of their mines, but are handicapped by the difficulty of securing electrical equipment and by the impossibility of securing sufficient power from the central stations supplying power to the anthracite regions.

The mines that have been electrified show some surprising results. The following figures are taken from a paper read before the A. I. M. E. (Economy of Electricity over steam for Power Purposes in and about Mines, by R. E. Hobart, Feb. 18th, 1918.)

	Steam Operation April 1914 to April 1915	Electrical Operation Nov. 1916 to Nov. 1917
Cost of Power	$46,992	$21,590
Cost of Heating	Included in above	8,700
Total	$46,992	$30,290
Tons of coal mined	343,665	435,073
Cost per ton	$0.137	$0.0696

When it is remembered that the value of the coal burned with steam operation has more than doubled and that the output of coal per man per year employed has increased from 540 tons to 647 tons it will be seen the figures above, large as they are, represent only part of the saving to be effected.

Some other figures obtained are interesting as showing costs of electrification, power used, etc.

Item	Mine No. 1	Mine No. 2
Gross tons-yearly	641,533	670,000
Kw-hr. consumption	2,312,195	3,477,876
Kw. demand	870	1,135
Annual L. F.	30.4	35

FIG. 1—AN EARLY ANTHRACITE BREAKER—STILL IN OPERATION

[CRANE]

FIG. 2—A MODERN ANTHRACITE BREAKER—ELECTRICALLY EQUIPPED

Kw-hr. per ton mined....	3.6	5.2
Kv-a. trans. capacity....	2,000	3,500
H. P. connected.....	2,400	2,535
Cost of electrification....	$134,500	$325,000
Cost per h. p. connected.	56.04	128.20
Per cent kw-hr. per pumping.....		50
Average depth of mine...	650 ft.(198.1m.)	800 ft.(243.8m.)

Mine No. 2 had a boiler plant of 800 h. p., used 900 tons of coal per month and 11 men have been put to other work about the mine. The output will be largely increased this year as the above investment provides for additional equipment, not yet in operation.

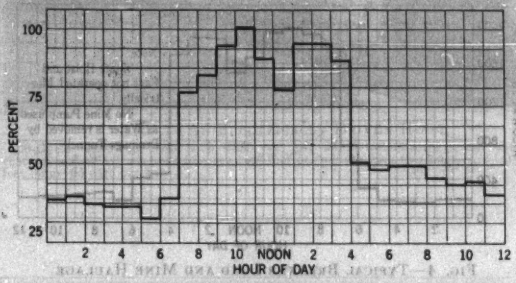

FIG. 3—TYPICAL ANTHRACITE MINE LOAD

Mine No. 1 has no pumping, hence the low kw-hr. per ton mined.

The best figures I have yet been able to obtain indicate that 12 kw-hr. per gross ton of coal mined is the average figure for an anthracite mine. This is of course subject to wide variations in individual cases on account of depth of mine, amount of air required for ventilation, quantity of water to be pumped as well as the head pumped against, and amount of work necessary to prepare coal for market. The writer, three years ago secured figures from over fifty bituminous mines and it is interesting to note the average kw-hr. per gross ton mined was 3.57.

The less power required for mining bituminous coal is due to various causes, among which may be mentioned:

Depth of Mine. Bituminous mines in many cases are situated above tipple, so that coal is brought to the surface on level tracks,

and from this point is loaded into the cars by gravity, while the anthracite mines are from 300 ft. (91.4 m.) to 1000 ft. (304.8 m.) in depth and all of the coal has to be raised to the surface and from there to the breaker.

Pumping of Water. The anthracite mines have more water to be taken care of and this water has to be elevated from the lowest level.

Ventilation. In spite of the fact that there is less volatile in the anthracite coal there is generally more gas to be taken from the mine and a larger amount of ventilation has to be provided for and the additional space used for shafts, etc. has to be ventilated.

Preparation. The breaker of the anthracite mine requires

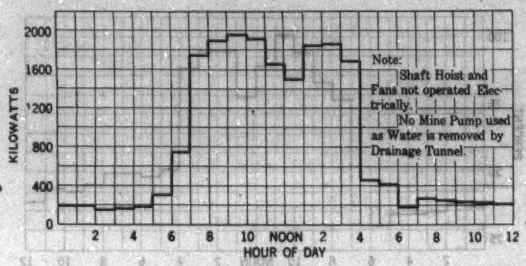

FIG. 4—TYPICAL BREAKER LOAD AND MINE HAULAGE

more power as the coal is sized to a greater extent than is bituminous coal.

One anthracite mine had meters installed for the different services for one year and the following average figures were obtained.

Operation	Kw-hr. per gross ton of coal produced
Haulage	1.73
Ventilation	1.62
Drainage	1.30
Lighting (inc. charging station)	0.12
Hoisting	1.02
Air compressor	2.11
Breaker	4.75
	12.65

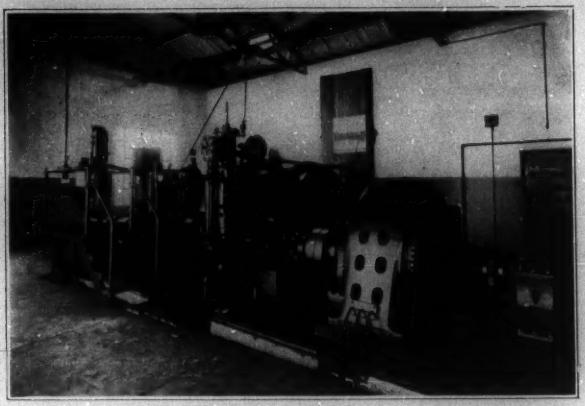

FIG. 5—AN ELECTRICALLY DRIVEN HOIST

60-Cycle—2200-volt—675 h. p.—450 rev. per min.—wound-rotor motor with liquid rheostat—double drum hoist running 1500 feet per minute with two 3-ton cars—slope 1100 feet long—95 cars per hour.

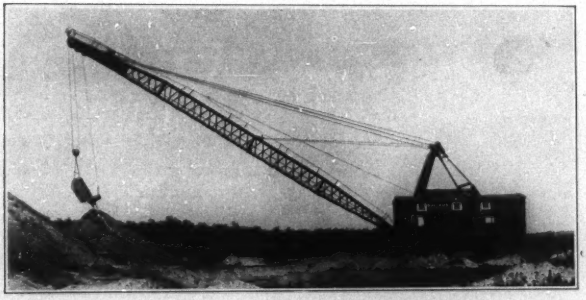

FIG. 6—AN ELECTRIC SHOVEL [CRANE]

Current supplied through a bank of three 100-kw-4000/400 volt transformers.

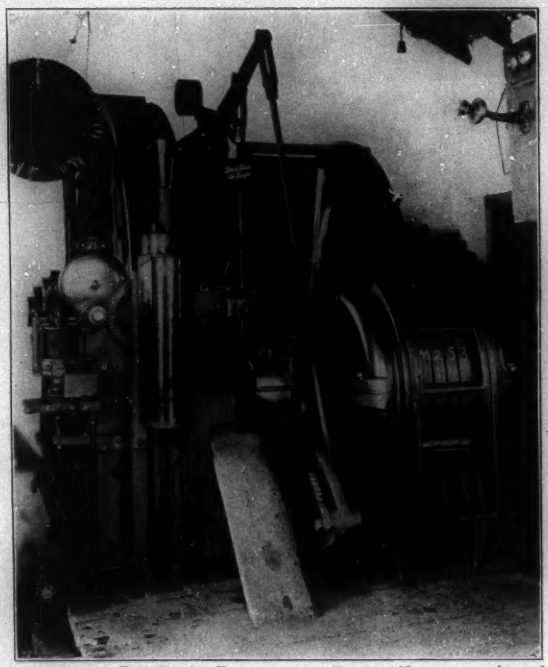

FIG. 7—AN ELECTRICALLY DRIVEN HOIST [CRANE]

60-cycle—440-volt—125-h. p.—500-rev per min.—wound-rotor motor—single drum hoist 900 feet per minute—one 3-ton car—slope 1000 feet long 52 degree pitch—18 cars per hour.

The above mine produced 558,394 tons for the year.

If we assume 12 kw-hr. per ton of coal produced there would have been required in 1917 if all the coal had been produced electrically..... 1,076,652,000 kw-hr.

There was actually used in 1917 in electrically operated anthracite mines from central station and mine plants..... 215,000,000

Total kw-hr. necessary to produce remainder of coal electrically..... 861,652,000

At the present time 10 per cent of the coal produced is used to provide power to mine the remainder. There are some very bad cases, at one colliery 400 tons of coal is burned in the boiler plant for every 1000 tons of coal shipped. The above 10 per cent does not include the coal used at central stations so that to produce the 89,720,982 tons in 1917 there was burned under boilers..... 8,972,000 tons

In large stations we can produce 1 kw-hr. for 2.5 lb. of small anthracite coal..... 965,000
(2.5 × 861,650 divided by 2240)

There would be released for sale..... 8,007,000 tons

At one colliery 2.5 per cent of the men were released by electrification. There are employed in the anthracite region 150,000 men. If 2.5 per cent are released by electrification this makes a total of 3,750 men additional, which would be put to mining coal. Each man produces 550 tons per year so we should get additional (550 × 3750)..... 2,062,500

Additional coal produced by total electrification..... 10,069,500

The cost of providing for this is estimated as follows:

Additional kw-hr. required..... 861,652,000

At 40 per cent load factor this represents a station capacity of..... 394,000 kw.

Cost of plant at $75 per kw..... $29,550,000

Transmission and distribution lines 250 miles at $5,000..... 1,250,000

$30,700,000

Substation and mine installations 900,000 h. p. at $40 per h. p.		$36,000,000
The savings to be effected would be 10,069,500 tons of coal at $2.00 per ton		20,139,000
Reduced mining cost, 700,000,000 tons at 7c. per ton		4,900,000
		$25,039,000
870,000,000 kw-hr. (861,652,000 plus losses) at 11 mills for current delivered at mine	$9,570,000	
Depreciation 5 per cent on $36,000,000	1,800,000	11,370,000
		$13,669,000

This represents 38 per cent on an investment of $36,000,000.

At the present time the mining companies are financially able and are installing as rapidly as possible electrically driven machinery for increasing production and cutting down the requirements for labor. They cannot at this time secure the material for power houses and the central stations are prevented from supplying this power by the impossibility of securing the necessary capital to add to their facilities. It is suggested that means should be found for supplying the amount required to finance additions to these plants so that the vital needs of this industry for power can be met.

Presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., December 13, 1918.

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DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

BY F. L. STONE

ABSTRACT OF PAPER

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

IN order to make the subject clear I propose to describe in a very general way why there is a problem connected with drum shape.

Coal in this country lies at a depth of from 40 ft. (12.1 m.) to 1800 ft. (548.6 m.) or more below the surface. There may be only one vein or there may be a number of veins lying over each other. In some instances the parent vein will split and form two or more veins. The pitch of these veins in any given direction may vary from horizontal to vertical.

In nearly all cases the veins outcrop on the surface somewhere though they may be so thin and pinched at the outcrop that it would be unprofitable to attempt mining operations at that point.

Possibly the most violent upheavals in coal measures have occurred in the anthracite mines in the Panther Creek Valley section. An approximate cross section of this locality is shown in Fig. 1. From this you will see how the measures twist and turn and what a different mining problem is found as compared with mining flat coal.

Fig. 2 shows a hard coal mine in which all conditions of mining are encountered. This picture tells in plain language the various methods involved. This model is 9 ft. (2.7m.) by 16 ft (4.8m.). The front portion about two thirds the area of the

model shows the workings of an 8-ft. (2.4m.) vein with all the pitches from flat to 80 deg. Back of this the overlying measures are shown in section with a shaft and a skip hoist or gun boat by means of which the coal is taken to two breakers. A conveyor is shown taking the coal from the shaft to a breaker. A stripping operation is also shown with two steam shovels in operation. The back ground is formed by a painting of characteristic coal region scenery.

The coal is mined in the various rooms and brought to the foot of the shaft or slope in mine cars which hold from one and one-half to five tons. It is usually gathered with small locomotives and made up into trips and taken to the foot by a large haulage locomotive. This locomotive returns with the empties.

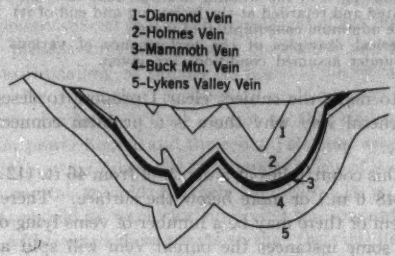
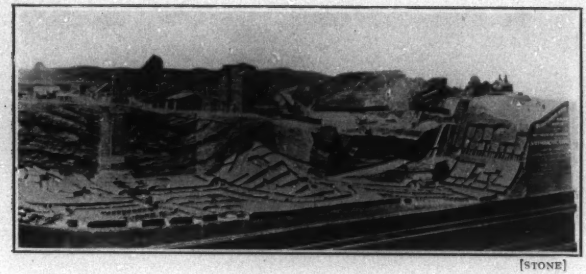


FIG. 1—SECTION OF COAL MEASURES IN PANTHER CREEK VALLEY

The coal is brought to the surface in a number of different ways. First the loaded car is put on the cage and hoisted to the landing while an empty is being lowered. At the landing the car is taken off the cage and sent to the breaker or tipple over the surface. The second method is that of using a dumping cage, that is to say the loaded car is taken to the surface and then the cage tilted and the car door opened and the contents dumped into a chute, the car never leaving the cage.

The third method is that of dumping the coal directly into a skip, the skip holding one or two mine car loads, the car never leaving the mine. This method involves an increase in the breakage of coal but not necessarily to any great extent. The fourth method is somewhat similar to the third except that the coal is dumped from the cars into a hopper and then loaded into the skip.



[STONE]

FIG. 2—MODEL SHOWING METHODS OF MINING ANTHRACITE COAL UNDER THE SEVERAL CONDITIONS OF ITS OCCURRENCE, ALSO TYPES OF MINING PLANTS

This method has many features to recommend it. Heavy loads can be lifted per trip at a comparatively low speed—for example if a given tonnage is required and it takes three trips per minute to get this output by the first or second method, a motor or engine of a h. p. X would be required. If the output could be obtained by the fourth method carrying heavy loads at low speed, the capacity of the drive would be reduced to $X/2$ or more. The mechanical parts, would, of course have to be stiffened up to take care of the increased rope stress.

From the foregoing remarks you will realize that standardization of mine hoists from the engineers standpoint is out of the question, since there are so many factors entering into the calculations for determining the proper size of motor and a change of any one may change the result very materially.

The following are the important factors which enter into the calculations:

Tonnage per hour
Weight of material per trip.
Total lift.
Rest period between trips.
Amount of material to be accelerated and retarded per trip.

You can readily see how at least one of these values will vary in every problem.

The problem is usually stated as follows:

Mine hoist is required to hoist—
Tons per day of eight hours a
Total lift . . . . . b
Weight of material per trip c
Weight of cages and cars . . . d
Rest between trips . . . . . e

If the average rope speed is comparatively high, say 20 ft. (6m.) per sec. or greater and the time of a trip comparatively short, say 20 sec. including rest period, then one may be fairly sure that the horse power required to accelerate the mass will be a large factor in determining the motor rating.

The ordinary horse power—time curve of a hoist using a cylindrical drum follows the general shape shown in Fig. 3. Motor starting from rest accelerates the total mass to running speed. This is held until the cage approaches a landing; then the

speed is reduced to rest. The cars are then changed and the cycle repeated.

The heating of the motor is, of course, proportional to the square of the current and the current is proportional to the torque, and therefore the r. m. s. value of the area from start to

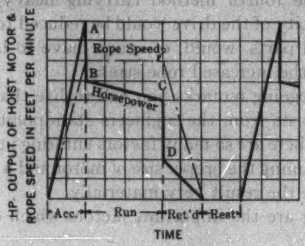


FIG. 3—TYPICAL MINE HOIST DUTY CYCLE

start (making proper allowances for the acceleration, retardation and rest periods) will be the rating of the motor.

You can appreciate the important part played by the accelerating area in the motor rating. It is this peak which, by the shape of the drum, we are trying to reduce.

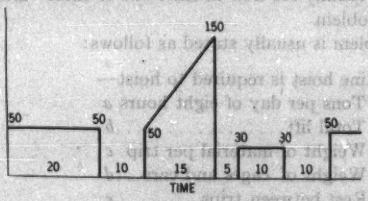


FIG. 4

It is obviously impossible to reduce the $\frac{m\,v^2}{2}$ but we can reduce the point B to which the h. p. of acceleration is added.

The depths I have selected are 300 (91.4m.) and 500 ft. (152.4m.). I have assumed the same tonnage output in each case.

In none of the examples chosen has the armature acceleration or retardation been included as this would only complicate

matters and would not change the relative result to any great extent.

The method of arriving at this motor rating is the usually accepted one of r. m. s. By this is meant the square root of the average squared ordinate of the entire area under the duty cycle. This is not strictly correct but sufficiently accurate for the normal hoist cycle.

I will illustrate the method by the following simple examples:

1st. A motor running at constant speed and the load varying rapidly. Fig. 4.

$$50^2 \times 20 = 50{,}000$$

$$\frac{(50^2 + 150^2 + 50 \times 150)}{3} 15 = 162{,}500$$

$$30^2 \times 10 = 9{,}000$$

$$221{,}500 = \text{Total squared area.}$$

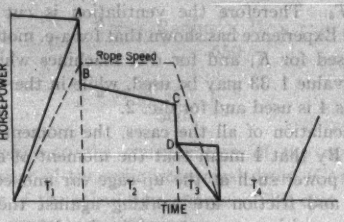


FIG. 5

$$\frac{221{,}500}{70} = 3164 = \text{mean squared ordinate.}$$

$$\sqrt{3164} = 56.2 \text{ r. m. s.}$$

If the values are amperes then the same heating would be obtained in the motor as if it were running continuously at 56.2 amperes. The same is equally true if the values are in horse power or kw. direct current, or kv-a. alternating current.

Now take the case of an intermittent running motor such as a hoist drive. Assume a cycle similar to Fig. 5.

$$\sqrt{\frac{(A^2 \times T_1) + \left(\frac{B^2 + C^2 + B\,C}{3}\right) T_2 + D^2 \times T_3}{\frac{T_1}{K} + T_2 + \frac{T_3}{K} + \frac{T_4}{L}}} = \text{r. m. s.}$$

Since the torque must be held constant or nearly so during acceleration the current corresponding to A horse power is flowing in the motor during the time T_1, but since the speed is O at start the horse power must be O also. For the slightly cumbersome expression $\left(\frac{B^2 + C^2 + B\,C}{3}\right) T_2$ may be substituted the expression $\left(\frac{B + C}{2}\right)^2 \times T_2$ if the difference between B and C is not greater than 10 or 15 per cent. The line joining B^2 and C^2 is in reality a curve and the area under this curve is represented actually by the expression above. The same remarks regarding A apply to D, the torque must be held constant from the time retardation begins until the hoist comes to rest. In getting the average square ordinate it would be incorrect to use the full value of T_1, T_3 or T_4 as in the first two cases the motor starts from rest and only reaches full speed at end of T_1 and is at rest at end of T_3. Therefore the ventilation is cut down quite materially. Experience has shown that for a-c. motors the value 2 may be used for K; and for d-c. machines which are more exposed the value 1.33 may be used, while in the case of L_1 for a-c. machines 4 is used and for d-c. 2.

In the calculation of all the cases, the moment method has been used. By that I mean that the moment of all the forces that require power such as the up-cage car and coal, up-rope, acceleration and friction are working against the motor and considered as positive moments while the down-cage and car, down-rope, and retardation are assisting the motor and therefore considered negative. The total negative values are subtracted turn for turn from the total positive values and the net moment found. From the shape of the drum and the depth, the total turns may be readily found.

From the total turns may be calculated the r. p. s. as follows:

$$\text{R. P. S.} = \frac{\text{Turns}}{T - \frac{t_a + t_r}{2}} = V$$

Where T = total running time, t_a = time of acceleration, t_r = time of retardation.

In regard to the friction value which has been assumed as 25 per cent of the useful work done in all cases. This

value covers the mechanical losses in hoist from motor coupling to the actual work done in the shaft, including gear and bearing losses, rope bending, guide friction and cage windage. There is, of course, no absolute method of predetermining this value and an intelligent guess is about the best method for practical cases. Many formulas have been evolved for the calculation of this value but they are based on assumptions which may or may not be correct.

Having the total moment including friction the horse power can be found by substituting in the following simple formula:

$$\text{H. P.} = \frac{2\pi\, r M}{550} \quad (r = \text{r. p. s.},\ M = \text{Moment.})$$

This formula is derived as follows:

at any point $M = W \times$ radius

Distance traveled in one second $= 2\pi \times$ radius $\times$ r. p. s. $= V$.

$V \times W$ = ft. lb. per sec.

$$\frac{V \times W}{550} = \text{h. p.} = \frac{2\pi \times \text{Radius} \times W \times \text{r. p. s.}}{550}$$

$$= \frac{2 \times M \times \text{r. p. s.}}{550}$$

We now come to the consideration of the cases selected. Only in the case of the cylindro-conical drum (Case 4) are the actual figures of the calculation shown. This case will serve to illustrate the general method used.

Weight of coal hoisted	5,000	lb.
Total lift	300	ft.
Weight of cage	11,000	lb.
Weight of car	4,000	lb.
Rope	1.25	in.
Number of trips per min.	3	
Rest between trips	4	sec.

Case 1.—Cylindrical drum.
Case 2.—Conical drum.
Case 3.—Cylindro-conical with long cone.
Case 4.—Cylindro-conical with short cone.

One of the most important points to be determined in the calculation of these cycles is the weight and radius of gyration of the rotating parts of the hoist. The figures used have been

secured from various hoist builders at various times and are believed to be fairly accurate.

Calculation of Case 4:—

Acceleration in 5 sec., run 6 sec., retard 5 sec., rest 4 sec.

To find number of turns on small drum to take up acceleration.

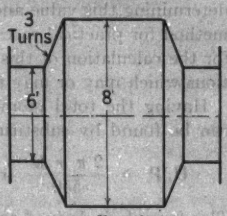


FIG. 6

x = r. p. s.

$\frac{5x}{2}$ = Turns during acceleration.

$\frac{5x}{2} \times \pi \times 6$ = length of rope wound on during acceleration. (1)

$\frac{5x}{2} \times \pi \times 8$ = length of rope wound on during retardation. (2)

$3 \times 7\pi$ = length of ropewound on cone. (3)

$6x$ = turns during constant running.

$6x - 3$ = turns on large diameter during constant run.

$(6x - 3)\,8\pi$ = Rope wound on. (4)

$$\frac{5x}{2} \times 6\pi + \frac{5x}{2} \times 8\pi + 21\pi + (6x - 3)\,8\pi = 300$$

$$30\pi x + 40\pi x + 42\pi + 96\pi x - 48\pi = 600$$

$$166\pi x - 6\pi = 600$$

$$166\pi x = 618.9$$

$$x = \frac{618.9}{166.\pi} = 1.187 \text{ r. p. s.}$$

Turns acceleration $\frac{1.187 \times 5}{2}$	2.97
Turns on cone	3
Turns on large diam. $6 \times 1.187 - 3$	4.11
Turns retard large diam. $\frac{1.187 \times 5}{2}$	2.97
Total Turns	13.05

Distance passed over.

Acceleration	$2.97 \times 6\pi$	56	ft.
Up cone	$3 \times 7\pi$	66	ft.
Large diam. constant speed	$4.11 \times 8\pi$	103.3	ft.
Large diam. retard	$2.97 \times 8\pi$	74.7	ft.
Total Lift		300.0	

Moments.

Up load

11,000	cage
5,000	coal
4,000	car
20,000	total.

Turns	Weight		Radius		Moment.
0	20,000	×	3	=	60,000
2.97	20,000	×	3	=	60,000
5.97	20,000	×	4	=	80,000
10.08	20,000	×	4	=	80,000
13.05	20,000	×	4	=	80,000

Down Load

11,000	cage
4,000	car
15,000	total.

Turns	Weight		Radius		Moment.
0	15,000	×	4	=	60,000
2.97	15,000	×	4	=	60,000
7.08	15,000	×	4	=	60,000
10.08	15,000	×	3	=	45,000
13.05	15,000	×	3	=	45,000

Up-rope (1.25 in. diam. weight 2.45 lb. per foot)

Turns	Weight		Radius		Moment.
0	300 × 2.45	×	3	=	2,205
2.97	244 × 2.45	×	3	=	1,795
5.97	178 × 2.45	×	4	=	1,743
10.08	74.7 × 2.45	×	4	=	732
13.05	0				0

Down-rope

Turns						
0	0					0
2.97	74.7	×	2.45	× 4	=	732
7.08	178.	×	2.45	× 4	=	1,743
10.08	244.	×	2.45	× 3	=	1,795
13.05	300	×	2.45	× 3	=	2,205

Plot all curves on coordinate paper—Moments against turns.

Sum of Moments.

			UP			
Turns.....	0	2.97	5.97	7.08	10.08	13.05
Up-load.....	60,000	60,000	80,000	80,000	80,000	80,000
Rope	2,200	1,790	1,740	1,500	730	0
Total up ...	62,200	61,790	81,740	81,500	80,730	80,000
			DOWN			
Down-load .	60,000	60,000	60,000	60,000	45,000	45,000
Rope	0	730	1,500	1,740	1,790	2,200
Total	60,000	60,730	61,500	61,740	46,790	47,200
Total up ...	62,200	61,790	81,740	81,500	80,730	80,000
Total down.	60,000	60,730	61,500	61,740	46,790	47,200
Net	2,200	1,060	20,240	19,760	33,940	32,800
Friction....	4,600	4,600	4,600	4,600	4,600	4,600
Gross M ...	6,800	5,660	24,840	24,360	38,540	37,400

Friction Determination.

$$\frac{2200 + 1060}{2} \times 2.97 = 4,830$$

$$\frac{1060 + 20,240}{2} \times 3 = 31,950$$

$$\frac{20,240 + 19,760}{2} \times 1.11 = 22,200$$

$$\frac{19,760 + 33,940}{4} \times 3 = 80,550$$

$$\frac{33,940 + 32,800}{2} \times 2.97 = 99,000$$

Area in moment turns 238,530

Average ordinate

$$\frac{238,530}{13.050} = 18,300$$

$$\frac{18,300}{0.80} = 22,900$$

Friction ordinate

$$22,900 - 18,300 = 4600$$

Acceleration and retardation of loads in shaft due to cone.

$$a = \frac{v_1 - v_0}{T}$$

Acceleration up-load $v_0 = 6\,\pi \times 1.187 = 22.4$

$v_1 = 8\,\pi \times 1.187 = 29.85$

$$T = \frac{3}{1.187} = 2.53 \text{ sec.}$$

$$a = \frac{7.45}{2.53} = 2.93$$

Allowing 750 lb. for rope in shaft.

$$F \text{ of acceleration} = \frac{20{,}750 \times 2.93}{32.2} = 1887.$$

Moment at 2.97 turn = 1,887 × 3 = 5661.
Moment at 5.97 turn = 1,887 × 4 = 7548.

Retard down-load.

$$F \text{ of retardation} = \frac{15{,}750 \times 2.93}{32.2} = 1435$$

Moment at 7.08 turn = 1435 × 4 = 5740
Moment at 10.08 turn = 1435 × 3 = 4305

Acceleration of up-load from start to full speed of drum.

$$v = 6\,\pi \times 1.187 = 22.4 \qquad a = \frac{22.4}{5} = 4.48.$$

$w = 20{,}000 + \text{Rope } 750.$

$$F = \frac{20{,}750 \times 4.48}{32.2} = 2870 \qquad M = 2870 \times 3 = 8600.$$

Acceleration down-load.

$$v = 8\,\pi \times 1.187 = 29.85 \qquad a = 5.97$$

$w = 15{,}000$

$$F = \frac{15{,}000 \times 5.97}{32.2} = 2780$$

$$M = 2780 \times 4 = 11{,}120$$

Drum and 1/2 rope.

$WR^2 = 350{,}000$, W at 3 ft. = 38,900

$V = 22.4 \qquad a = 4.48$

$$F = \frac{38{,}900 \times 4.48}{32.2} = 5410$$

$$M = 5410 \times 3 = 16{,}200$$

Total acceleration moment

Up-cage, car, coal and rope	8,600
Down-cage and car	11,120
Drum	16,200
	35,920

Retardation

Up-load $V = 29.85$ $a = 5.97$

$$M = \frac{20{,}000 \times 5.97}{32.2} \times 4 = 14{,}900$$

Down-load and rope $V = 22.4$ $a = 4.48$

$$M = \frac{15{,}750 \times 4.48}{32.2} \times 3 = 6560$$

Drums as before 16,200

Total retard moment up	14,900
Down	6,560
Drums	16,200
	37,660

SUM OF MOMENTS—FINAL

Turns	0	2.97	2.97	5.97	5.97	7.08	7.08	10.08	10.08	13.05
Gross	6,800	5,660	5,660	24,840	24,840	24,360	24,360	38,540	38,540	37,400
Acc.	35,920	35,920	...	...	...	...	...	...	...	...
Acc. on cone	...	...	5,661	7,548	...	...	...	...	...	...
Retd. on cone	...	...	...	...	...	...	−5,740	−4,305	...	...
Retd.	...	...	...	...	...	...	...	...	−37,660	−37,660
Total	42,720	41,580	11,321	32,388	24,840	24,360	18,620	34,235	880	−260
Time	0	5	5	7.53	7.53	8.47	8.47	11	11	16
H. P.	580	564	154	438	338	330	253	466	11.9	3.5

R. M. S.

$$\left(\frac{580 + 564}{2}\right)^2 \times 5 = 1{,}640{,}000$$

$$\left(\frac{154^2 + 438^2 + 154 \times 438}{3}\right) \times 2.53 = 239{,}000$$

$$\left(\frac{338 + 330}{2}\right)^2 \times .94 = 110{,}500$$

$$\left(\frac{253^2 + 466^2 + 253 \times 466}{3}\right) \times 2.53 = 338{,}000$$

$$\left(\frac{11.9 - 3.5}{2}\right)^2 \times 5 = 88$$

2,327,588

$T_1 = 5$	$K = 2$	Equiv. Time =	2.5
$T_2 = 6$		" "	6.
$T_3 = 5$	$K = 2$	" "	2.5
$T_4 = 4$	$L = 4$	" "	1.
Total Equiv. Time.....			12.0

$$\frac{2{,}327{,}588}{12} = 194{,}000$$

$$\sqrt{194{,}000} = 440.$$

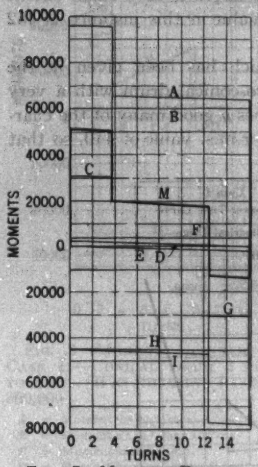


FIG. 7—MOMENT DIAGRAM—CASE I—SINGLE CYLINDRICAL DRUM—$W\ R^2$ ROTATING PARTS 200,000

A = Total moment up load
B = Moment up cage car and ore
C = Acceleration moment
D = Up rope moment
E = Down rope moment
F = Friction moment
G = Retardation moment
H = Down cage and car moment
I = Total down moment
M = Total net moment

Looking at Case I, Figs. 7 and 8, you will note the acceleration peak goes to 825 h. p. and the retardation goes below the zero line which indicate the motor must be plugged or the mechanical brakes applied to stop the hoist. This in the case of an induction motor drive is not conducive to good operation as the operator will invariably plug, which is expensive from a power standpoint, or he will take longer to retard which reduces the output. The r. m. s. value of this cycle is 600 h. p.

Case II is that of a conical drum 6 to 8 ft. in diameter. It will be noted the acceleration peak is considerably less in this than in Case I. This is brought about by the fact that the point *B* in the master diagram, has been reduced to approximately 200 h.p. Here, again during retardation the value becomes negative and as in Case I the motor must be plugged or stopped by means of mechanical brakes, though not to any such great extent as in Case I. The r. m. s. value of this cycle is 482 h. p. This is shown in Figs. 9 and 10.

Case III is that of a cyclindro-conical drum in which the

cylindrical parts are just large enough to take the turns during acceleration and retardation. The turns on the large diameter are common to both ropes, the drum being symmetrical around the center line. Here we have reduced the accelerating h. p. to approximately 600, and the retarding h. p. above the line which indicates that power must be kept on to a very small extent during retardation. The r. m. s. value in this instance is 452 h. p. Shown in Figs. 11 and 12.

Case IV, the calculation of which has been given in the previous pages is that of a cylindro-conical drum with a very steep pitch or spiral cone. This gives a good many of the characteristics found in Case III and a r.m.s. value of 440, so that

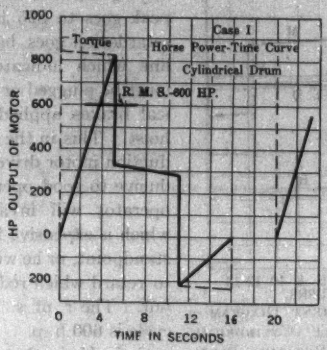


FIG. 8

apparently little is gained in the use of the steep pitch cone over the medium pitch cone of Case III and there is always some question in the minds of some operators, whether or not the pitch cone is a very desirable design. Personally I am not experienced enough with their operation and life to express an opinion, but feel that if the same result can be obtained with a moderate pitched cone, I personally would prefer the latter. Figs. 13 and 14 show results clearly.

In Cases VI, VII, VIII and IX the same tonnage output has been used, but the depth of the shaft increased to 500 ft. This, of course, means a very much more rapid speed of hoisting.

Case VI is that of a cylindrical drum which shows a very undesirable cycle from an induction motor standpoint. It has

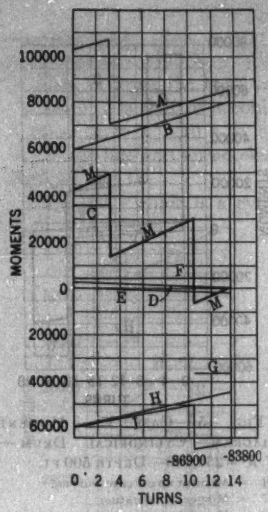


FIG. 9—MOMENT DIAGRAM—CASE II—CONICAL DRUM 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

- *A* = Moment up load, total
- *B* = Moment up car, cage and ore
- *C* = Moment acceleration
- *D* = Moment up rope
- *E* = Moment down rope
- *F* = Moment friction
- *G* = Moment retardation
- *H* = Moment down car and cage
- *I* = Moment total down side
- *M* = Moment total net

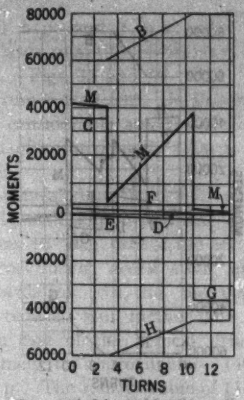


FIG. 11—MOMENT DIAGRAM—CASE III — CYLINDRO-CONICAL DRUMS 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

- *B* = Moment up car, cage and ore
- *C* = Moment acceleration
- *D* = Moment up rope
- *E* = Moment down rope
- *F* = Moment friction
- *G* = Moment retardation
- *H* = Moment down cage and car
- *M* = Total net moment

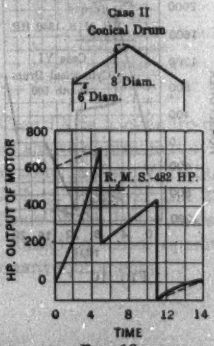


FIG. 10

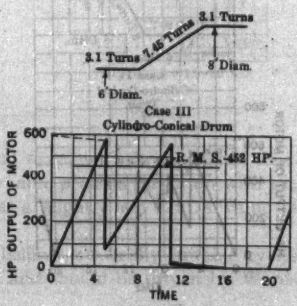


FIG. 12

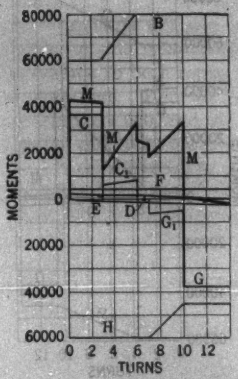


FIG. 13—MOMENT DIAGRAM—CASE IV—CYLINDRICAL CONICAL DRUM—3 TURNS ON CONE—WR^2 ROTATING PARTS 350,000

B = Moment up cage, car and ore
C = Acceleration
C_1 = Acceleration up load on cone
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
G_1 = Retardation down load on cone
H = Moment down cage and car
M = Moment total net

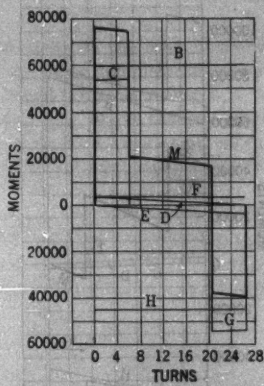


FIG. 15—CASE VI—MOMENT DIAGRAM—CYLINDRICAL DRUM—WR^2 = 250,000—DEPTH 500 FT.

B = Moment up car cage and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moments

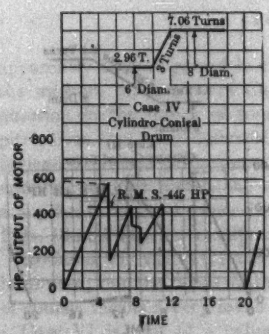


FIG. 14

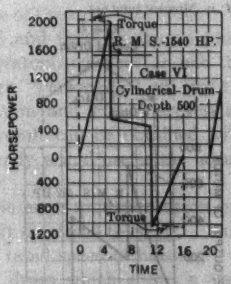


FIG. 16

an acceleration peak of over 2000 h. p. and r. m. s. value of approximately 1540 h. p. with a large quantity of braking to be done to bring the hoist to rest. Figs. 15 and 16.

Case VII is that of a conical drum 6 to 9 feet. It shows a r. m. s. value of 1210 h. p. and a considerable amount of area below the line and an acceleration of approximately 1800 h. p. Figs. 17 and 18.

Case VIII, which is a cylindro-conical drum 6 to 9 ft. with only sufficient cylindrical portions to take care of the active turns during acceleration and retardation. In this the peak is reduced still further and is the case of the conical drum, namely to approximately 1460 h. p. The r. m. s. value of 1128 h. p. and incidently the negative area has been considerably reduced. Figs. 19 and 20.

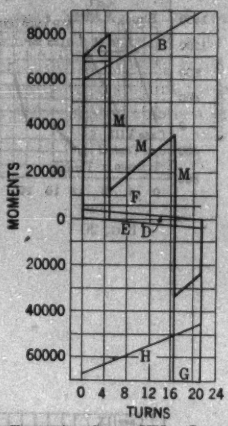


FIG. 17—CASE VII—CONICAL DRUM 6 FT. TO 9 FT.—DEPTH 500 FT.—$W R^2 = 400{,}000$

B = Moment up cage car and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moment

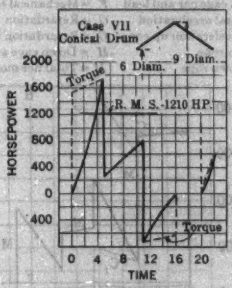


FIG. 18

Case IX is a cylindro-conical drum with steep pitched cone. This shows approximately the same peak as the drum with the long cone, in Case VIII. but has a slightly less r. m. s. value of 1070 h. p. Figs. 21 and 22.

None of the latter cases, I believe, are really suitable cycles for induction motor operation and should be driven by Ward-Leonard equipments.

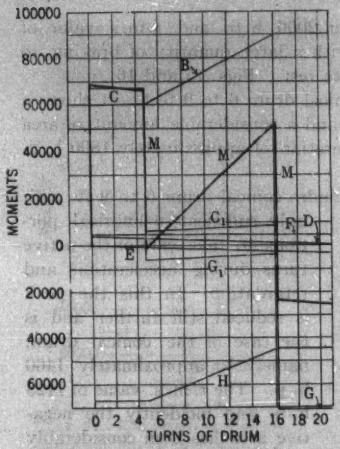


FIG. 19—MOMENT DIAGRAM—CASE VIII — CYLINDRICAL-CONICAL DRUM — $W R^2$ 400,000

B = Up cage car and load
C = Total acceleration
C_1 = Acceleration up cone
D = Up rope
E = Down rope
F = Mechanical friction
G = Retardation
G_1 = Retardation down cone
H = Down cage and car
M = Total net moments

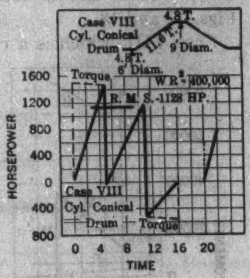


FIG. 20

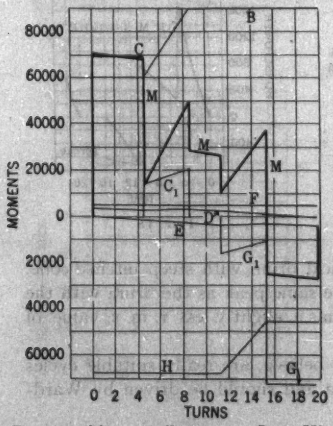


FIG. 21—MOMENT DIAGRAM—CASE IX—CYLINDRICAL-CONICAL DRUM

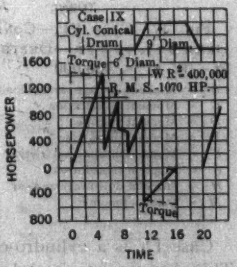


FIG. 22

I realize, of course, that in this short paper I have only touched the high spots in the problem of drum shapes, but I have, I hope selected sufficient examples to show that there is in the proper selection of drum an opportunity for some real engineering.

I have kept away from the problem of drum shapes connected with hoisting from great depths as in this class of problem the rope weight is usually the reason for any departure from the cylindrical drum. The reduction of the acceleration peak in deep hoisting is, of course, an advantageous point which comes automatically with the coning of the drums. The weight of the hoisting cable in some of the deep shafts exceeds by a considerable amount the weight of the active material hoisted. As before stated this is a problem by itself.

DISCUSSION ON "THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL" (CRANE) AND "DRUM SHAPE AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING" (STONE), PHILADELPHIA, PA., DECEMBER 13, 1918.

Graham Bright: In regard to Mr. Crane's paper, I think that one of the real reasons why the change to electrical operation has been more general in bituminous mines than in anthracite mines has been due to the fact that the electrification for the average bituminous mine costs less money and takes less apparatus than the average anthracite mine. This is due to the fact that many bituminous mines are drift mines and the apparatus generally consists of a small rotary or motor-generator set, two or three locomotives, a few small electric pumps and a fan. This equipment does not run into a great deal of money while in the case of the anthracite mines, the coal is taken out from several levels requiring a fairly large hoist and heavy pumping. The fan and breaker also require considerable power and a large amount of electrical equipment.

Mr. Crane mentions that the margin of profit is a little more in the case of the anthracite mine than for the average bituminous mine, in fact the profit has been so low in normal times in the bituminous mines that in order to compete and make any money at all, it was necessary to take every advantage available, and since it has been well proven that electrification cuts down the cost of operation to quite an extent, this accounts for the fact that many bituminous mines have electrified their operation during the last few years.

The difficulty of obtaining electrical equipment has been rather a bar to electrification in the last two or three years. This situation should, however, be greatly improved in the near future so that little difficulty should be experienced in obtaining equipment with fairly good delivery.

Mr. Crane mentions some figures taken from a paper of Mr. R. E. Hobart presented before a meeting of the American Institute of Mining Engineers last February. The results shown in that paper were taken from actual tests and are certainly startling. In discussing this paper when delivered, I made the statement that if the electrical manufacturer had gone to the mine operator and made any such claims as are substantiated in Mr. Hobart's paper, he would have probably been thrown out of the office and not given any consideration at all.

Mr. Crane mentions the fact that the value of coal is about double what it was a few years ago. When the price of coal was low, it was generally conceded that the electrification of a mine, especially by the use of central station power, would show considerable saving over steam operation. The price of coal has now been doubled but the price of power from a central station has not, so that if there was a decided advantage before, there should be a greater advantage at present with the high price of coal and central station power at a price not greatly in excess of what it was formerly.

There is certainly a marked difference between the amount of power used per ton of coal mined between the bituminous and anthracite mines. In a number of bituminous mines in Ohio and West Virginia, the kilowatt-hour consumption per ton of coal mined will average from 2 ½ to 3, while in the average anthracite mine the kilowatt-hours per ton are about 12. It is true that some of the bituminous mines in central Pennsylvania require from 5 to 8 kilowatt-hours per ton due to heavy pumping, heavy fan loads and long hauls. Mr. Crane points out that the breaker load alone on the average anthracite mine requires 4 kilowatt-hours per ton, so that the preparation of the coal after it is brought to the surface requires more power than the total amount required in the average bituminous mine.

In some of the anthracite mines, it is necessary to pump from 12 to 15 tons of water to the surface for every ton of coal hoisted. The result is that the load factor in an anthracite mine should average very much higher than in a bituminous mine. In some of our bituminous mines, the load factor will range from 15 to 25 per cent while in the average anthracite mine a load factor of over 50 per cent is frequently obtained. For this reason the cost of power per kilowatt-hour should be less in an anthraceit mine than for the average bituminous mine where the cost of producing the power is about the same.

The average mine operator is an experienced man in the mining and shipping of coal and as a rule does not know much about the production of power. This results occasionally in such condition as shown by Mr. Crane where for every 1000 tons mined, 400 tons were burned under the boiler. A condition of this kind can frequently be greatly improved by the use of central station power where the mine operator can secure the advice of the engineers of the power company who can frequently show him a great many ways of economically using his power. We have all been led to believe that there is considerable coal in Alaska, but I was talking to an engineer today who has spent considerable time in Alaska and he stated that he knew of a mine in Alaska where they had to burn wood under the boilers because they could not get sufficient coal from the mine to run their own plant.

The figures given by Mr. Crane regarding the quantity of coal and labor released, provided the anthracite mines should electrify, are certainly interesting. If by electrifying the anthracite mines we could save a total of 10,000,000 tons per year, this would mean considerably over 10 per cent of the total amount of anthracite coal mined in this country, and this would be available for other uses.

In regard to Mr. Stone's paper, the question of hoisting coal by means of skips is one about which there has been a wrong impression, especially in regard to the breakage of coal. It has always been thought that handling coal in skips would greatly increase the breakage. In talking with some of the operators

who have tried the use of skips, I find that the general impression is that there would be less breakage provided the skips are of fairly large capacity. It seems to be the experience that the coal moving in large masses protects the individual lumps so that there is very much less breakage than would be anticipated. It is my opinion that in the future we will see more hoisting by means of skips which will eliminate the high-speed cycles.

Mr. Stone mentioned that his method of figuring the r. m. s. as given in his paper would not be exactly correct for large motors using forced ventilation. The formula will be correct if the total rest period is used instead of a fraction of the rest period as shown. It has been found that with large low-speed motors, we cannot get sufficient ventilation by the ordinary fan blades which are placed on the rotating part. A considerable increase in capacity can be obtained by means of forced ventilation and it is probable that in the future all low-speed motors of large capacity will be equipped with forced ventilation. A comparatively small amount of power is required for this ventilation, considering the large advantage obtained. Forced ventilation also permits the customer to put in an air conditioning plant, which will allow clean air to be used in ventilating motors, and this will in turn cut down the maintenance and lessen the repairs.

If we calculate the capacity of motor and generator required for a large hoist using the Ward-Leonard system of control by formula as shown by Mr. Stone, we will find that if the rest period is very long and natural ventilation is used, the hoist motor must have a larger capacity than the generator which supplies power to the hoist motor. This is on account of the generator operating continuously and getting full ventilation during the entire period, while the motor receives only part ventilation during a considerable portion of the entire cycle.

In regard to the shape of the drums, it will be found that a great many operators still favor the old plain cylindrical drum which, after all, has some advantages over the conical and cylindro-conical drums which are some times called "freak" shaped drums. The advantage of the cylindro-conical drum is shown clearly in the curves submitted, but as stated by Mr. Stone these curves are based on certain inertia values given by the hoist builders. The straight cylindrical drum is a very simple piece of apparatus and its weight and inertia can be accurately predetermined. In building such drums as the cylindro-conical type, there is considerable risk due to cracking caused by shrinkage strains, so that the designer is apt to make these drums considerably heavier than necessary. This extra weight tends to counteract the good effects obtained by the shape of the drum. There is a particular case I have in mind of a hoist furnished in the Middle West which gave considerable trouble with the resistance of the controller when operation was started. It was found that the peak loads were much heavier than had been calculated and it was necessary to furnish heavier

resistance to take care of these loads. We then started to locate just what the trouble was and found that the weight of the drums was just twice what the hoist manufacturer had said it was going to be. For this reason, it is best not to take too low a value for the inertia of a "freak" shape drum.

A second disadvantage of the special drum is that the minimum diameter should be the same as the diameter of the cylindrical drum that would be used. This means that the average diameter of the special shaped drum is much greater than the cylindrical drum, which in turn means that its speed and revolutions per minute will be lower. This in turn means a lower speed motor which not only costs more, but also has a poorer efficiency and power factor than the higher speed motor.

The third disadvantage of the cylindro-conical drum is especially true when a short cone is used. The rope travels up the cone in the shape of a spiral and the rope wear on this spiral is much greater than on the cylindrical portions of the drum. This means that the rope is worn out and ready to scrap when a large portion of it is still in very good shape.

The real advantage which we obtain by the use of a cylindro-conical drum is lower peak loads and a slightly smaller capacity of equipment, but we do not save much on power consumption unless as shown we have a very fast cycle where we have high retarding rates which require plugging the motor or using up the power in brakes.

There are some cases where the cylindro-conical drum will be an advantage, but I do not think that they should be used indiscriminately. There are many cases where the straight cylindrical drum is really the best drum to use.

The question of friction is one which, as Mr. Stone says, we could argue on all day, as it is difficult to test just what the friction of a hoist is. I think that 25 per cent of the actual work done is rather low value for the friction. The total efficiency of a hoist generally runs about 50 per cent. This means that we are putting in twice as much power as we are getting out and taking 25 per cent of the actual work as friction means that one-eighth of the power going into the hoist motor is allowed for friction of the hoist. However, if this value of 25 per cent was actually doubled, it would practically make no difference in the capacity of equipment required to meet a certain set of conditions.

The cycles that Mr. Stone shows for a depth of 500 ft. are certainly not suitable for induction motors, and I should hardly want to use them even with a direct-current motor using the Ward-Leonard system. The average rope speed is 2740 ft. per min. which is not a practical operating speed for a depth of 500 ft. The accelerating and retarding rates are about 9 ft. per sec. per sec., and it will be found that for good practise we should not go much above 5 or 6 ft. per sec. per sec. In fact, the cycle although possible, shows that we are getting on dangerous

ground. It means that the operator must be on his toes at all times and the slightest trouble in the safety device or error in judgment of the operator may bring about disastrous results, and while it might be physically possible to operate for a short time under such a cycle, with the Ward-Leonard system, it is certainly not good practise. Where the output requires any such cycle, I agree with Mr. Stone that the only safe method is the installation of skips handling a much heavier quantity of coal for each trip, which results in a much lower rate of speed and a considerable lengthening of the cycle.

C. W. Parkhurst: Mr. Crane's paper clearly brings out the need for more complete electrification of anthracite mines. I notice in the paper, he has given 3.57 kw-hr. per ton mined as an average of 50 bituminous mines.

It has never been my good fortune to be connected with any bituminous mine where the coal came out of the ground so easily. The lowest power consumption per ton mined that I have ever seen, was 4.4 kw-hr. per ton, and the highest well over 20 and the average around 9 kw-hr.

I know of a great number of mines where the power used for ventilating the mine is over 5 kw-hr. I hastily gathered some figures together of three big bituminous mines, averaging about 300,000 tons per month, and for five years back the kw-hr. consumption was as follows: 5.5, 6.6, 7.9, 8.9, and 8.2.

Except for the last year, there was a continuous increase, and that is accounted for by more complete electrification each succeeding year, and because of the fact that as the mine was getting bigger, ventilation became a more serious problem each year, more water had to be pumped and the haulage distance was increased. It shows that the kw-hr. per ton is going to increase as the mine is developed.

I know of a big mine, with a consumption of 10.15 kw-hr. per ton, of which 5.74 kw-hr. is for ventilation. That mine, when first started, had a 15 h. p. motor on the fan, that was changed to 30 h. p. to 75 h. p., to 125 h. p., to 300 h. p., and the last change was to a 750 h. p. motor, with a 300 h. p. motor in reserve for Sunday and night work. The big motor shuts down at night.

I have some power consumption per ton figures for bituminous mines as follows:

A group of five drift mines, averaging about 51,000 tons per month, has a consumption of 6.8 kw-hr. per ton; a shaft mine, 9.2 kw-hr. per ton; a group of four drift mines averaging about 14,000 tons per month 9.08 kw-hr per ton; a group of six drift mines, mining together about 50,000 tons a month, 8.99 kw-hr. per ton.

F. L. Stone: I will say that this matter is fairly well covered in the paper, where it says: The coal is brought to the surface in a number of different ways. First the loaded car is put on the cage and hoisted to the landing while an empty is being lowered. At the landing the car is taken off the cage and sent to the breaker

or tipple over the surface. The second method is that of using a dumping cage, that is to say the loaded car is taken to the surface and then the cage tilted and the car door opened and the contents dumped into a chute, the car never leaving the cage.

The third method is that of dumping the coal directly into a skip, the skip holding one or two mine car loads, the car never leaving the mine. This method involves an increase in the breakage of coal but not necessarily to any great extent. The fourth method is somewhat similar to the third except that the coal is dumped from the cars into a hopper and then loaded into the skip.

E. J. Cheney: In addition to Mr. Stone's illumination of the subject, I think some people may not understand the general system of mine hoisting, which may be explained by means of Fig. 1.

There are two drums, on which the ropes are wound, these drums being driven by engines or motors. The ropes go from

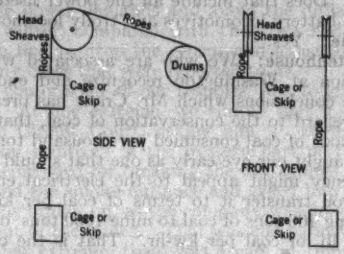


FIG. 1

the drums to the head sheaves at the top of the shaft and then down the shaft. At the end of each rope is suspended a cage or skip, as the case may be. Nearly all mine hoists are of this double type, ordinarily known as balanced hoists.

When one skip or cage is at the bottom, the other is at the top. The two balance each other, and the only unbalanced load is that due to the coal or ore, or whatever it is you are hoisting. As the empty skip, or the cage with the empty car goes down, the loaded one comes up. The difference due to the unbalancing of the load is constant, but the weight of the rope becomes a very serious factor in the case of a deep mine, so that actually, when you go to depths of 1000 ft. (304.8 m.), or more, the light side, so-called, may be the heavier; in other words, the weight of rope hanging in the shaft may exceed the coal or ore you have on the loaded side. The curves in Mr. Stone's paper will be more readily understood if you have in mind the fact that there are those two systems, one

going up and the other going down, and that you have an enormous amount of dead-weight, in drums, sheaves, skips, and ropes, to start and stop every time, whereas you are actually handling a comparatively small amount of live load.

W. I. Slichter: I would like to ask whether the fundamental principles governing the design of the shape of the drum are: that for the most economical use of power in accelerating any mass, a large force should be applied at the beginning, at standstill and at low speeds, and then as the speed increases this force should decrease, thereby limiting the power, the product of force and speed, to some definite reasonable value. It appears to me that the object of giving the drum a particular shape is to accomplish what the inherent characteristics of the series motor accomplishes in railway work.

Peter Kain: I would like to know if the figures of kw-hr. per gross ton of coal under "haulage" include the two types of locomotives used. The figure given is 1.73 kw-hr. gross ton charge for hauling. Does that include all the power used for hauling, both storage battery locomotives and trolly locomotives.

J. B. Crane: Yes.

L. H. Rittenhouse: We who are associated with the Fuel Administration at Washington recognize and endorse a great many of the conclusions which Mr. Crane has presented in his paper. In regard to the conservation of coal, that one glaring case of 400 tons of coal consumed per thousand tons mined is a case which caught our eye early as one that should be remedied. The inefficiency might appeal to the electrical engineer more forcibly if you transfer it to terms of coal per kilowatt-hour. That is, taking 400 tons of coal to mine 1000 tons of coal, would be 70 to 75 lb. of coal per kw-hr. That is the equivalent of what they were using.

Another advantage, conservation of men, also appealed to us very strongly, when men were the all-important article, are today, and will be in the future; the saving in men, or manpower, appealed to us very strongly.

There is another saving which the author mentioned in his paper, only from the investment point of view, and that is the iron and steel conservation. The War Industries Board was especially interested in the conservation of every pound of iron possible, and as the iron and steel go much further in the central station power plant than in the isolated plant, this was an additional argument in favor of the large station.

The ordinary advantages of the central station apply in the coal field, as in any other application of the central station, but, of course, there are some peculiar advantages which are brought out in the paper, among these being the smaller amount of coal used.

Still another point which the author might properly have indicated, and which I wish especially to emphasize, is that the central stations are truly a national resource. We, at the Fuel

Administration, felt very strongly about it at the time, because we realized the stations are really an asset of the country, and we felt the lack of these stations when we were looking around for them. The speaker was associated with the Production Bureau, in that section which concerned itself with the continuity of electric power for coal production, and when we were getting telegrams every day—"Why don't we get power for digging coal."—we looked around for the central stations and found they were loaded to the limit, and there was really no way by which these coal operators who wanted the additional power could get it.

As I say, we felt that it would be a national resource to have a few more central stations available, and it appealed to us from that angle. There were a few breakdowns, and we looked up the question of supplying power, and in the meantime we took certain steps to anticipate some of these breakdowns, if possible; in other words we started a census of coal mines purchasing power from central stations, so that we could anticipate some of these power demands and avoid the difficulties during the coming winter, compared with those we had last winter. However, we had to drop the census and assist and cooperate with the many cases that were reported to us.

I will tell you of some of our troubles, just to give an idea of their range. At one extreme, we simply sent a letter to the judge of the Circuit Court, asking him to be lenient with a central station company in the western part of Pennsylvania, which was being sued for breaking the law against the emission of cinders, and the smoke nuisance and asked him to be lenient and to put the brakes on, as the company was doing all it could to get coal out of the mines. That was comparatively simple.

On the other extreme, there was a case in Illinois where we assisted the superintendent of a large plant in operating a train of 30 tank cars, operating those tank cars every day, carrying water for condensing purposes to a station 20 miles away, located alongside a stream which happened to go dry. Incidentally that misfortune curtailed production by tens of thousands of tons of coal.

The point I am making is that these central stations are really a national asset, and it was brought home to us, who were trying to cooperate with them, very forcibly this summer, and I think an organization like this cannot be better employed than to engage in missionary work of this kind and help the central stations along.

There was a case in which one central station did have extra power. The New York Edison Company had about 20,000 kw. We had our eye on that, and I believe the Fuel Administration of the City of Philadelphia also had its eye upon it. But Newark "beat us to it" and they started to put cables under the Hudson River to connect up with the New York Edison Co. to help out the Newark situation, there being a lack of power facilities in that important manufacturing district.

That indicates the great advantage of the resources of these stations, and I think this institute should take this matter up in a proper way to bring it before the authorities. A number of well designed central stations with ample reserve, efficiently managed and strategically located provides a national asset in times of peace as well as of emergencies, that should not be overlooked. We think that the Government should be advised of the possibilities of such situations. We were doing our part. The natural question arises—Why did not the Government help? As Mr. Crane mentioned, they started to do something, and they were cooperating in regard to the financial questions he raised in his paper, and we were doing as much missionary work on Capitol Hill as possible. Of course, very properly, the members of Congress were watching out for the interests of the Government's purse strings—that is proper, and their function—, but the money was not going to be given away, on the contrary it would have been of great assistance, and in passing, I may mention that every dollar of it was being taken care of to the extent of the ability of the different government departments that were connected with the project.

Supplementing Mr. Crane's paper, I have a few figures in regard to the census of mines purchasing power. After the armistice was signed, we took time to add up the returns we had received. As far as we could discover, there were 107 central stations supplying approximately 1000 coal mines with electric power.

In regard to the question of the kw-hrs. per ton, we were vitally interested in that, because we wanted to conserve power in the mining operations and make it go further than it had in the past. Mr. Crane gave a figure in his paper of an average which was 3.57 kw-hr. per ton of coal mined, which figure was criticized later in the discussion.

I picked up some figures we had available from one of these central stations, and it is interesting to note that the average I obtained was 3.58. The figures I have given are for one station supplying 56 mines, and he said his was for a station supplying a little over 50. I wonder if both figures related to the same group. This particular group to which I refer was located west of the Mississippi.

J. B. Crane: My figures related to a group of mines in West Virginia.

L. H. Rittenhouse: An entirely different set of mines. But as was pointed out by a speaker, in the discussion a few moments ago, the conditions differ and I think possibly that 3.58 kw-hr. per ton is low, in other words, as far as my investigation carried me, the limit of the 56 mines ran from a fraction of a kw-hr., showing the power was used exclusively for electric lights, up to 53 kw-hr. per ton, with a foot-note calling attention to the fact that this mine had recently shut down. No wonder! These figures, however, account for the averages. We found it very

difficult to get exact figures, since in many mines connected to central station service some of the power used in developing coal was not electric. In other words, some operators may still retain a few mules, as well as auxiliary steam, and these do not figure in the kw-hr. unit.

I would like to ask Mr. Crane, in connection with mine No. 1 and Mine No. 3, 3.6 kw-hr. and 5.2 kw-hr., were those figures for an anthracite or a bituminous mine.

J. B. Crane: That was for an anthracite mine, fully electrified.

C. A. Adams: This is one of those numerous cases where the emergency situation produced by the war has forced us to the adoption of methods of cooperation which have proved so immensely effective and economical.

I do hope that this institute and the other engineering societies, and all other such bodies, will use their best efforts to help the nation perpetuate these advantages which have been forced upon us by the war during this reconstruction period and afterwards.

F. L. Stone: Since scarcity of power seems to be one of the limiting features in connection with the electrification of the anthracite field, I would like to ask Mr. Crane if everything has been done that is possible to secure the advantage of the diversity factor that exists between many of the smaller plants located in the region. I know that in some instances plants have tied together and thereby made available, for the use of both, several thousand kilowatts. It would seem to me that a great deal could be accomplished along this line.

L. S. Randolph: There is one phase of electrical operation which it is almost impossible to put in dollars and cents, viz., the convenience. Anyone who has put a 4 in. to 8 in. steam-pipe with exhaust pipe into a mine, and then puts a centrifugal pump with two small wires, sees where this comes in.

There is no chance of coming in the morning and finding a drunken fireman has burned a crown sheet the night before, and as a consequence shut the mine down for three or four days. One does not have any of that trouble.

Those are factors which cannot be put in dollars and cents, and yet they are tremendously valuable.

E. J. Cheney: There is one point these papers bring out, and that is the tremendous advantage which electrification produces in the case of mining work, as in almost every other instance, in that you are able to tell, in the first place what you want to do, and afterwards what you are doing.

So far as I know, in the case of steam-driven hoists, the hoist is simply designed according to the ideas of the man who gets up the drum part, and then a steam engine is put on that is big enough to drive it, and the hoist is put in and operated.

When you come to install an electric hoist, you have people like Mr. Stone, and the others who are figuring on electrical equipment, to calculate the effect of weight and inertia, and

drum size, and drum shape, and they go back to the hoist makers and say:—"See here, if you do so and so, that means a big difference in the result." A steam hoist is operated without much attention being paid to economy. It is hard to find out whether the inefficiency is in the boiler plant, or in the engine, or in the shaft, or somewhere else, and so it is usually not done; but in the electrical hoist, a few indicating and recording wattmeters tell the story all the time. You can look at them and see what is going on, and you can do it day after day, checking up watthours against tons of coal mined, and you can find out whether there is anything wrong and tell which type of installation is giving the best results. You are working with your eyes open all the time, and are not in the dark about anything.

Graham Bright: A few years ago I used to indicate steam hoists in order to find out just what work was being done, so that the proper size of motor could be determined where it was desirable to change a steam hoist to electric drive. I soon found out that these results were of very little value because in every case a square steam card was obtained which showed that no cut off whatever was used. In practically all cases the operator who would open up the throttle wide, used no cut off, and when the accelerating period was over, would throw off and drift so that at no time during the operation was there any cut off, so that you can see that there would be very little use in taking indicator cards from such an operation.

We can determine much more regarding the electrical equipment necessary if the operator will tell us how much material he wants to hoist each trip, how many tons he wants to get out in a given time, what his rest periods are and the weights and diameters of his drums, cages or skips, cars and size of rope. By having this information, we can figure out closely the electrical equipment necessary, but the steam engine itself is no guide whatever in determining the proper electrical equipment to put on a hoist. Regarding the steam engine, it is necessary that either side of the engine be capable of lifting the maximum weight on account of the fact that one side may be on dead center and unable to exert any torque at the start. This means, as a rule, that the engine is considerably larger than necessary and the result is that the operator uses very high rates of acceleration when a considerably lower rate would be ample to obtain the output required. The average operator thinks that he requires this high rate of acceleration and also thinks it necessary to have high rope speed. At the average mine, a trip of cars comes to the bottom of the shaft and the operator hoists this to the surface as rapidly as possible and then takes a rest. In a large majority of the cases, by using slower rates of acceleration and slower rope speeds, he could get out as much coal as the mine could produce except that he would not have so much time to loaf. In many cases, if we were to attempt to exactly duplicate the steam operation of a hoist, it would require an unneces-

sarily large and expensive electrical equipment which is not justified when a smaller and less expensive equipment would be ample to hoist all of the coal that the mine can produce in a day's time.

If the operator wants to produce a certain amount of coal in eight hours, it is not safe to figure out a cycle which will require the full eight hours. It is much better to estimate a cycle which would take the coal out in from six and one-half to seven hours if a mine works an eight hour day, which allows an hour to an hour and a half for the usual delays which are bound to occur. It is practically impossible to operate on a given cycle for a period of eight hours without delay. There are so many things about a mine to cause delays that in order to be safe, this margin should be anywhere from one to two hours per day.

J. B. Crane: I would like to bring out a few points. Mr. Bright mentioned the fact that the central station men were a little modest in their claims for the advantages of the use of central station power. That fact was brought home to me forcibly about three years ago. I had the pleasure of addressing the West Virginia Mining Institute, and they asked me to prepare a paper on "The Use of Central Station Power in Mines from the Central Station Standpoint." The man who was asked to prepare a paper to be presented at the same session, from the mine user's standpoint, was a man we had worked with for quite some time, to get him to see the advantages of electric power, etc., and I remember at one time one of our men went into his office and he asked him who he was with, and he told him he was with the power company, this man said to him, "Well, you might as well be on your way, no use stopping here." We naturally expected, when he read the paper we would get many knocks, but instead of that, his paper was strongly in favor of the use of central station power, and we were most surprised at his change of attitude.

Mr. Parkhurst brought out the question of the kilowatt hours per ton of coal mined, and his figures for the bituminous coal were larger than those I had given. These figures will vary with different districts. But I never heard of any bituminous mine which used as much power per ton of coal mined as those which he mentions, and I secured figures from several mines, and they were around the ones I secured in West Virginia as indicating the amount of power it takes to operate the mine, and the deeper the mine becomes, the more power it takes.

I had the pleasure last year of visiting the Maror Velho mine in Brazil. They went down 7000 feet, and intended to go another 1000 feet. They were using 4000 h. p. to operate the mine for the first 7000 feet. In going down the additional 1000 feet, they would require 4000 h. p. additional power for ventilation.

The conditions which they would then encounter with regard to ventilation, etc., would prevent them from going any deeper,

because the cost of power would run into such large figures that it would not pay.

Mr. Rittenhouse brought out the fact that power stations have been greatly overloaded during the period just passed, and, that this condition still exists.

Regarding the diversity factor which Mr. Stone mentioned, we have in our territory two power stations in the coal field, one good for about 16,000 kilowatts, and another one good for 30,000 kilowatts, and at a nearby point we have another power station good for 20,000 kilowatts, and one small station, or rather several small stations, good for 3000 or 4000 kilowatts.

The peak which we were getting from all these stations was 55,000 kilowatts. We connected the ones in the coal field with the station at Allentown, which is the one I have just referred to as being nearby, with a capacity of 20,000 kilowatts, and we have been going out and taking power from these three stations, amounting to 69,000 kilowatts.

In burning small size anthracite fuel, we are getting beyond an economic boiler rate. With stokers this rate is 175 per cent. We have been running them as high as 300 per cent, and averaged for the month over 200 per cent boiler rating. As a consequence, we have had to replace a boiler arch every other day, and have all kinds of trouble, but we have been able to carry the load in our particular territory.

The thing which Mr. Stone mentions, and has brought out forcibly, is the fact that in the coal fields the peak load is from 7 o'clock in the morning until 4 o'clock in the afternoon, and then we have about 10,000 kilowatts to dispose of between that time and 7 o'clock the next morning.

We are only 90 miles from Philadelphia, and they have a peak in the evening of something like 20,000 kilowatts above the daily load. You can see what that would mean if a transmission line could be run from our central power system to Philadelphia. The same equipment which takes care of the coal mines in the day time would take care of the lighting load in Philadelphia at night.

I know of one case in Texas where they connected in several stations, and in that case they were able to sell 10,000 kilowatts additional by connecting the central stations together and getting the benefit of the diversity factor.

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AMERICAN ENGINEERING RESEARCH

BY W. R. WHITNEY

THE purpose of this paper is to call attention to our national condition in relation to engineering research, in the hope that the desire among engineers to advance the sciences may be discussed and crystallized into the most acceptable form. The subject must have some sharpness of definition, and so I will give the meanings I attach to the title.

I am using the word research, to cover painstaking investigations which *add* to knowledge. Research alone may be too general a term to interest all engineers, so I limit my use of the term by the adjective "engineering", and mean by that, the study of matter. Much research may be done in fields not connected with engineering, but I am not referring now to archaeological, geographical, psychological research, or other similar fields where study and experiment also add to human knowledge. For the sake of a simple and clear field I am omitting such sciences. I use the prefix American, to call attention to one of the most burning questions which confronts us today. We have been so keen in engineering that American engineering is unsurpassed in the world, but we never made a fair start in American engineering research. Moreover, we have no plan for producing research men, though we have plenty of plans for consuming them.

Research is the foundation of engineering, whether it be electrical, physical, chemical, biological, or what not. It sometimes seems as though much of our best engineering is only very slightly removed from the original research in time or space. If the research forms the foundation, then engineers quickly add the first story. When the investigators showed the value of typhus vaccination, the Army doctors used it on a million soldiers. General Gorgas recently wrote Dr. Keen that of over 1½ million soldiers, there were only 18 deaths from this disease in 1917. At the Civil War rate there would have been 7200 deaths. That is good American engineering research. But

who is responsible for the research men? How may we procure and support them? A part of this responsibility is your own.

During a certain period of our American history there was a Puritan atmosphere in research. It hung like a fog over the territory of the Pilgrims, and even now it clouds the issues between engineering and research. It meant purity to the extent of sterility. It was with interest that I recently read, with the above geographical thought in mind, one of Emerson's Essays. He puts our duty in such a clear light that, before going into problems of detail, I want to quote him. His use of exactly the right word to give clearness to his thought seems very wonderful and his words will bear a more careful analysis than anything I can say.

> The destiny of organized nature is amelioration and who can tell its limits? It is for man to tame the chaos. On every side while he lives, to scatter the seeds of science and of song, that climate, corn, animals and men may be milder and the germs of love and benefit may be multiplied.

If we could all learn that by heart, I could well stop my talk at this point.

The experience of the past few months developed in the mind of practically every intelligent man a desire to do something to advance the country's interest. This was not confined to scientists and engineers, but was marked in those cases. The number of scientific and engineering organizations which evinced this desirable spirit is too long to read. Our leading societies united in committees representing all the sciences, and they in turn formed many sub-committees. The National Research Council enlisted in one committee or another almost all the scientists of the country. The Council of National Defense established scores of committees on engineering and allied lines. Inventories of everything conceivable were taken, including most of the material and personnel of our colleges and engineering schools.

Now there is one defect in this system of rapid utilization of our resources. It does not produce any new material, though it may organize the old. We needed organized engineering research long before the war. We needed it more during the war, and we shall certainly need it after the war. It is the duty of engineers to study this matter and take active steps to correct it. The problem is: how shall we insure the preparation of plenty of American men of science by some system which makes us independent of foreign assistance? My belief is that it can

well be done in our existing colleges by greater attention to new material truth. Experimental work in the sciences best prepares and preserves those desirable qualities in teachers: interest and enthusiasm. Any plan which would insure properly supported investigators whose vocation was experiment and whose avocation was teaching, where students could *work* with them, would be a most natural development. It is a way already thoroughly tested abroad and would be a step of the greatest value to this country.

It is unfortunate that we have no generally acceptable plan for insuring our scientific work and our engineering research. There has been little better than chaos on this subject thus far in America: No one's life work was research, though to a few it was a by-product. Within the past few years, and in a few lines in certain universities, investigations into the laws or properties of matter have been under way. This has been poorly appreciated. The *exceptions* are those successes which led to rapid physical amelioration, as of death or pain (naturally the first places for such appreciation). The value of research on the hook worm is readily understood. Heredity-studies in corn or cattle are recognized as worth while, but there are many researches undertaken whose value is not so apparent on the surface, and yet whose ultimate potency is warrant for their consummation. But for us at present, while the product of research work is desirable, the preparation of the men is of greater importance.

Because our condition has been described before and remedies suggested, is no reason why attention should not be called to it again. Something should be done. Important steps in a democracy call for repeated attempts to inform the interested people. Perhaps scores of addresses on general preparedness delivered before the war were wasted, but it was safer to assume that the reiteration was necessary.

It is owing to our Yankee inventiveness that I am interested in seeing much greater attention given to the problems of our engineering research. We are inclined to be so elated by our apparent successes in industry, that we imagine that there is some way to build without foundation, to practise without experience, to reap where we have never sown. Hundreds of thousands of inventors are choking our patent offices with ideas which can have but one end, the unhappy end of the inventor. This condition is wrong. We cannot wish to perpetuate it. It

may be greatly ameliorated by careful planning of engineering and research education in America. This planning falls as a duty on the engineers themselves.

There is a general opinion that discovery of truth is very prevalent here, and that somewhere in our national archives there lie buried a lot of wonderful but undeveloped possibilities. You have probably received notices from different groups of people who want to bring to light and life some such desirable inventions. As a matter of fact, there are probably very few such things in existence. One reason is that there is actually more capital to develop new ideas and more capitalists who see possibilities which do not exist, than are justified by works. In fact, while we lack training and knowledge as inventors, we more sorely lack men who are sufficiently expert to pass with certainty on suggestions of abnormal minds. The reason the steam engine and the submarine were not earlier recognized was due to the scarcity of men who knew science. When we had one Langley on aeronautics we needed 100. Paucity of knowledge is our present trouble, not superabundance of inventions. The point in this illustration is that we badly need more highly trained quantitative men of science. Our National planning for research, except for research in agriculture (which planning, strange to say, was done during the Civil War), has been entirely neglected. Agriculture is not our only interest, but even in that field, the future lies in the hands of the young research chemists and engineers who should do the work on potash and fixed nitrogen.

It is on this point of the special need for planning that I want to lay stress. Ways to accomplish the end have been tested and proved. Particularly during the past century there have appeared in other countries men who devoted their entire time to study in fields of new knowledge which by contact had attracted them. We might possibly have called them specialists, high brows, hobbyists, or men of large interrogation. They were very valuable.

These men were found in many walks of life, but by far the greater part of them were, or soon became, attached to foreign universities. They gave fully of their products to one another and took extreme and careful steps to give to the world accurate and detailed reports of their researches. Davy and Faraday were of this type, Liebig and Wöhler were like them, and it is safe to say that such benefactors of the race could be cited by

the hundred. In most cases their entire lives were spent in their laboratories and all that they learned was freely published. Still more important; in the laboratories of these men, particularly promising students were allowed to work, and these teachers illustrated for them scientific research by example. They did not confine their teaching to talks about the work of others. The men referred to, and others, such as Helmholtz, Kelvin, Berthelot, and more recent prototypes, such as Ramsay, Moissan, and Haber, were alike in these respects. We have produced altogether too few of them in America, and supported none. They became unusual men, but they started much like others. They were developed by a system or plan. As they developed in interest, in knowledge of new things, and grew abnormal in this respect, the opportunity for further development was provided, usually by the state. There is no reason to assume that the product they represent cannot be grown on American soil. They were the result of a policy almost identical in all cases. Each was freed by some planned system to evolve himself and follow his studies along the lines he chose, and he was given the assistance of students who were at the same time learning the value and rare power of careful thought and painstaking investigation. They created atmospheres of scientific pioneering. The teacher was the leader. He was exploring. His interest was transferred to others by that most direct way—close contact. The lines of scientific endeavor were not chosen by some remote director, or committee of experiment, but the subject itself led the investigator along, and new truth was the sole aim. The professor then became the leader in the university, because he was following a new line of action and not because, at the request of a faculty committee, he was perhaps trying to solve some particular indigenous problem, or reducing the cost of some existing factory product. He was being led by Nature, instead of trying, as the industrial laboratories so often do, to force Nature along lines which we think most desirable.

Someone has suggested that possibly our industrial research laboratories could supply the country's deficiency in scientific research and its teaching. Many industrial laboratories have been established recently, but, to my mind, there does not seem the slightest possibility of thus meeting the real need of the country. It is neither natural, nor desirable, and most recent writers on the subject have been clear on this point. The interesting papers by Colonel Carty, Dr. Jewett, and Mr.

Skinner, in the TRANSACTIONS of the A. I. E. E., are in good accord on this point.

It may be asked, do you consider it a good plan to arrange our colleges so that every teacher with a hobby may develop it by the aid of the students? If all our teachers had enjoyed good training in science and were good scientific investigators, the answer would certainly be "yes", but this is not always the case. Therefore we must lend our aid in helping to establish a condition which will insure a greater number of scientific teachers who are also trained as productive scientists. This is in reality the method employed in those countries where scientific education has been most advanced.

In other words, our great American engineering problem today is not, how shall we make use of our trained investigators? Shall we organize a few large centers of research and solve there the industrial problems of the country, or, shall we encourage and assist the industries to give more attention to research work? But it is rather: how may we produce annually active young students of matter at a greater rate but of higher quality than any other country? We must think in terms of thousands per year of the type of trained men represented, say by the doctor's degree, or by two or more years of individual experimental work.

The direct object of this paper is not to ask for the appointment of some committee to cooperate in planning American engineering training and knowledge. We do not need more committees for this work, but more constructive activity on a dozen existing committees. In other words, enough machinery already exists to do the work, and we need but a clear view of the necessary steps. Probably no one person has completed a perfect plan, but many have advanced suggestions, all of which seem to have failed to meet sufficient acceptance to move the wheels. My own ideas are not presented as new and original. They are plans of others, but they seem most natural and desirable because they accept higher education as our common right. They seem democratic.

Several years ago the American Association for the Advancement of Science appointed a Committee of One Hundred to bring about cooperation between universities and industries along the line of improving our work in science, its growth, and its use. Other organizations undertook similar studies, but at that time most of the effort was expended in arranging for the

use of, rather than for the production of facts, or of useful men. It was a shortsighted plan, helpful but not broad enough. In several colleges, research fellowships have been established, but these were generally for the solution of definite manufacturing problems where private interests seemed justified in paying for the work. This was done because scientific research which was aimed solely towards new physical truth could not be thus supported. Such methods are doubtless useful, but their aim is low. They advance single industries but are not intended to establish and make known new truths. Such organizations will only rarely become research centers where new leads in science will be persistently followed by groups of intensely interested investigators. There is greater value and attraction in disclosure of new physical phenomena than in any probable financial returns, and the atmosphere in a laboratory of scientific research is a thing by itself. But it cannot evolve of itself, and it is for practical beneficiaries, such as engineers and the public, to find the way.

Central bureaus of science and research have been repeatedly advocated and many are still being planned. The plans vary from separate departmental government laboratories like those of the Department of Agriculture, the Bureau of Mines, and the Bureau of Standards, to those of separate groups of colleges, groups of industries, societies, and finally, groups of men. One such is the most recent plan of the National Institute of Inventors. None of these is probably perfect. Most of them would consume (possibly ossify), but not produce men. Perhaps all of them should be studied by engineers and some of them developed, but if many were carried out, it would immediately become evident that our crying need is for general college research work to prepare trained investigators.

A few years ago a bill was prepared for Congress which had for its object the establishment of engineering research work in the federally assisted colleges of the country, and this called for additional federal support. One of the criticisms of the plan was that it did not include all colleges. It had been thought that the precedent having been established in 1862 of giving governmental aid to these institutions for their research and teaching work in agriculture, it might not be difficult to extend this line of activity to other scientific research work. One of the reasons this bill was not sufficiently supported was that a large number of struggling state universities and other colleges

objected to the distinction which would be made. This finally led to other proposals being drafted, and one, known as the Smith-Howard Bill, seems to be the most satisfactory suggestion yet made. Others are also appearing. It is not my intention to go into the details of this bill here. But its existence and its fundamental concept deserves your careful attention and your active interest. It called for funds for research in the colleges; $30,000. for a college in each state. Fifteen minutes of our present year's war cost would care for that plan a whole year. The plan seems a good experiment, but needs support. This line of thought raises the question whether our democracy should pay for its education and its standing in science and engineering, or whether it should be left to accidental philanthropy, to the begging ability of college presidents, or be neglected altogether.

President Shurman, of Cornell, said a few months ago:

> Germany has the advantage over all her opponents, especially Great Britain and the United States, in the solid fabric she has constructed during the last generation by means of generous government encouragement of, and appreciation for, scientific research and instruction. Our Government today, instructed by the war, is ready to spend billions of dollars for new scientific truths in which, prior to the war, it felt no concern and for which it would make no provisions.

This same thought has been expressed by others in many ways. The facts are pretty well known, but are still not entirely digested. We are such creatures of habit that we may never awaken to the fact that we are a slow but sanguine people. Slow, because our rate of advance as measured over the last century is less than Japan's; sanguine, because we hope for the best without being willing to provide for it.

If you consult the university president or engineering educator who is well acquainted with higher education in America and ask him why we have been so backward in the highest class of scientific teaching, he will generally reply, "lack of money". We are forced to train so many men to reach a certain stage of preparedness for standard vocations that we cannot afford to encourage advanced work nor can we obtain teachers who are primarily workers in their science. Our conditions in America are still in that stage of teaching where our energies must be exclusively used to repeat to students annually certain knowledge already acquired, but we cannot provide for acquirement of new knowledge nor for the training of men to obtain it. As far as our collective powers and plans go, the sciences would remain stationary.

Apparently persons of highest intelligence believe that while our schools should be supported by taxation, this should apply only up to a certain grade. Above that grade we should beg private interests for our advance in knowledge and for our most highly trained men, for our studies of disease and our search for cures. This fact accounts for the difficulty of the college president. It is merely a bad habit. Certainly no one really approves of it. Everyone who thinks carefully can find plenty of fault with it, but the engineering of a better plan is left undone. That is an engineer's job. In the past few decades intelligent men of wealth have fortunately recognized some of the needs of higher education in our country where our pioneer activities have not developed any satisfactory system. By this means, great good has certainly been accomplished, but the philanthropic method can scarcely be a strengthening one. It cannot lead to proper appreciation of the value of the advancement of knowledge by the people.

One of our greatest dangers is that, knowing how Germany arranged her higher education, quickly advanced and reaped those returns which were so harmful to us, we may spurn the adoption of some similar scheme because of the degraded utility she finally made of it.

What do we want? We want our democracy to do as well in science as German autocracy did, until we learn to do better. I do not care to compare the scientific work of different countries, but I want to call attention to the peculiar way science was treated in Germany for the half century preceding the war. I want to show that some of her ablest men were life-long workers in and teachers of science; that they were leaders in their civil groups, that they were appreciated by the country and given the widest support and honor. We have too little to compare with it, and we collectively, or our Government, are not making sufficient effort to support new scientific work.

Let me once more warn you against the fear that material truth and experimental knowledge are a menace. Only indolent people think this, but it is frequently expressed. Professor Richards, of Harvard, says on this point (1916):

> Science has recently been blamed by superficial critics, but she is not at fault if her great potentialities are distorted to serve malignant ends. Is not this calamity (war) due rather to the fact that the spiritual enlightenment of humanity has not kept pace with the progress of science? The study of nature can lead an upright and humane civilization ever higher

and higher to greater health and comfort and a sounder philosophy, but that same study can teach the ruthless and selfish how to destroy more efficiently than to create.

There are many European cities which owe their reputations to their scientific work. Most of them are university towns. Their renown is not based on the nine, nor on the eleven, nor on the local glee club. Freiburg, Bonn, Heidelberg, Jena, Charlottenburg, Leipzig, Halle, Karlsruhe, Gottingen, are such places. They recall scientific accomplishments. Some of our corresponding towns are Columbus, Ithaca, New Haven, Urbana, Madison, Princeton, Lawrence, Pittsburg, Burlington, Ames, and Lincoln. Does each remind you of scientific work? We are not yet doing our duty by them. Can we not help the scientific work in such towns?

Good scientists grow in all countries, but the support which investigators received in Germany is a point I want to emphasize. To the average physicist, chemist, or biologist, most of the following teachers recall fundamental advances in the sciences. I refer to men like Pettenkeffer, Liebig, Wöhler, Voit, Ludwig, Kühne, Hoppe-Seyler, Weber, Helmholtz, du Bois Reymond, Fechner, Engelmann, Pfluger, Hertwig, Leuckart, Ehrlich, Virchow, Hering, Emil Fisher, Buchner, Pfeffer, Weissman, Koch, Quincke, Hofmeister, Pringsheim, Boveri, Richter, Kekulé, Wislicenus, Kolbe, Hofman, Bunsen, Victor, Lothar and J. R. Meyer, Bayer, Clausius, Ostwald, Kolrausch, Planck, Nernst, Hertz, Boltzmann, Roentgen, Wien, and Haber.

Are we to suppose that America could not do as well? In my opinion, we should try the method already tested, and modify it from time to time as we see indicative developments.

The German universities operate under charters granted by the Government, which supports them. The faculty practically manages the institution, though the Government is represented by the curator, who manages the finances. The full professors regulate the internal affairs by their votes, and they elect their various officers, the rector, deans, etc.

On the other hand, we in America ask certain unprepared civilians to serve as our college trustees. They soon find that the condition of finances calls for help, and, together with the president, they must solve the almost insoluble problem of the year's budget. There is little chance for new work. Practically every college has all it can possibly do to meet the demands of large classes of undergraduate students, and the trustees,

despite their sympathy with scientific developments, cannot possibly care for demands which are less dominant. We have about 500 universities or colleges in this country. Many of them are equipped with everything necessary for good scientific work except the encouragement. Many laboratories are better equipped than those abroad. Would it not be worth while to arrange for the modest addition to our educational expenses which a fair trial of American university research would entail?

The Students' Army Training Corps has recently thrown all our colleges into an unsatisfactory and an unstable condition. Had that work continued, it is possible that permanent improvements in education would have resulted. It was too radical to continue, and the pressing need has disappeared, but we have all seen the possibilities of quickly effecting such changes, and we ought now to work and hope for some justifiable improvements. Is it good engineering to force the presumed leaders of our highest education to spend their best energies in perpetual scrambles for funds, instead of directing education? Probably no one thinks so, but everyone waits for the change to be forced by someone else.

The recent report on Engineering Education, published through the Carnegie Foundation, to which we are greatly indebted, leaves me with the thought that the past aim of our engineering education was to teach carefully selected men to merely repeat certain standard processes and thereby earn a livelihood. This is not enough. Watch good, experienced engineers at work and see them plan. Is their work repetition of old steps they were taught? It is nearer right to say that they were always trying experiments. We read daily of the newest and largest building, the most novel or the longest bridge, new types of tunnels or subways, longer range guns and new armor plates, lighter aero engines, larger turbines, heavier locomotives, Armco iron, Tantiron, Duriron, Monel, Stellite, etc., etc. There is certainly no limit to this line. Therefore I would not teach fixity, but right change, and do it by precept and *example*. I would increase for the engineer, during his college years, the facility with which he could begin to learn the elasticity and nobility of his calling, and that by contact with men interested in experiment.

What will be done to continue more men in advanced scientific lines in college? Can you engineers not help here? Is it not a good plan to arrange for the highest possible training on the

part of a selected few? Is it not the duty of organized engineers and scientists to take part in activities which influence legislation and to make our country the foremost for new finds in nature and in the development of men? Engineers will then be co-products with investigators.

I would be better satisfied with this suggestion if I did not know that very few realize how much the strength of a whole people depends on its scientific novelties. The facts are frequently something like this: some searcher for new truth in natural phenomena uncovers a fact. He gets a few dollars or a little mental satisfaction out of it. A few industries then base the work and the living of many people upon it, and its value, there, may be expressed in, say, many thousands of dollars. But it is not so readily seen that the people at large get most of the return and are therefore most highly interested in the advance. Their earnings in such cases are in millions and their advantages incalculable. The telegraph and telephone, the gas engine, the aeroplane, wireless, artificial lights, and all engineering improvements have this in common. The ratios may be 1 to 1000 to 1,000,000 for the three groups, but the largest group does least planning. It is not organized for it. Engineering societies might do this.

Large central laboratories of research recently suggested for industrial chemistry are worth good trial, and are being tried in England, but to be most effective they would have to follow somewhat the lines of other smaller laboratories. We have knowledge in such cases which may be of value. Several now exist in which scores of experimenters are employed. A group of one hundred working on miscellaneous research would probably represent between one-half and one million dollars a year outlay. The return would be problematical and cannot be estimated, but would seem very promising in various lines of federal interest. One result is, however, certain. Manned at once with any hundred investigators now available, it would be very much less useful than if manned with well trained research men, of whom we have too few. Our already extensive use of trained men such as electricians, chemists, and physicists is exceeding the supply.

That the young men of our colleges and engineering schools early quit school and went into war work is not a cause for complaint. That in itself was a splendid thing and accomplished wonders. Over a thousand young chemists have done the work

of our gas mask development, our poisonous gases, our smoke screens, our new explosives, etc., and the physicists went into the wireless, the submarine defense work, and similar problems in large numbers and with marked success. They have done things which older men would not have undertaken. Therefore this is rather a complaint that we had so few young engineers, and is a petition that in the future the supply be insured and the energies directed to peace time needs.

I do not see any possible way of insuring our future commercial and engineering supremacy that can compare with utilizing our colleges more perfectly. When barbarous war methods are forever stopped, science and engineering should be the main factors recognized by the League of Nations as the legitimate ground for a nation's supremacy. How can we better insure our position than by taking care of that amelioration which tames the chaos for all the people?

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RESEARCH IN AMERICA AFTER THE WAR

BY R. A. MILLIKAN

IT takes no statement from the chairman nor admission from myself, to apprise most of you of the fact that I belong to the genus "Damn professor," as we are affectionately called in Washington.

It may interest you, however, and perhaps be a little diverting too, to know how nearly we professors have to come to acting like real soldiers once in a while when we get on suitable clothes. About four weeks ago I chanced to go from Waterbury, Conn., to New Haven, on a trolley, starting about half past eleven o'clock at night. At a small station between the two towns, there boarded the trolley a couple of well-filled soldiers. They took seats on opposite sides of the car and began quarreling loudly. The conductor was quite unable to cope with the situation; finally the men arose and began hammering each other back and forth across the car. The passengers were in an uproar, when someone called "Here's an officer, let him attend to it." So with the aid of a couple of privates, I had to take part in getting these men into their seats, a task which we succeeded in accomplishing, thanks to the respect inspired by my uniform, and a certain muscular efficiency which the privates showed. We later succeeded in getting the drunks off the train.

Another incident, which has elements almost as heroic, occurred one night last summer when I was on the way from Washington to New York. I stopped off in Baltimore, for no ulterior or interior reasons, although it might have looked as if I had such motives from the fact that I missed the train at Baltimore and had to sit down in the station and wait for another train. In order to utilize my time, I took out my brief case and began writing. In doing so, I pulled my fountain pen from my pocket, and the cap fell off on the floor. Before I had time to recover it, an elderly lady, about sixty years

old, rushed from her seat and brought it back to me saying, "Oh, let me do something for a soldier."

To experiences like these should be added the sweet delight which a soldier hero feels as he passes through Indiana and Illinois (as I did yesterday) of not only being permitted, but of actually being forced to take from fair young hands reached up to the windows of the train, sandwiches made of thin slices of bread and thick slices of chicken, with a dill pickle added, all free to the man in uniform.

But the tragedies and the comedies of the war are ended. To use the memorable words of the official announcement of the British War Office—words so brief, so modest, so tactful that they contrast strangely with the vulgar boastings of both our press and our statesmen. "In the fifty-second month of a war without precedent in history, the French Army, aided by the Allies, has achieved the defeat of the enemy." And with the ending of the war all of our war activities either have ceased already or will soon do so.

But there are things that the war has done which should not, which fortunately will not, cease. Men like Dr. Whitney and like myself have been pulled out of the ruts in which our lives had been running and have been thrown into completely new surroundings; men from England have come to this country to teach our young soldiers, and to participate in our war researches; millions of our own men have gone to France and Italy, and to England, so that, if ever there was a time in the history of the world, when men were in a position to begin to see things as the other fellow sees them, to get a little larger outlook than they normally have, that time is surely the present. The country is just now in a state of fluidity, and before the crystallization process has set in great things may be accomplished. Movements which the inertia of mankind make it normally impossible to bring to fruition in any reasonable time, can now be put across with relative ease; for we are all seeing how things look from some other angle than that from which we have been in the habit of looking at them. So that the next few months, so it seems to me, are going to be pregnant months for the history of the United States and of the world.

I wish, then, to call attention to a few of the lessons which from my point of view, the war has taught, and I trust I shall be able to do something more than to make a mere academic

recital of them, for I hope that I may bring something before you which will inspire to action.

In the research activities in the aid of the war in which I have been associated, one thing has stood out exceedingly strikingly; namely, that the distinctions between the man whom you commonly call the pure scientist and the man whom you call the applied scientist have absolutely disappeared. In every one of the eighty or ninety researches with which I have had connection there have been associated physicists from the colleges, engineers from the General Electric Company, the Westinghouse Electric and Manufacturing Company, the Western Electric Company, and other companies, and they have all worked as one, and you could no more make one of these men say, "This problem belongs to physicists," or, "This belongs to engineers," than you could say that a man's head belongs to one being and his body to another. The pure and applied scientists have functioned as one single group as they have never done before, and I hope very much that we may be able to do something right now to keep bridged the chasm between the academic group and the industrial group. This chasm is unquestionably a detriment to both groups and an effective bar to progress—a bar which Germany had removed in some fields, at least, much better than we have. If the process of forming societies has already gone so far in this country that for the purposes of organization and classification—which are to be looked upon always as necessary evils—the Physical Society and the Engineering Societies must continue to exist as distinct units, at any rate I certainly feel that it is possible to bring about such an interlocking of committees within these societies that the two organizations may act in many respects as one, and I hope that this possibility will soon become a reality.

You noticed in Dr. Whitney's address that engineering research was defined as "anything which had to do with matter." That is fairly broad, isn't it, and it wipes out completely all of these distinctions about which I have been speaking. I am quite willing to accept his definition, and I would not object to being called an engineer if that is what it means to be one. As a matter of fact, I presume I should have had more satisfaction in my war work if I could have been called an engineer instead of a "damn scientist." You certainly cannot draw any distinction between scientists and engineers on the basis of the utilities of their work, and you all know that you cannot.

I know of hardly an investigation in so-called pure science which has not sooner or later found its utility, nor do I know of any so-called pure scientist whose motives are not utilitarian, in the sense that he believes that he is doing something which will sooner or later contribute to human good. But what I am aiming at is after all not mere words. A change of name alone is of no importance. What I want is rather an actual fusion of groups of men who are now further apart than they ought to be.

My second lesson is one of even greater importance. If there is anything which has been demonstrated by the experiences not only of America, but of all other countries in this war, it is that one cannot safely depend upon the so-called undirected inventive genius of a people to obtain large results. Advances have been made in this war almost exclusively by one method, namely that of bringing together groups of well trained thoroughly competent men and setting them upon the problems needing solution. In other words large and rapid progress has been made in this war and in my judgment it can in general be made only by adopting the strictly research method of approach. In this war every one of the major belligerent countries has had its Board of Inventions, and it has been the uniform experience in all of them, in Italy, France, England, and the United States, that not one suggestion in ten thousand which is of any value comes in through these bodies. Further, the chance is about 100 to 1 that the good suggestion which does occasionally come in this way has already been thought of, and worked upon, by the groups of trained men who have been giving their attention and skill exclusively to these research matters.

But how much in the past have we used this method? How much have we done in this country during the last hundred years in organizing such research groups? Dr. Remsen told me the other day that in 1872, when he was first called to Johns Hopkins, there was not a place in the United States where there was any research atmosphere or where any serious research work was being done. Perhaps some one may wish to raise a question as to the literal accuracy of that statement, but on the whole it gives a correct picture of the research development and the research interest in this country at the time of which Dr. Remsen spoke. I would not cast any dimness over the luster of names like those of Franklin and Henry and men of that type whom this country had produced

before that time, but, after all, these were individual geniuses. They were not the leaders of research groups and yet they accomplished wonderful things. But we have passed from the stage of the world's development in which we could depend solely on individual geniuses for advancing the wheels of progress. If these wheels are to move forward rapidly we must do in peace exactly as we have done in this war, namely, call together groups of research men in our industries, in our educational institutions—I do not care where—give them the problems, and let them push them forward. In this way these wheels will move forward ten times as rapidly, yes, a hundred times as rapidly, as will be the case if we depend upon the exceptional inventive genius of the country, however valuable that may have been to us in the past.

These are the two lessons which I would draw from the war.

Let me next ask you to consider how we may profit by them, and perhaps the path which we ought to take may be made more clear if we analyze ourselves somewhat objectively, and try to look, for the time being at least, not at our virtues but at our weaknesses, however disagreeable such introspection may be. Let me begin by asserting that in spite of our boasted inventiveness, we are as yet, from the scientific standpoint, a second rate people, whether we look at our accomplishments in war or in peace. I am certainly well within the bounds of the truth when I say that the contributions of the United States to this war in research and development lines have been less, far less, than is proportionate to our resources and our population. I think Dr. Whitney will agree that that is not an overstatement, and he knows fairly well what we have been doing, for he has been associated with much of it. England and France have both done far, far more effective war research than we have done. True, they have had more time, but they have not had either the men or the facilities which we have had. With almost every available man in France at the front, with most of her manufacturing facilities in the hands of the enemy, is it not an extraordinary tribute to the French scientific mind that the two most novel and most promising weapons which we had at the close of the war for combating the submarine came out of France. We have done some creditable work in this country on submarine detection, but in comparison with the number of men engaged on the work and the facilities at command, both France and England did vastly more than we. The other day an officer who had

had exceptional opportunity to appraise our contribution to the war was in my office and I was bewailing the fact that a particular device which we had been developing had not been completed in time to be of actual use on the front. But said I—"I do not know that I ought to complain, because I presume that it is in the same boat with approximately 96 per cent of all American effort." Said he—"I think you overestimate the amount that got across."

Now that is an impression which one does not gain from the newspapers. I would not unduly belittle the American effort, but I would like to do something to combat the lethal poison which is being introduced into the American system by the vain-glorious boasting in which our newspapers and politicians are indulging. We did indeed cast the deciding vote. It is true that save for us the Kaiser might not perhaps have been so easily persuaded to call out "Don't shoot Boss, I'll come down," but let us at least remember that 20 Frenchmen gave up their lives for every American boy who died, that more than ten Englishmen "died for France" to one American. These figures show the real pressure of our contribution, and let us not forget them.

Again I state that whether one measures our position by our contributions to the war, or, what is more to the point to-night, by our accomplishments in science in times of peace, I think that every American who is perfectly honest with himself will admit that we have no claim as yet to being anything more than a second rate people. Second rate whether you take as the criterion of first rate-ness either the number of men of outstanding ability and reputation which we possess, or the total output of the country in proportion to our population, or the general appreciation and respect in which science is held in the community at large. Look at the first criterion; who would for a moment claim that we can show a list of names comparable with these? Faraday, Maxwell, Kelvin, Stokes, Rayleigh, Thomson, Larmor, Schuster, Rutherford, Jeans. Or going to the field of chemistry, in which we have done as well perhaps as in any science, who would undertake to present such a list of illustrious American names as that which Dr. Whitney has just presented for Germany?

When we look at the second criterion and reflect that we have 100,000,000 population against England's 45,000,000 and Holland's 4,000,000, no man who is at all familiar with our relative outputs would claim for a moment our equality with

either of them. It will be generally admitted that as measured by this criterion we cannot think of claiming to be in the first rank. Such a statement may not be palatable to our pride, but it is true in spite of the fact that you see so much in our newspapers and elsewhere which might lead you to think otherwise. Here is something fairly typical. There is a tablet erected in Dayton, Ohio, which gives a list of twenty of the world's greatest inventions, and attributes thirteen of them to the United States, just one to England, the locomotive, one to Scotland, the steam engine, two to Germany, the Fahrenheit thermometer and the X-Ray, only one to France, the daguerreotype, but two to Italy, the pendulum clock and wireless, and two to Dayton, the cash register and the airplane. Such claims need no comment. Unfortunately they are more or less typical. I wish they were not typical, and perhaps we may do something to make them less so.

As for the third criterion, the public appreciation of science in any country may be gaged first by the general respect and honor in which the calling of the scientist is held, and second by the character of the general scientific journals which that country supports. Now it is a matter of common knowledge that in all European countries the scientist is held in greater respect than in this country, and as for journals, let me just mention the names of three English publications in different fields and see what kind of comparisons they bring up in your minds. The *Philosophical Magazine, The London Electrician and Nature.* Have we journals which are of a full equality with any of these?

I am presenting these unpleasant truths not because I think our situation is hopeless, but just exactly because I think these conditions are remediable, and because I think that you and I can assist in changing them. I say them because I believe in all my heart in the effectiveness in the long run of our American democracy. I say them because I believe with all my heart that when you once get the public opinion of the United States aroused it will set about changing bad conditions and will do it rapidly. You and I have already seen it doing certain things which a few years ago nobody believed a democracy could do. We have seen it handling the liquor problem for example in a better way, as it seems to me, than that in which it is being handled in any country in the world, and this is the last problem which you would expect that a democracy could handle. I am pointing out conditions which ought not to exist because I

believe at the bottom of my heart that our method, the democratic method, is capable of rectifying bad conditions more rapidly on the whole than is any other method, if only we can once get the real situation and the real need before the consciousness of the American people.

What we need then if we would become a first-rate rather than a second-rate people from the standpoint of our scientific development is, first, the development of more highly trained and more able scientific personnel in this country than we now have. Second, we need the development of a new public opinion on scientific matters. We need appreciation of what science is and what it can do. We do not need any more so-called "Popular Science Monthlys," but we do need a better educated, scientific intelligence than we have at the present time.

How are we going to get these things? Dr. Whitney has said that we have enough organizations. Maybe we have, at any rate we have not enough of concerted action among existing organizations. This is precisely what the National Research Council is endeavoring now to bring about, and if the Research Council cannot succeed in this work which it has been organized to do, then some other organization must arise which will do it. In other words, we must in some way get the scientific men of the country pulling together. I think this can be done and that it is the biggest, most worth while enterprise which any of us have ever had an opportunity to take part in. The problem is to persuade the engineering societies, and the physical societies, and the biological societies, and the medical fraternities, no longer to work all by themselves, but instead to pull all together in one common effort, first to educate the American public to a larger appreciation of the possibilities of science, second, to make the present American scientist more effective than he now is, and third and most important, to find means by which we may develop more and better scientific men.

The education of the public is of course a slow process, but it can be accomplished by directed effort and concerted action such as the Council is now attempting to bring about. The stimulation to greater effectiveness of existing scientific men also cannot fail to result from all of these new activities. But the producing of a continuous new crop of able scientific men is a problem to which I should like to give a little more detailed consideration. *The result aimed at can be attained only through the agency of the American Universities.* These however can-

not possibly make the kind of men Dr. Whitney wants out of poor material. Something can be done to a man by education alone, and yet there is no one who knows as well as the college professor how little merely going through the routine of getting an education amounts to. Our primary need is to find a way of getting a better *selection* of men than we now have, that is, a way of diverting the brains that now exist in the country into research channels. How is that to be done? The answer is to my mind very direct though perhaps not so simple, since it involves funds. It is to deliberately and consciously set about the task of creating more opportunities for research careers than we now have, and giving greater dignity to such careers. In connection with American Universities, I see ways in which this can be done, first by persuading philanthropic men and institutions to endow a considerable number of scholarships similar to the so-called 1851 Exhibition Scholarship of the British Universities, second, by the creation through private means of many more partial research professorships than we now have. I say partial research professorships because I think that the research man needs the broadening influence of a small amount of advanced teaching even for the greatest effectiveness in his research work. The creation of more pure research institutes like the Rockefeller Institute for Medical Research or the various research laboratories of the Carnegie Institution of Washington, may also be encouraged though I am myself a strong believer in building up research enterprises in connection with educational institutions so that the atmosphere of research may be inhaled by the young blood which is continually passing through these institutions. Create a real research atmosphere in a score of American Universities and a new generation of research men will quickly be created which will place America in the front rank in the fields of science. I should regard it as a calamity if the research funds available in America went exclusively or even primarily into the founding of central research laboratories. This country already has scores of magnificently equipped laboratories which produce one tenth as much as European laboratories one fourth as well equipped. We do not want more brick and mortar, more physical facilities, *we want more and better men.* As Dr. Whitney has so well said, *we want a man-producing program, not merely a man-consuming program. The man-producing agencies are primarily the universities.*

Funds which are donated to colleges or universities for

research work should be devoted by the terms of gift to research purposes and made altogether undivertable into the purely educational budget. One of the chief aims of the Research Council has been from the start to stimulate this sort of giving. The finest outgrowth of our democratic, individualistic civilization has been the development of a great multitude of public spirited men who devote their means to public ends and it will probably continue to be true in the future as it has in the past that our biggest and most important public enterprises will be inaugurated not by public but by private initiative. I see no reason therefore for not using the method which has been most successful in the past in many fields and which is most characteristic of the genius of Anglo-Saxon institutions —viz., the method of the private foundation of research scholarships and professorships.

But I agree with Dr Whitney that in a country like this, there is no reason why the federal government, as well as private persons and institutions, should not promote the development of research. The whole world has been aroused by the war to the importance of this sort of activity. England has appropriated governmental funds for scientific and industrial research, and Canada has done the same, and so has Australia, and Italy, and I think Japan also. The United States is almost the only large country which has not thus far done anything in this direction. But there is now before Congress a bill—the so-called Smith-Howard Bill—which in its original form had most unfortunate features, but which is apparently now in process of being changed into a really valuable measure. I am permitted to say that the authors and sponsors of this bill recently asked the National Research Council to appoint a committee to give it thorough study and make possible recommendations. A fairly representative committee of some ten men, including Dr. Whitney and the Director of the Bureau of Standards and the author of the bill, Mr. P. V. Stephens, has met and reported unanimously upon certain types of modifications in that bill. The most essential of these changes consist in the setting up of the principle, different from that incorporated in the original bill, that the funds appropriated by the federal government to each state shall be allotted to institutions within the state *by a board composed of at least five of the most eminent available scientists and engineers of the state, upon the basis of programs of research submitted annually by these institutions to this scientific board.* It was

the judgment of the Committee, and also of the executive board of the National Research Council which unanimously approved this report, that if the allottment of research funds within each state can be placed in the hands of men who know what research is, and if the institutions of a state can be stimulated to a rivalry in the development of research programs there will develop in existing laboratories of which we have already an abundance, an atmosphere of research which is now wanting. *It is only through the creation of such atmospheres that research men can be developed.* This sort of a plan seems to make it possible to plant men all over the country who will be creating such atmospheres. It provides for the stimulation of research throughout the country in both industrial and pure science lines much better than any scheme for federal aid which I have seen discussed.

There is very grave danger, however, that the desire of public officials, legislators and private individuals to leave brick and mortar *monuments* to themselves will divert funds which ought to go to the development of men to the creation of more elaborately equipped laboratories without men, such as already constitute a reflection upon American judgment and American brains. If the Smith-Howard Bill modified in some such vital way as I have indicated does not pass then I have little doubt that the National Research Council will in time itself establish within the states research boards which may then obtain state funds to be used in some such way. California has already such a State Board, working in cooperation with the National Research Council and this board has within the past year received and appropriated more than $50,000 for research within California.

I have said enough to suggest many ways in which research in industrial and pure science lines is likely to be stimulated in the near future. Let me close with a brief word on the organization of the National Research Council which is occupying itself with these large problems of the promotion of research in the United States by a method which is compatible with the genius of our Anglo-Saxon institutions, namely, by the democratic method of the voluntary associations of research men, societies and institutions into a great body which can effectively create public sentiment and stimulate in a large way research enterprises.

The Council is, in a word, simply a federation of the research organizations and activities of the United States for the purpose

of obtaining concerted action, mapping out new lines of progress, establishing the cross connections between different fields, acting as a central bureau of information and coordination, and promoting research in general. It has for the past year had a so-called war organization, because one of the most important problems which confronted our country was to get the scientific and engineering brains of the country effectively at work upon its war problems. The National Research Council has been one of the influential and effective agencies which has been doing this and it has thrown all its energies into the task. Now the time has come when it must reorganize so as to adapt itself as completely as possible to the problems of peace, which are more important even than those of war. The present tentative plan is to have the Council a thoroughly representative body in that at least a majority of its membership is essentially elected by the constitutent societies for a period of say three years. Each of the seven or eight divisions, namely, the division of Engineering, the division of Physics, Geophysics, Astronomy, Mathematics, the division of Chemistry and Chemical Technology, the division of Medicine and related sciences, the division of Biology, the division of Geology and related sciences, etc., to consist of 25 to 35 men presided over by a chairman elected by the group. The most essential feature of the proposed organization is the Executive Board which consists of these eight chairmen, eight vice-chairman and say ten members at large. The Board would meet at least once a month in Washington. The eight chairmen are to give up, in general, all other activities for their term of office, of probably a year's duration, to be paid suitable salaries, say $6,000 a year, are to be resident in Washington, and to devote their whole time while in office to the promotion of research in the United States. With eight of the best and most influential scientific men in the country devoting themselves each year to the large problems of surveying, stimulating, and coordinating the research activities of the country, the United States can I believe very soon be placed in the front of the nations of the earth in scientific discovery and its application to the arts and industries.

The National Research Council bespeaks your active cooperation and support in this great new democratic and characteristically American program.

DISCUSSION ON "AMERICAN ENGINEERING RESEARCH" (WHITNEY) AND "RESEARCH IN AMERICA AFTER THE WAR" (MILLIKAN), PHILADELPHIA, PA., DECEMBER 13, 1918.

Chas. E. Skinner: I made a plea from this very platform, not a great while ago, for the training of research men, and if iteration and re-iteration of the necessity of the case will help any, I am glad to lend my voice to a further plea along the lines that have been so ably indicated here this evening by Dr. Whitney and Col. Millikan.

I made a plea for the training of research men in the universities. I did not have at that time a well-organized plan in my mind for carrying out such training but plans are now well under way by the National Research council.

I have been asked a number of times recently to lend my support to the Smith-Howard bill. I have not been entirely satisfied with the bill, as presented in the early drafts because I felt that there would be a very considerable waste of money in arbitrarily selecting certain places and giving those places a fixed amount of money without any program to work on. I am more than pleased to hear Col. Millikan outline certain modifications which have been proposed to the bill which completely remove any objections along that line which I may have had in the past.

I have felt throughout the war that our program of research after the war was possibly of more importance to us than our program during the war, if we are to hold our place among the nations of the world. To carry out that program we need, above all things, a knowledge on the part of our people as to what properly directed research can accomplish. We need trained research men in all phases of our work, and particularly in connection with engineering research. If the desired results can be brought about by the plans proposed here this evening and I believe they can, these plans should have our heartiest support. In closing I wish to express my admiration at the way this subject has been presented, and to say that I feel we are, well on the road to establishing a plan of action for caring for this very important matter.

E. E. F. Creighton: There is nothing so valuable to our country at the present time as the consummation of the plans involved in the presentations of our eminent speakers this evening. Both Dr. Whitney and Col. Millikan have been able by their own efforts to show the enormous value and wealth to the country that accrue from research. So much can be obtained in this line of endeavor that the proposed appropriations of Congress seem far too small. Dr. Whitney has shown in more than a few instances that pure research will lead to the most valuable results for the whole of civilization. The mention of even one of them argues completely the justification of research: Langmuir's gas-filled lamp and the resulting saving in coal, or Coolidge's X-Ray tube and its benefits to the medical profession,

to say nothing of its uses in other fields. In some cases, I understand, years of pure research were carried on before a utilitarian result was reached.

The subject on which I wish to make my appeal I surmise has been characterized in Col. Millikan's great contribution to the electron theory in the mental processes preceding the counting of these tiny particles one by one. It appears to me like an invention to conceive of the mechanism of a floating particle of oil which, by the natural movements of electrons, would finally be struck and electrified. The presence of the electron was made known by observing the particle of floating oil between two plates which could be electrically changed in direction and intensity as desired. How much research preceded this invention we do not know, but it would seem that the valuable part of the research succeeded it. At any rate, the assumption that the invention could precede all of the research, the invention being founded on well-known principles of attraction and repulsion in the electrostatic field, illustrates the point I have in mind.

My interest for many years was in the development of invention as an activity distinct from research, although not separated from it. One may approach a problem from either the purely research standpoint or from an inventive standpoint. Research is seldom useless in that, if carried on properly, it gives more information on the laws of nature or on the characteristic of some material, but it is liable to be mostly lost unless it leads to some immediate useful end. On the other hand, invention is, in general, futile unless accompanied by a considerable amount of research. This would easily account for the low percentage of suggestions by inventors which proved valuable to the Naval Consulting Board. If one approaches the problem with an inclination toward the process of mind of invention before any great amount of research, one invention in ten would be a high percentage of success, but since the nine may be discarded with brief analysis, rapid advancement is possible. On one hand, in the extreme, there may be a blind research for whatever results may come from the investigation. On the other hand, research may be carried along specific lines to discover materials or characteristics of materials or laws which will fulfil the conditions of the invention. The difference between the two is the efficiency of directed force as compared to more or less chaotic force.

While America's list of workers in the pure research is comparatively small and our inventors have lost much by not harnessing research to their inventions, still we have much to be proud of in the line of invention. It is necessary only to mention the name of the greatest of all our inventors—Mr. Edison. Mr. Edison has done much research but as I understand his work, his research was usually preceded by an actual invention. It is customary to think and speak of Mr. Edison as

a genius, and the methods used by him unavailable to the rest of us. While it may be a long time before society is fortunate enough to find again the combination of ingenuity, interest, perseverance, enthusiasm, and assiduous application that has made it possible for Mr. Edison to so enrich the world, I can state by actual experience there is no reason why the methods cannot be taught to any student with the taste and the desire to become an inventor, with the results as good, on the average, I think, as obtained in the other professions. In brief, I believe inventing is just as much a profession as medicine, or law, or engineering. Invention results exactly and naturally from a particular view point or cultivated attitude of mind. There is much more which should be said of details.

The remarks of the speakers of the evening bring forth the plea for the desirability of drawing into the proposed research laboratories that body of chaotic American inventors who need the stabilizing effect of the exact knowledge of research; furthermore, to cultivate among the research students the utilitarian spirit which leads to invention.

P. G. Agnew: Coming from a National Laboratory, I can only say I hope some mechanism can be evolved for closer cooperation between the research agencies and these National Laboratories.

C. H. Sharp: I want to emphasize the point which has been brought out by the two eminent speakers of the evening, that research is more a matter of men than it is of material means. A great research worker can get results with inferior means that an inferior research worker cannot get with the best means in the world at his disposal. As Dr. Whitney said, we have in this country laboratories more splendid than can be found elsewhere, but we are not turning out men to get the results out of these laboratories.

An effort towards education of research men and towards stimulation of research work is being made in the Smith-Howard bill. The amendments which Dr. Millikan tells us have been proposed by the National Research Council to this bill have removed some of the very serious objections to that bill. We should bear in mind that while this bill calls for an appropriation of $15,000 per year to each state and territory, that figure obtains only for one year. It is increased year by year until in the fourth year, it becomes $30,000 per state and territory, and remains at that figure.

The proposition is, then, to subsidize each of the states and territories in the amount of $30,000 per year. This amounts to something over one million and a half dollars for the country.

Now, that is really not an excessive amount for the Federal Government to appropriate to this purpose, if there is a reasonable prospect that it will be wisely and economically expended. The British Government has recently appropriated One Million Pounds, Sterling, for the stimulation of research during the

year. However, the one million pounds includes the support of the National Physical Laboratory which the British Government has recently taken over, and the proposition in England is in other respects not parallel with the one in this country. It is proposed over there to subsidize individual research workers, men who show competency and talent for research, and thus give them an adequate training and a chance to develop their abilities. It is also proposed to subsidize Research Associations organized by various industries and chartered by the Board of Trade for the purpose of carrying out in cooperation, researches covering the great problems which are common to the industry as a whole.

Moreover the British Government is appropriating this money on top of a huge war debt and in the face of enormous taxes, such as we do not know and probably never shall know, in this country. A million and a half dollars therefore while by no means excessive is enough money to make it worth our while to see that it is expended in the way which will yield the best results.

A feature of this bill is that it denominates the Bureau of Standards as the co-ordinating agency for the direction of the research work in these various institutions which are so subsidized. This throws upon the Bureau of Standards an enormous responsibility, a responsibility which was never anticipated when the Bureau of Standards was organized, and which comes to it on top of the other greatly increased labors which have accrued to it as a result of the development during the years which have elapsed since the time of its organization.

Of course, this proposal at once suggests the thought that the Bureau of Standards thereby becomes the all-powerful agency in directing research in this country. This being the case, it is necessary that we should inquire whether this agency has directed its researches along such lines, and will probably direct the research contemplated on the Smith-Howard bill along such lines, as to yield the most useful results for the public which it is intended to benefit. This is a very serious question.

Now, we are all very strong supporters of the Bureau of Standards. We believe that it is an institution of inestimable value in this country as a scientific agency. As such we want to see it prosper and grow great, and we rejoice in the added prosperity which has come to it in recent years. I think, however, some of us would feel a little better about it, if we had a little more assurance that the lines of progress of the Bureau, that is to say, the lines which it attacks in its endeavors, the research problems which it takes up, are problems which are such, as it, of all agencies, is the most competent to tackle, problems which it alone of all agencies is able to solve, and that it is never wasting its time or its endeavor in assailing problems which can be assailed, perhaps, almost as well by other agencies; that it is maintaining its original status as a scientific institution, and is

not diverting its energies into other channels which, however important they may be in themselves, cannot be considered to lie in the domain of the physical sciences to which, as we conceive it, the activities of a Bureau of Standards should be confined. Certainly, if this added responsibility is to be put upon the Bureau of Standards, we ought to have some assurance along these lines, such as we have not had in the past.

To state the matter in general terms, when the future of this country as far as the guidance of industrial research is concerned is entrusted, to any agency, we who represent engineering and industry must make sure that safeguards are established such that there is no chance for this agency to fail to perform its functions to the most advantageous result for the public good.

E. P. Hyde: I was very delighted to hear Col. Millikan speak of the closer cooperation between the physical societies and the engineering societies. That has been a hobby of mine. It was not always a hobby. I grew up in an institution, the Johns Hopkins University, to which Col. Millikan referred in his address, in which pure science was placed on a very sacred pedestal, and the first two or three years of my connection with the Bureau of Standards were a veritable hell, because I was called upon to do something that was of some use to somebody, and I was prompted to go off and do something for the sake of pure science. I felt for a long while that the physical society was a group of moss-backs, who gathered periodically in some cobwebby room, in some University to discuss academic scientific subjects, and I think in a small way I was partly responsible for having the Physical Society break away from that custom, and meet once at an industrial institution, and that proved so satisfactory that they planned to do it each year, and I am only too sorry that some situation prevented the program which had been planned for a joint meeting of the American Physical Society with the American Institute of Electrical Engineers last October.

The second thing to which I would like to refer is getting the proper men to develop into research men. I do not think we would expect to draw strong men into a line of work which offers neither remuneration nor appreciation. This may not be the place to say this, but I am not at present connected with an educational institution, and therefore I am free to speak.

I think a thing we can all do,—and I am talking to men who are not in the universities now—is to help place scientific research in our institutions on a higher plane, and if we cannot induce the strongest men to go into research work, in the first place, as leaders, as guides, as pioneers, to inspire young men to follow in their footsteps, we cannot expect to get strong men to engage in research undertakings.

The process is cumulative. If we have no appreciation and no remuneration that is commensurate with the work that is

done, we will get comparatively weak men in our institutions, and the prevalence of these weak men in the institutions will tend to degrade the men who enter upon research work as a life work. We must increase the appreciation, and I think we must increase the income, incident to this work. As I say, it is not our place to do that, it is not our responsibility, possibly, but I have felt some of the responsibility, and I think, if more of us felt some of that responsibility as individuals, if we saw to it that there was more remuneration for men who do that work, and if we saw to it that there was great appreciation of what they did, we would only be performing our duty.

Can you imagine this situation occurring in this country. It happened to me in Karlsruhe. I had made an engagement with Herr Gehimrat Professor Doctor Bunte, Technische Hochschule, at Karlsruhe. I had made no provision for hotel accommodations, and when I reached the town in the evening I found there was some fete in progress and the hotel was filled. I secured an attic room that must have been used for the lowest grade of menial. It had absolutely no conveniences. I had to call over the banister for a maid if I wanted a glass of water. The next morning after breakfast I went to the laboratory of the Hochschule. I was met by a rather austere gentleman who wanted to know my business. I told him that Doctor Bunte had made an engagement to see me, and as I was at the hotel I would see him there if that was agreeable to him. I went back to the hotel and I said to the manager, Herr Professor Doctor Bunte is going to meet me here, is there some place where I can receive him, and he showed me into a private sitting room with a great deal of deference, and when in a few moments Doctor Bunte came, the proprietor at the hotel was at the entrance, to receive the doctor, and was making obeisance to the doctor all the way coming in, and the doctor was poking him aside with his cane, and after Doctor Bunte left, the proprietor of the hotel found that he had a very good room on the *premiere etage*. for me. Can you imagine anything like that happening in any of our small towns to which Dr. Whitney referred to, such as Columbus or Princeton, or anywhere else. It indicates a great deal, I think, and we must in this country establish that appreciation, and with that must come reasonable remuneration for our men who are training the young men, a reasonable remuneration for those men who are to inspire the young men in research work, if we are to hope to accomplish what has been accomplished in Germany, and, of course, it is our hope to go far beyond that.

L. T. Robinson: The Smith-Howard Bill which has been referred to seems to me to be a good move in the right direction. I say this because I did not know until tonight that it had been modified or that a somewhat similar bill had been prepared by others. It is the general idea that appeals to me as being good.

Such an arrangement will attract the right kind of professors and give them opportunities to carry on much research work

that could not be supported by present methods of financing. The arrangement will also be of benefit to students.

The only fear seems to be that the bill contemplates giving the Bureau of Standards too much responsibility.

F. A. Millikan: That feature was eliminated at the request of Dr. Stratton.

L. T. Robinson: I am rather sorry for that because I have confidence in the Bureau. I think all of us should have confidence in it. There must be some directing and coordinating force somewhere.

However, I do appreciate the fact that when you make the Bureau responsible for coordinating all this proposed work, that you are placing a tremendous burden of responsibility on the organization.

It seems to me that if the Bureau is to be the agency to do this thing—and I can think of no better one—we ought to make an effort to help them.

The arrangement that suggests itself to my mind is that an advisory board consisting of representatives from the four National Engineering Societies, The American Physical Society (which has tonight become an engineering society), The American Chemical Society (which is also an engineering society as defined tonight), together with representatives of the industries and business—covering the whole field that is and should be served by the Bureau of Standards—be created in some way. Probably a good way would be for the President or Secretary of Commerce to appoint representatives nominated by the various societies.

I am assuming that some such arrangement would be agreeable to all parties concerned. Such a board might be a branch of the National Research Council or perhaps it might act in the same capacity to the National Research Council and to the Bureau. The details of the arrangement have not occurred to me and are, at this time, relatively unimportant. I can at this time only express my belief that a non-partisan, and non-political advisory board created from engineering and scientific societies from the industries, public utilities, etc., could be formed that would be productive of great benefit to the Bureau and to the country as a whole.

We have seen recently in this country good results, accomplished by boards appointed almost haphazard. If you can get results with boards appointed in that way, how much better results could you get with a board created by nomination from the national engineering, technical and business associations, to help this great movement along and to support and advise the Bureau whose work we all regard so highly.

M. W. Franklin: There are three points in the addresses of Dr. Whitney and Prof. Millikan which have interested me particularly. In the first place, I refer to Dr. Whitney's disclaimer of any desire to form a board, a new committee, to take hold of this matter, and the assumption is made that there are enough committees already in existence.

In the second place, I want to refer to the premise of Prof. Millikan that the research of the present day is becoming pragmatic—practical—that is, it is having a utilitarian purpose, and that this is dominating the present-day research, and to a good end, and the other remark of Prof. Millikan, that he prefers the democratic method as against the autocratic method, and thinks it will produce better results.

I happen to be in a position to say something very cogent on the subject of research in this country at this time, and because of the interest that these three points have had for me, I am saying it. There is an engineering society in this country, if not the most obscure, perhaps one of the most obscure, a society with about 750 members, a society which until within the last five years, has been scarcely an engineering society in anything but name a society which had its inception in an aggregation of salesmen of a utilitarian article, and which has been struggling along but which has done something in connection with this matter of research recently, that merits your attention and your commendation, because I think it is a model for all the greater societies on which they can base their endeavors.

I refer to the American Society of Heating and Ventilating Engineers. I happen to be a member of that society. I say that advisedly—there was a time in my ill-starred past when I had some vicarious connection with the ventilating business—I do not boast about it—and I got into the society at that time and in spite of several resignations presented to the society, I am still a member of it. In fact, I have just been nominated as one of the vice-presidents, and in the course of events, in the irony of fate, I will be the president of the society if I do not die or get fired. I think it was due to another activity, equally unfortunate, in which I was interested, the institution of the research movement.

There was a group of boiler salesmen in the first place, who made up that society and who did not know anything about research matters. They could measure a house and guess how much radiation surface was needed. They carried a two-foot rule, and they carried some pipe and some radiator units, and one thing and the other, and they were great engineers, or thought they were.

About five years ago I got very busy in connection with the subject of research, I do not know why, and they took it seriously and got started on the research proposition.

Recently, since the war, on account of the different bureaus constructing battleships and barracks, and other things, in which human beings have to live for a certain length of time, they discovered that they needed something like precise knowledge on the subject of ventilation. They found that there was an enormous amount of data on the subject of ventilation, and that most of it was incorrect. That was discovered by accident when they began to ask different experts how to construct the

living spaces in battleships and barracks, and they found also that there were plenty of opinions, no two of which coincided in any respect whatever.

So then the American Society of Heating and Ventilating Engineers found itself in a very unenviable position—you might say in an equivocal position, they were looked up to and bowed down to, but in the secret recesses of their own hearts, they realized that it was camouflage in the most fully developed sense of the term.

They got together this research data, which had died a natural death sometime before, and got busy on the subject of getting something that was reliable in the way of research, and they have organized a series of sections, one of which I have been called upon to organize myself, and they have appointed a committee in each one of the twenty-nine sections or local chapters, as they call them. I will repeat, that the whole membership in the aggregate amounts to fewer than 800 persons.

They had organized in each of the 29 chapters a Local Research Committee, and each Local Research Committee went out to get the money and to get something done in the way of research, that will put on the records for all time reliable information about ventilating subjects, which are important subjects, and in connection with which this research study is especially valuable.

I was much interested in what Mr. Louis Robinson just said and I cannot help but admire his practical good sense.

These committees to which I have referred got together, in the first place, on the pragmatic basis, the practical basis, and they said:—"Let us do something that will be of value to somebody, and let us do it in a practical way. We will get together and make a list of all the manufacturers of apparatus in the United States, who can even remotely be interested in processes of manufacturing ventilating equipment, and in precise knowledge on the subject of heating and ventilating. We will get together, all the engineers in the United States, who remotely, at least, can be interested in this subject, always financially interested, don't forget that. That is pragmatic, that is higher philosophy according to Ole Yonson.

Then they said:—"Let us get together all the contracting engineers who can be even remotely interested in operations in which heating and ventilating play a part. We will make a list and divide them up according to locality. We will 'sic' the members of these societies on these men, and make them sign something which looks like a liberty bond voucher, and make them sign it to be good for a period of five years, to support this research bureau."

We made arrangements at the Bureau of Mines at Pittsburg, which has put at the disposal of this little society its beautiful research laboratory and facilities free of charge, and they are delighted to have the members of the society come in and do

something. The members of this society are live wires. They get on the job. We call all the members in, and hold a meeting, and 80 per cent of our membership comes to that meeting. I will admit that the methods of getting the members to the meetings are sometimes questionable. On one occasion they promised the members that I was going to read a paper on "The Control of Fumes and Noxious Gases,"—a subject I know little about—and the members came, and we sprung this on them. I really intended to speak on the subject but when we got on Research there was no time for the paper. We took our apportionment cards with a list of some 400 firms on them, and we read off the names and made the members volunteer to take some of these cards and go to the firms and "get it," the idea being to: "get it," which is again pragmatic, the means being subsidiary in every case.

We scorned the easier method of getting these names and going to two or three of the larger firms and having them subsidize this project. We could have done that. I could have picked out five firms in the United States that would have given us $35,000 or $40,000 a year for five years to have the work done.

We preferred to choose the method of democratizing the scheme, and get everybody into it. Almost every member contributed ten dollars a year for five years, and some contributed more, and everybody did his bit, and even those who are remotely interested, from monetary points of view, if they happen to have knowledge of the subject, in a general way, they put something in, and we are going to appoint two research men, and we will be consumers of research men, and incidentally producers of research men, and we will put these men at the Bureau of Mines at Pittsburgh and they will do the research. Every one of these manufacturers will have, without fear or favor, the right in a democratic way to say:—"This is a great problem, something of great importance, the transmission of heat through buildings and material, the transmission of heat through covered pipes, the efficiencies of certain types of boilers, certain types of grates." Without fear or favor, they will present their views and preferences for research subjects, and these will be carried out to a hard-boiled finish, and the results will be printed by the American Society of Heating and Ventilating Engineers, and when these things are published, in the course of five years, this research bureau will be a big thing and a live thing, and the Government will be ashamed not to be behind it, and will subsidize it, and pay for all the great work it is going to do, and the American Society of Heating and Ventilating Engineers will get the credit for this work. "Out of the mouths of babes and sucklings,"—not meaning me—but the great American Society of Heating and Ventilating Engineers, you can get something.

C. A. Adams: Dr. Franklin's emphasis on the pragmatic or dollar and cents side of this subject recalls what I have said on

many similar occasions in the past, namely that whereas some manufacturers are inclined to look askance at cooperative research since they cannot control the results but must share them with the other cooperators, the gains certainly far outweigh any loss due to sharing the results. After the results have been obtained there still remains ample opportunity for competition of intelligence and resourcefulness in their application. The best minds will win out in any case. Moreover cooperative industrial research means not only a great saving to the individual manufacturer but also the possibility of getting together the foremost experts in each field and a vastly greater probability of successful outcome.

John B. Taylor: Since the war came on, there has been much talk of research, much more than before, and through the newspapers and popular magazines, the public have at least learned the word, even if they do not know what research is. They are asking now, "What is Research." and they are receiving many replies.

To my mind research is not a college degree, or a laboratory, nor an equipment. It is an attitude of mind. You can find that attitude of mind, of course, in the colleges and in the laboratories, where there is equipment, and you can also find it on the farm, and in other places.

G. Francis Gray (communicated after adjournment): The notable addresses to which we have listened will leave a profound impression on all who are interested in research either in pure science or in the industries, and the closeness of these two to each other has been strikingly indicated by the fact that in these addresses the pure-science representative has advocated the union of science and engineering while the director of a great industrial laboratory has plead the cause of pure science.

In the discussion of the Smith-Howard bill it seems to me there is danger of confusion as to the exact purpose of this particular legislation. The four things generally spoken of are

a. Benefit to the industries from the results of research.

b. The training of advanced college students to be research experts.

c. Stimulating popular appreciation of the value of research work.

d. Developing college professors who are primarily leaders in science rather than college executives or professional educators.

Obviously a scheme that is best for one of these is likely to be somewhat unsuitable for the others, and unless the issue is clearly defined, discussion is likely to be at cross purposes. It is my opinion that the practicable plan for the present is to concentrate on a program designed to definitely and promptly benefit the industries of the several States, for the reason that this can readily be done, and well done, with people already available. Moreover the process will at the same time serve to train men by experience in productive work, and will provide

the material for the most effective kind of publicity, since every business man definitely helped by the results of research work will become its strong advocate. As for developing the real leaders in science, that will be possible only when our colleges awake to their responsibility and when the positions they have to offer carry adequate salaries, freedom from academic duties and the social and business prestige they deserve. This bill alone can not hope to accomplish such a revolution in our present conditions. It seems much better therefor not to expect too much of it and not to hamper it by too ambitious plans, but to recognize its limitations, while appreciating its very real possibilities, and by concentration on a small and definite part of the big program really make a beginning at once along lines in which success is practically certain.

STANDARDIZATION RULES

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Approved by the Board of Directors, November 8, 1918

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OTHER APPROVED STANDARDIZATION RULES

The following resolution, adopted by the Standards Committee, was approved by the Board of Directors on April 14, 1916:

"The Standards Committee, with the approval of the Board of Directors, recommends the use of the following rules and standards as adopted by other societies. These have been formally presented to the Standards Committee by the societies concerned and are found not to be incompatible with the Standardization Rules."

Standardization of Service Requirements for Motors, as printed in the 1915 report of the National Electric Light Association.

Standardization of Sizes, Voltages and Taps for Transformers, as printed in the 1916 report of the Electrical Apparatus Committee of the National Electrical Light Association.

Standard Specifications for Magnetic Tests of Iron and Steel, of the American Society for Testing Materials.

Report of the Joint Rubber Insulation Committee, published in the April, 1917, PROCEEDINGS of the American Institute of Electrical Engineers.

Accuracy Specifications in Sections IV and V of the **Joint Meter Code** of the Association of Edison Illuminating Companies and of the National Electric Light Association.

Accuracy Specifications in Section II of Circular 56 of the Bureau of Standards entitled **Standards for Electric Service.**

Report of the **Boiler Code Committee** of the American Society of Mechanical Engineers.

Suggested Safety Rules for Installing and Using Electrical Equipment in Bituminous Coal Mines, issued as Technical Paper 138 by the Bureau of Mines.

PREFACE

In framing these rules, the chief purpose has been to define the terms and conditions which characterize the rating and behavior of electrical apparatus, with special reference to the conditions of acceptance tests.

It has not been the purpose of the rules to standardize the dimensions or details of construction of any apparatus, lest the progress of design and production should be hampered.

NOTE.

The Standards Committee takes this occasion to draw the attention of the membership to the value of suggestions based upon experience gained in the application of the Rules to general practise.

Any suggestions looking toward improvement in the Rules should be communicated to the Secretary of the Institute, for the guidance of the Standards Committee in the preparation of future editions.

COOPERATING SOCIETIES

The following societies directly and through the committees named, have given helpful cooperation in the preparation of these Rules:

American Society for Testing Materials,
Committee B-1.

Association of Edison Illuminating Companies,
Committee on Meters.

Illuminating Engineering Society,
Committee on Nomenclature and Standards.

Electric Power Club,
Committee on Engineering Recommendations; Standardization Committee.

National Electric Light Association
Committee on Meters.
Committee on Apparatus.

Association of Railway Electrical Engineers
Committee on Wires and Cables.

American Electric Railway Engineering Association,
Committees on Equipment and Distribution.

Institute of Radio Engineers,
Committee on Standardization.

Society of Automobile Engineers,
Standards Committee

Railway Signal Association.

SCOPE OF THE 1918 REVISION

This edition of the Standardization Rules does not differ radically from the 1916 edition, although a number of changes, deletions and additions have been made.

Historical matter which previously had not been carried beyond 1916 is omitted from this printing. There is also omitted the By-Laws of the Standards Committee of the A. I. E. E. as these are undergoing revision.

The chapter on **Transformer Connections** and **Markings of Leads,** (sections **600-617**) has been modified to accord with current American practise.

The chapter on **Controllers, Circuit Breakers, Switches, Fuses,** and **Accessories** (sections **700** *to* **744**) has been extended, rearranged, and considerably altered. The chapter on **Telephony** and **Telegraphy** (sections **910** *to* **963**) has been extended and revised. And, in addition to minor typographical corrections, there have been the following:

Revisions: Sections **17, 50, 52, 152, 164, 171, 172, 301, 307, 320, 348** (footnote), **405, 406, 502, 504, 765** (footnote), Table **III.**

Deletions: Sections **169, 170, 657, 1017.**

Additions: Sections **17a, 84, 85, 142** *to* **147c, 237** *to* **244, 407** *to* **409, 1034** *to* **1036.**

STANDARDIZATION RULES

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

DEFINITIONS

NOTE. The following definitions are intended to be practically descriptive, rather than scientifically rigid.

CURRENT, E.M.F. and POWER.

(The definitions of currents given below apply also, in most cases, to electromotive force, potential difference, magnetic flux, etc.)

1 **Direct Current.** A unidirectional current. As ordinarily used, the term designates a practically non-pulsating current.

2 **Pulsating Current.** A current which pulsates regularly in magnitude. As ordinarily employed, the term refers to unidirectional current.

3 **Continuous Current.** A practically non-pulsating direct current.

4 **Alternating Current.** A current which alternates regularly in direction. Unless distinctly otherwise specified, the term "alternating current" refers to a periodic current with successive half waves of the same shape and area.

5 **Oscillating Current.** A periodic current whose frequency is determined by the constants of the circuit or circuits.

6 **Cycle.** One complete set of positive and negative values of an alternating current.

7 **Electrical Degree.** The 360th part of a cycle.

8 **Period.** The time required for the current to pass through one cycle.

9 **Frequency.** The number of cycles or periods per second. The product of 2π by the frequency is called the *angular velocity* of the current.

10 **Root-Mean-Square** or **Effective Value.** The square root of the mean of the squares of the instantaneous values for one complete cycle. It is usually abbreviated r.m.s. Unless otherwise specified, the numerical value of an alternating current refers to its r.m.s. value. The r.m.s. value of a sinusoidal wave is equal to its maximum, or crest value, divided by $\sqrt{2}$. The word "virtual" is sometimes used in place of r.m.s., particularly in Great Britain.

11 **Wave-Form** or **Wave-Shape.** The shape of the curve obtained when the instantaneous values of an alternating current are plotted against time in rectangular co-ordinates. The distance along

the time axis corresponding to one complete cycle of values is taken as 2π radians, or 360 degrees. Two alternating quantities are said to have the same wave-form when their ordinates of corresponding phase (see § 13) bear a constant ratio to each other. The wave-shape, as thus understood, is therefore independent of the frequency of the current and of the scale to which the curve is plotted.

12 **Simple Alternating or Sinusoidal Current.** One whose wave-shape is sinusoidal.

Alternating-current calculations are commonly based upon the assumption of sinusoidal currents and voltages.

13 **Phase.** The distance, usually in angular measure, of the base of any ordinate of an alternating wave from any chosen point on the time axis, is called the phase of this ordinate with respect to this point. In the case of a sinusoidal alternating quantity, the phase at any instant may be represented by the corresponding position of a line or *vector* revolving about a point with such an angular velocity ($\omega = 2\pi f$), that its projection at each instant upon a convenient reference line is proportional to the value of the quantity at that instant.

14 **Non-Sinusoidal Quantities.** Quantities that cannot be represented by vectors of constant length in a plane. The following definitions of phase, active component, reactive component, etc., are not in general applicable thereto. Certain "equivalent" values, as defined below, may, however, be used in many instances, for the purpose of approximate representation and calculation.

15 **Crest-Factor or Peak-Factor.** The ratio of the crest or maximum value to the r.m.s. value. The crest factor of a sine-wave is $\sqrt{2}$.

16 **Form Factor.** The ratio of the r.m.s. to the algebraic mean ordinate taken over a half-cycle beginning with the zero value. If the wave passes through zero more than twice during a single cycle, that zero shall be taken which gives the largest algebraic mean for the succeeding half-cycle. The form factor of a sine-wave is 1.11.

17 **Deviation Factor of a Wave.**—The deviation factor of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave of equal length to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible. (See §406)

17a **Interference Factor of a Wave.**—The interference factor of a wave is a measure of its departure from the sinusoidal shape based upon its disturbing effects when acting by induction on other circuits. (See §§407 to 409).

18 **Equivalent Sine Wave.** A sine wave which has the same frequency and the same r.m.s. value as the actual wave.

*19 **Phase Difference: Lead and Lag.** When corresponding cyclic values of two sinusoidal alternating quantities of the same frequency occur at different instants, the two quantities are said to differ in phase by the angle between their nearest corresponding

*See note next page.

values; *e.g.*, the phase angle between their nearest ascending zeros or between their nearest positive maxima. That quantity whose maximum value occurs first in time is said to lead the other, and the latter is said to lag behind the former.

*20 **Counter-Clockwise Convention.** It is recommended that in any vector diagram, the leading vector be drawn counter-clockwise with respect to the lagging vector, † as in the accompanying diagram, where OI represents the vector of a current in a simple alternating-current circuit, lagging behind the vector OE of impressed e.m.f.

*21 **The Active or In-Phase Component** of the current in a circuit is that component which is in phase with the voltage across the circuit; similarly the active component of the voltage across a circuit is that component which is in phase with the current. The use of the term *energy component* for this quantity is disapproved.

*22 **Reactive or Quadrature Component of the current in a circuit.** That component which is in quadrature with the voltage across the circuit; similarly, the reactive component of the voltage across the circuit is that component which is in quadrature with the current. The use of the term *wattless component* for this quantity is disapproved.

*23 **Reactive Factor.** The sine of the angular phase difference between voltage and current; *i. e.*, the ratio of the reactive current or voltage to the total current or voltage.

*24 **Reactive Volt-Amperes.** The product of the reactive component of the voltage by the total current, or of the reactive component of the current by the total voltage.

*25 **Non-Inductive Load and Inductive Load.** A *non-inductive* load is a load in which the current is in phase with the voltage across the load. An *inductive* load is a load in which the current lags behind the voltage across the load. A *condensive* or *anti-inductive* load is one in which the current leads the voltage across the load.

26 **Power in an Alternating-Current Circuit.** The average value of the products of the coincident instantaneous values of the current and voltage for a complete cycle, as indicated by a wattmeter.

27 **Volt-Amperes or Apparent Power.** The product of the r.m.s. value of the voltage across a circuit by the r.m.s. value of the current in the circuit. This is ordinarily expressed in kv-a.

28 **Power Factor.** The ratio of the power (cyclic average as defined in §26) to the volt-amperes. In the case of sinusoidal current and voltage, the power factor is equal to the cosine of their difference in phase.

29 **Equivalent Phase Difference.** When the current and e.m.f. in a given circuit are non-sinusoidal, it is customary, for pur-

*Note: Definitions 19, 20, 21, 22, 23, 24, 25 refer strictly only to cases where the voltage and current are both sinusoidal (see §11 and 12).

†See Publication of 12 the International Electrotechnical Commission (Report of Turin meeting, Sept. 1911, p. 78).

poses of calculation, to take as the "equivalent" phase difference, the angle whose cosine is the power factor (see §28) of the circuit. There are cases, however, where this equivalent phase difference is misleading, since the presence of harmonics in the voltage wave, current wave, or in both, may reduce the power factor without producing a corresponding displacement of the two wave forms with respect to each other; *e.g.*, the case of an a-c. arc. In such cases, the components of the equivalent sine waves, the equivalent reactive factor and the equivalent reactive volt-amperes may have no physical significance.

30 **Single-Phase.** A term characterizing a circuit energized by a single alternating e.m.f. Such a circuit is usually supplied through two wires. The currents in these two wires, counted positively outwards from the source, differ in phase by 180 degrees or a half-cycle.

31 **Three-Phase.** A term characterizing the combination of three circuits energized by alternating e.m.f.'s. which differ in phase by one-third of a cycle; *i.e.*, 120 degrees.

32 **Quarter-Phase, also called Two-Phase.** A term characterizing the combination of two circuits energized by alternating e.m.f's. which differ in phase by a quarter of a cycle; *i.e.*, 90 degrees.

33 **Six-Phase.** A term characterizing the combination of six circuits energized by alternating e.m.f's. which differ in phase by one sixth of a cycle; *i.e.*, 60 degrees.

34 **Polyphase.** A general term applied to any system of more than a single phase. This term is ordinarily applied to symmetrical systems.

Per Cent Drop.

50 In electrical machinery, the ratio of the **internal resistance drop** at 75°C. to the terminal voltage, expressed in percent, is called the "**percent resistance drop.**"

51 Similarly the ratio of the **internal reactance drop** to the terminal voltage, expressed in per cent, is called the "**per cent reactance drop.**"

52 Similarly the ratio of the **internal impedance drop** at 75°C. to the terminal voltage, expressed in percent is called the "**percent impedance drop.**"

Unless otherwise specified, these percent drops shall be referred to rated load and rated power factor.

53 **In the case of transformers,** the per cent drop will be the sum of the primary drop (reduced to secondary turns) and the secondary drop, in per cent of secondary terminal voltage.

54 **In the case of induction motors,** it is advantageous to express the drops in per cent of the internally induced e.m.f.

55 **The Load Factor of a machine, plant or system.** The ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day, a month, or a year, and the maximum is taken as the average over a short interval of the maximum load within that period.

In each case, the interval of maximum load and the period over

which the average is taken should be definitely specified, such as a "half-hour monthly" load-factor. The proper interval and period are usually dependent upon local conditions and upon the purpose for which the load factor is to be used.

56 **Plant Factor.** The ratio of the average load to the rated capacity of the power plant, *i.e.*, to the aggregate ratings of the generators.

57 **The Demand of an Installation or System** is the load which is drawn from the source of supply at the receiving terminals averaged over a suitable and specified interval of time. Demand is expressed in kilowatts, kilovolt-amperes, amperes, or other suitable units.

58 **The Maximum Demand of an Installation or System** is the greatest of all the demands which have occurred during a given period It is determined by measurement, according to specifications, over a prescribed time interval.

59 **Demand Factor.** The ratio of the maximum demand of any system or part of a system, to the total connected load of the system, or of the part of system under consideration.

60 **Diversity Factor.** The ratio of the sum of the maximum power demands of the subdivisions of any system or parts of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

61 **Connected Load.** The combined continuous rating of all the receiving apparatus on consumers' premises, connected to the system or part of the system under consideration.

62 **The Saturation Factor of a machine.** The ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. Unless otherwise specified, the saturation factor of a machine refers to the no-load excitation required at normal rated speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

63 **The Percentage Saturation** of a machine at any excitation may be found from its saturation curve (generated voltage as ordinates, against excitation as abscissas), by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in percent, is the percentage saturation, and is independent of the scales selected for excitation and voltage. This ratio, as a fraction, is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity; or, if f be the saturation factor and p the percentage saturation,

$$p = 100\left(1 - \frac{1}{f}\right)$$

64 **Magnetic Degree.** The 360th part of the angle subtended, at the axis of a machine, by a pair of its field poles. One **mechanical degree** is thus equal to as many magnetic degrees as there are pairs of poles in the machine.

65 The Variation in Prime Movers which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360 degrees.

66 The Variation in Alternators or alternating-current circuits in general, is the maximum angular displacement, expressed in electrical degrees, (one cycle = 360 deg.) of corresponding ordinates of the voltage wave and of a wave of absolutely constant frequency equal to the average frequency of the alternator or circuit in question, and may be due to the variation of the prime mover.

67 Relations of Variations in Prime Mover and Alternator. If p is the number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct-connected, and pn times the variation of the prime mover if rigidly connected thereto in such a manner that the angular speed of the alternator is n times that of the prime mover.

68 The Pulsation in a Prime Mover, or in the alternator connected thereto. The ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

80 Capacity. The two different senses in which this word is used sometimes lead to ambiguity. It is therefore recommended that whenever such ambiguity is likely to arise, the descriptive term *power capacity* or *current capacity* be used, when referring to the power or current which a device can safely carry, and that the term "*Capacitance*" be used when referring to the electrostatic capacity of a device.

81 Resistor. A device, heretofore commonly known as a resistance, used for the operation, protection, or control of a circuit or circuits See § 740.

82 Reactor. A coil, winding or conductor, heretofore commonly known as a reactance coil or choke coil, possessing inductance the reactance of which is used for the operation, protection or control of a circuit or circuits. See also §§ 214 and 736.

83 Efficiency. The efficiency of an electrical machine or apparatus is the ratio of its useful output to its total input.

84 Base Speed of an Adjustable-Speed Motor.—That speed of the motor obtained with full field under full load with no resistor in the armature circuit.

85 Electric Circuit.—An electric circuit is a path in which an electric current may flow. Strictly speaking, an electric circuit is a complete circulatory path, but the term circuit is commonly employed to designate a specific part of a complete path. When part of a complete path is referred to, such as a branch circuit, a derived circuit, or a conductor, the terminals of that path should be specified in order to avoid ambiguity: e. g., the circuit *a-b-c*. When the whole circuit is referred to, it may be designated as a complete or closed circuit.

90

SYMBOLS AND ABBREVIATIONS

TABLE I.

Name of Quantity.	Symbol for the Quantity.	Unit.	Abbreviation for the Unit.
Electromotive force, abbreviated e.m.f.	E, e	volt	....
Potential difference, abbreviated p.d.	V, v or E, e	"	....
Voltage	E, e or V, v	"	....
Current	I, i	ampere	....
Quantity of electricity	Q, q	coulomb, ampere-hour	....
Power	P, p	watt	....
Electrostatic flux	Ψ	....	....
Electrostatic flux density	D	....	....
Electrostatic field intensity	F		
Magnetic flux	Φ, φ	maxwell	....
Magnetic flux density	$B, \mathfrak{B}$	gauss	....
Magnetic field intensity	$H, \mathfrak{H}$	gilbert per centimeter or gauss†	gilbert per cm. ....
Magnetomotive force, abbreviated m.m.f.	$\mathfrak{F}$	gilbert*	....
Intensity of magnetization	J	....	....
Susceptibility	$\kappa = J/H$	....	....
Permeability	$\mu = B/H$	....	....
Resistance	R, r	ohm	....
Reactance	X, x	"	....
Impedance	Z, z	"	....
Conductance	g	mho	....
Susceptance	b	"	....
Admittance	Y, y	"	....
Resistivity	ρ	‡ohm-centimeter	ohm-cm

†The gauss is provisionally accepted for the present as the name of both the unit of field intensity and flux density, on the assumption that permeability is a simple numeric.

*An additional unit for m. m. f. is the "ampere-turn," for flux the "line," for magnetic flux-density "maxwells per sq. in."

‡*Note.* The numerical values of these quantities are *ohms resistance* and *mhos conductance* between two opposite faces of a cm. cube of the material in question, but the correct names are as given, not ohms and mhos per cm. cube, as commonly stated

Conductivity	γ	‡mho per centimeter	mho per cm.
Dielectric constant	ϵ or k	....	....
Reluctance	$\mathcal{R}$	....	....
Capacitance (Electrostatic capacity)	C	farad	....
Inductance (or coefficient of self induction)	L	henry	....
Mutual Inductance (or coefficient of mutual induction)	M	henry	....
Phase displacement	θ, φ	degree or radian	°
Frequency	f	cycle per second	~
Angular velocity	ω	radian per second	....
Velocity of rotation	n	revolution per second	rev. per sec.
Number of conductors or turns	N	convolution or turns of wire	
Temperature	T, t, θ	degree centigrade	C
Energy, in general	U or W	joule, watt-hour	....
Mechanical work	W or A	joule, watt-hour	....
Efficiency	η	per cent	....
Length	l	centimeter	cm.
Mass	m	gram	g
Time	t	second	sec.
Acceleration due to gravity	g	centimeter per second per second	cm. per sec. per sec.
Standard acceleration due to gravity (at about 45 deg. latitude and sea level) equals 980.665†	g_0	centimeter per second per second	cm. per sec. per sec.

91 E_m, I_m and P_m should be used for maximum cyclic values, e, i and p for instantaneous values, E and I for r.m.s. values (see §10) and P for the average value of the power, or the active power. These distinctions are not necessary in dealing with continuous-current circuits. In print, vector quantities should be represented by boldface capitals.

‡See note preceding page

†This has been the accepted standard value for many years and was formerly considered to correspond accurately to 45° Latitude and sea level. Later researches, however, have shown that the most reliable value for 45° and sea-level is slightly different; but this does not affect the standard value given above.

CLASSIFICATION OF MACHINERY

100 The machinery under consideration in these rules may be classified in various ways, these various classifications overlapping or interlocking in considerable degree. Briefly, they are Direct-Current or Alternating-Current, Rotating or Stationary. Under Rotating Apparatus there are two principal classifications: *First*, according to the function of the machines; Motors, Generators. Boosters, Motor-Generators, Dynamotors, Double-Current Generators, Converters and Phase Advancers; *Second*, according to the type of construction or principle of operation; Commutating, Synchronous, Induction, Unipolar, Rectifying. Obviously, some of these machines could be rationally included in either classification, *e.g.*, Motor-Generators and Rectifying Machines.

In the following, self-evident definitions have for the most part, been omitted.

ROTATING MACHINES.

FUNCTIONAL CLASSIFICATION OF ROTATING MACHINES.

101 **Generator.** A machine which transforms mechanical power into electrical power.

102 **Motor.** A machine which transforms electrical power into mechanical power.

103 **Booster.** A generator inserted in series in a circuit to change its voltage. It may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.

104 **Motor-Generator Set.** A transforming device consisting of a motor mechanically coupled to one or more generators.

105 **Dynamotor.** A transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

106 **A Direct-Current Compensator** or **Balancer** comprises two or more similar direct-current machines (usually with shunt or compound excitation) directly coupled to each other and connected in series across the outer conductors of a multiple-wire system of distribution, for the purpose of maintaining the potentials of the intermediate wires of the system, which are connected to the junction points between the machines.

107 **A Double-Current Generator** supplies both direct and alternating currents from the same armature-winding.

108 **A Converter** is a machine employing mechanical rotation in changing electrical energy from one form into another. There are several types of converters as follow:

109 A **Direct-Current Converter** converts from a direct current to a direct current, usually with a change of voltage. Such a machine may be either a motor-generator set or a dynamotor.

110 A **Synchronous Converter** (sometimes called a Rotary Converter) converts from an alternating to a direct current, or vice-versa. It is a synchronous machine with a single closed-coil armature winding, a commutator and slip rings.

111 A **Cascade Converter**, also called a **Motor Converter**, is a combination of an induction motor with a synchronous converter, the secondary circuit of the former feeding directly into the armature of the latter; *i.e.*, it is a synchronous converter concatenated with an induction motor.

112 A **Frequency Converter** converts the power of an alternating-current system from one frequency to another, with or without a change in the number of phases, or in the voltage.

113 A **Rotary Phase-Converter** converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

114 A **Phase Advancer** is a machine which supplies reactive volt-amperes to the system to which it is connected. Phase advancers may be either synchronous or asynchronous.

115 A **Synchronous Condenser** or **Synchronous Phase Advancer** is a synchronous machine, running either idle or with load, the field excitation of which may be varied so as to modify the power-factor of the system, or through such modification to influence the load voltage.

CONSTRUCTIONAL CLASSIFICATION OF ROTATING MACHINES

Commutating Machines

130 **Direct-Current Commutating Machines** comprise a magnetic field of constant polarity, an armature, and a commutator connected therewith. These include: Direct-Current Generators; Direct-Current Motors; Direct-Current Boosters; Direct-Current Motor-Generator Sets and Dynamotors; Direct-Current Compensators or Balancers; and Arc Machines.

131 **Alternating-Current Commutating Machines** comprise a magnetic field of alternating polarity, an armature, and commutator connected therewith.

132 **Synchronous Commutating Machines** include synchronous converters, cascade-converters, and double-current generators.

133 **Synchronous Machines** comprise a constant magnetic field and an armature receiving or delivering alternating-currents in synchronism with the motion of the machine; *i.e.*, having a frequency strictly proportional to the speed of the machine. They may be sub-divided as follow:

134 **An Alternator** is a synchronous alternating-current generator, either single-phase or polyphase.

135 **A Polyphase Alternator** is a polyphase synchronous alternating-current generator, as distinguished from **a single-phase alternator.**

136 **An Inductor Alternator** is an **Alternator** in which both field and armature windings are stationary, and in which masses of iron or inductors. by moving past the coils, alter the magnetic flux through them. It may be either single-phase or polyphase.

137 **A Synchronous Motor** is a machine structurally identical with an alternator, but operated as a motor.

138 **Induction Machines** include apparatus wherein primary and secondary windings rotate with respect to each other; *i.e.*, induction motors, induction generators, certain types of frequency converters and certain types of rotary phase-converters.

139 **An Induction Motor** is an alternating-current motor, either singlephase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotating member. The secondary winding receives power from the primary by electromagnetic induction.

140 **An Induction Generator** is a machine structurally identical with an induction motor, but driven above synchronous speed as an alternating-current generator.

141 **Unipolar or Acyclic Machines** are direct-current machines. in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

DEFINITIONS OF TERMS USED IN DESCRIBING ALTERNATING-CURRENT COMMUTATOR MOTORS.

142 It is not intended that the use of any one term will exclude, or render unnecessary, the use of any one or more of the other terms. Each term is intended to refer to a certain group of alternating-current commutator-motors. An alternating-current commutator motor logically may be classified under a number of different, but non-conflicting groups.

Classification by Phases of Energy Supply:

143 **Single-Phase Commutator Motor.**—A single-phase commutator motor is one that receives the whole of its energy from only one phase of an alternating-current supply system, without requiring external phase-converting apparatus.

143a **Polyphase Commutator Motor.**—A polyphase commutator motor is one that receives its energy from a plurality of phases of an alternating-current supply system, or from a single-phase system through phase-converting apparatus external to the motor.

144 Classification by Speed Characteristics.—For convenience, alternating-current commutator motors may be classified with reference to their speed characteristics as (1) constant-speed motors, (2) multi-speed motors, (3) adjustable-speed motors, and (4) varying-speed motors. Definitions of these terms as given in **§§151** to **154** for motors in general, should be adopted for alternating-current commutator motors, in so far as they are applicable.

Classification by Excitation:

145 Stator-Excited Commutator Motor.—A stator-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the stator.

145a Rotor-Excited Commutator Motor.—A rotor-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the rotor.

145b Stator- and Rotor-Excited Commutator Motor.—A stator- and rotor-excited commutator motor is one in which the torque-producing field* is due to currents in windings located on the stator and on the rotor.

145c Constant-Field Commutator Motor.—A constant-field commutator motor is one in which the torque-producing field* remains practically constant, independent of the load.

Alternating-current commutator motors of this class will in general have load-speed characteristics similar to those of the direct-current shunt motor, but not all alternating-current commutator motors having such load-speed characteristics are constant-field machines.

145d Varying-Field Commutator Motor.—A varying-field commutator motor is one in which the torque-producing field* varies in some proportion with the current in the armature (which latter is generally the rotor).

Such a motor will in general have load-speed characteristics similar to those of the direct-current series motor.

Classification by Neutralization and Compensation:

146 Neutralized Commutator Motor.—A neutralized commutator motor is one in which use is made of a winding for producing a magnetizing force which at each instant and at each point in the air-gap under the pole face is practically equal and opposite to the magnetizing force due to the armature current.

146a Compensated Commutator Motor.—A compensated commutator motor is one in which means, other than a neutralizing winding, are provided within the motor for improving the power-factor.

Classification by Energy Reception:

147 Conduction Commutator Motor.—A conduction commutator motor is one in which the working energy† is supplied to only one of the members, and is conveyed to it by conduction.

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*The term "torque-producing field" is intended to describe that component of the magnetic field which, with the in-phase component of current, produces the torque of the motor.

†See note next page.

147a **Transformer Commutator Motor.**—A transformer commutator motor is one in which the working energy† is transmitted from one member to the other by transformer action.

A motor in which the energy required by its armature (which is generally the rotor) is conveyed to it by electromagnetic induction or transformer action, may properly be referred to either as an "induction motor" or as a "transformer motor". Although it is equally applicable to a motor having a commutator, the term "induction motor" is usually applied to a motor without a commutator. The term "transformer commutator motor" is therefore recommended for use with motors of the induction, or transformer type, having commutators.

147b **Transformer-Conduction Commutator Motor.**—A transformer-conduction commutator motor is one in which the energy required by its armature (which is generally the rotor) is conveyed to it by both conduction and electromagnetic induction.

147c **Repulsion Commutator Motor.**—A repulsion commutator motor is a transformer commutator motor in which use is made of brushes for short-circuiting a number of coils of the commutated winding.

SPEED CLASSIFICATION OF MOTORS.

150 Motors may, for convenience, be classified with reference to their speed characteristics as follow:

151 **Constant-Speed Motors,** whose speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors.

152 **Multispeed Motor (Change Speed Motor).**—A multispeed motor (change speed motor) is a motor which can be operated at any one of several distinct speeds (these speeds being practically independent of the load), but which cannot be operated at intermediate speeds.

153 **Adjustable-Speed Motors,** in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load; such as shunt motors designed for a considerable range of speed variation.

154 **Varying-Speed Motors,** or motors in which the speed varies with the load, ordinarily decreasing when the load increases; such as series motors, compound-wound motors, and series-shunt motors. As a sub-class of varying-speed motors, may be cited, **adjustable varying-speed motors,** or motors in which the speed can be varied over a considerable range at any given load, but when once adjusted, varies with the load; such as compound-wound motors arranged for adjustment of speed by varying the strength of the shunt field.

†The term "working energy" is intended to describe the energy which is directly converted into mechanical energy, and which includes the shaft energy output plus core losses and friction.

CLASSIFICATION OF ROTATING MACHINES RELATIVE TO THE DEGREE OF ENCLOSURE

160 The following types are recognized:

Open
Protected
Semi-enclosed
Enclosed
Separately ventilated
Water-cooled
Self-ventilated
Drip-proof
Moisture-resisting
Submersible
Explosion-proof
Explosion-proof slip-ring enclosure

161 An **"open" machine** is of either the pedestal-bearing or end-bracket type where there is no restriction to ventilation, other than that necessitated by good mechanical construction.

162 A **"protected" machine** is one in which the armature, field coils, and other live parts are protected mechanically from accidental or careless contact, while free ventilation is not materially obstructed.

163 A **"semi-enclosed" machine** is one in which the ventilating openings in the frame are protected with wire screen, expanded metal, or other suitable perforated covers, having apertures not exceeding ⅛ of a square inch (3.2 sq. cm.) in area.

164 A **"Totally Enclosed" machine** is one so enclosed as to prevent circulation of air between the inside and the outside of the case, but not sufficiently to be termed "air-tight."

165 A **" separately ventilated " machine** has its ventilating air supplied by an independent fan or blower external to the machine.

166 A **" water-cooled " machine** is one which mainly depends on water circulation for the removal of its heat.

167 A **" self-ventilated " machine** differs from a **separately** ventilated machine only in having its ventilating air circulated by a fan, blower, or centrifugal device integral with the machine.

If the heated air expelled from the machine is conveyed away through a pipe attached to the machine, this should be so stated

168 A **"drip-proof " machine** is one so protected as to exclude falling moisture or dirt. A "drip proof" machine may be either "open" or "semi-enclosed", if it is provided with suitable protection integral with the machine, or so enclosed as to exclude effectively falling solid or liquid material.

171 An **"explosion-proof" machine** (or **"flame-proof" machine**) is a machine in which the enclosing case can withstand, without injury, any explosion of gas that may occur within it, and will not transmit the flame to any inflammable gas outside it.

172 A machine in which slip rings and brushes alone are included within an explosion-proof case should not be described as an explosion proof machine, but as a machine "with **explosion-proof slip-ring enclosure.**"

STATIONARY INDUCTION APPARATUS

200 **Stationary Induction Apparatus** changes electric energy to electric energy, through the medium of magnetic energy, without mechanical motion. It comprises several forms, distinguished as follow:

201 **Transformers**, in which the primary and secondary windings are ordinarily insulated one from another.

202 The terms "**high-voltage**" and "**low-voltage**" are used to distinguish the winding having the greater from that having the lesser number of turns. The terms "**primary**" and "**secondary**" serve to distinguish the windings in regard to energy flow, the primary being that which receives the energy from the supply circuit, and the secondary that which receives the energy by induction from the primary.

203 The **rated current of a constant-potential transformer** is that secondary current which, multiplied by the rated-load secondary voltage, gives the kv-a. rated output. That is, a transformer of given kv-a. rating must be capable of delivering the rated output at rated secondary voltage, while the primary impressed voltage is increased to whatever value is necessary to give rated secondary voltage.

The **Rated Primary Voltage** of a constant-potential transformer is the rated secondary voltage multiplied by the turn ratio.

204 The **ratio of a transformer,** unless otherwise specified, shall be the ratio of the number of turns in the high-voltage winding to that in the low-voltage winding; *i.e.*, the "**turn-ratio**"

205 The **voltage ratio** of a transformer is the ratio of the r.m.s. primary terminal voltage to the r.m.s. secondary terminal voltage, under specified conditions of load.

206 The "**current ratio**" of a current-transformer is the ratio of r.m.s. primary current to r.m.s. secondary current, under specified conditions of load.

207 The "**marked ratio**" of an instrument transformer is the ratio which the apparatus is designed to give under average conditions of use. When a precise ratio is required, it is necessary to specify the voltage, frequency, load and power factor of the load.

208 **Volt-Ampere Ratio of Transformers.**

The volt-ampere ratio, which should not be confused with real efficiency, is the ratio of the volt-ampere output to the volt-ampere input of a transformer, at any given power factor.

209 **Auto-transformers** have a part of their turns common to both primary and secondary circuits,

210 **Voltage Regulators** have turns in shunt and turns in series with the circuit, so arranged that the voltage ratio of the transformation or the phase relation between the circuit-voltages is variable at will. They are of the following three classes:

211 **Contact Voltage Regulators,** in which the number of turns in one or both of the coils is adjustable.

212 **Induction Voltage Regulators,** in which the relative positions of the primary and secondary coils are adjustable.

213 **Magneto Voltage Regulators,** in which the direction of the magnetic flux with respect to the coils is adjustable.

214 **Reactors,** heretofore commonly called **Reactance Coils,** also called **Choke Coils**; a form of stationary induction apparatus used to supply reactance or to produce phase displacement. **See also §§82 and 736.**

METERS AND INSTRUMENTS

225 Although the terms **Instruments** and **Meters** are frequently used synonymously in referring to electrical measuring devices, the meter departments of manufacturing and operating companies commonly use the word "meters" in the collective sense to designate only those devices which register the total energy or quantity of electricity consumed in or supplied to a circuit, and reserve the term "instruments," in the collective sense, for all other electrical measuring or indicating devices.

226 In general, the names of meters and instruments are self-defining, particularly when considered in connection with existing definitions. The following terms are preferred to other terms sometimes used for the same devices: **Reactive-Factor Meter, Power-Factor Meter, Watt-hour Meter,** etc.

227 **Crest Voltmeter.** A voltmeter depending for its indications upon the crest, that is the maximum value of the voltage of the system to which it is connected. These instruments are so calibrated that they indicate the r. m. s. value of the sinusoidal voltage having the same crest value.

228 **Synchronoscope** (also called a Synchroscope or Synchronism Indicator). A device which in addition to indicating synchronism between two machines, shows whether the speed of the incoming machine is fast or slow.

229 **Reactive-Volt-Ammeter** (also called a Reactive-Volt-Ampere Indicator). An instrument which indicates the reactive volt-amperes of the circuit to which it is connected.

230 **Line Drop Voltmeter Compensator.** A device used in connection with a voltmeter which causes it to indicate the voltage at some distant point of the circuit.

231 **Recording Ammeters, Voltmeters, Wattmeters, etc.,** are instruments which record graphically upon time-charts the values of the quantities they measure.

232 **A Demand Meter** is a device which indicates or records the demand or maximum demand (See §§**57** and **58**). In practice two types are recognized:

233 **An Integrated-Demand Meter** is one which indicates or records the maximum demand obtained through integration.

234 **A Lagged-Demand Meter** is one in which the indication of maximum demand is subject to a characteristic time lag.

235 **Errors of Indicating Instruments.** In specifying the accuracy of an indicating instrument, the error at any point on the scale shall be expressed as a percentage of the full scale reading.

236 **Torque** of meters and instruments shall be expressed in milli-meter-grams.

237 Period of an Instrument.—The period of an instrument, sometimes called the "periodic time," is the time taken for the pointer to make one complete oscillation (two consecutive swings). A swing is a complete movement in either direction.

In strongly damped instruments, the period is influenced by the amplitude of the movement.

238 Damping.—The damping of an instrument shall be expressed in terms of the following quantities, all three of which are essential to a complete description of the behavior:

a. The number of swings taken by the pointer in coming to rest.
b. The time, in seconds, required for the pointer to come to rest.
c. The overshooting, in per cent of the angular displacement due to the disturbance.

These quantities shall be measured by suddenly applying and maintaining a load which will give a steady deflection of one-half full angular scale. The pointer shall be at zero when the load is applied.

239 Rating Limitation of the Circuits in Meters and Instruments.—The rating of any circuit in a meter or instrument shall be no greater than that corresponding to the load to which it may be continuously subjected.

The full scale marking of an instrument does not necessarily correspond to its rating. Where the rating differs from the full scale marking, the rating shall be marked on the instrument.

240 Standard Temperature of Reference for Meter and Instrument Characteristics.—The standard temperature of reference for meter and instrument characteristics shall be 20° C. For purposes of rating, the standard ambient temperature shall be 40°C.

241 Temperature Rise and Ultimate Temperature of Shunts.—Shunts shall be so rated that the rise of temperature at the hottest spot shall not be greater than 80°C., and their use so restricted that the ultimate temperature (hottest spot) shall not exceed 120°C.

Exception is made of shunts having no soldered joint and made of material which is not permanently changed in resistance if continuously subjected to higher temperature. Hottest spot temperature should be measured in such a manner as not to cause undue local rise of temperature.

243 Marking of Switchboard Shunts.—Marking of switchboard shunts shall include the rating in amperes, the drop in volts at that rating, and, if the shunt is calibrated in connection with a particular instrument, the serial number of such instrument.

When shunts are designed to be used with devices taking sufficient current to be an appreciable portion of the whole, this fact shall be indicated. For example, a 100-ampere device having a drop of 0.050 volts, will have its shunt marked:

Volts 0.050
Amperes 100—10,

indicating that with 90 amperes in the shunt and 10 amperes in the measuring circuit, the drop across the shunt will be 0.050 volts.

244 Grounding of Meters and Instruments.—To avoid errors due to electrostatic action, the covers of meters and instruments which are used with current and potential transformers shall be connected to the secondary circuits of the transformers and grounded.

INSTRUMENT TRANSFORMERS

See §§741 to 744.

STANDARDS FOR ELECTRICAL MACHINERY

250 *The expressions "machinery" and "machines" are here employed in a general sense, in order to obviate the constant repetition of the words "machinery or induction apparatus."*

251 *All temperatures are to be understood as centigrade.*

252 *The expression "capacity" is to be understood as indicating "capability", except where specifically qualified, as, for instance, in the case of allusions to electrostatic capacity, i. e., capacitance.*

253 *Wherever special rules are given for any particular type of machinery or apparatus (such as switches, railway motors, railway substation machinery, etc., these special rules shall be followed, notwithstanding any apparent conflict with the provisions of the more general sections. In the absence of special rules on any particular point, the general rules on this point shall be followed.*

260 **Objects of Standardization.** To ensure satisfactory results, electrical machinery should be specified to conform to the Institute Standardization Rules, in order that it shall comply, in operation. with approved limitations in the following respects, so far as they are applicable.

Operating temperature
Mechanical strength
Commutation
Dielectric strength
Insulation resistance
Efficiency
Power factor
Wave shape
Regulation

261 **Capacity or Available Output of an Electrical Machine.** So far as relates to the purposes of these Standardization Rules, the Institute defines the Capacity of an Electrical Machine as the load which it is capable of carrying for a specified time (or continuously), without exceeding in any respect the limitations herein set forth.

Except where otherwise specified, the capacity of an electrical machine shall be expressed in terms of its available *output*. For exceptions see §§**277** and **802**.

262 **Rating of an Electrical Machine.** Capacity should be distinguished from Rating. The Rating of a machine is the output marked on the Rating Plate, and shall be based on, but shall not exceed, the maximum* load which can be taken from the machine under prescribed conditions of test. This is also called the rated output.

*The term "maximum Load" does not refer to loads applied solely for mechanical commutation, or similar tests.

263 **The Principle upon which Machine Ratings** are based, so far as relates to thermal characteristics, is that the rated load, applied continuously or for a stated period, shall produce a temperature rise which, superimposed upon a standard ambient temperature, will not exceed the maximum safe operating temperature of the insulation.

264 **A. I. E. E. and I. E. C. Ratings.** When the prescribed conditions of test are those of the A. I. E. E. Standardization Rules, the rating of the machine is the Institute Rating. (See **§620**). When the prescribed conditions of the test are those of the I. E. C.† Rules, the rating of the machine is the I. E. C. rating. A machine so rated in either case may bear a distinctive sign upon its rating plate.

265 **Standard Temperature and Barometric Pressure for Institute Rating.** The Institute Rating (See **§262**) of a machine shall be its capacity when operating with a cooling medium of the ambient temperature of reference (40° for air or 25° for water, see **§§305** and **309**) and with barometric conditions within the range given in **§308**. See **§§305A, 307, 320** and **321.**

266 **The Temperature Rises Specified in these Rules** apply to all ambient temperatures up to and including, but not exceeding, 40°C. for air and 25°C. for water. (For definition of ambient temperature see **§303.**)

267 **Any Machinery Destined for Use with Higher Ambient** temperatures of cooling mediums, and also any machinery for operation at altitudes for which no provision is made in **§308**, should be the subject of special guarantee by the manufacturer. The methods of test and performance set forth in these Rules will, however, afford guidance in such cases.

UNITS IN WHICH RATING SHALL BE EXPRESSED

274 The rating of **Direct-Current Generators,** shall be expressed in kilowatts (kw.) available at the terminals at a specified voltage.

275 The rating of **Alternators** and **Transformers,** shall be expressed in kilovolt-amperes (kv-a.) available at the output terminals, at a specified voltage and power factor.

276 It is strongly recommended that the **rating of motors** shall be expressed in kilowatts* (kw.) available at the shaft. (An ex-

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†I.E.C. stands for "International Electrotechnical Commission This rating has not yet been established.

*Since the input of machinery of this class is measured in electrical units and since the output has a definite relation to the input, it is logical and desirable to measure the delivered power in the same units as are employed for the received power. Therefore, the output of motors should be expressed in kilowatts instead of in horse power. However, on account of the hitherto prevailing practice of expressing mechanical output in horse power, it is recommended that for machinery of this class the rating should, for the present, be expressed both in kilowatts and in horse power; as follows:

kw.————approx. equiv. h.p.————

For the purposes of these Rules the horse-power shall be taken as 746.0 watts.

In order to lay stress upon the preferred future basis, it is desirable that on Rating plates, the Rating in kilowatts shall be shown in larger and more prominent characters than the rating in horse power.

ception to this rule is made in the case of Railway motors, which, for some purposes, are also rated by their *input*, see **§802.**)

277 **Auxiliary machinery,** such as regulators, resistors, reactors, balancer sets, stationary and synchronous condensers, etc., shall have their ratings appropriately expressed. It is essential to specify also the voltage (and frequency, if a-c.,), of the circuits on which the machinery may appropriately be used.

KINDS OF RATING

There are various kinds of rating such as:

281 **Continuous Rating.** A machine rated for continuous service shall be able to operate continuously at its rated output, without exceeding any of the limitations referred to in **§260.**

282 **Short-Time Rating.** A machine rated for short-time service; (*i.e.* service including runs alternating with stops of sufficient duration to ensure substantial cooling), shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding any of the limitations referred to in **§260.** Such a rating is a **short-time rating.**

283 **Nominal Ratings.** For railway motors and sometimes for railway substation machinery, certain nominal ratings are employed. See **§§765** and **800.** Nominal ratings for automobile propulsion motors and generators are not recommended; see **§837.**

284 **Duty-Cycle Operation.** Many machines are operated on a cycle of duty which repeats itself with more or less regularity. For purposes of rating, either a continuous or a short-time equivalent load, may be selected, which shall simulate as nearly as possible the thermal conditions of the actual duty cycle.

285 Standard durations of equivalent tests shall be for machines operating under specified duty-cycles as follow (see also **§836**):

5 minutes
10 "
15 "
30 "
60 "
120 "
and continuous.

Of these the first six are short-time ratings, selected as being thermally equivalent to the specified duty cycle.

When, for example, a short-time rating of 10 minutes duration is adopted, and the thermally equivalent load is 25 kw. for that period, then such a machine shall be stated to have a 10-minute rating of 25 kw.

286 In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are **within 5°C of the ambient temperature** at the time of starting the test.

287 In the absence of any specification as to the kind of rating, the continuous rating shall be understood.*

*An exception is made in the case of motors for railway service, where in the absence of any specification as to the kind of rating, the "nominal rating" as defined in §319 and 415 shall be understood.

288 Machines marked in accordance with **§264** shall be understood to have a continuous rating, unless otherwise marked in accordance with **§285**.

HEATING AND TEMPERATURE

300 **Temperature Limitations of the Capacity of Electrical Machinery.** The capacity, so far as relates to temperature, is usually limited by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. When the safe limits are exceeded, deterioration is rapid. The insulating material becomes permanently damaged by excessive temperature, the damage increasing with the length of time that the excessive temperature is maintained, and with the amount of excess temperature, until finally the insulation breaks down.

301 The result of operating at temperatures in excess of the safe limit is to shorten the life of the insulating material. This shortening of life is, in certain special cases, warranted, when necessary for obtaining some other desirable result, as, for example, in some instances of railway and other motors for propelling vehicles, in providing greater power within a limited space. See **§804**. The design of coils used in control and switching apparatus, such as contactors, controllers, induction starters, and in other operating mechanisms as arc lamps, magnet windings, etc., is such that the effect of higher temperature in the coils is less serious owing to the fact that these devices usually have shorter life than electrical machinery in general. Consequently, higher temperatures for the windings are recommended, in accordance with **§717a**.

302 There does not appear to be any advantage in operating at lower temperatures than the safe limits, so far as the life of the insulation is concerned. Insulation may break down from various causes, and when these breakdowns occur, it is not usually due to the temperature at which the insulation has been operated, provided the safe limits have not been exceeded.

303 **The Ambient Temperature** is the temperature of the air or water which, coming into contact with the heated parts of a machine, carries off its heat. See **§§309, 310** and **314**.

304 **The cooling fluid may** either be led to the machine through ducts, or through pipes, or merely surround the machine freely. In the former case the ambient temperature is to be measured at the intake of the machine itself. In the latter case see **§314**.

305 **Ambient Temperature of Reference for Air.** The standard ambient temperature of reference, when the cooling medium is air, shall be 40°C.

305A. **Whatever may be the Ambient Temperature** when the machine is in service, the limits of the maximum observable temperature or of temperature rise specified in the rules should not be exceeded in service; for, if the maximum temperature be exceeded, the insulation may be endangered, and if the rise be exceeded, the excess load may

lead to injury, by exceeding limits other than those of temperature; such as commutation, stalling load and mechanical strength. For similar reasons, loads in excess of the rating should not be taken from a machine.

306 The permissible rises in temperature given in column 2 of table III page 38 have been calculated on the basis of the standard ambient temperature of reference, by subtracting 40° from the highest temperatures permissible, which are given in column 1 of the same table.

307 A machine may be tested at any convenient ambient temperature, preferably not below 10°C., but *whatever be the value of this ambient temperature, the permissible rises of temperature must not exceed those given in column* 2 *of table III page* 38.

308 **Altitude.** Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters (3300 feet.) For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. See **§267.** It is recommended that when a machine is intended for service at altitudes above 1000 meters (3300 ft.) the permissible temperature rise at sea level, until more nearly accurate information is available, shall be reduced by 1 per cent for each 100 meters (330 ft.) by which the altitude exceeds 1000 meters. Water cooled oil transformers are exempt from this reduction.

309 **Ambient Temperature of Reference for Water-Cooled Machinery.** For water-cooled machinery, the standard temperature of reference for incoming cooling water shall be 25° C, measured at the intake of the machine.

310 In the testing of water-cooled transformers, it is not necessary to take into account the surrounding-air temperature, except where the cooling effect of the air is 15 per cent or more of the total cooling effect, referred to the standard ambient temperature of reference of 25°C. for water and 40°C. for air. When the effect of the cooling air is 15 per cent. or more of the total, the temperature of the cooling water should be maintained within 5°C. of the surrounding air. Where this is impractical, the ambient temperature should be determined from the change in the resistance of the windings, using a disconnected transformer, supplied with the normal amount of cooling water, until the temperature of the windings has become constant.

311 In the case of rotating machines, cooled by forced draught, a conventional weighted mean shall be employed, a weight of four being given to the temperature of the circulating air supplied through ducts (see **§304**), and a weight of one to the surrounding room air. In the case of air-cooled transformers, see "exception" **§321.**

312 **Machines Cooled by Other Means.** Machines cooled by means other than air or water shall receive special consideration.

313 **Outdoor Machinery Exposed to Sun's Rays.** Outdoor machinery not protected from the sun's rays at times of heavy load, shall receive special consideration

314 **Measurement of the Ambient Temperature During Tests of Machinery.** The ambient temperature is to be measured by means of several thermometers placed at diffe ent points around and half-way up the machine, at a distance of 1 to 2 meters (3 to 6 feet), and protected from drafts, and abnormal heat radiation, preferably as in **§316**.

315 The value to be adopted for the ambient temperature during a test, is the mean of the readings of the thermometers (placed as above), taken at equal intervals of time during the last quarter of the duration of the test.

316 In order to avoid errors **due to the time lag** between the temperature of large machines and the variations in the ambient air, all reasonable precautions must be taken to reduce these variations and the errors arising therefrom. Thus, the thermometer for determining the ambient temperature shall be immersed in a suitable liquid, such as oil, in a suitable heavy metal cup. This can be made to respond to various rates of change, by proportioning the amount of oil to the metal in the containing cup. A convenient form for such an oil-cup consists of a massive metal cylinder, with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. The larger the machine under test, the larger should be the metal cylinder employed as an oil-cup in the determination of the ambient temperature. The smallest size of oil cup employed in any case shall consist of a metal cylinder 25 mm. in diameter and 50 mm. high (1 in. in diameter and 2 in. high).

317 **Thermometers used for taking temperatures of Machinery** shall be covered by felt pads 3 mm. ($\frac{1}{8}$ inch) thick and 4 x 5 cm. wide ($1\frac{1}{2}$" x 2"), cemented on; oil putty may be used for stationary and small apparatus.

318 **In Transformer Testing,** and sometimes in testing other machines, it may be desirable to avoid errors due to time lag in temperature changes, by employing an idle unit of the same size and subjected to the same conditions of cooling as the unit under test, for obtaining the ambient temperature as described in **§310.**

319 Where **machines are partly below the floor line** in pits, the temperature of the rotor shall be referred to a weighted mean of the pit and room temperatures, the weight of each being based on the relative proportions of the rotor in and above the pit. Parts of the stator constantly in the pit shall be referred to the ambient temperature in the pit.

320 **Correction for the Deviation of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test, from the Standard Ambient Temperature of Reference.** Numerous experiments have shown that deviation of the temperature of the cooling medium

from that of the standard of reference, at the time of the heat run, has a negligible effect upon the temperature rise of the apparatus; therefore, no correction shall be applied for this deviation. It is, however, desirable that tests should be conducted at ambient temperatures not lower than 10°C.

321 *Exception*—A Correction shall be applied to the observed temperature rise of the windings of *Air-blast transformers*, due to difference in resistance, when the temperature of the ingoing cooling air differs from that of the standard of reference. This correction shall be the ratio of the inferred absolute ambient temperature of reference to the inferred absolute temperature of the ingoing cooling air, *i. e.* the ratio $274.5/(234.5 + t)$; where t is the ingoing cooling-air temperature.

Thus, a cooling-air room temperature of 30°C. would correspond to an inferred absolute temperature of 264.5° on the scale of copper resistivity, and the correction to 40°C. (274.5° inferred absolute temperature) would be 274.5 / 264.5 = 1.04, making the correction factor 1.04; so that an observed temperature rise of say 50°C. at the testing ambient temperature of 30°C. would be corrected to 50 × 1.04 = 52°C. this being the temperature rise which would have occurred had the test been made with the standard ingoing cooling-air temperature of 40°C.

322 **Duration of Temperature Test of Machine for Continuous Service.** The temperature test shall be continued until sufficient evidence is available to show that the maximum temperature and temperature rise would not exceed the requirements of the rules, if the test were prolonged until a steady final temperature was reached.

323 **Duration of Temperature Test of Machine with a Short-Time Rating.** The duration of the temperature test of a machine with a short-time rating shall be the time required by the rating. (See §§285 and 286).

324 **Duration of Temperature Test for Machine having more than One Rating.** The duration of the temperature test for a machine with more than one rating shall be the time required by that rating which produces the greatest temperature rise. In cases where this cannot be determined beforehand, the machine shall be tested separately under each rating

325 **Temperature Measurements during Heat Run.** Temperature measurements, when possible, shall be taken during operation, as well as when the machine is stopped. The highest figures thus obtained shall be adopted. In order to abridge the long heating period, in the case of large machines, reasonable overloads of current, during the preliminary period, are suggested for them.

TEMPERATURE MEASUREMENTS

340 **The Life of the Insulation of a Machine** depends in great measure upon the actual temperatures attained by the different parts, rather than on the rises of temperature in those parts.

341 **The Temperatures in the Different Parts of a Machine** which it would be desirable to ascertain, are the maximum temperatures reached in those parts.

343 **As it is Usually Impossible to Determine the Maximum Temperature** attained in insulated windings, it is convenient to apply a correction to the observable temperature, which approximates the difference between the actual maximum temperature and the observable temperature by the method used. This correction, or margin of security, is provided to cover the errors due to fallibility in the location of the measuring devices, as well as inherent inaccuracies in measurement and methods.

344 **In Determining the Temperature of Different Parts of a Machine** three methods are provided.

345 **Method No 1. Thermometer Method.**

This method consists in the determination of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest accessible part of the *completed* machine, as distinguished from the thermocouples or resistance coils embedded in the machine as described under Method No. 3.

346 **When Method No. 1 is Used,** the hottest-spot temperature for windings shall be estimated by adding a hottest-spot correction of 15°C to the highest temperature observed, in order to allow for the practical impossibility of locating any of the thermometers at the hottest spot.

347 *Exception.* When the thermometers are applied directly to the surfaces of bare windings, such as an edgewise strip conductor, or a cast copper winding, a hottest-spot correction of 5°C., instead of 15°C, shall be made. For commutators, collector rings, bare metallic surfaces not forming part of a winding, or for oil in which apparatus is immersed, no correction is to be applied.

348 **Method No. 2. Resistance Method.**

This method consists in the measurement of the temperature of windings by their increase in resistance, corrected* to the instant of

*Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied, so as to obtain as nearly as practicable the temperature at the instant of shut-down. This can sometimes be approximately effected by plotting a curve, with temperature readings as ordinates and times as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied. In transformers of 200 kv-a. and less the measured temperature shall be increased one degree for every minute between the instant of shut-down and the time of the final temperature measurement, provided this time does not exceed three minutes.

In cases where successive measurements show *increasing* temperatures after shut-down, the highest value shall be taken.

In testing air blast transformers the air supply should be shut off immediately at the end of the heat run and the air duct closed to prevent a further circulation of air. In checking the temperature ascertained by the Resistance Method, the reading of all the thermometers in contact with coils should be noted carefully and the maximum temperature reached by any of them, if higher than that determined by resistance, should be taken as the maximum observable temperature of the windings. When the above procedure has been followed, a hottest spot correction of 5° shall be applied.

shut-down when necessary. In the application of this method, *thermometer measurements shall also be made* whenever practicable without disassembling the machine† in order to increase the probability of revealing the highest observable temperature. Whichever measurement yields the higher temperature, that temperature shall be taken as the "highest observable" temperature and a hottest-spot correction of 10°C added thereto.

349 **The Temperature Coefficient of Copper** shall be deduced from the formula $1/(234.5 + t)$. Thus, at an initial temperature $t = 40°C.$, the temperature co-efficient of increase in resistance per degree centigrade rise, is $1/(274.5) = 0.00364$. The following table, deduced from the formula, is given for convenience of reference.

TABLE II.

Temperature Coefficients of Copper Resistance.

Temperature of the winding, in degrees C. at which the initial resistance is measured.	Increase in resistance of copper per °C., per ohm of initial resistance.
0	0.00 427
5	0.00 418
10	0.00 409
15	0.00 401
20	0.00 393
25	0.00 385
30	0.00 378
35	0.00 371
40	0.00 364

350 **In Circuits of Low Resistance,** other than transformer windings (see **§351**), where joints and connections form a considerable part of the total resistance, the measurement of temperature by the resistance method shall not be used.

351 **The Temperature of the Windings of Transformers** is always to be ascertained by Method 2. In the case of air-blast transformers, it is especially important to place thermometers on the coils near the air outlet. See **§348.**

352 **Method No. 3. Embedded Temperature-Detector Method.**

This method consists in the use of thermo-couples or resistance temperature detectors, located as nearly as possible at the estimated hottest spot. When method No. 3 is used, it shall, when required, be checked by method No. 2; the hottest spot shall

†As one of the few instances in which the thermometer check cannot be applied in Method No. 2, the rotor of a turbo-alternator may be cited

then be taken to be the highest value by either method, the required correction factors (§348 and §356) being applied in each case.

353 **By Building into the Machine** suitably placed temperature detectors, a temperature not much less than that of the hottest spot will probably be disclosed. When these devices are adopted for such temperature determinations, a liberal number shall be employed, and all reasonable efforts, consistent with safety, shall be made to locate them at the various places where the highest temperatures are likely to occur.

354 **Temperature-Detectors** should be placed in at least two sets of locations. One of these should be between a coil-side* and the core, and one between the top and bottom coil-sides where two coil-sides per slot are used. Where only one coil-side per slot is used, one set of detectors shall be placed between coil-side and core, and one set between coil-side and wedge.

355 Method No. 3 should be applied to all stators of machines with cores having a width of 50 cm. (20 in.) and over. It should also be applied to all machines of 5000 volts and over, if of over 500 kv-a., regardless of core width. This method is not required for induction-regulators, which shall be tested as transformers.

356 **Correction Factor for Method No. 3.**—In the case of two-layer windings, with detectors between coil-sides, and between coil-side and core, add 5° C to the highest reading. In single-layer windings, with detectors between coil-side and core and between coil-side and wedge, add to the highest reading 10° C. plus 1° C. per 1000 volts above 5000 volts of terminal pressure.

TEMPERATURE LIMITS

375 Table III, page 38, gives the limits for the hottest-spot temperatures of insulations. The permissible limits are indicated in column 1 of the Table. The limits of temperature rise permitted under rated-load conditions are given in column 2, and are found by subtracting 40° C. from the figures in column 1. Whatever be the ambient temperature at the time of the test, the rise of temperature must never exceed the limits in column 2 of the table. The highest temperatures, and temperature rises, attained in any machine at the output for which it is rated, must not exceed the values indicated in the Table and clauses following.

376 **Permissible Temperatures and Temperature Rises For Insulating Materials.** Table III (see next page) gives the highest temperatures and temperature rises to which various classes of insulating materials may be subjected, based on a standard ambient temperature of reference of 40°C.

377 NOTE. The Institute recognizes the ability of manufacturers to employ Class B insulation successfully at maximum temperatures of 150° C. and even higher. However, as sufficient data covering experience over a period of years at such temperatures are at present

*A coil side is one of the two active sides of the coil lying in a slot.

unavailable, the Institute adopts 125° C as a conservative limit for this class of insulation, and any increase above this figure should be the subject of special guarantee by the manufacturer.

TABLE III.

Permissible Temperatures and Temperature Rises for Insulating Materials.

Class	Description of Material	1 Maximum Temperature to which the material may be subjected	2 Maximum Temperature Rise
A.	Cotton, silk, paper and similar materials, when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil; also enameled wire*	105°C	65°C
B.	Mica, asbestos and other materials capable of resisting high temperatures, in which any Class A. material or binder is used for structural purposes only, and may be destroyed without impairing† the insulating or mechanical qualities of the insulation	125°C	85°C
C.	Materials capable of resisting higher temperatures than Class B, such as pure mica, porcelain, quartz, etc.	No limits specified.	

*For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be 10°C below the limits fixed for Class A, in Table III.

†The word impair is here used in the sense of causing any change which would disqualify the insulation for continuous service.

378 When a lower-temperature class material is comprised in a completed product to such an extent, or in such ways, that its subjection to the temperature limits allowed for the higher-temperature class material, with which it is associated, would affect the integrity of the insulation either mechanically or electrically, the permissible temperature shall be fixed at such a value as shall afford ample assurance that

no part of the lower-temperature class material shall be subjected to temperatures higher than those approved by the Institute and set forth above.

379 **TABLE IV**

Permissible Hottest Spot Temperatures and Limiting Observable Temperature Rises in Other than Water-Cooled Machinery‡

		Class	A*	B
		Permissible Hottest-Spot Temperature	105°	125°
METHOD I THERMOMETER ONLY See §§346 to 347		Hottest Spot Correction	15°	15°
		Limiting Observable Temperature	90°	110°
		Limiting Observable Temperature Rise	50°	70°
METHOD II RESISTANCE See §348		Hottest Spot Correction	10°	10°
		Limiting Observable Temperature	95°	115°
		Limiting Observable Temperature Rise	55°	75°
METHOD III EMBEDDED TEMPERATURE DETECTORS See §§ 352 to 356	Double-Layer Windings. For all Voltages	Hottest Spot Correction	5°	5°
		Limiting Observable Temperature	100°	120°
		Limiting Observable Temperature Rise	60°	80°
	Single-Layer Windings. For 5000 volts or less	Hottest Spot Correction	10°	10°
		Limiting Observable Temperature	95°	115°
		Limiting Observable Temperature Rise	55°	75°
	Single-Layer Windings. For more than 5000 Volts	Hottest Spot Correction	10° + (*E*—5)†	10° + (*E*—5)
		Limiting Observable Temperature	95° — (*E*—5)	115° — (*E*—5)
		Limiting Observable Temperature Rise	55° — (*E*—5)	75° — (*E*—5)

*For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be 10°C. below the limits fixed for class A.

†In these formulas, *E* represents the rated pressure between terminals in kilovolts. Thus for a three-phase machine with single-layer winding, and with 11 kilovolts between terminals, the hottest-spot correction to be added to the maximum observable temperature will be 16°C.

‡For exceptions to these limitations see §§301, 717a, 721, 726, 729, 731, 740a, and Special Cases §§385 to 392.

Special Cases of Temperature limits.

385 Temperature of Oil. The oil in which apparatus is permanently immersed shall in no part have a temperature, observable by thermometer, in excess of 90°C.

386 Water-Cooled Transformers. In these the hottest-spot temperature shall not exceed 90°C.

386A Enclosed Motors and Generators. In an enclosed machine (see **§164**) the limiting observable temperature and the limiting observable temperature rise shall be taken as 5°C. higher than in Table IV. This is not to be interpreted as an increase in the permissible hottest spot temperature, but is in recognition of the lesser difference between the hottest spot temperature and the observable temperature within an enclosed machine. This rule does not apply to those types of machines defined in **§§163, 165** and **167**.

387 Railway Motor Temperature Limits, see **§804** and **805**

387A. Automobile Propulsion Motors and Generators, see **§838.**

388 Squirrel-Cage and Amortisseur Windings. In many cases the insulation of such windings is largely for the purpose of making the conductors fit tightly in their slots, and the slightest effective insulation is ample. In other cases, there is practically no insulating material on the windings. Consequently, the temperature rise may be of any value such as will not occasion mechanical injury to the machine.

389 Collector Rings. The temperature of collector rings shall not be permitted to exceed the " hottest-spot " values set forth in **§376** and **379** for the insulations employed either in the collector rings themselves or in adjacent insulations whose life would be affected by the heat from the collector rings.

390 Commutators. The observable temperature shall in no case be permitted to exceed the values given in **§376** and **379** for the insulation employed, either in the commutator or in any insulation whose life would be affected by the heat of the commutator. These temperature limits are intended only to protect the insulation of the commutator and of the adjacent parts, and are not intended as a criterion of successful commutation. See **§402**.

391 Cores. The temperature of those parts of the iron core in contact with insulating materials must not be such as to occasion in those insulating materials temperatures or temperature rises in excess of those set forth in **§376** and **379**.

392 Other parts, (such as brush-holders, brushes, bearings, pole-tips, cores, etc.) All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material, may be operated at such temperatures as shall not be injurious in any other respect.

METHODS OF LOADING TRANSFORMERS FOR TEMPERATURE TESTS

393 Whenever practicable, transformers should be tested under conditions that will give losses approximating as nearly as possible to those obtained under normal or specified load conditions, maintained for the required time (See **§322** to **324**). The maximum temperature rises measured during this test should be considered as the observable temperature rises for the given load.

An approved method of making these tests is the "**loading-back**" method. The principal variations of this method are:

394 **With duplicate single-phase transformers.**

Duplicate single-phase transformers may be tested in banks of two, with both primary and secondary windings connected in parallel. Normal magnetizing voltage should then be applied and the required current circulated from an auxiliary source. One transformer can be held under normal voltage and current conditions, while the other may be operating under slightly abnormal conditions.

395 **With one three-phase transformer.**

One three-phase transformer may be tested in a manner similar to **§394** provided the primary and secondary windings are each connected in delta for the test. Normal three-phase magnetizing voltage should be applied and the required current circulated from an auxiliary single-phase source.

396 **With three single-phase transformers.**

Duplicate single-phase transformers may be tested in banks of three, in a manner similar to **§395** by connecting both primary and secondary windings in delta, and applying normal three-phase magnetizing voltage and circulating the required current from an auxiliary single-phase source.

397 NOTE:— Among other methods that have a limited application and can be used only under special conditions may be mentioned—

(1) Applying dead load by means of some form of rheostat.

(2) Running alternately for certain short intervals of time on open circuit and then on short circuit, alternating in this way until the transformer reaches steady temperature. In this test, the voltage for the open-circuit interval and the current for the short-circuit interval shall be such as to give the same integrated core loss, and the same integrated copper loss, as in normal operation.

ADDITIONAL REQUIREMENTS

398 **Short-Circuit Stresses.**

The Institute recognizes the self-destructibility, both mechanical and thermal, of certain sizes and types of machines, when subjected to severe short-circuits, and recommends that ample protection be provided in such cases, external to the machine if necessary.

Over-Speeds.

399 **All Types of Rotating Machines** shall be so constructed that they will safely withstand an over-speed of 25 per cent, except in the case of steam turbines, which, when equipped with emergency governors, shall be constructed to withstand 20 per cent over-speed.

400 **In the case of Series Motors,** it is impracticable to specify percentage values for the guaranteed over-speed, on account of the varying service conditions.

401 **Water-wheel Generators** shall be constructed for the maximum runaway speed which can be attained by the combined unit.

Momentary Loads.

402 **Continuously Rated Machines** shall be required to carry momentary loads of 150 per cent of the amperes corresponding to the continuous rating, keeping the rheostat set for rated load excitation, (See §§**281**, **764** and **803**.) and commutating machinery shall commutate successfully under this condition. Successful commutation is such that neither brushes nor commutator are injured by the test. In the case of direct-connected generators, this clause is not to be interpreted as requiring the prime mover to drive the generator at this overload.

403 **Machines for duty-cycle operation** shall be rated according to their equivalent load, either on the short-time or continuous basis, but if intended for operation with widely fluctuating loads, shall commutate successfully under their specified operating conditions. See §§**284** and **285**.

404 **Stalling Torque of Motors**

Motors for continuous service shall, except when otherwise specified, be required to develop a running torque at least 175 per cent of that corresponding to the running torque at their rated load, without stalling.

Obviously, duty-cycle machines must carry their peak loads without stalling.

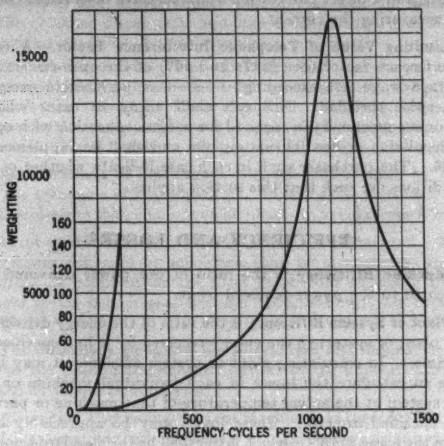
WAVE SHAPE

405 **The Sine Wave** shall be considered as standard, except where the departure therefrom is inherent in the operation of the system of which the machine forms a part.

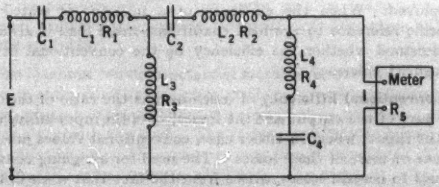
406 **Deviation Factor.**—A deviation factor (See §17) of the open-circuit terminal voltage wave of all synchronous machinery not exceeding 10% (0.10) is permissible except when otherwise specified.

407 **Telephone Interference Factor.**—The telephone interference factor is the ratio of the square root of the sum of the squares of the weighted values of all the sine-wave components (including in alternating waves both fundamental and harmonics) to the r.m.s. value of the wave. (See §17a.)

408 Relative Weighting of Components.—The weighting of the sine-wave components of different frequencies is as given in the following curve. (For trial only.)



The telephone interference factor of a voltage wave, corresponding to this weighting, may be measured by the use of the network shown in the following diagram.



Network Constants

$C_1 = 0.9$ mf.		$L_1 = 0.023$ henry		$R_1 = 5$ ohms $\pm 2\%$
$C_2 = 0.9$ "	$\pm 0.5\%$	$L_2 = 0.0205$ "	$\pm 0.5\%$	$R_2 = 12$ " $\pm 2\%$
		$L_3 = 0.068$ "		$R_3 = 73$ " $\pm 1\%$
$C_4 = 7.5$ "		$L_4 = 0.019$ "		$R_4 = 22.5$ " $\pm 2\%$
				$R_5 = 43$ " $\pm 1\%$

With this network the telephone interference factor is the ratio of the current I in micro-amperes in the meter branch of the network to

the voltage E applied to the external terminals of the network. The measurement may be made on the low tension side of a potential transformer. A sensitive vacuum thermocouple provided with a shunt, and a direct current mil-ammeter have been found convenient for measuring the current.

409 **Limiting Value of Telephone Interference Factor.**—A telephone interference factor (See §§**17a** and **407**) of the open-circuit terminal voltage wave not exceeding*———— is permissible except when otherwise specified. This rule shall apply in cases where synchronous machinery is required for use in connection with open-wire transmission or distribution circuits and shall be supplementary to §**406**. The purchaser shall in each case indicate whether or not the conditions are such that this section applies.

EFFICIENCY AND LOSSES

420 **Machine Efficiency** is the ratio of the power delivered by the machine to the power received by it.

421 **Plant or System Efficiency** is the ratio of the energy delivered from the plant or system to the energy received by it in a specified period of time.† In calculating plant or system efficiency it may be desirable to calculate the losses in each individual machine or part of the system at the actual temperature of that machine or part during the specified interval. These losses may be appreciably different from the losses at 75°C., which latter shall be the standard temperature of reference for all efficiency guarantees. See §**432**.

422 **In the Case of Machinery two Efficiencies are Recognized,** conventional efficiency (see §**423**) and directly measured efficiency. Unless otherwise specified, the conventional efficiency is to be employed. When the efficiency of a machine is stated without specific reference to the load conditions, rated load is always to be understood whether the efficiency be the conventional or directly measured efficiency.

423 **Conventional Efficiency** of machinery is the ratio of the output to the sum of the output and the losses; or of the input minus the losses to the input; when, in either case, conventional values are assigned to one or more of these losses. The need for assigning conventional values to certain losses, arises from the fact that some of the losses in electrical machinery are practically indeterminable, and must, in many cases, either be approximated by an approved method of test, or else values recommended by the Institute and designated "conventional" values shall be employed for them in arriving at the "conventional efficiency."

425 **Directly-Measured Efficiency.** Input and output determinations of efficiency may be made directly, measuring the output by brake,

*The appropriate limiting value of this factor has not yet been determined, but will be recommended after further experience shall have been obtained.

†An exception should be noted in the case of the efficiency of storage batteries.

or equivalent, where applicable. Within the limits of practical application, the circulating-power method, sometimes described as the Hopkinson or "loading-back" method, may be used.

426 **Values of the Indeterminate Losses** may also be obtained by brake or other direct test, and used in estimating actual efficiencies of similar machines, by the separate-loss method.

427 **Normal Conditions.** The efficiency shall correspond to, or be corrected to, the normal conditions herein set forth, which shall be regarded as standard. These conditions include voltage, current, power-factor, frequency, wave-shape, speed, temperature, or such of them as may apply in each particular case.

428 **Measurement of Efficiency.** Electric power shall be measured at the terminals of the apparatus. In polyphase machines, sufficient measurements shall be made on all phases to avoid errors of unbalance.

429 **Point at Which Mechanical Power Shall be Measured.** Mechanical power delivered by machines, shall be measured at the pulley, gearing, or coupling, on the rotor shaft, thus excluding the loss of power in the belt or gear friction. See, however, an exception in §800.

430 **The Efficiency Specified for Alternators and Transformers** shall be of the ratio of the kilowatt output to the kilowatt input at the rated kv-a. and power factor.

431 **Efficiency of Alternating-Current Machinery in regard to Wave Shape.** In determining the efficiency of alternating-current machinery, the sine wave is to be considered as standard, unless a different wave form is inherent in the operation of the system. See §405.

432 **Temperature of Reference for Machine Efficiency.** The efficiency, at all loads, of all apparatus, shall be corrected to a reference temperature of 75°C, but tests may be made at any convenient ambient temperature, preferably not less than 15°C. See §§348 and 445.

433 **The losses in constant-potential machinery,** either of the stationary type, or of the constant-speed rotary type, are of two classes; namely, those which remain substantially constant at all loads, and those which vary with the load. The former include iron losses, windage and friction, also I^2R losses in any shunt windings. The latter include I^2R losses in series windings. The constant losses may be determined by measuring the power required to operate the machine at no load, deducting any series I^2R losses. The variable loss at any load may be computed from the measured resistance of the series windings and the given load current.

434 **Stray Load Losses.** The above simple method of determining the losses and hence the efficiency is only approximate, since the losses which are assumed to be constant do actually vary to some extent with the load, and also because the actual loss in the copper windings is sometimes appreciably greater than the calculated I^2R loss. The difference between the approximate losses, as above de-

termined, and the actual losses, is termed the "stray load losses"*. These latter are due to distortions in electric or magnetic fluxes from their no-load distributions or values, brought about by the load current. They are usually only approximately measurable, or may be indeterminable.

TABLE V

Classification of Losses in Machinery

435 Losses in machinery may be classified as follows:

Accurately Measurable or Determinable	*Approximately Measurable or Determinable*	*Indeterminable*
a. No-Load Core Losses including eddy-current losses in conductors at no-load	*c*. Brush Friction Loss	*h*. Iron Loss due to flux distortion.
b. Load I^2R losses in windings No-Load I^2R losses in windings	*d*. Brush-Contact Loss	*i*. Eddy-Current losses in conductors due to transverse fluxes occasioned by the load currents.
	e. Losses due to windage and to bearing friction	*k*. Eddy-Current losses in conductors due to tooth saturation resulting from distortion of the main flux.
	f. Extra copper loss in transformer windings, due to stray fluxes caused by load currents	*l*. Tooth-frequency losses due to flux distortion under load.
	g. Dielectric Losses.	*m*. Short-Circuit Loss of Commutation.

436 **Evaluation of Losses.** The larger individual losses are either accurately or approximately determinable, but certain of the indeterm nable losses reach values in various kinds of machinery which require that they should be taken into account.

LOSSES TO BE TAKEN INTO ACCOUNT IN VARIOUS TYPES OF MACHINES

440 **Direct-Current Commutating Motors and Generators.**

Core losses. See §452.

I^2R loss in all windings.

Brush contact I^2R loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; *i. e.*, 2 volts

*In Table V, the stray load losses include f, h, i, k, l and m; but do not include increased core losses due to increased excitation for compensating internal drop under load.

for total brush drop, either carbon or graphite brushes. (See §§454 and 819). In the case of low voltage automobile propulsion motors and generators this loss should be determined experimentally; see §839.

Friction of bearings and windage.

Rheostat losses, when present.

Brush friction.

All indeterminable load losses (including stray-load iron losses) which may be important, which vary with the design, and for which no satisfactory method of determination has been found, shall be included as zero per cent in estimating conventional efficiency.

441 Synchronous Motors and Generators.

Core losses. See §452.

I^2R loss in all windings, based upon rated k-va. and power factor.

Stray load-losses. In approximating these losses, the method described in §458 shall be employed.

Friction of bearings and windage.

Brush friction and brush-contact loss is negligible, except in the case of revolving armature machines.

Rheostat losses, when present, corresponding to rated kv-a. and power factor.

442 Induction Machines.

Core losses. See §452.

I^2R losses in all windings.

Stray load-losses. In approximating these losses, the method described in §459 shall be employed

Brush friction when collector rings are present.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §454.

Friction of bearings and windage.

443 Commutating A-C. Machines

Core losses. See §452.

I^2R losses in all windings.

Brush friction.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §454 and 819.

Friction of bearings and windage.

Short-circuit loss of commutation. Iron loss due to flux distortion. Eddy-current losses due to fluxes varying with load and saturation.	The Institute is not at this time prepared to make recommendations for approximating these losses.

444 Synchronous Converters.

Core losses. See §452.

I^2R losses in all windings, based on rated kw. and unity power factor. The I^2R losses in the armature winding shall be derived from those corresponding to its use as a direct-current generator, by using suitable factors.

Brush friction.

Rheostat losses when present, corresponding to rated kw. and unity power factor.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §454.

Short-circuit loss of commutation.
Iron loss due to flux distortion when present.
Eddy-current losses due to fluxes varying with load and saturation.
} These losses, while usually of low magnitude, are erratic, and the Institute is not at this time prepared to make recommendations for approximating them.

Friction of bearings and windage.

For the booster type of synchronous converter, where the booster forms an integral part of the unit, its losses shall be included in the total converter losses in estimating the efficiency.

445 Transformers

No-load losses. These include the core loss, and the I^2R loss due to the exciting current, also the dielectric loss in the insulation. (See §470).

Load losses. These include I^2R losses, and stray load-losses due to eddy currents caused by fluxes varying with load. (See §471).

DETERMINATION OR APPROXIMATION OF LOSSES IN ROTATING MACHINERY

450 Bearing Friction and Windage may be determined as follows. Drive the machine from an independent motor. the output of which shall be suitably determined. The machine under test shall have its brushes removed and shall not be excited. This output represents the bearing friction and windage of the machine under test.

The bearing friction and windage of induction motors may be measured by running motors free at the lowest voltage at which they will rotate continuously at approximately rated speed; the watts input, minus I^2R loss, under these conditions being taken as the friction and windage.

In the case of engine-type generators, the windage and bearing friction loss is ordinarily very small,amounting to a fraction of one per cent of the output. In these rules this loss is neglected owing to its small value and the difficulty of measuring it.

451 **Brush Friction of Commutator and Collector Rings.** Follow the test of **§450,** taking an additional reading with the brushes in contact with the commutator or collector rings. The difference between the output obtained in the test in **§450** and this output shall be taken as the brush friction. Note: The surfaces of the commutator and brushes should already be smooth and glazed from running when this test is made.

452 **Core Loss.** Follow the test of **§451** with an additional reading taken in a similar manner, except that the machine is to be excited so as to produce at the terminals a voltage corresponding to the calculated internal voltage for the load under consideration. The difference between the output obtained by this test and that obtained by test under Sec. **451** shall be taken as the core loss.

For Synchronous Machines, the internal voltage shall be determined by correcting the terminal voltage for the resistance drop only.

The Core Loss of Induction Motors may be determined by measuring the watts input to the motor when running free at rated voltage and frequency and subtracting therefrom the no-load copper loss, bearing friction and windage.

454 **The Brush-Contact I^2R Loss** depends largely upon the material of which the brush is composed. As indicating the range of variation the following table will be of interest:

TABLE VI.

Brush-Contact Drop.

Grade of Brush	Volts drop across one brush-contact. (Average of positive and negative brushes)
Hard Carbon	1.1
Soft Carbon	0.9
Graphite	0.5 to 0.8
Metal-Graphite types	0.15 to 0.5 (The former for largest proportion of metal)

One volt drop per brush shall be considered as the Institute Standard drop corresponding to the I^2R brush-contact loss, for carbon and graphite brushes. Metal-graphite brushes shall be considered as special. See **§819.**

455 **Field-Rheostat Losses** shall be included in the generator losses where there is a field rheostat in series with the field magnets of the generator, even when the machine is separately excited.

456 **Ventilating Blower.** When a blower is supplied as part of a machine set, the power required to drive it shall be charged against the complete unit; but not against the machine alone.

457 Losses in Other Auxiliary Apparatus. Auxiliary apparatus, such as a separate exciter for a generator or motor, shall have its losses charged against the plant of which the generator and exciter are a part, and not against the generator. An exception should be noted in the case of turbo-generator sets with direct-connected exciters, in which case the losses in the exciter shall be charged against the generator. The actual energy of excitation and the field-rheostat losses, if any, (see §455) shall be charged against the generator.

458 Stray Load-Losses in Synchronous Generators and Motors. These include iron losses, and eddy-current losses in the copper, due to fluxes varying with load and also to saturation.

Stray load-losses are to be determined by operating the machine on short circuit and at rated-load current. This, after deducting the windage and friction and I^2R loss, gives the stray load-loss for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to specify a method for measuring them.

459 Stray Load-Losses in Induction Machines.

These include eddy-current losses in the stator copper, and other eddy-current losses due to fluxes varying with the load. In windings consisting of relatively small conductors, these eddy-current losses are usually negligible.

With rotor removed, measure the power input to the stator with different values of current at the rated frequency. The curve plotted with these values gives the combined I^2R and stray load-losses due to eddy currents in the stator copper. Deduct the I^2R loss determined from the resistance, and the difference will represent the stray load-losses corresponding to the various currents. While this method is not accurate for some types of motors it usually represents a sufficiently good approximation.

460 Polyphase Induction-Motor Rotor I^2R Loss. This should be determined from the slip, whenever the latter is accurately determinable, using the following equation:

$$\text{Rotor } I^2R \text{ loss} = \frac{\text{Output} \times \text{slip}}{1 - \text{slip}}$$

In large slip-ring motors, in which the slip cannot be directly measured by loading, the rotor I^2R loss shall be determined by direct resistance measurement; the rotor full-load current to be calculated by the following equation:

$$\text{Current per ring} = \frac{\text{watts output}}{\text{rotor voltage at stand-still} \times \sqrt{3} \times K}$$

This equation applies to three-phase rotors. For rotors wound for two phase, use 2 instead of the $\sqrt{3}$. K may be taken as 0.95 for motors of 150 kw. or larger. The factor K usually decreases as the size of motor is reduced, but no specific value can be stated for smaller sizes.

DETERMINATION OR APPROXIMATION OF LOSSES IN TRANSFORMERS

470 No-Load Losses. These shall be measured with open secondary circuit at the rated frequency, and with an applied primary voltage giving the rated secondary voltage plus the IR drop which occurs in the secondary under rated load conditions.

471 Load Losses. These include I^2R and stray load-losses. They shall be measured by applying a primary voltage, preferably at rated frequency, sufficient to produce rated load current in the windings, with the secondary windings short circuited.

DIELECTRIC STRENGTH TESTS OF MACHINERY

480 Basis for Determining Test Voltages. The test voltage which shall be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the machinery, and its normal operating voltage, upon the nature of the service in which it is to be used, and upon the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended, have been determined as reasonable and proper for the great majority of cases, and are proposed for general adoption, except when specific reasons make a modification desirable.

481 Condition of Machinery to be Tested. Commercial tests shall, in general, be made with the completely assembled machinery and not with individual parts. The machinery shall be in good condition, and high-voltage tests, unless otherwise specified, shall be applied before the machine is put into commercial service, and shall not be applied when the insulation resistance is low owing to dirt or moisture. High-voltage tests shall be made at the temperature assumed under normal operation or at the temperature attained under the conditions of commercial testing. High-voltage tests to determine whether specifications are fulfilled, are admissible on new machines only. Unless otherwise agreed upon, high-voltage tests of a machine shall be understood as being made at the factory.

482 Points of Application of Voltage. The test voltage shall be successively applied between each electric circuit and all other electric circuits and metal parts grounded.

483 Interconnected Polyphase Windings are considered as one circuit. All windings of a machine except that under test, shall be connected to ground.

484 Frequency, Wave Form and Test Voltage. The frequency of the testing circuit shall not be less than the rated frequency of the apparatus tested. A sine-wave form is recommended. See §405. The test shall be made with alternating voltage having a crest value equal to $\sqrt{2}$ times the specified test voltage. In d.c. machines, and in the general commercial application of a.c. machines, the testing frequency of 60 cycles per second is recommended.

485 Duration of Application of Test Voltage. The testing voltage for all classes of apparatus shall be applied continuously for a period of 60 seconds. See exception, **485A.**

485A *Exception*—**For All Standard Devices Produced in Large Quantities** and with a standard test pressure of 2500 volts or less, a test pressure applied for one second 20 per cent higher than the one-minute test pressure will be satisfactory.

486 Apparatus for Use on Single-Phase, 3-Phase-Delta or 3-Phase-Star Circuits. Apparatus, such as transformers, which may be used in star connection on three-phase circuits, shall have the delta voltage of the circuits on which they may be used indicated on the rating plate and the test shall be based on such delta voltage

VALUES OF A-C. TEST VOLTAGES

500 The Standard Test for All Classes of Apparatus, Except‡ as Otherwise Specified, Shall be Twice the Normal Voltage of the Circuit to Which the Apparatus is Connected, Plus 1000 Volts.

501 *Exception*—**Alternating-Current Apparatus connected to Permanently Grounded Single-Phase Systems, for use on Permanently Grounded Circuits of more than 300 Volts,** shall be tested with **2.73** times the voltage of the circuit to ground + **1000** volts. **This** does not refer to three-phase apparatus with grounded star neutral.

502 *Exception*—**Distributing Transformers.** Transformers for primary pressures from 550 to 4500 volts, the secondaries of which are directly connected to consumers' circuits and commonly known as distributing transformers, shall be tested with 10,000 volts from primary to core and secondary combined. The secondary windings shall be tested with twice their normal voltage plus 1,000 volts.

503 *Exception*—**Auto-Transformers** used for starting purposes, shall be tested with the same voltage as the test voltage of the apparatus to which they are connected.

504 *Exception*—**Household Devices.**—Apparatus taking not over 660 watts† and fractional horsepower a-c. and d-c. motors in capacities of less than ½ h. p., or 373 watts, intended solely for operation on supply circuits not exceeding 275 volts, shall be tested with 900 volts, except in the case of heating devices, which latter shall be tested with 500 volts at operating temperature.

505 *Exception*—**Apparatus for use on Circuits of 25 Volts or Lower,** such as bell-ringing apparatus,* electrical apparatus used in automobiles, apparatus used on low-voltage battery circuits, etc., shall be tested with 500 volts.

506 *Exception*—**Field Windings of Alternating-Current Generators** shall be tested with 10 times the exciter voltage, but in no case with less than 1500 volts nor more than 3500 volts.

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†The present National Electric Code power limit for a single outlet.

*This rule does not include bell-ringing transformers of ratio 125 to 6 volts. See National Electric Code.

‡Exceptiocs §§501 to 512.

507 *Exception*—Field Windings‡ of Synchronous Machines, including motors and converters which are to be started from alternating-current circuits, shall be tested as follows:

a. When machines are started with fields short-circuited they shall be tested as specified in **§506**.

b. When machines are started with fields open-circuited and sectionalized while starting, they shall be tested with 5000 volts.

c. When machines are started with fields open-circuited and connected all in series while starting, they shall be tested with 5000 volts for less than 275-volt excitation and 8000 volts for excitation of 275 volts to 750 volts.

508 *Exception.* **Phase-Wound Rotors of Induction Motors.** The secondary windings of wound rotors of induction motors shall be tested with twice their normal induced voltage, plus 1000 volts. By normal induced voltage is here meant the voltage between slip rings on open circuit at standstill with normal voltage impressed on the primary.

When induction motors with phase-wound rotors are reversed, while running at approximately normal speed, by reversing the primary connections, the test shall be four times the normal induced voltage, plus 1000 volts.

509 *Exception*—**Switches and Circuit Control Apparatus** above 600 volts, shall be tested with 2½ times rated voltage, plus 2000 volts. See **§§720** to **759b**.

510 *Exception*—**Assembled Apparatus.** Where a number of pieces of apparatus are assembled together and tested as an electrical unit, they shall be tested with 15 per cent lower voltage than the lowest required on any of the individual pieces of apparatus.

510A *Exception.* **Meters and Instruments.** The Institute is not at present in a position to make a recommendation in regard to the dielectric tests of meters and instruments.

511 **Testing Transformers by Induced Voltage.** Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings, in place of using a separate testing transformer. By "required voltage", is meant a voltage such that the line end of the windings shall receive a test to ground equal to that required by the general rules.

512 **Transformers with Graded Insulation** shall be so marked. They shall be tested by inducing the required test-voltage in the transformer and connecting the successive line leads to ground.

Transformer windings permanently grounded within the transformer shall be tested by inducing the required test voltage in such windings. (See **§500**).

‡Series field coils should be regarded as part of the armature circuit and tested as such.

MEASUREMENT OF VOLTAGE IN DIELECTRIC TESTS OF MACHINERY

530 **Use of Voltmeters and Spark-Gaps in Insulation Tests.** When making insulation tests on electrical machinery, every precaution must be taken against the occurrence of any spark-gap discharges in the circuits from which the machinery is being tested. A non-inductive resistance of about one ohm per volt shall be inserted in series with one terminal of the spark gap. If the test is made with one electrode grounded, this resistance shall be inserted directly in series with the non-grounded electrode. If neither terminal is grounded, one-half shall be inserted directly in series with each electrode. In any case this resistance shall be as near the measuring gap as possible and not in series with the tested apparatus. The resistance will damp high-frequency oscillations at the time of breakdown and limit the current which will flow. A water tube is the most reliable form of resistor. Carbon resistors should not be used because their resistance may become very low at high voltages.

531 **For Machinery of Low Capacitance.** When the machinery under test does not require sufficient charging current to distort the high-voltage wave shape, or change the ratio of transformation, the spark gap should be set for the required test voltage and the testing apparatus adjusted to give a voltage at which this spark gap just breaks down. This adjustment should be made with the apparatus under test disconnected. The apparatus should then be connected, and with the spark gap about 20 per cent longer, the testing apparatus is again adjusted to give the voltage of the former breakdown, which is the assumed voltage of test. This voltage is to be maintained for the required interval.

532 **For Machinery of High Capacitance.** When the charging current of the machinery under test may appreciably distort the voltage wave or change the effective ratio of the testing transformer, the first adjustment of voltage with the gap set for the test voltage should be made with the apparatus under test connected to the circuit and in parallel with the spark gap.

When making arc-over tests of large insulators, leads, etc. partial arc-over of the tested apparatus may produce oscillations which will cause the measuring gap to discharge prematurely. The measured voltage will then appear too high. In such tests the "equivalent" ratio" of the testing transformer should be measured by gap to within 20% of the arc-over voltage of the tested apparatus with the tested apparatus in circuit. The measuring gap should then be greatly lengthened out and the voltage increased until the tested apparatus arcs over. This arc-over voltage should then be determined by multiplying the voltmeter reading by the equivalent ratio found above. Direct measurement of the spark-over voltage over one gap by another gap should always be avoided.

533 **Measurements with Voltmeter.** In measuring the voltage with a voltmeter, the instrument should preferably derive its

voltage from the high-tension circuit, either directly, or by means of a voltmeter coil placed in the testing transformer, or through an auxiliary *ratio transformer*. It is permissible to measure the voltage at other places, such as the transformer primary provided corrections can be made for the variations in ratio caused by the charging current of the machinery under test, or provided there is no material variation of this ratio. In any case, when the capacitance of the apparatus to be tested is such as to cause wave distortion, the testing voltage must be checked by a spark gap as set forth in §538, or by a crest voltage meter. If the crest-voltage meter is calibrated in crest volts, its readings must be reduced to the corresponding r. m. s. sinusoidal value by dividing with $\sqrt{2}$.

534 **Measurements with Spark Gaps.** If proper precautions are observed, spark gaps may be used to advantage in checking the calibration of voltmeters when set up for the purposes of high-voltage tests of the insulation of machinery.

535 **Ranges of Voltages.** For the calibrating purposes set forth in §534 the sphere-gap shall be used for voltages above 50 kv., and is to be preferred down to 30 kv. The needle spark-gap may, however, be used for voltages from 10 to 50 kv.

536 **The Needle Spark Gap.** The needle spark gap shall consist of new sewing needles, supported axially at the ends of linear conductors which are at least twice the length of the gap. There must be a clear space around the gap for a radius of at least twice the gap length.

537 The sparking distances in air between No. 00 sewing needle points for various root-mean-square sinusoidal voltages are as follows:

TABLE VII.

Needle-Gap Spark-Over Voltages

(At 25°C and 760 mm. barometer).

R M S Kilovolts	Millimeters	R.M.S. Kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33	50	90
30	41		

The above values refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

538 **The Sphere Spark-Gap.** The standard sphere spark-gap shall consist of two suitably mounted metal spheres. When used as speci-

fied below, the accuracy obtainable should be approximately 2 per cent.

No extraneous body, or external part of the circuit, shall be nearer the gap than twice the diameter of the spheres. By the "gap" is meant the shortest path between the two spheres.

The shanks should not be greater in diameter than 1/5th the sphere diameter. Metal collars, etc., through which the shanks extend, should be as small as practicable and should not, during any measurement, come closer to the sphere than the maximum gap length used in that measurement.

The sphere diameter should not vary more than 0.1 per cent and the curvature, measured by a spherometer, should not vary more than 1 per cent from that of a true sphere of the required diameter.

539 **In using the spherometer** to measure the curvature, the distance between the points of contact of the spherometer feet should be within the following limits:

TABLE VIII

Spherometer Specifications

Diameter of Sphere in m.m.	Distance between contact points in mm.	
	Maximum	Minimum
62.5	35	25
125	45	35
250	65	45
500	100	65

539A **In using Sphere Gaps** constructed as above, it is assumed that the apparatus will be set up for use in a space comparatively free from external dielectric fields. Care should be taken that conducting bodies forming part of the circuit, or at circuit potential, are not so located with reference to the gap that their dielectric fields are superposed on the gap; *e.g.*, the protecting resistance should not be arranged so as to present large masses or surfaces near the gap, even at a distance of two sphere diameters.

In case the sphere is grounded, the spark point of the grounded sphere should be approximately five diameters above the floor or ground

540 **The sparking distances between different spheres** for various r.m.s. sinusoidal voltages shall be assumed to be as follows:

TABLE IX.

Sphere-Gap Spark-Over Voltages

(At 25°C and 760 mm. barometric pressure)

Kilovolts	Sparking Distance in Millimeters.							
	62.5 mm. spheres		125 mm. spheres		250 mm. spheres		500 mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2	..	..	..	..	..	..
20	8.6	8.6	..	..	..	..	..	..
30	14.1	14.1	14.1	14.1	..	..	..	..
40	19.2	19.2	19.1	19.1	..	..	..	..
50	25.5	25.0	24.4	24.4	..	..	..	..
60	34.5	32.0	30.	30.	29	29	..	..
70	46.0	39.5	36	36	35	35	..	..
80	62.0	49.0	42	42	41	41	41	41
90		60.5	49	49	46	45	46	45
100			56	55	52	51	52	51
120			79.7	71	64	63	63	62
140			108	88	78	77	74	73
160			150	110	92	90	85	83
180				138	109	106	97	95
200					128	123	108	106
220					150	141	120	117
240					177	160	133	130
260					210	180	148	144
280					250	203	163	158
300						231	177	171
320						265	194	187
340							214	204
360							234	221
380							255	239
400							276	257

The sphere gap is more sensitive than the needle gap to momentary rises of voltage and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap. Open arcs should not be permitted in proximity to the gap during its operation, as they may affect its calibration.

AIR-DENSITY CORRECTION-FACTORS FOR SPHERE GAPS

541 **The Spark-Over Voltage,** for a given gap, decreases with decreasing barometric pressure and increasing temperature. This correction may be considerable at high altitudes.

The spacing at which it is necessary to set a gap to spark over at some required voltage, is found as follows: Divide the required voltage by the correction factor given below in Table X. A new

voltage is thus obtained. The spacing on the standard curves obtained from Table IX, corresponding to this new voltage, is the required spacing.

The voltage at which a given gap sparks over is found by taking the voltage corresponding to the spacing from the standard curves of Table IX, and multiplying by the correction factor.

When the variation from sea level is not great, the relative air density may be used as the correction factor; when the variation is great, or greater accuracy is desired, the correction factor corresponding to the relative air density should be taken from Table X below, in which

$$\text{Relative air density} = \frac{0.392\ b}{273 + t}$$

b = barometric pressure in mm.
t = temperature in deg. C.

Corrected curves may be plotted for any given altitude, if desired.

Values of relative air density and corresponding values of the correction factor are tabulated below. It will be seen that for values above .9, the correction factor does not differ greatly from the relative air density.

TABLE X.

Air-Density Correction Factors for Sphere Gaps

Relative air density	Diameter of standard spheres in mm.			
	62.5	125	250	500
0.50	0.547	0.535	0.527	0.519
0.55	0.594	0.583	0.575	0.567
0.60	0.640	0.630	0.623	0.615
0.65	0.686	0.677	0.670	0.663
0.70	0.732	0.724	0.718	0.711
0.75	0.777	0.771	0.766	0.759
0.80	0.821	0.816	0.812	0.807
0.85	0.866	0.862	0.859	0.855
0.90	0.910	0.908	0.906	0.904
0.95	0.956	0.955	0.954	0.952
1.00	1.000	1.000	1.000	1.000
1.05	1.044	1.045	1.046	1.048
1.10	1.090	1.092	1.094	1.096

INSULATION RESISTANCE OF MACHINERY

550 **The insulation resistance of a machine** at its operating temperature shall be not less than that given by the following formula:

$$\text{Insulation Resistance in megohms} = \frac{\text{voltage at terminals}}{\text{rated capacity in kv-a.} + 1000}$$

The formula only applies to dry apparatus. Such high values are not attainable in oil-immersed apparatus.

Insulation resistance tests shall, if possible, be made at a d.c. pressure of 500 volts. Since the insulation resistance varies with the pressure, it is necessary that, if a pressure other than 500 volts is to be employed in any case, this other pressure shall be clearly specified.

The order of magnitude of the values obtained by this rule is shown in the following table:

TABLE XI.

Insulation Resistance of Machinery

Rated Voltage of machine	Megohms		
	100 kv-a.	1000 kv-a.	10,000 kv-a.
100	0.091	0.05	—
1,000	0.91	0.50	0.091
10,000	9.1	5.0	0.91
100,000	—	50	9.1

551 It should be noted that the insulation resistance of machinery is of doubtful significance by comparison with the dielectric strength. The insulation resistance is subject to wide variation with temperature humidity and cleanliness of the parts. When the insulation resistance falls below that corresponding to the above rule, it can, in most cases of good design and where no defect exists, be brought up to the required standard by cleaning and drying out the machine. The insulation-resistance test may therefore afford a useful indication as to whether the machine is in suitable condition for the application of the dielectric test.

REGULATION

DEFINITIONS

560 **Regulation.** The regulation of a machine in regard to some characteristic quantity (such as terminal voltage or speed) is the change in that quantity occurring between any two loads. Unless otherwise specified, the two loads considered shall be zero load and rated load, and at the temperature attained under normal operation. The regulation may be expressed by stating the numerical values of the quantity at the two loads. or it may be ex-

pressed by the "percentage regulation", which is the percentage ratio of the change in the quantity occurring between the two loads, to the value of the quantity at either one or the other load, taken as the normal value. It is assumed that all parts of the machine affecting the regulation maintain constant temperature between the two loads, and where the influence of temperature is of consequence, a reference temperature of 75°C shall be considered as standard. If change of temperature should occur during the tests, the results shall be corrected to the reference temperature of 75°C.

The normal value may be either the no-load value, as the no-load speed of induction motors; or it may be the rated-load value, as in the voltage of a.c. generators.

It is usual to state the regulation of d-c. generators by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads.

561 The **Regulation of d-c. Generators** refers to changes in voltage corresponding to gradual changes in load and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

In determining the regulation of a compound-wound d-c. generator, two tests shall be made, one bringing the load down and the other bringing the load up, between no-load and rated load. These may differ somewhat, owing to residual magnetism. The mean of the two results shall be used.

562 In **constant-potential a-c. generators,** the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in per cent of normal rated-load voltage.

563 In **constant-current machines,** the regulation is the ratio of the maximum difference of current from the rated-load value (occurring in the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.

564 In **constant-speed direct-current motors,** and induction motors, the regulation is the ratio of the difference between full-load and no-load speeds to the no-load speed.

565 In **constant-potential transformers,** the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage.

566 In **converters, dynamotors, motor-generators and frequency converters,** the regulation is the change in the terminal voltage of the output side between the two specified loads. This may be expressed by giving the numerical values, or as the percentage of the terminal voltage at rated load.

567 In **transmission lines, feeders etc.,** the regulation is the change in the voltage at the receiving and between rated non-inductive load and no load, with constant impressed voltage upon the sending

end. The percentage regulation is the percentage change in voltage to the normal rated voltage at the receiving end.

568 In **steam engines, steam turbines and internal combustion engines,** the percentage speed regulation is usually expressed as the percentage ratio of the maximum variation of speed, to the rated-load speed in passing slowly from rated load to no load (with constant conditions at the supply.)

569 If the test is made by passing suddenly from rated load to no load, the immediate percentage speed regulation so derived shall be termed the **fluctuation.**

570 In **a hydraulic turbine,** or other water motor, the percentage speed regulation is expressed as the percentage ratio of the maximum variation in speed in passing slowly from rated load to no load (at constant head of water), to the rated-load speed.

571 In **a generator unit,** consisting of a generator combined with a prime mover, the speed or voltage regulation should be determined at constant conditions of the prime mover; *i.e.* constant steam-pressure, head, etc. It includes the inherent speed variations of the prime mover. For this reason, the regulation of a generator unit is to be distinguished from the regulation of either the prime mover, or of the generator combined with it, when taken separately.

CONDITIONS FOR TESTS OF REGULATION

580 **Speed and Frequency.** The regulation of generators is to be determined at constant speed, and of alternating-current apparatus at constant frequency.

581 **Power Factor.** In apparatus generating, transforming or transmitting alternating currents, the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is to a load in which the current is in phase with the *e.m.f.* at the output side of the apparatus.

582 **Wave Form.** In the regulation of alternating-current machinery receiving electric power, a sine wave of voltage is assumed, except where expressly specified otherwise. See **§405.**

583 **Excitation.** In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, as well as in alternating-current generators, the regulation is to be determined under such conditions as to maintain the field adjustment constant at that which gives rated-load voltage at rated-load current, as follows:

(1) In the case of separately-excited field magnets—constant excitation.

(2) In the case of shunt machines, constant resistance in the shunt-field circuit.

(3) In the case of series or compound machines, constant resistance shunting the series-field windings.

584 Tests and Computation of Regulation of A-C. Generators.

Any one of the three following methods may be used. They are given in the order of preference.

Method a.

The regulation can be measured directly, by loading the generator at the specified load and power factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators, and it becomes necessary to determine the regulation from such other tests as can be readily made.

585 Method b.

This consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero-power-factor saturation curve. The latter curve, or one approximat-

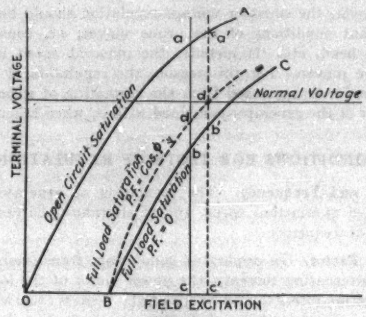


Fig. 1

ing very closely to it, can be obtained by running the generator with over excitation on a load of idle-running under-excited synchronous motors. The power factor under these conditions is very low and the load saturation curve approximates very closely the zero power factor saturation curve. From this curve and the open circuit curve, points for the load saturation curve, for any power factor, can be obtained by means of vector diagrams.

To apply Method b, it is necessary to obtain from test, the open-circuit saturation curve OA, Fig. 1, and the saturation curve BC at zero power factor and rated-load current. At any given excitation Oc, the voltage that would be induced on open circuit is ac, the terminal voltage at zero power factor is bc, and the apparent internal drop is ab. The terminal voltage dc at any other power factor can then be found by drawing an e.m.f. diagram* as in Fig. 2, where ϕ is

*Method b, for deducing the load saturation curve, at any assigned power factor, from no-load and zero power-factor saturation curves obtained by test, must be regarded as empirical. Its value depends upon the fact that experience has demonstrated the reasonable correctness of the results obtained by it.

an angle such that $\cos \phi$ is the power factor of the load, be the resistance drop (IR) in the stator winding, ba the total internal drop, and ac the total induced voltage; ba and ac being laid off to correspond with the values obtained from Fig. 1. The terminal voltage at power factor $\cos \phi$, is then cb of Fig. 2, which, laid off in Fig. 1, gives point d. By finding a number of such points, the curve Bdd' for power factor $\cos \phi$ is obtained and the regulation at this power factor (expressed in per cent) is $\frac{100 \times a'd'}{d'c'}$, since $a'd'$ is the rise in voltage when the load at power factor $\cos \phi$ is thrown off at normal voltage $c'd'$.

Generally, the ohmic drop can be neglected, as it has very little influence on the regulation, except in very low-speed machines where the armature resistance is relatively high, or in some cases where regulation at unity power factor is being estimated. For low power factors, its effect is negligible in practically all cases. If

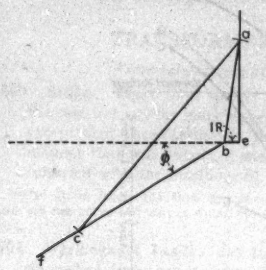


FIG. 2

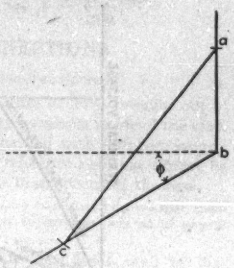


FIG. 3

resistance is neglected, the simpler e.m.f. diagram, Fig. 3, may be used to obtain points on the load saturation curve for the power factor under consideration.

586 Method c.

Where it is not possible to obtain by test a zero-power-factor saturation curve as in Method b, this curve can be estimated closely from open-circuit and short-circuit curves by reference to tests at zero power factor on other machines of similar magnetic circuit. Having obtained the estimated zero-power-factor curve, the load saturation for any other power factor is obtained as in method b.

Thus Method c is the same as Method b; except that the zero-power-factor curve must be estimated. This may be done as follows. In Fig. 4 OA is the open-circuit saturation curve and OE the short-circuit line as shown by test. The zero-power-factor curve corresponding to any given current BF will start from point B, and for machines designed with low saturation and low reactance, will follow parallel to OA as shown by the dotted curve BD, which is OA shifted horizontally parallel to itself by the distance OB. In high-speed machines, or in others

having low reactance and a low degree of saturation in the magnetic circuit, the zero-power-factor curve will lie quite close to BD, particularly in those parts that are used for determining the regulation. This is the case with many turbo-generators and high-speed water-wheel generators. In many cases, however, the zero-power-factor curve will deviate from BD, as shown by BC, and the deviation will be most pronounced in machines of high reactance, high saturation, and large magnetic leakage. The position of the actual curve BC with relation to BD, can be approximated with sufficient exactness by investigating the corresponding relation as obtained by test at zero power factor on machines of similar characteristics and magnetic circuit. Or curve BC can be calculated by methods based on the results of tests at zero power factor. After BC has been obtained, the saturation curve and regulation for any other power factor can be derived as in Method (b).

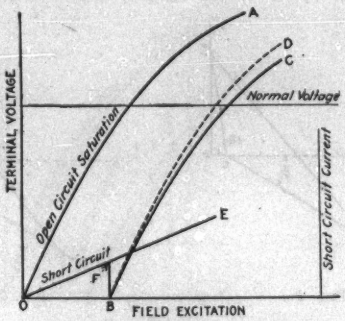


FIG. 4

587 Tests and Computation of Regulation for Constant-Potential Transformers.

The regulation can be determined by loading the transformer and measuring the change in voltage with change in load, at the specified power factor. This method is not generally applicable for shop tests, particularly on large transformers.

The regulation for any specified load and power factor can be computed from the measured impedance watts and impedance volts, as follows:

Let:

P = impedance watts, as measured in the short-circuit test and corrected to 75°C.

E_s = impedance volts, as measured in the short-circuit test.

IX = Reactance Drop in Volts.

I = Rated Primary Current.

E = Rated Primary Voltage.

q_r = percent drop in phase with current.
q_x = percent drop in quadrature with current

$$IX = \sqrt{E_z^2 - \left(\frac{P}{I}\right)^2}$$

$$q_r = 100\frac{P}{EI}$$

$$q_x = 100\frac{IX}{E}$$

588 Then—

1. For **unity power factor**, we have approximately:-

$$\text{Per cent regulation} = q_r + \frac{q_x^2}{200}$$

589 2. For **inductive loads** of power-factor m and reactive-factor n,

$$\text{Per cent regulation} = mq_r + nq_x + \frac{(mq_x - nq_r)^2}{200}$$

TRANSFORMER CONNECTIONS

(*These rules do not apply to auto-transformers*)

600 **Scope.** These rules specify the markings of leads brought out of the case but not the markings of winding terminals inside of the case, except that these terminals shall be marked with numbers in any manner that will permit of convenient reference and that cannot be confused with the markings of the leads brought out of the case.

NOTE—It is recognized that special cases will arise from time to time that these rules will not cover and that it would be very difficult to cover by any set of general rules.

601 **Markings of Leads.** (a) **In General.** The leads shall be distinguished from one another by marking each lead with a capital letter followed by a number. The letters to be used are: H for high-voltage leads, X for low-voltage leads and Y for tertiary-winding leads. The numbers to be used are **1, 2, 3,** etc.

NOTE.—By "tertiary winding" is meant a third winding that, compared with both of the other two windings, has smaller kv-a. rating than either or, if the kv-a. rating is the same as one or both of the other two, has lower voltage.

E. g., if a transformer has three separate windings, one for 1000 kv-a., 33,000 volts, one for 600 kv-a., 550 volts and one for 400 kv-a. 6,600 volts, the 400-kv-a. winding is the tertiary winding.

Or, if a transformer has three separate windings each with a capacity of 1,000 kv-a., and with voltages or 33,000, 6,600 and 550 respectively, the 550 volt winding is the tertiary winding.

According to this definition neither one of two similar windings arranged for series-parallel connection is to be classed as a tertiary winding.

(b) **A neutral Lead** shall be marked with the proper letter followed by O, *e.g.*, HO, XO.

EXCEPTION.—A lead brought out from the middle of a winding for some other use, than that of neutral lead, *e.g.*, a 50 per cent starting tap, shall be marked as a tap lead.

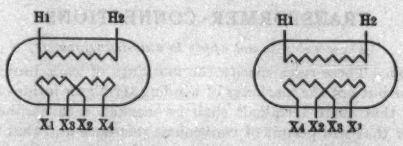
602 **Diagrammatic Sketch of Connections.** The manufacturer shall furnish with each transformer a complete diagrammatic sketch showing the leads and internal connections and their markings and the voltages obtainable with the various connections.

This sketch should preferably be on a metal plate attached to the transformer case.

SINGLE PHASE TRANSFORMERS

603 Order of Numbering Leads in any Winding. The leads of any winding (high-voltage, low-voltage or tertiary) brought out of case shall be numbered 1, 2, 3, 4, 5, etc., the lowest and highest numbers marking the full winding and the intermediate numbers marking fractions of winding or taps. All numbers shall be so applied that the potential difference from any lead having a lower number toward any lead having a higher number shall have the same sign at any instant.

If a winding is divided into two or more parts for series parallel connections, and the leads of these parts are brought out of case, the above rule shall apply for the series connection with the addition that the leads of each portion of winding shall be given consecutive numbers.



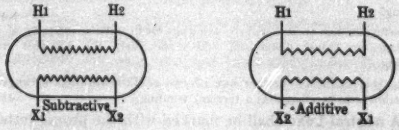
604 Relation of Order of Numbering Leads of Different Windings. The numbering of the high-voltage and low-voltage leads shall be so applied that when H_1 and X_1 are connected together and voltage applied to the transformer, the voltage between the highest numbered H lead and the highest numbered X lead shall be less than the voltage of the full high-voltage winding.

The same relation shall apply between high-voltage and tertiary and low-voltage and tertiary winding.

605 Polarity. When leads are marked in accordance with the above rules, the polarity of a transformer is

Subtractive when H_1 and X_1 are adjacent.

Additive when H_1 is diagonally located with respect to X_1.



606 Location of H_1 Lead. To simplify the work of connecting transformers in parallel it is recommended that the H_1 lead shall be brought out on the right hand side of the case, facing high-voltage side of the case.

607 Parallel Operation. Transformers having leads marked in accordance with these rules may be operated in parallel by connecting

similarly marked leads together, provided their ratio, voltages, resistances and reactances are such as to permit parallel operation.

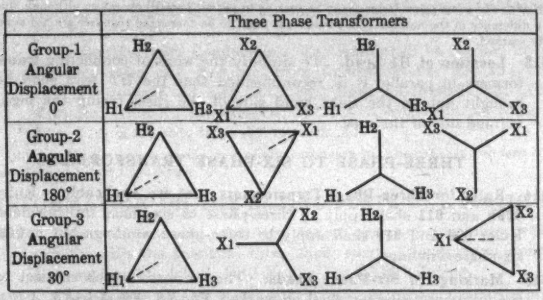
In some cases design may be such as to permit parallel operation, although due to the difference in the number of tap leads, the leads to be connected together may not have the same number.

THREE PHASE TRANSFORMERS

608 Marking of Full Winding Leads. The (3) high-voltage leads and the (3) low-voltage leads which connect to the full-phase windings, shall be marked H_1, H_2, H_3, and X_1, X_2, X_3. The full-phase winding of a tertiary winding shall be marked Y_1, Y_2, Y_3.

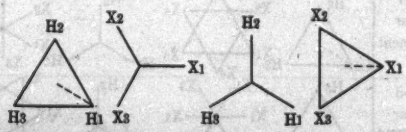
609 Relation between High- and Low-Voltage Windings. (a) The markings shall be so applied that if the phase sequence of voltage on the high voltage side is in the time order H_1, H_2, H_3 it is in the time order of, X_1, X_2 X_3 on the low-voltage side and, Y_1, Y_2 Y_3 for a tertiary winding.

(b) **Angular Displacement.** In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the three three-phase groups as shown. The angular displacement between the high-



voltage and low-voltage windings is the angle in each of the voltage vector diagrams between the lines passing from its neutral point through H1 and X1 respectively.

Group (3) may also be represented by the diagrams herewith and transformers marked according to them will parallel with those given above by connecting similarly marked leads. They are, therefore, superfluous but are given to simplify the application of the rule.



610 Tap Leads. (a) **Where tap leads** are brought out of the case (neutral lead excepted) they shall be marked with the proper letter

followed by the figures 4, 7, etc., for one phase, 5, 8, etc., for another phase and 6, 9, etc., for the third phase.

(b) **Star Connection.** The order of numbering tap leads shall be as follows: 4, 7, etc., from lead 1 towards neutral; 5, 8, etc., from lead 2 towards neutral; and 6, 9, etc., from lead 3 towards neutral.

611 Interphase Connection made Outside of Case. Where the interphase connections are made outside of case, the leads will be marked with the proper letter followed by the numbers 1, 4, 7, 10, etc., for one phase; 2, 5, 8, 11, etc., for the second phase; and 3, 6, 9, 12, etc., for the third phase.

The markings shall be so applied that when a star connection is made by joining together the highest numbered leads of each phase, all rules here given, excepting rule (**601b**) apply.

612 Parallel Operation. Transformers having leads marked in accordance with these rules may be operated in parallel by connecting similarly marked leads together provided their angular displacements are the same and provided also their ratios, voltages, resistances, and reactances are such as to permit parallel operation.

NOTE.—In some cases designs may be such as to permit parallel operation although, due to a difference in the number of tap leads, the leads to be connected together are not similarly marked.

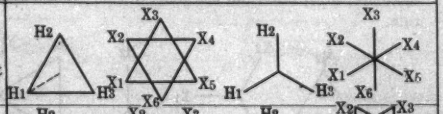
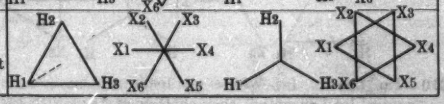
613 Location of H1 Lead. To simplify the work of connecting transformers in parallel it is recommended that the *H1* lead shall be brought out on the right hand side of the case, facing the high-voltage side of the case.

THREE-PHASE TO SIX-PHASE TRANSFORMERS

614 Rules for Three-Phase Transformers that are Applicable. Rules **609b** and **611** shall apply to three-phase to six-phase transformers. Rules **608** and **610** shall apply to three-phase windings but not to six-phase windings.

615 Markings of Six-Phase Leads. The six leads which connect to the full-phase windings shall be marked *X1*, *X2*, *X3*, *X4*, *X5*, *X6*.

616 Relation between Three-Phase and Six-Phase Windings. (a) The markings shall be so applied that if the phase sequence of voltage on the three-phase side is in the time order *H1*, *H2*, *H3*, it is in the time order of *X1*, *X2*, *X3*, *X4*, *X5*, *X6* on the six-phase side.

	Six Phase Transformers
Group-4 Angular Displacement 0°	
Group-5 Angular Displacement 30°	

(b) **Angular Displacement.** In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the two six-phase groups shown. The angular displacement between the high-voltage and low-voltage windings is the angle in each of the voltage vector diagrams from its neutral through *H1* and *X1* respectively.

617 Tap Leads. Where tap leads from low-voltage windings are brought out of the case (neutral lead excepted), they shall be marked as follows:

(a) **Diametrical Connection** tap leads shall be marked from the two ends of each phase winding towards the middle or neutral point in the following order; *X7*, *X13*, etc., from *X1* towards neutral; *X8*, *X14*, etc., from *X2* towards neutral; *X9*, *X15*, etc., from *X3* towards neutral; *X10*, *X16*, etc., from *X4* towards neutral; *X11*, *X17*, etc., from *X5* towards neutral; *X12*, *X18*, etc., from *X6* towards neutral.

A tap from the middle point of any phase winding, not intended as a neutral, shall be given a number determined by counting from *X1*, *X2* or *X3* and not from *X4*, *X5*, or *X6*; *e.g.*, if the only taps brought out are 50 per cent starting taps, they shall be numbered *X7*, *X8*, and *X9*.

(b) **Double Delta Connection.** Tap leads shall be marked in the following order; *X7*, *X13*, etc., from *X1* towards *X3*; *X8*, *X14*, etc., from *X2* towards *X4*; *X9*, *X15*, etc., from *X3* towards *X5*; *X10*, *X16*, etc., from *X4* towards *X6*; *X11*, *X17*, etc., from *X5* towards *X1*; *X12*, *X18*, etc., from *X6* towards *X2*.

NOTE.—For starting purposes it is generally customary to bring out only two taps from one delta and start three-phase.

INFORMATION ON THE RATING PLATE OF A MACHINE

620 It is recommended that the rating plate of machines which comply with the Institute rules shall carry a distinctive special sign, such as "A.I.E.E. 1918 Rating" or "A18" Rating.

621 The absence of any statement to the contrary on the rating plate of a machine implies that it is intended for continuous service and for the standard altitude and ambient temperature of reference. See **§§287, 305, 308** and **309.**

622 The rating plate of a machine intended to work under various kinds of rating must carry the necessary information in regard to those kinds of ratings.

STANDARDS FOR WIRES AND CABLES

TERMINOLOGY*

635 **Wire.**—A slender rod or filament of drawn metal.

The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition, the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire; while primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated, the term "wire" will be understood to include the insulation.

636 **Conductor.**—A wire or combination of wires not insulated from one another, suitable for carrying a single electric current.

The term "conductor" is not to include a combination of conductors insulated from one another, which would be suitable for carrying several different electric currents.

Rolled conductors (such as busbars) are, of course, conductors, but are not considered under the terminology here given.

637 **Stranded Conductor.**—A conductor composed of a group of wires, or of any combination of groups of wires.

The wires in a stranded conductor are usually twisted or braided together.

638 **Cable.**—(1) A stranded conductor (single-conductor cable); or (2) a combination of conductors insulated from one another (multiple-conductor cable).

The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering. The first kind of cable is a single conductor, while the second kind is a group of several conductors. The term "cable" is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of "cable." The term "cable" is a general one, and, in practise, it is usually applied only to the larger sizes. A small cable is called a "stranded wire" or a "cord," both of which are defined below. Cables may be bare or insulated, and the latter may be armored with lead, or with steel wires or bands.

639 **Strand.**—One of the wires, or groups of wires, of any stranded conductor.

640 **Stranded Wire.**—A group of small wires, used as a single wire.

A wire has been defined as a slender rod or filament of drawn metal. If such a filament is subdivided into several smaller filaments or strands, and is used as a single wire, it is called a "stranded wire." There is no sharp dividing line of size between a "stranded wire" and a "cable." If used as a wire, for example in winding inductance coils or magnets, it is called a stranded wire and not a cable. If it is substantially insulated, it is called a "cord," defined below.

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*From Circular No. 37 of the Bureau of Standards.

641 **Cord.**—A small cable, very flexible and substantially insulated to withstand wear.

There is no sharp dividing line in respect to size between a "cord" and a "cable," and likewise no sharp dividing line in respect to the character of insulation between a "cord" and a "stranded wire." Rubber is used as the insulating material for many classes of cords.

642 **Concentric Strand.**—A strand composed of a central core surrounded by one or more layers of helically-laid wires or groups of wires.

643 **Concentric-Lay Cable.**—A single-conductor cable composed of a central core surrounded by one or more layers of helically-laid wires.

644 **Rope-Lay Cable.**—A single-conductor cable composed of a central core surrounded by one or more layers of helically-laid groups of wires.

This kind of cable differs from the preceding in that the main strands are themselves stranded.

645 **N-Conductor Cable.**—A combination of N conductors insulated from one another.

It is not intended that the name as here given be actually used. One would instead speak of a "3-conductor cable," a "12-conductor cable" etc. In referring to the general case, one may speak of a "multiple-conductor cable" (as in definition §638 above.)

646 **N-Conductor Concentric Cable.**—A cable composed of an insulated central conducting core with $(N-1)$ tubular stranded conductors laid over it concentrically and separated by layers of insulation.

This kind of cable usually has only two or three conductors. Such cables are used in carrying alternating currents. The remark on the expression "N-conductor" given for the preceding definition applies here also.

647 **Duplex Cable.**—Two insulated single-conductor cables, twisted together.

They may or may not have a common insulating covering.

648 **Twin Cable.**—Two insulated single-conductor cables laid parallel, having a common covering.

649 **Triplex Cable.**—Three insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

650 **Twisted Pair.**—Two small insulated conductors, twisted together, without a common covering.

The two conductors of a "twisted pair" are usually substantially insulated, so that the combination is a special case of a "cord."

651 **Twin Wire.**—Two small insulated conductors laid parallel, having a common covering.

SPECIFICATION OF SIZES OF CONDUCTORS

652 The sizes of solid wires shall be stated by their diameter in mils, the American Wire Gage (Brown and Sharpe) sizes being taken as standard. The sizes of stranded conductors shall be stated by their cross-sectional area in circular mils. For brevity, in cases where the most

careful specification is not required, the sizes of solid wires may be stated by the gage number in the American Wire Gage, and the sizes of stranded conductors smaller than 250,000 circular mils (*i.e.*, No. 0000 A.W.G. or smaller) may likewise be stated by means of the gage number in the American Wire Gage of a solid wire having the same cross-sectional area. Futhermore, an exception is made in the case of "Flexible Stranded Conductors," for which see §655 below. In stating large cross-sections, it is sometimes convenient to use a circular inch (507 sq. mm.) instead of 1,000,000 circular mils.

STRANDING

653 Cables not requiring special flexibility shall be stranded in accordance with the following table.

TABLE XII

Standard Stranding of Concentric Lay-Cables

SIZE (See note 1.)	Number of Wires (See note 2)	
	A Bare, insulated or weather-proof cables for aerial use.	B Insulated cables for other than aerial use.
2.0 Cir. Inches	91	127
1.5 "	61	91
1.0 "	61	61
0.6 "	37	61
0.5 "	37	37
0.4 "	19	37
0000 A. W. G.	19 or 7 (See note 3.)	19
00 "	7	19
2 "	7	7
7 and smaller	..	7

1. For intermediate sizes, use stranding for next larger size.
2. Conductors of 0000 A. W. G. and smaller are often made solid and this table of stranding should not be interpreted as excluding this practice.
3. Class A cable, sizes 0000 and 000 A. W. G., is usually made of 7 strands when bare and 19 strands when insulated or weatherproof.

654 **Sectional Area of Cables.** The cross-sectional area of a cable shall be considered to be the sum of the cross-sectional areas of its component wires, when measured perpendicular to their axes.

655 Flexible Stranding. Conductors of special flexibility should ordinarily be made with wires of regular A.W.G. sizes,* the number of wires and size being given. The approximate gage number or approximate circular mils of such flexible stranded conductors may be stated. The stranding of standard flexible cables is given in Table XIII and a tentative stranding for apparatus cable in Table XIII-A.

TABLE XIII

Stranding of Flexible Cables

Nearest A.W.G. size (see Note 1)	Circular mils (see Note 3)	Diam. of cable. Mils	No. of wires	Size of each wire		Make-up (see Note 2)
				A.W.G.	Diam. Mils	
..	2039000	1885.	703	15.5	53.9	37 × 19
..	1816000	1779.	"	16.0	50.8	"
..	1617000	1679.	"	16.5	48.0	"
..	1440000	1584.	"	17.0	45.3	"
..	1284000	1496.	"	17.5	42.7	"
..	1103000	1372.	427	16.0	50.8	61 × 7
..	874600	1222.	"	17.0	45.3	"
..	693600	1088.	"	18.0	40.3	"
..	550000	969.	"	19.0	35.9	"
..	436200	863.	"	20.0	32.0	"
..	345900	768.	"	21.0	28.5	"
..	274300	684.	"	22.0	25.3	"
..	264600	671.	259	20.0	32.0	37 × 7
0000	209800	598.	"	21.0	28.5	"
000	171300	538.	133	19.0	35.9	19 × 7
00	135900	479.	"	20.0	32.0	"
0	107700	427.	"	21.0	28.5	"
1	82780	332.	91	20.5	30.2	Concentric
2	65650	295.	"	21.5	26.9	"
3	52060	263.	"	22.5	23.9	"
4	39190	228.	61	22.0	25.3	"
5	31080	203.	"	23.0	22.6	"
6	24650	181.	"	24.0	20.1	"
8	17410	152.	"	25.5	16.9	"
10	10560	118.	37	25.5	16.9	"
12	6642	94.	"	27.5	13.4	"
14	4176	74.	"	29.5	10.6	"
Smaller	....	....	To equal Required Size	30.0	....	Bunched

NOTE 1. The A.W.G. sizes except for 61 strands are approximated within 2 per cent. In the case of 61 strand cables the approximation is 6 per cent.

NOTE 2. "61 × 7" in the description of a rope-lay cable signifies 61 strands of 7 wires each.

NOTE 3. Circular mils are based on theoretical diameters of A.W.G. sizes which vary above or below values given in table by less than 0.1 mil.

*Where necessary to closely approximate a regular size cable, the strands may be made of half-size wires from No. 15 to No. 30 A. W. G.

TABLE XIII-A

Proposed Standard Stranding of Apparatus Cables

(This table is offered for consideration but will not be recommended for final adoption until ratified by other societies interested.)

Nearest A.W.G. size (see Note 1)	Circular mils (see Note 3)	Diameter of cable Mils	No. of wires	Size of each wire		Make-up
				A.W.G	Diam. Mils	
..	2053000	1900.	2257	20.5	30.2	61 × 37
..	1828000	1793.	"	21.0	28.5	"
..	1628000	1692.	"	21.5	26.9	"
..	1450000	1597.	"	22.0	25.3	"
..	1291000	1500.	"	22.5	23.9	"
..	1150000	1422.	"	23.0	22.6	"
..	1054000	1357.	1159	20.5	30.2	61 × 19
..	938900	1282.	"	21.0	28.5	"
..	836100	1209.	"	21.5	26.9	"
..	744600	1141.	"	22.0	25.3	"
..	663100	1076.	"	22.5	23.9	"
..	590500	1016.	"	23.0	22.6	"
..	525900	959.	"	23.5	21.3	"
..	451900	887.	703	22.0	25.3	37 × 19
..	402200	837.	"	22.5	23.9	"
..	358200	790.	"	23.0	22.6	"
..	319000	746.	"	23.5	21.3	"
..	284000	704.	"	24.0	20.1	"
..	253000	664.	"	24.5	19.0	"
0000	217600	609.	427	23.0	22.6	61 × 7
000	172500	543.	"	24.0	20.1	"
00	136800	483.	"	25.0	17.9	"
0	104600	422.	259	24.0	20.1	37 × 7
1	82990	376.	"	25.0	17.9	"
2	65820	335.	"	26.0	15.9	"
3	52200	298.	"	27.0	14.2	"
4	42610	268.	133	25.0	17.9	19 × 7
5	33800	239.	"	26.0	15.9	"
6 see Note 2.	26800	213.	"	27.0	14.2	"

NOTE 1. The A.W.G. sizes are approximated within 3 per cent.

NOTE 2. For sizes smaller than No. 6 see table XIII.

NOTE 3. Circular mils are based on theoretical diameters of A.W.G. diameters of the component wires.

656 Correction for Lay. The resistance and mass of a stranded conductor are greater than in a solid conductor of the same cross-sectional area, depending on the lay (*i.e.*, the pitch of the twist of the wires). Two per cent shall be taken as the standard increment of resistance and of mass. In cases where the lay is definitely known, the increment should be calculated and not assumed.

The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

CONDUCTIVITY OF COPPER.

675 The following I. E. C. rules are adopted:*

The following shall be taken as normal values for standard annealed copper:

(1) At a temperature of 20°C., the resistance of a wire of standard annealed copper one meter in length and of a uniform section of 1 square millimeter is 1/58 ohm = 0.017241 . . . . ohm.

(2) At a temperature of 20°C., the density of standard annealed copper is 8.89 grams per cubic centimeter.

(3) At a temperature of 20°C, the "constant mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is 0.00393 = 1/254.45 . . . . per degree centigrade.

(4) As a consequence, it follows from (1) and (2) that, at a temperature of 20 °C. the resistance of a wire of standard annealed copper of uniform section, one meter in length and weighing one gram, is (1/58) × 8.89 = 0.15328 . . . . . ohm.†§

676 **Copper Wire Tables.** The copper-wire Tables published by the Bureau of Standards in Circular No. 31 are adopted. These Tables are based upon the I. E. C. rules stated in **§675**.

HEATING AND TEMPERATURE OF CABLES.

677 **Maximum Safe Limiting Temperatures.**

The maximum safe limiting temperature in degrees C. at the surface of the conductor in a cable shall be:—

For impregnated paper insulation	(85—E)
" varnished cambric	(75—E)
" rubber insulation	(60—0.25E)

where E represents the r.m.s. operating e.m.f. in kilovolts between conductors.

Thus, at a working pressure of 3.3 kv., the maximum safe limiting temperature at the surface of the conductor, or conductors, in a cable would be:—

For impregnated paper	81.7°C.
" varnished cambric	71.7°C.
" rubber insulation	59.2°C.

ELECTRICAL TESTS.

678 **Lengths Tested.** Electrical tests of insulation on wires and cables shall be made on the entire lengths to be shipped.

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*See I. E. C. Publication No. 28 "International Standard of Resistance for Copper" March 1914.

†Paragraphs (1) and (4) of § 675 define what are sometimes called "volume resistivity," and "Mass resistivity" respectively. This may be expressed in other units as follows:—volume resistivity = 1.7241 microhms-cm. (or microhms in a cm. cube) at 20°C. = 0.67879 microhm-inch at 20°C., and mass resistivity = 875.20 ohms (mile, pound) at 20°C.

§For detailed specifications of commercial copper, see the "Standard Specifications" of the American Society for Testing Materials.

679 Immersion in Water. Electrical tests of insulated conductors not enclosed in a lead sheath, shall be made while immersed in water after an immersion of twelve (12) consecutive hours, if insulated with rubber compound, or if insulated with varnished cambric. It is not necessary to immerse in water insulated conductors enclosed in a lead sheath.

In multiple-conductor cables, without waterproof overall jacket of insulation, no immersion test should be made on finished cables, but only on the individual conductors before assembling.

680 Dielectric-Strength Tests. Object of Tests. Dielectric tests are intended to detect weak spots in the insulation and to determine whether the dielectric strength of the insulation is sufficient for enabling it to withstand the voltage to which it is likely to be subjected in service, with a suitable factor of assurance.

The initially-applied voltage must not be greater than the working voltage, and the rate of increase shall not be over 100 per cent in 10 seconds.

681 Factor of Assurance. The factor of assurance of wire or cable insulation shall be the ratio of the voltage at which it is tested to that at which it is used.

682 Test Voltage. The dielectric strength of wire and cable insulation shall be tested at the factory, by applying an alternating test voltage between the conductor and sheath or water.

683 The Magnitude and Duration of the Test Voltage should depend upon the dielectric strength and thickness of the insulation, the length and diameter of the wire or cable, and the assurance factor required, the latter in turn depending upon the importance of the service in which the wire or cable is employed

684 The following test voltages shall apply unless a departure is considered necessary, in view of the above circumstances. Rubber covered wires or cable for voltages up to 7 kv. shall be tested in accordance with the National Electric Code. Standardization for higher voltages for rubber insulated cables is not considered possible at the present time.

Varnished cambric and impregnated paper insulated wires or cables shall be tested at the place of manufacture for five (5) minutes in accordance with the Table XIV below.

TABLE XIV

Recommended Test Kilovolts Corresponding to Operating Kilovolts

Operating kv.	Test kv.	Operating kv.	Test kv.
Below 0.5	2.5*	5	14
0.5	3	10	25
1	4	15	35
2	6.5	20	44
3	9	25	53
4	11.5		

*The minimum thickness of insulation shall be $\frac{1}{16}$ in. (1.6 mm.)

Different engineers specify different thickness of insulation for the same working voltages. Therefore, at the present time the test kv. corresponding to working kv. given in Table XIV are based on the **minimum** thickness of insulation specified by engineers and operating companies.†

685 **The Frequency of the Test Voltage** shall not exceed 100 cycles per second, and should approximate as closely as possible to a sine wave. The source of energy should be of ample capacity.

686 **Where Ultimate Break-Down Tests** are required, these shall be made on samples not more than 6 meters (20 ft.) long. The maximum allowable temperature at which the test is made for the particular type of insulation and the particular working pressure, shall not be greater than the temperature limits given in **§ 677**.

687 **Multiple-Conductor Cables.** Each conductor of a multiple-conductor cable shall be tested against the other conductors connected together with the sheath or water.

INSULATION RESISTANCE

688 **Definition.** The insulation resistance of an insulated conductor is the electrical resistance offered by its insulation, to an impressed voltage tending to produce a leakage of current through the same.

689 **Insulation Resistance** shall be expressed in megohms for a specified length (as for a kilometer, or a mile, or one thousand feet), and shall be corrected to a temperature of 15.5° C. using a temperature coefficient determined experimentally for the insulation under consideration.

690 **Linear Insulation Resistance,** or the insulation resistance of Unit Length, shall be expressed in terms of the megohm-kilometer, or the megohm-mile, or the megohm-thousand-feet.

691 **Megohms Constant.** The Megohms Constant of an insulated conductor shall be the factor "K" in the equation

$$R = K \log_{10} \frac{D}{d}$$

where R = The insulation resistance, in megohms, for a specified unit length.

D = Outside diameter of insulation.

d = Diameter of conductor.

Unless otherwise stated, K will be assumed to correspond to the mile unit of length.

692 **Test.** The apparent insulation resistance should be measured after the dielectric-strength test, measuring the leakage current after a one-minute electrification, with a continuous e.m.f. of from 100 to 500 volts, the conductor being maintained negative to the sheath or water.

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†The Standards Committee does not commit itself to the principle of basing test voltages on working voltages, but it is not yet in possession of sufficient data to base them upon the dimensions and physical properties of the insulation.

693 Multiple-Conductor Cables. The insulation resistance of each conductor of a multiple-conductor cable shall be the insulation resistance measured from such conductor to all the other conductors in multiple with the sheath or water.

CAPACITANCE OR ELECTROSTATIC CAPACITY

694 Capacitance is ordinarily expressed in microfarads. Linear Capacitance, or Capacitance per unit length, shall be expressed in Microfarads per unit length (kilometer, or mile, or one thousand feet) and shall be corrected to a temperature of 15.5° C.

695 Microfarads Constant. The Microfarads Constant of an insulated conductor shall be the factor "K" in the equation

$$C = \frac{K}{\text{Log}_{10} \frac{D}{d}}$$

where C = the capacitance in microfarads per unit length.

D = the outside diameter of insulation

d = the diameter of conductor.

Unless otherwise stated, K will be assumed to refer to the mile unit of length.

696 Measurement of Capacitance. The Capacitance of low-voltage cable, shall be measured by comparison with a standard condenser. For long lengths of high-voltage cables, where it is necessary to know the true capacitance, the measurement should be made at a frequency approximating the frequency of operation.

697 Paired Cables. The capacitance shall be measured between the two conductors of any pair, the other wires being connected to the sheath or ground.

698 Electric Light and Power Cables. The capacitance of low-voltage cables is generally of but little importance. The capacitance of high-voltage cables should be measured between the conductors, and also between each conductor and the other conductors connected to the lead sheath or ground.

699 Multiple-Conductor Cables (not paired). The capacitance of each conductor of a multiple-conductor cable shall be the capacitance measured from such conductor to all of the other conductors in multiple with the sheath or the ground.

STANDARDS FOR CONTROLLERS, CIRCUIT-BREAKERS, SWITCHES, FUSES AND ACCESSORIES*

GENERAL DEFINITIONS

The following definitions are tentative. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed.

700 **Conducting Parts.**—Those parts designed to carry current or which are conductively connected therewith.

701 **Contact.**—The surface common to two conducting parts, united by pressure, for the purpose of carrying current.

702 **Grounded Parts.**—Those parts which may be considered to have the same potential as the earth.

703 **"Air" as a Prefix.**—The prefix "air" applied to a device which interrupts an electric circuit indicates that the interruption occurs in air.

704 **"Oil" as a Prefix.**—The prefix "oil" applied to a device which interrupts an electric circuit indicates that the interruption occurs in oil.

705 **Fume-Resisting.**—Apparatus is designated as fume-resisting when so constructed that it will not be readily injured by the specified fumes.

706 **Drip-Proof.**—Apparatus is designated as drip-proof when so protected as to exclude falling moisture or dirt. Drip-proof apparatus may be either open or semi-enclosed, if it is provided with suitable protection integral with the apparatus, or so enclosed as to exclude effectively falling solid or liquid material.

707 **Dust-Proof.**—Apparatus is designated as dust-proof when so constructed or protected that the accumulation of dust within or without the device will not interfere with its successful operation.

707a **Dust-Tight.**—Apparatus is designated as dust-tight when so constructed that the dust will not enter the enclosing case.

708 **Explosion-Proof.**—Apparatus is designated as explosion-proof when so constructed that explosions of gas within the casing will not injure it or ignite inflammable gas outside it.

*The "National Electrical Code" of the National Fire Protection Association deals with certain capacities of circuit-breakers up to 550 volts and of switches and fuses up to 600 volts.

The question of establishing uniform rules for apparatus coming under this classification is under consideration by the A. I. E. E. Standards Committee and the Electrical Committee of the N. F. P. A.

709 **Gas-Proof.**—Apparatus is designated as gas-proof when so constructed or protected that the specified gas will not interfere with its successful operation.

709a **Gas-Tight.**—Apparatus is designated as gas-tight when so constructed that the specified gas will not enter the enclosing case.

710 **Moisture-Resisting.**—Apparatus is designated as moisture-resisting when so constructed or treated that it will not be readily injured by moisture. (Such apparatus shall be capable of operating in a very humid atmosphere, such as found in mines, evaporating rooms, etc.)

711 **Splash-Proof.**—Apparatus is designated as splash-proof when so constructed or protected that external splashing will not interfere with its successful operation.

712 **Submersible.**—Apparatus is designated as submersible when so constructed that it will operate successfully when submerged in water under specified conditions of pressure and time.

713 **Sleet-Proof.**—Apparatus is designated as sleet-proof when so constructed or protected that the accumulation of sleet will not interfere with its successful operation.

714 **Under-Voltage or Low-Voltage Release.**—A term applied to a device which on the reduction or failure of voltage operates to cause the interruption of power to the main circuit, but which does not prevent the re-establishment of the main circuit on return of voltage.

714a **Under-Voltage or Low-Voltage Protection.**—A device which, on the reduction or failure of voltage, operates to cause and maintain the interruption of power to the main circuit, is said to provide under-voltage or low-voltage protection.

715 **Phase-Failure Protection.**—A device which, on the failure of power in one wire of a polyphase circuit, operates to cause and maintain the interruption of power on the remaining circuits, is said to provide phase-failure protection.

715a **Phase-Reversal Protection.**—A device which, on the reversal of the phase relations in a polyphase circuit, operates to cause and maintain the interruption of power in all of the circuits, is said to provide phase-reversal protection.

CONTROLLERS

716 **Electric Controller.**—A device, or group of devices, which serves to control in some predetermined manner the operation of the apparatus to which it is connected.

717 **Contactor.**—A device for repeatedly establishing and interrupting an electric circuit under **normal** conditions.

717a **Temperature of Contactors.**—

Operating Coils for Contactors.—The limiting observable temperature rise of operating coils for magnetic contactors shall be 70°C. when measured by thermometer.

Temperature of Contacts.—The limiting observable temperature rise for the contacts of magnetic contactors shall be the following:

65° C. rise for laminated contacts
100° C. " " solid "

Current Carrying Parts Insulated With Asbestos or Other Fire Proof Insulation.—The limiting observable temperature rise for current carrying parts insulated with asbestos or other fire proof material shall be 150° C.

CIRCUIT-BREAKERS.

718 **Circuit-Breaker.**—A circuit-breaker is a device (other than a fuse) constructed primarily for the interruption of a circuit under **infrequent abnormal** conditions.

719 **Rating.**—The rating of a circuit-breaker or switch includes (1) the normal r. m. s. current which it is designed to carry; (2) the normal r. m. s. pressure (voltage) of the circuit on which it is intended to operate; (3) the normal frequency of the current; (4) the interrupting capacity of the device (see **§720**).

720 **Interrupting Capacity.** By interrupting (breaking or rupturing) capacity is meant the highest r. m. s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times.

721 **Temperature Tests.**—Rated current at rated frequency shall be applied continuously until the temperature becomes constant. The maximum temperatures of the various parts shall not exceed the following when the ambient temperature of reference is 40° C.:

Contacts in air when clean and bright.. 70° C.
Oil and contacts therein..... 70° C.
Coils (see **§§376** to **379** incl.)
Other parts (see **§392**)

The Institute calls attention to the inherent decrease in capacity of switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit-breakers is, therefore, based on sufficient maintenance to keep the temperature within the specified limits.

722 **Dielectric Tests.**—For apparatus of 600 volts or less see **§§482, 484, 485,** and **500.** Apparatus above 600 volts shall be tested at 2½ times rated voltage, plus 2,000 volts, at a specified altitude.

As a supplementary test, devices for outdoor use should be capable of withstanding for 10 seconds a dielectric wet test at twice rated voltage plus 1,000 volts. This assumes a precipitation downward of 0.1 inches per minute at an angle of 45° from the perpendicular, with water having a resistivity as low as 7,000 ohm-centimeters.

SWITCHES

723 **Switch.**—A switch is a device for making, breaking or changing the connections in an electric circuit.

724 **Master-Switch.**—A master-switch is a device which serves to govern the operation of contactors and auxiliary devices of an electric controller.

24a **Control Switch.**—A control switch is a switch for controlling electrically-operated switches and circuit breakers.

724b **Auxiliary Switch.**—An auxiliary switch is a switch actuated by some main device, for signalling, interlocking, etc.

725 **Rating.**—Same as for circuit-breakers; see §§719 and 720.

726 **Tests.**—Same as for circuit-breakers; see §§721 and 722.

FUSES

727 **Fuse.** A fuse is an element designed to melt or dissipate at a predetermined current value, and intended to protect against abnormal conditions of current.

Any terminals, tubes, etc., integral with this element are included as part of the fuse.

Fuses may be divided into two classes:

(A) Those designed to protect the circuit and apparatus both against short-circuit and against definite amounts of overload (*e.g* fuses of the National Electric Code which open on 25 per cent overload.)

(B) Those designed to protect the system only against short-circuits; (*e.g.* expulsion fuses, which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed.

728 **Continuous Current Carrying-Capacity of a Fuse.**—Fuses shall be so constructed that they will carry continuously 110 per cent of their rated current.

729 **Temperature.**—The temperature of coils or windings (such as accompany fuses of the magnetic blow-out type) shall not exceed the limits set for machine coils having the same character of insulation. (See §§376 to 379). The highest temperature for the fuse proper should not exceed the safe limit for the material employed.

RELAYS

730 **Relay.**—A relay is a device by means of which contacts in one circuit are operated by change in conditions in the same or other circuits.

731 **Temperature Tests.**—Same as § 721.

732 **Dielectric Tests.**—Same as § 722.

LIGHTNING ARRESTERS

733 **Definition.** A lightning arrester is a device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration.

734 Rating. Arresters shall be rated by the voltage of the circuit on which they are to be used.

Lightning arresters may be divided into two classes:

(a) Those intended to discharge for a very short time.

(b) Those intended to discharge for a period of several minutes.

735 Performance and Tests. Dielectric Test same as **§722.**

The resistance of the arrester at double potential and also at normal potential, shall be determined by observing the discharge currents through the arrester.

(c) In the case of any arrester using a gap, a test shall be made of the spark potential on either direct-current or 60-cycle a-c. excitation.

(d) The equivalent sphere gap under disruptive discharge shall also be measured, using a considerable quantity of electricity.

(e) The endurance of the arrester to continuous surges shall be tested.

PROTECTIVE REACTORS

736 Definition. A reactor (See **§§82** and **214**) is a device for protecting circuits by limiting the current flow and localizing the disturbance under short-circuit conditions.

737 Rating.

(a) In kilovolt-amperes absorbed by normal current.

(b) By the normal current, frequency and line (delta) voltage for which the reactor is designed.

(c) By the current which the device is required to stand under short-circuit conditions.

738 Performance and Tests.

The Heat Test shall be made with normal current and frequency applied until the temperature is constant. The temperature should not exceed the safe limits for the materials employed. See **§§376** to **379.**

739 Dielectric Test. $2\frac{1}{2}$ times line voltage plus 2000, for one minute, from conductor to ground.

NOTE. The reactor shall be so designed as to be capable of withstanding, without mechanical injury, rated current at normal frequency, suddenly applied.

RESISTOR OR RHEOSTAT

740 Definition. Any device heretofore commonly known as a resistance, used for operation or control. (See **§81**) See National Electric Code.

740a Temperature of Resistors.—

Cast Grid Resistors.—The limiting observable temperature rise for cast grids used as resistors shall be 350° C.

INSTRUMENT TRANSFORMERS†

[For **Meters and Instruments** see §§**225** to **244.**]

741 **Instrument Transformer.**—An instrument transformer, is a transformer suitable for use with measuring instruments; that is, one in which the conditions of phase and of current or potential in the primary circuit, are represented with acceptable accuracy in the secondary circuit. An instrument transformer may be either an instrument current transformer or an instrument potential (voltage) transformer.

742 **Instrument Current Transformers on Open Secondary Circuit.**—Under conditions of open secondary circuit, current transformers shall be capable of carrying continuously rated primary current without damage to the primary insulation and without interruption of service.

743 **Instrument Current Transformers on Closed Secondary Circuit.**—Under conditions of closed secondary circuit, current transformers shall withstand 40 times rated current applied for 1 second, without injury.

744 For further definitions relative to instrument transformers, see §§ **205** to **207**. For dielectric tests of instrument potential (voltage) transformers see § **500**. The dielectric tests of instrument current transformers shall be the same as specified in § **509**.

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†The terms "load," "secondary load" and "secondary connected load" have been used in connection with the constants of the circuit connected to the secondary of a current transformer. It is suggested that the term "secondary burden" be used as a general term to designate this external resistance and inductance and shall be expressed quantitatively in terms of ohms and henrys.

STANDARDS FOR ELECTRIC RAILWAYS

DEFINITIONS

760 **Transmission System:** When the current generated for an electric railway is changed in kind or voltage, between the generator and the cars or locomotives, that portion of the conductor system carrying current of a kind or voltage substantially different from that received by the cars or locomotives, constitutes the *transmission system.**

761 **Distribution System:** That portion of the conductor system of an electric railway which carries current of the kind and voltage received by the cars or locomotives, constitutes the *distribution system.**

762 **Substation:** A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system.

RATING OF RAILWAY SUBSTATION MACHINERY

763 **Continuous Rating.** The rating of a substation machine shall be the kv-a. output at a stated power factor input, which it will deliver continuously with temperatures or temperature rises not exceeding the limiting values given in Sections **376** and **379** and also fulfilling the other requirements set forth in these rules and summarized in Section **260**.

764 **Momentary Loads.** These machines should be capable of carrying a load of twice their rating for one minute, after a continuous run at rated load, without disqualifying them for continuous service.

765† **Nominal Rating.** Where the continuous rating is inconvenient, the following nominal rating may be used. The nominal rating of a substation machine shall be the kv-a. output at a stated power factor input, which, having produced a constant temperature in the machine may be increased 50 per cent for two hours, without producing temperatures or temperature rises exceeding by more than 5°C. the limiting values given in §**376** and **379**. These machines should be capable of carrying a load of twice their nominal rating for a period of one minute, without disqualifying them for continuous service. The name plate should be marked "nominal rating."

CONDUCTOR AND RAIL SYSTEMS.

766 **Contact Conductors.** That part of the distribution system other than the traffic rails, which is in immediate electrical contact with

*These definitions are identical in sense, although not in words, with those of the Interstate Commerce Commission, as given in their Classification of Accounts for Electric Railways.

†The nominal rating should be applied only to machinery and apparatus carrying traction loads.

the circuits of the cars or locomotives, constitutes the contact conductors.

767 **Contact Rail:** A rigid contact conductor.

768 OVERHEAD CONTACT RAIL: A contact rail above the elevation of the maximum equipment line.†

769 THIRD RAIL: A contact conductor placed at either side of the track, the contact surface of which is a few inches above the level of the top of the track rails.

770 CENTER CONTACT RAIL: A contact conductor placed between the track rails, having its contact surface above the ground level.

771 UNDERGROUND CONTACT RAIL: A contact conductor placed beneath the ground level.

772 GAGE OF THIRD RAIL: The distance, measured parallel to the plane of running rails, between the gage line of the nearer track rail and the inside gage line of the *contact surface* of the third rail.

773 ELEVATION OF THIRD RAIL: The elevation of the contact-surface of the third rail, with respect to the plane of the tops of running rails.

774 STANDARD GAGE OF THIRD RAILS: The gage of third rails shall be not less than 26 inches (66 cm.) and not more than 27 inches (68.6 cm.).

775 STANDARD ELEVATION OF THIRD RAILS: The elevation of third rails shall be not less than 2¾ inches (70 mm.), and not more than 3½ inches (89 mm.).

776 THIRD RAIL PROTECTION: A guard for the purpose of preventing accidental contact with the third rail.

777 **Trolley Wire:** A flexible contact conductor, customarily supported above the cars.

778 **Messenger Wire or Cable:** A wire or cable running along with and supporting other wires, cables or contact conductors.

A primary messenger is directly attached to the supporting system. A secondary messenger is intermediate between a primary messenger and the wires, cables or contact conductors.

779 **Classes of Construction:** Overhead trolley construction will be classed as *Direct Suspension* and *Messenger or Catenary Suspension.*

780 DIRECT SUSPENSION: All forms of overhead trolley construction in which the trolley wires are attached, by insulating devices, directly to the main supporting system.

781 MESSENGER OR CATENARY SUSPENSION: All forms of overhead trolley construction in which the trolley wires are attached, by suitable devices, to one or more messenger cables, which in turn may be carried either in *Simple Catenary, i.e.*, by primary messengers, or in *Compound Catenary, i.e.*, by secondary messengers.

782 SUPPORTING SYSTEMS shall be classed as follows:

783 SIMPLE CROSS-SPAN SYSTEMS: Those systems having at each support a single flexible span across the track or tracks.

†The contour which embraces cross-sections of all rolling stock under all normal operating conditions.

784 MESSENGER CROSS-SPAN SYSTEMS: Those systems having at each support two or more flexible spans across the track or tracks, the upper span carrying part or all of the vertical load of the lower span.

785 BRACKET SYSTEMS: Those systems having at each support an arm or similar rigid member, supported at only one side of the track or tracks.

786 BRIDGE SYSTEMS: Those systems having at each support a rigid member, supported at both sides of the track or tracks.

787 STANDARD HEIGHT OF TROLLEY WIRE ON STREET AND INTERURBAN RAILWAYS: It is recommended that supporting structures shall be of such height that the lowest point of the trolley wire shall be at a height of 18 feet (5.5m.) above the top of rail under conditions of maximum sag, unless local conditions prevent. On trackage operating electric and steam road equipment and at crossings over steam roads, it is recommended that the trolley wire shall be not less than 21 feet (6.4m.) above the top of rail, under conditions of maximum sag.

RAILWAY MOTORS†

RATING

800 **Nominal Rating**: The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90 °C. at the commutator, and 75 °C. at any other normally accessible part after one hour's continuous run at its rated voltage (and frequency in the case of an alternating-current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower. The rise in temperature as measured by resistance, shall not exceed 100 °C.*

801 The statement of the nominal rating shall also include the corresponding voltage and armature speed.

802 **Continuous Rating:** The continuous ratings of a railway motor shall be the *inputs* in amperes at which it may be operated continuously at $\frac{1}{2}$, $\frac{3}{4}$ and full voltage respectively, without exceeding the specified temperature rises (see §805), when operated on stand test with motor covers and cooling system, if any, arranged as in service. Inasmuch as the same motor may be operated under different conditions as regards ventilation, it will be necessary in each case to

* This definition differs from that in the 1911 edition of the Rules, principally by the substitution of a kilowatt rating for the horse-power rating and the omission of a reference to a room temperature of 25°C. For the purposes of these Rules the horse-power shall be taken as 746.0 watts. On account of the hitherto prevailing practise of expressing mechanical output in horse-power, it is recommended that, for the present the capacity be expressed both in kilowatts and in horse-power, a double rating, namely,

kw.————approx. equiv. h.p.————

In order to lay stress upon the preferred future basis, it is desirable that on rating plates, the rating in kilowatts shall be shown in larger and more prominent characters than the capacity in horse power.

†See also Appendix II.

define the system of ventilation which is used. In case motors are cooled by external blowers, the flow of air on which the rating is based shall be given.

803 Maximum Input. The subject of momentary loads for railway motors is under investigation.

TEMPERATURE LIMITATIONS

804 The allowable temperature in any part of a motor in service will be governed by the kind of material with which that part is insulated. In view of space limitations, and the cost of carrying dead weight on cars, it is considered good practice to operate railway motors for short periods at higher temperatures than would be advisable in stationary motors. The following temperatures are permissible:

TABLE XV

Operating Temperatures of Railway Motors

Class of Material See §376 to 379.	Maximum Observable Temperature of windings when in continuous service.	
	By Thermometer See §345	By Resistance
A	85	110
B	100	130

For infrequent occasions, due to extreme ambient temperatures, it is permissible to operate at 15° higher temperature.

805 With a view to not exceeding the above temperature limitations, the continuous ratings shall be based upon the **temperature rises** tabulated below:

TABLE XVI

Stand-Test Temperature Rises of Railway Motors*

Class of Material See §376 to 379	Temperature Rises of windings	
	By Thermometer See §345	By Resistance
A	65	85
B	80	105

*The temperature rise in service may be very different from that on stand test. See § 1104 for relation between stand test and service temperatures, as affected by ventilation.

806 **Field-Control Motors.** The nominal and continuous ratings of field-control motors shall relate to their performance with the operating field which gives the maximum motor rating. Each section of the field windings shall be adequate to perform the service required of it, without exceeding the specified temperature rises.

CHARACTERISTIC CURVES

810 **The Characteristic Curves** of railway motors shall be plotted with the current as abscissas and the tractive effort, speed and efficiency as ordinates. In the case of a-c. motors, the power factor shall also be plotted as ordinates.

811 **Characteristic curves of direct-current motors** shall be based upon full voltage, which shall be taken as 600 volts, or a multiple thereof.

812 **In the case of field-control motors,** characteristic curves shall be given for all operating field connections.

EFFICIENCY AND LOSSES

815 **The efficiency** of railway motors shall be deduced from a determination of the losses enumerated in **§816** to **820.** (See also **§ 1100** and **1101.**)

816 **The copper loss** shall be determined from resistance measurements corrected to 75° C.

817 **The no-load core loss, brush friction, armature-bearing friction and windage** shall be determined as a total under the following conditions:

In making the test, the motor shall be run without gears. The kind of brushes and the brush pressure shall be the same as in commercial service. With the field separately excited, such a voltage shall be applied to the armature terminals as will give the same speed for any given field current as is obtained with that field current when operating at normal voltage under load. The sum of the losses above-mentioned, is equal to the product of the counter-electromotive force and the armature current.

818 The **core loss** in d-c. motors shall be separated from the friction and windage losses above described by measuring the power required to drive the motor at any given speed without gears, by running it as a series motor on low voltage and deducting this loss from the sum of the no-load losses at corresponding speed. (See **§1101** for alternative method).

The friction and windage losses under load shall be assumed to be the same as without load, at the same speed.

The core loss under load shall be assumed as follows:

TABLE XVII

Core Loss in D-C. Railway Motors at Various Loads.

Per cent of Input at Nominal Rating	Loss as Per cent of No-load Core Loss
200	165
150	145
100	130
75	125
50	123
25 and under	122

Note:—With motors designed for field control the core losses shall be assumed as the same for both full and permanent field. It shall be the mean between the no-load losses at full and permanent field, increased by the percentages given in the above Table.

819 **The brush-contact resistance loss** to be used in determining the efficiency, may be obtained by assuming that the sum of the drops at the contact surfaces of the positive and negative brushes is three volts.

820 The losses in gearing and axle bearings for single-reduction single-geared motors, varies with type, mechanical finish, age and lubrication. The following values, based on accumulated tests, shall be used in the comparison of single-reduction single-geared motors.

TABLE XVIII

Losses in Axle Bearings and Single-Reduction Gearing of Railway Motors.

Per Cent of Input at Nominal Rating	Losses as Per Cent of Input
200	3.5
150	3.0
125	2.7
100	2.5
75	2.5
60	2.7
50	3.2
40	4.4
30	6.7
25	8.5

NOTE:—Further investigation may indicate the desirability of giving separate values of the losses for full and tapped fields, or low- and high-speed motors.

ELECTRIC LOCOMOTIVES

830 **Rating.** Locomotives shall be rated in terms of the weight on drivers, nominal one-hour tractive effort, continuous tractive effort and corresponding speeds.

831 **Weight on Drivers.** The weight on drivers, expressed in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

832 **Nominal Tractive Effort:** The nominal tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers, when the motors are operating at their nominal (one-hour) rating.

833 **Continuous Tractive Effort.** The continuous tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full-voltage continuous rating, as indicated in **§802**.

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for $\frac{1}{2}$ or $\frac{3}{4}$ voltage, but in such cases the voltage shall be clearly specified.

834 **Speed:** The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

See also Appendix II on Additional Standards for Railway Motors.

RATING OF AUTOMOBILE PROPULSION MOTORS AND GENERATORS

(ROAD VEHICLES)

835 **Continuous Rating.** Automobile propulsion motors and generators shall be given a continuous rating, expressed in kilowatts output available at the shaft at specified speed. The machines shall be able to operate continuously at their rated outputs without exceeding any of the limitations referred to in **§260**.

836 **Short-Time Rating.** Owing to the variety of services which road vehicles are called upon to perform, no single standard period for short-time ratings is recommended.

837 **Nominal Rating.** No special nominal rating is required for automobile propulsion motors or generators.

838 **Temperature Rises.** Owing to space limitations and the cost of carrying dead weight on automobiles, it is considered good practice to operate the propulsion machinery at higher temperatures than would be advisable in stationary machines. The rating of automobiles motors and generators shall be based upon temperature rise, on a stand test and with motor covers arranged as in service, fifteen degrees by thermometer or twenty-five degrees by resistance, above those of **§379**.

839 **Efficiency and Losses.** Unless otherwise specified the efficiency of automobile propulsion machines shall be based upon the output at the shaft, using conventional losses as tabulated in **§440**. When such machines are of low voltage, the great influence of brush-contact losses on the efficiency requires that these losses be determined experimentally for the type of brush used.

DEFINITIONS and UNITS for ILLUMINATION AND PHOTOMETRY

The following Sections, **850** to **895**, are abstracts from the rules of the Nomenclature and Standards Committee of the Illuminating Engineering Society. They are here included by permission.

850 **Luminous Flux** is radiant power evaluated according to its capacity to produce the sensation of light.

851 **The stimulus coefficient** K_λ for radiation of a particular wave-length, is the ratio of the luminous flux to the radiant power producing it.

852 **The mean value of the stimulus coefficient,** K_m, over any range of wave-lengths, or for the whole visible spectrum of any source, is the ratio of the total luminous flux (in lumens) to the total radiant power (in ergs per second, but more commonly in watts).

853 **The luminous intensity** of a point source of light is the solid angular density of the luminous flux emitted by the source in the direction considered; or it is the flux per unit solid angle from that source.

Defining equation:

Let I be the intensity, F the flux and ω the solid angle.

Then if the intensity is uniform,

$$I = \frac{F}{\omega},$$

854 **Illumination**, on a surface, is the luminous flux-density over that surface, or the flux per unit of intercepting area.

Defining equation:

Let E be the illumination and S the area of the intercepting surface.

Then when uniform,

$$E = \frac{F}{S},$$

855 **Candle,** the unit of luminous intensity maintained by the National Laboratories of France, Great Britain, and the United States.[1]

856 **Candle-power,** luminous intensity expressed in candles.

857 **Lumen,** the unit of luminous flux, equal to the flux emitted in a unit solid angle (steradian) by a point source of one candle-power.[2]

858 **Lux,** a unit of illumination equal to one lumen per square meter. The C. G. S. unit of illumination is one lumen per square centimeter. For this unit Blondel has proposed the name "Phot." One millilumen per square centimeter (milliphot) is a practical derivative of

1 This unit, which is used also by many other countries, is frequently referred to as the international candle.

2 A uniform source of one candle emits 4π lumens.

the C. G. S. system. One foot-candle is one lumen per square foot and is equal to 1.0764 milliphots.

860 **Specific luminous radiation,** the luminous flux-density emitted by a surface, or the flux emitted per unit of emissive area. It is expressed in lumens per square centimeter.

Defining equation:

Let E' be the specific luminous radiation.

For surfaces obeying Lambert's cosine law of emission.

$$E' = \pi b_0.$$

863 **The Lambert, the C. G. S. Unit of Brightness,** the brightness of a perfectly diffusing surface radiating or reflecting one lumen per square centimeter. This is equivalent to the brightness of a perfectly diffusing surface having a coefficient of reflection equal to unity and illuminated by one phot.

864 **For most purposes, the millilambert** (0.001 lambert) is the preferable practical unit. A perfectly diffusing surface emitting one lumen per square foot will have a brightness of 1.076 millilamberts.

865 **Brightness expressed in candles per square centimeter** may be reduced to Lamberts by multiplying by π.

Brightness expressed in candles per square inch may be reduced to foot-candle brightness, by multiplying by the factor $144\pi = 452$.

Brightness expressed in candles per square inch may be reduced to lamberts by multiplying by $\pi/6.45 = 0.4868$.

In practise, no surface obeys exactly Lambert's cosine law of emission; hence the brightness of a surface in lamberts is, in general not numerically equal to its specific luminous radiation in lumens per square centimeter.

866 **Coefficient of reflection,** the ratio of the total luminous flux reflected by a surface to the total luminous flux incident upon it. It is a simple numeric. The reflection from a surface may be regular, diffuse or mixed. In perfect regular reflection, all of the flux is reflected from the surface at an angle of reflection equal to the angle of incidence. In perfect diffuse reflection, the flux is reflected from the surface in all directions, in accordance with Lambert's cosine law. In most practical cases, there is a superposition of regular and diffuse reflection.

867 **Coefficient of regular reflection** is the ratio of the luminous flux reflected regularly to the total incident flux.

868 **Coefficient of diffuse reflection** is the ratio of the luminous flux reflected diffusely to the total incident flux.

Defining equation:

Let m be the coefficient of reflection (regular or diffuse).

Then, for any given portion of the surface,

$$m = \frac{E'}{E}$$

869 **Lamp,** a generic term for an artificial source of light.

870 **Primary luminous standard,** a recognized standard luminous source reproducible from specifications.

871 **Representative luminous standard,** a standard of luminous intensity adopted as the authoritative custodian of the accepted value of the unit.

872 **Reference standard,** a standard calibrated in terms of the unit from either a primary or representative standard and used for the calibration of working standards.

873 **Working standard,** any standardized luminous source for daily use in photometry.

874 **Comparison lamp,** a lamp of constant but not necessarily known candle-power, against which a working standard and test lamps are successively compared in a photometer.

875 **Test lamp,** in a photometer,—a lamp to be tested.

876 **Performance curve,** a curve representing the behavior of a lamp in any particular (candle-power, consumption, etc.) at different periods during its life.

877 **Characteristic curve,** a curve expressing a relation between two variable properties of a luminous source, as candle-power and volts, candle-power and rate of fuel consumption, etc.

878 **Horizontal Distribution Curve.** A polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane perpendicular to the axis of the unit, and with the unit at the origin.

879 **Vertical Distribution Curve.** A polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane passing through the axis of the unit, and with the unit at the origin. Unless otherwise specified, a vertical distribution curve is assumed to be an *average* vertical distribution curve, such as may in many cases be obtained by rotating the unit about its axis and measuring the average intensities at the different elevations. It is recommended that in vertical distribution curves, angles of elevation shall be counted positively from the nadir as zero, to the zenith as 180 degrees. In the case of incandescent lamps, it is assumed that the vertical distribution curve is taken with the tip downward.

880 **Mean horizontal candle-power** of a lamp,—the average candle-power in the horizontal plane passing through the luminous center of the lamp.

It is here assumed that the lamp (or other light source) is mounted in the usual manner, or, as in the case of an incandescent lamp, with its axis of symmetry vertical.

881 **Mean spherical candle-power** of a lamp,—the average candle-power of a lamp in all directions in space. It is equal to the total luminous flux of the lamp, in lumens, divided by 4π.

882 **Mean hemispherical candle-power** of a lamp (upper or lower),—the average candle-power of a lamp in the hemisphere considered. It is equal to the total luminous flux emitted by the lamp, in that hemisphere, divided by 2π.

883 **Mean zonal candle-power** of a lamp,—the average candle-power of a lamp over the given zone. It is equal to the total luminous flux emitted by the lamp in that zone, divided by the solid angle of the zone.

884 **Spherical reduction factor** of a lamp,—the ratio of the mean spherical to the mean horizontal candle-power of the lamp.[3]

885 **Photometric Tests** in which the results are stated in candle-power should be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made in the measurement of lamps with reflectors, the results should always be given as "apparent candle-power" at the distance employed, which distance should always be specifically stated.

886 **The output of all illuminants** should be expressed in lumens.

887 **Illuminants should be rated** upon a lumen basis instead of a candle-power basis.

888 **The specific output of electric lamps** should be stated in lumens per watt; and the specific output of illuminants depending upon combustion should be stated in lumens per b.t.u. per hour. The use of the term "efficiency" in this connection should be discouraged.

When auxiliary devices are necessarily employed in circuit with a lamp, the input should be taken to include both that in the lamp and that in the auxiliary devices. For example, the watts lost in the ballast resistance of an arc lamp are properly chargeable to the lamp.

889 **The Specific Consumption** of an electric lamp is its watt consumption per lumen. "Watts per candle" is a term used commercially in connection with electric incandescent lamps, and denotes watts per mean horizontal candle-power.

890 **Life Tests. Electric Incandescent Lamps** of a given type may be assumed to operate under comparable conditions only when their lumens per watt consumed are the same. Life-test results, in order to be compared, must be either conducted under, or reduced to, comparable conditions of operation.

891 **In Comparing Different Luminous Sources,** not only should their candle-power be compared, but also their relative form, brightness, distribution of illumination and character of light.

892 **Lamp Accessories. A reflector** is an appliance, the chief use of which is to redirect the luminous flux of a lamp in a desired direction or directions.

893 **A Shade is** an appliance, the chief use of which is to diminish or to interrupt the flux of a lamp in certain directions, where such flux is not desirable. The function of a shade is commonly combined with that of a reflector.

894 **A Globe is** an enclosing appliance of clear or diffusing materials, the chief use of which is either to protect the lamp, or to diffuse its light.

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3 In the case of a uniform point-source, this factor would be unity, and for a straight cylindrical filament obeying the cosine law it would be $\pi/4$.

895

TABLE XIX.

Photometric Units and Abbreviations.

Photometric quantity	Name of unit	Abbreviations, Symbols and defining equations
1. Luminous flux	Lumen	F, Ψ
2. Luminous intensity	Candle	$I = \frac{dF}{d\omega}, \Gamma = \frac{d\Psi}{d\omega}$, cp.
3. Illumination	Phot., foot-candle, lux	$E = \frac{dF}{dS} = \frac{I}{r^2}\cos\theta.\beta$
4. Exposure	Phot-second	$t \quad E$
5. Brightness	Apparent candles per sq. cm.	$b = \frac{dI}{dS \cos\theta}$
	Apparent candles per sq. in.	
	Lambert	$L = \frac{dF}{dS}$
6. Normal brightness	Candles per sq. cm.	$b_0 = \frac{dI}{dS}$
	Candles per sq. in.	
7. Specific luminous radiation	Lumens per sq. cm.	
	Lumens per sq. in.	$E' = \pi b_0 \beta'$
8. Coefficient of reflection	—	$m = \frac{E'}{E}$

9. Mean spherical candlepower — scp
10. Mean lower hemispherical candlepower — lcp
11. Mean upper hemispherical candlepower — ucp
12. Mean zonal candlepower — zcp
13. 1 lumen is emitted by 0.07958 spherical cp.
14. 1 spherical candlepower emits 12.57 lumens.
15. 1 lux = 1 lumen incident per square meter = 0,0001 phot = 0.1 milliphot.
16. 1 phot = 1 lumen incident per sq. cm. = 10.000 lux = 1000 milliphot.
17. 1 milliphot = 0.001 phot = 0.929 foot-candle.
18. 1 foot-candle = 1 lumen incident per square foot = 1.076 milliphot = 10.76 lux.
19. 1 lambert = 1 lumen emitted per square centimeter.*
20. 1 millilambert = 0.001 lambert.
21. 1 lumen, emitted, per square foot* = 1.076 millilambert.
22. 1 millilambert = 0.929 lumen, emitted, per square foot*.
23. 1 lambert = 0.3183 candle per sq. cm. = 2.054 candles per sq. in.
24. 1 candle per sq. cm. = 3.1416 lamberts.
25. 1 candle per sq. in. = 0.4868 lamberts = 486.8 millilamberts.

*Perfect diffusion assumed.

DEFINITIONS FOR TELEPHONY AND TELEGRAPHY

(See also §§407 to 409.)

910 Many of the following definitions are tentative and not yet fully established. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed. Some of the definitions are specific to telephony, and differ in detail from similar definitions appearing in other parts of the rules.

911 **Damping of a Circuit.** The damping at a given point in a circuit from which the source of energy has been withdrawn, is the progressive diminution in the effective value of electromotive force and current at that point resulting from the withdrawal of electrical energy.

912 **Damping Constant.**—The damping constant of a circuit is a measure of the ratio of the dissipative to the reactive component of its admittance or impedance.

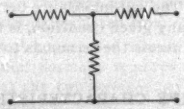
Applied to the admittance of a condenser or other simple circuit having capacity reactance, the damping constant for a harmonic electromotive force of given frequency is the ratio of the conductance G, of the condenser or simple circuit at that frequency to twice the capacitance, C, of the condenser at the same frequency, $(G/2C)$.

Applied to the reactance of a coil or other simple circuit having inductive reactance, the damping constant for a harmonic current of a given frequency is the ratio of the resistance, R, of the coil or circuit at that frequency to twice the inductance, L, at the same frequency $(R/2\,L)$.

913 **Equivalent Circuit.**—An equivalent circuit is a simple network of series and shunt impedances, which, is the approximate electrical equivalent of a complex network.

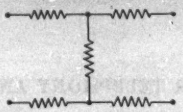
914 **"*T*" Equivalent Circuit.**—A "*T*" equivalent circuit is a triple-star or "*Y*" connection of three impedances externally equivalent to a complex network.

Symbol:



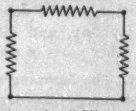
914a **"*I* Equivalent Circuit."** An "*I*" equivalent circuit is a connection of five impedances in the form shown below, which is externally equivalent to a complex network. It differs from the "*T*" equivalent circuit in that the impedances are arranged symmetrically on the

two sides of the circuit which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.

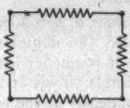


915 "π" **Equivalent Circuit.**—A "π" equivalent circuit is a delta connection of three impedances externally equivalent to a complex network. It is also called a "*U*" equivalent circuit.

Symbol:



915a "*O*" **Equivalent Circuit.**—An "*O*" equivalent circuit is a connection of four impedances in the form shown below, externally equivalent to a complex network. It differs from the π equivalent circuit in that the impedances are arranged symmetrically on the two sides of the circuit, which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.



NOTE: As ordinarily considered, the simple networks as defined, are the electrical equivalents of complex networks only with respect to definite pairs of terminals.

IMPEDANCE

916 **Mutual Impedance.**—The mutual impedance for single frequency alternating currents, between a pair of terminals and a second pair of terminals of a network, under any given condition, is the negative ratio* of the electromotive force produced between either pair of terminals on open circuit to the current flowing between the other pair of terminals. A receiving-end impedance is an example of a mutual impedance.

917 **Self-Impedance.**—The self-impedance between a pair of terminals of a network, under any given condition, is the ratio* of the electromotive force applied across the terminals to the entering current.

LINE CHARACTERISTICS

918 **Characteristic Impedance.**—The characteristic impedance of a line is the ratio* of the applied electromotive force to the resulting steady-

*Single frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

state current upon a line of infinite length and uniform structure, or of periodic recurrent structure.

NOTE: In practice, the terms (1) line impedance, (2) surge impedance, (3) iterative impedance, (4) sending-end impedance, (5) initial sending-end impedance, (6) final sending-end impedance, (7) natural impedance and (8) free impedance, have apparently been more or less indefinitely and indiscriminately used as synonyms with what is here defined as "characteristic impedance."

919 **Sending-End Impedance.**—The sending-end impedance of a line is the ratio* of the applied electromotive force to the resulting steady-state current at the point where the electromotive force is applied.

NOTE: See note under "Characteristic Impedance". In case the line is of infinite length of uniform structure or of periodic recurrent structure, the sending-end impedance and the characteristic impedance are the same.

920 **Propagation Constant.**—The propagation constant of a uniform line, or section of a line of periodic recurrent structure, is the natural logarithm of the ratio* of the steady-state currents at various points separated by unit length in a uniform line of infinite length, or at successive corresponding points in a line of recurrent structure of infinite length. The ratio is determined by dividing the value of the current at the point nearer the transmitting end by the value of the current at the point more remote.

921 **Attenuation Constant.**—The attenutation constant for a single frequency is the real part of the propagation constant taken at that frequency.

922 **Wave-Length Constant.** The wave-length constant is the imaginary part of the propagation constant.

923 **Standard Cable.**—A standard cable is an ideal uniform line in terms of which the attenuation of a line or network may be specified. It is characterized by the following constants: Linear resistance, 88 ohms per loop mile (54.7 ohms per loop km.). Linear capacitance between wires 0.054 microfarad per loop mile (0.03355 microfarad per loop km.). Linear inductance and linear leakance, 0.

LINE CIRCUITS

930 **Ground-Return Circuit.** A ground-return circuit is a circuit consisting of one or more metallic conductors in parallel, with the circuit completed through the earth.

931 **Metallic Circuit.** A metallic circuit is a circuit of which the earth forms no part.

932 **Two-Wire Circuit.**—A two-wire circuit is a metallic circuit formed by two parallel conductors insulated from each other.

933 **Superposed Circuit.** A superposed circuit is an additional circuit obtained from a circuit normally required for another service, and in such a manner that the two services can be given simultaneously without mutual interference.

934 **Phantom Circuit.** A phantom circuit is a superposed circuit, each side of which consists of the two conductors of a two-wire circuit in parallel.

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*See note preceding page.

935 **Side Circuit.** A side circuit is a two-wire circuit forming one side of a phantom circuit.

936 **Non-Phantomed Circuit.** A non-phantomed circuit is a two-wire circuit, which is not arranged for use as the side of a phantom circuit.

937 **Simplexed Circuit.**—A simplexed circuit is a two-wire telephone circuit, arranged for the superposition of a single ground-return signalling circuit operating over the wires in parallel.

938 **Composited Circuit.** A composited circuit is a two-wire telephone circuit, arranged for the superposition on each of its component metallic conductors, of a single independent ground-return signalling circuit.

939 **Quadded or Phantomed Cable.**—A quadded or phantomed cable is a cable adapted for the use of phantom circuits.

940 **Simplex Circuit.**—A simplex circuit in telegraphy is one arranged for operation in one direction at one time.

941 **Duplex Circuit.**—A duplex circuit in telegraphy is one arranged for simultaneous operation in opposite directions.

942 **Diplex Circuit.**—A diplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in the same direction.

943 **Quadruplex Circuit.**—A quadruplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in each direction.

944 **Multiplex Circuit.**—A multiplex circuit in telegraphy is one arranged for the simultaneous transmission of several messages in one direction.

945 **Interference.**—Interference (electrical) is an electrical disturbance introduced into a communication system by an extraneous source. Examples of interference are lightning interference. interference between radio stations and interference from power systems into telegraph and telephone systems. (See §§**407** to **409**). Interference due to the effect of one telephone line on another is usually referred to as "cross-talk."

946 **Linear Electrical Constants.**—The linear electrical constants of a line are the electrical constants per unit length of the line, e. g. linear resistance, linear inductance etc.

947 **Smooth Line.**—A smooth line is a line whose electric elements are all continuously and uniformly distributed throughout its length.

948 **Periodic Line.**—A periodic line is a line consisting of successive similar sections in each of which one or more electric elements are not distributed uniformly. As examples of periodic lines are (1) loaded lines and (2) artificial lines consisting of successive similar sections of lumped constants.*

949 **Equivalent Smooth Line.**—An equivalent Smooth Line of a periodic line is a smooth line having the same electrical behavior as the periodic line, at a given single frequency, when measured at terminals or at corresponding section junctions. Similarly,

*The term periodic in this definition refers to the line constants and not to time relations.

949a Equivalent Periodic Line.—An equivalent periodic line of a smooth line is a periodic line having the same electrical behavior, for an assumed single frequency, as the smooth line, when measured at terminals or at corresponding section junctions. The terms conjugate smooth line and conjugate periodic line are also sometimes used.

949b Composite Line.—A composite line is a line consisting of a plurality of successive sections having different linear electrical constants, as in the case where an underground cable section is joined to an overhead open-wire section.

949c Amplifier.—See §1034.

LOADING

950 Loaded Line.—A loaded line is one in which the normal reactance of the circuit has been altered for the purpose of increasing its transmission efficiency.

951 Series Loaded Line.—A series loaded line is one in which the normal reactance has been altered by reactance serially applied.

952 Shunt Loaded Line.—A shunt loaded line is one in which the normal reactance of the circuit has been altered by reactance applied in shunt across the circuit.

953 Continuous Loading. A continuous loading is a series loading in which the added inductance is uniformly distributed along the conductors.

954 Coil Loading. A coil loading is one in which the normal inductance is altered by the insertion of lumped inductance in the circuit at intervals. This lumped inductance may be applied either in series or in shunt.

NOTE: As commonly understood, coil loading is a series loading, in which the lumped inductance is applied at uniformly spaced recurring intervals

MISCELLANEOUS

955 Microphone. A contact device designed to have its electrical resistance directly and materially altered by slight differences in mechanical pressure.

957 Resonance.—Resonance of a harmonic alternating current of given frequency, in a simple series circuit, containing resistance, inductance and capacity, is the condition in which the positive reactance of the inductance is numerically equal to the negative reactance of the capacity. Under these conditions, the current flow in the circuit with a given electromotive force has its highest r.m.s. value.

958 Retardation Coil. A retardation coil is a reactor (reactance coil) used in a circuit for the purpose of selectively reacting on currents which vary at different rates.

NOTE: In telephone and telegraph usage, the terms "impedance coil," "inductance coil," "choke coil" and "reactance coil" are sometimes used in place of the term "retardation coil."

959 **Skin Effect.**—Skin effect is the phenomenon of the non-uniform distribution of alternating current throughout the cross-section of a linear conductor, occasioned by variations in the intensity of the magnetic field due to the current in the conductor.

960 **Telephone Receiver.** A telephone receiver is an electrically operated device designed to produce sound waves or vibrations which correspond in form to the electromagnetic waves or vibrations actuating it.

961 **Telephone Transmitter.** A telephone transmitter is a sound-wave or vibration-operated device designed to produce electromagnetic waves or vibrations which correspond in form to the sound waves or vibrations actuating it.

962 **Coefficient of Coupling of a Transformer.**—The coefficient of coupling of a transformer at a given frequency is the ratio* of the mutual impedance between the primary and secondary of the transformer, to the square root of the product of the self-impedances of the primary and of the secondary.

963 **Repeating Coil.** A term used in telephone practice meaning the same as transformer, and ordinarily a transformer of unity ratio.

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*Single frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

APPENDIX I.

DEFINITIONS FOR RADIO COMMUNICATION

The following Sections **1000** to **1033** have been abstracted from the report of the Standardization Committee of the Institute of Radio Engineers, and are here included by permission as an Appendix, until further revised. For full particulars, see the I.R.E. Standardization Committee report.

1000 Acoustic Resonance Device. One which utilizes, in its operation, resonance to the audio frequency of the received signals.

1001 Antenna. A system of conductors designed for radiating or absorbing the energy of electromagnetic waves.

1002 Atmospheric Absorption. That portion of the total loss of radiated energy due to atmospheric conductivity.

1003 Audio Frequencies. The frequencies corresponding to the normally audible vibrations. These are assumed to lie below 10,000 cycles per second.

1004 Capacitive Coupler. An apparatus which, by electric fields, joins portions of two radio frequency circuits, and which is used to transfer electrical energy between these circuits through the action of electric forces.

1005 Coefficient of Coupling (Inductive). The ratio of the effective mutual inductance of two circuits to the square root of the product of the effective self-inductances of each of these circuits.

1006 Direct Coupler. A coupler which magnetically joins two circuits having a common conductive portion.

1007 Counterpoise. A system of electrical conductors forming one portion of a radiating oscillator, the other portion of which is the antenna. In land stations a counterpoise forms a capacitive connection to ground.

1008 A Damped Alternating Current is an alternating current whose amplitude progressively diminishes.

1009 The Damping Factor of an exponentially damped alternating current is the product of the logarithmic decrement and the frequency.

Let I_0 = initial amplitude
I_t = amplitude at the time t
ϵ = base of Napierian logarithms
a = damping factor

Then: $I_t = I_0\, \epsilon^{-at}$

1010 Detector. That portion of the receiving apparatus which, connected to a circuit carrying currents of radio frequency, and in conjunction with a self-contained or separate indicator, translates

the radio frequency energy into a form suitable for operation of the indicator. This translation may be effected either by the conversion of the radio frequency energy, or by means of the control of local energy by the energy received.

1011 Electromagnetic Wave. A periodic electromagnetic disturbance progressing through space.

1012 Forced Alternating Current. A current, the frequency and damping of which are equal to the frequency and damping of the exciting electromotive force.

1013 Free Alternating Current. The current following any electromagnetic disturbance in a circuit having capacity, inductance, and *less* than the critical resistance.

1014 Critical Resistance of a Circuit. That resistance which determines the limiting condition at which the oscillatory discharge of a circuit passes into an aperiodic discharge.

1015 Group Frequency. The number per second of periodic changes in amplitude or frequency of an alternating current.

NOTE 1. Where there is more than one periodically recurrent change of amplitude or frequency, there is more than one group frequency present.

NOTE 2. The term "group frequency" replaces the term "spark frequency."

1016 Inductive Coupler. An apparatus which, by magnetic forces, joins portions of two radio frequency circuits and is used to transfer electrical energy between these circuits, through the action of these magnetic forces.

1018 Logarithmic Decrement of an exponentially damped alternating current is the logarithm of the ratio of successive current amplitudes in the same direction.

NOTE: Logarithmic decrements are standard for a complete period or cycle.

Let: I_n and I_{n+1} be successive current amplitudes in the same direction.

d = logarithmic decrement

Then: $$d = \log_e \frac{I_n}{I_{n+1}}$$

1019 Radio Frequencies. The frequencies higher than those corresponding to the normally audible vibrations, which are generally taken as 10,000 cycles per second. See also Audio Frequencies.

NOTE: It is not implied that radiation cannot be secured at lower frequencies and the distinction from audio frequencies is merely one of definition based on convenience

1020 Resonance of a circuit to a given exciting alternating e.m.f. is that condition due to variation of the inductance or capacity in which the resulting effective current (or voltage) in that circuit is a maximum.

1024 A Standard Resonance Curve is a curve the ordinates of which are the ratios of the square of the current at any frequency to the

square of the resonant current, and the abscissas are the ratios of the corresopnding wave length to the resonant wave length; the abscissas and ordinates having the same scale.

1026 Sustained Radiation consists of waves radiated from a conductor in which an alternating current flows.)

1027 Tuning. The process of securing the maximum indication by adjusting the time period of a driven element. (See Resonance.)

1028 A Wave-Meter, is a radio frequency measuring instrument, calibrated to read wave lengths.

1030 Decremeter. An instrument for measuring the logarithmic decrement of a circuit or of a train of electromagnetic waves.

1031 Attenuation, Radio. The decrease with distance from the radiating source, of the amplitude of the electric and magnetic forces accompanying (and constituting) an electromagnetic wave.

1032 Attenuation, Coefficient of (Radio). The coefficient, which, when multiplied by the distance of transmission through a uniform medium, gives the natural logarithm of the ratio of the amplitude of the electric or magnetic forces at that distance, to the initial value of the corresponding quantities.

1033 Coupler. An apparatus which is used to transfer radio-frequency energy from one circuit to another by associating portions of these circuits.

1034 Amplifier.—An amplifier is an instrument which modifies the effect of a local source of energy in substantial accordance with the waveform of the received energy, and gives out a wave of greater amplitude than that which it receives.

1035 Interference.—See **§§17a, 407, 408, 409** and **945.**

1036 Phase Angle Defect.—The phase angle defect of a condenser is the departure from quadrature of the phase difference between potential and current at terminals. This is sometimes called the phase angle of a condenser: although strictly speaking the phase angle of a condenser is 90° less the phase angle defect, and is therefore exactly 90° when the phase angle defect is zero.

APPENDIX II.

ADDITIONAL STANDARDS FOR RAILWAY MOTORS*

1100 In comparing projected motors, and in case it is not possible or desirable to make tests to determine mechanical losses, the following values of these losses, determined from the averages of many tests over a wide range of sizes of single-reduction single-geared motors, will be found useful, as approximations. They include axle-bearing, gear, armature-bearing, brush-friction, windage, and stray-load losses.

TABLE XX
Approximate Losses in D-C. Railway Motors.

Input in per cent of that at nominal rating	Losses as per cent of input
100 or over	5.0
75	5.0
60	5.3
50	6.5
40	8.8
30	13.3
25	17.0

1101 The core loss of railway motors is sometimes determined by separately exciting the field, and driving the armature of the motor to be tested, by a separate motor having known losses and noting the differences in losses between driving the motor light at various speeds and driving it with various field excitations.

1102 Selection of Motor For Specified Service

The following information relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(a) Weight of total number of cars in train (in tons of 2000 lb.) exclusive of electrical equipment and load.
(b) Average weight of load and durations of same, and maximum weight of load and durations of same.
(c) Number of motor cars or locomotives in train, and number of trailer cars in train.
(d) Diameter of driving wheels.
(e) Weight on driving wheels, exclusive of electrical equipment.
(f) Number of motors per motor car.
(g) Voltage at train with power on the motors—average, maximum and minimum.

*See also Sections 800 et seq.

(h) Rate of acceleration in mi. per. hr. per second.
(i) Rate of braking (retardation in m. per hr. per second).
(j) Speed limitations, if any (including slowdowns).
(k) Distances between stations.
(l) Duration of station stops.
(m) Schedule speed including station stops in m.p.h.
(n) Train resistance in pounds per ton of 2000 pounds at stated speeds.
(o) Moment of inertia of revolving parts, exclusive of electrical equipment.
(p) Profile and alignment of track.
(q) Distance coasted as a per cent of the distance between station stops.
(r) Time of layover at end of run, if any.

1103 Stand-Test Method of Comparing Motor Capacity with Service Requirements: When it is not convenient to test motors under actual specific service conditions, recourse may be had to the following method of determining temperature rise.

1104 The essential motor losses affecting temperatures in service are those in the motor windings, core and commutator. The mean service conditions may be expressed, as a close approximation, in terms of that continuous current and core loss which will produce the same losses and distribution of losses as the average in service.

A stand test with the current and voltage which will give losses equal to those in service, will determine whether the motor has sufficient capacity to meet the service requirements. In service, the temperature rise of an enclosed motor (**§164**), well exposed to the draught of air incident to a moving car or locomotive, will be from 75 to 90 per cent (depending upon the character of the service) of the temperature rise obtained on a stand test with the motor completely enclosed and with the same losses. With a ventilated motor (**§165 and §167**), the temperature rise in service will be 90 to 100 per cent of the temperature rise obtained on a stand test with the same losses.

1105 In making a stand test to determine the temperature rise in a specific service, it is essential in the case of a self-ventilated motor (**§ 167**), to run the armature at a speed which corresponds to the schedule speed in service. In order to obtain this speed it may be necessary, while maintaining the same total armature losses, to change somewhat the ratio between the I^2R and core-loss components.

1106 Calculation for Comparing Motor Capacity with Service Requirements. The heating of a motor should be determined, wherever possible, by testing it in service, or with an equivalent duty cycle. When the service or equivalent duty-cycle tests are not practicable, the ratings of the motor may be utilized as follows to determine its temperature rise.

1107 The motor losses which affect the heating of the windings are as stated above, those in the windings and in the core. The former are proportional to the square of the current. The latter vary with the voltage and current, according to curves which can be supplied by the manufacturers. The procedure is therefore as follows:

1108 (a) Plot a time-current curve, a time-voltage curve, and a time-core loss curve for the duty cycle which the motor is to perform, and calculate from these the root-mean-square current and the average core loss.

1109 (b) If the calculated r.m.s. service current exceeds the continuous rating, when run with average service core loss and speed, the motor is not sufficiently powerful for the duty cycle contemplated.

1110 (c) If the calculated r.m.s. service current does not exceed the continuous rating, when run with average service core loss and speed, the motor is ordinarily suitable for the service. In some cases, however, it may not have sufficient thermal capacity to avoid excessive temperature rises during the periods of heavy load. In such cases a further calculation is required, the first step of which is to compute the equivalent voltage which, with the r.m.s. current, will produce the average core-loss. Having obtained this, determine, as follows, the temperature rise due to the r.m.s. service current and equivalent voltage.

Let t = temperature rise
$p_0 = I^2R$ loss, kw.
p_c = core loss, kw.
} with r.m.s. service current, and equivalent service voltage.

T = temperature rise
$P_0 = I^2R$ loss, kw.
P_c = core loss, kw.
} with continuous load current corresponding to the equivalent service voltage.

Then

$$t = T\,\frac{p_0 + p_c}{P_0 + P_c}, \text{ approximately.}$$

1111 (d) The thermal capacity of a motor is approximately measured by the ratio of the electrical loss in kw. at its nominal (one-hour) capacity, to the corresponding maximum observable temperature rise during a one hour test starting at ambient temperature.

1112 (e) Consider any period of peak load and determine the electrical losses in kilowatt-hours during that period from the *electrical* efficiency curve. Find the excess of the above losses over the losses with r.m.s. service current and equivalent voltage. The excess loss, divided by the co-efficient of thermal capacity, will equal the extra temperature rise due to the peak load. This temperature rise added to that due to the r.m.s. service current, and equivalent voltage, gives the total temperature rise. If the total temperature rise in any such period exceeds the safe limit, the motor is not sufficiently powerful for the service.

1113 (f) If the temperature reached, due to the peak loads, does not exceed the safe limit, the motor may yet be unsuitable for the service, as the peak loads may cause excessive sparking and dangerous mechanical stresses. It is, therefore, necessary to compare the peak loads with the short-period overload capacity. If the peaks are also within the capacity of the motor, it may be considered suitable for the given duty cycle.

APPENDIX III.

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REPORT OF THE BOARD OF DIRECTORS

FOR FISCAL YEAR ENDING APRIL 30, 1918

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-fourth Annual Report, for the fiscal year ending April 30, 1918. A General Balance Sheet showing the condition of the Institute's finances on April 30, 1918, together with other detailed financial statements, is included herein.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year and one adjourned meeting. Eight meetings were held in New York, one in Philadelphia in October, and one in Cleveland in March. The adjourned meeting was held in New York in March.

The Executive Committee held two meetings in New York, on January 5 and January 22, 1918. Both of these meetings were held for the purpose of considering matters relating to the Engineering Council.

In accordance with the practise established years ago, the Board has endeavored to keep the membership informed of its proceedings by the monthly publication of a résumé of the business transacted at each meeting. These notices have appeared in each issue of the PROCEEDINGS, but they are not, however, a complete report of the work done by the Board at any one meeting, for the reason that many important matters are referred to committees for further consideration, and publicity in such cases must generally be deferred pending their final disposition. Information relating to such matters is usually published in subsequent issues.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 18, 1917. The Annual Report of the Board of Directors for the fiscal year ending May 1, 1917, was presented as published in full in the June 1917 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1917.

The principal feature of the meeting was the presentation of the Edison Medal to Nikola Tesla; this ceremony following the business meeting.

Annual Convention.—The Thirty-fourth Annual Convention was originally scheduled to be held at Hot Springs, Va., June 26-29, 1917. In view of the national crisis which developed in April it was decided by the Board of Directors at the May meeting to cancel the Convention and to hold instead a special meeting in New York City, for the presentation of

the papers originally scheduled for presentation at the Convention. This special meeting was held on June 27-28.

Pacific Coast Convention.—The Pacific Coast Convention for 1917 was to have been held under the auspices of the Los Angeles Section in September 1917. At the May meeting of the Board of Directors, upon the recommendation of the Los Angeles Section and the Meetings and Papers Committee, the Convention was cancelled.

Philadelphia Meeting.—The first Institute meeting of the administrative year of 1917-18 was held in Philadelphia on October 8, 1917. There were two technical sessions, at which three papers were presented.

Three-City Meeting.—A Three-City Meeting was held in Boston, January 8, New York, January 11, and Chicago January 14, 1918, the same paper being presented at all three meetings. This innovation was tried for the purpose of providing an opportunity for the local members in all three cities to participate in the discussion, all of which will be published in the TRANSACTIONS.

Midwinter Convention, New York.—The Sixth Midwinter Convention was held in New York on February 15 and 16, 1918. Four technical sessions were held. The attendance was about 400. On the evening of February 15 an informal dinner was held which was attended by 225 members and guests.

Cleveland Meeting.—An inter-section meeting in which the Toledo, Toronto, Detroit, Pittsburgh and Cleveland Sections participated, was held in Cleveland, Ohio, on March 8, 1918. Four technical papers were presented at two technical sessions. An informal dinner was held between the afternoon and evening sessions. The latter session being held jointly with the Association of Iron and Steel Electrical Engineers, one of the four papers was presented on behalf of the A. I and S. E. E. The meeting was attended by Presidents E. W. Rice, Jr., of the A. I. E. E., and C. A. Mink, of the A. I. and S. E. E., and members were present from all the sections mentioned above. The attendance at this meeting was about 150.

Pittsburgh-New York Meeting.—The Institute meeting for April was held in Pittsburgh on April 9 and in New York on April 12, under an arrangement similar to the meetings held in Boston, New York and Chicago in January 1918. The same two papers were presented at both the New York and Pittsburgh meetings.

New York Meetings.—In addition to the meetings referred to above, Institute meetings were also held in New York in November and April. The attendance at the November meeting was 175 and one paper was presented; the attendance at the April meeting was 250 and two papers were presented. The April meeting was preceded by an informal buffet supper under the auspices of the New York Membership Acquaintance committee.

National Defense.—In the annual reports for 1916 and 1917 the published references to the Institute's connection with matters pertaining to National Defense were limited to the various actions taken during the year covered by each report. It has been thought, in view of the entry of the United States into the war, that the membership will gain a better perspective of the services actually rendered by the Institute if in preparing the present report there be included not only a record of the developments of the past year, but a summary of all of the defense work done during the three years from the time when the co-operation of the engineering societies was first suggested in connection with national preparedness. It is felt that such a statement will also be useful as a record for future reference, and the following has been prepared with these objects in view.

Engineer Officers' Reserve Corps.—Early in 1915, when the subject of an Officers' Reserve Corps in connection with the Army reorganization plan was under consideration in Congress, the suggestion was made that the National Engineering Societies tender their assistance to the War Department in the formation of an Engineer Reserve, there being no adequate provision at the time for any large body of engineers in the Army. Acting on this suggestion the Institute and other national engineering societies formed a joint committee for the purpose of taking such steps as seemed desirable to assist in the organization of such a corps as a part of the regular Army. Conferences were held with the Secretary of War, officers of the General Staff and the War College, and the chairmen of the House and Senate Committees on Military Affairs. Largely as a result of the work of this joint committee, the National Defense Act of 1916, which became effective on July 1st of that year, embodied a provision for an Engineer Reserve Corps as part of the Officers' Reserve Corps. A circular containing an abstract of the engineer section of this Act was mailed to all members of the Institute on July 5, 1916.

Upon completion by the War Department of the details of the requirements and qualifications for commissions in the Officers' Reserve Corps as provided in the new law, a second circular was issued by the Institute to the membership under date of September 12, 1916, giving complete information regarding the qualifications for commissions in the Reserve, thus enabling interested members to apply for commissions promptly. Many applications were filed at that time by members of the Institute who are now in active service.

Naval Consulting Board.—In July 1915 the Institute was invited by the Honorable Josephus Daniels, Secretary of the Navy, to select two members for appointment by him upon a proposed Naval Advisory Board, to be presided over by Mr. Thomas A. Edison, and to be composed of men recognized throughout the country for their inventive genius and engineering achievements, to assist the Navy Department, instructively and critically, in the development of such new ideas for naval advance as might be found worthy of consideration. The underlying idea was to make available to the Navy Department the latent inventive and engineering genius of the country to improve the Navy and to bring the officers of the service into more intimate contact with the industrial resources of the

country. Similar invitations were extended to ten other scientific and engineering organizations.

Industrial Inventory.—The good work of the Institute's representatives upon the Naval Consulting Board is attested to in the following letter and invitation received by the Institute from the President of the United States under date of January 13, 1916:

"The White House
Washington. January 13, 1916.

My dear Sir:

The work which the American Institute of Electrical Engineers has done through its members on the Naval Consulting Board is a patriotic service which is deeply appreciated. It has been so valuable that I am tempted to ask that you will request the Institute to enlarge its usefulness to the Government still further by nominating for the approval of the Secretary of the Navy a representative from its membership for each state in the Union to act in conjunction with representatives from the American Society of Mechanical Engineers, the American Society of Civil Engineers, the American Chemical Society, and the American Institute of Mining Engineers, for the purpose of assisting the Naval Consulting Board in the work of collecting data for use in organizing the manufacturing resources of the country for the public service in case of emergency. I am sure that I may count upon your cordial cooperation.

With sincere regard,

Cordially yours,
(Signed) WOODROW WILSON.

Mr. John J. Carty, President,
American Institute of Electrical Engineers,
New York City."

This invitation was accepted by the Board of Directors of the Institute at a special meeting called on January 21, 1916, and the nominees selected by the Institute were subsequently appointed by the Secretary of the Navy. The resulting organization became officially known as the *State Directors of the Organization for Industrial Preparedness and Associate Members of the Naval Consulting Board of the United States.* These State Directors made a canvass of all the industrial establishments in their respective states by means of a confidential industrial inventory form giving in great detail data regarding their manufacturing and producing resources. The statistics obtained by the Government as the result of this work fully justified the confidence shown by the President of the United States in entrusting to engineers a task of such magnitude and detail.

General Co-operation with Government.—On February 5, 1917, during the diplomatic exchanges between the American and German Governments, resulting from the announced policy of the latter to sink merchant shipping in certain prescribed zones without warning, the following joint telegram, signed by the Presidents of the Civil, Mining, Mechanical and Electrical Engineers, was sent to the President of the United States:

"To the President,
Executive Mansion,
Washington, D. C.

We, the presidents of the National Societies of Civil, Mining, Mechanical and Electrical Engineers, and of the United Engineering Society, with a membership of thirty thousand,

cordially unite in supporting Congress and the Administration in the stand for freedom and safety on the seas, and we are confident that we represent the membership of the four societies in offering to assist toward the organization of engineers for service to our country in case of war."

This was followed by a special meeting of the Executive Committee on February 6, at which it was decided to issue immediately, on behalf of the Board of Directors, to all Institute members in the United States, a circular letter calling attention to this telegram and to the opportunities existing for patriotic service to the nation in case of emergency. This circular letter was issued under date of February 8, 1917, and was accompanied by a simple classification sheet which members were requested to fill out and return to Institute headquarters, indicating whether or not they would be available for military or naval service if required. Over two thousand of these classification sheets were filled out and returned to Institute headquarters and subsequently were placed at the disposal of the War Department. A considerable number of the members who sent in data sheets was communicated with by the branches most urgently in need of their services.

On April 2, 1917, in co-operation with several other engineering societies, the Institute issued to the membership two pamphlets containing instructions and methods of procedure for engineers who desired to offer their services in the Army or Navy of the United States. The pamphlets also contained all information available regarding all of the branches of the Army and Navy in which the services of electrical engineers could be utilized to good advantage. The circulation of these pamphlets aroused great interest and resulted in the filing of a great many more applications by Institute members for commissions in the Officers' Reserve Corps and in the Naval Reserve, many of which were favorably acted upon.

During the past year the Institute has maintained its connection with the Government through a number of agencies, among which may be mentioned: the Naval Consulting Board, the Subcommittee on Wires and Cables of the Standards Committee, the National Research Council, the Engineering Council, and the General Engineering Committee of the Advisory Commission of the Council of National Defense.

Personnel.—In the work of obtaining technically trained men and men of special qualifications for Government service, the Institute has been able to render considerable assistance to the Government. It has been called upon repeatedly by various branches of the Army, and the Navy for technical men for special services, also by industrial corporations and various Government bureaus, to recommend individuals or small groups of men for special work.

On September 20, 1917, Rear-Admiral L. C. Palmer, Chief of the Bureau of Navigation, Navy Department, in a letter addressed to the President of the Institute, informed him that the Secretary of the Navy had authorized the commissioning of one hundred graduate electrical engineers as Lieutenants, junior grade, in the U. S. Naval Reserve Force, that three organizations, namely, the Naval Consulting Board, the National Research Council and the American Institute of Electrical Engineers, were each requested to nominate eighty-five men possessing certain specified

qualifications, and that from the total of two hundred and fifty-five candidates thus nominated, a Board of Naval Examiners would select the one hundred men to be commissioned. A circular containing this information was mailed to the entire membership of the Institute in the United States, so that every member might have an opportunity to apply for one of these commissions or to recommend desirable candidates.

At the Directors meeting held on October 8, 1918, a special committee was appointed to examine the applications received by the Secretary and to select therefrom the Institute's eighty-five nominees. This committee, consisting of five members of the Board of Directors, met at Institute headquarters on October 16 and 17, and from the applications received up to that date, selected, solely upon their merits, eighty-five nominees whose names were transmitted to Rear-Admiral Palmer on October 18. The Naval Consulting Board and the National Research Council transmitted their nominations upon the same date.

Of the eighty-five men nominated by the Institute, thirty-eight were included in the first hundred commissioned. It was learned later from reports received from individual members of the Institute that in addition to the one hundred men who were originally commissioned a considerable number of other applicants received commissions.

Under date of March 22, 1918, the Institute was requested by Rear-Admiral Palmer to nominate, in groups of twenty-five each, specially qualified electrical engineers, for training as submarine officers in the Naval Reserve Force. The successful candidates are to be commissioned as Ensigns, and after satisfactorily completing a special technical course of instructions they will become part of the active Submarine Officer Complement of the Navy. The first group of nominations was submitted to the Navy Department, early in April.

This request was supplemented several weeks later with a call for an additional 50 nominees for the rank of Ensign in general service in the Naval Reserve Force. Work on the selection of these nominees is now progressing.

Other requests received by the Institute for co-operation in obtaining the services of technical men include the following:

Navy Department: Bureau of Navigation, candidates for aviation inspection duty for training as Ensigns in the Naval Reserve Flying Corps; Bureau of Yards and Docks, candidates for appointment in the Civil Engineer Corps of the Naval Reserve Force, for the ranks of Ensign, Lieutenant (junior grade) and Lieutenant.

Bureau of Chemistry: electrical engineers for special investigations in grain-dust explosions in mills throughout the country.

War Department: Signal Corps, electrical men for radio division, radio operators and mechanicians for Air Service, and graduate electrical engineers for the aviation section; Coast Artillery Corps, electrical graduates for training school at Ft. Monroe, Va. with opportunity for commissions; Ordnance Department, Trench Warfare Section, technical men for commissions in Ordnance Officers Reserve Corps; Corps of Engineers, enlisted men for engineering and railway units and replacement regiments of engineers.

Eight hundred and twenty-four members of the Institute are now serving with the uniformed forces in the Army or Navy of the United States, and in addition, a large number are serving the Government in various civilian capacities. A great many members are also giving their services as members of various committees engaged in war activities.

General Engineering Committee.—Shortly after the entry of the United States into the war and at the suggestion of President Wilson, each of the four National Engineering Societies designated two representatives upon a committee of the Advisory Commission of the Council of National Defense under Dr. Hollis Godfrey, Commissioner of Engineering and Education, as chairman, to serve as the medium through which the engineering societies might serve the Government. In September last this committee was reorganized as the "General Engineering Committee," with Prof. C. A. Adams as chairman.

Besides acting on minor questions referred to it by the Government, this committee was active on four important tasks: 1. the preparation of a scheme of organization for war purchases, which has since partly been put into operation; 2. electric welding in shipbuilding; 3. development of cast steel anchor chain; 4. specifications for shipboard cable for the Navy Department. (For details regarding the three latter items see statement herein relating to the Standards Committee.)

The General Engineering Committee dissolved when the Council of National Defense discontinued all of its advisory committees. The work was then reorganized and continued under other agencies.

National Research Council.—The National Research Council was formed at the request of the President of the United States by the President of the National Academy of Sciences for the purpose of furthering scientific research in its broadest aspects. Its field includes educational institutions, technical, scientific and medical organizations, governmental departments and the industries. Upon invitation the Institute designated representatives upon the Engineering Committee of the Council. This committee was organized early in 1917 with offices in Washington and New York. It is in contact with the work of the other committees of the Council and since its organization it has been engaged on a wide range of engineering problems brought to its notice by the several branches of the Government. Weekly meetings have been held and weekly reports of its activities have been sent to each member of the committee. The confidential nature of these reports and of the problems which have been under consideration has rendered it inexpedient to publish detailed statements regarding the work which has been carried on.

Membership Service Classification.—Early in February 1917 the Institute offered its services to the Government in case of emergency. It soon became evident that in order to extend the fullest possible cooperation and to respond promptly and effectively to the numerous demands which were being made upon the Institute by the different governmental departments and industrial establishments engaged in Government work, for technical men of special qualifications, it would be necessary to have available for immediate reference at Institute headquarters de-

tailed information regarding the many members who were willing and anxious to serve the Government. After conferences with the other National Engineering Societies, the matter was brought to the attention of the Board of Directors of the Institute at its meeting of October 8, 1917, at which meeting a proof of a suggested form of questionnaire which had been prepared was submitted with a request for an appropriation sufficient to distribute the form to the membership and to compile an index of the replies. Recognizing the importance of such a classification, the Board granted an appropriation of $1,000 for this purpose and authorized the Secretary to proceed with the work after opportunity had been given for final revision of the form. A special committee was appointed to have supervision of the classification and indexing.

Although primarily intended as an aid in selecting technical men for service in the present emergency, it is proposed to maintain a permanent file of this data at Institute headquarters to be revised from year to year for use after peace has again been restored. Such a classification has been contemplated for several years in connection with the regular work of the Institute and its committees.

Employment.—It is over three years since the Institute first initiated the plan for assisting its members in obtaining employment and employers in obtaining desirable employees. It has not attempted to conduct an employment department in the generally accepted sense of the term, but has simply acted as a medium for placing men in touch with opportunities, through the publication of announcements in the montly Institute PROCEEDINGS, under the heading of "Engineering Service Bulletin."

While the service might have been developed to a greater extent had it been possible to appropriate funds for carrying on the work, the results attained since the plan was inaugurated are, nevertheless, very gratifying. There is no doubt that it offers a wide sphere of usefulness, and that the service is highly appreciated; not only by the individual member who is seeking a position, but also by the many corporations and men in posts of responsibility who have been assisted by the Institute in obtaining the services of high-class technical men. The co-operation of the membership in bringing vacancies to the attention of the Secretary will be very helpful in the future successful conduct of this work.

United Engineering Society.—In the Directors' Annual Report for 1917 reference was made to the addition of three stories to the Engineering Societies Building as a result of an agreement whereby the American Society of Civil Engineers was to be admitted into the fraternity of founder societies and take up its headquarters in the building.

Full details regarding this agreement were published in the PROCEEDINGS for September 1916.

This work was completed in the Fall of 1917 and on December 7, 1917, a joint meeting was held under the auspices of the Institute, the American Society of Mechanical Engineers, and the American Institute of Mining Engineers, at which the American Society of Civil Engineers was formally welcomed into the fraternity of the founder societies and the occupancy of its quarters in the enlarged Engineering Societies Building. These

four great National Engineering Societies are thus brought together under one roof with all of the obvious advantages for closer cooperative action, realizing more fully the purpose of the donor of the building, Mr. Andrew Carnegie, that it should be the home and headquarters of the engineering profession of America.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by four of the leading national engineering societies—the American Society of Civil Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers and the American Institute of Electrical Engineers, to establish a central body to deal with matters of common interest to engineers and to serve as a connecting medium between the engineer on the one hand and the public welfare on the other so far as such matters relate to the engineering profession, in order that united action may be possible.

The first meeting of the Council was held on June 27, 1917. The Annual Meeting of the Council was held on February 21, 1918. Officers were elected and the following committees were appointed: Executive, Finance, Rules, Public Affairs, American Engineering Service, War, Fuel Conservation and Patents.

For details regarding future plans of the Council, its field, and aims, members are referred to the abstracts from a statement issued by the Council in February 1918, published in the Institute PROCEEDINGS for April 1918.

Sections Committee.—The encroachment of the many demands of these unprecedented times has been reflected in the activities of the Sections and Branches. Many of the leading men have been called away on war duties, while others have assumed additional burdens at home. The number of meetings held during the past year has for these reasons been reduced, yet it is gratifying to observe that the total attendance in the Section meetings has actually increased.

This is undoubtedly due to the fact that these meetings have been arranged to present topics of current interest. It has been evident to any one who has seen the notices of these various meetings over the country that the thought of service and knowledge of present day problems were uppermost in the mind of the engineer.

The list of Sections has been increased by the addition of Erie, Pa., which qualified on January 11, 1918, under conditions which inspire confidence in its future.

The Branches have, however, been severely affected by the influences of war. There our young engineers have forfeited their present educational advantages and have gone into the ranks of the service in large numbers. Several Branches have suspended activity, temporarily, while two, the Queens University Branch, Kingston, Ontario, and the Michigan Agricultural College Branch, East Lansing, Michigan, were added, respectively, on January 11 and March 15, 1918. Two other Branches, the Iowa State College Branch, and the Rhode Island State College

Branch, were terminated by the Board of Directors at their request. The tabulation following shows the reduction in Branch activity.

	For Fiscal Year Ending					
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918
SECTIONS						
Number of Sections	29	30	31	32	32	34
Number of Section meetings held	244	233	246	251	265	245
Total Attendance	22,825	22,626	23,507	28,553	31,299	34,614
BRANCHES						
Number of Branches	47	47	52	54	59	59
Number of Branch meetings held	357	306	328	360	368	268
Attendance	11,808	11,617	12,712	15,166	16,107	10,683

On the whole the situation in the Sections and Branches is one for encouragement rather than otherwise. It augurs well for the participation of the engineer in the country's large affairs and insures the continued maintenance of the activities of the Institute.

Standards Committee.—The Standards Committee has held monthly meetings throughout the year except in January and the summer months.

Owing to the numerous demands made upon many members of the committee by war work, all non-essential subcommittees were allowed to mark time thus reducing the number of active subcommittees from over 40 to about 20.

Subcommittee on Wires and Cables.—The Subcommittee on Wires and Cables of the Standards Committee was appointed by the Board of Directors in response to the request of Rear-Admiral Griffith, Chief of the Bureau of Steam Engineering, to assist the Navy Department in the solution of problems relating to wires and cables with special reference to the high-tension cables to be used on the new electrically-driven warships.

This committee, after conducting a considerable number of investigations and much experimental research, has sent reports to the Navy Department on several subjects. The committee is still co-operating with the Navy Department and expects to extend its activities to other departments of the Government.

Electric Welding in Ship Construction. Another important piece of war work which originated in the Standards Committee is the application of electric welding to shipbuilding. This was started in August 1917 as the Electric Welding Subcommittee of the Standards Committee, and was adopted by the General Engineering Committee of the Council of National Defense in September. Finally, after the Council of National

Defense had dropped all of its advisory committees, this subcommittee was appointed in February by the U. S. Shipping Board, Emergency Fleet Corporation, as its Electric Welding Committee, with C. A. Adams as chairman and with ample financial support. An enormous amount of valuable work has been and is being done by this committee, which includes representatives of: The Emergency Fleet Corporation, the Classification Societies, (Lloyds Register of Shipping and the American Bureau of Shipping), U. S. Navy, Bureau of Standards, electric welding manufacturers and users, electrical engineers and metallurgists.

The Research Subcommittee of the Electric Welding Committee is also attached to the Engineering Division of the National Research Council.

The principal object of the Electric Welding Committee is to save time, labor, material and expense by the extension of the application of electric welding to shipbuilding. This is being done not only by the extension of the application to minor parts of ships, but also by demonstrating its suitability and economy on the capital parts of ships.

British Conference. As a result of an invitation from the Engineering Standards Committee of Great Britain, Mr. H. M. Hobart was sent to London in September 1917 as a delegate from the Standards Committee to a joint conference on standards for electrical machinery. Mr. Hobart brought back numerous suggestions which have been acted upon by the Standards Committee.

Mr. Hobart also investigated the electric welding situation in England and rendered a report which has proved very satisfactory in the work of the Electric Welding Committee.

Another result of this visit was that the Emergency Fleet Corporation, engaged Captain James Caldwell, in charge of the electric welding work for the British Admiralty, to spend a couple of months here and to give the Electric Welding Committee the benefit of the experience of Great Britain. This visit has proved most helpful all around.

Standardization Rules. The result of the year's work of the Standards Committee as far as the Standardization Rules are concerned, does not involve many radical changes, although some of the additions are distinctly valuable. The changes and additions will be published in a revised supplement. A complete revision of form and arrangement is under way and will be completed for a new edition in 1919.

American Engineering Standards Committee.—In January 1917 a joint committee was appointed with three representatives from each of the Four National Engineering Societies, and the American Society for Testing Materials, to prepare a plan of organization for an American Engineering Standards Committee. This committee completed its work on June 19, 1917, and rendered its report to the five societies interested.

The governing boards of the four National Engineering Societies, A. S. C. E., A. S. M. E., A. I. M. E., and A. I. E. E., approved the plan of organization and appointed three representatives each upon the proposed American Engineering Standards Committee. The executive committee of the A. S. T. M., presented certain suggestions and additions. The organization committee was therefore continued and proceeded to consider the suggestions of the A. S. T. M.

A revision of the constitution and rules of procedure governing the proposed American Engineering Standards Committee, satisfactory to the representatives of all the societies, has finally been completed and a report has been submitted to the governing boards of the respective societies for approval.

Meetings and Papers Committee.—The Meetings and Papers Committee has held six monthly meetings during the past year, at which the programs of the Institute meetings and conventions during the season were arranged. One meeting was held in Philadelphia in October 1917, four in New York during the winter, and one in Cleveland in March 1918.

The policy of the Institute of holding a number of its regular monthly meetings outside of New York City was thoroughly discussed at the opening meeting and while there was considerable sentiment in favor of continuing this policy as in the previous year, the difficulty of railroad travel and the fact that the present year was an extremely busy one for a large proportion of the Institute membership led the committee to limit the meetings outside of New York to two; one of which was held in Philadelphia in October, and the other in Cleveland in March.

About 80 papers have been considered by the committee during the year and owing to the abnormally high cost of printing and publishing at the present time the selection of papers has been made with unusual care. Every paper which has been offered during the past year has been reported upon by one or two members of the technical committee to which it belonged before final action has been taken by the Meetings and Papers Committee.

In order to bring the papers presented at regular meetings before as large a number of the Institute members as possible, a new arrangement for meetings has been devised which has been called Inter-section Meetings, in which one paper is presented as nearly simultaneously as possible before several different Sections. Two of these meetings have been held this year with considerable success; one in January 1918 in Boston, New York and Chicago, and another in April in Pittsburgh and New York. The meeting at Cleveland was held under the joint auspices of the Cleveland, Pittsburgh, Toledo, Toronto and Detroit Sections, and a joint technical session was held with the Association of Iron and Steel Electrical Engineers. This participation by the different Sections in regular Institute meetings is believed to have added considerable stimulus to the activities of the Sections participating and has been generally beneficial to the Institute.

Editing Committee.—The Editing Committee, which now has entire supervision of the TRANSACTIONS, announced last year that future issues of TRANSACTIONS would be published semi-annually. Owing to war considerations, however, which resulted in a considerable reduction in the amount of material published, it was found that the papers for the entire year would make only one volume of the usual size and it was subsequently decided to publish the 1917 TRANSACTIONS in one volume. This plan was followed to avoid the publication of two small books instead of one of the usual size, and it also results in a very considerable reduction in the expense of binding and distribution.

The volume is now completed and is being distributed approximately five months earlier than in previous years.

The system of handling papers and discussions for the TRANSACTIONS has been gradually evolved from experience with different methods. It has now been practically standardized for several years and appears to meet with general satisfaction.

All authors and discussors have the opportunity of revising their contributions both in the manuscript of the stenographer and in the proof. Each author has the privilege of reading the proof of all discussion before the final revision of his closure.

All papers published in the PROCEEDINGS are not necessarily published in the bound TRANSACTIONS. The responsibility for a decision in this matter now rests with the Editing Committee.

The committee suggests that papers of the character that are accepted for publication without presentation be printed only in the Annual TRANSACTIONS.

Public Policy Committee.—The Public Policy Committee held one formal meeting during the year at which a communication regarding the bill to incorporate the American Academy of Engineers, referred to the committee by a vote of the Board of Directors, was considered. A report upon this matter was subsequently made to the Board of Directors by the committee.

A majority of the members of the Public Policy Committee, being members also of the Engineering Council, official action by the committee on matters other than that above referred to has not seemed desirable, as such matters now come within the scope of the Engineering Council, thus resulting in a large measure in cooperation between the four National Engineering Societies.

Code Committee.—The Code Committee has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association and representatives of the committee attended the biennial meeting of the National Fire Protection Association. The name of this committee was recently changed by an amendment to the by-laws of the Institute from "Code Committee" to "Committee on Safety Codes."

U. S. National Committee, International Electrotechnical Commission.—Two meetings were held by this committee during the year, both at Institute headquarters; the first on December 14, 1917, the second on March 13, 1918, the latter meeting being held jointly with the Institute's Standards Committee.

The International Electrotechnical Commission has been prevented by the War from holding any international gathering, but it has been the expressed desire of the U. S. National Committee that an attempt should be made to hold such a meeting as soon as conditions will permit, as the lack of intercommunication between the various national committees of the allied countries since 1913 has naturally served to retard generally electrotechnical development.

At the March meeting attention was given to certain inquiries made by the French National Committee concerning such changes as may have

taken place in the A. I. E. E. Standardization Rules since the last convention of the International Electrotechnical Commission in 1913. The necessity for such explanations would not have arisen if the work of the Commission had not been held in abeyance by the world-war. Some progress, however, has been made independently among the various individual national committees.

Committee on Code of Professional Conduct.—The only matter which this committee acted upon during he year was the formulation of definitions of an "Electrical Engineer." A report embodying several definitions has been prepared and filed at Institute headquarters.

Board of Examiners.—The Board of Examiners held 18 meetings during the year, averaging about three hours each. It considered and referred to the Board of Directors with its recommendations a total of 1889 applications of all kinds. In addition to these the Board reviewed or reconsidered 33 applications for a second and third time.

The demand upon the time of the Board has been greater during the past year than in any previous year, owing to the large number of applications filed for admission or transfer to the higher grades. Such applications are considered in great detail and all of the evidence submitted by the applicants, including the record and communications from references and others, is read by the Board.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION.

Recommended for grade of Associate	1036	
Not recommended for grade of Associate	10	1046
Recommended for grade of Member	74	
Not recommended for the grade of Member	50	124
Recommended for grade of Fellow	11	
Not recommended for Fellow	5	16
Recommended for enrolment as students	576	576

APPLICATIONS FOR TRANSFER.

Recommended for grade of Member	66	
Not recommended for grade of Member	41	107
Recommended for grade of Fellow	11	
Not recommended for grade of Fellow	9	20
Total number of applications considered		1889
Applications reconsidered		33
Total		1922

Membership Committee.—The work of the committee has resulted in the filing of 1235 new applications. Much credit for the success of the committee's work is due to the loyal support of the Sections and more particularly the chairmen of the local membership committees.

The following tabulated statement shows the number of members in each grade, the total membership of the Institute on April 30, and the additions and deductions which have been made during the year. It is not intended, however, to show the number of applications received, as a considerable number is still in the preliminary stages and cannot, therefore, be embodied in a list of the members of the Institute.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1917.	4	455	1223	7028	8710
Additions:					
Transferred	....	14	76	....	....
New Members Qualified	2	3	66	937	....
Reinstated	....	2	8	54	....
Deductions:					
Died	....	4	7	26	....
Resigned	....	1	3	111	....
Transferred	....	....	4	84	....
Dropped	....	5	27	318	....
Membership, April 30, 1918	6	464	1332	7480	9282

Net increase in membership during the year 572

Deaths.—The following deaths have occurred during the year:

Fellows.—Albert F. Ganz, F. B. H. Paine, John K. Robinson, Karl Von Krogh.

Members.—Harry Bottomley, Eugene F. Roeber, Henry R. Ford, J. G. Lorrain, Osborn P. Loomis, E. W. Stevenson, E. P. Warner.

Associates.—L. R. Pomeroy, S. H. Harvey, Stuart A. Nims, W. K. Kretsinger, P. H. Goodwin, W. S. Horry, Arthur Gunn, William G. Bee, T. Ohta, John Sachs, W. H. Peberdy, George Scharfe, Percy L. Cobb, John Gilmartin, R. C. Carter, Charles O. Smith, William Duddell, John Hesketh, John H. Goehst, Robert L. Stevenson, O. Zell Howard, Bernard W. Capen, Cyril F. Mickler, St. John Chilton, James A. Barkley, Henry N. Brooks.

Total deaths, 37.

Edison Medal.—The Edison Medal for 1916, which had been awarded to Nikola Tesla by the Edison Medal Committee in December of that year "for his early original work in polyphase and high frequency electric currents" was presented to Mr. Tesla with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 18, 1917.

The Edison Medal for 1917 has been awarded to Col. John J. Carty "for his work in the science and art of telephone engineering," and the

presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 17, 1918.

John Fritz Medal.—The John Fritz Medal for 1917 was awarded to Dr. Henry Marion Howe "for his investigations in metallurgy, especially in the metallography of iron and steel." The presentation was made in the Engineering Societies Building in New York on May 10, 1917.

The John Fritz Medal for 1918 was presented to Mr. J. Waldo Smith in the Engineering Societies Building, New York, at a joint meeting held on April 17, 1918, for "achievement as engineer in providing the City of New York with a supply of water."

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein. It will be noted that in the report a readjustment of the Institute's equity in the property held in trust by the United Engineering Society has been made, due to the admission of a fourth founder society to the United Engineering Society.

In company with the Secretary, the Treasurer, and a representative of Haskins and Sells, the chairman of the committee examined the securities held by the Institute and found them to be as stated in the accountants' report.

NEW YORK
CHICAGO
DETROIT
ST. LOUIS
CLEVELAND
BALTIMORE
PITTSBURGH

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

SAN FRANCISCO
LOS ANGELES
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1918, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1918, that the Statement of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 14, 1918.

EXHIBIT A.

AMERICAN INSTITUTE OF
GENERAL BALANCE SHEET

ASSETS.

REAL ESTATE:			
One-fourth Interest in United Engineering Society's Real Estate, No. 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment, etc.		14,292.79	
Total Real Estate			$486,792.79
EQUIPMENT:			
Library—Volumes and Fixtures		$ 40,031.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,274.65	
Total		$ 55,307.55	
Less Reserve for Depreciation		9,419.77	
Remainder—Equipment			$ 45,887.78
INVESTMENTS:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,834.19	
United States Liberty Loan, 4¼% Bonds		10,000.00	
Total Investments			25,834.19
WORKING ASSETS:			
Publications Entitled "Transactions," etc.		$ 14,049.00	
Paper and Cover Paper		1,046.28	
Badges		676.70	
Total Working Assets			15,771.98
CURRENT ASSETS:			
Cash		$ 12,270.07	
Accounts Receivable:			
Members for Past Dues		9,559.82	
Advertisers		769.70	
Miscellaneous		470.76	
Accrued Interest on Investments		56.25	
Accrued Interest on Bank Balances		358.66	
Total Current Assets			23,485.26
FUNDS:			
Life Membership Fund:			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 943.99		
New York City Bonds, 4½%, 1957, Par Value $2,000.00	2,248.71		
Accrued Interest	45.00	3,237.70	
MAILLOUX FUND:			
Cash	$ 167.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50		
		1,189.85	
Midwinter Convention Fund—Cash		163.58	
International Electrical Congress of San Francisco—Cash		40.50	
Total Funds			$ 9,972.38
DUES PAID IN ADVANCE—INTERNATIONAL ELECTROTECHNICAL COMMISSION, LONDON, ENGLAND			500.00
Total			$608,244.38

ELECTRICAL ENGINEERS.

APRIL 30, 1918.

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 6,304.01	
Due United Engineering Society Account Building Addition	10,000.00	
Dues Received in Advance	2,382.87	
Entrance Fees and Dues Advanced by Applicants for Membership	179.50	
Total Current Liabilities		$ 18,866.38
FUND RESERVES:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,237.70	
Mailloux Fund	1,189.85	
Midwinter Convention Fund	163.58	
International Electrical Congress of San Francisco	40.50	
Total Fund Reserves		$ 9,972.38
SURPLUS: Per Exhibit "B"		$579,405.62
Total		$608,244.38

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Statement of Income and Profit and Loss.

For the Year Ended April 30, 1918.

Exhibit B.

Revenue:			
Entrance Fees		$ 5,545.00	
Dues		96,897.93	
Student's Dues		3,771.00	
Transfer Fees		940.00	
Advertising		8,189.02	
Subscriptions		3,286.20	
Sales of "Transactions," etc		2,574.24	
Badges Sold	$2,253.75		
Less Cost	1,992.12		
		261.63	
Interest on Investments		675.00	
Interest on Bank Balances		1,027.43	
Exchange		30.74	
Total			$123,198.19
Expenses:			
Meetings and Paper Committee:			
Salaries		$ 5,560.00	
Binding and Mailing Proceedings		4,384.97	
Printing Proceedings		6,052.07	
Engraving Proceedings		1,523.22	
Paper and Cover Paper		4,581.05	
Envelopes		723.20	
Stationery and Miscellaneous Printing		177.96	
General Expenses		250.35	
Meetings		3,193.14	
Editing Committee:			
Volume No. 34		136.22	
Volume No. 35		12,370.15	
Volume No. 36		5,372.29	
Total		$ 44,324.62	
Deduct Increase in Inventory of Publications:			
April 30, 1917	$12,884.25		
April 30, 1918	14,049.00	1,164.75	$43,159.87
Executive Department:			
Salaries		$ 17,698.50	
General Expenses		2,737.33	
United Engineering Society—Assessments		4,800.00	
Express		311.13	
Postage		1,980.65	
Advertising		2,623.74	
Stationery and Miscellaneous Printing		3,027.68	
Year Book and Catalgue		3,487.90	
Office Furniture and Fixtures—Discarded		51.20	
Total			36,718.13
Sections Committee:			
Section Meetings		$ 6,162.15	
Branch Meetings		208.85	
Salaries, New York Office		2,340.00	
Stationery and Printing, New York Office		698.70	
Express on Advance Copies		8.09	
Total			$ 9,417.79
Forward			$ 89,295.79
Revenue—(Forward)			$123,198.19

REVENUE—(Forward)			$123,198.19
EXPENSES—(Forward)			$ 89,295.79
General:			
Library Committee	$ 4,000.01		
Membership Committee	1,079.38		
Finance Committee	150.00		
Standards Committee	1,221.72		
Code Committee	30.00		
Annual Dues, International Electrotechnical Commission	250.00		
National Defense—Miscellaneous Expenses	1,488.42		
John Fritz Medal Award	94.00		
Honorary Secretary	4,000.00		
United Engineering Society, Engineering Council	1,600.00		
American Engineering Standards Committee	359.37		
Membership Classification Service	776.65	15,049.55	
Total		$104,345.34	
Deduct:			
Decrease in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:			
May 1, 1917—As Adjusted	$ 7,869.81		
April 30, 1918	6,304.01	1,565.80	
Total Expenses			$102,779.54
NET REVENUE			$ 20,418.65
PROFIT AND LOSS CREDITS:			
Accessions to Library Volumes and Fixtures	$ 151.75		
Refund of Unexpended Balance of Contribution of American Institute of Electrical Engineers to proposed International Electrical Congress 1915	321.99		
Total		473.74	
GROSS SURPLUS FOR THE YEAR			$ 20,892.39
PROFIT AND LOSS CHARGES:			
Uncollectible Dues Written Off	$ 4,003.75		
Provision for Depreciation of Furniture and Fixtures	1,393.49		
Amortization of Premium on City of Wilmington, Delaware—4½% Bonds of 1934	52.14		
Total		5,449.38	
NET SURPLUS FOR THE YEAR			$ 15,443.01
SURPLUS, May 1, 1917	$623,016.43		
ADD—REAL ESTATE EQUIPMENT AND PRELIMINARY EXPENSES CREDITED TO THE INSTITUTE BY THE UNITED ENGINEERING SOCIETY BUT NOT HERETOFORE SHOWN ON THE BOOKS	15,710.44		
Total	$638,726.87		
DEDUCT:			
Value of Equity Adjusted to One-fourth Interest:			
Real Estate Equipment, etc.	$ 4,764.26		
Land and Building	132,500.00		
Total	$137,264.26		
Less Increase in Equity in Building by Addition of Three Stories	62,500.00		
Net Adjustment of Equity Value		74,764.26	
Surplus as Adjusted			563,962.61
SURPLUS, APRIL 30, 1918			$579,405.62

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Cash Receipts and Donations for Designated Purposes. Also Disbursements, for the Year Ended April 30, 1918.

Exhibit C.

Receipts:	
Life Membership Fund	$217.68
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	93.40
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Contributions and Interest	74.53
Total	$430.61
Disbursements:	
Life Membership Fund	217.68
Midwinter Convention Fund	6.75
Total	$224.43

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	1911	1912	1913	1914	1915	1916	1917	1918
Membership, April 30, each year	7117	7459	7654	7876	8054	8212	8710	9282
Receipts per Member	$13.37	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17
Disbursements per Member	11.03	12.44	15.57	12.86	13.54	13.74	12.75	11.99
Credit Balance per Member	$2.34	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 17, 1918.